

The visco-elastic properties of the arterial wall.

Bergel, D.H.

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THE VISCO-ELASTIC PROPERTIES OF THE ARTERIAL WALL

D.H. BERGEL, M.B.,B.S.

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ABSTRACT

The visco-elastic properties of the arterial wall are of importance in determining the behaviour of the circulatory system for the pulse wave is a relatively rapid event. It has been reported that arteries show very little change in length with each heart beat. There are no reports of studies on arterial behaviour carried out under conditions that take account of these two factors. Accordingly an investigation has been made of the pressure-diameter relations of various types of dogs' arteries under static and dynamic conditions. All experiments were performed with the vessel held at the length which it occupied in vivo. Measurements were also made of the relative wall thickness and of the amount of shortening which occurred when an artery was excised.

Under static conditions all arteries show an elastic modulus which increases with distension; this is less marked, in the low pressure range, for the elastic thoracic aorta. This increasing modulus results both from the presence of various wall constituents arranged in parallel and from the non-linear elastic behaviour of these elements themselves.

Under dynamic conditions the artery shows increased stiffness, the magnitude of this increase is proportional to the muscularity of its wall. This increase occurs at low frequencies and is compatible with what is known of the time-dependent properties of smooth muscle. Little further increase occurs at frequencies between 2 and 20 c/s. The dilatation also lags behind the pressure changes by a small amount. These factors will influence both the velocity and the attenuation of the pulse wave. The variation with frequency of these two factors has been calculated using the data obtained in these experiments. The results of this calculation stand comparison with published values for the pulse wave velocity.

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INTRODUCTION

The equations describing the behaviour of fluids in elastic tubes contain parameters relating to the mechanical properties of the wall. In particular, the well known Moens-Korteweg formula deals with the velocity of transmission of pressure waves, and this is found to depend on the dimensions and elastic modulus of the tube as well as the density of the transmitting fluid. The analysis of circulatory biophysics which has been developed in the last few years by workers in this laboratory (see Womersley, 1957a; Taylor, 1957a, b; McDonald, 1960) has called for rather more precise knowledge of the properties of the arterial wall than has previously been available.

While a very considerable volume of work has been carried out on the elastic properties of blood vessels, most of the previously published observations fail, for a variety of reasons, to provide adequate information on the wave transmission characteristics to be expected in the intact circulatory system.

In the first place the majority of workers have studied pressure-volume relationships only, and while the wave velocity can be determined from such data, all too frequently important quantitative details are not available. Furthermore such studies have been almost entirely confined to the thoracic aorta and little is known of the smaller vessels.

A second major deficiency in previously published work lies in the universal neglect of the length changes in the excised artery. It has long been known that an artery will shorten on removal from the body, and that when such a specimen is closed off and inflated it will lengthen. It is obvious that this is not the case in vivo, and the neglect of

these two factors renders much of the earlier data doubtfully valid in reference to circulatory mechanics.

Thirdly it must be remembered that the heart beat is a rapid and repetitive event. The undoubted presence in the arterial wall of elements that react to a force in a manner comparable to that of a viscous fluid makes it likely that length-tension, or pressure-volume, relationships obtained by means of slow extensions and inflations will be different from those performed on a time scale comparable with that of the pulse-wave. The time dependent properties of the arterial wall have been shown to be complex (e.g. Zatzman et al., 1954). Although some determinations have been made of the response of strips of aortic wall to oscillating tensions by various workers whose findings will be fully discussed later, there appears to be no information on the dynamic properties of any type of fluid filled artery.

It is for these reasons that the present work has been performed. An attempt has been made to measure the elastic behaviour of various types of arteries under static and dynamic conditions and to predict therefrom the behaviour of the circulatory system in so far as it is determined by the visco-elastic properties of the arterial wall.

CHAPTER 1. THEORETICAL DISCUSSION

(a) General elastic theory

The theory of elasticity deals with the relations between the forces applied to a body and its consequent deformation. If there exists a constant relation between these two factors, and if the deformation is fully reversible when the force is removed, the body is said to be perfectly elastic. An imperfectly elastic solid may show some residual deformation when the force is removed; such a material is said to exhibit plastic deformation. Such behaviour is in marked contrast to that shown by a fluid, which undergoes viscous flow at a rate determined by the applied force. However a large number of substances exhibit properties appropriate to both an elastic solid and a viscous liquid, and the deformation suffered by such a material will depend both on the magnitude of the force and on the time for which it was in operation.

It is with the properties of this latter group of visco-elastic materials that the present work is concerned. Before dealing with these substances a brief account of classical elastic theory and of various modifications that have been proposed to cover specific situations will be made.

We owe the first description of the relation between force and deformation to Robert Hooke. In 1676 he published his discoveries in the form of an anagram in order to retain certain patent rights.

The anagram read "CEIINOSSSTUV", and two years later he revealed it as "UT TENSIO SIC VIS", "as the (ex)tension so the force". Materials such as metals which show this direct proportionality between force and extension are spoken of as perfectly elastic or Hookean substances, but even these only exhibit a linear force-extension relationship at small extensions; if the extension is too great the relationship changes irreversibly and some degree of plastic set will remain when the force is removed.

The classical theory of elasticity stems from Hooke's Law and makes two further basic assertions; that the extensions considered are infinitesimally small, and that the structure of the material considered is continuous. Full theoretical development is given by Love (1927) and by Southwell (1941), from whom the following brief treatment has been derived.

Strain

When a body of length l_0 is extended to length l , the relative increase in length is referred to as the longitudinal strain. A strain exists in any axis when the distance between any two points on that axis is altered. Positive strain indicates an increased, and negative strain a decreased separation. We may further distinguish three types of strain.

Tensile strain; change in length.

Shear strain; the displacement of two points in parallel planes in a direction parallel to those planes

Compressive strain; change in volume.

Since strain is a ratio of length, it is dimensionless. We are only concerned here with tensile strain, which is given by

$$\epsilon_1 = \frac{L_1 - L_0}{L_0}$$

A body undergoing an extension will show simultaneous changes in its width and breadth (a and b) and we may call these strains ϵ_2 and ϵ_3

$$\epsilon_2 = \frac{a_1 - a_0}{a_0} = -\sigma_2 \epsilon_1,$$

$$\epsilon_3 = \frac{b_1 - b_0}{b_0} = -\sigma_3 \epsilon_1,$$

The symbol σ , the ratio of transverse to longitudinal strain, is known as Poisson's ratio. This ratio is a characteristic property of a material and in classical elastic theory it is constant, but there is no a priori justification for assuming that $\sigma_2 = \sigma_3$. Similarly primary extensions of the nature ϵ_2 and ϵ_3 must indicate the existence of four more Poisson's ratios, making six in all; whether or not any or all of these have the same value will depend on the nature of the material considered.

Consider now an infinitesimally small cube within the substance of any object with faces parallel to the primary axes x, y and z. The strain at any face can be resolved into three components in directions at right angles to each other, one tensile and two shearing strains. These may be written e_{xx} , e_{xy} , and e_{xz} , where e_{xx} indicates a strain separating two points on the x axis in the direction of the x axis, and e_{xy} a strain separating these two points in the direction of the y axis. The convention is that strains on a plane in the direction

inside-to-outside are positive. We thus have nine components of strain at a point;

$$e_{xx}, e_{xy}, e_{xz}, e_{yx}, e_{yy}, e_{yz}, e_{zx}, e_{zy}, e_{zz}$$

It may be shown (Love) that strain components of the form e_{xy} and e_{yx} are equal in magnitude. We may therefore eliminate three of them and are left with the six independent components of strain;

$$e_{xx}, e_{xy}, e_{yy}, e_{yz}, e_{zz}, e_{zx}.$$

To these correspond six independent Poisson's ratios;

$$\sigma_{yx} \sigma_{zx} \sigma_{xy} \sigma_{zy} \sigma_{xz} \sigma_{yz}.$$

It can be shown that $-1 < \sigma < 0.5$. Although it is theoretically possible, no material is known which possesses a negative Poisson's ratio. In practice σ lies between 0 and 0.5 (for most metals the value of σ is in the region of 0.3).

Consider now a cube of dimensions x, y, z , subjected to a tensile strain e_{xx} : the volume when unstrained (V_0) was $x_0 y_0 z_0$, after deformation the dimensions are:

$$\begin{aligned} x_1 &= x_0 (1 + e_{xx}) \\ y_1 &= y_0 (1 - \sigma_{yx} e_{xx}) \\ z_1 &= z_0 (1 - \sigma_{zx} e_{xx}) \end{aligned}$$

The new volume (V_1) is given by

$$V_1 = x_1 y_1 z_1,$$

and the change in volume (Δ) is

$$\Delta = V_1 - V_0 = x_0 (1 + e_{xx}) y_0 (1 - \sigma_{yx} e_{xx}) z_0 (1 - \sigma_{zx} e_{xx}) - x_0 y_0 z_0.$$

Assuming that e_{xx} is very small we may neglect all of its powers greater than unity, thus

$$\Delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \epsilon_{xx} (1 - \sigma_{yx} - \sigma_{zx})$$

Making the additional assumption that the material is homogeneous, that is that its properties are the same when studied from any direction, we may write

$$\sigma_{yx} = \sigma_{zx} = \sigma_{xy} = \sigma_{zy} = \sigma_{xz} = \sigma_{yz} = \sigma$$

It is at once obvious that when $\sigma = 0.5$, $\Delta = 0$. It can then be seen that if a homogeneous body undergoes no volume change on being strained, provided that the strain is small, then $\sigma = 0.5$. Furthermore if the material is not homogeneous but still isovolumetric we may still take all values of σ to be equal to 0.5, since were some values of σ less than this they would require to be balanced by one or more greater values, that is to say that some dimensions would have to be increased under compression, and no material is known which behaves in this way.

Stress

Stress is the intensity of force acting across a given plane in a body, if the force F is uniformly distributed over the area A then stress = F/A . The units of stress are force per unit area, or in absolute units (mass) \times (length)⁻¹ \times (time)⁻² (ML⁻¹T⁻²)

The stress acting on a point in a given plane may be resolved into normal (positive tensile and negative compressive) stresses and tangential (shearing) stresses. The stresses acting on a point may also be resolved into components along the three axes x , y , and z . These, following Love, are designated

$$X_x, X_y, X_z, Y_x, Y_y, Y_z, Z_x, Z_y, Z_z$$

where the capital letter indicates the direction of the component and the subscript the plane across which it acts. It can also be shown that the stress components such as X_y and Y_x , though quite distinct, must be equal in magnitude; were this not so rotating moments would exist at each point. We are therefore left with six independent components of stress

$$X_x, X_y, Y_y, Y_z, Z_z, Z_x$$

The relationship between stress and strain

Hooke's law states that within certain limits stress is proportional to strain. This proposition may be further elaborated as the Generalised statement of Hooke's law:- "each of the six independent components of stress may be expressed as a linear function of the six components of strain, and conversely." Thus we write

$$X_x = a_{11}\epsilon_{xx} + a_{12}\epsilon_{yy} + a_{13}\epsilon_{zz} + a_{14}\epsilon_{xy} + a_{15}\epsilon_{yz} + a_{16}\epsilon_{zx}$$

$$\epsilon_{xx} = b_{11}X_x + b_{12}Y_y + b_{13}Z_z + b_{14}X_y + b_{15}Y_z + b_{16}Z_x$$

where $a_{11} \dots b_{11} \dots$ are constants of the material. There are thus 36 such constants to consider but 15 of these are shown to be interrelated; the 21 remaining constants of proportionality can be reduced to 2 providing a further assumption is made.

This assumption is that the material shall be homogeneous or isotropic; materials which exhibit 'grain' are said to be anisotropic or anisotropic. Most substances are anisotropic on the microscopic scale, for example metals have a crystalline structure, but they can be adequately treated as isotropic for most purposes. The materials studied in this work are macroscopically anisotropic and this must lead to much uncertainty in the description of their behaviour.

It has been said that the assumption of homogeneity allows the number of elastic constants to be reduced to two, these constants are λ and μ , and the three well-known elastic constants or moduli can be expressed in terms of these. The three moduli describe the behaviour of an isotropic material under longitudinal tension, shear, or compression provided always that the strains produced are small and that there is no other stress operating. All these moduli, being the ratio of stress to strain, have the same dimensions as stress, i.e. $M L^{-1} T^{-2}$, or Force $\times L^{-2}$ usually expressed as dynes cm^{-2} .

Young's modulus E is the ratio of tensile stress to strain

$$E = \frac{X_x}{\epsilon_{xx}}$$

The Shear modulus or modulus of rigidity μ (sometimes written as C) is the ratio of shear stress to strain

$$\mu = C = \frac{X_y}{\epsilon_{xy}}$$

(shear strain is measured as the ratio of the displacement of a point parallel to a plane to the perpendicular separation from that plane).

The bulk modulus K is the ratio of compressive stress to strain, that is to the relative change in volume. Consider a cube under the compressive stresses $-X_x$, $-Y_y$, $-Z_z$. The mean stress is

$$-(X_x + Y_y + Z_z)/3$$

Suppose this stress to be produced by a hydrostatic pressure $-p$, then

$$X_x = Y_y = Z_z = p$$

We have seen that the cubical dilatation is given by

$$\Delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

The volumetric strain is $-\Delta/V_0$

$$\text{and } k = \frac{pV_0}{\Delta} = \frac{(X_x + Y_y + Z_z)V_0}{3(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz})}$$

The five elastic constants that have now been mentioned for an isotropic substance are so related that only two need be evaluated independently, each may be described in terms of any other two.

$$k = \frac{2}{3} \frac{(1 + \sigma)N}{3(1 - 2\sigma)} = \frac{E}{3(1 - 2\sigma)} \quad \dots\dots(1)$$

$$E = 2(1 - \sigma)\mu = \frac{9k\mu}{3k + \mu} = \frac{9kN}{3k + N} \quad \dots\dots(2)$$

$$\sigma = \frac{1}{2} \frac{3k - 2\mu}{3k + \mu} \quad \dots\dots(3)$$

$$\lambda = \frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)} \quad \dots\dots(4)$$

$$\mu = c = \frac{E}{2(1 + \sigma)} \quad \dots\dots(5)$$

(b) The behaviour of an elastic tube of finite wall thickness under a distending pressure

It is relatively simple to predict the behaviour of a material subjected to a single stress. When more than one stress is applied the strains produced can be considered additive assuming that the first strain does not significantly alter the stress produced by a second force, and so on. Each element in the wall of a pressurised tube is

under stresses of all three types. If the material is isotropic, the tube of uniform bore and wall thickness, and the strains small the stresses can be resolved as shown by Love (1927). Consider such a tube of internal and external radius R_1 and R_0 , subjected to internal pressure P_1 and external pressure P_0 ; the stresses are

$$\hat{r}r = \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2} - \frac{P_1 - P_0}{R_0^2 - R_1^2} \cdot \frac{R_0^2 R_1^2}{R^2} \quad \dots\dots(6)$$

$$\hat{\theta}\theta = \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2} + \frac{P_1 - P_0}{R_0^2 - R_1^2} \cdot \frac{R_0^2 R_1^2}{R^2} \quad \dots\dots(7)$$

$$\hat{z}z = \frac{\lambda}{\lambda + \mu} \cdot \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2} + e \frac{3(\lambda + 2\mu)\mu}{\lambda + \mu} \quad \dots\dots(8)$$

where $\hat{r}r$ = radial stress

$\hat{\theta}\theta$ = circumferential stress

$\hat{z}z$ = longitudinal stress

R = any chosen radius, such that $R_1 < R < R_0$

e = longitudinal extension

If the tube is closed by a flat plate then the forces acting on that plate will be balanced by the forces due to the longitudinal tension

$$\pi(R_0^2 - R_1^2) \hat{z}z = \pi(R_1^2 P_1 - R_0^2 P_0) \quad \dots\dots(9)$$

This gives for e

$$e = \frac{1}{3\lambda + 2\mu} \cdot \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2}$$

Substituting values for λ and μ given by equations (4) and (5)

$$e = \frac{3E\sigma}{(1+\sigma)(1-2\sigma)} + \frac{2E}{2(1+\sigma)} \cdot \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2} = \frac{1-2\sigma}{2E} \cdot \frac{P_1 R_1^2 - P_0 R_0^2}{R_0^2 - R_1^2} \quad (10)$$

Thus it can be seen that a closed isotropic tube made of material in which $\sigma = 0.5$ (i.e. $1 - 2\sigma = 0$) will remain at the same length when (moderately) distended; conversely if such a tube does lengthen then either, or both, of the two assumptions made about the material are false.

The behaviour of the tube is governed both by the transmural pressure ($P_1 - P_0$) and by the absolute magnitude of P_1 and P_0 , that is the tube is distended and the wall is radially compressed. If the wall is essentially incompressible we may ignore the second set of strains. Writing $P_0 = 0$ and $P_1 = P$, and forcing the condition that $e = 0$ we may simplify equations (6), (7), (8)

$$\hat{r}_R = P \frac{R_1^2}{R_0^2 - R_1^2} - P \frac{R_0^2 R_1^2}{R_0^2 - R_1^2} \frac{1}{R^2} \quad (6a)$$

$$\hat{\theta}_\theta = P \frac{R_1^2}{R_0^2 - R_1^2} + P \frac{R_0^2 R_1^2}{R_0^2 - R_1^2} \frac{1}{R^2} \quad (7a)$$

$$\hat{z}_z = \frac{\lambda}{\lambda + \mu} P \frac{R_1^2}{R_0^2 - R_1^2} \quad \dots (8a)$$

The greatest stress is circumferential, when $R = R_1$, at the inner surface

$$\hat{\theta}_\theta = P \frac{R_1^2 + R_0^2}{R_0^2 - R_1^2}$$

and the least stress is also circumferential at the outer surface

$$\hat{\theta}_\theta = P \frac{2R_0^2}{R_0^2 - R_1^2}$$

The expression for the radial displacement U of a point on a shell of radius R is of the form

$$U = AR + \frac{B}{R} \quad \dots\dots(11)$$

where

$$A = \frac{P_1 R_1^2 - P_0 R_0^2}{2(\lambda + \mu)(R_0^2 - R_1^2)} - \frac{\lambda e}{2(\lambda + \mu)} \quad \dots\dots(12)$$

$$B = \frac{(P_1 - P_0) R_0^2 R_1^2}{2\mu(R_0^2 - R_1^2)} \quad \dots\dots(13)$$

substituting again for λ and μ to give an equation in E and σ we have

$$\begin{aligned} 2(\lambda + \mu) &= 2 \frac{E\sigma}{(1+\sigma)(1-2\sigma)} + \frac{E}{2(1+\sigma)} \\ &= \frac{E}{(1+\sigma)(1-2\sigma)} \end{aligned}$$

Returning to equation (11), let us substitute these values, and the following

$$P_1 = P$$

$$P_0 = 0$$

$$e = 0$$

$$R = R_0 \text{ — i.e. taking the displacement at the outer wall.}$$

$$\begin{aligned} U &= \frac{P R_1^2 (1+\sigma)(1-2\sigma) R_0}{E (R_0^2 - R_1^2)} + \frac{P R_0^2 R_1^2 2(1+\sigma)}{2E (R_0^2 - R_1^2)} \frac{1}{R_0} \\ &= \frac{P R_1^2 2(1-\sigma^2) R_0}{E (R_0^2 - R_1^2)} \quad \dots\dots(14) \end{aligned}$$

E is therefore given by

$$E = \frac{P R_1^2 2(1-\sigma^2)}{R_0^2 - R_1^2} \frac{R_0}{U} \quad \dots\dots(15)$$

And, when $\sigma = 0.5$

$$E = \frac{1.5 P R_0^2}{R_0^2 - R_1^2} \cdot \frac{R_0}{U} \dots\dots(16)$$

Before passing to other matters it is pertinent to examine some of the assumptions which have been made in arriving at equation (16). In fact these assumptions present great, but not insuperable difficulties, and an entirely different way of handling the whole problem will be considered. In particular the question of anisotropy and the significance of Poisson's ratio must be considered.

(c) Anisotropy

There is no a priori reason why the individual elements of which the arterial wall is composed should be isotropic, bearing in mind their largely fibrous nature: it is to be presumed that the tissue itself is certainly anisotropic. When it is recalled that up to 15 independent constants of proportionality may be needed to describe anisotropic behaviour it is not to be expected that measurements of pressure-radius relationships will allow them all to be determined.

Lambossy and Müller (1954) were concerned to discover the influence of wall anisotropy on the propagation of pressure waves in a tube. After making some simplifying assumptions they derived an equation showing the relation between E_1 , the measured modulus of radial expansion, and E_R and E_T , the radial and tangential Young's moduli. Their work is of interest in that they show that the longitudinal modulus (E_L) has no effect on wave velocity provided the expansion is so small as to involve negligible longitudinal strain.

This condition holds in the case of the artery in situ (Lawton and Green, 1956) and is the basis of a fruitful mathematical treatment (Womersley, 1957) which will be further considered later. The modulus E_1' which governs the pulse wave velocity is the one measured here, isotropy being assumed in the derivation of Eqn. 16.

More recently Faucett (1957) has published equations describing the behaviour of anisotropic tubes which made use of six constants, three Young's (tensile) moduli, and three Poisson's ratios. All these must be determined independently. In addition Faucett considered the change in length on inflating a tube with closed ends. If such a tube is made of isotropic and incompressible ($\sigma = 0.5$) material the forces on the occluding disc will be exactly balanced by the shortening force developed as the radius increases, and no change in length will occur (see Eqn. 10). Fenn (1957) has in fact measured the longitudinal expansion which occurs when a closed arterial specimen is inflated. He finds the lengthening to be large and this implies either anisotropy or compressibility of the wall, or both. (Many other authors, e.g. Simon and Meyer (1958), report similar or greater length changes.) Fenn concludes from these experiments that it is more reasonable to assume incompressibility (as shown by Lawton, 1954) and that therefore the arterial wall is not isotropic.

(d) Poisson's Ratio

It will be remembered that Poisson's ratio is the ratio of lateral contraction to longitudinal expansion, and that, provided the strains involved are extremely small the change in volume is negligible if $\sigma = 0.5$. There is good evidence that elastomers undergo large extensions virtually

isovolumetrically. Holt and Macpherson (1946) strained rubber up to 400% and could detect no volume change, while Lawton (1954) studied aortic strips and stated that ordinarily the volume changes observed were well within the range of experimental error. The largest change he observed was a compression of 0.3% at an extension of 50%. However such small volume changes are of doubtful significance considering the method used, and it is reasonable to conclude that at extensions of around 100% volume changes are negligibly small. This assumption has been made in the present work.

It should be appreciated that the statement that finite strains are isovolumetric does not mean that $\sigma = 0.5$. In fact if no volume change occurs σ is related to the strain by the expression

$$\sigma = \frac{1}{\epsilon} \left(1 - \frac{1}{\sqrt{1+\epsilon}} \right) \dots\dots(17)$$

where ϵ is the strain. Only for extremely small strains does approach 0.5, and for example with an isovolumetric strain of 100%

$$\sigma = 0.293.$$

These facts further emphasise that the concepts of classical elastic theory cannot be rigorously applied where large strains are involved, but it also seems unreasonable to employ a value of σ appropriate to the extension for the computation of Young's modulus which itself is a measure of resistance to infinitesimal extensions. One possible approach to this problem is to make use of the methods developed by the rubber technologists to deal with large strains. For reasons which will be given this line of attack has not been used here, but the ideas inherent in it deserve consideration.

(e) Rubber-like elasticity

"The classical theory of elasticity is based on the postulate, which is a generalisation of Hooke's Law, that in any type of deformation the components of stress are linear functions of the components of strain. This postulate is valid only in the limiting case where the strains are infinitesimally small. In the case of finite or large strains this postulate is no longer applicable and entirely different mathematical techniques for handling problems of this type must be introduced."

(Treloar, 1958)

Much attention has recently been paid to the mechanical properties of the elastomers. This class of substance includes rubber and the synthetic rubber-like polymers which share two important properties; a high degree of reversible extensibility (extensions up to 500-1,000%) and a relatively low Young's modulus in the region of 10^6 - 10^7 dynes/cm² (c.f. the modulus of metals which is ca. 10^{11} dynes/cm²). It is claimed that these properties are shared by soft body-tissues (Guth, 1947; King & Lawton, 1950).

Rubber-like elasticity is conceived as being conferred by the molecular architecture of the elastomers and has been successfully treated in molecular terms, postulating the following features;

the presence of long chain molecules with fully rotating links, weak secondary forces between molecules (as opposed to primary forces within the molecule), and the interlocking of the molecules at a few places along their length to form a three dimensional network.

The work done on straining an elastomer is stored in two forms, as an actual separation of neighbouring atoms in the molecular chain (internal

energy) and as an imposed orientation of the chains in the direction of the strain. The molecular chains are conceived as being acted upon by Brownian forces which will be maximally effective in directions normal to the chain axis and they will therefore tend to coil themselves into a random configuration. This tendency is related to the principle of 'maximum mechanical chaos' or entropy, and the forces acting to this end will be proportional to the absolute temperature. In a 'perfect' elastomer all the work of deformation would go to decrease the entropy with no change in internal energy. No real systems are 'perfect', even rubber, but in rubber the internal energy changes are small in comparison with the entropy changes at extensions above 100%; at smaller extensions however the changes are of the same order of magnitude.

The entropy contribution changes sign at extensions corresponding to the thermo-elastic inversion point (the point at which a change of temperature has no effect on tension; in rubbers at a strain of about 10%). Lawton (1954) has investigated the thermo-elastic behaviour of aortic strips and finds that the retractile force is primarily the result of entropy change, and that the internal energy contribution is largely constant but negative, i.e., tending to produce extension. At large extensions he finds that this term changes and becomes positive. He attributes the negative internal energy to the complex structure of elastin fibres, and the later changes to collagen which is considered to bear an increasing part of the load at high extensions.

It follows from the basic thermo-dynamic equations that any changes in internal energy observed on straining an elastomer are associated with changes in volume. The work done by the compressive component of stress

is smaller by a factor of about 10^5 than the work done by the shear components, this being the order of difference of the shear and bulk moduli of elastomers. Thus the volume may be considered to be unchanged by extension. The effect of this is (for small deformations at least) that the elasticity of rubber arises primarily from entropy changes, and Lawton (1954) has shown that aortic strips show basically similar properties. This implies that there is no change in volume on extension. Therefore the elastic force developed by rubber derives from changes in entropy, but at extreme extensions the internal energy term begins to become significant, that is, the molecules themselves are coming under strain, and some change in volume occurs.

Equations have been developed which seek to predict the stress-strain relationships of rubber by reference to entropy changes. The original elementary statistical theory, due to Kuhn, postulated that the degree of coiling of the molecular chains, measured as their end-to-end separation, was distributed in a Gaussian fashion and that these separations were altered by deformation exactly in proportion to the deformation undergone by the material which they form. The properties of rubber were found to be fairly well predicted by the use of this model which sets up a single elastic modulus G . The use of a single modulus stems from the assumption that no volume changes occur (Treloar, 1958).

At large extensions the observed behaviour of rubber differs from that predicted by this simple theory, but a model has been developed, based on a more complex distribution of chain lengths, which predicts the behaviour remarkably closely. This model has been adapted to animal tissues with success by King (1946a, b, 1947a, b) although he is careful to state that only an approximate description of the observed elastic characteristics

should be obtained. King and his collaborator Lawton have been able to fit these equations to the elastic diagrams of a number of tissues. Three parameters appear, β , N and L_0 . The latter represents the unstretched length of the constituent molecular chains, while N is the number of cross links per chain; β is a figure which is found to alter with the age of the specimen. Although this approach is somewhat attractive it has not been followed here for a number of reasons. In the first place this analysis stems from the assumption that the tissue has a structure basically similar to rubber. This may well be true of collagen and elastin fibres themselves but it is manifestly not the case with arterial wall. These parameters are only defined in terms of the model chosen. Outside the model one may doubt their significance, in particular the interpretation of changes in β needs to be approached with care. Unless these factors present a more meaningful picture of tissue properties than can be got from the familiar elastic moduli there seems little to gain in using them.

A more practical objection may also be raised. These equations contain parameters relating to the unstressed dimensions of the material. It will be seen that, in the case of blood vessels at least, it is somewhat difficult to define this unstressed state.

Lastly, the necessary computations are tedious and time consuming. They involve the inverse Langevin function, and tables of this are difficult to obtain. In a more descriptive work of this nature some approach which does not involve any assumptions about the wall structure is needed. We will therefore return to the classical concepts and see whether some compromise cannot be reached. What is needed is a simple and comprehensible measure of the relation between force and extension under any particular

circumstances, and one that can reasonably be used in the description of large strains.

The position may be clarified by considering the definition of strain. In any situation involving finite strains three different tensile moduli of elasticity may be computed. In the first case total stress is related to total strain at a particular load; we may here speak of a 'total' modulus.

The second convention is also well known. The tangent modulus is a measure of the slope of the stress-strain curve at any point, that is the ratio of stress change to the resulting change in strain. The definition of strain is, of course, tied to the unstressed length.

This tie is abandoned by using what I shall term the incremental modulus, following Krafka (1939). Here we compare increments of stress and strain, but strain is taken as the ratio of the length change to the immediately preceding length. In effect the material is treated as if with each increment in length a new substance with different properties had been produced. This would seem to be a fair statement of fact, the molecular configuration of a finitely strained material has been altered and one might expect to find changed elastic properties.

By this third approach the concept of unstressed length is abandoned. In fact it is not possible to measure this dimension in this type of work. When an artery is split open longitudinally it will unroll itself to a varying degree to the form of a flat ribbon. This surely indicates some degree of stress even when there is no distending pressure. In addition it will be seen that all vessels have here been studied under longitudinal stress, they are stretched beyond their excised length and this stretch

alters all dimensions. It will also be repeatedly remarked that the radius of any specimen at zero pressure is not constant; owing to the time dependent behaviour of the wall large changes are seen in the course of an experiment.

All these factors combine to make the recognition of unstressed state difficult to say the least. It would seem, incidentally, that this fact throws some doubt on the practical value of King's approach which leans rather heavily on measurements of unstressed dimensions.

Strains computed in this way are somewhat nearer in size to those dealt with by classical theory. Consequently it seems admissible to employ as Poisson's ratio the figure 0.5 for a material isovolumetric under strain; and this has been done here.

In order to make the method quite clear an example will be given. Consider a tube whose external radius at internal pressures $P_{(1)}$, $P_{(2)}$, $P_{(3)}$, is $R_{o(1)}$, $R_{o(2)}$, $R_{o(3)}$. Now, using Eqn. (16), the incremental modulus at the pressure $P_{(2)}$ (or, more meaningfully, at the radius $R_{o(2)}$) is

$$E_{inc} R_{o(2)} = \frac{(P_{(2)} - P_{(1)}) 2(1 - \sigma^2) R_{i(2)}^2}{R_o^2 - R_i^2} \frac{R_{o(2)}}{R_{o(3)} - R_{o(1)}} \quad (18)$$

The tangential modulus will, of course, be somewhat smaller

$$E_{Tan} R_{o(2)} = \frac{(P_{(2)} - P_{(1)}) 2(1 - \sigma^2) R_{i(2)}^2}{R_o^2 - R_i^2} \frac{R_{o(2)}}{R_{o(3)} - R_{o(1)}} \quad (19)$$

(For comparison, the total modulus is given by

$$E_{Total} R_{o(2)} = \frac{P_2 \cdot 2(1 - \sigma^2) R_{i(2)}^2}{R_o^2 - R_i^2} \frac{R_{o(2)}}{R_{o(2)} - R_{o(0)}} \quad 19a)$$

It might therefore be objected that the use of an incremental modulus to describe the properties of a (hypothetical) substance exhibiting long range Hookean behaviour would be misleading. Such a material might be expected to respond to equal increments of tension by equal changes in length, thus the strain calculated as in Eqn. (18) would become progressively smaller and the incremental modulus correspondingly larger. Were such a material known this would indeed be so, however the increasing stiffness would be in a sense real. Once the domain of the classical theory has been quitted and large strains are being considered, the properties of a material must be expected to change as its dimensions are changed.

It was of course questions of this nature that led to the development of special methods of handling highly extensible substances. These have been referred to and have not been adopted, mainly for practical reasons. The present compromise approach seems well enough adapted to the particular problem at hand.

(f) Visco-elastic behaviour

The response of a purely elastic solid to an imposed stress may be likened to that of a spring; the extension observed is proportional to the force and remains constant for so long as the force is maintained. On the other hand a force applied to a fluid causes flow, the rate of which is proportional to the force and which continues at a steady rate for so long as the force is maintained.

Living tissues and other substances exhibit the properties both of solids and of fluids in their response to stresses and a useful mechanical model may be used to assist the understanding of such behaviour. Elastic behaviour is considered as characterised by a spring, with elastic constant E . A dashpot is taken to embody viscous flow; this is a vessel of fluid

containing a movable vane, the whole having a coefficient of viscosity η

Thus the response of a spring of unloaded length L to a tension T is

$$\frac{\Delta L}{L} = \frac{T}{E}$$

while that of a dashpot is

$$\frac{\delta L}{\delta t} = \frac{T}{\eta}$$

where t = time. It is to be noted that these models merely exemplify elastic and viscous behaviour and that they should not be imagined as having any structural significance.

By combining these units in various ways it is possible to produce models showing all the properties of visco-elastic behaviour. A spring and a dashpot in series is known as a Maxwell element. On suddenly extending such an element the spring only will extend, the tension being $\frac{\Delta L}{L} \cdot E$. This force will gradually extend the dashpot, at the same time causing the tension and the rate of viscous flow to diminish in an exponential manner to zero, according to the formula

$$T = T_0 e^{-\frac{Et}{\eta}} \quad \dots(19)$$

where T_0 = initial tension

e = the base of the natural logarithm

t = time

After time equal to η/E the tension will have fallen to $1/e$ of its original value, and this characteristic parameter η/E is termed the relaxation time τ of the element. If a second spring, modulus E_2 , is placed in parallel with the dashpot the tension will not decay to zero but will tend towards a value where

$$T = \frac{A L}{L} \cdot \frac{E_1 E_2}{E_1 + E_2} \dots\dots(20)$$

This phenomenon of tension decay at constant (overall) length is known as stress-relaxation and it is well known that it occurs in biological substances (Zatzman, Stacy, Randall & Eberstein, 1954). Previous work on this subject will be discussed later, but one point will be considered here; this is that there is general agreement among workers in this field that stress-relaxation is not a simple exponential function. To represent such behaviour a more complex model is necessary. This must incorporate one or more of the following properties.

- (1) Non Newtonian viscous flow
- (2) Non Hookean elastic extension
- (3) In the place of a single relaxing element a whole family may be substituted, these must possess a suitable spectrum of relaxation times.

Each of these approaches will be found suitable. A full account of the whole subject is given by Alfrey (1948) but the most satisfactory way of treating the problem has up to now been based on the third. The best way of determining the relaxation-time spectrum of a material is by subjecting it to oscillatory stresses over a very wide frequency range, as has been done for rubber by Nolle (1956); where only a narrow range has been studied, as in the present work, rather less information on the distribution of relaxing elements can be obtained.

The Maxwell element embodies two separate forms of extension; these may be considered separately. The total deformation is the sum of the viscous and elastic deformations and the stress acts equally on both components.

The different property of retarded elastic response is characterised by the Voigt element. This consists of a spring and dashpot connected in parallel so that neither can move independently; each unit undergoes the same extension, and the stress is distributed between both. Such an element cannot respond instantaneously; when a force is applied it will extend exponentially according to the expression

$$\frac{\Delta L}{L} = \frac{T}{E} \left(1 - e^{-\frac{Et}{\eta}} \right) \quad \dots\dots(21)$$

In this case the quantity η/E is termed the retardation time, the time taken to extend $1/e$ of its final travel.

Arterial wall displays both stress relaxation (Zatzman et al., 1954) and retarded elastic response, commonly referred to as creep (Remington, 1955), and a suitable model must therefore include both Maxwell and Voigt elements. In the last analysis, however, these two models can be considered as extreme examples of a single model (Buchthal & Kaiser, 1951). The common unit contains a spring in series with a parallel spring-dashpot configuration. If the series spring is infinitely compliant we are left with a Voigt model, if it is the parallel spring which is so altered the final result is a Maxwell element. Such a three element model is necessary to describe the visco-elastic materials here considered. This may be considered as a 'lumped model' in which each element represents the combined responses of a wide range of separate elements. The nature of a material represented by such a model can be obtained from a study of stress-relaxation and creep curves, but such information is often difficult to analyse. The direct interest of the visco-elastic properties of the arterial wall lies in discovering how

this material responds to the varying stresses to which it is subjected in life and this can be done by subjecting it to suitable oscillating stresses, in the present case to oscillating hydrostatic pressures. The time-scale of such stresses is obviously of paramount importance. At very low frequencies the most sluggish dashpot will be able to move freely in and out, and the model will behave like two springs in series, that is as a spring more compliant than either. At very high frequencies, whose repetition rate is shorter than the shortest retardation time in the system, the Voigt element will not be deformed at all and the model will behave as the single spring, and will thus be stiffer than before. At intermediate frequencies the model will be of intermediate stiffness but its response will be retarded and will lag behind the driving force to a varying extent. The work here presented has therefore been performed by subjecting arterial specimens to sinusoidally varying pressures and observing the resulting changes in diameter. The interpretation of the results has been made much simpler by employing an electrical analogue and using the alternating current theory which is easier to handle than complex differential equations.

Certain expressions that relate the excitation and response of mechanical elements have the same form as the expressions that relate the excitation and response of electrical elements. This allows the behaviour of mechanical circuits to be analysed after substituting the appropriate electrical components for their mechanical equivalents. In the system here used the electrical analogue of viscous behaviour is

resistance, while that of compliance is capacitance; voltage corresponds to tension and current to velocity, thus the analogue of displacement is charge. A Maxwell element, in which each unit bears the whole tension, is therefore represented by a capacitance and resistance in (electrical) parallel, and a Voigt element by the same components in series. The problem is to define the relation between voltage and charge of a suitable circuit in terms of its components; this will be a simple function of the impedance (voltage/current) which is the alternating current equivalent of resistance. When dealing with oscillatory behaviour it must be realised that these related quantities are all complex, that is to say that they represent both the length, or modulus, and the angle of rotation, or phase, of a rotating vector.

A complex number may be written mathematically in various ways which are interchangeable, thus the complex number X is defined as follows:

$$X = A + iB \quad \dots\dots(22)$$

In this convention the vector is fixed by the Cartesian coordinates of one end, while the other is pivoted as it were at the intercept of the primary axes. The figure i is the square root of minus one, which serves only to distinguish the projection of the vector on the Y axis which is known as the imaginary part, from the real part, the projection on the X axis.

The same number may also be written in exponential form

$$X = r e^{i\theta} \quad \dots\dots(23)$$

$$\text{where } r = \sqrt{A^2 + B^2}$$

$$\text{and } \theta = \text{Tan}^{-1} \frac{B}{A}$$

This statement of modulus and phase may also be put in the following form

$$X = r \text{Cos } \theta + ir \text{Sin } \theta \quad \dots\dots(24)$$

If the resistance and capacitance (and inductance) of the units which make up a network are known, its impedance may be calculated in the same manner as is used to calculate resistances in D.C. work.

The impedance of Z_1 and Z_2 , if in series, is then $Z_1 + Z_2$, while if they are in parallel it is $\frac{Z_1 Z_2}{Z_1 + Z_2}$.

The impedance of a resistance (R ohms) is R, and of a capacitance (C farads) is $\frac{1}{i\omega C}$, where ω is the circular frequency of the alternating potential, 2π times the frequency.

Thus the analogue of a Voigt element is a capacitance and resistance in series. The total impedance, Z, is therefore

$$\begin{aligned} Z &= R + \frac{1}{i\omega C} \\ &= R - \frac{i}{\omega C} \end{aligned}$$

Consider the response of this circuit to an alternating voltage V, where

$$v = |V| e^{i\omega t + \theta}$$

where $|V|$ = modulus of voltage

t = time, hereafter in seconds

θ = any angle

The charge, Q, taken up by such a circuit will vary sinusoidally at the same frequency, but will lag behind the voltage by some angle

$$Q = |Q| e^{i\omega t + \theta - \phi}$$

The current, I, is given by

$$I = \frac{\delta Q}{\delta t} = i\omega |Q| e^{i\omega t + \theta - \phi}$$

Now

$$Z = \frac{V}{I} = \frac{|V| e^{i\omega t + \theta}}{i\omega |Q| e^{i\omega t + \theta - \phi}} = \frac{1}{i\omega} \frac{|V|}{|Q|} e^{i\phi}$$

$$Z = \frac{1}{i\omega} \left[\frac{|V|}{|Q|} \cos \phi + i \frac{|V|}{|Q|} \sin \phi \right]$$

$$= \frac{1}{\omega} \left[\frac{|V|}{|Q|} \sin \phi - i \frac{|V|}{|Q|} \cos \phi \right]$$

We have seen that for this circuit

$$Z = R - \frac{i}{\omega C}$$

which may be written

$$Z = \frac{1}{\omega} (R\omega - \frac{i}{C})$$

If two complex numbers are equal their real and imaginary parts are separately equal, thus we can see, using Eqn.(24)

$$\frac{V}{Q} \sin \phi = R\omega$$

$$\frac{V}{Q} \cos \phi = \frac{1}{C}$$

Returning to the mechanical circuit, the analogous quantities are as follows:

Tension T corresponds to Voltage V

Displacement D corresponds to Charge Q

Compliance $1/E$ corresponds to Capacitance C

Viscosity η corresponds to Resistance R

$$\therefore \frac{|T|}{|D|} \sin \phi = \eta\omega, \quad \frac{|T|}{|D|} \cos \phi = \underline{E} \quad (25) \quad (26)$$

Following Dillon, Prettyman and Hall (1944) we may introduce the terms E_{dyn} and $\eta\omega$ for the real and imaginary parts respectively of the mechanical impedance, or the complex Young's Modulus E

$$E = E_{dyn} + i\eta\omega \quad (26a)$$

In the case of a substance whose properties are represented by the Voigt model these two terms separate the viscous and elastic components in a simple manner, with other models they become more complex.

The model which will best represent the frequency dependent behaviour of the arterial wall will be discussed when the results have been presented.

In order to determine these properties it is necessary to measure simultaneously the tension and length changes in a specimen. It is of course necessary to know the modulus and phase of both of these quantities.

In the experiments here presented the relation of pressure P to external radius R_o is measured. In fact we have two changing quantities ΔP and ΔR_o superimposed upon the steady values. The respective moduli are $|\Delta P|$ and $|\Delta R_o|$ and the phase difference between pressure (leading) and radius (lagging) is ϕ

By combining Eqn. (15), (25) and (26) we have

$$E_{dyn} = \frac{|\Delta P|}{|\Delta R_o|} \frac{2(1-\sigma^2)R_i^2}{R_o^2 - R_i^2} \cdot R_o \cos \phi \quad (27)$$

$$\eta\omega = \frac{|\Delta P|}{|\Delta R_o|} \frac{2(1-\sigma^2)R_i^2}{R_o^2 - R_i^2} \cdot R_o \sin \phi \quad (28)$$

Having thus defined the problem, the next section will constitute a historical survey of previous work on the mechanical properties of blood vessels. Following this, the method here used will be described.

CHAPTER 2. HISTORICAL SURVEY
Structure of arterial wall

The mechanical properties of a heterogenous tissue will be determined by the arrangement and properties of the materials of which it is composed. These two matters will be considered before dealing with the properties of arterial wall itself.

The general pattern of the structure of the larger blood vessels is well known and is described in the standard textbooks (e.g. Cowdry, 1938; Maximow & Bloom, 1958). On histological grounds these vessels are divided into three groups, elastic, muscular, and intermediate, on the basis of the dominant component. The 'elastic' arteries include the aorta and its major branches, smaller vessels such as the femoral and brachial are classed as muscular; intermediate, or mixed, structure is found in the common iliacs and the bifurcation region of the carotids. There is, however, some difference of opinion on this point; for example the common carotid is classified as elastic by Benninghof (1930) and Cowdry (1938), and as intermediate by Sato (1926) and by Maximow & Bloom (1958). Blood vessel walls are divided into three zones, the tunica intima, media and adventitia. The separation between these areas is most clearly marked in the muscular arteries where the dense internal and external elastic laminae sharply define the boundaries of the tunica media.

The intima consists of the vascular endothelium and a thin layer of delicate elastic and collagen fibres by which it is anchored to the internal elastic lamina; in the dog this layer is very thin. The adventitia is a region of collagen and elastic tissue which merges with the surrounding connective tissue and contains vasa vasorum, nerves, lymphatics and some fatty cells.

It is by the structure of the tunica media that the different types of arteries are defined. In the elastic arteries the media is a thick layer consisting of 40-60 concentric shells of elastic tissue; the outer layers are somewhat condensed to form the external lamina while a less marked internal lamina also exists. The internal elastic lamina is a complex structure consisting of a fenestrated membrane lined on the intimal side by a coarse fibrous network. Dees (1923). The intermediate layers have a fibrous structure, the fibres running circularly or in a tight helix. Between these layers lie smooth muscle cells, mostly parallel to the elastic fibres though some are orientated longitudinally, and these lie in a network of fine collagen and elastic fibres which pass between adjacent elastic laminae and in addition form a well marked basket-like structure around each individual muscle cell. Collagen fibres are also found lying on either side of each elastic layer.

Benninghof (1927, 1930) distinguished between "spanmuskeln" and "ringmuskeln"; the former are inserted into the adjacent fibrous layers, while the latter form complete muscular rings with no 'tendinous' insertion. He further stated that the latter are rarely found in the aorta. Hughes (1943) described a similar structure in the developing avian aorta with sheets of muscle forming a series of incomplete concentric laminae. According to Benninghof (1930) in the ox the spanmuskeln are inserted in direct continuity with short elastic fibres, while in the human aorta they pass obliquely between neighbouring laminae.

In the muscular arteries the elastic tissue of the media is largely

concentrated at its edges. The internal elastic lamina is a dense layer separating the media from intima, while the junction between media and adventitia is formed by the external lamina, a rather more diffuse zone consisting of several elastic layers. The space between these two is largely occupied by muscle among which run strands of circular and radial elastic and collagen fibres. Benninghof stated that the muscle is largely in the form of complete rings not directly inserted into fibrous tissue (ringmuskeln). At the outer margin of the media the muscle lies obliquely and the closed rings are replaced by a helix.

Strong (1938), who used a microdissection technique, found that the muscle layer of the smaller vessels was arranged as a continuous helical strip with a tight pitch. In arteries of mixed or intermediate type the media contains bands of muscle scattered irregularly within the elastic laminae (Cowdry, 1938).

Two factors serve to distort the histological appearance of excised arterial specimens. The first is the longitudinal retraction which occurs when such a vessel is removed from the body, the second is that they are normally fixed at zero distending pressure.

Concerning the first Hiller (1884) found the aortas of young adults retracted by 27% of their original length, and that this figure diminished with the age of the subject, being 2% for the 65-78 years age group. Scheele (1908) obtained similar results, as did Hesse (1926), who studied arm arteries and whose youngest subject was aged 12, in which she reported a retraction of 40%. Other authors (Fuchs, 1900; Remington & Hamilton, 1945) have found the behaviour of dogs' vessels to be essentially the same.

It is well known that the elastic fibres in the vessel wall, and particularly those of the internal lamina, when examined in routinely prepared sections are seen to be markedly wavy. Is this waviness the result of the elastic recoil of the fibres themselves, or is it due to the action of some other component of the vessel wall, in particular the muscle? Furthermore are the fibres this shape when the artery is distended? Writing at the turn of the century MacWilliam (1902) raised these questions and held that the wavy shape of the elastin was due to a strong agonal or post-mortem contraction of the vascular smooth muscle, and in a second paper (MacWilliam & Mackie, 1908) this contraction was studied further. Later workers (Nakonetschny, 1922; Galloway, 1936) were of the same opinion and they found that in vessels fixed while distended at a pressure of 80 mm Hg or more the elastic fibres were almost invariably quite unkinked. This however is no proof that muscular contraction is the cause because many authors (e.g. Reuterwall, 1921) mention that the elastic fibres in unstretched ligamentum nuchae are also markedly wavy, and Strong (1938) found the same appearance in isolated elastic fibres teased from arterial walls. This problem was also studied by Wolff (1930), who stated "wir glauben mit gewissen Einschränkungen diesen Zustand als die Ruhelage, den Entspannungszustand der elastischen Membranen deuten zu dürfen". The question is not yet settled, though it seems more reasonable to suppose that the waviness represents the natural unstretched form of the fibres.

The retraction on excision and the collapse on release of pressure will also alter the ratio of arterial wall thickness to vessel radius.

Nevertheless of four extensive investigations of this ratio (Scheile-Weigandt, 1880; Tschewsky, 1903; Kani, 1910; Hürthle, 1920) only Hürthle considered this factor and took precautions against it. Hürthle presented the findings of a colleague who had been killed in the war. In a very careful study he filled the entire vascular bed of an animal with fixative in situ at a pressure of 130 cm H₂O. Sections were then cut and optical measurements made of wall thickness (h) and external diameter ($2R_0$). He found that the ratio h/R_0 was remarkably constant throughout the whole system, with a mean value of 0.15 except for the ascending thoracic aorta where the figure lay between 0.16 and 0.22. It must be remembered, however, that these figures refer very largely (229 out of 288 measurements) to a single animal, a one year old dog.

Quantitative data on the relative amounts of the three main constituents of arterial wall is also remarkably scanty. Figures for the quantity of elastin and collagen were reported by Harkness, Harkness & McDonald (1957) for dogs using a chemical estimation. It was found that in all but the smallest vessels elastin and collagen together made up 50% of dry weight. It was also stated "on the basis of relative proportions of elastin and collagen the systemic arterial tree was found to be divided rather sharply into two regions; in the intrathoracic aorta there was about twice as much elastin as collagen in the wall; in all other vessels the relation was reversed". Concerning the amount of smooth muscle the information is even less in amount and appears much less reliable, since it has been derived from estimates of the amount seen in stained sections. Ducret (1931) produced a figure of 80% for the mesenteric artery, while Hürthle produced the following

estimates of the relative amounts of collagen, elastin and muscle:

<u>Vessel</u>	<u>Collagen</u>	<u>Elastin</u>	<u>Muscle</u>
Aorta	1/3-1/10	1/3-2/5	1/3-1/2
Carotid	1/8	1/10-2/5	1/2-3/4
Brachio-cephalic	1/6	1/2	1/3

It is difficult to have much confidence in these figures; certainly those for collagen and elastin bear little relation to those given by Harkness et al (1957). This is not surprising since estimates of this sort rely on the use of stains which are often variable in behaviour, not to mention the difficulty of measuring the area occupied by a long fibre. In particular there is still much uncertainty as to what is actually coloured by the usual elastic stains (Hall, Reed & Tunbridge, 1952); furthermore the apparent amount of this material can vary enormously between adjacent specimens.

It is also obvious that when dealing with such fibrous materials estimates of the amounts of each present in any tissue will give little information on the mechanical properties to be expected. It is equally important to know the precise orientation and linkages of the fibres.

The mechanical properties of collagen, elastin and smooth muscle

Before considering the behaviour of arterial wall as such it will be helpful to review what is known as to the properties of its three main constituents. At this point it is also pertinent to remember that there is in fact a fourth, the acellular ground substance, the behaviour of which is quite unknown.

The three other materials have each been studied in as pure a state as possible, and the results obtained are summarized in Table 1. For collagen various tendons have been used, for elastin the ligamentum nuchae of the ox, while smooth muscle from many species and situations provides information which may be of value to this study. As regards elastin the ligamentum contains some collagen whose presence modifies its behaviour (Meyer & Ferri, 1935), but which when extracted has been shown to be more extensible than tendon (Wood, 1954). Furchgott (1955) has provided pharmacological evidence that vascular smooth muscle differs in many respects from that found in other parts of the body.

Although not directly related to the properties of arteries, the early work of W. Weber (1835) deserves mention here, for he emphasized two facts which form the substance for much of the following discussion. Weber studied the properties of silk fibres and pointed out that they became steadily less compliant the more they were stretched. All subsequent workers are agreed that, with the possible exception of bone, animal tissues show non-Hookean elastic behaviour. Furthermore he observed continuing extension in his specimens under constant load, or as we would now term it, creep. Weber argued that this behaviour could be considered to indicate the presence within the fibre of a force resisting rapid extension which was analogous to the viscosity of fluids. In addition he pointed out that this force would tend to damp out any induced oscillations in such a material, and performed experiments to show that this internal viscosity was a much larger factor than the resistance of the air.

Ligamentum nuchae and tendon were studied by Haycroft (1904) who considered their behaviour to be Hookean; this work contains no quantitative data. Very adequate figures are, however, to be found in the work of Reuterwall (1921). This author found tendon to be relatively inextensible (up to 0.05 strain), and that it was non-Hookean, becoming less extensible with increasing load; I calculate from his data final incremental moduli of the order of $10^7 - 10^8$ dynes/cm². In the case of ligamentum nuchae he found that it was more extensible (strains of 1 to 1.5), markedly non-linear, and with smaller Young's modulus (10^6 dynes/cm²). He also showed that both these tissues showed hysteresis and creep, though not to so great an extent as artery itself. He laid much stress upon this latter phenomenon, referring to continuing extension under constant load as 'accommodation'. Very similar results were also obtained by Wohlisch et al. (1926).

A series of experiments on arteries, tendon, ligamentum nuchae, and taenia coli were reported by Krafka (1939, 1940). This author used a commercial apparatus (Scott serigraph) developed for the textile industry, which produces semi-automatically length-load diagrams for strips of tissue. This apparatus is convenient but its use seems to have led Krafka to compare the behaviour of tissues under the same load, with no reference to their cross-sectional area. This is very confusing and makes it difficult to make full use of his data. He states that ligamentum nuchae and tendon were Hookean (but gives figures which suggest otherwise); at loads of 250 G the moduli (total) he gives are

4 and 50×10^6 dynes/cm² respectively. It should also be mentioned that it was this author who suggested the term incremental modulus for use when comparing the properties of arterial wall with those of its constituents.

Burton (1954) has reviewed the whole field of arterial elasticity. He stresses the importance of dealing with data in terms of the tension-circumference relation rather than that of pressure and volume, since the wall tension depends on the radius as well as the pressure. Burton's ideas on the behaviour of the arterial wall will be considered later but he gives figures for some elastic moduli which should be mentioned here. For rats' tendon he quotes a Young's modulus of 1.3×10^9 dynes/cm² at a strain of 30%. In the case of smooth muscle he calculates a figure from the data of Bozler of 0.06×10^6 , and for elastic tissue (data from Hass, 1942) of 3×10^6 .

Thus there seems to be fairly general agreement on the properties of tendon and varieties of elastic tissue. Both show non-linear elastic behaviour, tendon is relatively inextensible with a high Young's modulus; the reverse is the case for elastic tissue.

In the case of smooth muscle the position is very much more complex since the length-tension curve will depend on the state of activity of the tissue, and the speed of loading; in addition it has been known for a long time that it may react to a load with a myogenic contraction (W.M. Bayliss, 1902). In the case of resting molluscan muscle it has been shown (Bozler, 1936, 1941, 1957; Abbot & Lowy, 1958) that no elastic tension is developed until it is highly extended (to a length greater than is likely to occur in vivo); that is, it behaves

very much as a purely viscous substance. On being stretched to an extreme length it then behaves as a non-linear elastic substance. Working with the active dog's retractor penis muscle, Winton (1926) found that the length-tension curve was markedly concave towards the tension axis, but he gives no figures for cross-sectional area. This particular tissue is now known to be extremely unreliable but the same author (Winton, 1937) found similar behaviour of molluscan muscle. Krafka (1939) found that strips of dog taenia coli behaved much like ligamentum nuchae qualitatively and quantitatively.

There is much disagreement as to the tension that smooth muscle can exert or sustain. The figures given (in dynes/cm²) range between 1×10^3 (Burton, 1954), 8×10^5 (Fischer, 1944), 1×10^6 (Ducret, 1931) and 1×10^7 (Abbot & Lowy, 1958) (see Table 1). The figure taken from Ducret is the only one that refers specifically to vascular smooth muscle; it was derived from the study of a dog's mesenteric artery under the influence of adrenalin. The amount of muscle in the specimen was estimated histologically at 80%; as was mentioned earlier this method cannot be a very accurate one.

There is of course no simple relation between the tension exerted by a muscle and the Young's modulus. Since the latter is the force necessary to produce unit strain it must at least be greater than the tension, but it is hard to say how much greater it will be. These figures are therefore included to give some idea of the likely magnitude of the modulus of active smooth muscle, various direct and indirect measures also being mentioned in Table 1.

Although collagen and elastin both show some degree of elasticity after extension, or creep, this is much less than that seen in arteries and in smooth muscle. This fact has been pointed out by many workers, e.g. Reuterwall (1921), and is the basis for the belief that the time dependent properties of the arterial wall are determined by the muscle it contains.

Winton (1930) appears to have been the first to have studied the elasto-viscous properties of smooth muscle. In this study he used dog's retractor penis, which, as already stated, is now known to be unsatisfactory in that it is too thick to be adequately oxygenated in the usual organ bath. Winton also (1937) found similar behaviour in molluscan (*mytilus edulis*) muscle. For both muscles he used a three element model to represent the elastic behaviour; this consisted of a dashpot (viscous response), a Voigt element (retarded elastic response) and a spring (instantaneous elastic response). We have seen that the first element allows steady viscous extension up to a relatively great length in the relaxed muscle; Winton found similar behaviour when active. The spring was found to be distinctly non-Hookean, becoming markedly stiffer at great extensions. The time course of the retarded elastic element was stated to be exponential, but it is not certain to what extent this is an approximation. It is worth remembering that one might not expect the lengthening under steady load of such a tissue to be truly exponential, since the area over which the force is applied will become less as extension proceeds.

The relaxation time of this element was greatly altered by adrenalin and by stimulation with alternating and direct currents (Winton, 1937). The 90% relaxation time (isometric) after A.C. stimulation was 30 seconds, when D.C. was used it was of the order of 60 minutes. Long relaxation times of this magnitude are also produced by acetylcholine (Lowy & Millman, 1959). Such properties could permit this muscle to maintain a steady tension with the minimal expenditure of energy.

Winton (1937) also performed the interesting experiment of studying the response of molluscan muscle to oscillating stress. He found that the muscle became stiffer when stretched at increasing rates, above 2 c/s no further increase in stiffness occurred, with unstimulated muscle the increase was about 10-20%. A.C. stimulation produced little change in stiffness at 20 c/min. but at very low frequencies (0.6 c/min.) twice the extension was produced by the same force. That is to say that the stiffness was halved, due presumably to a diminution of viscosity.

Similar results were reported by Pryor (1950, 1952) who has also discussed the behaviour of smooth muscle. Winton's experiments on the dynamic properties of molluscan muscle were repeated and the critical frequency was determined, this is the frequency at which the apparent elastic modulus of a material changes from the static to the dynamic value. In the case of resting muscle this frequency was less than 0.02 c/s, with active muscle it was about 1 c/s, again suggesting a diminished viscosity.

Remington & Alexander (1955) and Alexander (1957) have put forward somewhat different views. Studying the pressure-volume relationships of kittens' bladders, they demonstrated the usual non-linear elastic behaviour

with separation of the inflation and deflation curves. The configuration of these hysteresis loops was not markedly altered by poisoning the viscus with cyanide. The slope of tension decay was also unchanged by death or by stimulation of the muscle. They therefore held that the contractile elements were not the same as those responsible for these aspects of viscous behaviour, thus disagreeing with Winton.

The hysteresis loops obtained by a series of inflations were examined in some detail. It was found that with repeated stretches these loops became progressively narrower, the curve of deflation approaching the constant curve of deflation. After a series of such cycles (usually 4 or 5) a relatively narrow 'stable loop' was obtained; a long time (30 minutes) was necessary for the tissue to return to its original 'unstretched' state. Again, identical behaviour was shown by the poisoned bladder.

The properties of excised gut rings were also described (Remington & Alexander, 1956). The elongation curves given by this tissue when loaded followed a roughly exponential course for about 10 seconds, thereafter the trace showed a slow extension at a steady rate which was unaltered by changing the load. This behaviour is very similar to that described by Winton in 1937, except for the fact that it was essentially unaltered by the administration of adrenalin and acetylcholine. When the specimen was subjected to a second stretch the rate of the logarithmic extension was found to be much greater; the effect of a stretch was thus to alter the muscle viscosity, recovery of the original state was very slow.

It will be realised that the hysteresis loop behaviour is not necessarily a reflection of viscosity in the accepted meaning of the term. The development of these hysteresis loops, whose width does not seem to

be markedly time dependent, may be considered as due to 'structural hysteresis' (Remington, 1957). This develops when the internal configuration of a material is altered by straining; when the load is removed different conditions exist and the course of unloading is likely to be different from that of loading.

These remarks do not apply to the experiments on the rate of tension decay which were unaltered by stimulation or death. The reason for this is uncertain; other workers, (Pryor, 1950; Winton, 1937) find a definite diminution in viscosity. Remington & Alexander were unable to study the full time course of tension decay as they could only maintain a contraction for 10-20 secs. Their measurements do not appear to be extremely precise; in another paper (Remington & Alexander, 1956) they found some evidence of viscosity change in specimens of ileum; the rate of elongation under constant load was altered by adrenalin and acetylcholine, but the changes were small.

The dynamic properties of smooth muscle have been the subject of a series of interesting experiments by Reichel (1952b). This worker used strips of molluscan muscle and compared their responses under static and dynamic (31.6 c/s) conditions. Because the specimens tended to stretch under a steady load he found that the stiffness was better related to the applied tension than to the length. The dynamic stiffness was always greater than the static, but the difference became less the more the specimen was loaded. At mean loads the ratio was 1.8:1. It is possible to calculate the corresponding elastic moduli from Reichel's data. The figures I obtain are static, 2.5, and dynamic, 4.5 dynes/cm² x 10⁶ (incremental moduli).

No details of behaviour at different frequencies are given. The author states however that the properties of this muscle were qualitatively the same as those of striated muscle. It had been found earlier (Fieper, Reichel & Wetterer, 1951) that the greatest increase in stiffness occurred below 0.5 c/s. At higher frequencies greater stiffness was found but the increments became progressively less up to 100 c/s. This behaviour was stated to be compatible with the Levin-Wyman model of muscle, which consists of a damped and undamped spring in series, provided that the springs were considered non-linear. The same statement was made in the case of smooth muscle but no exact analysis seems to have been made.

The work of Bozler (1936, 1941, 1957) has already been mentioned. This author has repeatedly maintained that the contractile elements of smooth muscle and those responsible for its viscous properties are one and the same. As evidence for this he has shown that the time course of isometric tension decay (stress relaxation) is identical whether the initial tension was produced actively or passively. With the use of more refined recording techniques these results have been challenged. The tension after isometric tetanus in the molluscan muscle (as studied by Bozler) does indeed decay exponentially with a half time of about 1 second (Abbot & Lowy, 1958b). Following a passive stretch, however (Abbot & Lowy, 1957) the fall in tension was found to be described by two exponential functions, one fast and one slow. These rate constants were, in the case of the mytilus posterior adductor, 5.8 and 68 seconds for 50% relaxation, and it was stated that they were unaffected by rate of stretch or temperature.

The evidence, therefore, would appear to allow one or two general conclusions on the viscoelastic properties of some forms of smooth muscle. It seems that the resting muscle behaves much as a purely viscous material until it has extended to relatively great lengths; when this point is reached non-linear elastic resistance is encountered. Tension decay follows a relatively simple course which may partly be the effect of relaxing elements separate from the contractile apparatus. There is little agreement on the amount of force smooth muscle can exert when stimulated, and relatively little evidence on the viscous state of the tissue when active.

The evidence quoted on the long relaxation times and low critical frequency, together with that concerning long range viscous deformation suggest that the difference between dynamic and static elastic properties of tissues rich in smooth muscle will be great, and that differences will become apparent at quite low frequencies.

The most satisfactory results derive from work on molluscan unstriated muscle. One must be wary in assuming that these findings apply in the case of mammalian vascular smooth muscle. It has been shown (Hanson & Lowy, 1957) that these molluscan muscles possess a double array of myofilaments such as have been described in striated muscle, and which form the basis of a current theory of muscular action (Huxley, 1957). No such structure has been demonstrated for mammalian smooth muscle (Lowy, 1959) and thus the only structural feature that these tissues share is the absence of cross striations, a somewhat negative correspondence. The first paper by Winton does suggest some degree of functional similarity, but in the present state of knowledge there are no grounds for supposing that it is very marked.

Studies on arterial tissues

The literature on the subject of static arterial elasticity is extremely copious and it has not been my intention to cover it in its entirety, but rather to select what have seemed to be the more important contributions. Full references may be gathered from a number of reviews on this matter (Reuterwall, 1921; Wezler & Sinn, 1953; Burton, 1954; Kapal, 1954; Sinn, 1956; Mayer, 1958). It will also be apparent that the methods used are various and include the study of vascular rings and strips, and of whole vessels which may or may not have been free to alter in length; in addition the experiments have been performed at greatly differing speeds: all these factors contribute to the difficulty of presenting an overall impression of the present state of knowledge.

The earliest experiments were performed by Wertheim (1847) who studied the length-load curves of various animal tissues. He found that arterial strips became stiffer with increasing extension and that the curve was somewhat like a hyperbola, but became even steeper at high loads.

The most important of the earlier workers was undoubtedly Roy (1880) whose beautifully performed experiments shed light on many aspects of the subject. He noted that arterial strips lengthened when cooled and became warm on being stretched, and that this behaviour was also seen in rubber but was contrary to that shown by metals. He was aware of the phenomenon of stress-relaxation and therefore performed his experiments as slowly as possible, using an ingenious apparatus to register automatically pressure-volume curves of his specimens which were, however, free to

elongate. These curves were S-shaped, showing first a low, then a high and finally a rapidly decreasing distensibility; the region of maximum distensibility fell within the normal range of blood pressure for the animal concerned. He studied the behaviour of strips and confirmed Wertheim's results and in addition showed that the behaviour of strips cut circumferentially and longitudinally was much the same, but that the latter exhibited a more abrupt increase of stiffness. Some years later (Thoma & Kaefer, 1889) somewhat different results were published. These workers used human external iliac arteries of various ages which were fixed in length and produced pressure-radius curves of the familiar shape; however, when pressure is plotted against radius squared (see Fig. 16) it can be seen that the volume-extensibility decreases steadily from the lowest value used (20 mm Hg). The reason for this difference (other workers had obtained curves of either form) appeared to be settled some years later (MacWilliam, 1902; MacWilliam & Mackie, 1908) on the basis of smooth muscle activity. It was found that in a vessel removed from the body the muscle was in a state of spasmodic contraction which might persist for several days. The pressure-volume curve of such a vessel was markedly sigmoid. When the spasm was released, which might be effected by distension and other manoeuvres, the type of curve published by Thoma & Kaefer would now be produced. The nature of this spasm was in doubt, but it was certainly extreme; "the artery becomes so stiff that a piece 7-8 cm long may often be held out (by one end) in an almost horizontal position."

The next major study was that of Reuterwall (1921, 1922). This author studied vessels both as strips and entire, and also tendons and

ligamentum nuchae; his work is especially valuable for the completeness of his quantitative data and a curve taken from his 1921 paper is given in Fig. . He was particularly concerned with the phenomenon of accommodation (creep) which he showed to be more noticeable in the muscular vessels, though it could be seen in ligamentum nuchae and even, to a very small extent, in tendon. In his experiments on the vessels they were held at a fixed length somewhat greater than excised length. Reuterwall's figures show steadily diminishing extensibility as pressure is increased, with a rather sharp inflection in the region of normal blood-pressure. On the basis of relative stiffness he concluded that the elements of the vessel wall were arranged in parallel, that is to say that the load was passed from one to the next as the circumference increased and the slack was taken up. This point is reached for elastic fibres at a pressure of 20-30 mm Hg, for collagen at 150-160; he was less sure as to the influence of the muscle except for its relation to accommodation. This conception has lasted to the present day, though later workers have tended to be more concerned with the effects of smooth muscle.

Similar conclusions were reached by Hwiliwitskaja (1926), who also showed that the modulus of volume elasticity increased with age. This author also put forward the idea that the effect of muscular contraction was to shift the load on to the more compliant muscle fibres, thus resulting in an increased distensibility.

Ten years later Hallock & Benson (1937) published a well-known paper on the effects of age on the properties of human aortas. They performed fast pressure-volume determinations on specimens at fixed length

and found that the distensibility decreased with age, and that only the youngest vessels gave a sigmoid curve. They found fair correlation between their results and average pulse wave velocities, but the figures derived from their elasticity measurements were about 10% too low; this they suggested might be due to the fact that they had removed the adventitia before testing. Wiggers (1938) attempted to obtain more directly physiological data by measuring aortic elasticity (pressure/volume) in situ and obtained sigmoid curves, but emphasised the variability of the results from dog to dog.

The first serious study of arterial wall in terms of elastic moduli was made by Krafka (1939, 1940) using vessel and tissue strips. He obtained total moduli of the order of 1×10^6 dynes/cm² and related them to similar measurements on muscle, tendon and ligamentum nuchae. He held that the distensibility of a vessel was better related to size than to load, and that an old vessel was bigger and therefore stiffer, this being due to the parallel arrangement of the wall elements, as postulated by Reuterwall. Similar ideas were put forward by Hass (1942, 1943), who showed that when the collagen was extracted with formic acid the length-tension curve of the remaining elastic fibres became more nearly linear, and rose much less steeply at the larger lengths; in addition hysteresis was diminished.

Since the end of the second world war there has been a revival of interest in the elastic characteristics of blood vessels which has been largely prompted by the desire to understand more fully the pressure-flow pattern of the arterial system, and to measure the stroke-volume from a

consideration of the pulse pressure wave. This work is due chiefly to the groups at Georgia and Munich, but many other schools have contributed (see for example Remington, 1954, for an account of the former group's approach).

In three papers (Remington & Hamilton, 1945; Hamilton & Remington, 1947; Remington et al., 1948) results were given of many measurements on rings of human and dog vessels. Pressure volume curves constructed from this data are very slightly sigmoid for the thoracic aorta (Fig. 16) while in the other vessels the curve is more nearly hyperbolic. In addition they compared the behaviour of human aortas of ages from 8-89 years and confirmed the findings of Hallock & Benson with the important proviso that the scatter was so great as to render the differences probably insignificant (the earlier authors only gave average results for each age group). It was also noted that there was little difference between the actual volume increments for each pressure change at the various ages, but that the relative distensibility decreased with age because the original volume tended to be greater. They felt that they could safely ignore changes in vascular 'tone' since they found that the curve given by a constricted vessel ring was parallel to that from a dilated ring, i.e. $\frac{\delta P}{\delta V}$ remained constant.

In 1951 there appeared the first of a series of papers from Munich dealing with arterial elasticity (Wagner & Kapal, 1951). A semi-automatic apparatus was described for producing pressure-volume curves of whole vessels free to lengthen on inflation. With this method they obtained, using cow's aortas, markedly sigmoid curves showing increased distensibility over a large pressure range (25-125 mm Hg). From this they deduced

that the pulse wave velocity should be substantially constant throughout the physiological blood-pressure range, and they devoted some time to the consideration of this aspect of the body's "economy". These experiments were performed very slowly but hysteresis was fairly marked during the first cycle of inflation and deflation; with repeated cycles however the ascending and descending curves approached one another until a constant, relatively narrow, loop was obtained.

Later the same authors (Wagner & Kapal, 1952) published similar results for human aortas. Sigmoid curves were only found in the youngest vessels, those from subjects older than 25-30 showed no double inflection.

Landgren (1952) considered the influence of muscular contraction on the elastic properties of the carotid sinus and found the vessel to be smaller and less distensible under the influence of adrenaline; after the application of nitrite the reverse situation was observed. This concept was challenged by Alexander (1954a) who found the aorta was constricted and became more distensible when the muscle was made to contract. The same behaviour was observed, somewhat indirectly, in the splanchnic venous bed. This idea is, of course, similar to that previously advanced by Hwiliwitskaja in 1926. In the same year this author (Alexander, 1954b) also studied the delayed compliance of the vascular bed and concluded that it was due to the smooth muscle.

The whole subject of the relation of vascular structure to function was reviewed by Burton (1954) who again emphasised that the tension in the vessel wall depended on both the internal pressure and the radius and showed that conversion of pressure-volume data into length-tension

diagrams was a necessary first step in the understanding of the problem. Believing that the tension exerted by smooth muscle was very much less than that to be expected in the walls of arteries he postulated various complex linkages between it and the other wall elements. That such an arrangement would confer a mechanical advantage (100:1) is undoubted, but at the same time extreme shortening would be required to bring about an appreciable constriction of the vessel. As was indicated earlier the evidence suggests that the tension developed by smooth muscle may be many times greater than was assumed by Burton, and this hypothesis is unnecessary.

Having compared the Young's moduli of collagen and elastin with those from arterial wall Burton revived the idea that the behaviour of vessels at low pressures is dominated by elastin, while at high loads the collagen comes into play. He seems to have been led to this conclusion by the belief that any homogeneous material should be Hookean in its properties. Later (Roach & Burton, 1957), the extraction experiments of Hass (1942) were repeated and extended. After removal of much of the collagen with formic acid, or of elastin with crude trypsin, the length tension curves of the treated vessel were compared with the normal. Undoubtedly the properties of the remaining structures will be altered by such treatment, but the results do show a similarity between the elastance of the collagen depleted specimen and the intact vessel at low pressures. Similarly the elastin depleted material had a stiffness similar to the distended artery. The relative amounts of either material under stress in the middle pressure range cannot be estimated, as the authors attempt to do, because of the marked changes in dimensions of the treated specimens.

The work of Remington & Alexander (1955) on the properties of the bladder has already been discussed; in the same year Remington (1955) reported similar behaviour in aortic rings. A series of stretches produced gradually narrowing hysteresis loops in both the 'living' and cyanide treated specimen, and indeed when the tissue had been left to putrefy. This behaviour was qualitatively similar to that found in the bladder, femoral and carotid arteries.

In a later discussion of this subject (Remington, 1957) the rate of stress relaxation in specimens of tendon, ligamentum nuchae, aorta, femoral artery and gut was compared. When arranged by amount and speed of stress relaxation these tissues came in the same order as given, the last named showing the greatest effects (but the next diagram presented appears to show no difference). Although it had been stated earlier that the rate of filling had no effect on the peak tension achieved in the bladder wall when a stable loop had been achieved (Remington & Alexander, 1955), the later paper by Remington contains a figure (Fig. 9) showing just such an effect. When a bladder was inflated in situ at a cycle length of three minutes a tension swing of 20 g/cm was seen; to produce the same volume change twice a second about three times this tension change was required. Unfortunately the same experiments were not performed on blood vessels.

In the same year it was shown (Leonard & Sarnoff, 1957) that helically cut strips of dog's vein became shorter and less extensible when the smooth muscle was made to contract with the drug 'aramine', and that the amount of stress-relaxation varied directly with the tension produced. However Bader & Kapal (1957a) published further pressure-volume curves in which the effect of muscular contraction was to decrease the

volume contained at zero pressure and that thereafter the traces were parallel. To account for this they have postulated (Kapal & Bader, 1958⁴) an arrangement of muscle fibres connecting the circumferential connective tissue fibres like the rungs of a ladder, a similar type of model to that proposed by Burton (1954). These same authors have also published a series of papers (Bader & Kapal, 1957b, 1957c, 1958, 1959) describing the behaviour of rubber tubes. They show that the distensibility always increases with inflation, resembling the behaviour of the aorta in the lower pressure range, but by surrounding one tube by a series of others or by using nylon jackets the resemblance can be extended to cover the whole physiological pressure range. The final paper shows that a rubber tube becomes more distensible when it is stretched longitudinally and discusses this interesting finding in relation to the retraction of excised arteries.

In addition Kapal & Bader (1958⁴) compared the behaviour of the thoracic and abdominal aortas and found that the latter was less distensible and gave a more sharply inflected curve. Using the same apparatus Karnbaum (1957a, b) confirmed the sigmoid pressure-volume curve for young human aortas but found that those from subjects of more than 40 years had no region of maximum distensibility. Simon & Meyer (1958), however, felt that this change occurred at about the age of 20. It is also of interest that the elasticity of the extra-pleural pulmonary vessels has been found (Meyer & Schollmeyer, 1957) to alter with age in the same manner as do the systemic vessels; the resting volume increased with age while the volume increments with pressure were the same.

It would seem that the following general conclusions may be drawn from the previous work on the static elastic properties of excised arteries which has been cited.

1. The arterial wall is non-Hookean in its elastic behaviour, becoming stiffer with increasing distension.
2. There is a tendency for the arteries to become relatively stiffer with increasing age, though this may be mainly a reflection of their increase in size.
3. There may or may not be a pressure region of maximal distensibility for young vessels, with increasing age the pressure-volume curve becomes hyperbolic (using the term loosely). Markedly sigmoid curves are possibly the result of smooth muscle spasm.
4. The effects of smooth muscle activity on vessel distensibility are disputed. The more muscle present in the wall the more pronounced will be its time dependent behaviour.
5. The collagen and elastin fibres are arranged in parallel in the arterial wall; the disposition of the smooth muscle is uncertain.

Representative values from the literature, together with my own, are shown in Table 2.

Relatively few studies have been made of the properties of arteries in vivo, except for more indirect methods based on the pulse wave velocity. Curiously enough the greater number of these reports refer to the dynamic properties, but a few are concerned with the more long-term relationships of pressure and volume.

The size of the aorta was measured with a mechanical aortagraph by Wiggers & Wegria (1938). When acute hypertension was induced either

by drugs or nervous stimulation the vessel first dilated, and then, while the pressure was still high, constricted. It was held that this latter effect, which outlasted the pressure rise for a few seconds, was due to an active muscular contraction. The distensibility of the vessel also changed, becoming smaller as the diameter increased, and vice versa. The pulsatile diameter changes, which were also recorded, were not much smaller than the longer term alterations, but no attempt was made to calibrate the records. The curves of pressure and dilatation reproduced are almost identical in shape, and appear to be synchronous.

In 1947 Katz et al. attempted measurement of pressure volume curves in vivo by comparing the records of mean pressure with X-ray photographs of the aorta outlined with injected contrast medium. Their results show a great deal of variation and it is difficult to draw conclusions from them. In addition they attempted to correlate these findings with pressure volume curves determined on the excised vessel. In these experiments the aorta was apparently free to change in length, and in fact it was found that the two methods gave different results.

More recently Richards (1953) has employed an ingenious method for estimating the elasticity of the entire femoral bed of a cat by following for about ten seconds the fall in pressure when the femoral artery was occluded. A pressure-volume curve of roughly sigmoid shape was obtained, showing maximal distensibility in the region of 40-50 mm Hg. It is somewhat difficult to interpret these results since the regions whose capacity changes is being measured may be expected to alter as the pressure in the main vessel falls. As Richards points out, this could result in a sigmoid curve even if the individual segments of the tree showed a linear pressure-volume relationship.

The dynamic properties of arteries have been studied both in vivo and in vitro, taking the term 'dynamic' to mean the length tension characteristics of the vessel wall measured on a time scale comparable to that of the pressure pulse wave. Studies in vivo have been largely confined to attempts to measure the change in diameter occurring at each pulse beat. Such changes as do occur are extremely small, as was remarked upon by Joseph Lister (1879); "If any increase do occur in the diameter of an artery in systole it is inappreciable by ordinary methods of measurement."

Since then the increase in diameter of an artery in systole has been measured on many occasions and it is indeed very small. These experiments have been mainly directed at discovering or disproving the existence of active pulsatory arterial behaviour (see for example Hess, 1916; Fleisch, 1920; Wehn, 1957). This question is outside the scope of the present work; a full review of the matter is given by the last-named author. Thus many studies were not quantitative, more attention being paid to the form of the pressure and diameter changes.

However, as might be expected, Hurthle has studied the matter, and he gives some figures (Hurthle, 1923). These derive from experiments in which the volume of a length of vessel was measured with a small plethysmograph, while a simultaneous record of blood-pressure was obtained from a nearby branch, or even from the contralateral vessel. He found that the femoral artery changed about 4% in volume for a pressure change of 100 cm water, while in the case of the carotid the figure was about 2%. Taking the relative wall thickness to be 15% (Hurthle, 1920) it was calculated that the Young's modulus for the arterial wall was in the ratio 1:6.6:10 for the aorta (abdominal) carotid and femoral arteries respectively.

Rushmer (1955) has also studied this problem. Using a small highly compliant strain gauge of a type earlier used for limb plethysmography (Whitney, 1953) tied around the aortic arch, he measured changes in circumference of approximately 2.5% (1 mm in 38) at blood pressures of 120/100 mm Hg. Lowering the pressure appeared to diminish the distensibility and vice versa, in contrast to the findings of Wiggers & Wegria (1938). Rushmer was concerned that the phase relationships of his records appeared to indicate an active supply of energy by the aortic wall to the blood, i.e. the diameter changes led the pressure changes. The phase lead was not very great, nothing like the figure of 180° reported by Wehn (1957) in the rabbit's femoral, but they were disturbing. It was suggested that three factors might be concerned, artefacts produced by the recording apparatus (it has recently been shown, Lawton & Collins (1959) that the dynamic behaviour of the strain gauge was satisfactory), active aortic systole, or length changes of the vessel due to cardiac movements. He was unable to decide between these, though he favoured the third explanation.

The changes in length and diameter of segments of abdominal aorta have also been studied (Lawton & Green, 1956; Lawton, 1957) using high-speed cinematography. Small glass beads were sewn on to the vessel and changes in their separation related to the simultaneously recorded pressure.

The results differed from Rushmer's in several respects, in particular no reversed hysteresis loops were observed between pressure and diameter. This record also indicated an increased distensibility when the pressure was lowered with acetylcholine, and the loop was wider

(increased damping). The reverse occurred following the use of adrenaline. The diameter changes in different conditions varied from 3% to 8%, length changes were found to be smaller ($\pm 1\%$) and not clearly related to the pressure level. In addition the length changes were 180° out of phase with the pressure, a finding that did not fit with any of the models previously suggested by one of the authors (King & Lawton, 1950).

There have not been many reports on the dynamic behaviour of the arterial wall in vitro. Some of these have been mentioned earlier (e.g. Remington, 1955) but I refer here to those studies in which a quantitative determination has been made, either directly or indirectly, of the frequency-dependant properties of this tissue.

There is no general agreement on these properties, except that they are too complex to be explained in any simple manner. Since this matter forms the substance of this work the literature will be discussed later in relation to my own findings.

Before passing on to the next section an indirect way of investigating this matter will be considered briefly.

Pulse-wave velocity

Although other factors are also important, the velocity of propagation of a pressure wave in an elastic tube is largely determined by the elasticity of the wall. Therefore measurements of pulse-wave velocity are to some extent measurements of the dynamic arterial behaviour.

Modern experimental studies of this nature may be said to start with the paper of Bramwell & Hill (1922). These authors showed that the figures of Roy (1880) predicted a rise in velocity with pressure but the figures were all lower than those they had observed in man (4-5 as against 6-7 m/sec). They attributed this to dynamic elastic effects in the artery (Roy performed his experiments very slowly). In an ingenious experiment they measured the velocity in a length of human carotid which was filled with mercury to slow down transmission. This showed clearly that the pulse wave velocity increased with pressure (though there was a minimum at 57 mm Hg) and they suggested that the relevant pressure was the diastolic. Further experiments (Bramwell, Downing & Hill, 1923) confirmed this, the mean velocity in the common carotid being around 10 m./sec. The relation between velocity and pressure was pursued to the point where it was suggested that the systolic peak of the pressure pulse might overtake the foot and form a breaker, as a wave on the sea (Bramwell & Hill, 1923). The analogy here appears merely verbal and has not been followed further. Bramwell, McDowall & McSwiney (1923) were able to alter the effective diastolic pressure in a length of vessel in vivo by surrounding the limb with an inflatable cuff. Measurements of pulse wave velocity in this situation showed a decreasing arterial extensibility from zero pressure upwards. Later experiments in which the mean pressure was altered by raising the arm (Hickson & McSwiney, 1925) or by pressure or suction applied through a cuff (Hemingway, McSwiney & Allison, 1928) provided yet further confirmation of this point.

Bazett & Dreyer (1922) studied the velocity in different parts of the body and found that it increased peripherally from about 4 m./sec in the aorta to $\times 8$ in the radial artery. This was confirmed by Fulton & McSwiney (1930).

In all the experiments already referred to the arrival of the pulse wave was detected externally by means of a tambour. Furthermore Bazett & Dreyer estimated central velocity by using a recorder over the apex beat and allowing for a slight lag in cardiac ejection, these figures therefore are not wholly above reproach. However Broemser & Ranke (1930), using optical manometers, recorded aortic velocities in dogs and cats of between 4.5 and 5 m/sec.

In 1934 Hallock studied the changes in arterial elasticity with age by means of measurements of the pulse wave velocity. He showed that the peripheral velocity was greater than the central and that both increased with age. While agreeing with Bramwell & Hill on the relation with pressure, he was unable to obtain a good correlation between velocity and diastolic pressure because of the age variation of his subjects. Bazett et al. (1935) confirmed the variation with age in an interesting paper in which they attempted to use the arterial distensibility derived from velocity measurements in the calculation of stroke volume.

Wezler & Boger (1936) also confirmed the peripheral increase in velocity in a study on human subjects. In addition they found that the aortic velocity was much increased when the blood pressure was raised with adrenaline.

A study of the velocity of transmission in various parts of the dog's arterial system, using intra-arterial recording (Daulton & Hamilton, 1939) gave the following figures, though the authors emphasised that the values vary

considerably from animal to animal: upper aorta 4-7, middle aorta 6-8, and iliac and femoral 8-12 (m./sec). Very similar figures have been published more recently (Laszt & Müller, 1952a, b).

The papers of Hamilton et al. (1945) and Remington et al. (1945) contain interesting discussions on the difficulties in correlating the experimentally determined velocity with that calculated from elasticity measurements. They emphasise that the viscous properties of the artery must be taken into account or the predicted velocity will be too low. In addition the stretch data to be used should not be taken from the first extension, in other words the vessel must be allowed to 'accommodate'. When these factors are allowed for their results suggested that the factor 0.9 should be inserted into the Moens-Korteweg equation to give the best fit. The measured pulse wave velocities were about 10% higher than those predicted from their determinations of elasticity in the lower pressure range, but the discrepancy increased at greater distension. These effects may be partly due to the use of rings in which changes of length in the long axis of the vessel must have been occurring.

An ingenious new approach to the measurement of pulse wave velocity has been devised by Landowne. This author has studied the transmission of pressure waves and spikes in rubber tubes and umbilical arteries (Landowne, 1957a) and in human radial arteries in vivo (Landowne, 1957b). Pressures were generated by impacts on the outside of the vessel from a small solenoid driven device. He found that the velocity increased steadily with frequency up to 50 c/s, and confirmed that it also increased with the blood pressure. The velocities he found in human vessels were rather high (e.g. 10 m/sec at 30 mm Hg, 30 m/sec at 200 mm Hg). This

may possibly be due to the fact that his subjects were old; one is described as 'a 91 year old man with marked calcific sclerosis'. If the velocity is also frequency dependent it might be that high frequency waves constituting these spikes were travelling faster on this account.

CHAPTER 3. METHODS

The method here detailed has been described briefly in a previous publication (Bergel, 1958). The arterial specimens used were all obtained from the cadavers of dogs which had been used as blood-donors in extra-corporeal circulation experiments. They were removed within an hour of death, and most commonly within half an hour. These animals were nearly all greyhounds, of around 25 kg body weight, and all that were used showed no signs of disease. The vessels of any hound showing pathological features, in particular scarred kidneys or rigid, brittle arteries, were discarded.

The portions of the vascular tree required were exposed and carefully mobilised by blunt dissection; this usually entailed opening the thorax and abdomen and exposing the entire aorta, iliac, femoral, brachio-cephalic and common carotid arteries, but in a few cases smaller portions were cleared. The selected vessels were then marked by inserting fine entomological pins tangentially through the adventitia, care being taken to avoid entering the lumen, and the separation of the pins was then measured, by means of dividers, to the nearest millimetre. When this was completed the entire specimen was cut free, leaving all branches, especially the very smallest, as long as possible, and placed in a large volume of isotonic (0.9%) NaCl.

The separation of the marker pins was then remeasured, and by this means the contraction of the vessel on excision could be calculated. Suitable lengths of artery, about 5-7 cm long, were now chosen, the aim being as far as possible to select a portion free from major branches;

obviously curved or kinked pieces were not used. The chosen segment was gently freed from all surrounding connective tissue; for this purpose fingers were found most suitable, since instruments were prone to tear off small branches close to the parent vessel.

A point was reached in this process at which a relatively smooth and compact layer free from loose strands and sheets of tissue was exposed. This was taken to be the outer limit of the arterial wall proper.

Cleaning being complete all branches were ligated and cut as close to the main branch as possible, using fine terylene thread. Finally the vessel was patted dry in a cloth and prepared for suspension on a balance by threading a short (3 cm) length of 36 gauge wire through the wall close to one end.

The specimen was weighed twice; in air, and suspended in distilled water. In the second weighing great care had to be taken to ensure that no air bubbles remained in the lumen; for this purpose a jet of water was passed through the vessel from a length of fine polythene cannula attached to a syringe. The volume of the specimen was obtained by Archimedes' principle, and assuming it to be a regular cylinder the relation between internal and external radius (R_i and R_o) at any length L could be easily found; $R_o^2 - R_i^2 = V/\pi L$, where V is the volume.

The outside of the artery was next coated with waterproof Indian ink to reduce reflections, and when dry it was replaced in saline for at least fifteen minutes before mounting in the apparatus.

The measurement of vessel diameter

It was decided that a method of measuring diameter changes which involved any restraint was undesirable, and since with a substance whose behaviour is non-linear it is essential to use displacements as small as possible, an optical device employing a photomultiplier was employed; this apparatus is detailed in Figs. 1-5.

The inked vessel was first attached securely to a pair of tubular holders of appropriate size which were threaded and provided with washers. These could be attached to two horizontal bars whose separation could be varied so as to extend the specimen to the length it occupied in vivo. The vessel was placed vertically in the path of a horizontal light beam, of depth $1/8$ inch and of variable width. The light source used was a 60 Watt single filament tungsten bulb driven by a 12 Volt accumulator; to avoid changes in light intensity the connections were made of aluminium strips which did not heat up and so alter in resistance. The battery was on continuous charge from a mains supplied rectifier. The beam was collimated by passage through two sets of adjustable slits at either end of a metal tube (bore 2 in, length 12 in). To reduce internal reflections an inner tube of corrugated rubber as used in anaesthetic apparatus was inserted, and the inner surface of this was smoked.

The shadow of the specimen was thrown on a diffusing screen. This was made from a 2 inch length of $1\frac{1}{4}$ inch diameter perspex rod which had been longitudinally bisected; the flat surface faced the light and was ground with the finest emery powder, while the cylindrical face concentrate

FIG. 1.

A schematic diagram of the apparatus used. On the right is the arterial specimen (A) held vertically between supports. The lower end is connected to the cam driven pump and to the mercury column manostat (C.P.). Light from the lamp (La) passes through the collimator (C), past the specimen and through the hemicylindrical lens (L), finally falling on the window of the photo-multiplier tube (P-M.T.). The capacitance manometer head (M.H.) lies above the specimen and is connected to its lumen by a long cannula. The recorded pressure changes are amplified by the manometer amplifier (Man.Amp.) and are fed to the Y plates of an oscilloscope (C.R.O.). Diameter changes are similarly displayed on the X axis of the tube. Also shown are the resolver and its synchronising switch-plate (R.S-P) and contacts (R.C). These are mounted on the cam shaft of the pump, as is the tachometer generator (Tacho).

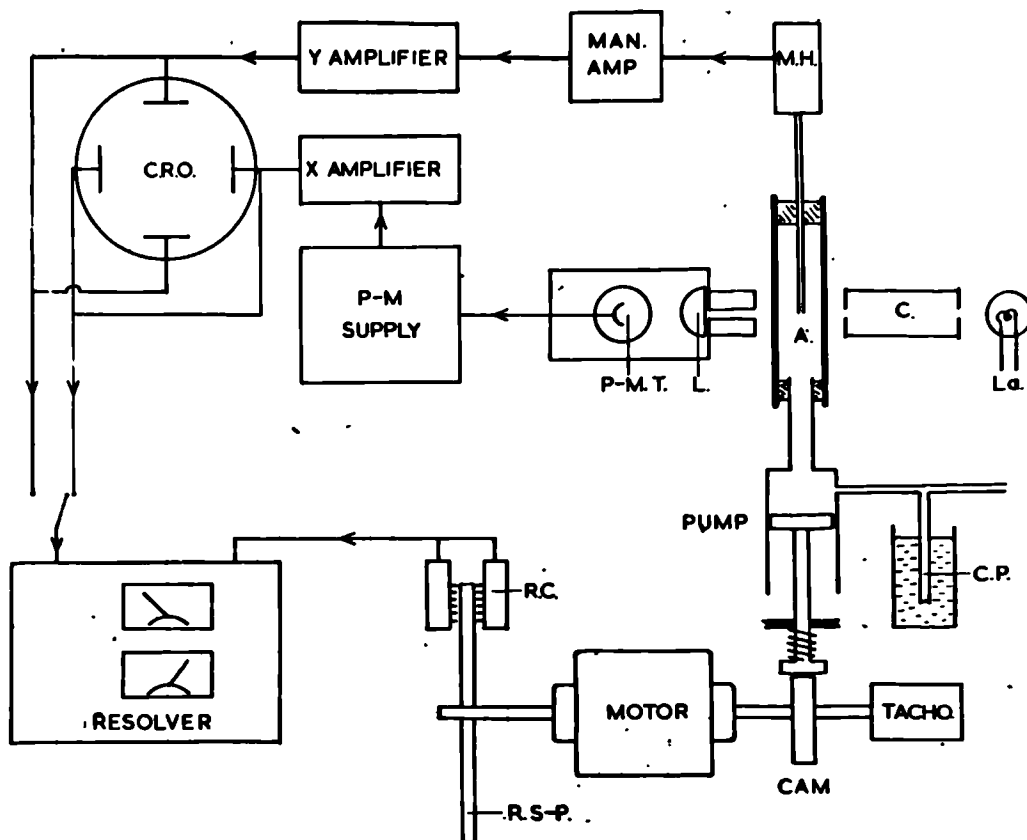


Fig. 1

FIG. 2.

The pump used is shown here. The barrel is in the upper right-hand side of the picture, and the tube leading to the specimen can just be seen. The pipe attached to one of the taps leads to the manostat. On the left is seen the eccentrically mounted ball race which bears on a flat plate connecting with the plunger of the pump. The plate is held against the cam by a strong spring. Behind the cam is the large switch-plate which runs within a yoke carrying an array of contacts. In the foreground is a motor and bevel gear, this has replaced the belt driven pulley wheel which was used in the experiments described.

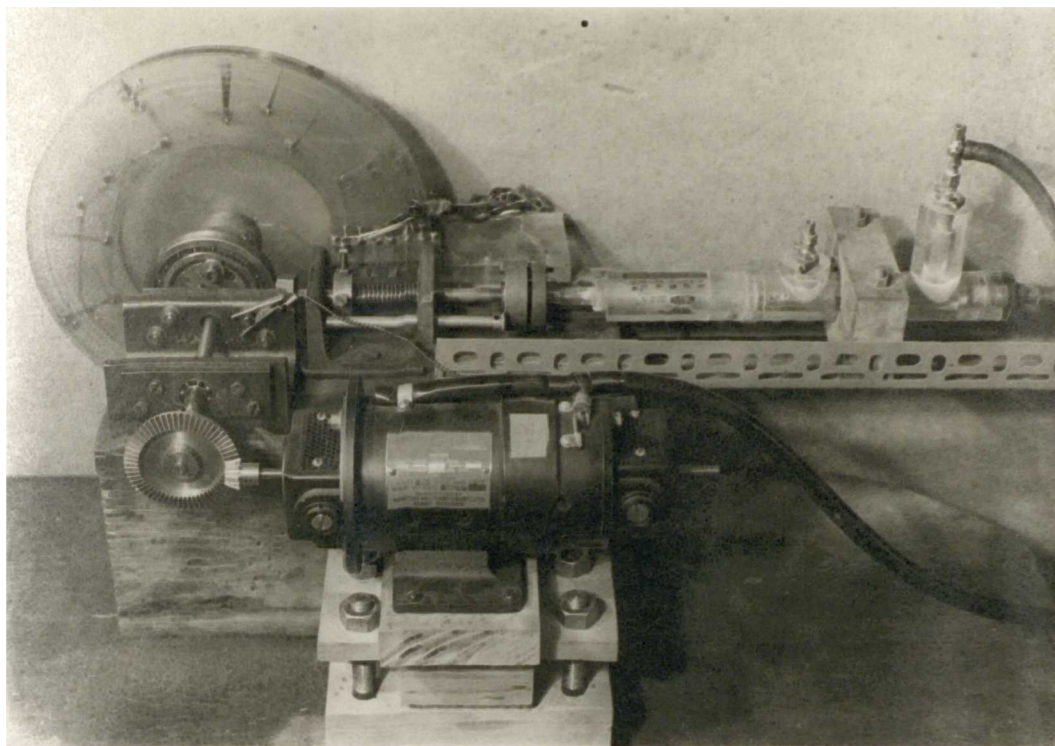


Fig. 2

FIG. 3

This shows the central part of the apparatus. On the extreme right is the light-tight box in which is the photo-multiplier tube attached to the thick cable. To the left of this the two specimen supports are seen; these are arranged as for an experiment but no specimen is mounted. The long cannula passes through the upper support to the level of the light beam, and is attached to the manometer transducer unit which is held above the specimen. Below is seen the line to the pump. At the left is the collimator and the lamp with its electrical leads. The flask behind the collimating tube is the saline reservoir, it is connected to the pump and also to the mercury column manostat (extreme left).

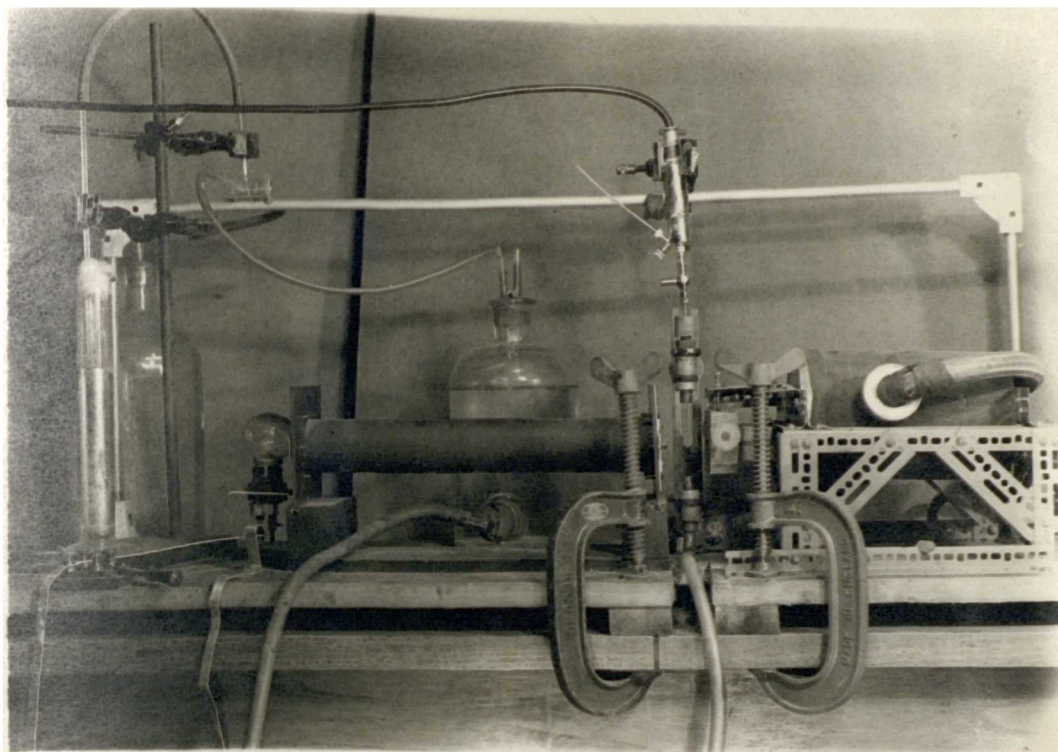


Fig. 3.

FIG. 4

A general view of part of the apparatus, showing the pump, and behind it the controls of the motor (this was not employed for most of the experiments described). To the left lies the apparatus seen in Fig. 3.

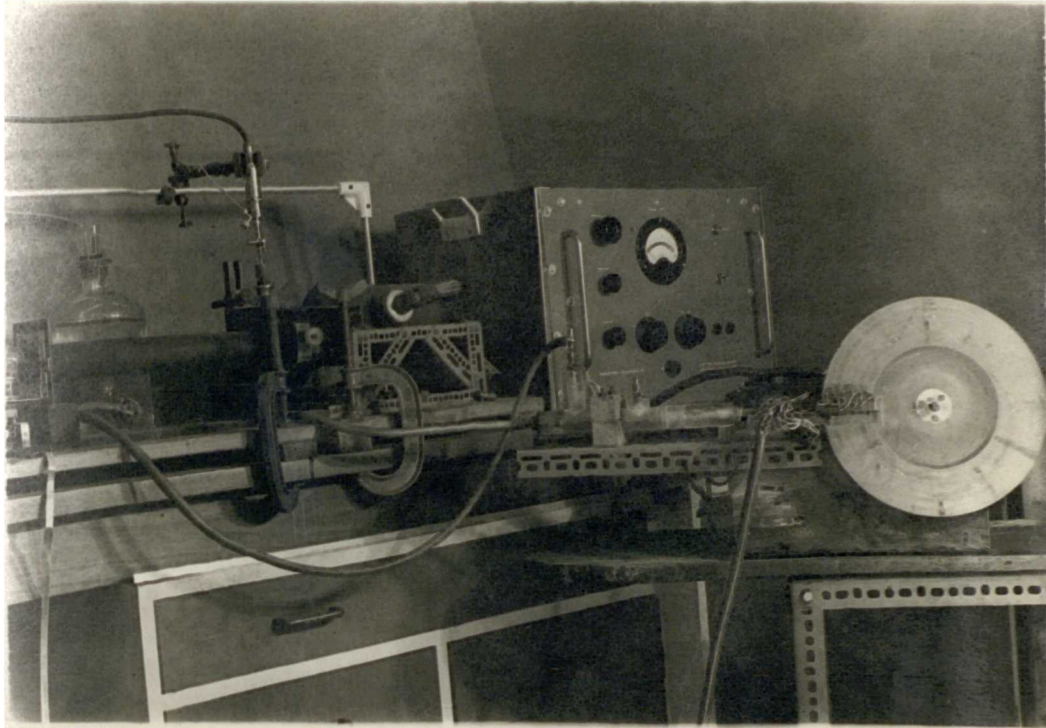


Fig. 4.

FIG. 5

This photograph shows the half of the apparatus not visible in the preceding Figure which extended as far as the manostat, here seen on the extreme right. Reading from right to left the following units are seen: the supply unit of the photo-multiplier, on top of which is the galvanometer used in radius calibration; the manometer amplifier; the resolver, above which are the tachometer dial and motor speed-control, and finally the oscilloscope.

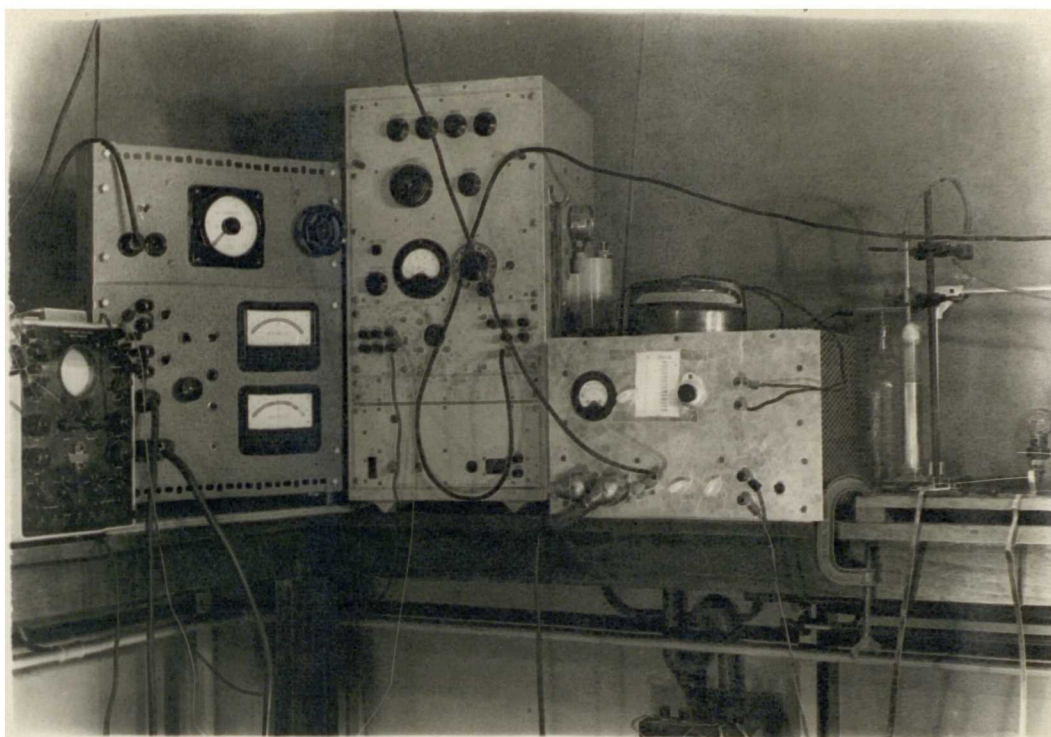


Fig. 5.

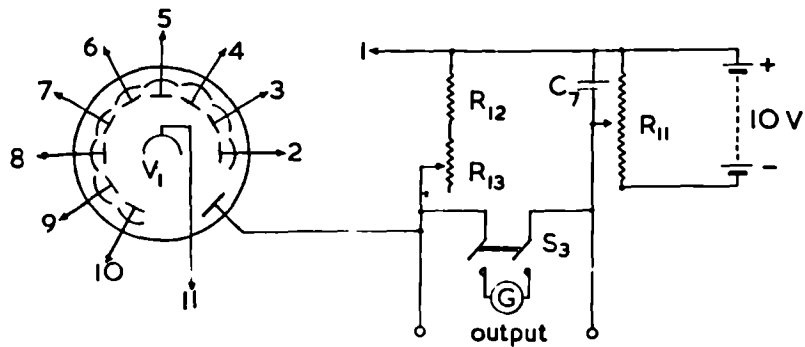
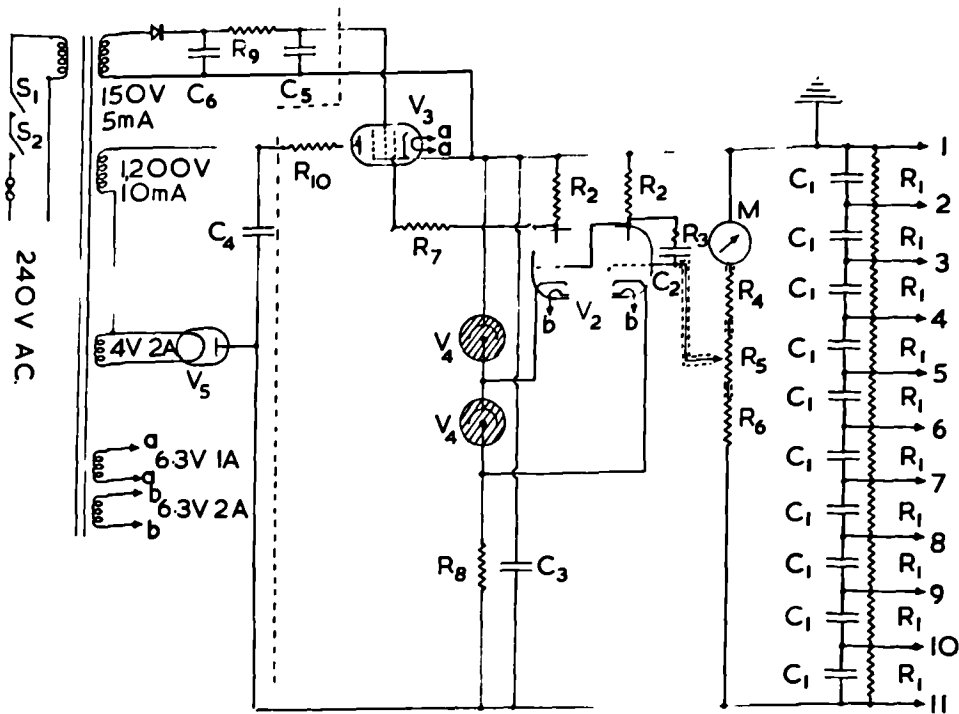
the light to some extent on the side window of the photomultiplier tube, (RCA 931A) mounted horizontally in a light-tight container 3 inches behind the lens.

The circuit of the power supply is shown in Fig. 6. The potential across the voltage divider (R_1) could be adjusted by means of the potentiometer R_5 ; in this work it was held in the region of 600 V: by under-running the photo-multiplier somewhat stability was improved. For safety's sake the photocathode was held at this high negative potential and the anode was then close to earth potential. ~~The current drawn from the anode:~~ The current drawn from the anode was as low as possible; the makers recommend that for maximum stability it should not exceed 10 microamperes. In this case the resistance to earth (R_{12}) was 10 M Ω while the input impedance of the Y amplifier of the oscilloscope used (Furzehill Laboratories Type 1684) was 3.3 M Ω ; as these two were in parallel the load impedance was 2.5 M Ω . Since the anode potential never dropped more than 10 V below earth optimum operating conditions were assured. Nevertheless it was found that on exposing the photomultiplier to a steady level of illumination the output would fall rapidly for a few minutes. After two hours, however, the rate of decline was very small. The rate of drift was also greater when the amount of light reaching the photocell was large.

The effects of this drift were reduced as far as possible by running the unit for at least 2 hours before an experiment, by calibrating immediately before and after each test, and by reducing the width of the light beam to the minimum practicable (see discussion of calibration).

FIG. 6

The circuit of the high tension supply of the photo-multiplier tube is shown in the upper part. Below is a diagram showing the connections to the photo cathode (11) and dynodes (10-2) of this tube. The anode (not numbered) is connected to the output terminal of the unit. The potential on this terminal may be balanced, after closing the switch S3, against the calibrating potentiometer R11.



V ₁ RCA. 931 A	R ₁ 220 K Ω	R ₆ 240 K Ω 1/2 w.w.	R ₁₁ 10 K Ω w.w.	C ₃ 0.25 μ F
V ₂ 6SN7	R ₂ 1 M Ω 5/8 w.w.	R ₇ 1 K Ω	R ₁₂ 10 M Ω	C ₄ 0.25 μ F
V ₃ EL38	R ₃ 47 K Ω	R ₈ 270 K Ω 5/8 w.w.	R ₁₃ 1 M Ω	C ₅ 8 μ F
V ₄ 85A1	R ₄ 57 Ω 1/2 w.w.	R ₉ 33 K Ω	C ₁ 0.01 μ F	C ₆ 4 μ F
V ₅ SU2150	R ₅ 100 K Ω	R ₁₀ 100 Ω	C ₂ 0.001 μ F	C ₇ 24 μ F

M 0-1,000 μ A 50 Ω

Fig. 6.

By these means the error over 2 hours in the estimation of diameter was found to be less than $\pm 4\%$, and as the slope of the calibration curve altered but little with drift the error in the measurement of diameter changes was substantially less than this. No experiment took longer than $1\frac{1}{2}$ hours.

Calibration of photo-multiplier unit

For this purpose a number of silver-steel rods whose diameter varied by steps of $1/16$ inch from $1/8$ to 1 inch, were used. These were painted matt black and measured with a micron meter. Using a lathe a shallow hole was cut axially in the base of each of these rods; by this means they could be supported vertically on a brass cone whose base was threaded to fit in the lower of the two horizontal specimen supports. In this position it was co-axial with the specimen. On referring again to Fig. 6 it can be seen that by closing the switch S 3 the photomultiplier anode is connected across the galvanometer G to the movable pole of R_{11} . This latter is a linear ten-turn potentiometer (10 K linearity $\pm 0.1\%$) with a graduated dial giving a complete range of 1,000 units. By balancing the galvanometer to zero deflection the output, in potentiometer units, corresponding to each of a suitable series of rods, of which three to five were used in each case, could be measured.

For convenience the voltage across the potentiometer was taken from the same accumulator as supplied the light source, and the balancing potential was also applied to the Y2 amplifier terminal to centre the trace. Since the batteries were under continuous charge a certain amount

of partly rectified 50 cycle voltage appeared on the tube; this in no way affected the reading given by the resolver, which will be described later, but to make the trace easier to observe it was smoothed out by the large capacitance C_7 . It must be emphasised that this capacitance is applied across a steady balancing potential and does not introduce any phase shift.

The calibration curve given by the unit over its whole range is reproduced in Fig. 7. It will be seen that the output is linear with diameter except for the very largest rods. At this size the amount of light passing around the edge was extremely small and it was felt that this effect was due to diffraction. As a result of this finding the width of the light beam was adjusted in each case to extend well beyond the vessel when maximally distended, but not so far as to allow serious output drift. With experience a satisfactory adjustment was easily attained.

Static pressure-radius measurements were made using a hand pump attached to a saline reservoir and a mercury manometer. The level of the fluid in the reservoir was kept about 5 cm above the middle of the specimen to avoid collapse at zero pressure; thus the actual pressures were in fact 5 cm H_2O higher than those recorded.

For the dynamic measurements sinusoidal variations in the internal volume of the vessel were imposed by a motor driven pump (Fig. 2). This was designed by M.G. Taylor (Taylor, 1959) and develops pressure oscillations containing less than 2% of higher harmonics. The pump itself was made from perspex tube into which was cemented the barrel of an all-glass syringe from which the barrel had been amputated. Taps

FIG. 7

Calibration of the photomultiplier unit.

Ordinate: output in arbitrary units. Abscissa: radius (of a series of metal rods). Note that this curve covers the whole range of sizes that the apparatus can accommodate.

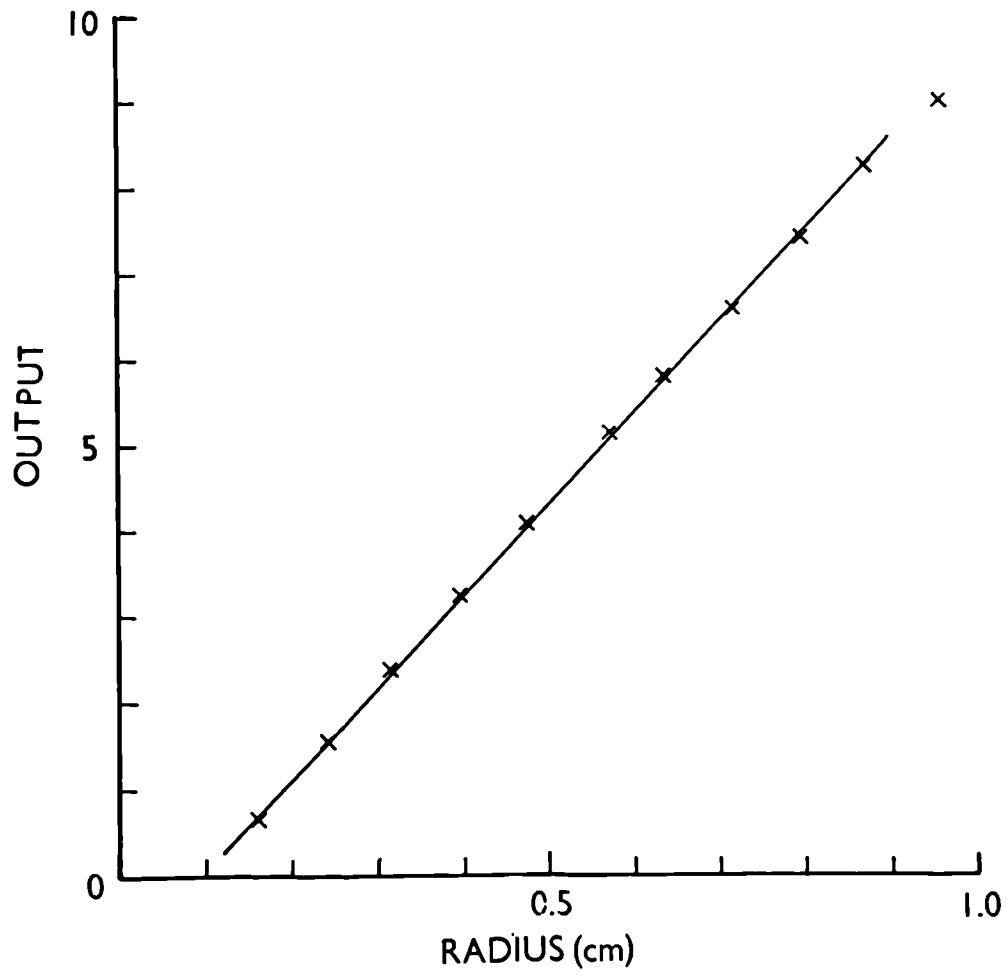


Fig. 7.

were attached for the removal of bubbles and to one of these the saline reservoir was attached. Fluid was led from the pump by a length of wide 'portex' tubing to a $\frac{3}{8}$ inch pipe brazed to the lower specimen support and thence to the vessel lumen. Three pumps were made, from 1, 5 and 20 ml. syringes; the eccentricity of the cam which moved the plunger could be varied and thus the stroke-volume could be adjusted to suit any specimen.

The cam-shaft was driven through a belt by a 200 V D.C. $\frac{1}{4}$ H.P. motor. Speed was controlled with a rheostat and recorded from a large milliammeter (0-10 mA, 9 inch scale) actuated by a tachometer generator attached to the pump-shaft. A perspex commutator disc which supplied triggering pulses to the resolver was also mounted on this shaft.

Oscillatory pressures were recorded by a capacitance manometer (Southern Instruments) through a cannula 10 cm long and c. 1 mm bore. This was inserted through a water-tight seal on the upper specimen support and its tip was adjusted to lie within the illuminated segment of vessel.

Measurement of pressure changes

The dynamic behaviour of the manometer used is of obvious importance. Although modern instruments are a great improvement in this respect on the older ones, some distortion of rapidly altering pressures is inevitable. This is especially important in the present instance where one is attempting to measure small phase shifts; very large phase distortions may be introduced by a manometer even though no error in the recorded amplitude can be detected.

The theory of manometers has been discussed by many authors. I have made use of the monograph of Hansen (1949) in which the matter is very fully discussed.

The performance of a manometer may be specified by two parameters, undamped natural frequency (f_0) and degree of damping. The latter, following McDonald (1960), will be referred to as β since the symbol α used by Hansen has another meaning in this laboratory. The frequency at which the system would oscillate if entirely undamped, f_0 , must be distinguished from f_d , the damped natural frequency, which is found when damping is present. With regard to β three conditions are possible. When $\beta < 1$ the system is said to be underdamped, following a sudden pressure change a series of oscillations (frequency f_d) which diminish exponentially in magnitude will occur. When $\beta = 1$ the system is critically damped and approaches the new equilibrium position exponentially. With higher values of β ('overdamped' manometer) the movement is slower.

These parameters are given by the following expressions

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{\pi r^2 E}{L}} \quad \dots\dots(29)$$

$$= \frac{4\mu}{r^3} \sqrt{\frac{L}{\pi E}} \quad \dots\dots(30)$$

where r and L are the radius and length of the manometer connections (these are largely determined by the dimensions of the cannula used), μ is the coefficient of viscosity of the transmitting fluid, and E is the modulus of volume elasticity, $\Delta P/\Delta V$, of the unit. The values of these may be measured directly, or the natural frequency and damping

may be calculated from the behaviour following an abrupt pressure change. One must determine the frequency of the oscillations, f_d , and the logarithmic decrement of their decay in amplitude, Λ . (If damping is great the situation is more complex but the necessary information may still be obtained (Warburg, 1950)). We then have,

$$f_o = f_d \sqrt{1 + \left(\frac{\Lambda}{2\pi}\right)^2} \quad \dots\dots(31)$$

$$\beta = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}} \quad \dots\dots(32)$$

From these values the error in amplitude and phase recorded at any frequency, f , may be found from the tables given by Hansen. When β is very small the amplitude will be enormously exaggerated as f approaches f_d , but the phase error (lag) will be small when the frequency is very small. If the instrument is critically damped there will be less amplitude distortion but the phase error will rise linearly with f , becoming $\pi/2$ radians when $f = f_o$. Consequently for present purposes it is necessary to use a manometer as little damped as possible and with a very high natural frequency.

The instrument used in the experiments was calibrated by recording its response to a sudden pressure change. It was found to have an undamped natural frequency of 89 c/s and a degree of damping of 0.102. From Hansen's formulae it would be expected to register a pressure change 5.2% too great with a phase lag of 0.048 radians at 20 c/s. A smaller phase error would have been desirable but this could not be achieved.

Hansen derived his expressions for fluid resistance from the Poiseuille formula. In the case of oscillatory pressures the flow will be modified by the frequency-dependent parameter α (McDonald, 1960).

Since fluid resistance increases with frequency the damping at low frequencies will be less than that predicted by the experiment described. Hansen used the following expression,

$$\beta_0 = \frac{4}{r^2}$$

this should be replaced by

$$\beta_0 = \frac{\mu}{2r^2} \frac{\alpha^2}{M'_{10}} \sin \epsilon_{10}$$

M'_{10} and ϵ_{10} depend on α , as f tends to 0, so α tends to 0 and α^2/M'_{10} tends to 8 while $\sin \epsilon_{10}$ becomes 1, thus at low frequencies the two expressions become identical. For the cannula used $\alpha = 13$ at 90 c/s and the resistive term is found to be about 1.5 times too great, consequently the phase error is likely to be overestimated by this amount.

In order to determine the calibrations to be used the behaviour of the manometer was estimated directly up to 20 c/s by comparing its output with an air-filled unit which had a natural frequency of nearly 1,000 c/s. The results (Figs. 8 and 9) fitted reasonably with those predicted from Hansen's theory and it did not seem that the damping had been overestimated. This is possibly due to errors in the reading of the record of the transient response but it seemed to be wisest to construct a calibration table from the simple theory which fitted the directly determined performance more closely. Thus the pressures recorded at 20 c/s were reduced by 5% and the phase lag between pressure and radius changes was increased by 0.048 radians. At lower frequencies these factors alter virtually linearly with frequency. The corrections used are shown in Table 5.

The modulus and phase of pressure and radius changes could have

FIG. 8

The closed circles show the amplitude distortion at different frequencies of the manometer used; the continuous line is the behaviour predicted by the equation

$$A = \frac{1}{(1-\gamma^2)^2 + (2\beta\gamma)^2}, \text{ where } A \text{ is the relative amplitude}$$

(Hansen, 1949). The other symbols are identified in the text. Predicted and observed values for two other manometers are also shown.

The inset shows diagrammatically the way in which the manometer being tested and the reference unit were mounted on the pump.

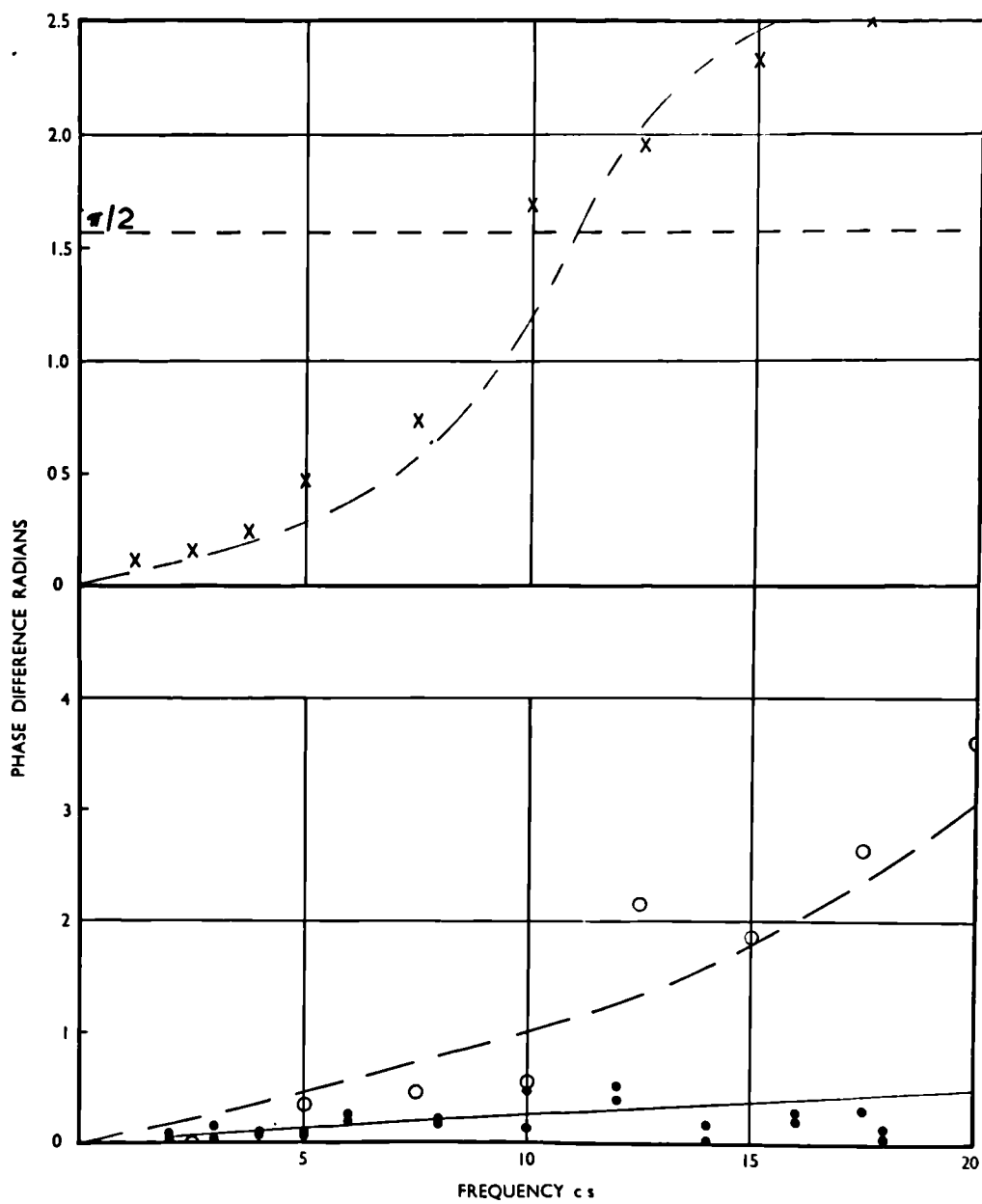


Fig. 8.

FIG. 9

Comparison of the calculated and measured phase distortion shown by various manometers in the experiment shown in Fig. 8. The closed circles in the lower part of the Figure refer to the manometer used in these experiments. The continuous curve shows the behaviour of a manometer with undamped natural frequency 89 c/s, and degree of damping 0.102. $\tan \psi = \frac{2\beta\gamma}{1-\gamma^2}$, where ψ is the manometer phase lag. See text, and also Table 5. The other points refer to similar experiments on other manometer units.

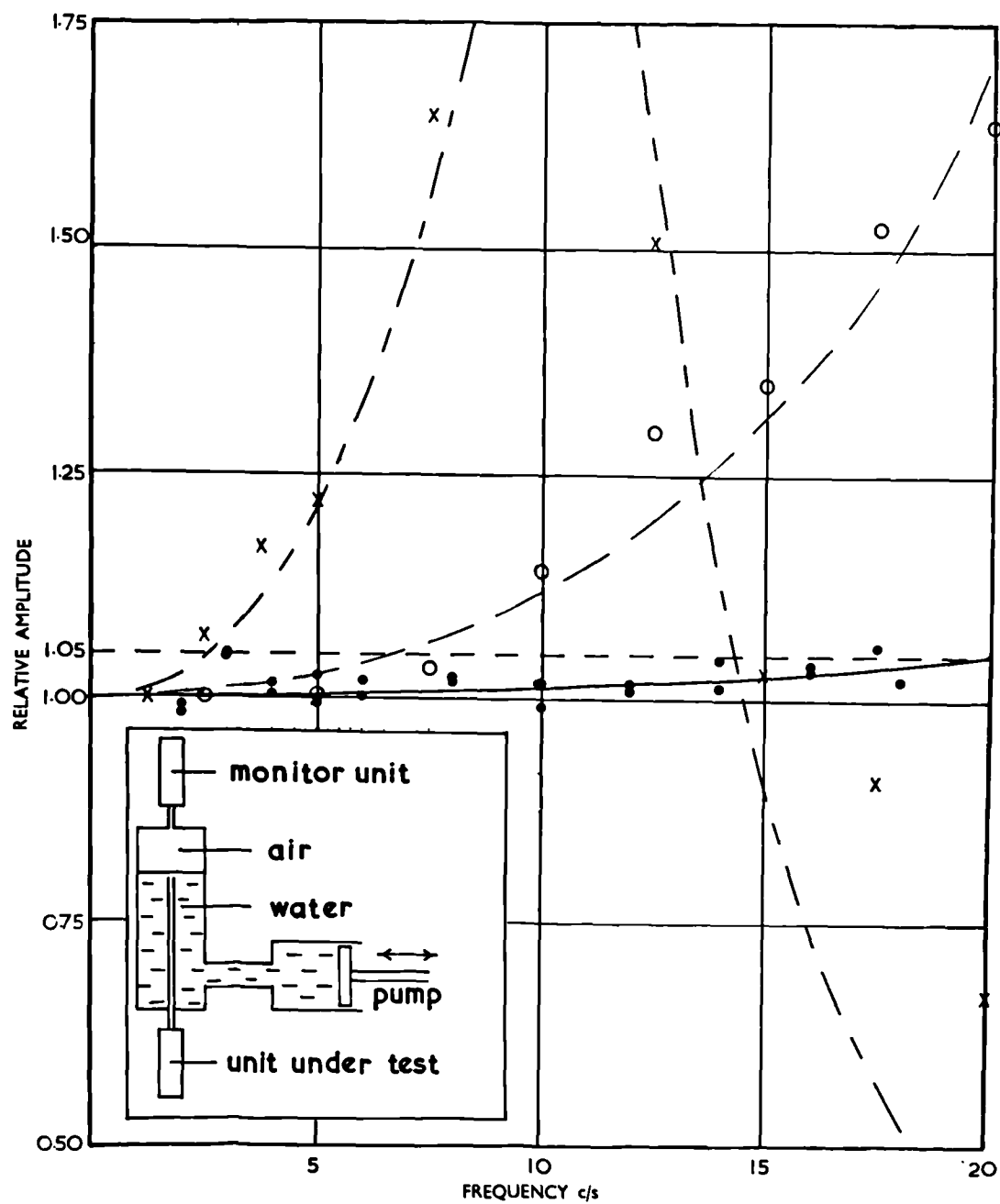


Fig. 9.

been obtained by Fourier analysis of the photographed oscilloscope trace; this is a time consuming process and the labour was enormously reduced by the use of an automatic Fourier resolver (Taylor, 1957b, 1959). By this means the Fourier coefficients of the first, second and third harmonics could be directly read from a pair of dials while the experiment was in progress. As a virtually pure sine wave was generated by the pump the fundamental only was recorded, but it was useful to be able to check that no distortion was occurring which would happen if, for example, the stroke was too large.

Procedure

Having calibrated the photo-multiplier output over a suitable range, the specimen was mounted vertically and stretched to its original length as previously described. The 'static' pressure-radius relationship was determined by inflating the vessel over the range 0-240 mm Hg in steps of 20 mm Hg, and measuring the radius at each stage by balancing the output potential against the graduated potentiometer R_{11} using the galvanometer (G). In order to permit as complete an extension as possible at least two minutes was allowed between each step, the cycle 0-240-0 mm Hg taking around fifty minutes. A full experiment was not carried out in all cases, in some the radius at pressures of 0, 90, 100 and 110 mm Hg only was determined.

The dynamic experiment was next performed. All these, except where otherwise stated, were carried out at a mean pressure of 100 mm Hg, the oscillations about this level were usually about ± 5 mm; they were never more than 15 mm Hg as, owing to the markedly non-linear behaviour of the

arterial wall, larger strokes would have generated distorted pressure waves. Also on account of this non-linearity great care had to be taken to maintain a constant mean pressure. This was eventually achieved, after some trials, by the simple expedient of leaving the connection between the pressure reservoir and the body of the pump open while the latter was running. Although this changed the absolute amplitude and phase of the pressure variations those of the radial dilatation were correspondingly altered and in experiments on rubber tubes the relationship between the two was found to be unaltered. The pressure in the reservoir was kept constant by a vertical glass tube inserted to a depth of 100 mm below the surface of a measuring cylinder of mercury and connected to the compressed air line. A large ($2\frac{1}{2}$ litre) air-filled flask served to damp out any small variations due to bubbling.

The frequency of the pump was varied over the range 2-20 c/s and held steady to within 0.1 c/s at each desired level while the two resolver readings of the pressure and radius variations were taken. At the end of the experiment the resolver was calibrated for both variables. A small change of pressure, measured by a mercury, or where necessary a water, manometer, was produced by hand and the deflection of the resolver needles was noted. Simulated radius changes were produced by rotating the graduated potentiometer on the photo-multiplier unit by a small amount, thus putting the resolver input out of balance, and similarly recording its reading (the gain of the Y_1 and Y_2 oscilloscope amplifiers was found to be identical).

Finally the vessel was removed and the steel rods again used to obtain a second calibration of radius. The mean of this determination

and that carried out before the experiment was used to construct the calibration curve, which was in fact always a straight line, to be used. All experiments were carried out at room temperature which was recorded and ranged between 15° and 24°C; the great majority of experiments were carried out at temperatures of 17-21°C.

CHAPTER 4. RESULTS AND DISCUSSION

The results may best be presented in three sections and each will be discussed in order. The first section will deal with some observations on arterial structure and dimensions. After this the static and the dynamic elastic properties of the vessels I have studied will be discussed.

Arterial dimensions and structure

1. Retraction

When removing specimens for study the opportunity was taken to measure the length of a large part of the arterial tree before and after excision. The measured region extended from the common carotid bifurcation, through the aorta and iliacs down to the femoral artery just above the knee. The subclavian and brachiocephalic were not studied, the latter vessel was found to be rather tensely stretched around the first rib when the animal was on its back with limbs tied down, and it seemed that any figures thus obtained would be suspect. For the purpose of the whole of this work the various vessels were defined in the following way. The common carotid (referred to as the carotid) lies between the bifurcation and the division of the brachiocephalic artery. The thoracic aorta was measured from the root of the right brachiocephalic to the coeliac axis; however the mechanical studies were carried out on the descending thoracic aorta below the third or fourth intercostal vessels. The aortic trifurcation separates the iliac artery from the abdominal aorta, while the inguinal ligament marks the transition from the former to the femoral artery. The latter vessel was taken to extend down to the point at which it penetrates the adductor muscles.

The shortening of 99 vessels was measured. The retraction, expressed as percentage loss of length, is detailed in Fig. 10. In addition this Figure contains the values for the relative wall thickness which will be referred to shortly.

Results of other workers

It will be seen that the amount of retraction which occurred is fairly large and differs from vessel to vessel. The differences are all highly significant, $p < 0.01$. These figures are of the same order as those previously recorded. *Remington & Hamilton* (1945) mention figures of 33% for the dog's aorta, thoracic and abdominal, while rather more comprehensive figures are given by Fuchs (1900). I have taken the following mean values from this author, who also worked with dog's vessels (the figures in parentheses give the number of specimens).

Thoracic aorta	21.0%	(11)
Abdominal aorta	33.3%	(9)
Carotid artery	33.6%	(4)
Femoral artery	37.6%	(4)

These figures are rather smaller than my own, but they agree with mine in that the thoracic aorta was found to retract the least, and the femoral the most. It would be interesting to speculate on the reason for the differences between the various groups, but it would be speculation only. One thing can be said, the high figure for the femoral is not an artefact produced by the position in which the animal lay when the length of the vessel was first measured, and this is obviously not the case for the vessel showing the second highest degree of shortening, the iliac artery.

FIG. 10

Mean values for relative wall thickness and amount of retraction in different dogs' arteries.

VESSEL	h/R _x 100 AT 100 mm.Hg. INTERNAL PRESSURE			RETRACTION SHORTENING AS % ORIGINAL LENGTH		
	No.	MEAN	S.E.M	No.	MEAN	S.E.M.
THORACIC AORTA	15	10.5	0.46	22	31.9	0.62
ABDOMINAL AORTA	9	10.5	0.61	17	34.0	0.63
FEMORAL ARTERY	11	11.5	0.96	22	42.0	0.89
CAROTID ARTERY	13	13.2	0.75	29	35.2	0.53
ILIAC ARTERY	-	-	-	9	39.5	0.74

Fig. 10

This was shown by a simple experiment; the femoral artery was cleaned of all surrounding connective tissue but the branches were left intact, this being normal procedure when making these measurements. No change in length of the artery was discernible when the leg was taken through a full range of movements. In fact it would seem that the arrangement of the major blood vessels is such that little change in length is to be expected in the normal range of body movements; they mostly lie close to the joints over which they pass.

The degree of shortening found corresponds to that reported in young subjects by other workers (Hiller, 1884; Scheele, 1908; Hesse, 1926) This might be expected since the animals used were all quite young; it appears that greyhound breeders discard any animals that seem unlikely to succeed on the racetrack and that this decision is taken when the animals are 3-4 years old. The comparative uniformity of the results is presumably a reflection of the uniform ages of the specimens. The occasional animal was found which appeared younger than average but the measured retractions were no greater than in the others. It may be that these degrees of shortening represent the maximum attained (for example, Hesse (1926) gave a figure of 40% for the upper arm vessels of a twelve year old child). On the other hand one dog was dissected the vessels of which were obviously hardened. This animal had grossly scarred kidneys and ascites and no figures obtained from it are included in the table. The amount by which this animal's vessels shortened was clearly less than average (e.g. thoracic aorta 20%, abdominal aorta 15%); this is interesting when it is recalled that Roy (1880) described the properties of the aorta from a starving and marasmic dog as those he would have expected from a very old

animal.

All previous workers are agreed that the amount of shortening decreases progressively with age. The most complete figures are those of Hesse (1926) who found a steady reduction from 40% at age 12 to 12% and nil at 65 and 67 respectively. It would in fact seem that this simple measurement might prove of use in the investigation of ageing processes.

The question of the age dependence of retraction raises a point of more direct importance in the present context. It has been previously mentioned that an artery lengthens when inflated, and this fact has been seen to be a strong argument that the vessel wall is anisotropic (Fenn, 1957). The amount by which a vessel lengthens in this way might be expected to decrease with age, and this is in fact so (Simon & Meyer, 1958). It follows that when the studied vessels are free to lengthen the form of the arterial pressure volume curve can be expected to alter with age on this account alone. This is of course a real alteration but hardly relevant to *in vivo* conditions. Even when the vessel is kept at constant length the same argument applies if the length concerned is the excised length. This was the case in the experiments of Hallock & Benson (1937). The influence of these factors on the behaviour of arterial rings is more difficult to assess. In such a specimen the length will become even less as it is stretched, for the ring is freed from all longitudinal restraint and should therefore show a lower circumferential modulus. The importance of determining the 'natural length' of arterial strips is obvious, but has been largely neglected (Roy, 1880; Krafka, 1939).

Enough has been said to indicate that experiments on excised

vessels in which the retraction has been neglected must be interpreted with great caution. It is rather surprising, in view of the fact that retraction has been known for at least 75 years, and that the amount of shortening may be very considerable, that no experiments have been discovered in the literature which have taken account of it.

Another point of interest in this respect is the cause of this retraction. It points to the existence of a certain longitudinal tension within the vessel wall in vivo. However, when a closed vessel is inflated it does undoubtedly recover its length to some extent. Fuchs (1900) found that the majority of vessels, arteries and veins, returned to their natural length at pressures of 70-100 mm Hg, but that some never did so no matter how high the pressure. This opinion is shared by Simon & Meyer (1958), but in their case the extension on inflation alone was measured and the amount of previous shortening was inferred from the figures of Hesse (1926), which makes their evidence unsatisfactory. It has been my experience that these vessels never return to their natural length at a pressure of 100 mm Hg, and that higher pressures produce virtually negligible further change. My impression, and that of McDonald (1960) is that, at this pressure the arteries have regained $2/3 - 3/4$ of the lost length, i.e. they are then at about 90% of their natural length. Thus some external force must be applied to restore natural length. This is particularly so in the case of the abdominal aorta. Whereas all other vessels may be stretched with ease to greater lengths than in situ, this artery is extremely difficult to return even to natural length. This makes it a most difficult vessel to work with since in many cases the attachment to the holding-pieces would slip before full extension had

been obtained. The necessary tension must be of the order of several kilograms per square centimetre. Why this should be so is difficult to say; this vessel is very firmly attached to the surrounding structures and the adventitia may contain a higher proportion of longitudinally arranged collagen fibres than others. There was no histological evidence to show that this layer was relatively thicker. It can be seen in Fig. 10 that the average amount of retraction shown by the vessel was less than for all but the thoracic aorta.

The question arises as to the origin of this tendency to shorten. One answer would be that it is due to differential growth between the vascular system and the rest of the body. Supposing that this differential process is most marked when the actual amount of overall growth is the greatest, we might expect to find that the amount of retraction at different ages would run parallel to the growth curve, falling off as the amount of vascular growth catches up with the body growth, or alternatively as the effect of long continued tension shows itself by some slow extension of the vessels. Another possibility would be that the newly laid down connective tissue fibres of the vessel wall become shorter some time later in a way analogous to the contraction of scar tissue. In this case the retraction would also be greatest during or just after the time of maximum body growth. There is at present nothing to suggest whether either of these alternatives is correct.

The phenomenon of retraction is interesting in its own right, but, for the present purpose it is important simply because it is so large. It is likely that any measured arterial properties will differ from those to be expected in vivo if retraction is ignored.

Relative wall thickness

The figures that have been obtained for the relative wall thickness of four different types of artery are given in Fig. 10. The differences between the femoral artery and the aorta on one hand and the carotid on the other are highly significant ($P < 0.01$).

It seems then that the arterial wall is rather thicker in the smaller vessels, but that the difference is small. In particular the amount by which the pulse wave velocity in the carotid would exceed that in the aorta is, on this account, only some ten percent. It was seen earlier that the increase is of the order of two or three times, and this must therefore be largely due to other factors.

The actual figures obtained are to some extent artificial, in that they depend on a rather arbitrary decision on the limits of the wall. These vessels were gently cleaned of connective tissue until there was no more that would easily separate. This seemed to give a reasonably uniform preparation and when some specimens were later examined histologically the adventitia was about 10% of the wall thickness. This figure is the same as that obtained by Hurthle (1920) which suggests that there may be some true limit to the arterial wall proper. However some specimens had very much thinner walls than the average. It was found that the distensibility of these thin vessels at about 100 mm Hg was not much different from the rest. This suggests that the outermost layers are slack at these pressures and only come into play at greater distensions.

Results of other workers

As was mentioned earlier the only comparable figures in the literature are those of Hürthle (1920) and it will be recalled that he found a figure of 15% for the ratio h/R for the whole arterial system at

pressure of 130 cm H₂O with the exception of the aortic root which was somewhat thicker (16-20%). Looking at his figures it can be seen that the scatter is wide, but there is no clear indication that the figures are larger for the smaller vessels, though it is possible that so small an increase as I have found would be missed. The figure of 15% would appear to have been obtained 'by eye', but it is clear that the figures are rather larger than mine; values above 20% are quite common. The difference between our findings is not very great and there are a number of reasons which might account for it. It is not due to the inclusion of different amounts of adventitia.

It is possible that the three animals used by Hürthle may have had rather thicker vessels; 229 out of the 288 sections measured were derived from one dog and the others were from rabbits. I have seen no evidence that any one animal tended to have generally thicker or thinner arteries. Histological methods are extremely difficult and time consuming, especially when the specimens measured are somewhat elliptical in cross-section, which is the rule rather than the exception. If, in addition, some degree of collapse occurred after a fixation period (which appears to have been only two hours) then there seem to be enough factors to account for the difference in our results.

It is emphasised however that my own figures depend on an accurate determination of specimen volume and on the assumption that arteries are regular cylinders. The second assumption is clearly to some degree unjustified. Hürthle found that arteries are rarely circular in cross-section and my own specimens confirm this, though it may be that the form

is distorted by fixation. If my specimens were markedly elliptical this might result in a gross asymmetry of expansion when inflated; this has not been observed.

Calculation of the wall volume depends on two measurements of weight, in air and in water. The second weighing is simple and, provided that it is reasonably quick, should not be noticeably influenced by any osmotic fluid imbibition. It is of course very important that no air bubbles be left clinging to the surface. This was assured by passing a stream of water down the lumen via a small catheter. It was obvious if any air remained since the wall thickness would later be found to be extremely large (for example these precautions were not employed in the weighing of an early specimen and the relative wall thickness was found to be 28.8%). The specific gravities of the whole series were determined and the mean found to be 1.0756 (s.e.m. 0.0019, n = 61). There was a tendency for the carotids to have a rather higher density, but the difference was not significant.

It is to be expected that measurements of relative wall thickness on undistended and retracted vessels would be greater than my own, but the difference should in fact not be so very great. On the whole it was found that the vessel diameter under these conditions was not far different to that measured at natural length and at 100 mm Hg. On retraction the radius becomes greater and so the two processes cancel themselves out. It follows that the wall thickness should be found to be greater than the 'natural' value in the ratio natural length/retracted length, i.e. between 1.5 and 2 times as great.

The figures given by Scheile-Weigandt (1880) and Tschewsky (1903) range between 15 and 20%, which is what one might expect. Kani (1910) however, gives rather higher values of about 25% and found that they tended to increase with age to 30-35% at 30 years. The method he used does not seem particularly delicate; the thickness was measured with an engineers micrometer and the width of the opened strip of vessel gave the circumference. It is therefore rather a pity that these particular results should have been used by Bazett et al. (1935), and also by King in his recent series of papers (e.g. King, 1947).

It is reasonable to suppose that, other things being equal, a change in the wall thickness of an artery indicates a change in its composition, for the wall stresses produced by the blood pressure will also be changed. There is of course some uncertainty about the other factors involved, for example it might be that the thoracic aorta is less supported by surrounding tissues and is therefore somewhat thicker than might otherwise be expected.

The most obvious fact seen here is that there is no change in thickness to correspond with the abrupt change in the collagen/elastin ratio reported by Harkness et al. (1957) at the junction between intra and extra thoracic vessels. Thus the stresses imposed by the blood pressure do not change across this boundary, nor is there any reason to suppose that the amount of dilatation changes so abruptly. One might imagine that, within the physiological pressure range at least, the elastin and collagen fibres are so extended that their Young's moduli are much the same. However the figures in the literature (e.g. Reuterwall, 1920; Krafka, 1939) which refer to tendon and ligamentum

nuchae show that even at its greatest extension the elastic modulus of the latter is less than one tenth that of unstretched tendon. There must then be some other explanation and this is most likely to lie in the arrangement of the fibres. If it be supposed that a greater proportion of the collagen in the abdominal aorta is arranged longitudinally, then the amount of each arranged circumferentially could be the same as in the intrathoracic vessel. It has been mentioned that it was impossible to stretch the abdominal specimens to a length appreciably above natural length, and this would again indicate considerable amounts of very strong material which become taut at this length. The explanation seems reasonable and serves to show that information on the arrangement, as well as on actual amounts of vessel wall constituents, is essential if one is to explain their mechanical properties convincingly.

There is however a highly significant ($P < 0.01$) increase in relative wall thickness in the femoral and carotid arteries as compared with the aorta, and the difference between these two types of vessel is similarly highly significant. This increase is not very great and would of itself account for only a very small increase in pulse wave velocity, but it must presumably reflect a change in the structure of the wall. The most reasonable assumption is that it is due to an increase in the amount of smooth muscle in the wall, for whatever the exact strength of this tissue it is certainly less than that of collagen, the major fibrous constituent of all extra-thoracic arteries. If this reasoning is correct it follows that the thickest walled vessels, the common carotid, contain the most smooth muscle. It was mentioned earlier that there is some disagreement among the histologists as to the classification of this vessel, it is stated to be either of elastic or mixed type, while all are agreed that

the femoral is a muscular type. This does not seem to be the case for the dog, in which both these types of vessel appear to be of muscular form, while the greatest amount of muscle is to be found in the common carotid. This statement is based on wall thickness measurements, microscopical examination of a few specimens, and on the static and dynamic elastic properties of these vessels. The functional evidence will be presented in due course; here I will describe briefly my histological observations.

Histological structure of the arterial wall

A small number of specimens of all four types studied were prepared for examination using the fixative described by Hürthle (1920) which he found to cause minimal shrinkage, the composition of which is as follows:

Zenker's fluid	100 parts
Glacial acetic acid	5 "
Formalin	10 "

These preparations were originally made so that a direct check could be carried out on the measurements of wall thickness; in consequence the vessels were held at their natural length and filled with fixative at a constant pressure of 100 mm Hg which was maintained overnight, the whole specimen was also immersed in the fixative. Determinations of wall thickness and radius made on the projected image of cut sections of these vessels agreed very well with the previously estimated values, and the indirect method of obtaining the wall thickness was confirmed as accurate.

All specimens were embedded in wax and sectioned at 7μ , the most useful stain was found to be Wiegert's, Verhoeff's and Van Giesen's.

When sections prepared in this way were examined it at once appeared that the internal architecture was very much more clear and easy to interpret than is the case with normally fixed specimens.

The medial coat of thoracic aorta consisted of a large number (50-60) of parallel elastic laminae which were, when prepared in the above way, entirely without any wavy form. These laminae were associated with a certain amount of fine collagenous fibres, arranged largely on the adventitial aspect of the elastic tissue. When seen in longitudinal section the elastic laminae were also straight; those in the outer $1/3$ rd were continuous while the inner layers were somewhat fragmented.

Between these fibrous layers lay smooth muscle cells; these were rather more numerous than was expected. At a rough estimate, about half of the area of the section was occupied by muscle (cf. Hürthle, 1920, who reported a similar proportion). The arrangement of this muscle was fairly sharply differentiated between the inner $2/3$ and outer $1/3$. In the former the muscle cells lay obliquely across the gap between adjacent laminae, and could occasionally be seen to be inserted into the collagenous sheath around the elastic fibres. Adjacent layers of muscle were arranged in alternate directions so that the whole area had a herring-bone pattern. The space between the laminae, which extended as much as one quarter round the vessel, averaged about 10μ , the length of the muscle cells was $30-60\mu$. When however the aorta was sectioned in the relaxed state the angle between the long axis of the muscle and a vessel radius was reduced from $70-80^\circ$ to zero, giving the appearance of a radial arrangement. It seems then that this arrangement is that termed by Benninghof 'spann-muskeln' (1927).

The muscle cells in the outer third of the thoracic aorta are arranged obliquely in a flat helix, being inserted into the connective tissue in the same manner as that in the internal layers. One specimen

was also examined before and after soaking for two hours in 25% KCNS. If there was any change in appearance it was very small, on the whole it was judged that the waves in the elastic tissue were very slightly less prominent after cyanide; the same observations were made on a piece of carotid artery with similar results. This suggests that active muscle spasm is not an important factor in the production of the corrugations of elastic tissue.

The appearance of the abdominal artery was similar. The number of elastic laminae was less (20-30) and these were rather less well defined and fragmented with many fine branches forming something of a network. The laminae were again quite straight. In this artery the collagen was rather more obvious; a well-marked fine tracery surrounded the elastic tissues, but it was most prominent in the adventitia. Here lay a dense layer of thick collagen bundles which were cut across in both transverse and longitudinal sections and thus were presumably arranged as a helix which appeared to have a broad pitch. The arrangement of the muscle, which seemed to be no more abundant than in the thoracic aorta, was very much the same as in that vessel, and was again orientated longitudinally in the outer part of the tunica media. These specimens thus support the hypothesis that the extra collagen in the abdominal vessel is to a large extent longitudinally arranged. It was also obvious that any quantitative estimate of the relative amounts of the tissues present was very difficult; it would not have been possible to say that the proportions of collagen and elastin were reversed between the thoracic and abdominal aorta as was found by Harkness et al. (1957). The observation of Benninghof (1927, 1930) that all the muscle in these vessel

was arranged as 'spann-muskeln' was confirmed, it appeared that its insertia into the elastic tissue was always via the sheath of collagen which surround the latter.

Both the femoral and the carotid arteries had the structure of typical muscular arteries; well marked internal and external elastic laminae separated by a thick band of muscle which contained a few strands of elastin and collagen. Once again the waviness of the elastic tissue was absent, though in the case of the carotid it had not entirely disappeared. The exact attachments of the muscle cells were not easy to make out. It appeared that they were largely arranged circumferential in bands several cells thick; it did not seem that these were continuous right around the vessel; strands of some length, several hundred μ at least, appeared to terminate by insertion in the connective tissue. Again there was a sub-adventitial zone in which the arrangement was longitudinal. The difference between these two types of vessel was one of degree only. The femoral artery wall was thinner and contained less muscle both absolutely and relatively; about the same number of muscular strands were seen, they appeared to be rather meagre and the cells less plump.

No arrangement was seen which might correspond to that described by Benninghof (1927, 1930) as 'ringmuskeln', but no great number of specimens were examined and it is extremely difficult to pick out the exact attachments of a smooth muscle cell. Nevertheless if such an arrangement were the rule, it should have been discernible. My impression is that in all the vessels studied the muscle cells lay obliquely between

adjacent connective tissue layers, the difference between the elastic and muscular vessels is in the relative amounts of muscle present and in the mechanical advantage with which it might work. The arrangement corresponds with neither a pure parallel nor a series arrangement but rather with something in between. The essential characteristic of a series disposition of muscle and connective tissue is that activity of the former will increase the tension on the latter, where the arrangement is a parallel one the converse will be the case. Thus the effect of muscular contraction on the mechanical properties of arteries will depend very much on the details of the anatomical linkages between the elements of the vessel wall.

In the case of the elastic artery the muscle obtains a strong mechanical advantage at the expense of a relatively short pull, and even when it is fully contracted the elastin can presumably be lengthened between the muscle attachments. The effect of muscular activity would be to take a series of short tucks in the elastic tissue; this would be done by a large number of cells acting in parallel with enhanced strength. The lengths of elastin left free to extend will be reduced and the stiffness of the wall should be increased.

The alternative arrangement allows the muscle a much greater length to act, but it can exert correspondingly less tension; one might expect that these vessels should be able actively to change their calibre to a great extent, assuming the muscle can develop active tension of the order of 10^6 dynes/cm² (see Table 1).

The arrangement found in the aorta is of the same sort as that postulated by Burton (1954) and by Bader & Kapal (1958), though it would

not confer anything like a mechanical advantage of 100:1 as in Burton's model. The diameter changes produced by smooth muscle action have been reported as very small (Wiggers & Wegria, 1938; Bader & Kapal, 1958) and this is to be expected from the structure. On the other hand there is good clinical evidence to show that the muscular vessels may go into a state of spasm which may even obliterate the lumen. The arrangement seen in these vessels, which is not quite the same as Benninghof's 'ringmuskeln', might be expected to allow this, as it is generally agreed that smooth muscle can contract by about $2/3$ of its length. The fact that many of the muscle fibres lie round the artery in a helix in these vessels (and in the smaller ones (Strong, 1938)) would result in greater actual radius changes; in addition these oblique fibres, which were more numerous at the outer margin of the media and tissue within this layer, would be squeezed together which would narrow the lumen still more.

I have no evidence on the effects of muscle contraction and cannot discuss the subject with any direct knowledge. Muscular contraction involves an increase in stiffness and this was seen in the experiments of Landgren (1952) (carotid sinus), and Patel, Schilder & Mallos (1960) (pulmonary artery). The changes to be expected in the distensibility of a vessel are less simple; if any appreciable alteration in diameter occurs this will be accompanied by an increase in wall thickness. Both these factors will reduce the wall tension per unit area, if the pressure remains the same, this will then allow a further narrowing and so on. (This is the argument behind Burton's 'critical closing pressure' hypothesis, Burton, 1951). The fact that graded arterial contractions appear to occur

means that the muscle tension must be reduced when the appropriate change in length has been achieved. When equilibrium is seen the wall tension will be that due to the pressure, and this will now be seen at a smaller vessel size which was previously appropriate to a smaller pressure. Therefore the elastic modulus of the wall at a specified vessel diameter must have been increased by muscular contraction. Alexander's contention that the distensibility of a vessel is increased by muscular activity may then be true; comparisons of pressure-volume curves are truly meaningful only if the resting vessel size is unchanged; his data indicate quite large changes and give no information on the length-tension relations of the wall (Alexander, 1954a).

An effect of this sort was recently shown in the pulmonary artery, and following administration of noradrenaline the stiffness of the wall was increased (Patel, Schilder & Mallos, 1960).

The static elastic properties of the arterial wall

These properties were investigated by inflating the specimens in a series of 20 mm Hg steps from zero to 240 mm Hg. After each step a pause of two minutes was allowed before the diameter was measured; at the end of this time there was usually no further increase in size to be seen. From time to time a specimen was found which took longer than this to accommodate fully and accordingly a greater time was allowed. Such behaviour was more common in the muscular vessels.

Before the experiment was started the specimen was inflated to about 200 mm Hg for a few minutes; this was done to ensure that there

were no leaks but it also served the purpose of giving the vessel a preliminary stretch which opened it up. It was found that a period of up to 30 minutes at such a pressure might be necessary to obtain full accommodation, since the vessel became stiffer as it enlarged a longer experiment might have resulted in higher values for the elasticity in some cases.

The values obtained for the incremental moduli of each arterial type are shown in Figs. 11, 12, 13 and 14. The line drawn through each Figure is the mean of those shown. In Fig. 15 the mean values are compared; these values also include a number of extra figures at 100 mm Hg which came from experiments in which a full static determination was not performed. This accounts for some irregularity of the figures at this pressure, which is particularly manifest for the abdominal aorta.

The pattern seen in all the vessels is clear; the incremental modulus mounts steadily with increasing distension and for most of the range there is no significant difference between the abdominal aorta and the femoral and carotid arteries. At the lower pressures the thoracic aorta shows a less marked increase in stiffness. The curve is almost flat and this presumably relates to the much higher proportion of elastin found in this vessel by Harkness et al. (1957).

Another interesting finding may be mentioned. It will be seen that with increasing pressures the thoracic aorta becomes markedly more stiff so that by 200 mm Hg it is indistinguishable from the others. However above this pressure it is now seen that the carotid artery shows no further increase in stiffness. In fact it appears to become less stiff: there is probably not an actual decrease, these values are the differentials

FIGS. 11, 12, 13 and 14.

The next four Figs. show the incremental modulus of the walls of the four different types of vessel studied, at different internal pressures. In these Figs. each specimen is identified by a symbol. The mean values for each type are indicated by the continuous lines. The Figures refer to thoracic aorta (T), abdominal aorta (A), femoral artery (F), and carotid artery (C) in that order; each is identified by a letter in the bottom right-hand corner.

Fig. 11 - Static elastic behaviour of thoracic aorta.

Fig. 12 - " abdominal aorta.

Fig. 13 - " femoral artery.

Fig. 14 - " carotid artery.

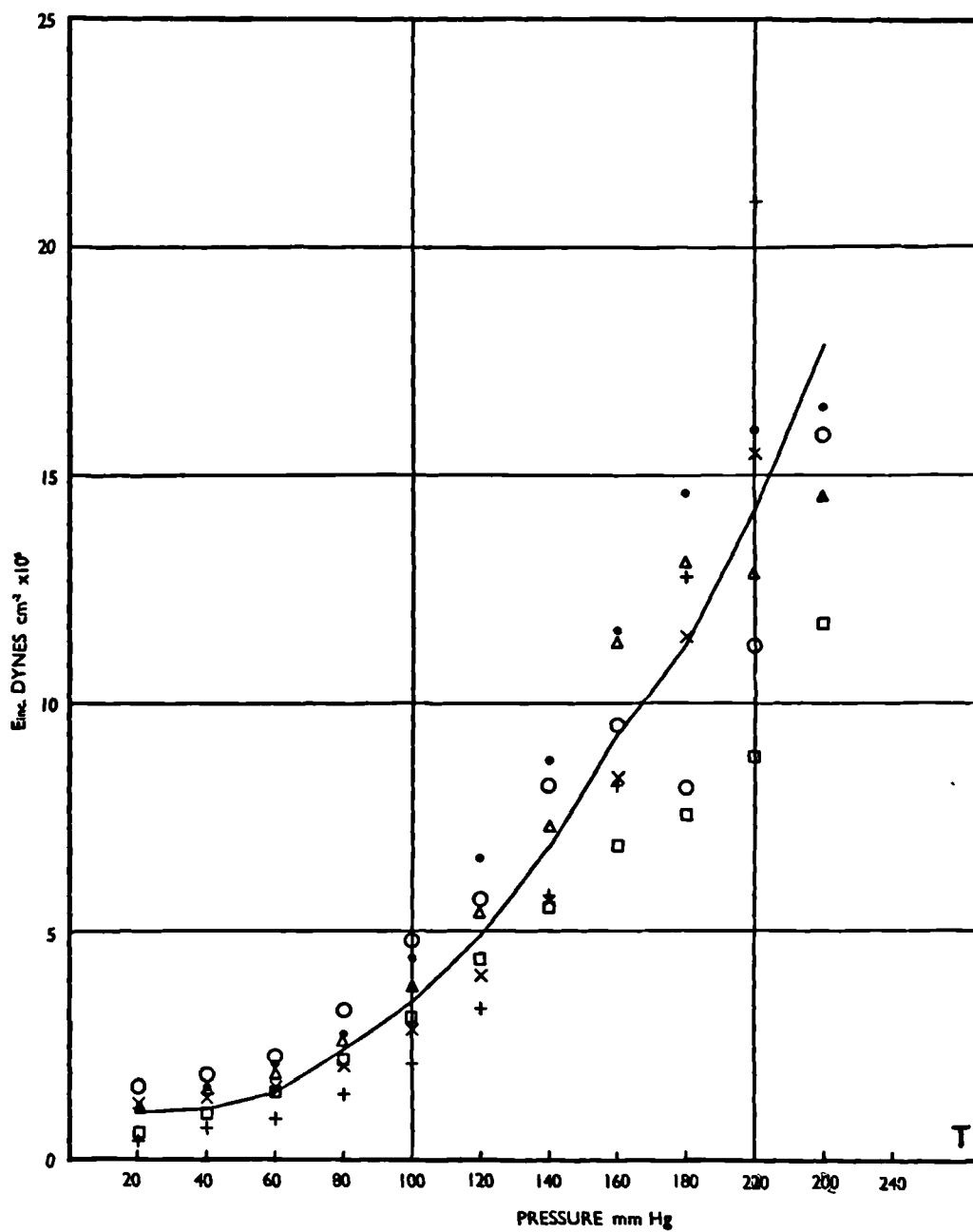


Fig. 11

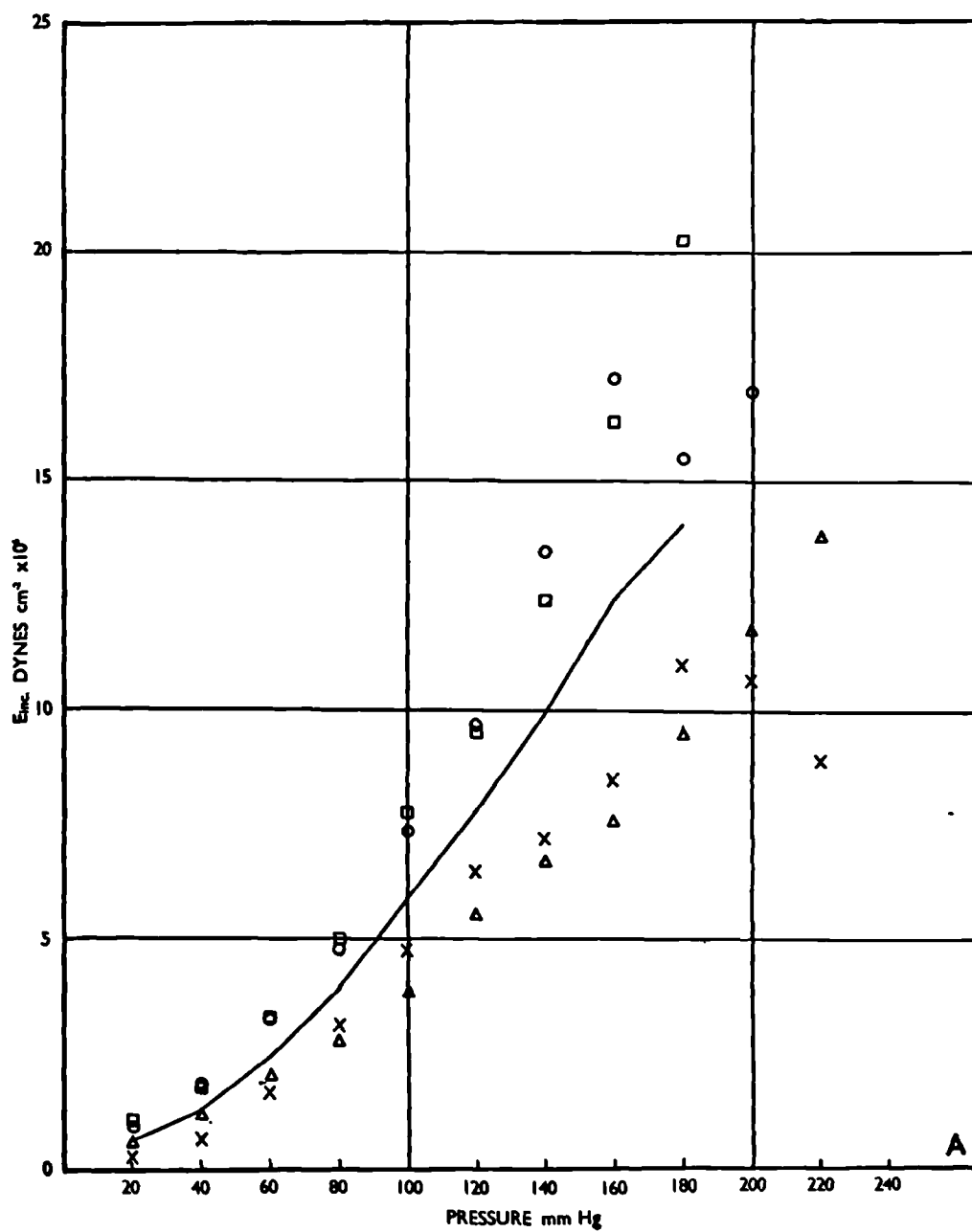


Fig. 12

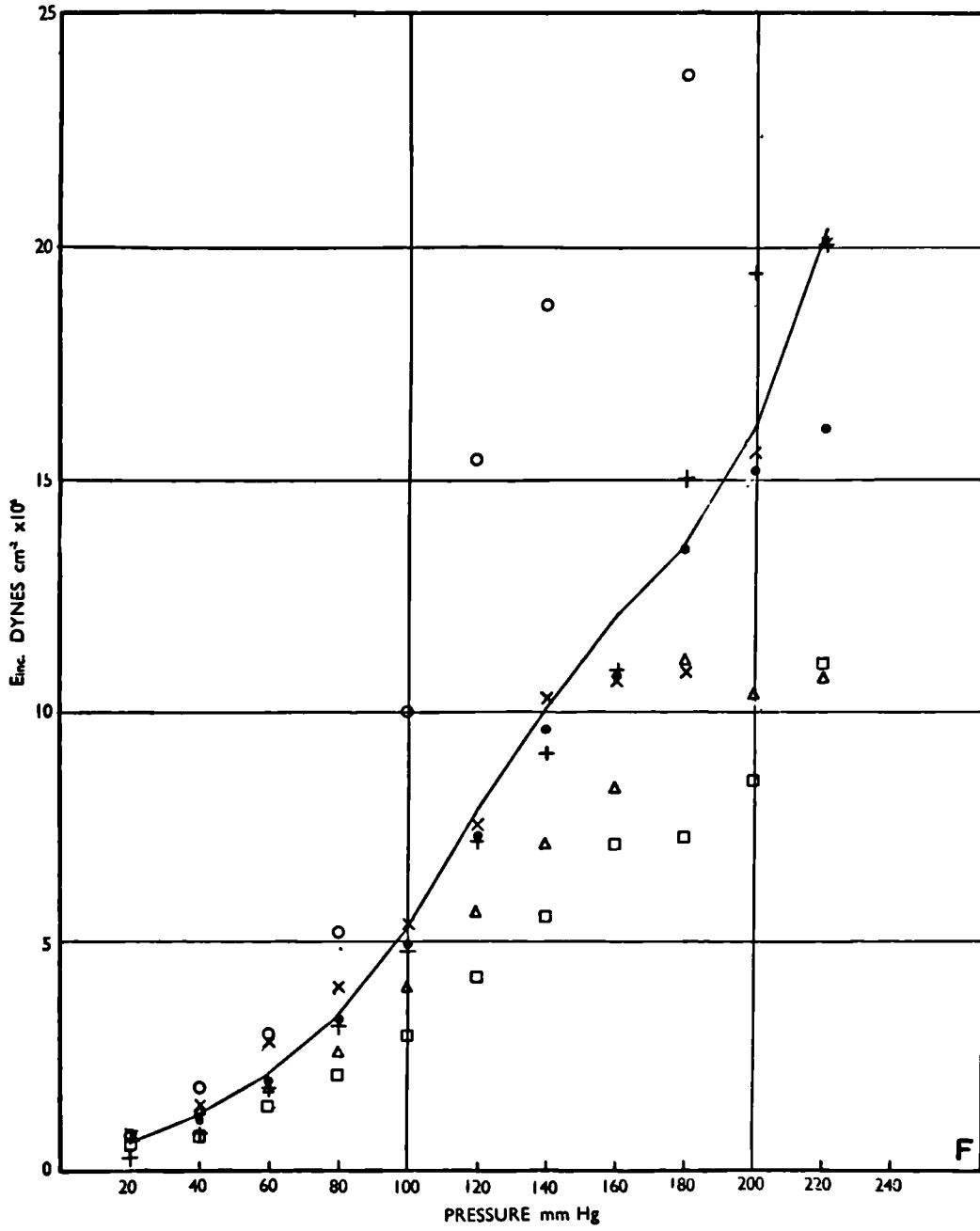


Fig. 13

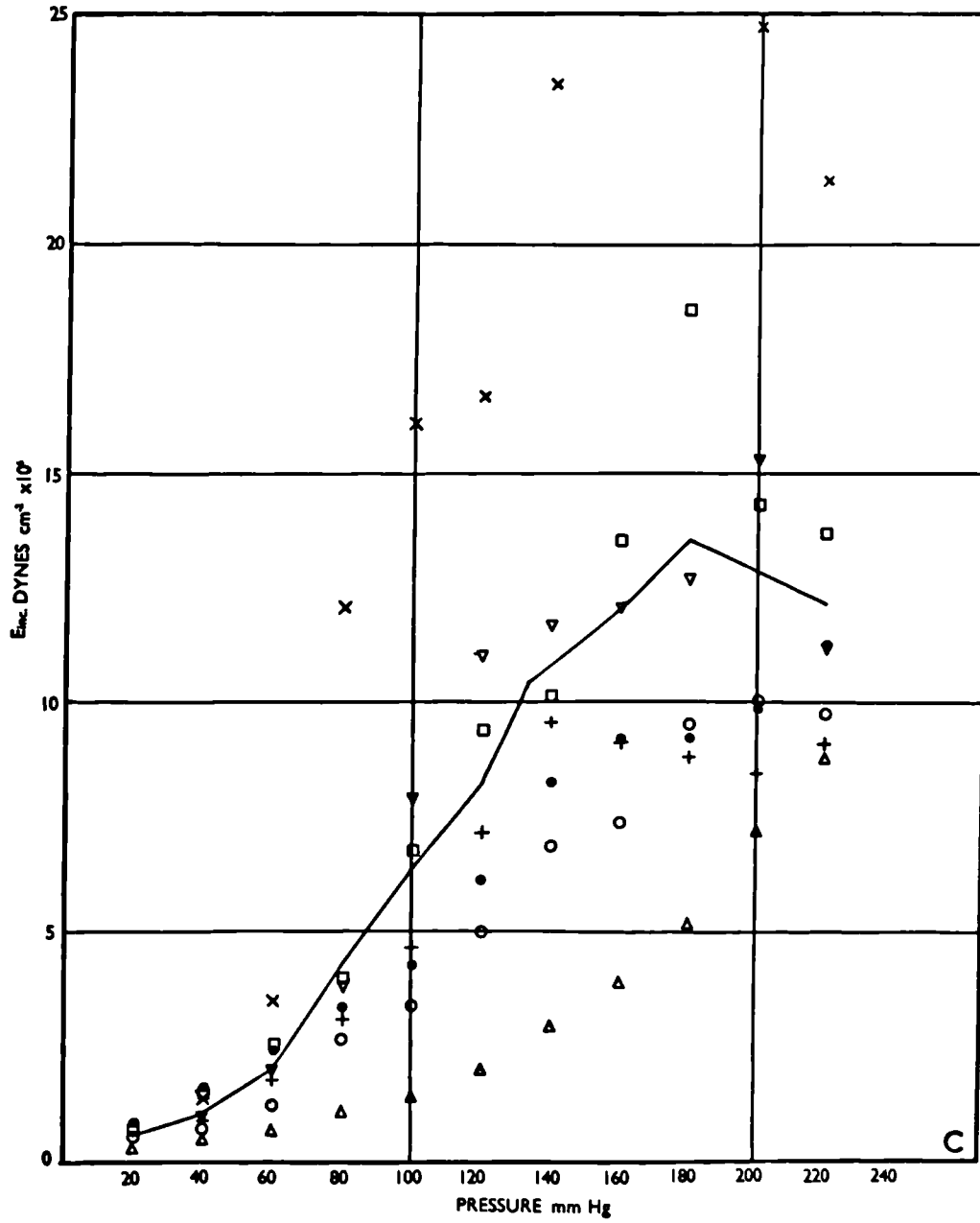


Fig. 14

FIG. 15

Here are shown the mean values at different pressures for the static incremental modulus of the vessels studied. The different types of artery are identified by the symbols shown. The Figure also includes a number of values which were obtained at pressures of 100 mm Hg only. Lines have been drawn connecting the points given by the thoracic aorta and carotid artery, the vertical lines show the standard errors in these vessels at pressures of 100 and 220 mm Hg.

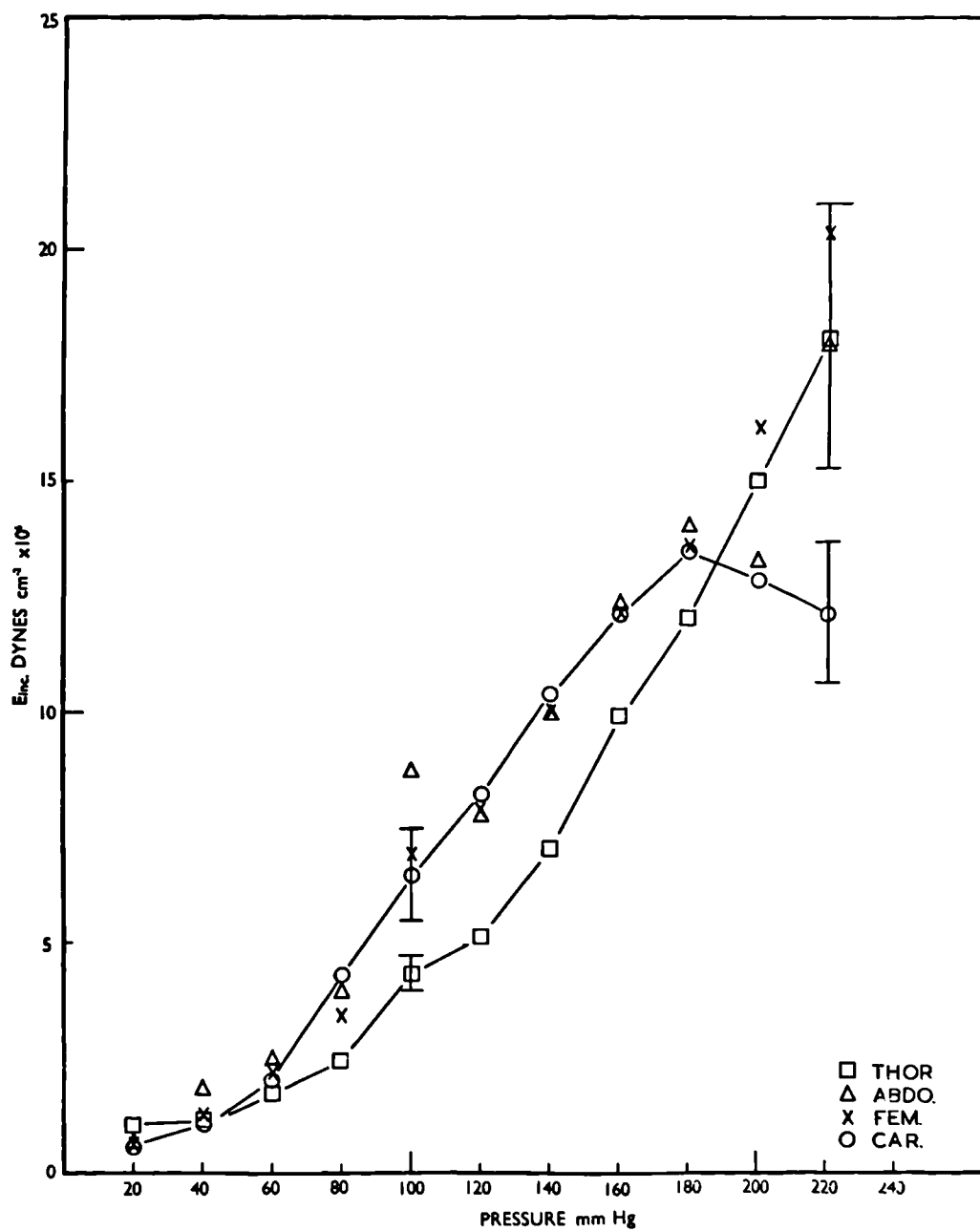


Fig. 15

of a very steep pressure-diameter curve and small variations may well be seen where the distensions measured are close to the resolution of the apparatus (3-4 μ). The difference between the modulus of carotid and thoracic aorta at 100 and 220 mm Hg is probably significant ($P < 0.05$).

There seems then to be a limiting modulus of about 13×10^6 dynes/cm² for the carotid wall, if there is a limit for the other vessels it has not been reached at the pressures used. The only reasonable explanation for this fact seems to be that the carotid must contain a relatively high proportion of some mechanically weak material, in this case the load at high pressures will be borne by a small number of stronger elements which have reached their maximal stiffness. The modulus of these elements is of course very much higher than that calculated for the whole wall. The only suitable candidate for the role of a relatively weak material is smooth muscle which, when relaxed, would appear from the evidence quoted to have a modulus lower than that of elastin and much lower than collagen. This suggests that the carotid arteries of these dogs at least contain more muscle in their wall than do the other vessels examined, and this is in line with the evidence quoted on the wall thickness and the histological appearance.

Smooth muscle cannot be so very much weaker, however, or one would expect to find some difference between the muscular vessels and the others in the low pressure-range, and this cannot be seen. There is less difference between these arteries than might have been expected in the amounts of muscle seen in cut sections, but there is no doubt that the smaller vessels contain a higher proportion of this tissue. In

addition the arrangement seen in the aorta might allow the elastin and collagen to extend with relatively little disturbance of the muscle and this should accentuate the differences. The values quoted in Table 1 suggest that elastin has a modulus not more than twice that of smooth muscle if molluscan muscle is at all similar to that studied here. It is worth remembering that Krafka (1939) found very little difference between ligamentum nuchae and taeniae coli, as samples of elastin and muscle respectively. The properties of the thoracic aorta suggest that elastin may even have the lowest modulus of all and one which changes relatively little with increasing length.

Harkness et al. (1957) found that the elastin in the wall of this vessel made up 1/3 of dry weight. If we assume that all three tissues contain the same amount of water, which is in fact unlikely, then the modulus of aortic elastin might be three times that of the wall, i.e. c. 12×10^6 dynes/cm². This is at least twice that found for the ligamentum nuchae, and the fact that muscle contains more water than does elastin would widen the difference. It must be that the collagen is making a significant contribution to the wall modulus even at low pressures. Certainly there was no obvious slack to be seen in the collagen in the sections of vessels fixed at a pressure of 100 mm Hg. Nor is there any sign that any vessel became suddenly stiffer at any point, the figures show a steady increase all the way. There is much evidence, both of my own and of other authors, to show that the collagen and elastin are arranged in parallel and it seems that each bears some of the load at all distensions. The evidence of Hass (1940) and of

Roach & Burton (1957) shows that elastin is mainly concerned at low and collagen at high pressures, but it is unreasonable to attempt to define the point at which one takes over from the other.

It is worth emphasising here that the modulus estimated for the wall relies on the assumption that the load is evenly distributed over the whole wall thickness. If the tissue contains relatively stiff elements these will bear a greater part of the tension. Furthermore a large proportion of a fibrous structure will be occupied by extracellular fluid and this will not take any share of the tension; this effect has been shown in models by Müller (1959). Therefore only very general conclusions can be made on the contribution of elastin, collagen or muscle to the stiffness of the whole tissue on the basis of its incremental modulus.

If it is agreed that muscle and elastin show fairly similar length-tension relationships then evidence of the sort that has been discussed up to now cannot differentiate between a series or a parallel arrangement of these tissues. Some findings which do enable a decision to be made are discussed below.

Another aspect of the subject may be dealt with now. The findings show that the wall becomes steadily less extensible when it is stretched; most work has up to now been presented in the form of pressure-volume diagrams and many workers have found that such diagrams are sigmoid in shape with a region of maximal volume distensibility at some point. These findings are not necessarily incompatible owing to the inter-relationships of pressure, radius and wall tension of a cylinder.

Since the tension in the wall of a cylinder is a function both of pressure and of radius, and since the area over which it acts diminishes with inflation, successive equal increments of pressure will impose progressively increasing wall stresses. If the material becomes stiffer as it is stretched the relation between pressure and volume will depend on which increases the fastest, the radius or the elastic modulus.

The relation between pressure change and radius increase can easily be obtained from Eqn. (15)

$$\frac{\Delta P}{\Delta R_0} = \frac{E(R_0^2 - R_1^2)}{2(1 - \sigma^2)R_1^2} \cdot \frac{1}{R_0}$$

As the wall itself does not change appreciably in volume (Lawton, 1954), we may write,

$$\Delta V = 2 \Delta R_0 \cdot R_0$$

and thus

$$\begin{aligned} \frac{\Delta P}{\Delta V} &= \frac{E(R_0^2 - R_1^2)}{2(1 - \sigma^2)R_1^2} \cdot \frac{1}{R_0^2} \\ &= K \frac{E}{R_1^2 R_0^2} \approx K \frac{E}{R^4} \end{aligned}$$

The pressure volume curve will then be linear if the wall modulus increases as the fourth power of the radius. Inspection of my records shows that this is something like the case for the thoracic aorta, but that all other vessels will show a diminishing volume distensibility.

In order to compare my results with those of others the pressure volume curve of a typical thoracic aorta was constructed using the square of the internal radius. This is seen in Fig. 16 as the curve labelled 6. It can be seen that there is a very slight inflection and the diagram is

very similar to those taken from Reuterwall (4), Thoma & Kaefer (5) and from Hamilton & Remington (7), but the inflection is very much less than was found by Roy (1), Wagner & Kapal (3), and Hallock & Benson (2). Those curves which most closely resemble my own were obtained from experiments in which the length of the specimen was not allowed to alter, or in which arterial rings were used. On the other hand Roy's specimens were free to lengthen and so were Wagner & Kapal's. Hallock & Benson maintained a constant length but were at pains to perform their experiments as fast as possible in order to approach in vivo conditions. It may be doubted how successful they were in this, the rate of pressure increase was about 2 mm Hg/sec, but these inflations were clearly much faster than anyone else's. Sulzer (1928) has shown theoretically that the fast extension of a visco-elastic substance would make the length tension curve more sigmoid than it would otherwise be, and this may explain these authors' findings.

The influence of length changes is more difficult to predict except that the overall distensibility will be exaggerated. Some figures given by Schluter (1958) show a decidedly sigmoid curve of arterial length against pressure, while the radius curve is hyperbolic. (Incidentally this author also finds some 3% wall volume diminution on inflation. This was estimated by subtraction of the internal and external volume changes of a vessel enclosed in a plethysmograph; it is very hard to believe that this method is sufficiently accurate to detect such small changes.) It seems quite possible that the changes in length are responsible for the difference between Roy, Wagner and Kapal and myself on this point. The latter authors only found a sigmoid curve in young

FIG. 16

A series of arterial pressure-volume curves taken from the papers of various authors. These curves have been separated vertically, the scale of volume-change is shown at the bottom right. Each curve is identified by a numeral which indicates its source. Except for curve No. 5 all these were obtained from specimens of the thoracic aorta. Curves Nos. 4 and 5 have been calculated from data on pressure-radius relationships.

1. Roy (1880) (rabbit).
2. Hall & Benson (1937) (human, aged 20-24).
3. Wagner & Kapal, (1951) (cow).
4. Reuterwall (1921) (woman, aged 34).
5. Thoma & Kaefer (1889) (human external iliac, man aged 25).
6. Bergel (dog).
7. Hamilton & Remington (1947) (dog, calculated from studies on rings)

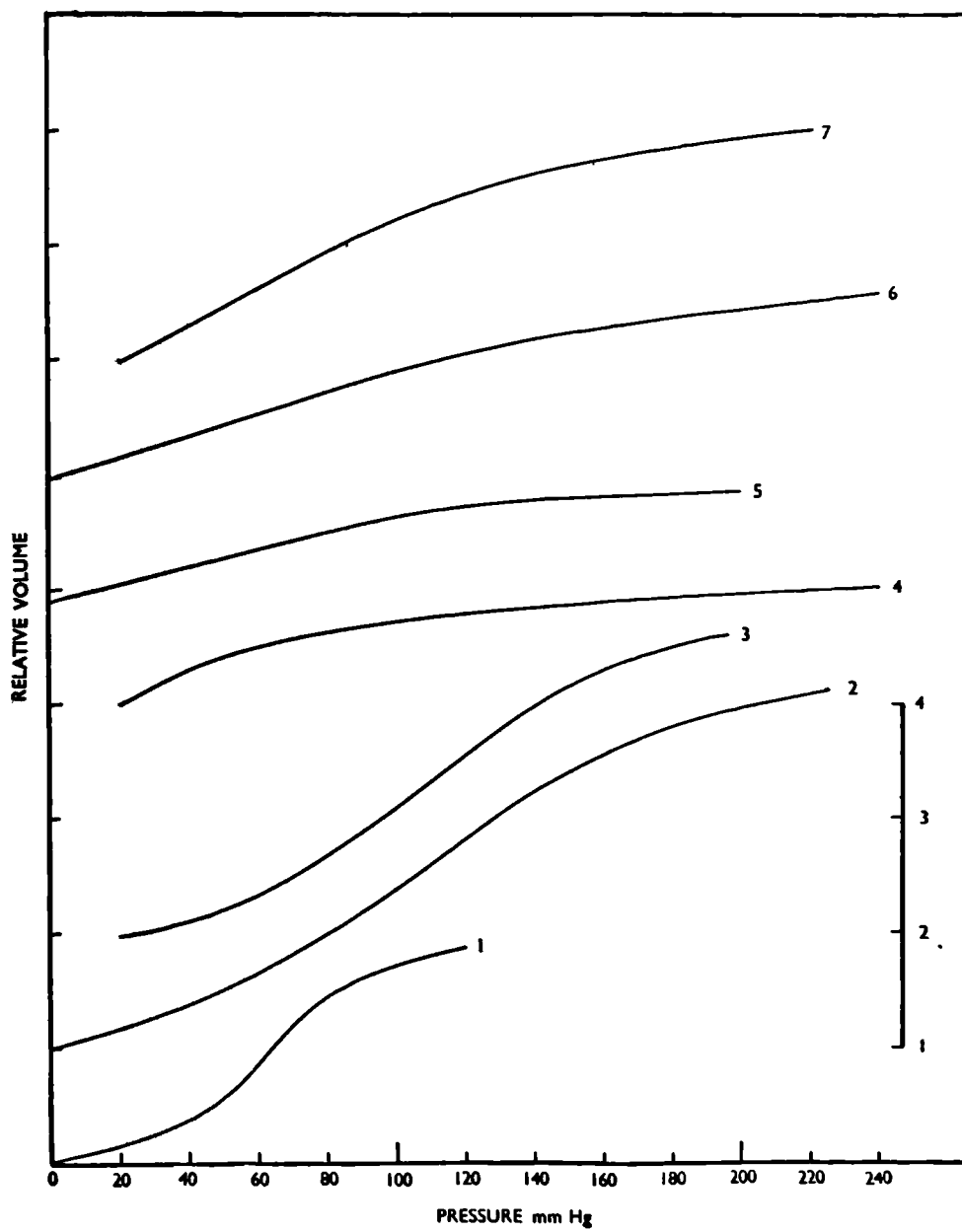


Fig. 16

specimens, but then my specimens were also from young animals.

To discover whether these very sigmoid curves are artifacts a simple experiment was performed. Pressure-volume curves were measured on a length of thoracic aorta which was made leak proof by the insertion of a fine tube of condom rubber which contained the fluid.

One end of the specimen was closed around a plug of appropriate size, the other connected through a tap to a manometer and a syringe with which accurate increments of isotonic saline could be added to the lumen. The starting volume was determined by rolling up the specimen to empty it and measuring the inflow under a very small pressure. The whole apparatus was placed in a water bath at a temperature of 37°C.

Three experiments were performed.

1. A slow inflation (two minutes between each step) with the vessel held at natural length.
2. A fast inflation (four minutes 0-300 mm Hg), cf. Hallock & Benson (1937) the specimen being at excised length (76% natural length).
3. A slow inflation, the vessel being free to lengthen, cf. Wagner & Kapal (1951).

These experiments were carried out in the order given. Before beginning, the specimen was inflated and deflated several times so that no curve represents the first, or virgin extension (Kapal, 1954). Two hours elapsed between each determination; at the end the first experiment was repeated with the same results as previously. The results are shown in Fig. 17 where it can be seen that curve 1 is slightly sigmoid, but the curvature becomes progressively more marked in curves 2 and 3. In addition curve 3 showed a greater over-all distensibility than the others

FIG. 17

Three pressure volume curves on the same specimen of dogs' thoracic aorta determined under different conditions (see text).

1. A slow inflation with the specimen held at natural length.
2. A fast inflation with the specimen held at excised length.
3. A slow inflation, during which the specimen was free to lengthen.

Note that the curves have been displaced horizontally in order to avoid confusion.

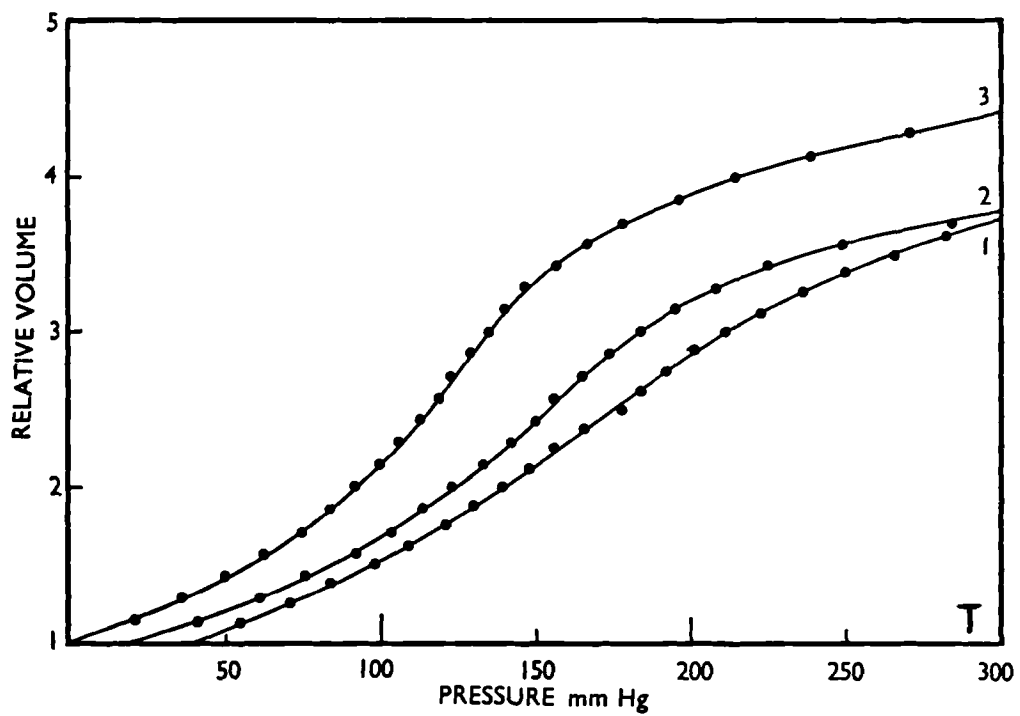


Fig. 17

and attained the lowest slope of all (in the range between 100-150 mm Hg). This curve is in fact quite similar to those given by Wagner & Kapal (1951).

It must be emphasised that many of my results for the thoracic aorta show a region of maximal distensibility (at about 50 mm Hg), but this feature was never very marked. I have never observed such extreme curvature as reported by Wagner & Kapal. For example these authors present a figure in which the expression $K (= V \cdot \Delta P / \Delta V)$ hardly alters in value between relative volumes of 1.5 and 3, corresponding to a pressure range of 25-125 mm Hg. If this were so the pulse-wave velocity in this vessel would be constant throughout this pressure range. There is no experimental evidence to support this. I feel that such floridly sigmoid curves are largely artifacts, either due to allowing the specimen to lengthen as it is inflated, or performing the experiment rapidly.

A rapid inflation might be thought to reproduce the effects of the heart beat more satisfactorily, though a rate of 2 mm Hg/sec (as in the case of Hallock & Benson (1937)) is far too slow. On the other hand the time necessary for the vessels to accommodate themselves to changes in mean pressure is much greater than this; it would appear that these investigators have got the worst of both worlds.

Certain other qualitative aspects of aortic elasticity were studied. It was confirmed that a static inflation and deflation curve form a hysteresis loop. The separation between the ascending and descending limbs was, however, extremely small and this is not illustrated for that reason. There was also a tendency, more noticeable in the muscular vessels, for a series of inflations to produce a further narrowing until a 'stable

loop¹ was achieved after 3-4 cycles. This occurred when the inflation curve approached the deflation curve, and this was accompanied by a slight (10%) increase in the radius at zero pressure. This latter observation agrees with those previously reported, for example by the Georgia school, though the width of the loops and the magnitude of the changes seem less than they obtained. All these phenomena were especially well seen in the carotid and femoral arteries.

Experiments were also carried out to discover the effect on their circumferential elasticity of holding specimens at different lengths. The results of one such experiment are shown in Figs. 18 and 19. The first figure shows the relation of the incremental modulus to pressure at lengths between 66% (excised length) and 110% of natural length.

When highly stretched, curve No. 6, the vessel was less stiff than it was at 75% (No. 3) and 90% (No. 4) natural length. The inflation at excised length (No. 2) could not be carried beyond 100 mm Hg because the specimen bowed and moved out of the light beam. This experiment suggests that the more a vessel is stretched in length the more distensible it becomes. Similar results have been reported by Bader & Kapal (1959) in rubber tubes.

At first sight this is paradoxical, but it must be remembered that stretching a vessel will reduce the radius at all pressures. When the same data are plotted in terms of modulus and radius (Fig. 19) this paradox is resolved. It can now be seen that the curves lie parallel to one another and that the effect of a longitudinal stretch is to shift them to the left, with little change in shape. The behaviour of the unstretched specimen is anomalous (curve No. 2); this is probably due to the bowing mentioned above.

FIG. 18

A series of determinations of the static incremental modulus of a thoracic aortic specimen held at different lengths. The experiments were performed in the order given. The lengths employed were as follows:

1. Natural length
2. 66% natural length (i.e. excised length)
3. 75% " "
4. 90% " "
5. 100% " "
6. 110% " "
7. 100% " "

In this Figure the modulus is plotted against internal pressure.

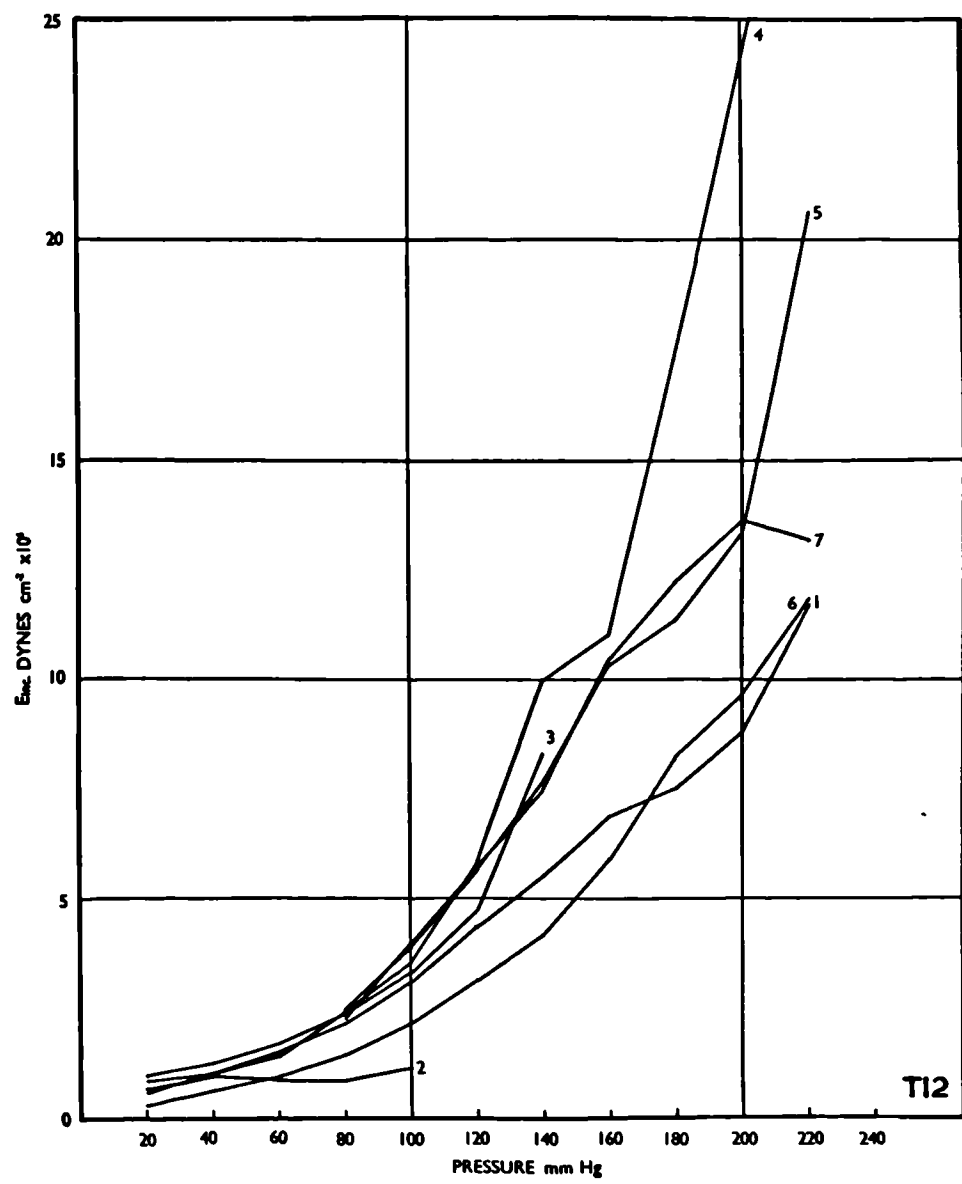


Fig. 18

FIG. 19

Here are shown the results of the same experiments in the previous Figure, but this time the modulus is plotted against external radius. Identifying numerals are explained in the legend to Fig. 18.

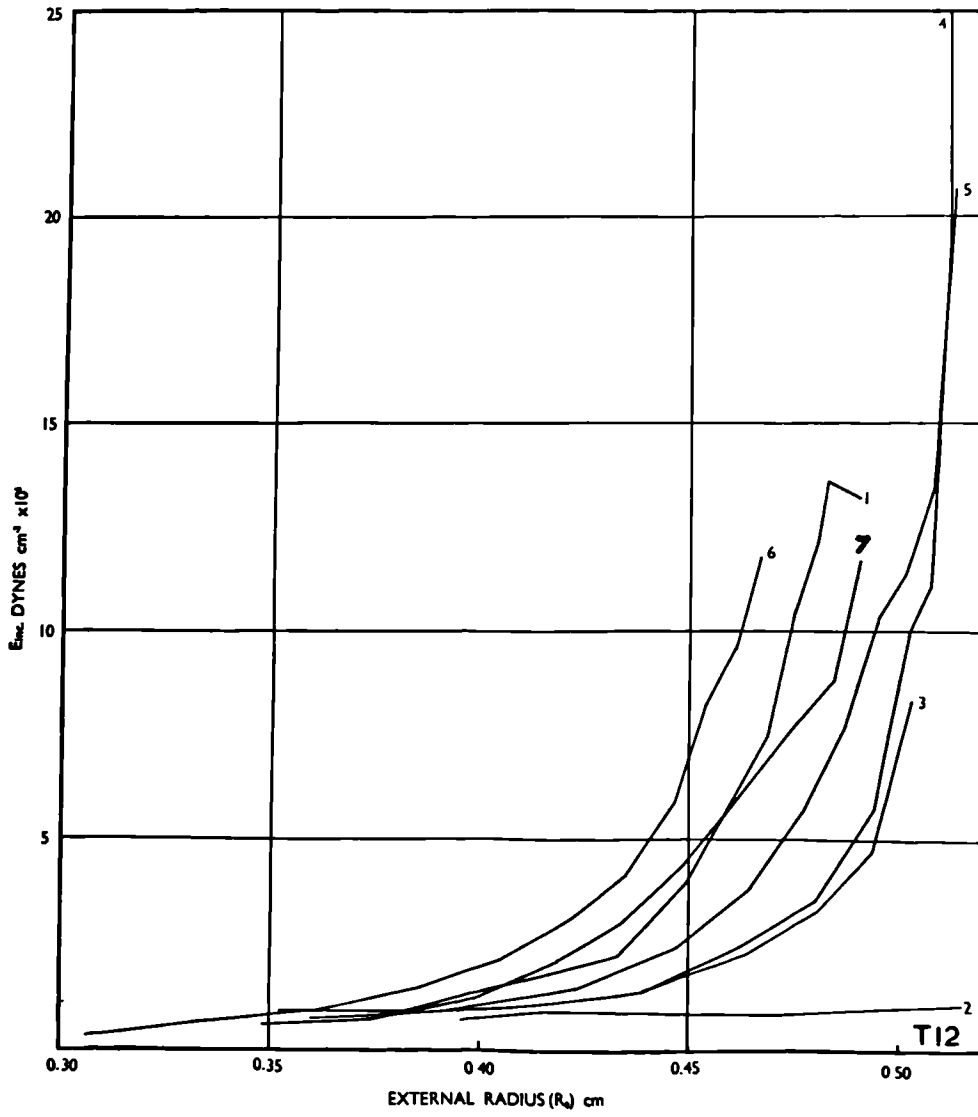


Fig. 19

FIG. 20

This Figure shows the results of a similar experiment to that shown in Fig. 19. The ordinate shows the incremental modulus of a specimen of latex rubber tube, the abscissa shows the different range of external radius covered after various degrees of longitudinal stretch. The stretch imposed was chosen to correspond with that used in the experiment on the thoracic aorta.

1. Unstretched (cf. excised length).
2. Equivalent to 75% natural length for the specimen described in Fig. 19.
3. " 90%
4. " 100%
5. " 110%

The lines shown have been drawn by eye.

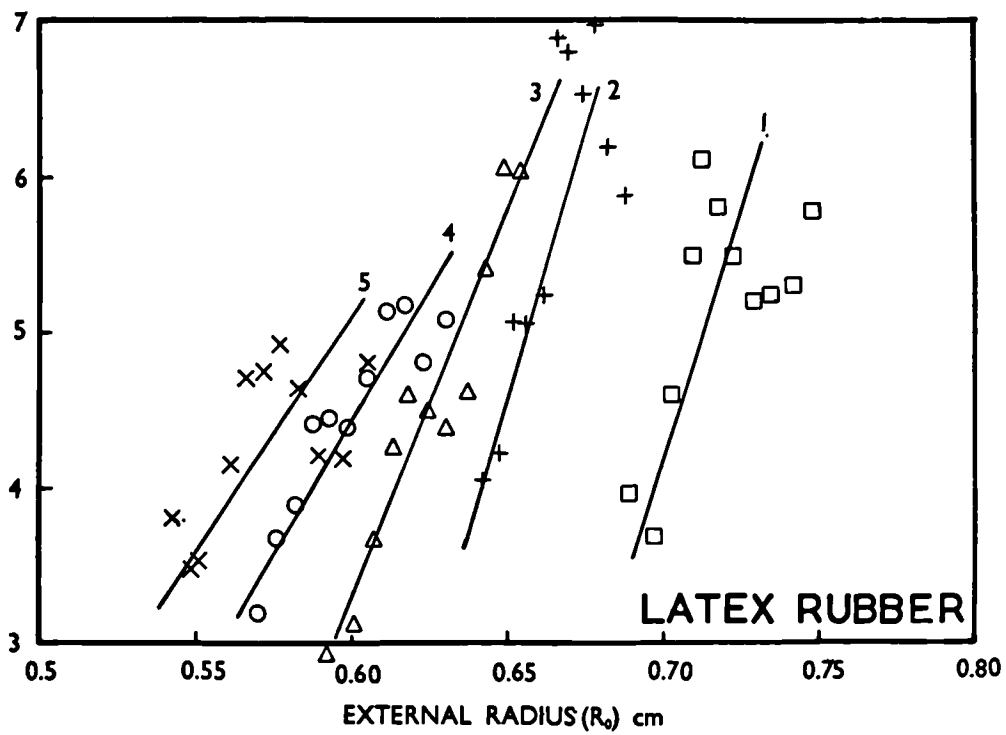


Fig. 20

The same experiment was performed on a length of latex rubber tube (Fig. 20). This was judged to be isotropic since no increase in length was observed on inflation to 250 mm Hg. The points are somewhat erratic, the changes in radius were very small, but the trend is clear.

At first glance these two Figures (Figs. 19 & 20) seem to be radically different, but this difference is largely a matter of the relative scales of the abscissae and ordinates. This experiment again shows a shift of all the curves to the left with increasing length.

By comparing the relative effects of longitudinal and circumferential extension one should be able to make a rough estimate of the ratio of the elastic moduli in these directions. Let us assume that the properties of the tube wall are represented by an elastic element arranged helically around the tube, and that the stiffness of this element is a function of its length. The ratio we seek will be the tangent of the angle this hypothetical element makes with the long axis of the tube. It is necessary to extrapolate the curves downwards to determine at what radius this element would have returned to its reference length when the starting radius was reduced by an increase in length. This extrapolation is obviously somewhat open to error and the result will not be very accurate.

It is therefore rather gratifying that a value of 43° was obtained for this 'pitch angle' in the rubber tube (a value of 45° would be expected if the wall were truly isotropic). In the case of the artery the best value (mean of three) of the ratio $E_{\text{longitudinal}} / E_{\text{circumferential}}$ was 0.68 and the pitch angle 34° .

This whole calculation rests on rather doubtful grounds and too much should not be made of it. In particular it assumes that there is only a quantitative difference between the elastic behaviour in the two directions. Roy (1880) compared the properties of strips of thoracic aorta cut along these axes and his illustrations show that the longitudinal strip was at first less stiff than the circumferential and later more so, i.e. the inflection of this curve was rather sharper. Some evidence along these lines may be gathered from my experiment. The ratio became rather greater the more the vessel was stretched; this would imply that the longitudinal modulus was increasing faster than the circumferential. Benninghof (1930) has found that the fibrous elements in the outer part of the tunica media are arranged helically at an angle of 30° to the longitudinal axis. This would seem to confirm these findings.

The dependence of the elastic modulus on the size of the vessel is well seen when the specimen shows any creep. This was especially noticeable in the muscular arteries, and in Fig. 21 the results of an experiment on a femoral specimen are shown. Care was taken not to inflate the vessel any more than was necessary to show that it was not leaking, consequently the first inflation showed a wide hysteresis loop and at the end the radius at zero pressure had increased by about 30%.

The artery was now held at a steady internal pressure of 150 mm Hg for 20 minutes. The pressure was then released and the radius at zero pressure was found to have increased from 1.3 to 1.4 mm. A second inflation and deflation cycle was performed and a much narrower loop resulted. Next followed the measurement of its dynamic properties during which the mean pressure was 100 mm Hg for a further 20 minutes. A very

slight increase in resting radius was found on deflation and the third pressure-volume curve was similar to the second except that the loop was closed.

These curves are very similar to those reported by Remington (1955) and other authors and they show the development of the 'stable loop'. In Fig. 22 the radius-modulus curves for the ascending limbs of these loops are shown. Those for the descending limbs are not plotted since they lay so close to the others that confusion would have resulted.

There is an almost constant relation here between radius and modulus; the first inflation was so different from the others because the vessel was so much smaller. The wide hysteresis loops seen, which were not obviously time dependent in the ordinary sense, are an example of what Remington has termed 'structural hysteresis' (Remington, 1957).

It is reasonable to suppose that the increase in size seen in this specimen was due to the forcible extension of smooth muscle. It can be seen that the first curve lies somewhat above the other two, i.e. the wall has become less stiff as a result of muscular relaxation. Other experiments gave the same results: this is what one should expect, for muscular contraction implies an increase in stiffness.

These results can only be explained on the assumption that the smooth muscle works in parallel with the other elements of the arterial wall. Although this time honoured concept has been tentatively questioned by Remington (1957) I have obtained no results which suggest that it is not true. The histological evidence which has been discussed is in agreement with these findings. It seems certain also that the collagen

FIG. 21

Three consecutive pressure-volume curves given by a femoral specimen (see text). The sequence of inflation and deflation is shown by the arrows (the last two deflations followed the same course).

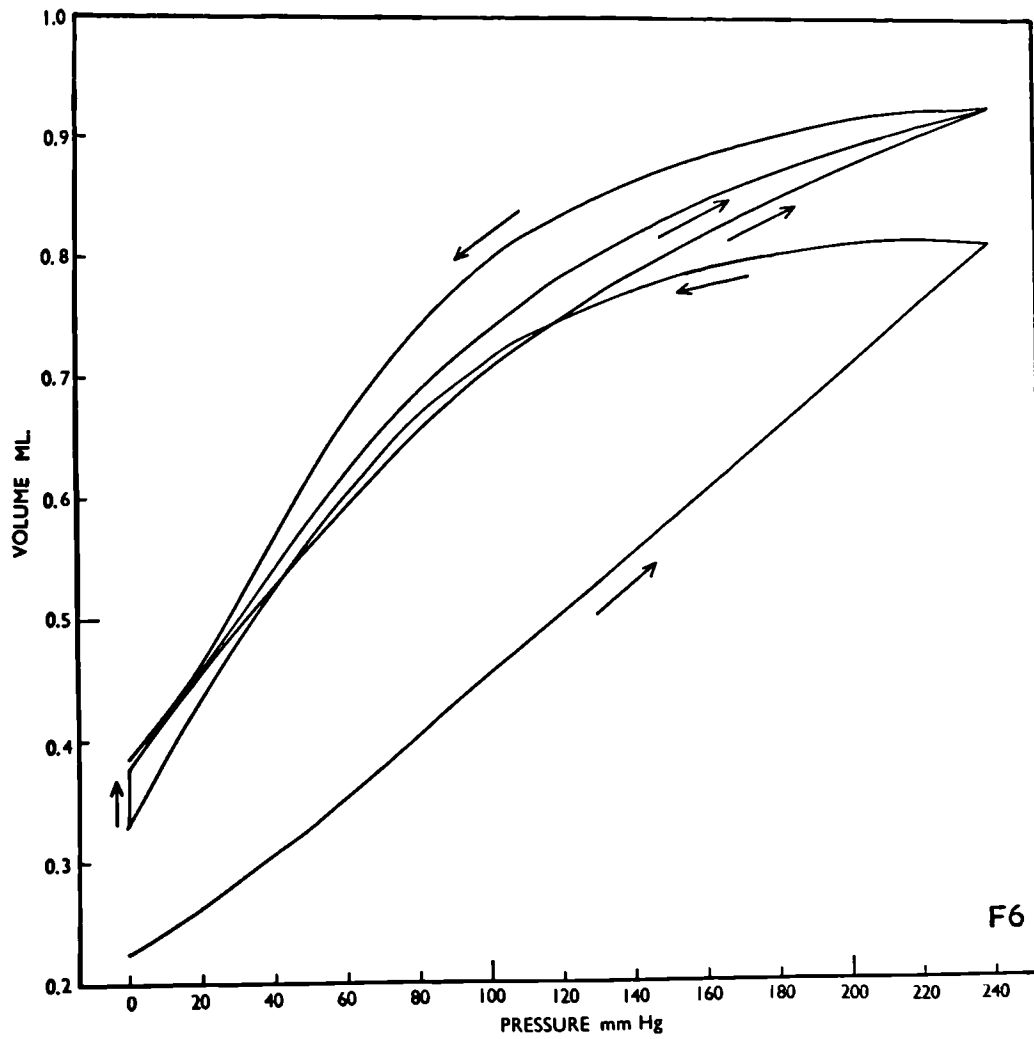


Fig. 21

FIG. 22

The relation between external radius and static incremental modulus for the inflation limbs of the three loops shown in the previous Figure.

Crosses, first inflation.

Closed circles, second inflation, after 20 min at an internal pressure of 150 mm Hg

Open circles, third inflation, after a further 20 min at 100 mm Hg

The vertical lines indicate the values obtained in each inflation cycle at a pressure of 100 mm Hg.

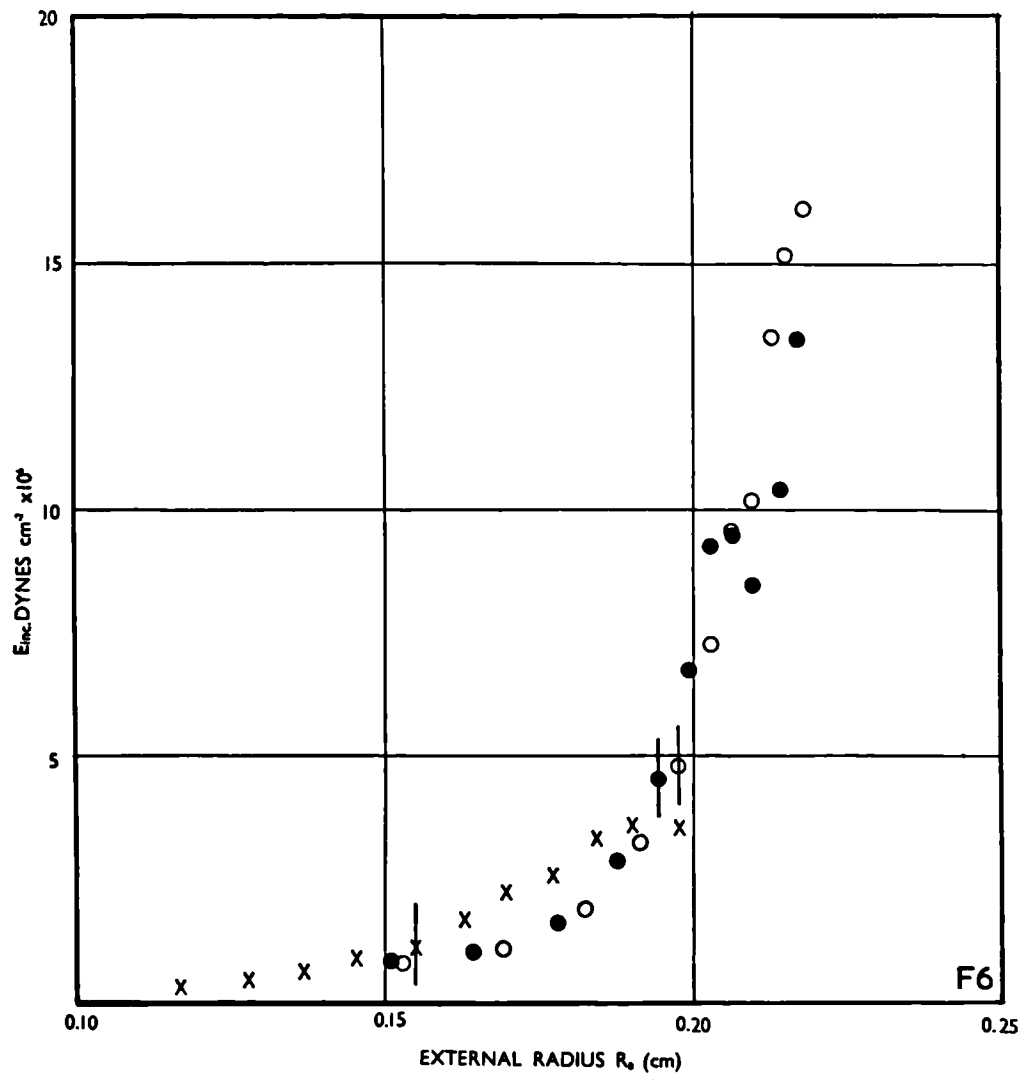


Fig. 22

and elastin are also arranged in parallel with each other. This follows from the histological evidence and from the fact that the highest modulus observed in elastin, even when contaminated by collagen, is much lower than that achieved by the maximally distended artery.

The experiments just described indicate that the elastic modulus of the arterial wall depends primarily on its length, i.e. on the vessel circumference. Nevertheless the relation between pressure and modulus is of more directly practical use and my results have been presented in this way. In Table 2 these results are compared with those in the literature.

Relatively little information can be found from past work on the elastic moduli of any vessels other than the thoracic aorta. For this vessel the figures quoted range between $\times 1$ and $6 \text{ dynes/cm}^2 \times 10^6$. The highest figure derives from Krafka (1939), from whose results I have calculated the incremental modulus; the lowest is one of Krafka's total moduli. It is impossible to infer which of his values corresponds to a pressure of 100 mm Hg or what were the natural lengths of his specimens.

With the exception of the figures of McDonald and those I have derived from Wagner & Kapal, all figures quoted refer to longitudinal moduli. We have seen that these will tend to be lower than the circumferential moduli, though one usually has no information on the amount of retraction that occurred when these specimens were prepared.

Nevertheless the agreement between these various figures is reasonable. At 100 mm Hg pressure most of my specimens had a radius of about 1.5 times that at zero pressure, consequently the incremental modulus will be this much bigger than the tangential. McDonald's figures

thus bear out mine, his measurements of circumferential moduli derive from measurements on photographs of vessels held at natural length at different pressures.

Lawton's and Hardung's figures for arterial strips are lower than mine. These are incremental moduli taken at approximately natural length. It seems reasonable to suppose that this bears out the conclusion drawn from the experiments quoted that the aorta is about 1.5 times as stiff circumferentially as longitudinally.

The figures from Wagner & Kapal are suspect in that they assume the relative wall thickness to be the same in man and the cow as for the dog. Whether this is so I have no information, but the discrepancy between their results and mine is not great.

By and large it seems that my figures for the thoracic aorta are perfectly consistent with those in the literature, and this suggests that my values for the other vessels (for which there are no other values) are reasonable.

The figures that I obtained for the abdominal aorta ranged between 3.7 and 12.8, with a mean of 9.2. At least the value calculated from Kapal & Bader is within this range.

In the case of the other vessels studied the results must be compared with figures derived from measurements of pulse-wave velocity. To make this comparison the dynamic moduli are necessary, and these will be considered in the section which follows.

The dynamic behaviour of the arterial wall

At this point it is convenient to consider published reports of the visco-elastic properties of arterial specimens. The reports are not numerous and are somewhat contradictory; in addition they relate, with one exception, to arterial strips.

The first serious approach to this subject was made by Ranke (1934). He considered the behaviour of the Levin-Wyman model which had been suggested (Levin & Wyman, 1927) as a unit to represent qualitatively the behaviour of striated muscle. This model consists of a damped spring in series with an undamped one, but Ranke found that the properties of aortic rings could not be adequately predicted by its use. Further experiments were performed on the free vibrations of air-filled carotids. At frequencies of 20-40 c/s the dynamic stiffness was about twice the static, and the logarithmic decrement suggested a phase lag of c. 45° in the wall's response to tension. It was also found that hypertonic saline solutions increased aortic viscosity markedly.

These experiments with air-filled vessels suspended in a sort of water-filled plethysmograph were probably much influenced by fluid inertia and viscosity. Nevertheless they do serve to show that the properties of these materials are complex and are unlikely to be explained in terms of simple models containing linear elements.

Some years later two papers appeared from Fribourg (Hardung, 1952, 1953) in which a sensitive apparatus was described for determining the stress-strain relationships of strips of arteries, and other substances, at frequencies from 0 to 20 c/s. The elastic modulus so derived was

treated as a complex number and resolved into real and imaginary parts E_{dyn} and $\eta\omega$ (see Eqns. 25, 26). It was found that the behaviour of aortic strips was similar to that of rubber; above frequencies of 2 c/s E_{dyn} was constant with frequency at a value of around 1.25 times the static modulus. On the other hand the viscous term, $\eta\omega$, rose linearly with frequency, though most steeply in the region between 0 and 1 c/s. The rate at which $\eta\omega$ rose was, however, not proportional to the increase in frequency; thus the coefficient of viscosity η decreased with increasing ω .

In addition Hardung investigated the behaviour of a number of rubbers and plastics. His results with these will be compared with some of my own.

A rather similar method to that of Hardung was employed by Kapal (1954) to study the dynamic behaviour of aortic rings. A cam driven lever was used to impose oscillating strains on the specimen. The stress at the other side of the ring was recorded by the movement of a small mirror mounted on an isometric spring. Preliminary reports of the use of this apparatus with specimens of striped and smooth muscle have been given by Reichel (1952a, b), who found the apparatus satisfactory. Kapal's results contrast with those of Hardung. He reported that the dynamic modulus increased somewhat above the static at 1 c/s and thereafter continued to rise. The mean results of all his experiments showed that the ratio $E_{dynamic}/E_{static}$ was 1.05 at 0.05 c/s, 1.1 at 1 c/s, thereafter rising linearly to 1.35 at 40 c/s. Kapal made no measurements of the phase difference between stress and strain. His results are also

of interest in that he showed that this ratio of dynamic to static stiffness was not related to the mean extension. This ratio had a constant value when measured at a mean load from 3 to 15 dynes/cm² x 10⁵.

An entirely different approach to the problem was described by Lawton (1955). Specimens of dogs' aortic strips were loaded with various weights and the resonant frequency of longitudinal oscillation was measured. The results suggest a dynamic modulus of twice the static at small extensions above excised length, but with increasing strain the ratio increased up to six times. Lawton uses the term 'dynamic modulus' to mean the complex modulus ($E_{dyn} + i\eta\omega$).

A rise in the modulus would be detected, in these experiments, as a rise in resonant frequency. One cannot say whether the increased stiffness Lawton found was due to the greater mean strain or to the increase in frequency. In either case the increase is very much greater than that found by Hardung and by Kapal, and it is difficult to find an adequate explanation for this.

The viscous behaviour of the specimens was also studied. It was found that viscous properties played a relatively small part; furthermore the coefficient of internal viscosity fell hyperbolically with increasing frequency (2-9 c/s), that is Lawton's term similar to Hardung's $\eta\omega$ had a constant value throughout this range.

More recently an attempt has been made (Cope, 1959) to determine the pressure volume relations of whole aortic specimens under dynamic conditions. The experimental method employed leaves much to be desired and the author was unable to be certain that his specimens showed increased

stiffness at 2 c/s as compared with the static measurements. Certainly no marked changes are apparent from the reproduced records.

The subject was studied in a different manner by Zatzman et al. (1954), who measured the time course of stress relaxation in arteries following a sudden injection of fluid sufficient to increase the circumference 5-10%. Two types of vessel were studied, carotid (dog) and umbilical (human); their behaviour differed only as to the relative amount of tension decay that occurred within ten seconds, about 20% and 90% respectively. The authors therefore concluded that the decay was a property of the wall musculature. The time course of the decay was not truly exponential, instead it followed a pattern seen in textiles and rubbers by other workers. Such behaviour may be obtained from a model containing a number of Maxwell elements, the relaxation times of which are distributed in a 'box-like manner'. Feeling that such an arrangement was somewhat unnatural Stacy (1957) put forward a theory in which the extension of muscle is governed by the rate at which molecular elements can shift between two possible states, 'short' and 'long'. When suitable time constants for the conversion and reversion were selected theoretical curves of stress relaxation were obtained which matched well with the experimental results. It is however difficult to derive from these equations an estimate of frequency dependent behaviour of the sort I have measured here, though an attempt on these lines will be mentioned later.

In addition it should be noted that the arterial specimens used in these experiments were free to lengthen. It was stated that a length increase of about 10% occurred on inflation, but that no discernible

shortening occurred while the stress relaxation was being observed. This is not surprising as the volume was held constant, but nevertheless the tension decay observed must have been influenced by alterations in longitudinal tension to an unknown extent.

The 'box like' distribution function of relaxation times is not in fact artificial, since the choice of relaxing elements containing linear elastic and viscous elements is no more than a mathematical convenience. No structural elements whatever are implied. A similar 'short-long' model was also in fact used for striated muscle by Buchthal & Kaiser (1951); it is difficult to see where this may be fitted into the currently favoured model of Huxley & Hanson (Huxley, 1957).

Apart from the general feeling that the behaviour is too complex to be explained by a simple model there is then little agreement on the dynamic properties of arteries among these workers. It has been found that the dynamic modulus is greater than the static, but estimates on the relative increase vary.

Before considering the properties shown by my specimens it might be as well to be reminded of the derivation of the two terms which are to be considered.

The complex elastic modulus has the form (Eqn. 26a)

$$E = E_{\text{dyn}} + i\eta\omega$$

This modulus possesses both amplitude and phase (ϕ); these are in fact measured and then resolved into elastic (E_{dyn}) and viscous ($\eta\omega$) components. The relation between these three is therefore:

$$|E| = \sqrt{E_{\text{dyn}}^2 + \eta\omega^2}$$

$$\text{Tan } \phi = \eta\omega / E_{\text{dyn}}$$

where $|E|$ is the amplitude (modulus) of the complex modulus.

First consider Fig. 23. Here are shown the results of two separate determinations on the same length of red rubber laboratory tubing. The repeatability of the results is satisfactory. There is a difference of about 3% in the static figures, when this is taken into account there is no great discrepancy between the two sets of dynamic measurements (a single calibration factor is computed and then used for each series). There is some widening of the gap above 15 c/s but nowhere does the variation exceed 5% for either the viscous or elastic term. It therefore seems that this figure of 5% should be a fair assessment of the error of the whole experiment.

The actual behaviour shown by this specimen is interesting in itself. In broad outline similar properties could be seen in the arteries as far as their viscous properties go. There is a rapid rise of $\eta\omega$ to a value of $0.2 \text{ dynes/cm}^2 \times 10^6$ at 2 c/s. Between this frequency and 20 c/s there is a much more gradual increase; it does in fact double in this range. Similarly E_{dyn} shows a jump in the lowest frequencies and then continues to rise slowly and roughly linearly.

It is at once obvious that this behaviour is not that of a simple parallel elastic and viscous element. E_{dyn} is not constant with a value equal to the static figure, nor does $\eta\omega$ rise linearly with frequency at a constant slope. However Hardung (1953) also found similar viscous properties in rubber though the rise in E_{dyn} was less. Different specimens of rubber do however give very variable behaviour, and these figures match reasonably well with those obtained by Dillon *et al.* (1944). These authors worked in the frequency range 25-280 c/s and found both

E_{dyn} and $\eta\omega$ to be constant with frequency, the former at a value of about twice the static.

Other rubber tubes were also studied. The experiment shown in Fig. 24 has been included because the wave propagation characteristics of a 15 m length of the same tube have been studied by my colleague M.G. Taylor (Taylor, 1959). In this Figure the closed circles are the results obtained by my method. The picture is much the same as before, though the increase in E_{dyn} is less marked. The vertical bars indicate the limits of the values obtained in a series of experiments, using fluids of different viscosity, by Taylor (his values for η have been multiplied by ω).

The agreement between the two entirely different experiments is very gratifying. Considering the difficulties inherent in the measurement of very small phase angles the agreement on $\eta\omega$ is excellent. There is, however, a small difference in the elastic component which is probably not significant. The increase in this term between 2 and 20 c/s that I measured was about 8%, the propagation constant measured by Taylor indicated a rather greater rise, around 12%.

Figures 25 and 26 are included largely to show that rather more pronounced changes of properties with frequency are satisfactorily measured with this apparatus. Here are shown the behaviour of polyvinylchloride and of clear latex rubber. Both these materials exhibit viscous properties similar to those expected from the simple model; the marked increase in the elastic term is of course quite a different matter. The properties of the p.v.c. tube are virtually identical with those found by Hardung.

FIG. 23

Two separate determinations of the visco-elastic properties of a rubber tube. The experiments are distinguished by the closed and open circles. Note that, in this and subsequent Figures, the scale of the ordinate has been expanded below to show better the pattern of the viscous component, $\eta\omega$.

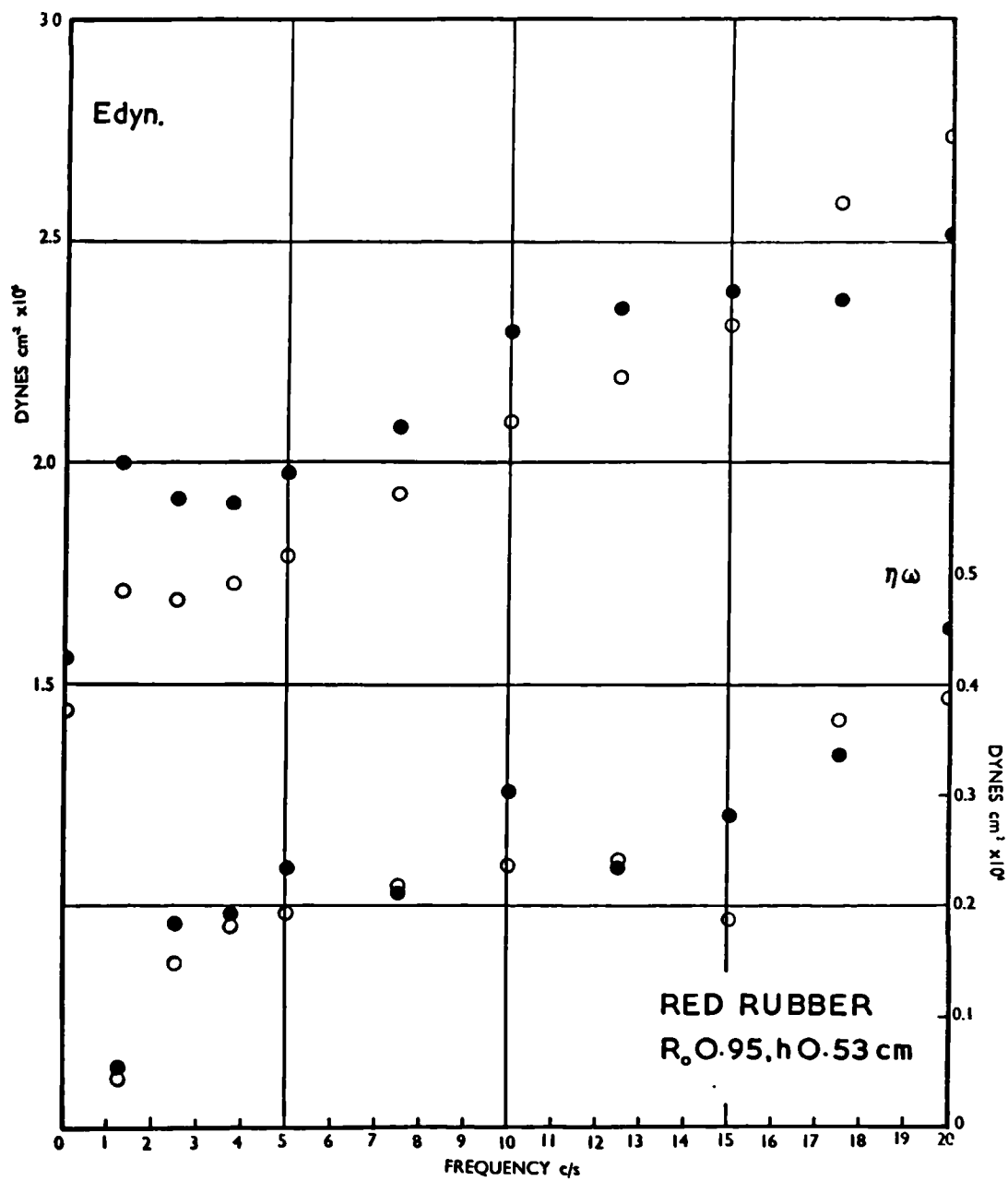


Fig. 23

FIG 24

In this Figure the closed circles show the values obtained by method for E_{dyn} and $\eta\omega$ in short specimens of rubber tube. The vertical lines indicate the range of a series of determinations made by Taylor (1959). These were computed from the wave propagation characteristics of a long length of the same tube.

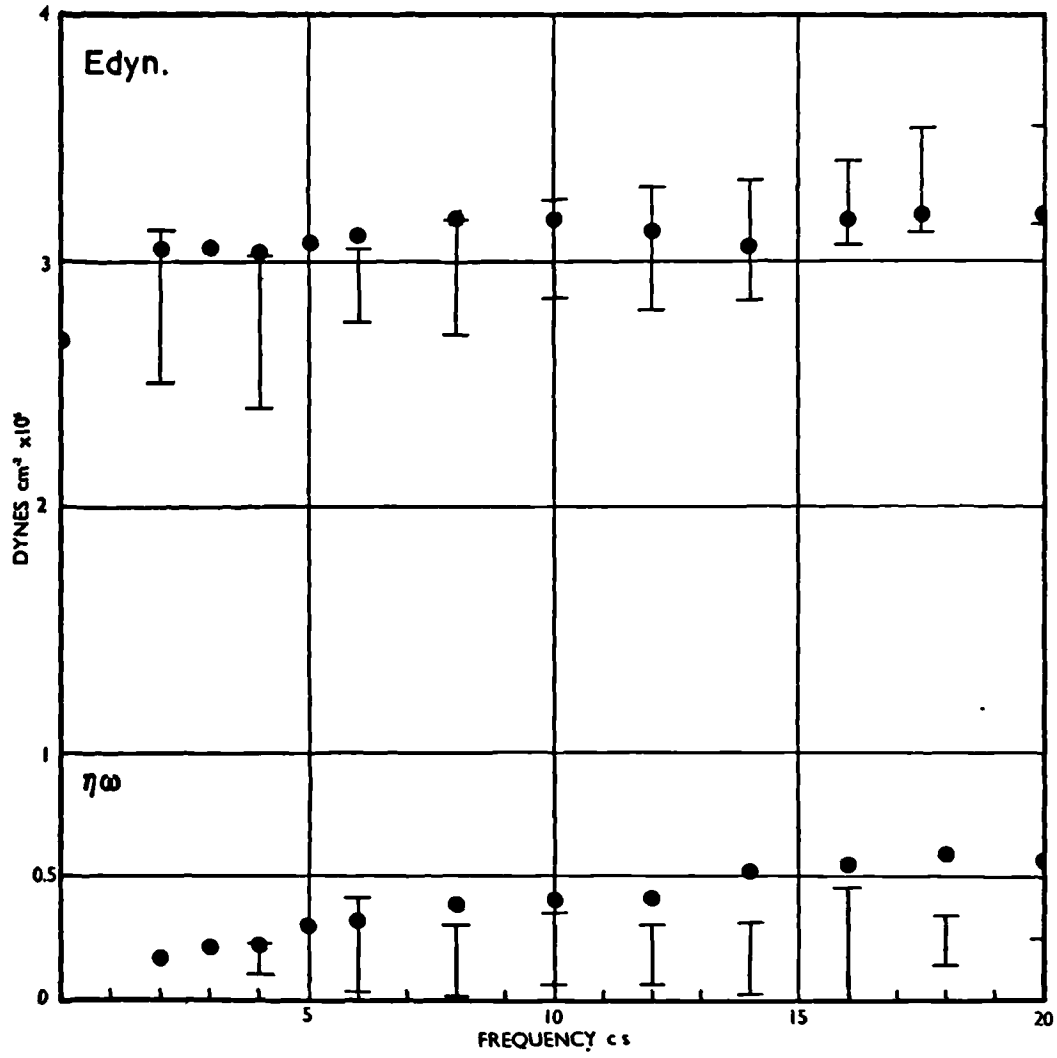


Fig. 24

FIG. 25

The visco-elastic properties of a specimen of poly-
vinylchloride laboratory tubing.

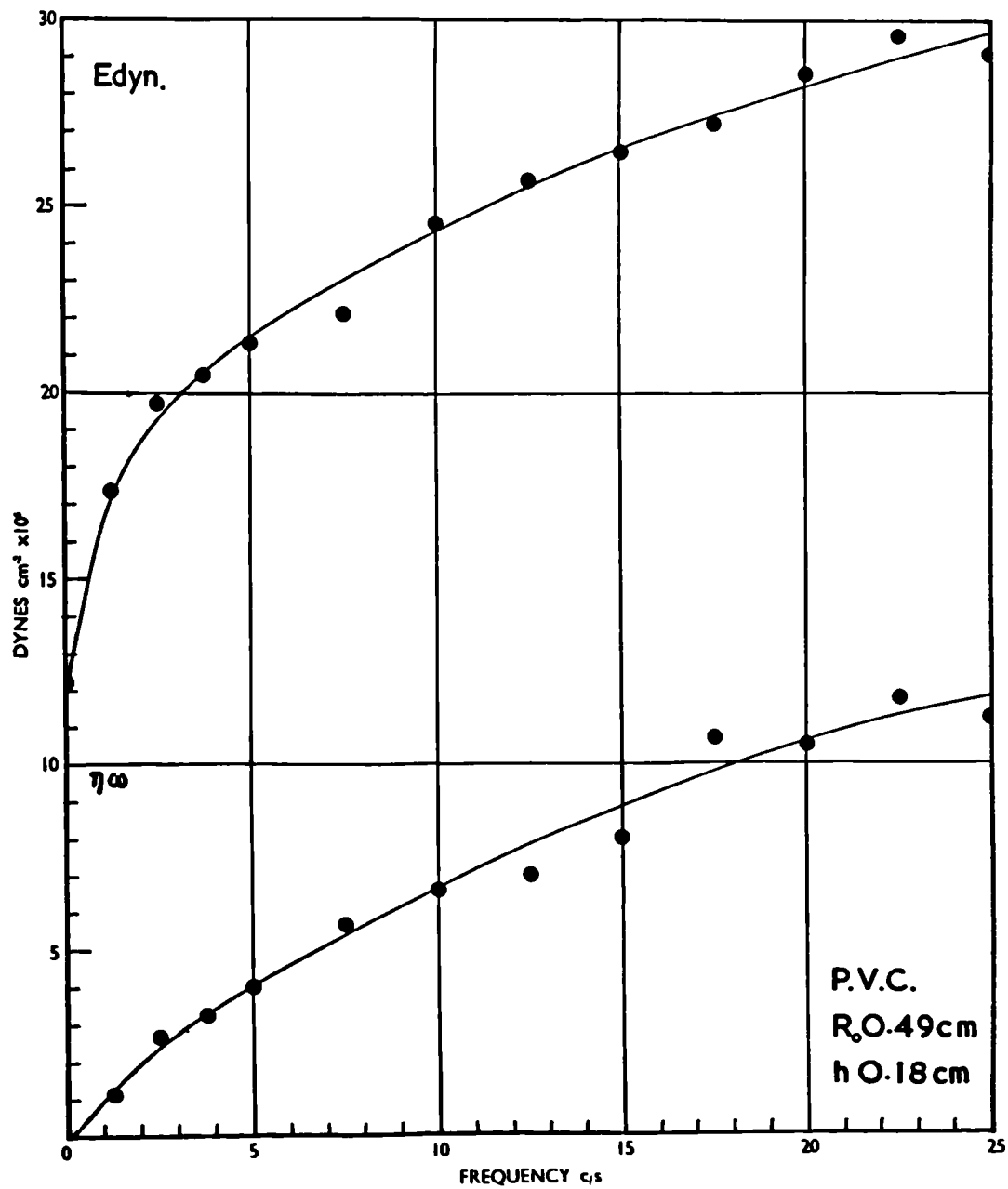


Fig. 25

FIG. 26

The vico-elastic properties of a specimen of latex-rubber tube.

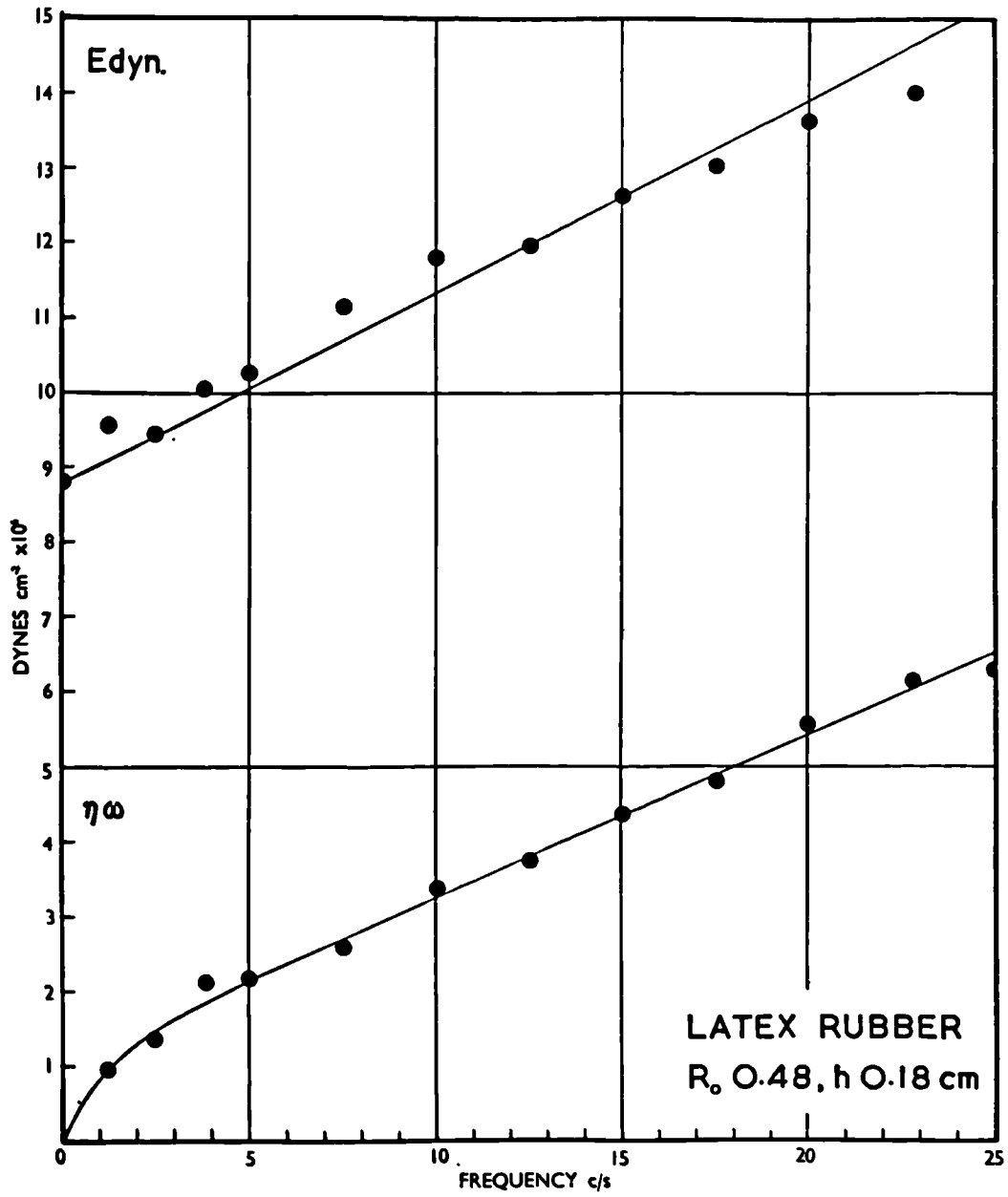


Fig. 26

It would seem then that the method of study used here satisfactorily measures the properties of the visco-elastic materials: we may now consider the behaviour of arteries.

The results are given graphically, though some of the information is repeated in tabular form. For each of the four vessels studied the figures (Figs. 27-34) present the following findings:-

E_{dyn}

$\eta\omega$

The ratio of E_{dyn} to the static modulus (E_{dyn}/E)

The product $E_{dyn}h/2R_0$

The values obtained for each of the specimens studied successfully are shown. Later Figures (Figs. 35, 36, 37) compare the mean values for each vessel type. The standard errors of the means are also indicated at various frequencies; the imprecision is fairly great as may also be seen in the variability shown in the earlier Figures.

Consider first the pattern of changes seen in the elastic component, E_{dyn} . One feature is at once obvious, this is the sharp increase in this term seen between zero frequency and 2 c/s. Something of the sort was seen in the artificial tubes but it is very much more obvious here. At higher frequencies the changes are very slight and somewhat variable. There is a tendency for the carotids to show a further rise, the thoracic and abdominal aorta remain essentially constant, while the femoral arteries actually show a decrease in stiffness, though the final value is still greater than the static figure.

It is easier to see the differences between the various types in Fig. 36, which shows the mean relative increase in modulus with frequency.

FIG. 27, 28, 29, 30, 31, 32, 33, 34

The following eight Figures deal with the visco-elastic properties of the arteries. There are two Figures for each type of vessel. The first shows all values obtained for the terms E_{dyn} and $\eta\omega$. In the second Figure of each pair the upper half shows the dynamic modulus as a ratio of the static value, E_{dyn}/E . The lower half shows the product of E_{dyn} and the factor $h/2R_0$, the ratio of wall thickness to the external diameter. All the experiments were carried out at a mean pressure of 100 mm Hg.

Fig. 27	E_{dyn} and	determined on thoracic aorta specimens
Fig. 29	"	" abdominal aorta "
Fig. 31	"	" femoral artery "
Fig. 33	"	" carotid artery "
Fig. 28	Upper part, E_{dyn}/E ; lower part $E_{dyn} h/2R_0$ for the	thoracic aorta *
Fig. 30	"	" abdominal aorta
Fig. 32	"	" femoral artery.
Fig. 33	"	" carotid artery*

*.

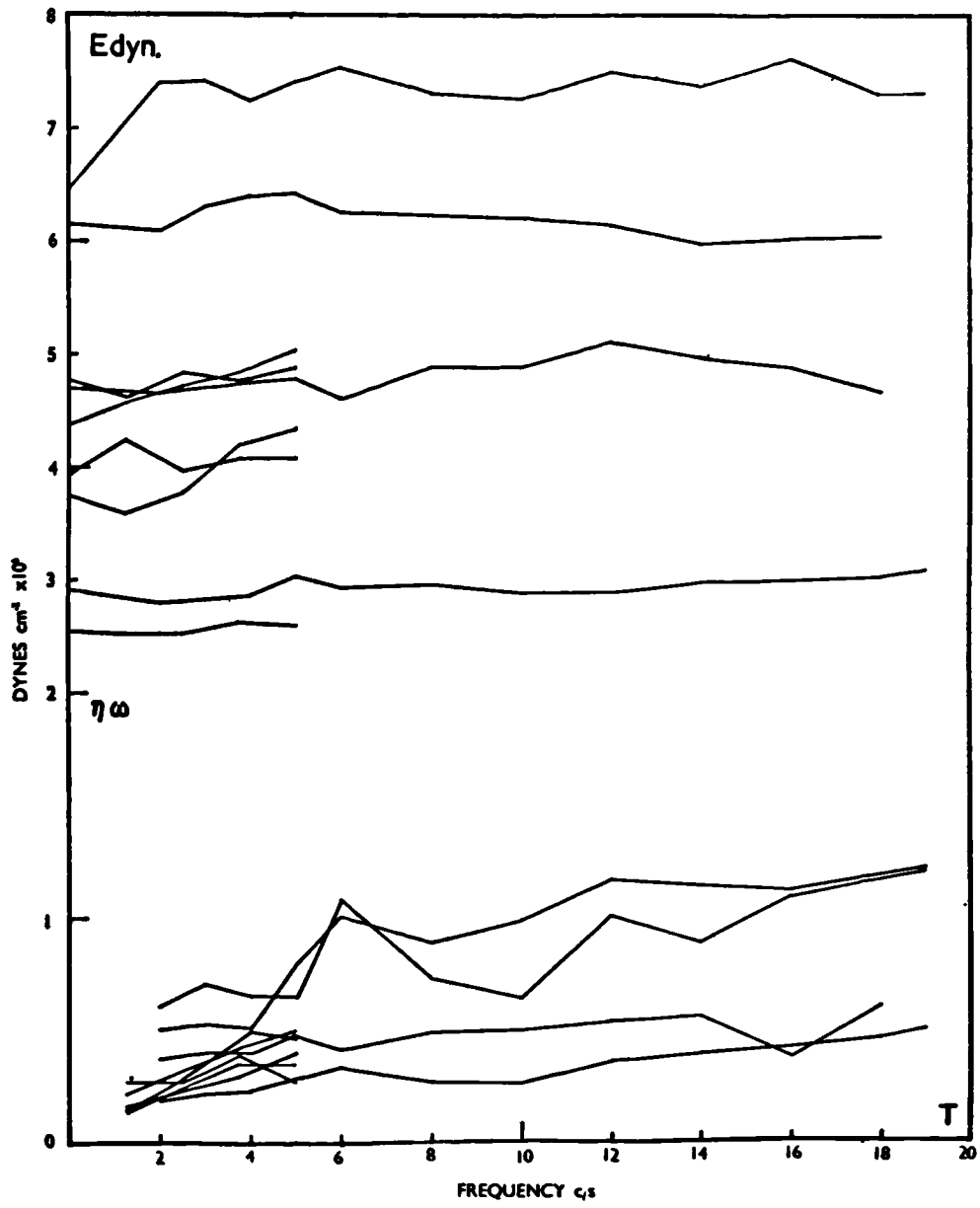


Fig. 27

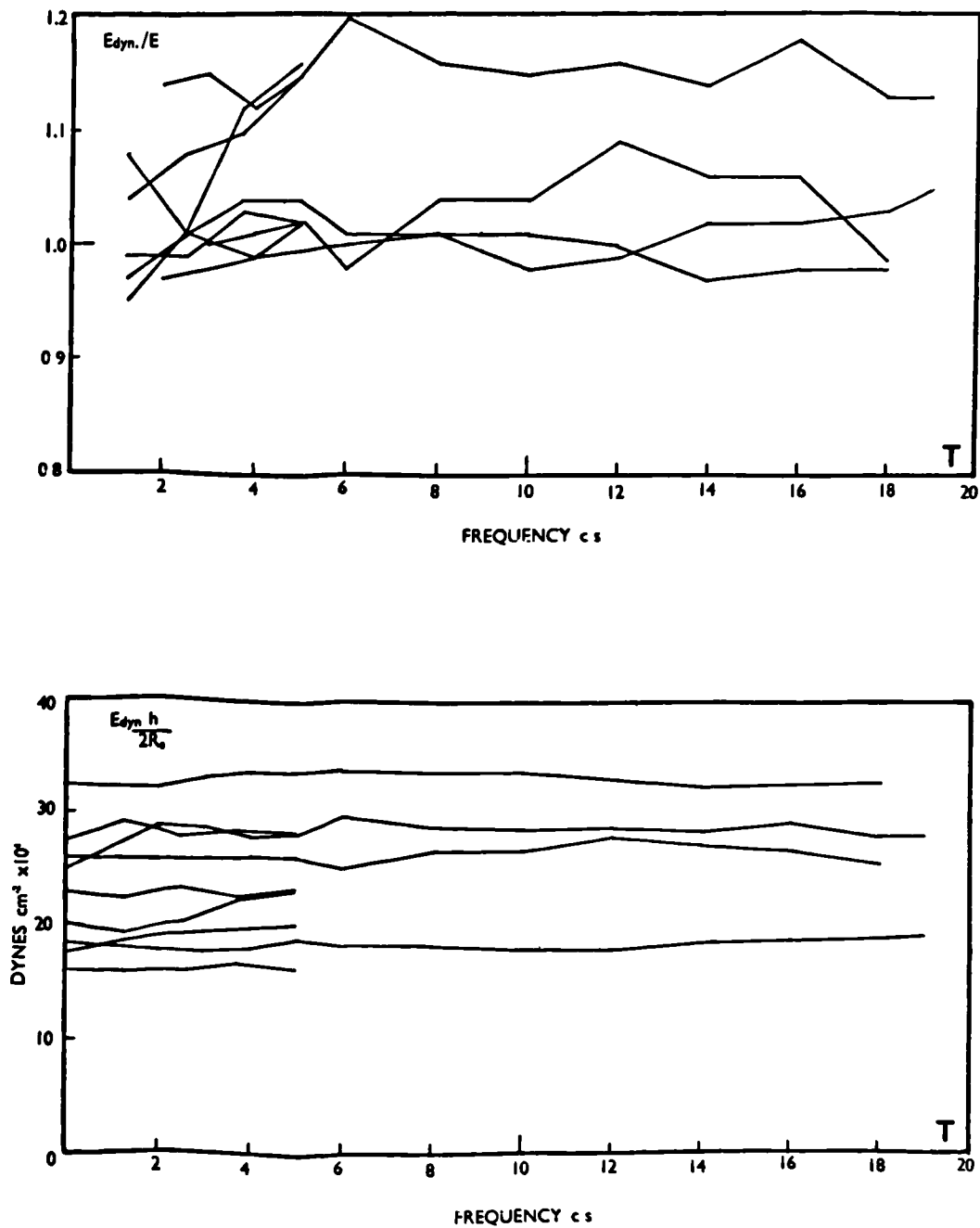


Fig. 28

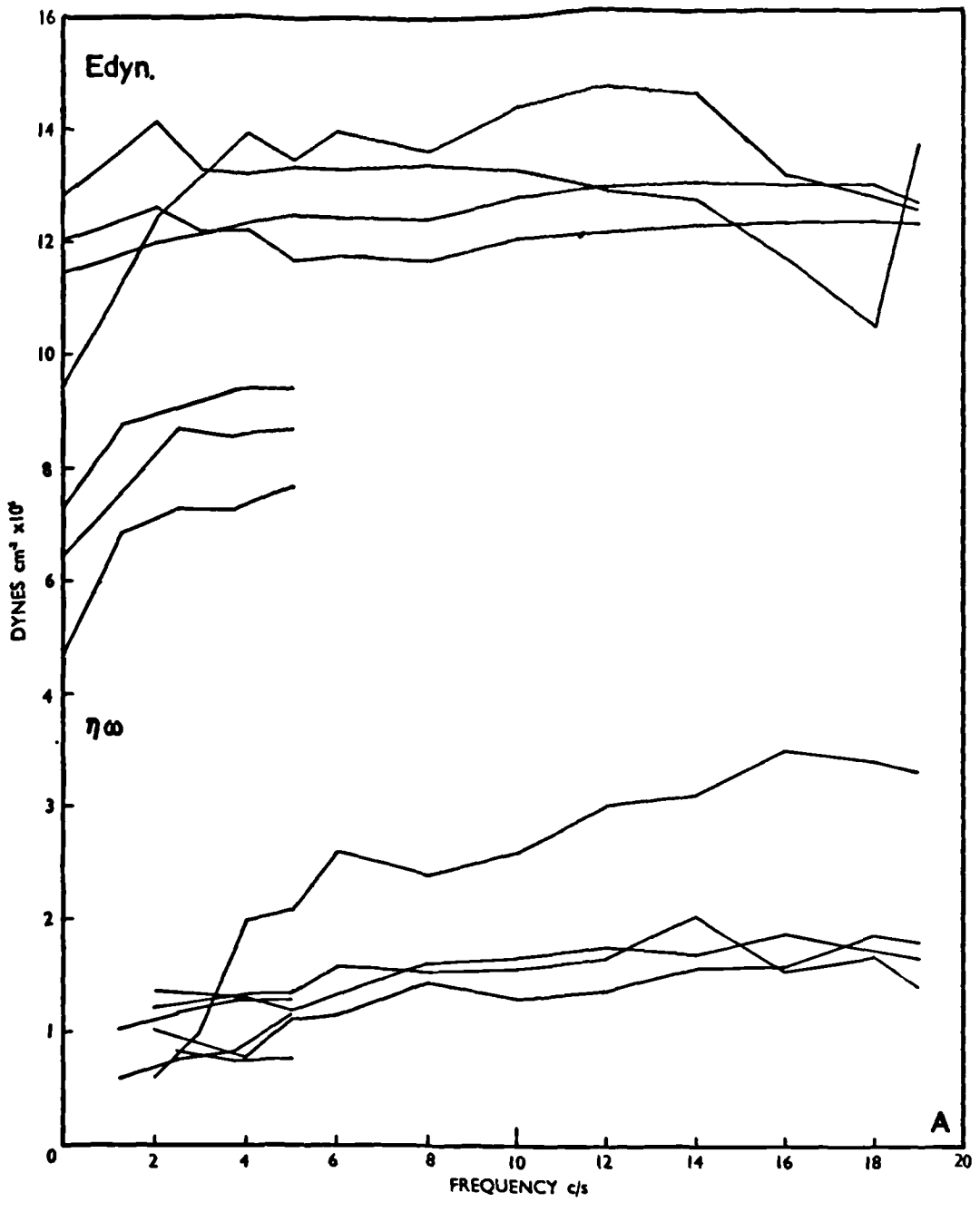


Fig. 29

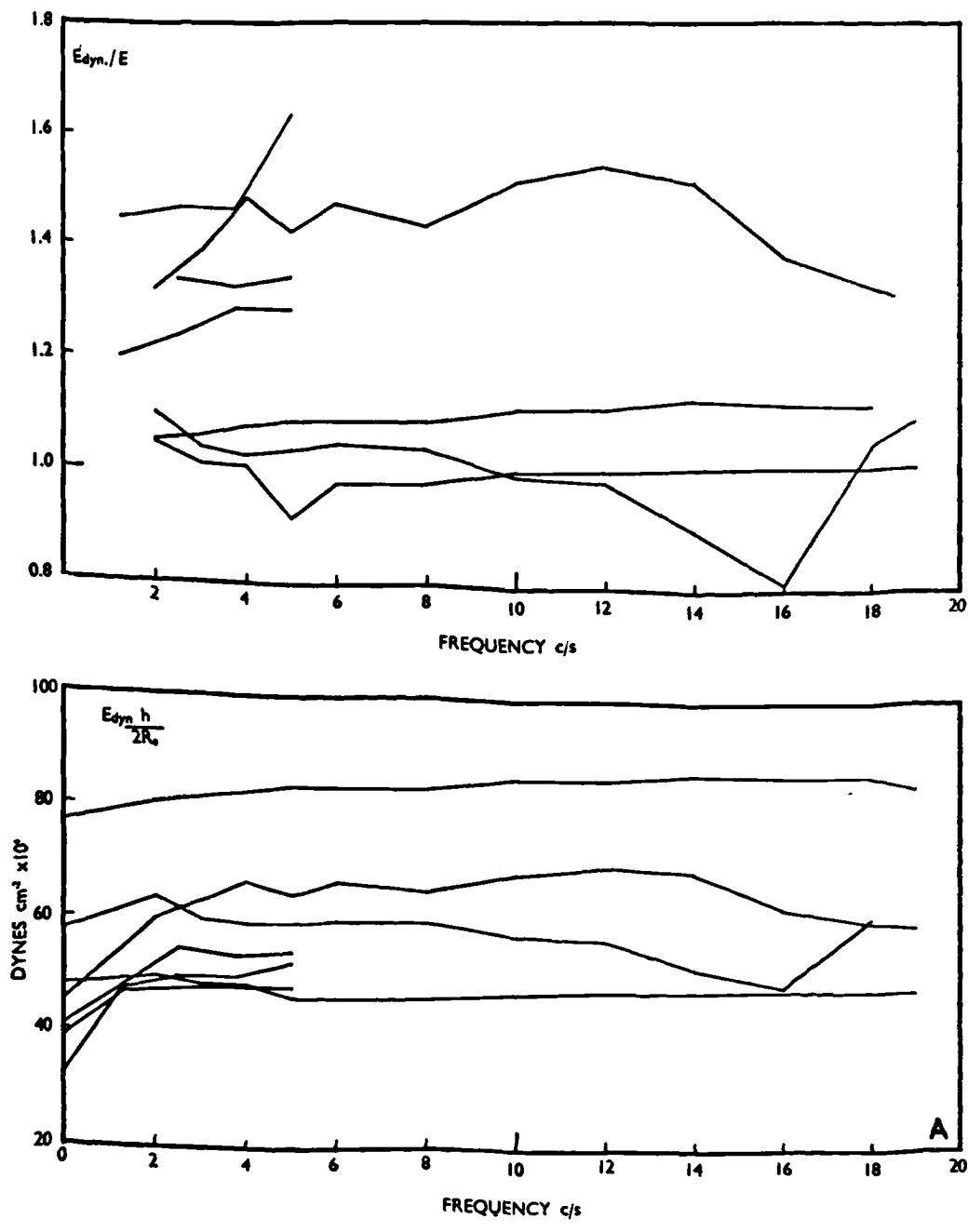


Fig. 30

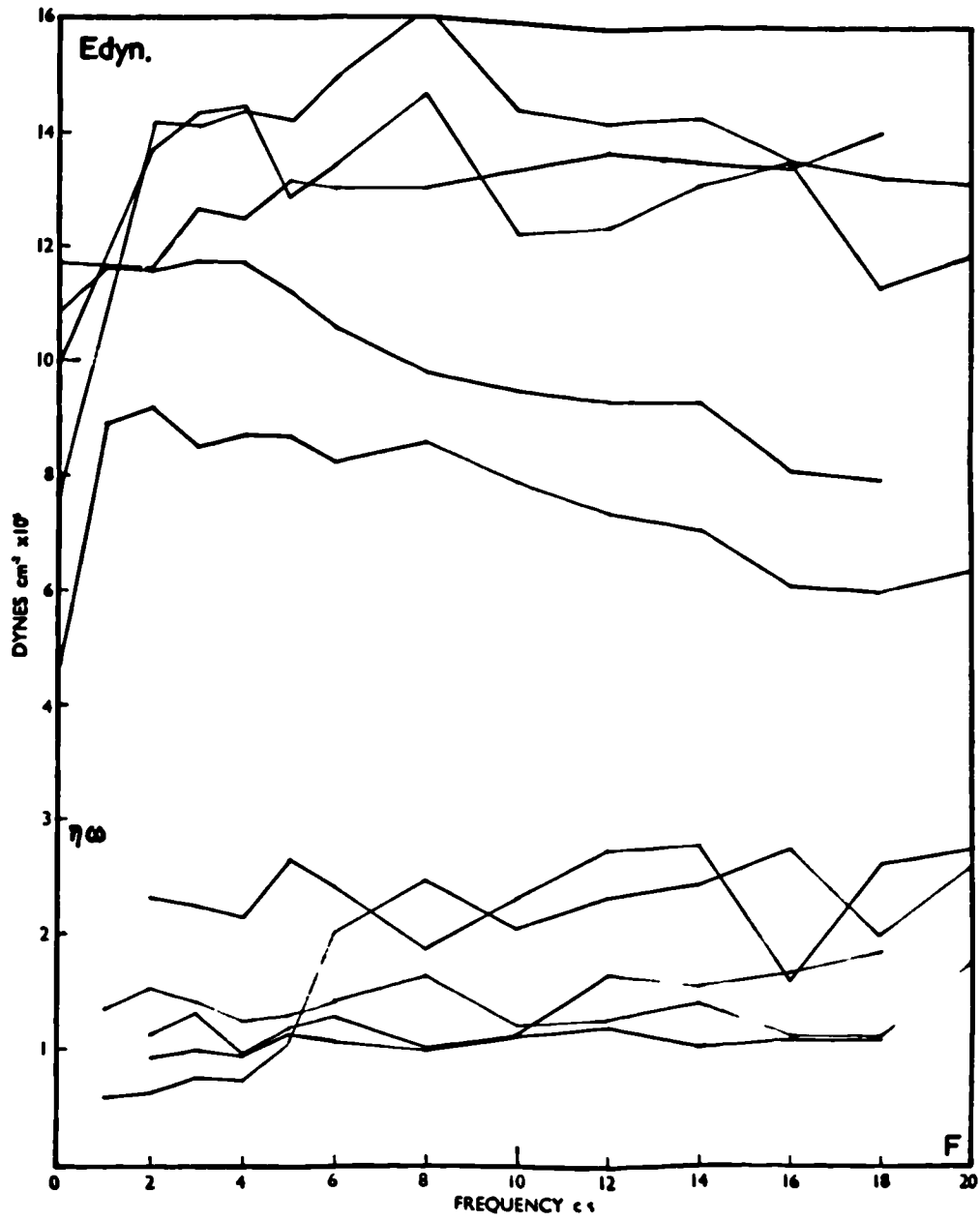


Fig. 31

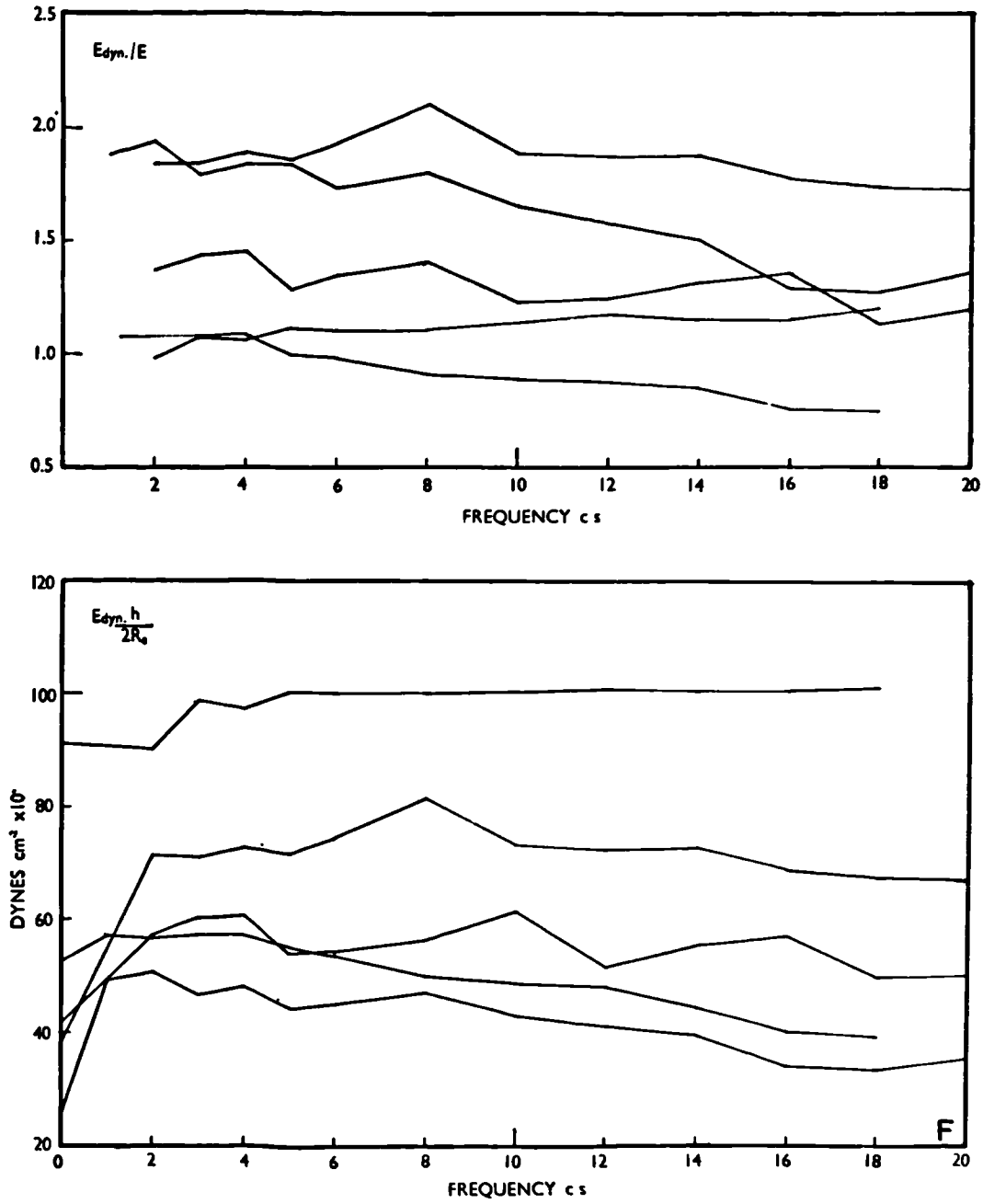


Fig. 32

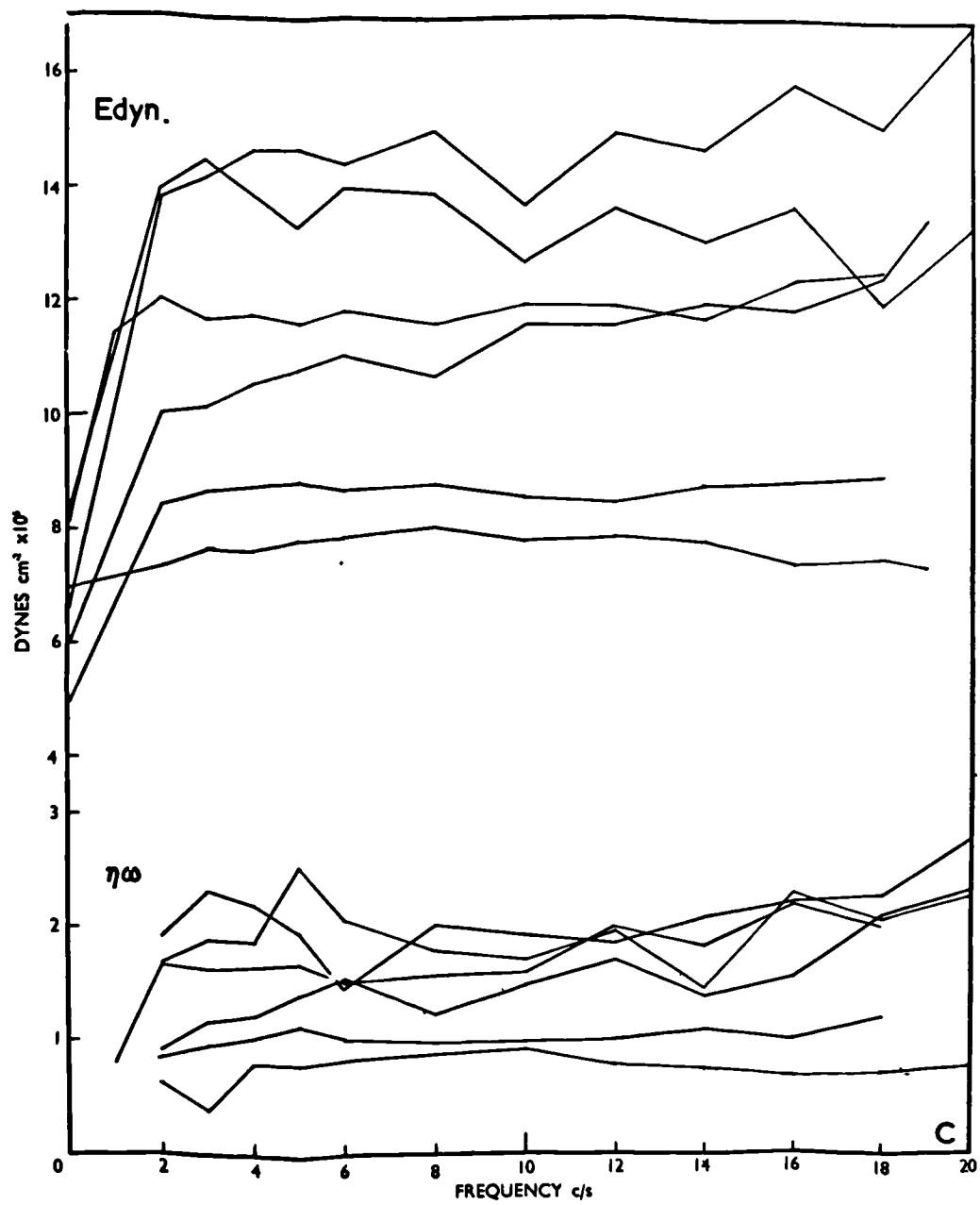


Fig. 33

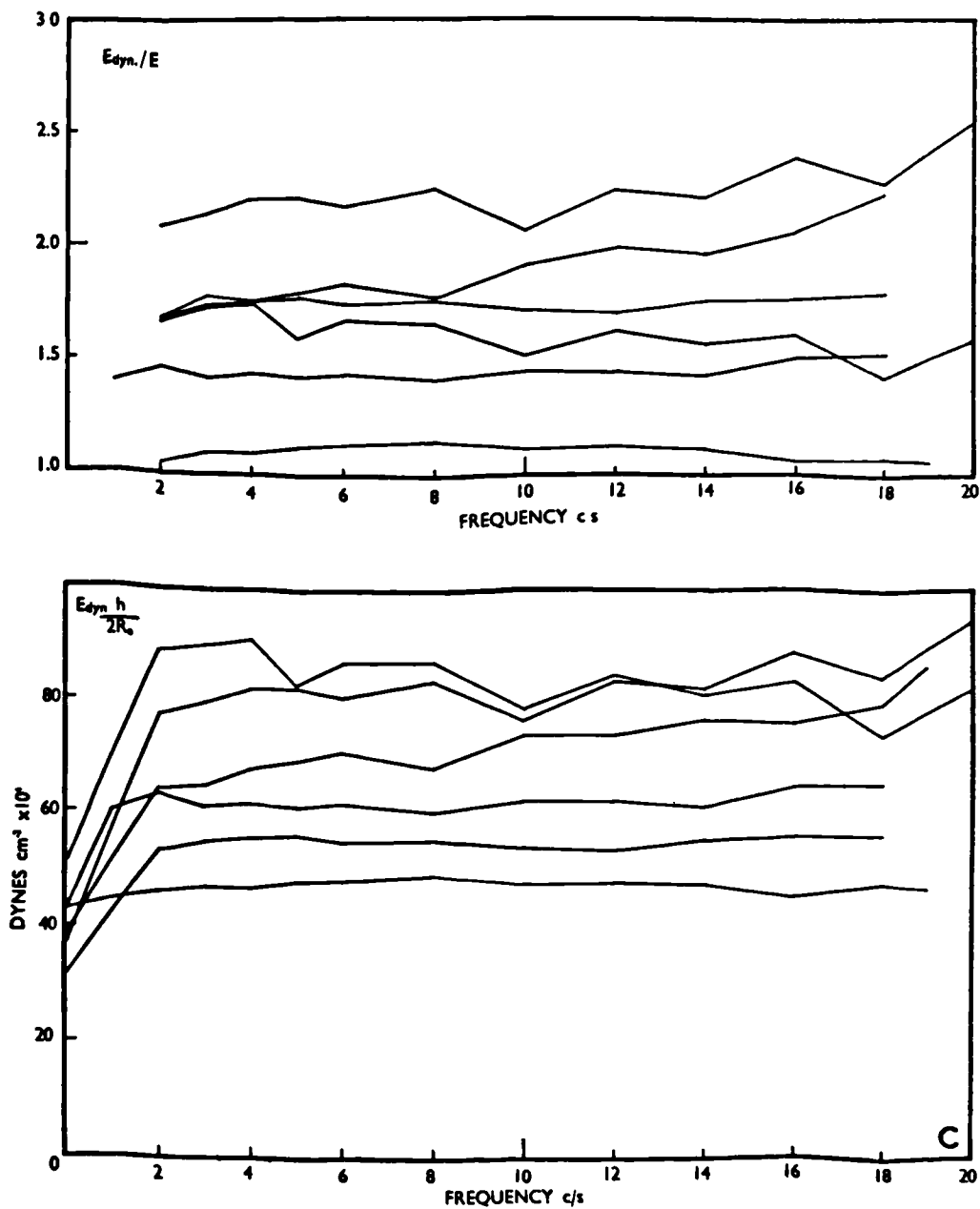


Fig. 34

FIG. 35

The mean values for the term E_{dyn} at different frequencies in four types of artery. These are identified by the symbols shown:- Squares - thoracic aorta; Triangles - abdominal aorta; Crosses - femoral artery; Circles - carotid artery.

These symbols have the same significance in all subsequent Figures. The vertical lines connecting the short horizontal bars show the S.E.M. for the thoracic aorta and the carotid artery. The errors for the abdominal and femoral vessels are not shown; they were of the same magnitude as for the carotid and there was thus no significant difference between these three types of artery.

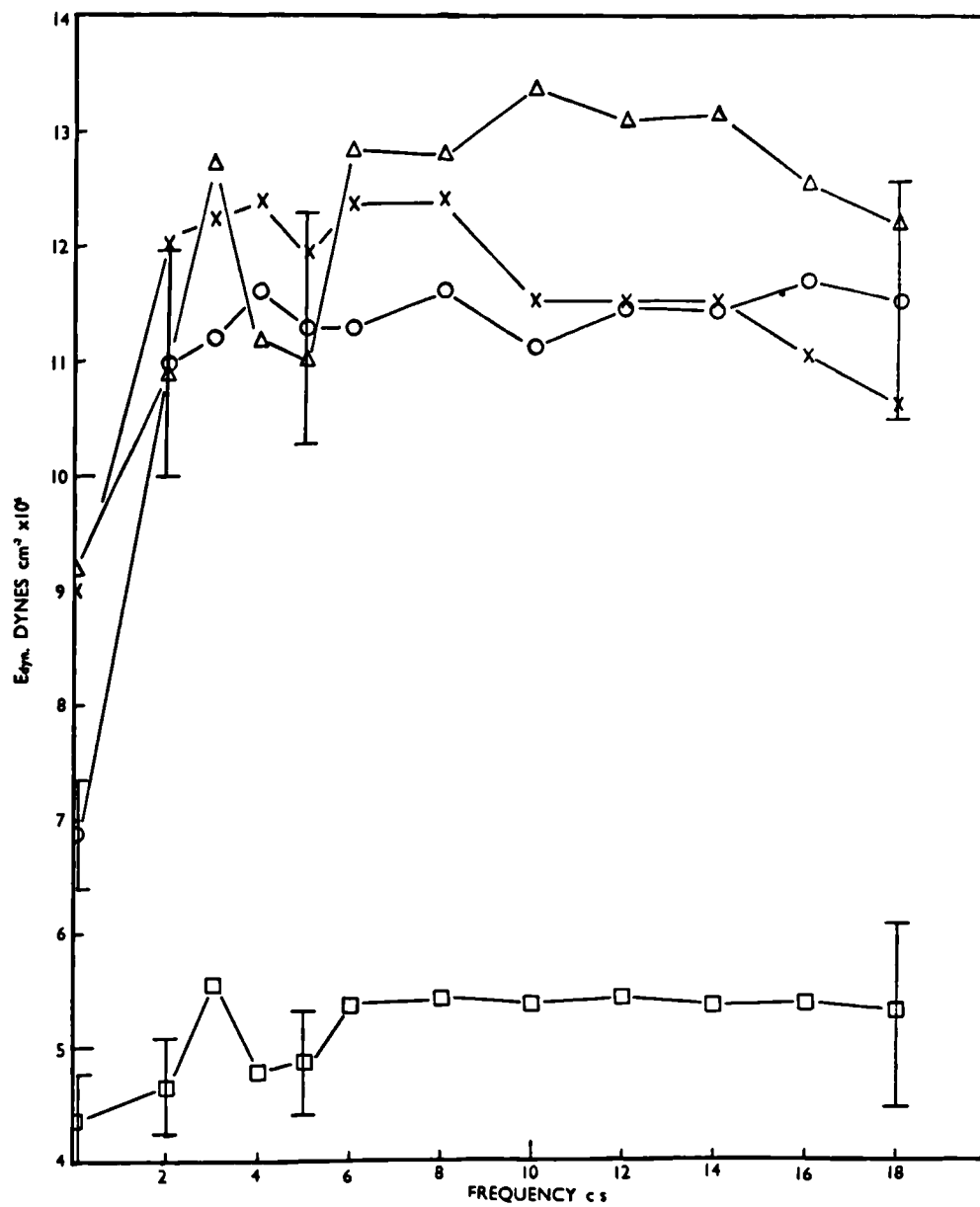


Fig. 35

FIG. 36

Mean values of the relative dynamic modulus, E_{dyn}/E , in different arteries. The standard errors are shown by the vertical lines at frequencies of 2, 5 and 18 c/s. The probability that the differences shown are due to chance are shown in the following table (Student's t test)

Difference between	Values of 'p'		
	2 c/s	5 c/s	18 c/s
Thoracic & abdominal	0.1	0.15	0.35
Abdominal & femoral	0.23	0.13	0.35
Femoral and carotid	0.1	0.1	0.05

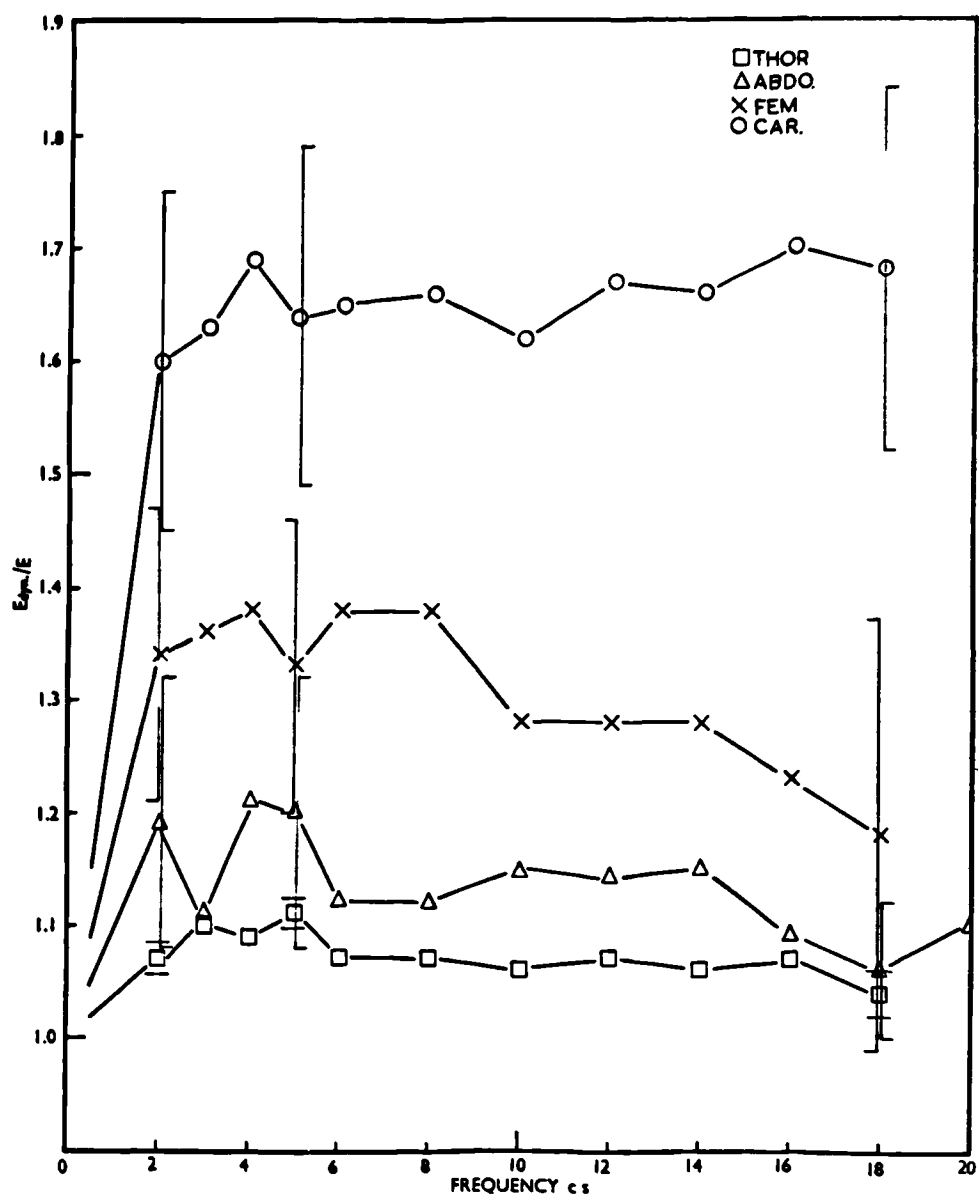


Fig. 36

FIG. 37

Mean values for the product $E_{\text{dyn}} h/2R_0$ for the four vessel types studied. This term is related to the pulse wave velocity by the Moens-Korteweg equation (Eqn. 29).

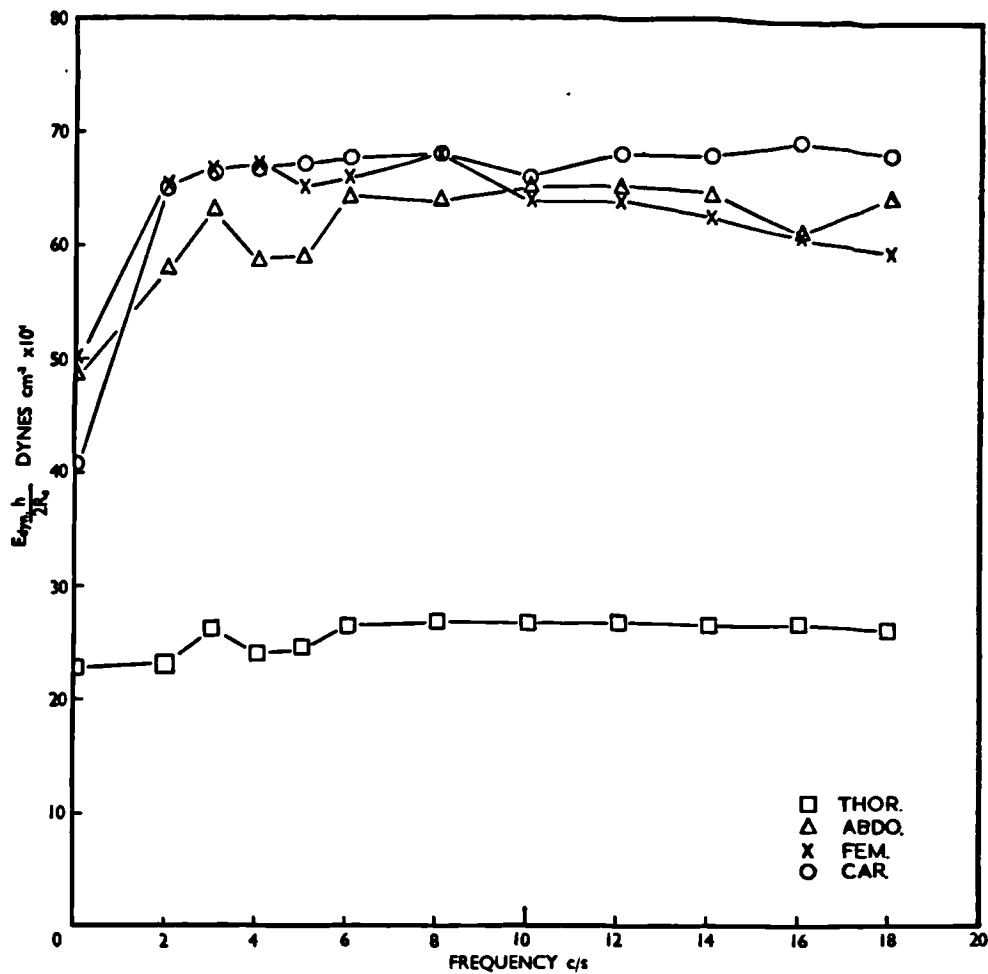


Fig. 37

It is now apparent that the vessels may be separated on the basis of the magnitude of this initial increase in stiffness. The thoracic aorta shows the least rise and the carotid the greatest, thus the order in which they lie is the same as that of the amount of smooth muscle seen in the specimens examined microscopically. This leads to the supposition that it might be the smooth muscle in the wall that is responsible for the low frequency changes. This implies that the muscle is substantially unable, by virtue of high internal viscosity, to follow stresses faster than 1-2 c/s. This would occur at frequencies whose repetition rate was of the same order as the relaxation time of this tissue.

We have seen that the relaxation time of resting smooth muscle is in fact measured in seconds and tens of seconds (Abbot & Lowy, 1957; Zatzman et al., 1954), though the behaviour is not simple and at least two exponentials are necessary to describe it.

Before this idea is pursued too enthusiastically it would be a good idea to examine the pattern of viscous behaviour and see whether or not so simple a conception as that outlined above would predict the behaviour of $\eta\omega$ that was actually seen.

In Figs. 27, 29, 31, 33 the values for $\eta\omega$ determined for the individual specimens are shown; it must be pointed out that in these figs. the scale has been expanded at the bottom to make things more clear. The behaviour of the viscous term is remarkably similar through the whole series. It will also be remembered that the pattern is not unlike that seen in the red rubber specimens: it may in fact be mentioned here that the similarity between these rubber tubes and blood vessels is considerable. rubber tubes, then, are not bad models of arteries.

In every case $\eta\omega$ rises somewhat between 2 and 5 c/s and little at higher frequencies. Between 0 and 2 c/s the value must therefore have increased from zero to a figure not much lower than that at 20 c/s. In the occasional specimen it was possible to make a reading at 1 or 1.25 c/s and it sometimes occurred that $\eta\omega$ was caught, as it were, halfway up the slope. It is however unfortunate that the method did not allow a proper exploration of the low frequency range. Since the rise in the higher frequencies was not proportional to ω ($= 2\pi f$) then the value of η would appear to be decreasing more or less hyperbolically all the time. This sort of behaviour was also shown by the rubber specimens, and any model proposed will have to take account of this.

In view of the well marked differences between the various arteries that were seen in the elastic term, these Figures show remarkably little to distinguish one from the other. There is a difference in the magnitude of $\eta\omega$ of the various vessels, but this is a reflection of the difference in magnitude of the whole modulus, $\eta\omega$ being defined as $E \cdot \sin \phi$. This factor may be eliminated by considering the actual phase angle between pressure and dilatation. Fig. 38 shows the mean phase difference for the various specimens. Above 5 c/s there is certainly nothing to separate one from the other. There is a difference however at 2 c/s but this is soon lost with increasing frequency. At the lowest rate the mean of the thoracic and abdominal aortas taken together show a smaller angle ($0.080r. \pm .016 n = 8$) than does the mean of the smaller vessels ($0.111r. \pm .011 n = 11$). To make the picture clearer the values have been calculated for the vessels in pairs, and so have the standard errors. This would seem reasonable to judge from the

FIG. 38

Upper half. Here are shown the mean values for the term $\gamma\omega$ over the frequency range used. Lower half. This Figure shows the mean values, in radians, of the phase difference between pressure and dilatation in the various arteries studied. The standard errors, shown by the vertical lines, have been computed for the 'muscular' and 'non-muscular' vessels, that is, the thoracic and abdominal aorta have been lumped together, as have the femoral and carotid artery. This was suggested by the great similarity shown at the lowest frequencies by these pairs of vessels.

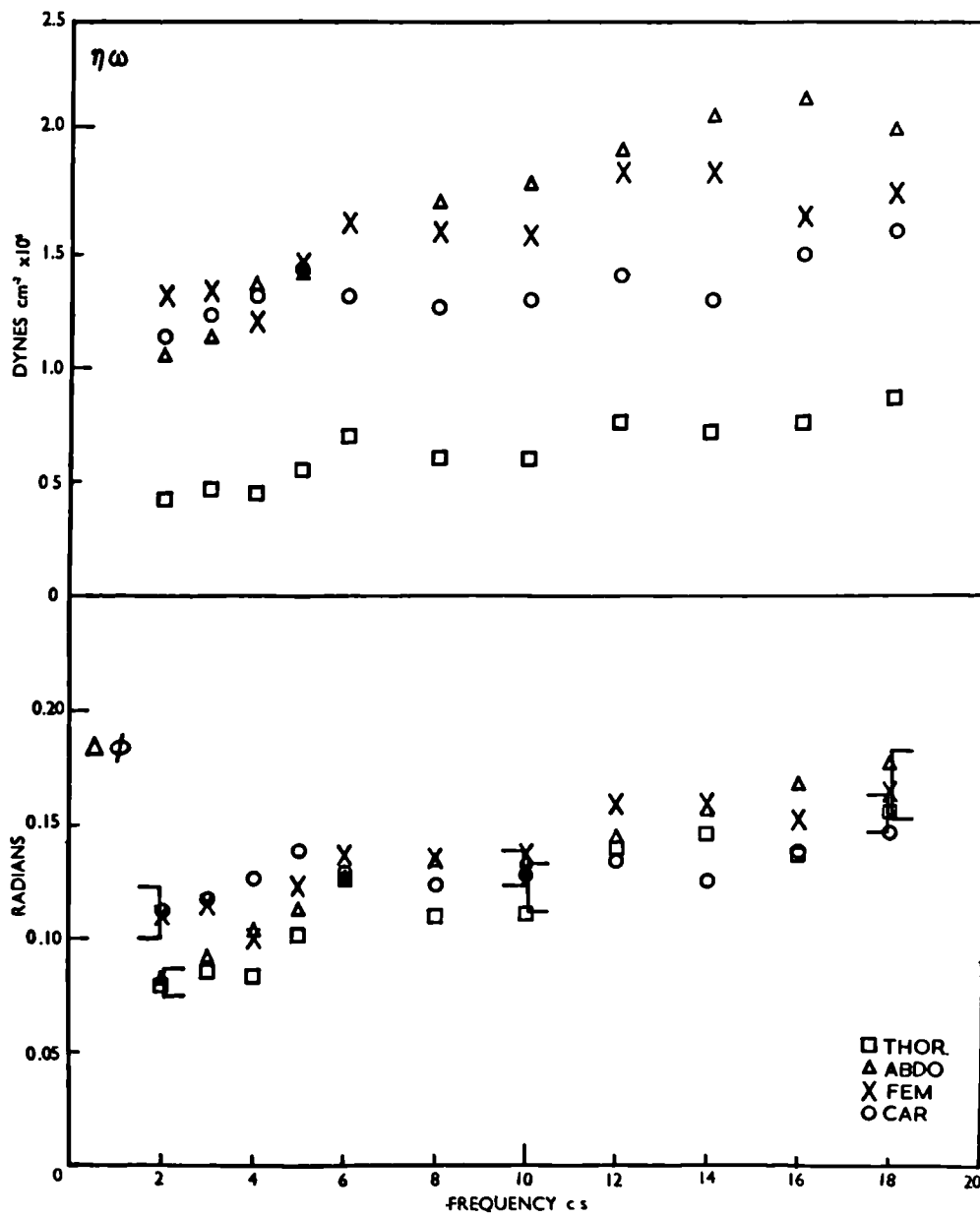


Fig. 38

pattern shown in Fig. 38. The difference between the two means is significant ($0.01 < P < 0.02$).

Another point worth mentioning here is that the angles measured were extremely small (between 5° and 10°). The ellipses seen on the oscilloscope were very narrow, and angles of this size could quite easily be missed unless the greatest care was taken. Even using a capacitance manometer the correction that had to be made for phase lag was a considerable size when compared with these angles. For instance, at 20 c/s the correction was $0.048r$, about a third of the angle to be measured. The calibration of the manometer is thus of paramount importance in this respect. Bearing in mind the difficulty of measuring angles of this size, the calibrations were satisfactory, but more precisely controllable methods of developing calibrating pressure than I have yet achieved would undoubtedly be an advantage. Even if the entire correction factor is removed it will be seen that the phase difference would still be increasing somewhat with frequency.

This point is of importance since it has been reported by Dillon *et al.* (1944) that the angle was constant in rubbers above 25 c/s, and my own results on rubber, shown in Fig. 24, indicate the same thing between 15 and 20 c/s. In this respect at least there is some difference between rubber and arteries, though it is not a very great one.

Two points which are of importance when considering the practical application of these results must be mentioned. Firstly, all the experiment described were performed at a mean pressure of 100 mm Hg. A very few determinations were made using higher and lower pressures; the results of these suggested that the increase in dynamic stiffness and the ratio

between E_{dyn} and $\eta\omega$ were not altered by changes in mean stress. Support for this view comes from Kapal (1954) who found the ratio of dynamic to static stiffness constant in aortic strips over a wide range of mean tensions. My experiments are not numerous enough definitely to confirm this, however the data given should apply in the normal blood-pressure range.

Temperature is possibly a more important factor. All my experiments were carried out at room temperature; owing to the method chosen to measure radius changes it was not possible to perform determinations at 37°C. Hardung (1953) found that the static tension in an aortic strip held at constant length increased some 10% when the temperature was raised from 20 to 40°C. The dynamic modulus, however, fell as the specimen was warmed and one experiment showed a decrease of about 25% in this temperature range.

Lawton's (1955) results indicate a much smaller change in stiffness than those of Hardung (1953), they also show for one specimen a fall in the viscous component of 47% with a change in temperature from 26 to 38°C.

Sarnoff & Berglund (1952) have also studied the effects of temperature changes on the static elasticity of the pulmonary artery and find very slight changes with a temperature change of 10°C.

It is possible that the values I have obtained for E_{dyn} would have been lower had the experiments been carried out at body temperature. It is not possible to say how large the change might be; if it were very large the predicted values for pulse-wave velocities would be too great and this does not seem to be so. It is also possible that the internal viscosity would be reduced by an increase in temperature. This matter is obviously important and ought to be investigated further.

There are few accounts in the literature relating to the phase retardation shown by animal tissues. Reichel's work (1952a, b) has already been referred to; working with resting striated muscle he found a phase lag of about 8° (0.14 r.) at frequencies around 50 c/s (these were measured from the displacement of the recorded maxima of length and tension). Although he states that the behaviour of smooth muscle was similar he does not, unfortunately, quote any measurements of phase lag.

Hardung's (1953) paper is of great interest in this respect. He quotes an experiment on a strip of ox aorta in which he found an almost linear increase of $\eta\omega$ up to 15 c/s, though there was a higher slope at low frequencies. In another experiment he measured the properties at very low frequencies; $\eta\omega$ rose sharply up to 0.5 c/s and thereafter continued to rise linearly at about half the rate. The angle he found was also greater than mine, at 15 c/s $\eta\omega$ had a value of 0.4 Edyn; thus the phase angle was $\tan^{-1} 0.4$, about 0.38 radians or rather more than twice the values I have found. Even at 2 c/s Hardung found an angle of about 0.1 r., as against my mean figure of 0.08 r., so there is a difference at all frequencies.

Kapal (1954) made no measurements of phase difference; from the original record reproduced in his paper a slight lag can just be detected, at 5 c/s this cannot exceed 10° (0.18 r.) and is probably much less.

Similarly, Lawton (1955) made no actual measurements of phase differences, but he was able to derive them from the internal damping shown by his specimens. His results show that the angle was independent of frequency over the range 3-15 c/s, though closer inspection suggests

that there was some tendency for the angles to increase very slightly with rise in frequency. These results were obtained under a wide range of mean extensions and so are not strictly comparable with my own, but they do seem to be more similar to mine than to Hardung's. The phase angle had a tangent of 0.077, i.e. $\phi = 4.5^\circ$, c. 0.08r.

There is then a difference here between my results and those of Hardung, the only other worker who actually measured phase angles. Other studies seem to support my findings, but this support is not strong. One possible cause of error which must be considered would be the presence of unrecognised inertial effects.

We have in these experiments a (partially) damped spring attached to a mass; the same situation exists for a manometer and we can expect similarities in behaviour with frequency between these two systems. The resonant frequency is given by $\omega^\circ = \sqrt{S/M}$, where S is the stiffness of the spring, and M the mass. Let us consider the thoracic aorta, the biggest and the most compliant vessel which will therefore have the lowest frequency.

The effective stiffness in this situation is P/R° , for the thoracic aorta a figure of 2×10^6 dynes/cm² seems a reasonable average. The relevant mass is difficult to assess; the arterial wall is moved and with it must go a certain amount of fluid, but the fluid movement is presumably zero at the axis and the exact equivalent mass is doubtful. Let us again take the worst possible case and assume that each square cm of vessel wall is accompanied by a mass of fluid of depth equal to the vessel radius, for which an average figure of 0.5 cm is reasonable.

We then have $\omega^\circ = (2 \times 10^6 / 0.5)^{1/2}$

$$f^0 = 2,000/2\pi$$

$$= c. 300 \text{ c/s}$$

Thus, even taking the worst possible case, we should expect an undamped natural frequency of about 300 cycles which is well above the highest frequencies used here. Whatever the degree of damping in this system it is unlikely that much change in E_{dyn} would be expected in my frequency range. The phase relationships between force and displacement are generally a more sensitive measure of behaviour in a resonant system. The exact phase behaviour will, of course, depend on the degree of damping. This could be computed, but it is enough to know that the system is underdamped, as witness the fact that Lawton was able to study the free vibrations of weighted strips. In fact the damping is very low, Lawton quotes logarithmic decrements of the order of 0.1, so that the phase lag will increase little with frequency. We might therefore expect that the worst phase error would not be greater than, say, 0.01 r. at 20 c/s. This would be a phase lag, so that my phase angles would be erroneously great by this amount. This factor cannot explain the discrepancy between my results and Hardung's because the effect is to widen the gap between us, as might be expected. The calculation, crude though it is, was performed to indicate that there seems no need to include inertial elements in any proposed model since the effect is certainly extremely small, and indeed, when considering the actual behaviour in vivo identical forces will be present. The possibility that inertial effects were responsible for the large phase angles Hardung found was eliminated, as he determined the mass of all moving parts of his apparatus and calculated a maximum error of 2% from this cause.

Another explanation must be sought. There does seem to be one possible factor. It was shown by Ranke (1934) that the internal viscosity of blood vessels was markedly increased by soaking them in hypertonic solutions. Although Hardung describes a waterbath to surround his specimens it appears that this was only used in experiments on the effects of temperature changes. If the other experiments were performed in air, as seems to have been the case, it is to be expected that they would become rather dehydrated and thus more viscous. My own specimens were filled with saline and there was always sufficient transudation of fluid to keep them moist throughout.

The close similarity in viscous behaviour between these vessels is not as surprising as it might at first seem. We have seen how the properties of a visco-elastic element are characterised by their retardation or relaxation times. These parameters describe the ratio between the viscous and elastic units in such an element, for example the relaxation time of a Maxwell unit is E/η , elastance over viscance. (These terms, elastance, compliance, etc. are convenient terms to cover the overall mechanical properties of a system without consideration of its dimensions and shapes. They were introduced to physiology by Bayliss & Robertson, 1939.) Thus the constant ratio of viscous to elastic effects seen in these specimens points to the influence of some common factor with the appropriate distribution of relaxation times. Once again the most likely material with properties of this sort would seem to be smooth muscle. One is, in fact, perpetually driven to the feeling that the properties of this tissue are of primary importance.

From the description of the experimental findings we may pass to

the consideration of possible models to interpret them. A word on the use of models is in place at this point. Two different types of models are frequently used. The first type represents an attempt to clarify knowledge of the properties and structure of a material by reference to some easily comprehended mechanical device. The units comprising such a model should then be equivalent to the various structures imagined to exist in the material studied. This then is a model in the dictionary sense of the word: we may take as an example the twin-filament model of striated muscle.

The second type includes all those to be considered here; these we may term pragmatic models. One attempts to set up some system which will have certain properties, and the units chosen for this will usually be those that can be readily described in mathematical terms. The ultimate sophistication of this form of device is an equation.

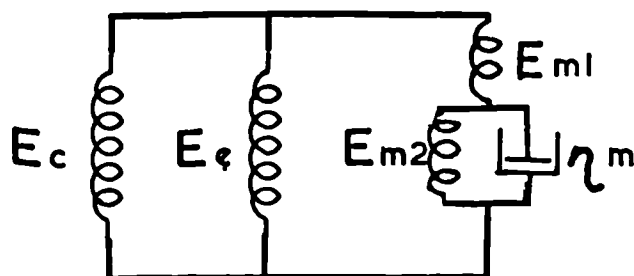
Although the first type can, and should, lead to the formulation of a mathematical expression the second is nothing more than the expression itself. This type of model carries no structural significance, for it is no more than a theoretical analogue to facilitate description.

It is conventional to discuss the mechanical properties of complex substances in terms of springs and dashpots. For mathematical simplicity these are taken to be Hookean and Newtonian in their behaviour. If a suitable arrangement can be devised which will predict the properties that have been found then no more has been done than to fit an equation: if the search is fruitless, or if the resulting model is no more comprehensible than the original data, one must either look for a different language or rest content with the results as they stand.

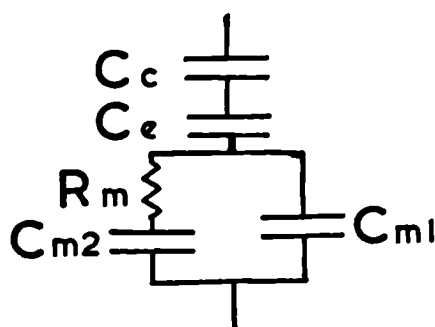
The unit commonly suggested to represent the properties of muscle is known as the Levin-Wyman model. This was proposed by these workers in 1927 to account for the behaviour shown by striated muscle allowed to shorten at controlled rates during contraction. This model in its original form consists of an undamped spring in series with a damped one. It is thus a mixture of both and Maxwell and Voigt elements. Winton's (1930, 1939) experiments suggested the addition of a further purely viscous element in series. Long term relaxation at a steady rate has been reported in smooth muscle by many other authors, but there are difficulties here since some shortening force has to be postulated to allow the specimen to eventually regain its original dimensions. It seems more reasonable to consider it in parallel with a highly compliant spring. It then becomes a constituent part of the single lumped Voigt element.

We have also seen, largely from the work of Reuterwall (1921), that elastin and collagen show poorly marked viscous behaviour and we shall for the present take these as represented by single springs. These are not of course linear, but we are dealing here with small strains and may neglect this factor.

To represent the wall of a muscular artery let us begin with the following model.



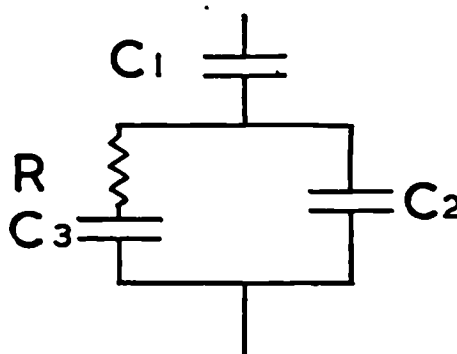
In this diagram E_c and E_e represent collagen and elastin; the unit at the right represents the parallel smooth muscle. The equivalent electrical unit is then as follows.



To simplify things we may combine the elastin and collagen capacitances into a single unit, where $1/C_1 = 1/C_e + 1/C_c \equiv E_c + E_e$.

We have the following equivalences,

$$\begin{aligned} 1/C_1 &\equiv E_c \text{ \& \ } E_e \\ 1/C_2 &\equiv E_{m1} \\ 1/C_3 &\equiv E_{m2} \\ R &\equiv \eta_m \end{aligned}$$



The impedance (real and imaginary parts) of this circuit can be expressed in terms of these elements. After a little arrangement it is found that

the following are the expressions for the 'elastic' and 'viscous' components of the complex modulus of this model

$$E_{\text{dyn}} = \frac{(c_2 + c_3)(c_1 + c_2 + c_3) + \omega^2 R^2 c_2 c_3 (c_1 c_3 + c_2 c_3)}{c_1 (c_2 + c_3)^2 + \omega^2 R^2 c_1 c_2^2 c_3^2}$$

$$\eta\omega = \frac{R\omega c_3^2}{(c_2 + c_3)^2 + \omega^2 R^2 c_2^2 c_3^2}$$

when $\omega = 0$, $\eta\omega = 0$ and $E_{\text{dyn}} = \frac{c_1 + c_2 + c_3}{c_1 (c_2 + c_3)}$ This is as would

be expected. When ω becomes infinite $\eta\omega$ is again zero and E_{dyn} reduces to that of the two undamped compliances, that is $c_1 + c_2 / c_1 c_2$

At intermediate frequencies the behaviour will depend on the values chosen, but the trend can be seen. E_{dyn} will rise towards its final value and $\eta\omega$ will increase only to fall off rapidly once a maximum is reached. In fact the second trace will have the form of the differential of the first. These expressions have been computed by Ranke (1934). It is at once obvious that this is not the pattern seen in the results which have been given.

Multiplying the number of damped elastic elements and producing a wide spectrum will not alter the form of the solution. An infinitely wide spectrum will result in a constant phase shift at all frequencies which would account for the viscous behaviour. On the other hand it would predict a continuously rising elastic term.

Only by building in some rather bizarre properties into the dashpots can this sort of model be made to fit the bill. A form of thixotropic behaviour is needed whereby the viscosity diminishes with frequency so

that the relaxation time shortens as the rate of loading is increased. Under these conditions the point at which the viscous element fails to respond will never be reached and by a suitable choice of elements something suitable could, it seems, be reached. Bearing in mind the purely pragmatic nature of this model such a scheme has little point as it would be no easier to comprehend than the original results.

No account is taken in these equations of inertial effects. It was shown earlier that it seems unlikely that these should be important here, but they must be considered. Assuming that we have an underdamped system (as Lawton, 1955, found) one would expect the stiffness to be lowered as the resonant frequency was approached, while at the same time the phase lag would be increased. If we suppose that at 5 c/s the critical frequency of smooth muscle has been passed then in the absence of inertial effects E_{dyn} would be virtually constant and η would diminish as the frequency of pumping was raised. If inertia was important then the phase lag might diminish less or even increase and E_{dyn} would decrease. .

This is not entirely unlike what was observed, though there is one important point against it. These resonant effects would be most marked for the vessel with the greatest mass and compliance. There is nothing in my results to show that the thoracic aorta was behaving in this way.

It would be possible to construct a model which possessed inertia, but the computations would be difficult. Two factors suggest that it would not be worth doing. Firstly these vessels are filled with blood in life, and if inertia is important in the experiments then similar

effects will be operating in vivo. These results, it is believed, represent to some extent the dynamic properties of the arterial system from whatever mechanisms they spring.

The stress-relaxation experiments of Zatzman et al (1954) provide a second reason for believing mass effects unimportant. It is possible to compute from their results the dynamic behaviour at the frequencies I have used. The necessary Fourier transforms were obtained through the courtesy of Mr. S. Michaelson of the Department of Mathematics, Imperial College, London, using the Pegasus computer. It was not possible to obtain enough points from the graphs published by Zatzman et al. (something like 100 would be needed) to give precise results, and the terms tend to oscillate, especially $\eta\omega$ which goes negative at times. However the figures produced are not grossly dissimilar to my own. In particular a sudden increase in stiffness is predicted below 2 c/s and the increase is about 150% of the static figure. It will be seen in Fig. 34 that one carotid specimen showed an increase of 120%. At higher frequencies there is possibly a tendency for a slight further increase. The $\eta\omega$ term oscillated about zero with no tendency upwards with increasing frequency, when it had a positive value this was about one tenth of E_{dyn} .

Although there was not really enough data to go on, this pattern is not too far from that shown in my experiments. Presumably inertial effects may be entirely discounted in these stress-relaxation experiments and thus this computation, as far as it goes, does not suggest that any new factor was at work in my experiments.

In conclusion then the results obtained, which have been shown, do not appear to be compatible with any reasonably simple model of a sort that bears some resemblance to the elements of the arterial wall. Until a great deal more is known on the dynamic properties of these elements there seems little point in devising highly complex models. It is perhaps worth mentioning that, as far as I know, no satisfactory model has been postulated by the rubber technologists for the dynamic properties of high polymers, which are much the same as those of these arteries..

The predicted pulse-wave velocity

The generally quoted equation for the wave velocity in an elastic tube, the Moens-Korteweg formula, relates the velocity (c) to the thickness (h) and Young's modulus (E) of the tube wall and to the density (ρ) of the contained fluid (but neglects its viscosity), and has the form

$$c = \sqrt{Eh/2R\rho} \quad \dots\dots(29)$$

(see Lambossy (1950) for an account of the development of this equation)

This expression may be derived from Newton's formula for the velocity of sound, as was briefly indicated by Thomas Young himself (1821, see Lambossy, 1951). Newton's formula is $c = \sqrt{E/\rho}$, where c is the velocity, E the coefficient of volume compressibility, and ρ the density. When we have to consider the behaviour of a fluid contained in a tube the bulk modulus of that fluid may be considered to be altered by the behaviour of the wall, and E is replaced by $Eh/2R$. Thus the tube wall is considered only as the boundary of the fluid and no account is taken of its other properties, in particular its Poisson's ratio; h ,

the thickness of the wall is also considered to be very small compared with R , the mean radius.

The Moens-Korteweg formula then relates to the transmission of pressure waves in an incompressible non-viscous fluid contained in a thin-walled tube. The radial movement of the fluid is neglected, and the relation only applies if the circumference of the tube is small compared with the wave-length (Lambossy, 1951). We must consider to what extent these simplifications are justified and what corrections must be applied to the simple equation to cover the situation in the body.

Firstly no fluid is absolutely incompressible, and to take account of this case the formula must be modified as follows (Lambossy, 1951),

$$c = \sqrt{\frac{1}{\frac{\rho_1}{K} + \frac{2R\rho_1}{Eh}}} \quad \dots\dots(30)$$

where ρ_1 and K are the density and bulk modulus of the fluid. If we take K to be 2×10^{10} dynes/cm² (Newman & Searle, 1948) we can see that there is very little error in neglecting this factor when the modulus of the wall is of the order of 10^6 - 10^7 .

The error in a tube of finite thickness and whose wall has a Poisson's ratio is more considerable. For convenience the wall thickness has been taken as the ratio h/R_0 , the external radius being the simplest to measure.

From Eqns. (11), (12), (13) the relation between internal radius and pressure may be obtained.

$$\Delta R_1 = \frac{\Delta P}{E} \left[\frac{R_1^3 (1+\sigma)(1-2\sigma) + R_0^3 R_1 (1+\sigma)}{R_0^3 - R_1^3} \right]$$

Now, providing that the length of the specimen does not alter, the relation between radius and volume is

$$\frac{\Delta V}{V} = \frac{2 \Delta R_1}{R_1}, \text{ for small radius changes.}$$

The pulse-wave velocity calculated from Eqn. (24) by substituting R_0 for R is designated c_0 . The velocity may also be calculated from the equation

$$C = \sqrt{V \cdot \Delta P / \Delta V \rho} \quad \dots (31)$$

which is more truly analogous to the Newton formula and was shown by Frank (1920) and Bramwell & Hill (1923) to be identical with Eqn. (29) when the wall is very thin. This velocity is indicated by C , and the relation between C and c_0 is shown below.

Rewriting Eqn. (31) in terms of E and R (but omitting ρ which appears in both equations and will be eliminated) we have

$$C^2 = \frac{E (R_0^2 - R_1^2)}{R_1^3 (1+\sigma)(1-2\sigma) + R_0^2 R_1 (1+\sigma)} \frac{R_1}{2}$$

Let $\gamma = h/R_0$, then $R_1 = R_0(1 - \gamma)$ and

$$\begin{aligned} C^2 &= \frac{E R_0^2 [1 - (1-\gamma)^2] R_0 (1-\gamma)}{2 [R_0^3 (1-\gamma)^3 (1+\sigma)(1-2\sigma) + (1-\gamma)(1+\sigma) R_0^3]} \\ &= \frac{E [1 - (1-\gamma)^2] (1-\gamma)}{2 [(1-\gamma)^3 (1+\sigma)(1-2\sigma) + (1-\gamma)(1+\sigma)]} \end{aligned}$$

Now, $C_0^2 = \frac{Eh}{2R_0} = \frac{E\gamma R_0}{2R_0}$ (again omitting ρ),

$$\text{Thus } \frac{C^2}{C_0^2} = \frac{E [1 - (1-\gamma)^2] (1-\gamma)}{2 [(1-\gamma)^3 (1+\sigma)(1-2\sigma) + (1-\gamma)(1+\sigma)]} \frac{2}{E\gamma}$$

which after a little rearranging becomes

$$\left(\frac{c}{c_0}\right)^2 = \frac{2 - \gamma}{2 - 2\gamma(1 - \sigma - 2\sigma^2) + \gamma^2(1 - \sigma + 2\sigma^2) - 2\sigma^2} \dots\dots(32)$$

Eqn. (32) is interesting, it shows that the estimated pulse-wave velocity will be too small, by a factor of $\sqrt{4/3}$ even when the wall is vanishingly thin, if σ is taken to be 0.5. Only when Poisson's ratio is put at zero does the discrepancy disappear, that is when the wall serves only to contain the fluid and is not itself changed on dilatation. Similarly with an infinitely thick wall, $\gamma = 1$, the correction becomes $\sqrt{2/3}$. Poisson's ratio appears as an important factor when considering tubes of arterial dimensions, because it is entirely neglected in the derivation of the Moens equation; the wall was imagined to be displaced outwards with no change in thickness, i.e. $\sigma = 0$.

For the dogs' vessels studied the ratio h/R_0 ($= \gamma$) has been discussed earlier. Using these values and taking σ as 0.5 the following are the corrections which should be applied to the pulse wave velocities calculated by the Moens-Korteweg formula (29)

Aorta ($\gamma = 0.105$)	$(1.895/1.5)^{\frac{1}{2}} = 1.124$
Femoral artery ($\gamma = 0.115$)	$(1.885/1.5)^{\frac{1}{2}} = 1.121$
Carotid artery ($\gamma = 0.132$)	$(1.868/1.5)^{\frac{1}{2}} = 1.116$

A further possible cause of error lies in neglecting the dilatation of the vessel. In fact there is a radial component of flow and hence also some inertial component due to the wall motion. The dilatation has been found to be small, 2-3% of the vessel radius (Rushmer, 1955; Lawton & Green, 1956) and my own figures would support this; for a pulse pressure

of 40 mm Hg I would expect a radial strain of 4% in the aorta and about half that in the small vessels. For this situation Womersley (1955) has calculated the necessary correction and finds it small, amounting to a few percent. The inertial terms are also small (Womersley, 1957b) and may be neglected. In any case such inertial forces there may be here are presumably present in the measurements I have made. This is one reason why these terms have not been extracted from my results; they would have to be replaced in the wave velocity equations, and it seems also that such effects are not large.

Next the influence of the steady blood-stream on the wave propagation needs consideration. The interaction between oscillatory and steady pressure gradients is complex and each influences the other. In the arterial system where $\bar{V} \ll c_0$ (\bar{V} is the mean steady flow) the oscillatory and steady terms sum algebraically (Womersley, 1957a); the wave velocity will thus be increased by the mean steady flow rate. This velocity is of the order of 15 cm/sec in the dogs' vessels (McDonald, 1960), so that the pulse wave velocity will be increased by some 1-3%.

The effect of wall anisotropy must also be considered. Lambossy & Müller (1954) showed that the apparent circumferential Young's modulus would be a function both of the true modulus and of that for the radial direction, but they also showed that the longitudinal modulus had no effect providing that no change in length occurred. This is the situation in the present experiments and also, as we shall see, in the body. Thus the effective modulus, which is measured here, is 'contaminated' only by the radial stiffness. There is nothing in the literature to suggest how big this radial modulus might be.

The fluid viscosity can, however, exert a great effect on the wave velocity. The analysis of the behaviour of a viscous fluid in an elastic tube has been undertaken by several authors with essentially the same results (see for example the discussion of this matter by McDonald, 1960); one study is due to the late J.R. Womersley and his treatment will be followed.

His first approach (Womersley, 1955) dealt with the behaviour of a thin walled elastic tube and gives equations containing the parameters K ($K = h/R$), σ (Poisson's ratio) and α .

α is a non-dimensional parameter determining kinematic similarity and it is defined as follows,

$$\alpha = R \sqrt{\omega/\nu} \quad \dots\dots(33)$$

where R is the vessel radius, ω is the 'circular' frequency and ν is the kinematic viscosity (μ/ρ). In the case of blood ν is taken as $0.04P/1.05 = 0.039$ stokes.

Womersley's equations allow one to predict the longitudinal motion of the wall that will result from the viscous drag in a perfectly elastic tube. This motion should be considerably greater in magnitude than the dilatation. It has been observed, however, by McDonald (1960) and by Lawton & Green (1956) that any longitudinal motion that occurs is extremely small (1% or less was found by the second authors). This led to the idea that the arterial wall was under longitudinal restraint and Womersley (1957a, b) analysed the propagation of waves in an elastic tube which was not free to move longitudinally.

This restraint may be due to both wall-properties and external causes. External restraint will result from the tethering action of branches and the surrounding connective tissue. After cutting across an artery full retraction is not seen until the connective tissue has been severed. Nevertheless some shortening does occur and I am inclined to think that the wall properties may be more important. This is suggested by the fact that the studies quoted above were done on vessels which had been to some extent dissected out to prepare them for photography and the longitudinal motion was still very small. The evidence considered in a previous section shows that the longitudinal modulus of the arterial wall at 'in situ' length is very high, which will act as a longitudinal constraint. Furthermore the wall is very non-linear in its elastic properties in the longitudinal direction (McDonald, 1960). Thus there will be considerable resistance to movement to any small segment since the viscous forces will always be pulling on a strong spring while slackening off a weaker one.

The experiments reported by Taylor (1959) on a rubber tube seem to bear this out. The results he obtained for wave transmission fitted very closely Womersley's equations for the tethered tube, yet his tube was merely restrained at the end and lay on a series of matchsticks lain across a trough. The restraint in these experiments must have been very largely due to some internal influence, though Taylor considered that the wall viscosity was the important factor.

With the constrained tube (Womersley, 1957b) the term K , which originally represented the relative wall thickness, was redefined to

include any possible mass-loading and the longitudinal restraint

$$K = \left(1 + \frac{h_1 R_1 \rho_1}{h R \rho} \right) \left(1 - \frac{m^2}{\omega^2} \right)$$

In this expression h_1 , R_1 and ρ_1 are the thickness, radius and density of the mass loading the tube, while the symbols without subscripts refer to the wall itself and the fluid as before. The loading mass develops no elastic tension, m is the natural frequency of the longitudinal constraint, so that when $m > \omega$, K becomes negative, and with extreme restraint $K \rightarrow -\infty$. When $m = \omega$ the wall is in a state of resonance and behaves as if it had no mass, K becomes zero and the equations of wave motion reduce to the Moens-Korteweg formula.

Tables showing the effect of changes in the value of K and have been prepared by Womersley (1957a).

In this treatment the wave velocity is complex,

$$\frac{c_0}{c} = X - iY$$

where c_0 is the velocity predicted by the Moens-Korteweg equation.

The phase velocity, c_1 , the velocity of a single frequency component, and the damping are determined by the real and imaginary parts of c , so that

$$\frac{c_1}{c_0} = \frac{1}{X}$$

and the attenuation per wave length is given by

$$\left[1 - \exp(-2\pi Y/X) \right]$$

These two functions are plotted in Figs. 39 and 40. It can be seen that as α increases ($\alpha \propto \sqrt{\omega}$ for any given tube) the phase velocity increases, when $K = 0$ the asymptotic value for $c_1/c_0 = 1.0$, however when

FIG. 39

This Figure and the following one have been redrawn from Womersley (1957b), and have been kindly provided by Dr. D.A. McDonald.

Fig. 39 shows the variation of phase velocity, c_1 (expressed as a ratio of c_0 , the velocity predicted by the Moens-Korteweg equation) with α for three different degrees of wall constraint. $K = 0$ indicates no constraint. When $K = -\infty$ the wall constraint is infinite; $K = -2$ indicates a moderately stiff constraint. See the text for a fuller explanation.

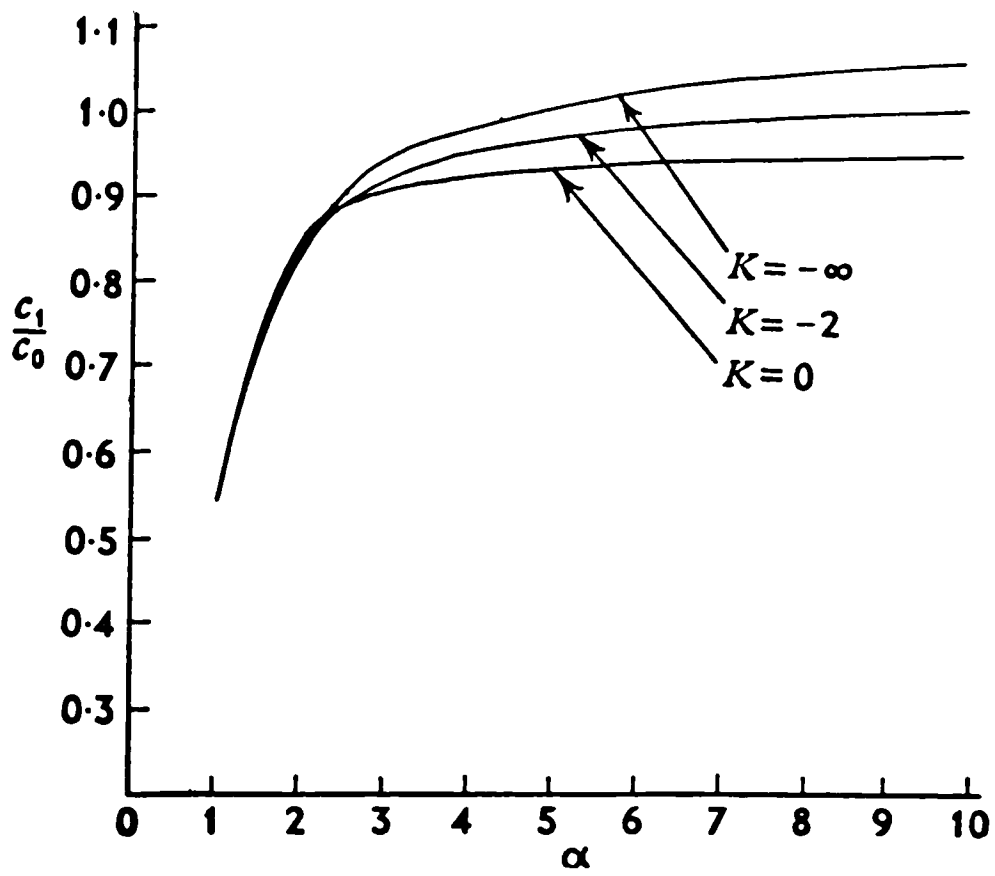


Fig. 39

FIG. 40

This Figure shows the variation of wave transmission per wavelength (expressed as the fraction of the wave amplitude which is transmitted) with change in α . Curves are shown for the same three degrees of wall constraint as were illustrated in Fig. 39.

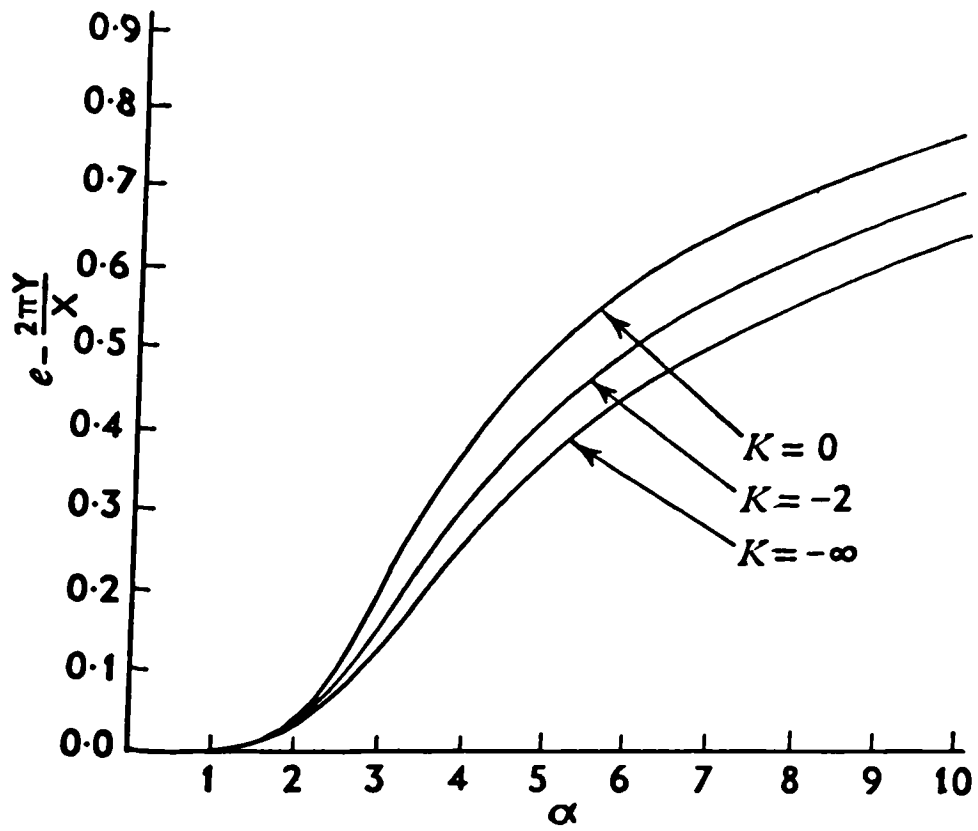


Fig. 40

$K = -\infty$ the value becomes $1/\sqrt{1-\sigma^2}$ (1.16 when $\sigma = 0.5$).

Fig. 40 plots the transmission of the wave and shows that the damping per wave-length decreases with increasing frequency (but as the wave-length falls with frequency the damping per unit length increases).

These figures predict the wave propagation characteristics of an elastic tube, but no account is taken of the visco-elastic properties of the wall material. In this situation the Young's modulus and Poisson's ratio become complex and Womersley (1957a) writes,

$$E_c = E(1 + i\omega\Delta E)$$

and

$$\sigma_c = \sigma(1 + i\omega\Delta\sigma)$$

At first glance the idea of a complex Poisson's ratio seems strange. It will be recalled that the various elastic constants are inter-related and that if one of these is to be treated as complex then they must all be. Eqn. 1 gives the relation between E , σ and K , the bulk modulus,

$$K = \frac{E}{3(1-2\sigma)}$$

$$\sigma = 0.5 - \frac{E}{6K}$$

While the bulk modulus may be to some extent influenced by frequency there is no evidence to suggest that the effects are great. Assuming then that K is all real we can relate ΔE and $\Delta\sigma$

$$\Delta\sigma = - \frac{\Delta E}{6K}$$

As the bulk modulus of water is 2×10^{10} and the Young's modulus of arterial wall in the region of 1×10^7 , it will be seen that $\Delta\sigma$ will be something like 10^4 times smaller than ΔE and it will therefore be neglected.

With complete longitudinal tethering and ΔE and $\Delta\sigma$ small Womersley gives

$$\frac{c_0}{c} = \left(\frac{1 - \sigma^2}{1 - F_{10}} \right)^{1/2} \left[1 - i\omega \left(\frac{\Delta E}{2} + \frac{\sigma^2}{1 - \sigma^2} \Delta\sigma \right) \right]$$

When $\sigma = 0.5$ this reduces to

$$\frac{c_0}{c} = \left(\frac{0.75}{1 - F_{10}} \right)^{1/2} \left[1 - i\omega \left(\frac{\Delta E}{2} + \frac{\Delta\sigma}{3} \right) \right]$$

The term $1 - F_{10}$ is a function of α and values for modulus and phase are given by Womersley (1957a).

The expression in square brackets relates to the visco-elasticity of the wall. As $c/c_0 = X - iY$ by definition we may rewrite this expression (Taylor, 1959), as

$$c_0/c = (X - iY)(1 - i\omega W)$$

where $W = \Delta E/2 + \Delta\sigma/3$

It was seen that $\Delta\sigma$ may be neglected. The complex Young's modulus used in my experiments was written $E = E_{\text{dyn}} + i\eta\omega$ (Eqn. 26a). Compared with the notation of Womersley and Taylor above it can be seen that

$$\omega \Delta E = \frac{\eta\omega}{E_{\text{dyn}}}$$

and

$$\omega W = \frac{\eta\omega}{2E_{\text{dyn}}}$$

In the presence of wall viscosity Taylor gives the phase velocity as

$$\frac{c_1}{c_0} = \frac{1}{\left[X \left(1 - \frac{X}{Y} \omega W \right) \right]}$$

and it is therefore greater in the visco-elastic than in the non-viscous tube. The transmission per wave-length is

$$\text{exp. } \left[-2\pi \frac{Y}{X} \left\{ \left(1 + \frac{X}{Y} \omega W\right) / \left(1 - \frac{Y}{X} \omega W\right) \right\} \right]$$

this will thus be decreased by wall viscosity, i.e. the damping is increased.

X and Y are functions of α and are tabulated by Womersley (1957a) for values of α up to 10. When $\alpha > 10$ they may be derived from the expressions that he gives.

The reason why the damping should be related to α and not merely to the frequency is not at once clear. However the damping is imposed by the interaction of the wall with the fluid and will depend on the form of the fluid motion and this, in its turn, depends on α .

As α becomes large Y/X becomes small and thus the phase velocity is raised little above that for the non-viscous tube (this, it will be recalled, approaches $c_0 \cdot 2/\sqrt{3}$); thus the wall viscosity makes little difference unless α is small.

The reverse is the case with the damping since the factor X/Y appears here and this becomes very large at high α 's. The influence of the wall viscosity on damping was studied by Taylor (1959), who used a rubber tube and found that the transmission was considerably reduced when α was high (> 5).

We are now in a position to predict the phase velocity and the damping in the vessels studied. The calculations involve the Moens-Korteweg values for c_0 , modified for finite wall thickness and Poisson's ratio. The ratio between phase velocity, c_1 , and c_0 depends on α and is

obtained from Womersley's tables, employing the equations involving the complex Young's modulus. The results of these computations are shown in Figs. 41 and 42. To derive the values of α the following mean internal radii were used; thoracic aorta 0.60 cm, abdominal aorta 0.45 cm, femoral artery 0.25 cm, and carotid artery 0.20 cm. Phase velocity and damping were calculated for four frequencies at which the following values of α were obtained.

Frequency c/s	Thoracic aorta	Abdominal aorta	Femoral artery	Carotid artery
2	10.9	8.2	4.6	3.6
5	17.2	12.9	7.2	5.7
12	26.7	20.1	11.1	8.9
18	32.7	24.6	13.6	10.9

The density of blood is taken as 1.05 g/cm^3 , and its viscosity as 0.04 Poises.

It can be seen (Fig. 41, upper part) that the effect of wall viscosity on phase velocity is predominantly due to the increase in stiffness below 2 c/s. At higher frequencies the fluid viscosity imposes a small but steady rise which is increased but little by the wall viscosity.

The amount of damping-per wavelength is considerably increased by the wall viscosity as can be seen in Fig. 42. Fig. 41 (lower half) shows the transmission to be expected over a distance of ten centimetres. The attenuation at the higher frequencies is surprisingly great and especially so in the thoracic aorta. This is a result of two factors, the high values of α in a large vessel and the comparatively short wave-length

FIG. 41

The upper half shows the mean values for the phase velocity at different frequencies in the four vessel types. The lines have been drawn through points computed for zero, 2, 5, 12 and 18 c/s. The lower part of this Figure shows the similarly computed wave transmission over a distance of 10 cm. These values have been calculated on the basis of the mean values of the complex elastic modulus found experimentally, using the equations given in the text. The letters at the end of each curve indicate the type of artery concerned.

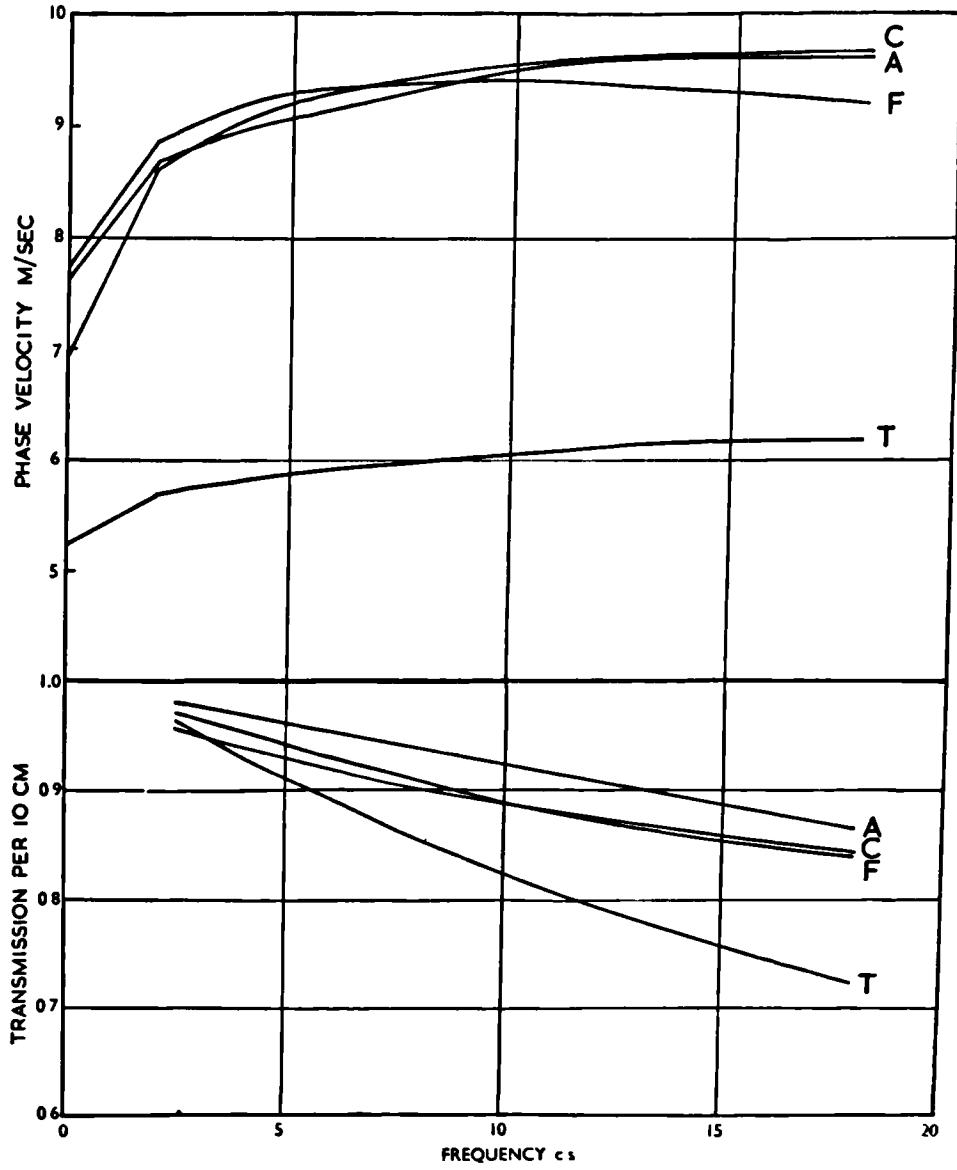


Fig. 41

FIG. 42

This Figure shows the influence of wall viscosity on wave-transmission. The thick line shows the transmission per wave-length in an infinitely tethered and loaded tube with variation in α , as given by Womersley (1957b). Also shown are the transmission expected from my results for the four types of artery, assuming infinite constraint. The calculations are explained in the text. Note how, on account of the difference in size of these vessels, the same frequency range (2-18 c/s) covers different values of α for each artery. It will also be noted that the predicted transmission per wave-length is greatest for the thoracic aorta, while the transmission per 10 cm is the least for this vessel (see the preceding Figure). This is because the phase velocity is relatively low in this artery and hence a distance of 10 cm represents a much greater fraction of a wave length.

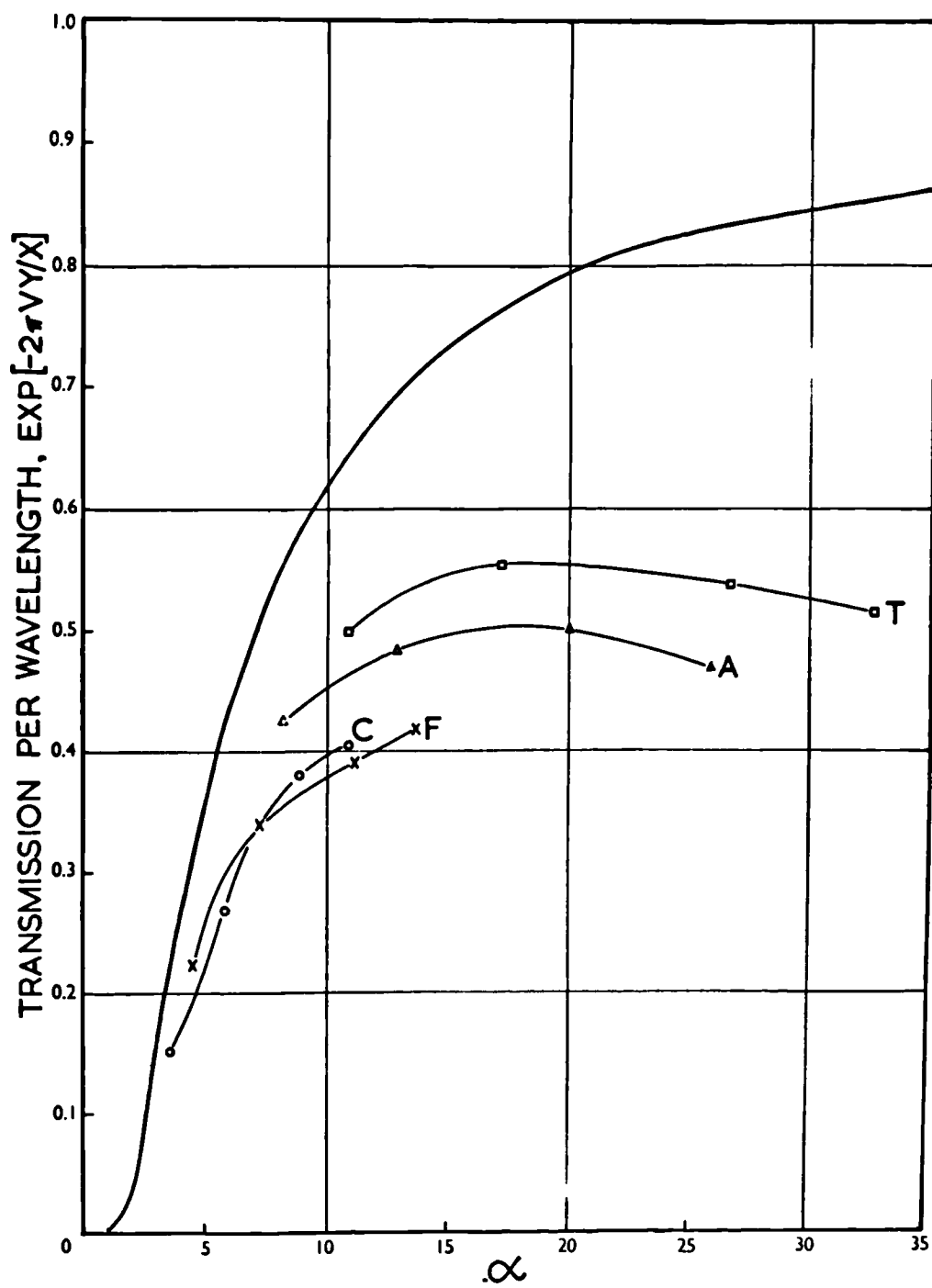


Fig. 42

here. The question of damping will be considered after the phase velocity has been discussed.

Up to now the discussion has concerned only the phase velocity, this is the velocity of a sinusoidal wave at the frequency chosen and may be measured from the change in phase of the oscillation between two points a known distance apart. The pulse wave is of compound shape and contains many frequency components, but the true phase velocity of these is not easy to ascertain. This difficulty comes from the presence of reflexions in the arterial system.

The influence of reflexions is complex and has been treated both theoretically and experimentally by Taylor (1957a, b). He has shown that the value obtained for the phase velocity will depend on the distance of the measuring point from the site of the reflexions. When this is a quarter wave-length this apparent phase velocity will be greater than the 'true' value. With increasing distance it will oscillate below and above the true value, the oscillations becoming small as the distance becomes greater. These effects can be seen in the arterial system (McDonald & Taylor, 1959).

The phase velocity must be measured, in practice, over a finite interval and this must be great enough to allow a reasonable phase change between the recording points. Moreover a large interval will give a better value for the phase velocity as the oscillations mentioned above will tend to average out. The dimensions of the vascular tree are such that a long interval, in terms of wave-lengths, can only be obtained for the high frequency components of the pulse-wave (wavelength varies inversely with frequency). The sharp inflexion at the foot of the pulse-wave is due

to the presence of these high frequency components and for these reasons the velocity is best assessed as the 'foot-to-foot' velocity. A full discussion of this matter will be found in the review by McDonald & Taylor (1959).

The foot-to-foot velocity can then be taken as the best available measurement and may be compared with the figures I would expect for the phase velocity at high frequencies (18 c/s). If this comparison is favourable the figures for the slower waves may be accepted as reliable. This would be of importance practically in the estimation of the amount of reflexion in the arterial bed, since one would then have some idea of the true phase velocity to compare with the apparent velocity actually measured. Values for the pulse wave velocity (foot-to-foot) from the literature are collected in Table 4.

My figures are all of the same order as those reported, though there is scanty evidence for the carotid artery. My figures show little difference between this vessel and the femoral, and the Table also suggests that the arteries of the arm are similar in their properties. The velocities reported in the thoracic aorta are similar to those indicated by my experiments, though the values obtained by Laszt & Müller are a little lower for both parts of the aorta.

My figures suggest velocities in the carotid and femoral at the lower limit of those reported. My explanation for this would be that my vessels were in a state of muscular relaxation, and that some amount of tone might exist in life and increase the modulus of the wall.

The interesting experiments of Landowne (1957a, b) on the propagation of externally generated pressure pulses have been described. The very

high velocities he reported in human brachial arteries, up to 30 m/sec, might in part be due to the increased phase velocity at high frequencies, though I do not think that this can be the whole explanation. It appears that his subjects may have been old men: possibly the spikes in the record he reproduces are overtaking the pulse-waves, but the difference in velocity does not seem great. One must conclude that many of these results relate to subjects in which the pulse-wave velocity was greater than in those studied by the authors cited in the Table. Landowne (1957a) also reported a clear effect of frequency on wave propagation, but below 10 c/s the velocity was less than that predicted from static stress strain values. A similar state of affairs was seen in rubber tubes. I can think of no explanation for this very odd finding unless it be that reflexions were altering the phase velocity. Human umbilical arteries were used which cannot have been very long, and the length of the rubber tubes was 86 cm. The actual velocities were not reported, but they may be calculated from the data given. It appears that the 'static' velocity in the rubber tube would be 4 m/sec, thus the tube represents a $1/4$ wavelength at 1.2 c/s. In this situation it is to be expected that the apparent phase velocity would alter markedly with frequency (Taylor, 1957) and the lowest frequency waves would have a velocity less than the value calculated from the static experiment. No doubt there are also effects from the visco-elastic nature of the rubber; no reflexions would be present when the transmission of single 'impact' waves was measured, and the velocity of these suggests a dynamic modulus about 1.5 times the static which is not unreasonable. Presumably similar factors were operating in the umbilical artery experiments which gave very much the same results.

Finally a few words must be said about the damping to be expected in the arteries. It has been shown (Figs. 41, 42) that this may be considerable and will be especially marked in the larger vessels at high frequencies. This is not because these arteries have a higher internal viscosity, but is a result of the high values of α in this situation.

In the presence of reflexions it is difficult to determine the amount of damping of a pressure oscillation unless it is so great that virtually no wave returns from the periphery. For the fundamental and the lower harmonics of the pulse wave the reflected wave will still be of considerable size if we assume a reflexion coefficient of 0.3 - 0.4 (McDonald & Taylor, 1959). The most rapidly damped oscillations will be those at high frequencies in the large vessels: certainly the aortic dirotic notch loses its sharp outlines very soon (see for example the tracings of Laszt & Müller, 1952a, b).

Landowne (1957a) reproduced a record showing the change in a sharp spike of pressure superimposed upon the radial pulse. In this vessel, with an α of about 50 for the spike I would expect (assuming similar visco-elastic properties to my specimens) a reduction in amplitude of around 60% over 30 cm. This was about the distance used by Landowne and his spikes are, in fact, reduced by 60% in his records.

In conclusion it would seem fair to say that the visco-elastic properties of the arterial wall, as determined in my experiments, are reasonably compatible with the evidence available on the behaviour of the pulse wave, but it is not possible to make very exact comparisons.

My results are somewhat different to those previously reported on aortic strips. Some, at least, of these differences may be due to the

fact that my experiments were performed on intact vessels. If this is so then my results might bear a closer relation to the behaviour of these types of arteries in vivo, though it remains to be seen to what extent this will be modified by activity of the vascular smooth muscle.

TABLE 1(a)
Elastic moduli of arterial wall components

Tissue	Author	Modulus dynes/cm ² x 10 ⁶	Type*	Comments
Tendon (maximal)	Reuterwall (1921)	100	I	Man. Calculated by Bergel.
"	Wohlisch <u>et al.</u> (1926)	30	Tan	Cow. (10% strain)
"	Krafka (1939)	30-100	T	Cow
"	Burton (1954)	1,000	T	Rat. (30% strain)
Ligamentum nuchae (maximal)	Reuterwall (1921)	1-6	I	Ox. Calculated by Bergel.
"	Wohlisch <u>et al.</u> (1926)	6	Tan	Cow
"	Krafka (1939)	4	T	Cow
Purified aorta	Hass (1942b)	3	T	Man. Calculated by Burton, 1954.
Smooth muscle	Bozler (1936)	0.1	Tan	Molluscan. Calculated from relaxation heat.
"	Krafka (1939)	8	T	Mammalian taeniae coli
"	Bozler (1941)	0.06	T	Resting molluscan muscle calculated by Burton, 1954.
"	Reichel (1952b)	2.5	I	Static. Stretched resting molluscan. Calculated by Bergel.
"	Reichel (1952b)	4.5	I	Dynamic. "

*The letters in this column refer to the type of modulus quoted.

I = Incremental; T = Total; Tan = Tangent moduli as defined by Eqns.
(18), (19) and (19a)

TABLE 1(b)

Maximal tension exerted by smooth muscle

Origin of tissue	Author	Tension dynes/cm ² x 10 ⁶	Comments
Mammalian	Ducret (1931)	0.8	Mesenteric artery
"	Fischer (1944)	0.65	Nictitating membrane
"	"	0.63	Ileum
"	"	0.44-0.65	Retractor penis
Molluscan	Abbot & Lowy (1958a)	3.5 - 8	These muscles have atypical structure (Lowy, 1959)

TABLE 2

Static elastic moduli of the arterial wall. (Wherever possible values have been chosen appropriate to a pressure of 100 mm Hg.)

Author	Modulus dynes/cm ² x 10 ⁶	Type*	Comments
<u>Thoracic aorta</u>			
Krafka (1939)	1.16	T	Cow, longitudinal strip.
"	6.5	I	Cow, calculated by Bergel.
"	1.54	T	Dog, longitudinal strip.
Hardung (1953)	1.6	I	Cow, strip c.1.25 x excised length.
Lawton (1955)	2-3	I	Dog, strips 1.5 c excised length.
McDonald, (1960)	2-3	Tan	Dog, longitudinal.
"	2	Tan	Dog, circumferential.
Bergel	4.33 ± 0.39	I	Dog, circumferential. Mean of 12.
+Wagner & Kapal (1951)	3	I	Cow, whole aorta.
+Wagner & Kapal (1952)	5	I	Human, aet. 16-30. Whole aorta.
<u>Abdominal aorta</u>			
+Kapal & Bader (1958b)	11	I	Human, aet. 25.
McDonald (1960)	2-60	Tan	Dog, longitudinal, from small to maximal extensions.
Bergel	8.68 ± 3.51	I	Dog, circumferential, mean of 8.
<u>Femoral artery</u>			
McDonald (1960)	2-60	Tan	Dog, longitudinal, from small to maximal extensions.
Bergel	6.90 ± 1.02	I	Dog, circumferential, mean of 9.
<u>Carotid artery</u>			
Bergel	6.43 ± 1.0	I	Dog, circumferential, mean of 12.

Notes

*The letters in this column refer to the type of modulus quoted.

T = Total; I = Incremental, Tan = Tangential moduli as defined by Eqns. (18), (19) and (19a)

+These figures have been calculated by Bergel, assuming that the relative wall thickness ($Eh/2R_0$) was 0.05, from the values for $V \, dP/dV$.

TABLE 3

Dynamic elastic behaviour of the arterial wall.

Author	Frequency c/s	Dynamic E_6 dynes/cm ² x 10 ⁶	$\eta\omega$	Comments
Ranke, 1934	0	100%	-	Carotids of pig and rabbit
"	15-40	180-230%	-	Phase-lag c. 45°
Hardung, 1953	0	1.6	0	Aortic strip, cow
"	2	2.0	0.2	" "
"	13	2.0	0.8	" "
Kapal, 1954	0	100%	-	Human aortic rings,
"	2	105%	-	mean of 22.
"	10	120%	-	"
"	40	130%	-	"
Lawton, 1955	0	2.89	-	Dog aortic strips; phase-
"	c. 55	5.76	-	lag 'usually less than 5°'. Mean of 15 observations, T = 37.7°C

Note: Ranke and Kapal quote relative values only.

TABLE 4. Reported values for the pulse-wave velocity

Author	Pulse-wave velocity (m/sec)	Species	Comments
<u>Thoracic aorta</u>			
Broemser & Ranke (1930)	4.5-5	Cat & Dog	Carotid to iliac
Hallock (1934)	5-6	Man aet. 25	Carotid to femoral
Wezler & Boger (1936)	5.5-6.5	Man	Carotid to femoral
Dow & Hamilton (1939)	4-7	Dog	Thoracic aorta
"	6-8	Dog	Abdominal aorta
Remington, Hamilton & Dow (1945)	7	Dog	Thoracic aorta
Laszt & Müller (1952a,b)	4.8-5.5	Dog	Thoracic aorta
"	5.5-8.5	Dog	Abdominal aorta
Bergel	5.7	Dog	Calculated phase velocity 2 c/s
"	6.1	Dog	" 18 c/s "
<u>Abdominal aorta</u>			
Bergel	8.7	Dog	" 2 c/s "
"	9.6	Dog	" 18 c/s "
<u>Femoral artery</u>			
Bazett & Dryer (1922)	9	Man 40	Femoral to dorsalis pedis
Bazett <u>et al.</u> (1935)	8.8	Man 40	"
Dow & Hamilton (1939)	8-12	Dog	Mean of 8
Laszt & Müller (1952a)	8.5-13	Dog	
Bergel	8.9	Dog	Calc. phase vel. 2 c/s
"	9.1	Dog	" 18 c/s
Kapal <u>et al.</u> (1951)	8.1	Man	Mean value for young adults
<u>Carotid artery</u>			
Bramwell, Downing & Hill (1923)	8-12	Man	In vitro
Bergel	8.6	Dog	Calc. phase vel. 2 c/s
"	9.7	Dog	" 18 c/s
<u>Arm arteries</u>			
Bazett & Dryer (1922)	8.5	Man	Brachial to radial
Hickson & McSwiney (1925)	5-6	Man	Carotid to radial
Hemingway <u>et al.</u> (1928)	6	Man	Carotid to radial
Hallock (1934)	6-7	Man aet. 25	Carotid to radial

TABLE 4 (continued)

Author	Pulse-wave velocity (m/sec)	Species	Comments
<u>Arm arteries (cont.)</u>			
Fulton & McSwiney (1930)	4.7	Man	Carotid to brachial. Mean of 14 young adults
"	8.7	Man	Brachial to radial. Mean of 14 young adults
Bazett <u>et al.</u> (1935)	9	Man < 40	Carotid to radial. Mean value
Wesler & Boger (1936)	7.5-9	Man	Carotid to radial. Young adults

TABLE 5

Dynamic behaviour of the manometer used in these studies.

Undamped natural frequency (f_0) 89 c/s. Degree of damping (β) 0.102

Frequency (c/s)	A	ψ (radians)
1	1.00	0.002
2	1.00	0.005
3	1.001	0.007
4	1.002	0.009
5	1.003	0.012
6	1.004	0.014
8	1.008	0.018
10	1.012	0.023
12	1.018	0.028
14	1.025	0.033
16	1.033	0.038
18	1.042	0.043
20	1.052	0.048

A is the ratio of recorded pressure amplitude to the true value, the recorded modulus of pressure is therefore multiplied by 1/A.

ψ is the manometer phase-lag, this angle is to be added to the measured phase angle between pressure and radius changes.

The values of these factors at any frequency f , where $\gamma = f/f_0$, are obtained from the expressions,

$$A = \frac{1}{(1-\gamma^2)^2 + (2\beta\gamma)^2}$$

$$\tan \psi = \frac{2\beta\gamma}{1-\gamma^2} \quad (\text{Hansen, 1949})$$

SUMMARY

1. The classical theory of elasticity has been discussed in relation to the description of highly extensible substances. After discussing various approaches to this subject a suitable treatment employing an 'incremental' modulus of elasticity has been proposed.
2. Previous studies of the mechanical properties of the arterial wall and of the materials of which it is composed have been reviewed and discussed.
3. A method whereby the pressure-diameter relationships of an arterial specimen may be determined both under static and dynamic conditions has been described.
4. Some observations on the dimensions and structure of the vessels studied have been reported. The ratio between wall thickness and radius was found to be fairly constant with a value of 0.10 - 0.13. The larger values were found in the smaller arteries. The amount of retraction occurring on excision was measured and found to be of the order of 30-40%, this was discussed in relation to previous reports of arterial properties. Some observations on the histology of various vessels have also been described and discussed.
5. The static elastic behaviour of four types of dogs' artery, namely the thoracic and abdominal aorta, femoral and carotid arteries, have been described. It was found that the incremental modulus of the vessel wall increases with distension; this increase is less marked in the thoracic aorta over the lower pressure range, and this is taken to be a reflection of the large proportion of elastin in its wall. At

higher pressures (220 mm Hg) all vessels, save only the carotid, show similar and relatively high moduli. It would seem that this is due to the presence of collagen which comes under load at increasing distension. The carotids were found, on the basis of several lines of evidence, to be the most muscular vessels of those studied. The failure of these arteries to maintain a high resistance to stretch at these pressures seems to be best explained by the relative weakness of smooth muscle.

The behaviour of a muscular vessel on repeated inflation leads to the conclusion that the muscle is arranged in parallel with the elastin and collagen fibres. Histological evidence bears this out and shows that the majority of the connective tissue fibres in the wall are also deployed in parallel.

Some evidence on the anisotropy of the arterial wall is provided by experiments in which the length of the specimen is altered. In the physiological range of distension the wall is about 1.5 as stiff circumferentially as longitudinally.

6. The dynamic elastic properties of the arterial wall have been described. All the vessels become stiffer at frequencies between 0 and 2 c/s, but there is little further increase at higher rates of straining. The amount of this increase appears to be determined by the amount of smooth muscle in the wall, and this would indicate that vascular muscle possesses a considerable elastic modulus. The amount of phase difference between pressure and dilatation appears also to depend to some extent on the smooth muscle: it was found that this phase shift did not increase very greatly with frequency.

Simple models of visco-elastic substances, composed of springs and dashpots, are discussed. The sort of behaviour seen here cannot be explained in terms of such models. The properties of rubber are not dissimilar to those of arteries; a complex statistical model containing long tangled molecular chains has been invoked to account for the behaviour of rubber, but no attempt has been made in this work to apply this treatment to the inhomogeneous arterial wall.

7. The results have been applied to the calculation of the wave velocity and damping to be expected in the arterial system. Both these are markedly influenced by the viscous properties of arteries and of the blood. An estimate of these factors may allow a better appreciation of the importance of other effects to be found in the intact circulatory system.

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ERRATA

p.18, 3 lines from bottom; for $E_{(2)}$ read $E_{(L)}$

p.38, line 14; for "linae" read laminae

p.67, line 3; omit "x"

p.86, line 9; for "Micron meter" read micrometer

pp.95 & 97. The Figure labelled Fig. 8 should be Fig. 9, and vice versa.

Fig. 9 (here labelled Fig. 8); the scale of the ordinate of the lower part is ten times too big. It should read from 0 to 0.4 radians.

p.119, line 12; omit "which".