

Adaptive filtering applications to satellite navigation

Zhahir, Md Amzari

The copyright of this thesis rests with the author and no quotation from it or information derived from it may be published without the prior written consent of the author

For additional information about this publication click this link. https://qmro.qmul.ac.uk/jspui/handle/123456789/364

Information about this research object was correct at the time of download; we occasionally make corrections to records, please therefore check the published record when citing. For more information contact scholarlycommunications@qmul.ac.uk

Adaptive Filtering Applications to Satellite Navigation

by

Md Amzari bin Md Zhahir

2010



A thesis submitted as part of degree of Doctor of Philosophy, School of Engineering and Material Science, Queen Mary, University of London

ABSTRACT

Differential Global Navigation Satellite Systems employ the extended Kalman filter to estimate the reference position error. High accuracy integrated navigation systems have the ability to mix traditional inertial sensor outputs with navigation satellite based position information and can be used to develop high accuracy landing systems for aircraft.

This thesis considers a host of estimation problems associated with aircraft navigation systems that currently rely on the extended Kalman filter and proposes to use a nonlinear estimation algorithm, the unscented Kalman filter (UKF) that does not rely on Jacobian linearisation. The objective is to develop high accuracy positioning algorithms to facilitate the use of GNSS or DGNSS for aircraft landing. Firstly, the position error in a typical satellite navigation problem depends on the accuracy of the orbital ephemeris. The thesis presents results for the prediction of the orbital ephemeris from a customised navigation satellite receiver's data message. The SDP4/SDP8 algorithms and suitable noise models are used to establish the measured data. Secondly, the differential station common mode position error not including the contribution due to errors in the ephemeris is usually estimated by employing an EKF. The thesis then considers the application of the UKF to the mixing problem, so as to facilitate the mixing of measurements made by either a GNSS or a DGNSS and a variety of low cost or high-precision INS sensors.

Precise, adaptive UKFs and a suitable nonlinear propagation method are used to estimate the orbit ephemeris and the differential position and the navigation filter mixing errors. The results indicate the method is particularly suitable for estimating the orbit ephemeris of navigation satellites and the differential position and navigation filter mixing errors, thus facilitating interoperable DGNSS operation for aircraft landing.

ACKNOWLEDGMENTS

First and foremost, I am thankful to Allah for His continuous help, giving me strength, enlightenment and motivation to work hard and smart towards my thesis.

I would like to express my sincere gratitude to my supervisor, Dr Ranjan Vepa for his valuable supervision, patience, enthusiasm and critical comments throughout the course of this programme.

Not forgetting my family, especially my parents, my friends and all those who have helped and supported me throughout this course, without naming names, but who will always remain in my heart and in my mind.

My hope is that anyone who encounters and gains knowledge from my work will remember me in his/her supplications.

This work was funded by the Malaysian Ministry of Higher Education and Universiti Putra Malaysia, to whom my deepest thanks.

TABLE OF CONTENTS

Title Page	1
Abstract	2
Acknowledgements	
Table of Contents	4
List of Figures	9
List of Tables	
List of Abbreviations	
List of Nomenclature	
List of Notations	
Chapter 1: Introduction	
1.1 Navigation Satellites Overview	
1.1.1 GPS	
1.1.2 GLONASS	
1.1.3 Galileo	
1.1.4 COMPASS/BeiDou	
1.2 GPS/GLONASS/Galileo System Comparison	
1.3 Navigation Satellite Augmentation Systems	
1.3.1 Differential Systems: DGPS Overview	
1.3.2 Ground-based Augmentation Systems	
1.3.3 Satellite-based Augmentation Systems	
1.3.4 Aircraft-based Augmentation Systems	
1.4 Interoperability	
1.5 Accuracy versus Integrity	
1.6 Motivation and Aims of Research	
1.7 Thesis Organisation	

Chapter 2: Real-Time Orbit Prediction and Pseudorange Measurement
2.1 Introduction to Orbital Dynamics
2.2 Orbit Position Determination: The Keplerian Elements
2.2.1 Transformation to Position and Velocity
2.2.2 The GPS Ephemeris
2.3 Orbit Prediction: Error Modelling
2.4 Short Term Prediction: The GLONASS Approach
2.5 NORAD Method for Orbit Propagation
2.5.1 NORAD TLE
2.5.2 NORAD SPACETRACK Propagation Models
2.6 Position Determination Fundamental
2.6.1 Satellite Signals Flow through a Receiver
2.6.2 Signal Tracking
2.6.3 Code-based Measurement
2.6.4 Carrier Phase Measurement (Real Time Kinematics)
2.7 Measurement of Pseudorange
2.7.1 Measurement Errors in Code-based Measurement
2.7.2 Measurement Errors in Carrier Phase-based Measurement

Chapter 3: Pseudorange Error Modelling for Position Estimation	68
3.1 Pseudorange Error Models	69
3.2 Clock Bias Errors	72
3.3 Receiver Dependent Errors	76
3.3.1 Multipath Error Model	76
3.3.2 Signal Strength and Noise	77
3.3.3 Constant Integer Phase Ambiguity	77
3.4 Common Mode Errors	78
3.4.1 Ionospheric Delay	78
3.4.2 Improved Modelling of the Ionosphere	80
3.4.3 Tropospheric Delay	83
3.4.4 Selective Availability	84
3.4.5 Errors in Ephemeris	85
3.5 Simulation of Pseudorange Errors	

3.6 Estimation and Prediction of Pseudorange	
--	--

Chapter 4: Differential Correction Estimation for Interoperability	
4.1 Standard Differential Satellite Navigation Reference Station Algorithms	
4.2 Kalman Filter of The Differential Correction	93
4.3 Introducing Interoperability	97
4.4 Ephemeris Conversion for Interoperability	
4.5 Ensuring Consistency of Error Models for Interoperability	103
4.6 Adaptive Kalman Filter	105
4.7 Filtering for Interoperability	109
4.8 Unscented Kalman Filtering	111

Chapter 5: Adaptive Orbit Estimation for Interoperability	
5.1 The Orbit Estimation Problem	116
5.2 Orbit Dynamic Propagation Models	117
5.3 UKF-based Orbit Estimation	118
5.4 Adaptive UKF-based Orbit Estimation	119
5.5 Simulation and Validation	

Chapter 6: Enhanced Accuracy Algorithms	131
6.1 Introduction	131
6.2 Solution to Integer Phase Ambiguity	132
6.3 Carrier Phase Smoothing applied to DGPS: The Hatch Filter	134
6.4 Modelling the Hatch Filter	135
6.5 Single and Multi-Frequency Filters	136
6.6 Multipath Estimation	138
6.7 WAAS Correction Algorithms	143
6.8 Adaptive Filtering for Interoperability	144
6.9 Integration and System Design	145
6.9.1 Approaches to Composite Error Estimation	145
6.9.2 Interoperable System: The Switching Mechanism	147
6.9.3 The Complete System Architecture	148

6.10 Recommendations on Imp	plementations
-----------------------------	---------------

Chapter 7: Adaptive Mixing Filters for Interoperability	150
7.1 Principles of the Mixing Filter	150
7.2 Adaptive Satellite Navigation-INS Mixing Filters	152
7.2.1 Overview of INS	154
7.2.2 Strapdown INS	155
7.2.3 Gyro-free Strapdown INS	155
7.2.4 High Precision INS	156
7.2.5 Process Modelling with Gyro-free Acceleration Measurements	158
7.2.6 Process Modelling with Carrier Phase Measurements	162
7.2.7 Customised Satellite Navigation Measurement Modelling	167
7.2.8 Formulation of the Satellite Navigation-INS Mixing Filter	169
7.3 Application of the Adaptive UKF to Satellite Navigation-INS Mixing Filter	172
7.3.1 Low Cost Satellite Navigation-INS Mixing Filter	173
7.3.2 High Precision Satellite Navigation-INS Mixing Filter	182
7.3.3 Doppler-Aided High Precision Satellite Navigation-INS Mixing Filter	190
7.4 Interoperable Satellite Navigation-INS Mixing Filters	197

Chapter 8: Conclusions	199
8.1 Summary of Findings	199
8.2 Thesis Contributions and Achievements	200
8.3 Recommendations for Future Work	

Appendix	203
Appendix A: The SDP Model	
Appendix B: Ionospheric Models	209
Appendix C: EGNOS Tropospheric Correction Model	217
Appendix D: Orbit Dynamic Propagation Models	220
Appendix E: Ambiguity Estimation and Resolution	230
Appendix F: Kalman Filter Variants	241
Appendix G: The Earth's Gravitational Potential	245

List of References	247
List of Publications	264

LIST OF FIGURES

Figure 1.1 GPS constellation	32
Figure 2.1 Diagram of orbital elements	46
Figure 2.2 Two-dimensional user position and three satellites positions	61
Figure 2.3 A simplified flowchart of signals flow through a receiver	62
Figure 2.4 Carrier and code tracking loops	63
Figure 3.1 General two-static model describing clock errors	72
Figure 3.2 Linear combinations of satellites and receivers	75
Figure 3.3 Chapman Profile of the ionosphere	79
Figure 3.4 Graph of receiver and satellite clock bias errors	86
Figure 3.5 Graph of selective availability and multipath errors	87
Figure 3.6 Graph of selective availability and multipath errors(zoomed-in)	87
Figure 3.7 Graph of ionospheric models based on Chapman Profile and Gauss-Mark	OV
	88
Figure 3.8 Graph of tropospheric delay models against elevation angle	88
Figure 3.9 Pseudorange errors without selective availability	89
Figure 3.10 Pseudorange errors with selective availability	89
Figure 4.1 Navigation satellite orbital determination and ephemeris conversion	
processes	98
Figure 4.2 An example of filtering a Gaussian prior propagated through an highly	
nonlinear function	113
Figure 5.1a GLONASS satellite normalised position prediction	122
Figure 5.1b GLONASS satellite normalised velocity prediction	122
Figure 5.1c GLONASS normalised satellite position prediction error	123
Figure 5.1d GLONASS normalised satellite velocity prediction error	123
Figure 5.2a GLONASS normalised satellite position prediction error (Lagrange	
equation)	124
Figure 5.2b GLONASS normalised satellite velocity prediction error (Lagrange	
equation)	124
Figure 5.3a GLONASS satellite UKF-based normalised position estimate error	125
Figure 5.3b GLONASS satellite UKF-based normalised velocity estimate error	125
Figure 5.4 GLONASS satellite UKF-based pseudorange estimate error	126

Figure 5.5a GLONASS satellite modified UKF-based normalised position estimate error	.126
Figure 5.5b GLONASS satellite modified UKF-based normalised velocity estimate	126
Figure 5.6 GLONASS satellite modified UKF-based pseudorange estimate error	.127
Figure 5.7a GLONASS satellite adaptive UKF-based normalised position estimate err	or
Figure 5.7b GLONASS satellite adaptive UKF-based normalised velocity estimate err	128 or
Figure 5.8 GLONASS satellite adaptive UKF-based pseudorange estimate error	.128
Figure 5.9a GLONASS satellite normalised position prediction comparison using UK	F
and EKF filters	.129
Figure 5.9b GLONASS satellite normalised velocity prediction comparison using UK	F
and EKF filters	129
Figure 6.1 The Wavelength-Gap-Bridging method	138
Figure 6.2 The process flowchart for multipath mitigation	139
Figure 6.3 Parallel approach to composite error estimation	146
Figure 6.4 Serial approach to composite error estimation	146
Figure 6.5 A switching mechanism for interoperability	.147
Figure 6.6 Complete system architecture	.148
Figure 7.1 Schematic form of a complementary filter	.151
Figure 7.2 The GYROCUBE: A sensor for inertial measurement	.156
Figure 7.3a Estimates of the LLA (low cost)	.174
Figure 7.3b Estimates of the NED velocities (low cost)	.175
Figure 7.3c Estimates of the horizontal velocities (low cost)	.175
Figure 7.3d Estimates of the vertical velocities (low cost)	.176
Figure 7.3e Estimates of the body angular velocities in body coordinates (low cost)	.176
Figure 7.3f Estimate of the user position error (low cost)	.177
Figure 7.3g Estimate of the pseudorange measurement error (low cost)	.177
Figure 7.4a Comparison of the LLA (low cost)	.178
Figure 7.4b Comparison of the NED translational velocities (low cost)	.179
Figure 7.4c Comparison of the horizontal velocities (low cost)	.179
Figure 7.4d Comparison of the vertical velocities (low cost)	.180
Figure 7.4e Comparison of the body angular velocities (low cost)	.180

Figure 7.4f Comparison of the user position errors (low cost)	181
Figure 7.4g Comparison of the pseudorange measurement errors (low cost)	181
Figure 7.5a Comparison of the LLA (high precision)	183
Figure 7.5b Comparison of the NED translational velocities (standard high precisio	n)
	184
Figure 7.5c Comparison of the body quarternion (standard high precision)	184
Figure 7.5d Errors in the UKF estimated pseudorange (standard high precision)	185
Figure 7.5e Errors in the UKF estimated three-axis user position components (stand	lard
high precision)	185
Figure 7.6a Comparison of the LLA (adaptive high precision)	186
Figure 7.6b Comparison of the NED translational velocities (adaptive high precisio	n)
	187
Figure 7.6c Comparison of the horizontal velocities (adaptive high precision)	187
Figure 7.6d Comparison of the vertical velocities (adaptive high precision)	188
Figure 7.6e Comparison of the body quarternion (adaptive high precision)	188
Figure 7.6f Errors in the UKF estimated pseudorange (adaptive high precision)	189
Figure 7.6g Errors in the UKF estimated three-axis user position components (adap	tive
high precision)	189
Figure 7.7a Comparison of the NED translational velocities (adaptive Doppler-aide	d)
	190
Figure 7.7b Comparison of the body quarternion (adaptive Doppler-aided)	191
Figure 7.8a Comparison of the LLA (adaptive Doppler-aided)	192
Figure 7.8b Comparison of the NED translational velocities (adaptive Doppler-aide	d)
	193
Figure 7.8c Comparison of the horizontal velocities (adaptive Doppler-aided)	193
Figure 7.8d Comparison of the vertical velocities (adaptive Doppler-aided)	194
Figure 7.8e Comparison of the body quarternion (adaptive Doppler-aided)	194
Figure 7.8f Comparison of errors in the UKF estimated pseudoranges (adaptive	
Doppler-aided)	195
Figure 7.8g Comparisons of errors in the UKF estimated three-axis user position	
components (adaptive high precision)	196
Figure D.1 Relative motion rotating Euler-Hill reference frame	220

LIST OF TABLES

Table 1.1 GPS, GLONASS and Galileo attribute comparison	34
Table 2.1 GPS ephemeris and clock correction elements	50
Table 2.2 A typical NORAD TLE element set	40
Table 3.1 Pseudorange error classification	71
Table 3.2 Typical Allan variance parameters for various timing standards	74
Table 3.3 Parameters for Gauss-Markov multipath error source	76
Table 3.4 Measurement noise parameters	77
Table 3.5 Parameter for Gauss-Markov ionosphere error source	80
Table 3.6 Parameters for Gauss-Markov troposphere error source	83
Table 3.7 Parameters for Gauss-Markov selective availability error source	85
Table 3.8 Parameters for Gauss-Markov ephemeris error source	85
Table 4.1 Real-time prediction of GPS BII-09 starting at epoch time	100
Table 4.2 Real-time prediction of GPS BII-09 starting at 4 th October 2006, 0 UTC	100
Table 4.3 Classification of GPS ephemeris	101
Table 4.4 SDP4 computation of approximated GPS ephemeris elements	102
Table 4.5 Comparison between direct SDP4 outputs, SDP-4 based GPS ephemeris	
computation for satellite GPS BII-09 (PRN 15) and the differences	103
Table B.1 Seasonal term to compute <i>foE</i> in the Titheridge's model	213
Table C.1 Average and seasonal variation values of the five meteorological parameter	ers
used by the EGNOS model	219

LIST OF ABBREVIATIONS

Follows are the list of abbreviations that has been used in this te	ext.
---	------

ABAS	Aircraft-based Augmentation Systems
ADC	Analog-to-Digital Converter
Beidou	People's Republic of China Satellite Navigation Augmentation System
CDMA	Code Division Multiple Access
CPGPS	Carrier Phase Enhancement GPS
COMPASS	People's Republic of China Global Satellite Navigation and Positioning
	System (also known as Beidou-2)
C/A	coarse-acquisition code; civilian GPS signal
CS	Control Segment
CONUS	Continental United States
DGPS	Differential GPS
DGNSS	Differential Global Navigation Satellite Systems
DME	Distance Measuring Equipment
DoD	United States Deparment of Defense
DOP	Dilution of Precision
ECEF	Earth-centered Earth-fixed frame
ECI	Earth-centred Inertial
EGNOS	European Geostationary Navigation Overlay Service
EKF	Extended Kalman Filter
ESA	European Space Agency
EU	European Union
FDMA	Frequency Division Multiple Access
FOC	Full Operational Capability
Galileo	Name of the European Navigation Satellite Systems
GAGAN	Indian GPS Aided GEO Augmented Navigation
GBAS	Ground-based Augmentation Systems
GIM	Global Ionospheric Map
GLONASS	Russian GNSS equivalent to United States GPS
GNSS	Global Navigation Satellite System
GPS	United States Global Positioning Systems (also known as NAVSTAR)

HAL	Horizontal Alert Limit		
ICAO	International Civil Aviation Organization		
ICG	International Committee on Global Navigation Satellite Systems		
IMO	International Maritime Organization		
IOC	Initial Operational Capability		
IORs	Interoperability Requirements		
IRI	International Reference Ionosphere		
IRNSS	Indian Regional Navigational Satellite System		
LAAS	United States Local Area Augmentation System		
LLA	Latitude, Longitude and Altitude		
LORAN-C	LOng Range Navigation type C		
LPE	Lagrange Planetary Equations of motion		
MA	Moving Average filter		
MATLAB	a numerical computing environment and programming language		
MMLE	Method of Maximum Likelihood Estimation		
MSAS	Japan's Multi-Functional Satellite Augmentation System		
NMEA	United States-based National Marine Electronics Association		
NST	Navigation Technology Satellite		
NORAD	North American Aerospace Defense Command		
PNT	Positioning, Navigation and Timing		
P(Y)	precise code; military GPS encrypted signal		
QZSS	Japan's Quasi-Zenith Satellite System		
RNP	Required Navigation Performance		
SA	Selective Availability		
SBAS	Space-based Augmentation Systems		
SDCM	Russian Federation's System of Differential Correction and Monitoring		
SDP	a NORAD SPACETRACK Orbit Propagation Model		
SDP4	a NORAD SPACETRACK Orbit Propagation Model for Deep-Space		
SDP8	a NORAD SPACETRACK Orbit Propagation Model for Deep-Space		
SGP4	a NORAD SPACETRACK Orbit Propagation Model for General		
	Perturbation		
SGP8	a NORAD SPACETRACK Orbit Propagation Model for General		
	Perturbation		
SNR	Signal-to-Noise Ratio		

SS	Space Segment
STM	State Transition Matrix
SV	Space Vehicle (for example, satellite or space shuttle)
SVD	Singular Value Decomposition
TEC	Total Electron Content
TIMATION	GPS predecessor
TLE	Two-Line Element (set of data)
UAV	Unmanned Aerial Vehicle
UKF	Unscented Kalman Filter
US	User Segment
UTC	Universal Time Coordinated
VAL	Vertical Alert Limit
VOP	Variation of Parameters
VOR	VHF Omnidirectional Range
WAAS	United States Wide Area Augmentation System
arctan	arctangent comparator
e/d select	selecting outputs to their proper connectors
lpf	low pass filter
osc	oscillator
sqrt	square root process

LIST OF NOMENCLATURE

The following is the list of nomenclature, which has been used throughout this text. Some of the notations have the same meaning, R and a are both Earth semi-major axis while some constants and variables have the same notations, but different interpretations, for example, f may be defined as the carrier frequency of a wavelength as well as the flattening factor of the Earth. In case of conflicts in the definition of parameters, please refer to the particular equation and take the definition within the context of the equation.

- 1. Constants Universal gravitation constant ($G = 6.67428 \times 10^{-11} m^3 kg^{-1} s^{-2}$) G J_n Standard zonal harmonic coefficient: $J_2 = 1.082616 \times 10^{-3}, J_3 = -2.53881 \times 10^{-6}, J_4 = -1.65597 \times 10^{-6}$ and to $J_{21} = 0$, $J_{22} = 1.86 \times 10^{-6}$, $J_{31} = 2.1061 \times 10^{-6}$ Earth mass ($M_0 = 5.9742 \times 10^{24} \ kg$) M_0 WGS-84 Earth semi-major axis (6378137.0 m) R a, R_{e} Earth's semi-major axis or equatorial radius (a = 6378137 m) Earth's semi-minor axis (b = 6356752.3142 m) b Speed of light ($2.99792458 \times 10^8 \text{ ms}^{-1}$) С Earth flattening factor (f = 298.257223563) fGravitational attraction the surface at of the Earth g $(9.812865328 \,\mathrm{ms}^{-2})$ Variations (m = 0.00344978600308) т The mean rotation rate of Earth $(7.2921151467 \times 10^{-5} \text{ rad s}^{-1})$ $\dot{\Omega}_{\rho}$ Equatorial acceleration due to gravity ($\gamma_a = 9.7803267715 \text{ ms}^{-2}$) γ_a Pi (3.1415926535898) π
- ρ Approximated air density at an altitude of 450 km, $ρ = 1.585 \times 10^{-12} \text{ kg m}^{-3}$

2. Variables

Α	Average cross-sectional or projected area of the satellite
A_N, A_E, A_D	North, east, down components of the measured acceleration in the
	n-frame which must be compensated for by adding the
	acceleration due to gravity g , in down direction
\mathbf{A}_{NED}	Vector of the north, east, down components of the measured
	acceleration in the <i>n</i> -frame
\mathbf{A}_{NED}^{T}	North, east and down accelerations
\mathbf{A}_{body}	Body components of the measured acceleration
\mathbf{A}_m	Actual measured acceleration vector obtained from a triad of
	pendulous accelerometers
B^{*}	Ballistic drag term
C_D	Drag coefficient
C_{is} , C_{ic}	Amplitudes of sine & cosine harmonics correction (short period)
	terms for inclination (GPS ephemeris elements)
C_{rs}, C_{rc}	Amplitudes of sine & cosine harmonics correction (short period)
	terms for semi-major axis of orbital radius (GPS ephemeris
	elements)
C_{us}, C_{uc}	Amplitudes of sine & cosine harmonics correction (short period)
	terms for argument of latitude (GPS ephemeris elements)
$\mathbf{C}^{k,N}_{v}$	Estimate of the covariance of the innovation
C_0 , C_i	Carriers to the noise power spectral densities of a satellite signal
C_{2i}	Oblateness parameter (for $i = 0, 2$, by r_{geo}^2)
${ ilde C}_{2i}$	Normalised oblateness coefficients
$\mathbf{D}_{b,I}$	Transformation from the inertial to the body fixed frame
$\mathbf{D}_{n,b}$	Transformation of the measured body acceleration components to
	the north, east, down components in the <i>n</i> -frame
Ε	Eccentric anomaly
E(t)	Error in the calculated ephemeris
G	Gravitational component of the acceleration in the body frame
$\mathbf{G}_{\scriptscriptstyle NED}$	North, east and down components of the gravity vector

Η	Measurement matrix
\mathbf{H}_{k}	$(m \times n)$ state to measurement distribution matrix
$\mathbf{H}_1, \mathbf{H}_2$	Measurement matrices for observable variables z_1 , z_2
\mathbf{H}_3 , \mathbf{H}_4	Measurement matrices for observable variables z_3 , z_4
I _a	Ionospheric effects
Κ	Weighted gain
\mathbf{K}_{k}	Optimal Kalman gain
L	Matrix of the three directions of sensitivity of the fibre-optic laser
	gyros
L	Dimension of random variable \mathbf{w}
L_{ϕ}	Total measurements are made between two successive code
	measurements
$\mathbf{L}_1, \mathbf{L}_2$	Output matrices for the range correction $y_1(t)$ and range rate
	correction $y_2(t)$
М	Mean anomaly
MP	Code multipath which is to be removed
MP_1 , MP_2	Code multipath refering to the first and second carrier frequency
Ν	Constant integer phase ambiguity (the whole number between the
	satellite and the receiver at initial time measurement)
N_{1}, N_{2}	Carrier integer ambiguity refering to the first and second carrier
	frequency
Р	atmospheric pressure P
$\hat{\mathbf{P}}_k^-$	Predicted state covariance matrix
\mathbf{P}_{k-1}	Previous state covariance matrix
$\mathbf{P}_{_{WW}}$	Covariance of random variable \mathbf{w}
\mathbf{P}_k^f , \mathbf{P}_k^h	Transformed covariance matrices
\mathbf{P}_{k}^{fh}	Transformed cross-covariance matrix
\mathbf{Q}_k	Covariance matrix associated with \mathbf{w}_k
\mathbf{Q}_k , \mathbf{R}_k	Covariance matrices of the process and measurement noise
	sequence respectively
$\mathbf{R}(au)$	Exponential autocorrelation function

R _I	Inertial acceleration of the origin of the body frame.
R_M , R_P	Radii of curvature in the meridian and prime vertical at a given
	latitude
\hat{R}_0	Estimation of the reference-to-satellite range
R_{0i}	Signal-to-noise ratio between two closely spaced antennae
S_{f}	Spectral amplitude associated with the white noise driving
	function u_{cb}
S _g	Spectral amplitude associated with the white noise driving
	function u_{cd}
Т	Orbital period of the planet
Т	Chip width in unit metres
Т	Temperature
TEC	Total electron content along the signal path
Тое	Time of the Ephemeris; Time of Ephemeris (GPS ephemeris
	elements); Time of the Ephemeris (GLONASS model)
T_{GD}	Satellite group delay differential (GPS ephemeris elements)
T_{ϕ}	Sampling interval
U_2	Earth's gravitational perturbation potential
V_N, V_E, V_D	North, east and down velocities in the local tangent plane, with
	reference to a local geodetic frame often referred to as the
	navigation frame (<i>n</i> -frame) or north-east-down frame
a	Semi-major axis
a	Acceleration of the correction
a _i	Scalar acceleration measurements
a_m	Measured acceleration vector
b	Measurement bias vector
$b_{REF,t}$	Error caused from the imperfect knowledge of reference position
	at time t
b_{ut}	User clock offset and bias
b ₁ , b ₂	Measurement bias and drift vectors
$\dot{\mathbf{b}}_1$, $\dot{\mathbf{b}}_2$, $\dot{\mathbf{b}}_3$, $\dot{\mathbf{b}}_4$	First order Gauss-Markov drift and bias vectors

С	Filtered correction (i.e. pseudorange error)
$c\Delta t$	Receiver clock bias difference
$c\Delta t_a$	Total of combined errors of ionospheric and tropospheric
	propagation delays
$c\Delta t_{ion}(t)$	Ionospheric delay
$c\Delta t_{trop}(t)$	Tropospheric delay
$c\delta t_{sv}$	Residual reference satellite clock error
$c\delta t_0$	Residual reference receiver clock error
d_i	Range between the i^{th} navigational satellite and a user receiver
$d\mathbf{F}_{drag}$	Drag acceleration on a satellite
dn	Mean motion difference (GPS ephemeris elements)
$\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$	First derivatives of x, y, z with respect to time
$\frac{d\dot{x}}{dt}, \frac{d\dot{y}}{dt}, \frac{d\dot{z}}{dt}$	Second derivatives of x, y, z with respect to time
$\frac{d\tilde{x}}{d\tau},\frac{d\tilde{y}}{d\tau},\frac{d\tilde{z}}{d\tau}$	Non-dimensional time derivatives (normalised) position
	components
$\frac{d\tilde{x}'}{d\tau},\frac{d\tilde{y}'}{d\tau},\frac{d\tilde{z}'}{d\tau}$	Non-dimensional time derivatives (normalised) velocity
	components
e	Eccentricity; Eccentricity (GPS ephemeris elements)
e	Partial pressure of water vapour
е	Difference between the rates of change of the code and carrier
	phase correction
$e_{cm}(t)$	Single term which collected all the common mode errors and the
	receiver clock bias
f	Satellite carrier frequency
\mathbf{f}^{UT} , \mathbf{h}^{UT}	Unscented transformations (UT) of the states
g	Gravity vector
h	Altitude
$h_0, \ h_{-1}, \ h_{-2}$	Typical Allan variance parameters
i	Orbital inclination

i_k	Inclination at epoch k		
i_0	Inclination (GPS ephemeris elements)		
i	Rate of inclination (GPS ephemeris elements)		
k	k th number of epoch elapsed, satellite, discrete time etc		
т	Mass of the satellite		
\hat{m}_k	Second state of the Hatch filter, the current pseudorange error due		
	to the multipath component		
mp(t)	Phase multipath error		
n	Measurement noise vector		
n ₁	Measurement white noise vector		
$\mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_5, \mathbf{n}_6,$	White noise vectors driving the $\dot{\mathbf{b}}_1$, $\dot{\mathbf{b}}_2$, $\dot{\mathbf{b}}_3$, $\dot{\mathbf{b}}_4$ processes		
	respectively		
р	Semi-latus rectum		
q_0 , s	Altitude parameters		
$q_{11}, q_{12}, q_{21}, q_{22}$	Elements of \mathbf{Q}_k covariance matrix		
ġ	Attitude quaternion		
r	Distance of the body centre of mass		
r	Radial distance from the centre of the Earth		
r and θ	Heliocentric polar coordinates for the planet		
\mathbf{r}^{i}	Position vector of the accelerometer location in the body fixed		
r _B	Navigation satellite's sight line vector at the current time		
$\vec{r}_c, \vec{r}_c, \vec{r}_c$	Range correction and its first two derivatives, excluding		
	ionospheric effect		
r _{geo}	Length scale normalisation		
r_k	Orbital radius at epoch k		
\mathbf{r}_k	Residual sequence		
r _s	Radius at a surface point of the flattened Earth ellipsoid		
r ₀	Navigation satellite's sight line vector at the initial reference time		
\hat{r}_{R}^{k}	Computed distance between the satellite k and a reference		
	position using the broadcast ephemeris,		

ĩ	Normalised orbital radius		
t	Time since periapsis (or perihelion if the Sun was the centre)		
t _k	Time since the ephemeris		
t_k	Discrete time		
$t_{oe}, a_{f0}, a_{f1}, a_{f2}$	Satellite clock corrections (GPS ephemeris elements)		
u _{cb}	White noise driving function for x_{cb}		
u _{cd}	White noise driving function for x_{cd}		
<i>u</i> _k	Latitude at epoch k		
\mathbf{u}_k	known input vector,		
v	Rate of change of the correction		
$\mathbf{v}(k)$	Measurement noise		
W	Random variable		
$\overline{\mathbf{W}}$	Mean of random variable \mathbf{w}		
\mathbf{w}_k	Discrete noise		
$W_{cb,k}$	Discrete white noise for $x_{cb,k}$		
$W_{cd,k}$	Discrete white noise for $x_{cd,k}$		
\mathbf{w}_k	Gaussian white noise		
\mathbf{w}_k , \mathbf{v}_k	uncorrelated Gaussian white noise sequences with zero means of		
	process and measurement respectively		
<i>x</i> , <i>y</i> , <i>z</i>	Position components in three-axis		
$\mathbf{x}(k)$	State of a system		
X _{cb}	Clock bias state		
$X_{cb,k}$	Discrete clock bias state error		
X _{cd}	Clock drift state		
$X_{cd,k}$	Discrete clock drift state error		
x_i, y_i, z_i	Position vector of the satellite in an Earth-centred inertial (ECI)		
	coordinate frame		
\mathbf{x}_k	Discrete state of the process; Discrete form of the clock error		
	model		
\mathbf{x}_k	Parameter being simulated		

\mathbf{x}_k	$(n \times 1)$ state vector		
$\hat{\mathbf{x}}_k$	State vector predicted from the previous corrected state vector		
$\mathbf{\hat{x}}_{k-1}$	Previous corrected state vector estimated		
x_{sv} , y_{sv} , z_{sv}	Known three-dimensional satellite position		
\hat{x}_{sv} , \hat{y}_{sv} , \hat{z}_{sv}	Estimation of a satellite positions in x , y , z components		
x_u , y_u , z_u	Unknown three dimensional user positions; User positions in x ,		
	y, z in components		
x_0, y_0, z_0	Position components in Cartesian ECEF coordinates (GLONASS		
	model); Known precisely surveyed location of a reference receiver		
<i>x</i> , <i>y</i> , <i>z</i>	Velocity components in three-axis		
\widetilde{x} , \widetilde{y} , \widetilde{z}	Normalised position components		
\widetilde{x}' , \widetilde{y}' , \widetilde{z}'	Normalised velocity components		
$\dot{x}_{0}, \dot{y}_{0}, \dot{z}_{0}$	Velocity components in Cartesian ECEF coordinates (GLONASS		
	model)		
$\ddot{x}_{res}, \ddot{y}_{res}, \ddot{z}_{res}$	Residual acceleration over the prediction interval, mainly due to		
	the gravitational effects of the Moon and Sun in Cartesian ECEF		
	coordinates (GLONASS model)		
У	Output of a nonlinear transformation function		
$\mathbf{y}(k)$	Output		
$y_1(t), y_2(t)$	Range and range rate corrections respectively		
\mathbf{y}_k	Output vector at time k .		
$\mathcal{Y}_{t,0}^k$	Corrected and linearised single difference of carrier phase		
	measurements from satellite k at two successive epochs t and 0		
Z	Measurement matrix		
\mathbf{Z}^{i}	Direction of sensitivity of the i^{th} accelerometer		
\mathbf{Z}_k	$(m \times 1)$ measurement vector		
Z_m	Discrete measurement of the error in the difference of the phase		
	differentials due to changes in the attitude		
<i>z</i> ₁ , <i>z</i> ₂ , <i>z</i> ₃ , <i>z</i> ₄	Observable variables		
diff of dat	Relative transit time, compared to the first satellite		

const	Arbitrary selected constant to make pseudoranges positive			
finetime	Use for obtaining time resolution better than 200 ns			
$\frac{1}{\beta}$	Time constant			
Γ	Input matrix			
$\hat{\Delta}_{ extsf{DGPS}}$	Basic range space differential correction (for each satellite)			
$\hat{\Delta}_{GPS}$	Broadcast corrections			
$\Delta MP(t)$	Code multipath error			
ΔN	Integer ambiguity difference			
ΔP	Measured code pseudorange single difference between antennae			
$\Delta SA(t)$	Selective availability error			
ΔT	Sample time			
Δn	Additive error in the mean motion			
Δt	Interval between each step			
$\Delta t_{ion}(t)$	Dispersive ionospheric errors			
$\Delta t_r(t)$	Receiver clock bias			
$\Delta t_{sv}(t)$	Satellite clock bias			
$\Delta t_{ir}(t)$	Nondispersive tropospheric error			
Δt_0	Bias in the reference receiver clock			
$\Delta \mathbf{x}$	Corrected error state			
$\Delta \Phi$	Measured carrier phase single difference between antennae			
$\Delta \Psi$	Multipath error for carrier phase			
$\Delta \Psi_{0i}$	Difference in carrier phase multipath error			
$\Delta arepsilon_p$	Receiver code noise difference			
$\Delta {m arepsilon}_{arphi}$	Receiver carrier phase noise difference			
$\Delta arepsilon_{Mp}$	Code pseudorange multipath error difference			
$\Delta arepsilon_{Marphi}$	Carrier phase multipath error difference			
$\Delta \widetilde{m{arepsilon}}_t^k$	Residual correction errors and higher order modelling errors due			
	to linearisation in addition to $\Delta \varepsilon_t^k$			
Δho	Range difference due to spatial separation between antennae			
$\Delta { au}_{0i}$	Difference in code multipath error 24			

$\Delta \phi_{_m}$	Difference in the measured phase differential
$\Delta \left(\widetilde{\phi} + N \right) \cdot \lambda$	Single difference of $(\widetilde{\phi} + N) \cdot \lambda$, either two receivers tracking the
	same satellite or one receiver tracking two satellites
$\nabla\Delta \left(\widetilde{\phi} + N \right) \cdot \lambda$	Double difference of $(\widetilde{\phi} + N) \cdot \lambda$
Ξ	Hour angle of the vernal equinox
Φ	System matrix
$\mathbf{\Phi}_{k-1}$	$(n \times n)$ transition matrix
Ω	Right ascension of the ascending node
$\Omega_{_0}$	Right ascension (GPS ephemeris elements)
Ω	Rate of right ascension (GPS ephemeris elements)
Ω_k	Corrected longitude of the ascending node
α	Scaling parameter between 0 and 1 for UKF filter
α	Reflection coefficient
α'	Correlation ratio
${\gamma}_0$	Reflected signal relative phase at the reference antenna
γ_i	Reflected signal relative phase at antenna i
δ	Latitude measured from $x - y$ plane
$\delta i_k(t)$	Short period correction of the inclination at epoch k
$\delta r_k(t)$	Short period correction of the orbital radius at epoch k
$\delta u_k(t)$	Short period correction of the latitude at epoch k
$\delta \overline{x}_{t,0}$	Relative position of a receiver from the position at time 0
$\eta(t)$	Random code measurement noise
$\zeta(t)$	Random phase measurement noise
ε	Single term by collecting carrier phase multipath, error in the
	calculated ephemeris and random measurement noise
$ heta_0$	Reflected signal elevation
$ heta_0$	Output of the complementary filter
θ_1, θ_2	Outputs of System 1 and System 2 respectively
K	Secondary scaling parameter for UKF filter
λ	Longitude measured from the long end of the body (15° west
	longitude in the case of the Earth)

Wavelength corresponding to the carrier frequency f	
Scaling parameter for UKF filter	
Geodetic latitude	
Geocentric latitude	
standard gravitational parameter ($\mu = GM$)	
Innovation sequence (A white Gaussian noise sequence with zero	
mean when the filter is optimal)	
white noise process, representing the multipath component of the	
noise in the i^{th} code pseudorange measurement	
Receiver random noise measurement	
First order Gauss-Markov multipath error	
Stationary white noise processes corresponding to \hat{v}_k^{ϕ} , \hat{m}_k and \hat{v}_k	
Third state of the Hatch filter, the current complete pseudorange	
error state representing the Hatch filter	
First state of the Hatch filter, the current pseudorange error due to	
ambiguity	
Zero-mean white noise process, representing the receiver noise	
True anomaly	
Elevation angle between the user receiver and the satellite	
Earth-centered, Earth-fixed position vector of the aircraft	
Actual magnitude of the pseudorange vector	
Atmospheric density which is assumed to satisfy a power-law	
function	
Estimation of the pseudorange	
Measurement of code pseudorange	
Measured i^{th} code pseudorange	
Estimate of the code pseudorange	
Reference value of the atmospheric density	
Noise of the smoothing code measurement	

σ_{ϕ} , $\sigma_{ ho_{\mathit{mc}(i)}}$	Standard deviations for the carrier phase measurement noise and		
	the code-based measurement noise respectively		
σ^2	Variance		
τ	Time normalisation		
τ	Period of integration		
$ au_a, \ au_M, \ au_i$	Correlation times of the acceleration, multipath and ionospheric		
	respectively		
$ au_{\phi}$	Filter time constant		
$\hat{ au}_c$	Multipath error for code		
ϕ	Common or geodetic latitude is the angle between the equatorial		
	plane and a line that is normal to the reference ellipsoid		
$\widetilde{\phi}$	Measurement of the full carrier phase		
ϕ_k	Argument of latitude at epoch <i>k</i>		
$\phi_{m(k)}, \phi_{m(k-1)}$	Measured carrier phase at epochs k and $k-1$ respectively		
φ	State transition matrix of clock error bias model		
φ	Rate of change of longitude		
$arphi_0$	Reflected signal azimuth		
χ	Matrix of $2L+1$ sigma vector		
$\overline{\chi}$	Mean of the sigma points vector χ		
$\boldsymbol{\omega} = \boldsymbol{\omega}_b$	Body angular velocity vector		
ω	Argument of perigee		
$\omega(k)$	Input		
ω_{ua}	Rotational angular velocity of the Earth's upper atmosphere,		
	which is assumed to be fixed		
$\mathbf{\omega}_{G}$	Angular velocity vector of the local geodetic frame or n frame		
$\boldsymbol{\omega}_m$	Actual measured angular velocities		
$\boldsymbol{\omega}_{s}, \ \boldsymbol{\omega}_{s}$	Earth angular velocity in the local geodetic frame; Angular		
	velocity of the Earth		
ω_0	Argument of perigee (GPS ephemeris elements)		
1_t^k	Line of sight vector to the satellite k at time t		

LIST OF NOTATIONS

The following is the list of notational conventions, which has been used in this text.

$E\{\cdot\}$	Expectation operator
x	Non-boldfaced variables denote scalar
X	Boldfaced variables denote vector or matrix quantities and also denote
	the actual value of x
Ŷ	Denotes the estimated value of \mathbf{x}
ĩ	Denotes the measured value of \mathbf{x}
$\overline{\mathbf{X}}$	Denotes the mean value of x

CHAPTER 1

INTRODUCTION

Navigation is a very ancient science, which began with human travel. Literally, navigation can be described as to determine the exact position, orientation and velocity of a moving object based upon the previous position (which is also known as dead-reckoning) or with the assistance of a map, celestial charts or any external information (termed as position-fixing) at a specified given time.

Nowadays, space technology has advanced to a stage where humans can use a consumer satellite receiver to pinpoint his/her position at virtually any place in the world. This is thanks to navigation satellites, and the advantages of this technology will be extended even further as the modernisation of the Global Positioning Systems (GPS) takes place as scheduled and the full deployment of GLONASS and Galileo satellite constellations are completed according to time and roadmap.

For civil aviation applications, more accurate and precise data are required. This is where augmentation systems are essential. One common problem faced by navigation satellite users for civil aviation purposes is availability. In order to increase the availability, this work proposes a way to interoperate between several Navigation Satellites, i.e. GPS, GLONASS and GALILEO, and possibly the Chinese COMPASS/Beidou and the Indian IRNSS.

Basic ideas about interoperability and necessary navigation performance are discussed. Also, in this chapter, the motivation and aims of the research are duly explained. Finally, the organisation of the thesis is elaborated.

29

1.1 GLOBAL NAVIGATION SATELLITE SYSTEMS OVERVIEW

An overview of Global Navigation Satellite Systems will be mentioned briefly before going into details in the following subsections. Simply abbreviated as GNSS, it provides autonomous geospatial positioning, navigation and timing (PNT) with worldwide coverage. A GNSS permits small electronic satellite receivers to determine their location (latitude, longitude and altitude or abbreviated as LLA) to within a few metres using electromagnetic signals transmitted along a line of sight by radio from the respective satellites. Fixed position receivers on the ground can be used to compute the precise time as a reference for scientific experiments such as study of earthquakes, and synchronisation of telecommunications networks. The International Committee on Global Navigation Satellite Systems (2007) or in short, ICG has identified and recognised four GNSSes as the current and planned systems providers, namely Global Positioning System (GPS), GLObal Navigation Satellite System (GLONASS), European Satellite Navigation System (Galileo) and COMPASS/BeiDou.

1.1.1 GPS

The GPS is currently the only fully functional GNSS in the world. It is officially named as NAVSTAR GPS, developed and operated by the United States Department of Defense (DoD), initially based on its predecessor, the TIMATION program which successfully launched two satellites named NST I and NST II (NST stands for Navigation Technology Satellite) in 1974 and 1977 respectively (Lasiter and Parkinson 1977; Easton 1980). These were the first satellites in the world to carry atomic clocks, a rubidium and caesium one in turn, and they are considered prototypes of its successor GPS satellites. The first GPS satellite was launched in February 1978. In 1993, initial operational capability (IOC) was formally declared. The constellation of 24 satellites in six orbital planes (Block I and Block II/IIA) was

completed a few months later in 1994. In 1995, full operational capability (FOC) was declared with 24 BLOCK II/IIA satellites launched.

GPS is a complex system divided into 3 segments, the space segment (SS), the control segment (CS), and the user segment (US). The space segment uses a constellation of 24 medium Earth satellites in six orbital planes. The six planes have an approximately 55° inclination (tilt relative to the Earth's equator) and are separated by 60° right ascension of the ascending node (angle along the equator from a reference point to the orbit's intersection). Each GPS satellite revolves approximately 2 complete orbits for each sidereal day. As of April 2007, there are 30 actively broadcasting satellites in the GPS constellation. The additional satellites improve the precision of GPS receiver calculations by providing redundant measurements. With the increased number of satellites, the constellation was changed to a non-uniform arrangement. Such an arrangement was shown to improve reliability and availability of the system, relative to a uniform system, in the event of multiple satellite failure. Massatt and Brady (2002) argue extensively that a single satellite failure for a best six-plane uniform constellation suffered significant losses of accuracy. Although uniform constellation configuration are effective at maximising the number of satellite in view, they do not effectively provide the best geometry to minimise position-estimate errors and actually prevented accurate ranging. In a non-uniform arrangement during satellite outage, degradation in accuracy is less severe while maintaining the availability of the system. Moreover, this asymmetric constellation is less sensitive to satellite drift unlike the uniform configuration.

GPS has a worldwide application as a navigation aid and has become a useful tool for map-making, land-surveying, hydrographic surveying, atmospheric modelling, and the latest application - aircraft structure health monitoring.

CHAPTER 1: INTRODUCTION

Figure 1.1 illustrates the formation of GPS constellation. Detail on the description of GPS is laid out in comparison to other systems as in Table 1.1.



Figure 1.1 GPS constellation (Public Domain Image courtesy of National Executive Committee for Space-Based Positioning, Navigation and Timing (PNT) (2010).)

1.1.2 GLONASS

GLONASS is the former Soviet Union's GNSS answer to GPS and was developed for the Cold War in the 1960s. Nowadays, GLONASS is operated for the Russian government by the Russian Space Forces.

The Russian Federation has proposed the world civil community to provide with a standard accuracy service through GLONASS and it has been officially accepted by the International Civil Aviation Organization (ICAO) and International Maritime Organization (IMO) in 1996. Apart from GPS, GLONASS plays it roles in providing

users with navigation service and precise timing. Although officially declared operational on September 24, 1993, IOC (Initial Operational Capability) by decree of the president of the Russian Federation (Prasad and Ruggieri 2005) and reaching full constellation of 24 satellites in 1995 (Alkan et. al. 2005), the GLONASS system was never brought to completion. This is mainly due to a lack of necessary funding after the collapse of the Soviet Union, while other reasons are insufficient motivation following the end of the Cold War, no solid prospect of civilian applications and related market opportunities. Currently, GLONASS is in the process of being restored to full operation.

1.1.3 Galileo

The Galileo positioning system is named after the Italian astronomer, Galileo Galilei. It is called Galileo to distinguish it from the United States GPS. Galileo will be the third GNSS in the world after GPS and GLONASS. Galileo is built by European Satellite Navigation Industries for the European Union (EU) and European Space Agency (ESA) and is expected to be fully operational by 2012.

1.1.4 COMPASS/BeiDou

China has stated the intention to expand its regional Beidou navigation system into a GNSS. It has been reported in Inside GNSS News (2008) that China is planning to launch 10 COMPASS satellites (Beidou-2) during the next two years in order to create a regional positioning, navigation, and timing (PNT) capability in the Asia-Pacific region by 2010 and turn it into a full-fledged GNSS system within a few years. The system is designed to consist of 5 geosynchronous satellites and 30 MEO spacecraft. By 2020, it is anticipated that COMPASS will reach its Full Operational Capacity (FOC).

1.2 GPS/GLONASS/GALILEO SYSTEM COMPARISON

	GPS	GLONASS	Galileo
Country	United States	Russia	European
Carrier Frequencies	L1: 1575.42	L1: 1602 + 0.5625n	E5a ,E5b: 1164-1215
(in MHz)	L2: 1227.60	L2: 1246 + 0.4375n	E6: 1215-1300
	L5: 1176.45	n is the frequency	E2-L1-E1: 1559-1592
		channel number	
		(n=0,1,)	
Channel Access	CDMA	FDMA (current)	CDMA
Method		CDMA (preliminary)	
Number of Satellites	24	24	30
in the Constellation			
Orbital Planes	6	3	3
Orbit Inclination	55°	64.8°	56°
Orbital Altitude	20200 km	19100km	23200 km
Orbital Period	11 hours and 58	11 hour and 15 minutes	14 hour and 4 minutes
	minutes		
Launch Vehicle	Delta 2-7925	Proton K/DM-2	Ariane V, Proton,
			Soyuz etc.
Coordinates system	WGS84	PZ90	WGS84
Number of	15 ephemeris + 1	9 ephemeris + 1 clock	-
Ephemeris Elements	clock data + 5 clock	data elements	
	correction elements		
Intersatellite Links	Yes	GLONASS: No	No
		GLONASS-M, -K: Yes	

The following Table 1.1 compares the characteristics of GPS, GLONASS and Galileo.

Table 1.1 GPS, GLONASS and Galileo attribute comparison.

The main difference between GLONASS compared with other GNSS systems is that GLONASS uses frequency division multiple access (FDMA) for channel access method, rather than code division multiple access (CDMA). FDMA gives users an individual allocation of one or several frequency bands or channels, while CDMA uses a special coding scheme (where each transmitter is assigned a code) to allow multiple users to be multiplexed over the same physical channel. Furthermore, CDMA is a form of spread-spectrum signalling where the modulated coded signal has a much higher data bandwidth than the data being communicated. The term 'Multiple Access' is referring to the coordination access between multiple users.

According to Hein (2006), it is this difference "which hinders its full integration into a future Global Navigation Satellite System of Systems". Since GPS, Galileo and COMPASS use the same signal access scheme; there is a huge possibility for integration at signal level, whereas, integration of these systems with GLONASS is only achievable at system level.

1.3 NAVIGATION SATELLITE AUGMENTATION SYSTEMS

The performance of a basic stand-alone GNSS receiver can be greatly enhanced by the technique of GNSS augmentation through the integration of external information into the calculation process. Several types of augmentation systems will be elaborated on this section.

1.3.1 Differential Systems: DGPS Overview

Differential Systems are supplementary navigation systems to the generic GNSS term for Differential GPS (DGPS) in that they use a network of fixed ground-based local reference stations to broadcast the correction between the instantaneous satellite systems positions and the precisely surveyed positions of the stations, which is then transmitted to user receivers. These stations broadcast the difference between the measured pseudoranges (calculated from relative pseudorange using the sampling frequency) and the actual pseudoranges (using the actual precise position of the station). The reference station computes differential corrections for its own location and time. The reference station will then remove the clock biases (satellite and receiver station clock) before transmitting the correction signal to user receivers. The distance between the users and the nearest station may be within 200 nautical miles radius (approximately 370 km), but as the distance increases, the accuracy of the
transmitted correction signal decreases. The problem becomes more cumbersome if both are not able to observe the same satellites.

Due to wide coverage of GPS, Differential GPS (DGPS) are commonplace. Currently nearly all commercial GPS units available on the market offer DGPS data inputs, which provide better positional accuracy.

1.3.2 Ground-based Augmentation Systems (GBAS)

Ground-based augmentation systems (GBAS) is a localised reference system within 20 km that supports navigation satellite augmentation through the use of terrestrial radio messages composed of an individual or a network of accurately surveyed ground stations, which take measurements concerning the GNSS, and one or more radio frequency signals, which transmit the information directly to the user receiver.

One example of GBAS implementations is the United States Local Area Augmentation System (LAAS). It is important to note that GBAS is a generic term, while DGPS is a differential system specifically designed for GPS.

1.3.3 Satellite-based Augmentation Systems (SBAS)

A satellite-based augmentation system (SBAS) is an augmentation system which consists of the satellite and a network of multiple precisely surveyed ground-based stations that supplements navigation aid to a specific region through additional satellite by broadcast messages. The ground stations take measurements of the observed GNSS satellites, the satellite signals or other environmental factors, which may affect the signal received by the users. Later these measurements are used to create the information messages and are sent to one or more SBAS satellites for broadcast to the users. A typical SBAS is normally a regional satellite system, which covers a regional area in comparison to a GNSS, which has a worldwide coverage. Several SBAS implementations namely according to International Committee on Global Navigation Satellite Systems (2007) are the United States' Wide Area Augmentation System (WAAS), the Russian Federation's System of Differential Correction and Monitoring (SDCM), the European Geostationary Navigation Overlay Service (EGNOS), the Indian GPS Aided GEO Augmented Navigation (GAGAN) system, Japan's Multi-Functional Satellite Augmentation System (MSAS) system and Quasi-Zenith Satellite System (QZSS).

1.3.4 Aircraft-based Augmentation Systems (ABAS)

The concept of aircraft-based augmentation systems is an integration of information acquired with GNSS and onboard aircraft information. The augmentation systems blend additional information from satellite navigation systems and other navigation aids, i.e. Distance Measuring Equipment (DME), VHF Omnidirectional Range (VOR), LORAN-C, inertial navigation sensors and other navigation sensors.

Mixing information between GPS and INS derived position and velocity can improve accuracy due to the fact that they have complementary characteristics. This simple scheme of combining deterministic signals is known as complementary filtering and feasible for ABAS.

1.4 INTEROPERABILITY

The Institute of Electrical and Electronics Engineers (1990) defines interoperability as:

The ability of two or more systems or components to exchange information and to use the information that has been exchanged.

Also, Pridmore and Rumens (1989) define interoperability as

The ability of systems, units or forces to provide services to and accept services from other systems, units or forces and to use the services so exchanged to enable them to operate together effectively.

In the same text, Pridmore and Rumens (1989) also outline the Interoperability Requirements (IORs) as

An operationally recognisable activity or sequence of activities that has a definable starting action, a definable concluding action, and which involves the exchange of information between two or more platforms. Such an information exchange may be interactive and may involve the use of more than one transfer medium, however, the information content on all transfer media must be definable.

Hein (2006) categorises the interoperability terms into two specific definitions, namely system interoperability – where different GNSS systems provide the same answer, within the specified accuracy of each individual system, and signal interoperability – in which different GNSS systems transmit signals allowing them to combine in a "simple" receiver for a combined PNT solution.

The research focus of this thesis is on the facilitation and integration of system interoperability. This work found that adaptive filtering provides the ability to operate in

the presence of uncertainties in the noise statistics of satellite-based positioning and consequently facilitates interoperability between various types of GNSS systems at a system level.

1.5 ACCURACY VERSUS INTEGRITY

For an aircraft to operate within a defined airspace, the aircraft must meet necessary navigation performance requirements or otherwise known as Required Navigation Performance (RNP). The International Civil Aviation Organization (ICAO) developed this approach in the early 1990s due to the strict necessities of aviation safety. These requirements are defined as follows (ICAO 1999):

- *Accuracy*: The degree of conformance between the estimated or measured position and/or the velocity of a platform at a given time and its true position and/or velocity. Radio navigation performance accuracy is usually presented as a statistical measure of system error and is specified as:
 - Predictable: The accuracy of a position in relation to the geographic or geodetic coordinates of the Earth.
 - *Repeatable*: The accuracy with which a user can return to a position whose coordinates have been measured at a previous time with the same navigation system.
 - *Relative*: The accuracy with which a user can determine one position relative to another position regardless of any error in their true position.
- *Integrity*: The ability of a system to provide timely warnings to users when the system should not be used for navigation. In particular, the system is required to deliver to the user an alert within the *time* to alert when an *alert limit* is exceeded. The alert limit is the maximum allowable in the user-computed

position solution; the alert limit can be specified in horizontal alert limit (HAL) and vertical alert limit (VAL).

- *Continuity*: The continuity of a system is the capability of the total system (including all elements necessary to maintain aircraft position within the defined airspace) to perform its function without non-scheduled interruptions during the intended operation. The continuity risk is the probability that the system will be unintentionally interrupted and will not provide guidance information for the intended operation. More specifically, continuity is the probability that the system will be available for the duration of a phase of operation, presuming that the system was available at the beginning of that phase of operation.
- Availability: The availability of a navigation system is the percentage of time that the system is performing a required function under stated conditions. Availability is an indication of the ability of the system to provide a usable service within the specified coverage area. Signal availability is the percentage of time that the navigation signals transmitted from external sources are available for use.

1.6 MOTIVATION AND AIMS OF RESEARCH

The aim of this research is to develop differential satellite navigation reference station algorithms as well as mixing filter algorithms to facilitate interoperability. Two and three frequency reference station algorithms are developed that may be employed with any navigation satellite. The motivation behind the design of the algorithms has been the need for reference station algorithms that can deal with an interoperable system of navigation satellites to obtain high accuracy positioning information local to the roving vehicle.

CHAPTER 1: INTRODUCTION

The main focus of this research is to utilise adaptive extended Kalman filter and adaptive unscented Kalman filter to process a variety of satellites based on Method of Maximum Likelihood Estimation. (MMLE). The filters designed are tested with simulated GPS data by adding error models into the predicted pseudorange. Adaptive extended Kalman filtering can also be employed to estimate the ephemeris parameters of the orbiting satellites, while the adaptive unscented Kalman filter based mixing filters can be used to develop a high-precision kinematics satellite aided inertial navigation satellite with a modern receiver that incorporates carrier phase smoothing and ambiguity resolution.

1.7 THESIS ORGANISATION

This thesis is organised in 8 chapters. The chapters are summarised as follows:

- In this chapter, several sections are presented, covering an overview of global navigation satellite systems, GPS/GLONASS/Galileo system comparison, navigation satellite augmentation systems, interoperability, accuracy versus integrity, motivation and aims of research, and finally the thesis organisation.
- Chapter 2 starts with an introduction to orbital dynamics, discusses the issues related to the prediction and transformation of orbital ephemeris in real time and specific elaboration about NORAD TLE and NORAD SPACETRACK propagation models.
- The emphasis of Chapter 3 is on estimating and measuring a position using satellite-based measurements.

CHAPTER 1: INTRODUCTION

- The focus of Chapter 4 is about error modelling and estimation for interoperability.
- Chapter 5 discusses the selection of an appropriate orbit dynamics propagation model for the purpose of orbit estimation and solves the orbit filtering problem by applying the unscented Kalman filter. A host of fixed and adaptive UKFbased orbit estimation methods are validated.
- Chapter 6 is on enhanced accuracy algorithms, which is about the use of carrier phase measurements or real-time kinematics to increase navigational accuracy.
- Chapter 7 covers the interoperable mixing filters whereby a generic satellite navigation system and INS measurement can be mixed to produce more accurate estimates of position and velocity.
- Chapter 8 is the conclusion of the thesis, and includes a summary of the findings made during the course of this research, recommendation for future work and the principal contributions and achievements of the thesis.

CHAPTER 2

REAL-TIME ORBIT PREDICTION AND PSEUDORANGE MEASUREMENT

There are two ways in which the direction of this research can be taken. The first is by taking the navigation satellite data from a real satellite receiver and using this data to feed into a program which calculates its positions and velocities instantaneously. Secondly, a more profound way is to simulate the satellite orbit itself given a set of parameters. This research has selected the second way as the direction to go forward in predicting any catalogued satellite orbiting in real time.

The purpose of this chapter is not to present an all-inclusive review of position location methods; rather it is to provide a basic understanding of orbital navigation techniques and how the theoretical aspect can be applied practically.

2.1 INTRODUCTION TO ORBITAL DYNAMICS

The first essential requirement in orbital determination is to establish the coordinate system and hence, defines its origin. In this work, since the satellites revolve around the Earth, the Earth's centre is taken as the origin of the 3-axes Cartesian coordinate.

Next comes the orientation of the object concerned. There are two ways to describe the orbit of a celestial body. Firstly, by using a state vector which has three parameters (x, y, z) for position and another three parameters $(\dot{x}, \dot{y}, \dot{z})$ for velocity. Secondly, an orbit can have a set of parameters which describe the size and the shape of the orbit, the

plane and the direction of rotation, angle between the equatorial plane and the plane of rotation, the time or angle for one revolution. This set of parameters is called the orbital elements. This will be discussed further in the next section.

2.2 Orbit Position Determination: The Keplerian Elements

The German astronomer Johannes Kepler [1571-1630] formulated three empirical laws of planetary motion based on astronomical data provided to him by the Danish astronomer Tycho Brahe in the late 1590's. The laws were published over a period spanning a decade at about the same time as Galileo was making his landmark astronomical observations. The laws and its mathematical formulae are:

i) The orbit of each planet is an ellipse with the Sun at one focus;

$$r = \frac{p}{1 + e\cos\theta} \tag{2.1}$$

where r and θ are heliocentric polar coordinates for the planet, p is the semi-latus rectum, and e is the eccentricity, which is less than one.

ii) The line joining the Sun to the planet sweeps out equal areas in equal lengths of time; and finally,

$$M = \frac{2\pi \cdot t}{T} \tag{2.2}$$

where M is the mean anomaly, T is the orbital period of the planet and t is the time since periapsis (or perihelion if the Sun was the centre).

iii) The squares of the orbital periods of the planets are proportional to the cubes of their mean distances from the Sun.

$$T^2 \propto a^3 \tag{2.3}$$

where T is the orbital period of the planet and a is the semi-major axis of the orbit. For satellites, whose mass is negligible compared to that of the Earth, the formula becomes

$$T = 2\pi \sqrt{\frac{a^3}{gR^2}}$$
(2.4)

where T and a are the same as in the previous equation, R is the radius of Earth and g is the gravitational attraction at the surface of the Earth.

Seven numbers, known as satellite orbital elements are required to define a satellite's orbit about a planet. This set of seven numbers is called the satellite's "Keplerian" orbital elements, or simply just elements. These numbers define an ellipse, its orientation about the planet and place the satellite on the ellipse at a particular time. In the Keplerian model, satellites orbit in an ellipse of constant shape and orientation. Uniquely associated with an ellipse are two *foci* and when these two foci coincide, the orbit is circular with a constant radius. The planet is at one focus of the ellipse, not the centre (unless the orbit ellipse is actually a perfect circle). The point on the orbit closest to this focus is the *perigee* while the farthest point is the *apogee*. The minimum separation between the satellite and the planet is said to be at *periapse* and the maximum at *apoapse*. The direction of a satellite or other body travelling in orbit can be direct, or <u>prograde</u>, in which the satellite moves in the same direction as the planet rotates, or <u>retrograde</u>, going in a direction opposite to the planet's rotation.

The primary orbital elements are numbers that: i) orient the orbital plane in space; ii) orient the orbital ellipse in the orbital plane; iii) specify its shape and size; iv) locate the satellite in the orbital ellipse. These elements are epoch, orbital inclination, right ascension of the ascending node, argument of perigee, eccentricity, mean motion, mean anomaly and drag, which will be defined in greater detail in subsequent paragraphs. The orbital elements is visualised by Figure 2.1.



Figure 2.1 Diagram of orbital elements.

Epoch: A set of orbital elements is a snapshot, at a particular time, of the orbit of a satellite. Epoch is simply a number, which specifies the time at which the snapshot was taken.

Orbital Inclination (*i*): The orbit ellipse lies in a plane known as the orbital plane. The orbital plane always goes through the centre of the planet, although it may be tilted at any angle relative to the equator. Inclination is the angle between the orbital plane and the equatorial plane. By convention, inclination is a number between 0 and 180 degrees. Orbits with inclination near 0 degrees are called equatorial orbits while orbits with an inclination near 90 degrees are called polar. The intersection of the planet's equatorial plane (ecliptic plane) and the orbital plane is a line which is called the line of nodes. <u>Nodes</u> are points where an orbit crosses a plane. When an orbiting body crosses the ecliptic plane going north, the node is referred to as the <u>ascending node</u>, while it is known as the <u>descending node</u> when it is south bound.

Right Ascension of the Ascending Node (Ω) is the second element that orients the orbital plane in space. Once the orbital inclination is defined there are an infinite number of orbital planes possible. Of the two nodes on the line of nodes one is the ascending node where the satellite crosses the equator going from south to north. The other is called the descending node where the satellite crosses the equator going from north to south. By convention, one specifies the location of the ascending node by the expression "right ascension of ascending node" which is an angle, measured at the centre of the planet, from the vernal equinox, a reference point in the sky where right ascension is defined to be zero, to the ascending node. It is an angle measured in the equatorial plane from the vernal equinox.

Argument of Perigee (ω): Argument is yet another word for angle. Once the orbital plane is oriented in space, it is essential to orient the orbit ellipse in the orbital plane. This is done by a single angle element known as the argument of perigee.

Eccentricity (e): In the Keplerian orbit model, the satellite orbit is an ellipse. Eccentricity tells us the "shape" of the ellipse. When e = 0, the ellipse is a circle. As e approaches 1, so the ellipse becomes longer and narrower. So far the orbital elements define the orientation of the orbital plane, the orientation of the orbit ellipse in the orbital plane, and the shape of the orbit ellipse. One still needs to define the "size" of the orbit ellipse.

Mean motion is usually given in units of revolutions per day. Kepler's third law of orbital motion gives us a precise relationship between the speed of the satellite and its distance from the planet. So by specifying the speed of the satellite or its mean motion, it is possible to define the size of the orbit. Sometimes the *semi-major axis* (a) is specified instead of mean motion. The semi-major axis is one-half the length (measured the long way) of the orbit ellipse, and is directly related to mean motion by a simple equation. It now remains to specify exactly where the satellite is on this orbit ellipse at a particular time.

Mean anomaly is simply an angle that marches uniformly in time from 0 to 360 degrees during one revolution. It is defined to be 0 degrees at perigee, and therefore is 180 degrees at apogee. It is related to the *true anomaly*, v_0 which is often employed as an alternate element. It is a term used to describe the locations of various points in an orbit. It is the angular distance of a point in an orbit past the point at periapsis, the point on the orbit referred to as the perigee, measured in degrees at the focus nearer to the perigee, which is also where the planet's centre is located. For example, a satellite might cross a planet's equator at 30° true anomaly, which defines where the satellite is on this orbit ellipse at a particular time.

There is one important, but optional secondary element: the *drag*. The drag orbital element defines the rate at which mean motion is changing due to drag or other related effects.

2.2.1 Transformation to Position and Velocity

If the six Keplerian orbital elements $(a, e, i, \Omega, \omega, \text{ and } v_0)$ and the standard gravitational parameter μ of a celestial body are available, the orientation representation can be transformed to a state vector consisting of position (x, y, and z) and velocity $(\dot{x}, \dot{y}, \text{ and } \dot{z})$ and vice-versa. The method used here is transformation from Keplerian orbital elements to Geocentric-Equatorial frame or ECI via the Perifocal frame.

$$p = a \cdot \left(1 - e^2\right) \tag{2.5}$$

$$\begin{aligned} r_{p} &= \begin{bmatrix} \frac{p \cdot \cos v_{0}}{(1 + (e \cdot \cos v_{0}))} \\ \frac{p \cdot \sin v_{0}}{(1 + (e \cdot \cos v_{0}))} \\ 0 \end{bmatrix} \end{aligned} \tag{2.6}$$

$$v_{p} &= \begin{bmatrix} \sqrt{\frac{\mu}{p}} \cdot \sin v_{0} \\ \sqrt{\frac{\mu}{p}} \cdot (e + \cos v_{0}) \\ 0 \end{bmatrix}$$

$$R &= \begin{bmatrix} (\cos(\Omega) \cdot \cos(\omega)) - (\sin(\Omega) \cdot \sin(\omega) \cdot \cos(i)) & -(\cos(\Omega) \cdot \sin(\omega)) - (\sin(\Omega) \cdot \cos(\omega) \cdot \cos(i)) & (\sin(\Omega) \cdot \sin(i)) \\ 0 & -(\cos(\Omega) \cdot \cos(\omega)) + (\cos(\Omega) \cdot \sin(\omega) \cdot \cos(i)) & -(\sin(\Omega) \cdot \sin(\omega)) + (\cos(\Omega) \cdot \cos(\omega) \cdot \cos(i)) & -(\cos(\Omega) \cdot \sin(i)) \\ \sin(\omega) \cdot \sin(i) & \cos(\omega) \cdot \sin(i) & \cos(i) \end{bmatrix}$$

$$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \times r_p$$
(2.9)

$$v = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = R \times v_p \tag{2.10}$$

2.2.2 The GPS Ephemeris

A GPS satellite transmits 16 ephemeris elements and 5 satellite clock correction elements. These elements are then used to define the position of a satellite at a certain time. Table 2.1 lists GPS ephemeris and clock correction elements into two separate categories.

Inclination (i_0) Right Ascension (Ω_0) Argument of Perigee (ω_0) Time of Ephemeris (<i>Toe</i>) Eccentricity (<i>e</i>) Mean Anomaly (M_0) Amplitudes of sine & cosine harmonics correction (Short Period) terms for:- • Semi-major axis (<i>a</i>) of orbital radius (C_{rs}, C_{rc}); • of inclination (C_{is}, C_{ic}); • of argument of latitude (C_{us}, C_{uc}); Mean Motion Difference (<i>dn</i>)	Satellite group delay differential (T_{GD}) Satellite clock correction (t_{oe}) Satellite clock correction (a_{f2}) Satellite clock correction (a_{f1}) Satellite clock correction (a_{f0})
Rate of Right Ascension ($\dot{\Omega}$) Rate of Inclination (\dot{i})	

Table 2.1 GPS ephemeris and clock correction elements.

2.3 Orbit Prediction: Error Modelling

Although the Keplerian elements allow for the accurate modelling of the orbit of a planetary satellite, they ignore all perturbation effects and the standard approach to include these effects is defined by the Lagrange Planetary equations. There are several forms of these equations in use but the primary form is the one that may be employed for both short term and long term orbit predictions.

Solutions to the Lagrange Planetary equations, applied to navigation satellites, have a generic structure. Thus errors to the Keplerian elements may be expressed by a set of equations as follows. The argument of the latitude is defined as:

$$\phi_k = \omega + \nu_k \tag{2.11}$$

The latitude, the orbit radius and the inclination may be expressed as the sum of the slowly varying and short period terms as:

$$u_k = \phi_k + \delta u_k(t) \tag{2.12}$$

$$r_{k} = a(1 - e\cos E_{k}) + \delta r_{k}(t)$$
(2.13)

$$i_k = i_0(t) + \dot{i}_0(t) \times t_k + \delta r_k(t)$$

$$(2.14)$$

where the short period corrections are given by,

$$\delta u_k(t) = C_{uc}(t) \cdot \cos 2u_k + C_{us}(t) \cdot \sin 2u_k \tag{2.15}$$

$$\delta r_k(t) = C_{rc}(t) \cdot \cos 2u_k + C_{rs}(t) \cdot \sin 2u_k \tag{2.16}$$

$$\delta i_k(t) = C_{ic}(t) \cdot \cos 2u_k + C_{is}(t) \cdot \sin 2u_k \tag{2.17}$$

Finally, the corrected longitude of the ascending node may be expressed as,

$$\Omega_k = \Omega_0(t) + \dot{\Omega}_0(t) \cdot t_k - \Omega_e \cdot t$$
(2.18)

where the time since the ephemeris is given by, $t_k = t - Toe$ and $\Omega_e = 7.2921151467 \cdot 10^{-5} \text{ rad s}^{-1}$, is the mean rotation rate of Earth.

Such a model reflects the fact that the slowly varying elements consist of the Keplerian terms, secular terms which are linear in t, and the long period variations which are modelled in terms of the time varying coefficients. Along with the additive error in the mean motion, Δn , and the time of the ephemeris, *Toe*, one has a set of 16 elements defining the orbit in this model. It is the primary model adopted in a GPS system where the GPS ephemeris (the 16 elements), for each satellite, is the core of the navigation message provided to the user at regular time intervals to facilitate the computation of the satellite's orbits. It constitutes one of the early error prediction models adopted for navigation satellite position prediction. It is important to note that the error prediction model must be compatible with the way the element sets are defined and therefore not easy to convert from one type of an error prediction model to another without further physical consideration. This issue will be discussed with an example in a later section.

2.4 SHORT TERM PREDICTION: THE GLONASS APPROACH

The GLONASS system, which was first established by the erstwhile Soviet Union, follows a totally different approach to the issue of error prediction modelling. The GLONASS satellites operate in a middle Earth orbit - a 19,100 km circular orbit, 64.8° inclination and a period of 11 hours and 15 minutes. Yet the error model used for these satellites was derived from the motions of satellites in the highly eccentric Molniya orbits which exhibit peculiar mean-motion behaviours. This is due to perturbations resulting from the Earth's oblateness. Satellites in a Molniya orbit are in a 2-to-1 mean-motion resonance with the Earth's spin, they follow the same ground track on each orbit, with two alternating geostationary perigees, separated by 180 degrees of terrestrial longitude, over the southern hemisphere. Although the peculiarities of the Molniya orbits are absent in the GLONASS satellites as they follow a circular orbit, the Molniya type error model was adopted for the GLONASS satellites and it is different in nature to the GPS ephemeris. The GLONASS navigation message consists, besides some other information, of 9 ephemeris states:

 x_0, y_0, z_0 : Position components in Cartesian Earth Centred Earth Fixed frame (ECEF),

 $\dot{x}_0, \dot{y}_0, \dot{z}_0$: Velocity components in Cartesian ECEF coordinates,

 $\ddot{x}_{res}, \ddot{y}_{res}, \ddot{z}_{res}$: Residual acceleration over the prediction interval, mainly due to the gravitational effects of the Moon and Sun in Cartesian ECEF coordinates and *Toe* is the reference time of ephemeris.

A broadcast message as described above is based on a dynamic model for the acceleration, referenced in an ECEF frame. In addition to the normal central force field and the acceleration components, the dynamics account for the primary Earth oblateness represented by the C_{20} coefficient. The equations of motion are then expressed as:

$$dx/dt = \dot{x}, \ dy/dt = \dot{y}, \ dz/dt = \dot{z},$$
 (2.19)

$$\frac{d\dot{x}}{dt} = -\frac{\mu}{r^3}x + \frac{3}{2}\left(\frac{\mu C_{20}}{r^5}\right)x\left(1 - 5\left(\frac{z}{r}\right)^2\right) + \Omega_e^2 x + 2\Omega_e \dot{y} + \ddot{x}_{res}$$
(2.20)

$$\frac{d\dot{y}}{dt} = -\frac{\mu}{r^3} y + \frac{3}{2} \left(\frac{\mu C_{20}}{r^5}\right) y \left(1 - 5\left(\frac{z}{r}\right)^2\right) + \Omega_e^2 y - 2\Omega_e \dot{x} + \ddot{y}_{res}$$
(2.21)

$$\frac{d\dot{z}}{dt} = -\frac{\mu}{r^3} z + \frac{3}{2} \left(\frac{\mu C_{20}}{r^5} \right) z \left(3 - 5 \left(\frac{z}{r} \right)^2 \right) + \ddot{z}_{res}$$
(2.22)

where μ is the gravity parameter. These equations are integrated numerically to obtain the orbit parameters. However there are two difficulties with the GLONASS error model in the context of interoperability. Firstly, the equations are not in a form that is easily integrable in real time. The orbit equations are fundamentally nonlinear and singular at certain points in the domain. It is often essential to predict and propagate a state vector where the initial data is relatively unbalanced. In such cases the numerical prediction is facilitated by a suitable scaling transformation of the state variables. As the equations are nonlinear, scaling has a dissimilar effect on each of the variables being propagated and appropriate scaling facilitates the prediction of the states with relatively uniform and small errors in all of them. Secondly the error model is inconsistent with other error models. Yet it is indeed possible to address both issues.

The solution to the inconsistency problem is to introduce the full Earth gravity model. The longitudinal variations of the Earth's gravitational perturbation potential are expressed by its tesseral coefficients. Because the resonance is 2-to-1, only the coefficients of even azimuthal numbers cause secular changes in mean motion. These coefficients, to a good approximation, describe an ellipsoidal, or "triaxial" potential, that is, one which may be described by three axes, two of which lie in the equatorial plane. The long axis of this geoidal model of the Earth lies along 15° west longitude and 165° east longitude. Introducing the Earth's full gravitational perturbation potential (refer to Appendix G), the last three equations in the GLONASS error model are expressed as,

$$\frac{d\dot{x}}{dt} = -\frac{\mu}{r^3}x + \frac{\partial U_2}{\partial x} + \Omega_e^2 x + 2\Omega_e \dot{y} + \ddot{x}_{res}$$
(2.23)

$$\frac{d\dot{y}}{dt} = -\frac{\mu}{r^3} y + \frac{\partial U_2}{\partial y} + \Omega_e^2 y - 2\Omega_e \dot{x} + \ddot{y}_{res}$$
(2.24)

$$\frac{d\dot{z}}{dt} = -\frac{\mu}{r^3} z + \frac{\partial U_2}{\partial z} + \ddot{z}_{res}$$
(2.25)

The Earth's gravitational perturbation potential can be expressed as,

$$U_{2} = \frac{\mu C_{20}}{r^{3}} \left(1 - \frac{3}{2} \cos^{2} \delta \right) + \frac{3\mu C_{22}}{r^{3}} \cos^{2} \delta \cos 2\lambda$$
(2.26)

where *r* is the distance of the body centre of mass, δ is the latitude measured from x - y plane, and λ is the longitude measured from the long end of the body (15° west longitude in the case of the Earth). In Cartesian coordinates the potential may be expressed as,

$$U_{2} = -\mu C_{20} \frac{\left(x^{2} + y^{2} - 2z^{2}\right)}{2r^{5}} + 3\mu C_{22} \frac{\left(x^{2} - y^{2}\right)}{r^{5}}$$
(2.27)

where $r = \sqrt{x^2 + y^2 + z^2}$, $\sin \delta = \frac{z}{r} = \frac{z_i}{r}$, $\tan \lambda = \frac{y}{x}$, $\tan(\lambda + \Omega_e t) = \frac{y_i}{x_i}$ and

 (x_i, y_i, z_i) is the position vector of the satellite in an Earth-centred inertial (ECI) coordinate frame.

To facilitate the easy numerical integration of these equations in real time, a length scale and a time normalisation defined by,

$$r_{geo} = \left(\frac{\mu}{\Omega_e^2}\right)^{\frac{1}{3}}$$
(2.28)

$$\tau = \Omega_e t \tag{2.29}$$

are introduced. The length scale r_{geo} is the resonance radius where the point mass gravitational attraction of the Earth equals centripetal acceleration with the satellite rotating at the Earth's rotation rate. The time parameter τ is equivalent to the rotation

angle of the Earth, with one full rotation every 2π radians. By normalising the distances by the resonance radius, r_{geo} , the velocities by $\Omega_e r_{geo}$ and the oblateness parameters C_{2i} , for i = 0, 2, by r_{geo}^2 , and by employing τ as the independent variable, the equations may be written in terms of the normalised position and velocity coordinates as,

$$d\tilde{x}/d\tau = x', \ d\tilde{y}/d\tau = y', \ d\tilde{z}/d\tau = z'$$
(2.30)

$$\frac{d\tilde{x}'}{d\tau} = -\frac{1}{\tilde{r}^3}\tilde{x} + \frac{\partial U_2}{\partial \tilde{x}} + \tilde{x} + 2\tilde{y}' + \tilde{x}''_{res}$$
(2.31)

$$\frac{d\tilde{y}'}{d\tau} = -\frac{1}{\tilde{r}^3}\tilde{y} + \frac{\partial U_2}{\partial \tilde{y}} + \tilde{y} - 2\tilde{x}' + \tilde{y}''_{res}$$
(2.32)

$$\frac{d\tilde{z}'}{d\tau} = -\frac{1}{\tilde{r}^3}\tilde{z} + \frac{\partial U_2}{\partial \tilde{z}} + \tilde{z}''_{res}$$
(2.33)

where,

$$\frac{\partial U_2}{\partial \widetilde{x}} = -\frac{\widetilde{C}_{20}\widetilde{x}}{\widetilde{r}^5} + \frac{5\widetilde{C}_{20}\widetilde{x}}{2\widetilde{r}^7} \left(\widetilde{x}^2 + \widetilde{y}^2 - 2\widetilde{z}^2 \right) + \frac{6\widetilde{C}_{22}\widetilde{x}}{\widetilde{r}^5} - \frac{15\widetilde{C}_{22}\widetilde{x}}{\widetilde{r}^7} \left(\widetilde{x}^2 - \widetilde{y}^2 \right)$$
(2.34)

$$\frac{\partial U_2}{\partial \tilde{y}} = -\frac{\tilde{C}_{20}\tilde{y}}{\tilde{r}^5} + \frac{5\tilde{C}_{20}\tilde{y}}{2\tilde{r}^7} \left(\tilde{x}^2 + \tilde{y}^2 - 2\tilde{z}^2\right) + \frac{6\tilde{C}_{22}\tilde{y}}{\tilde{r}^5} - \frac{15\tilde{C}_{22}\tilde{y}}{\tilde{r}^7} \left(\tilde{x}^2 - \tilde{y}^2\right)$$
(2.35)

$$\frac{\partial U_2}{\partial \tilde{z}} = 2 \frac{\tilde{C}_{20}\tilde{z}}{\tilde{r}^5} + \frac{5\tilde{C}_{20}\tilde{z}}{2\tilde{r}^7} \left(\tilde{x}^2 + \tilde{y}^2 - 2\tilde{z}^2\right) - \frac{15\tilde{C}_{22}\tilde{z}}{\tilde{r}^7} \left(\tilde{x}^2 - \tilde{y}^2\right)$$
(2.36)

where,

$$\begin{split} \widetilde{x} &= \frac{x}{r_{geo}}, \ \widetilde{y} = \frac{y}{r_{geo}}, \ \widetilde{z} = \frac{z}{r_{geo}}, \ \widetilde{r} = \frac{r}{r_{geo}}, \ \widetilde{x}' = \frac{\dot{x}}{\left(\Omega_e r_{geo}\right)}, \ \widetilde{y}' = \frac{\dot{y}}{\left(\Omega_e r_{geo}\right)}, \ \widetilde{z}' = \frac{\dot{z}}{\left(\Omega_e r_{geo}\right)} \end{split} \text{ and} \\ \widetilde{C}_{2i} &= \frac{C_{2i}}{r_{geo}^2}. \end{split}$$

The oblateness coefficients, C_{2i} , are also related to the principal moments of inertia of the Earth and could be expressed in terms of alternate relationships to the standard zonal harmonic coefficients, $J_2 = 1.082616 \times 10^{-3}$, $J_3 = -2.53881 \times 10^{-6}$, $J_4 = -1.65597 \times 10^{-6}$ and to $J_{21} = 0$, $J_{22} = 1.86 \times 10^{-6}$, $J_{31} = 2.1061 \times 10^{-6}$. This standard zonal harmonics

coefficients in the generic term, J_n reflect the mass distribution of the Earth independent of the longitude and dominates the gravitational pertubative influences of the Earth. These coefficients have mainly be determined from the motion of Earth-orbiting spacecraft (Fortescue, Stark and Swinerd 2003, pg. 95-96). More definition of the standard zonal harmonics and Earth's gravitational function is described in Appendix G.

MATLAB simulations of the above equations indicate that they are now in a form suitable for numerical integration. The period of integration is usually in the range of about, $\tau = 0.01 - 0.2$.

2.5 NORAD METHOD FOR ORBIT PROPAGATION

In the United States, it is the responsibility of North American Aerospace Defense (NORAD) Command to maintain general perturbation element sets on all resident space vehicles. These Two-Line-Element (TLE) data sets for a specific satellite each consist of two 69-character lines of data. These element sets are periodically updated so as to maintain a reasonable prediction capability on all space vehicles. In turn, these element sets are made available to others, providing them with a means of propagating these element sets in time to obtain a position and velocity of a specific space vehicle of particular interest by applying the relevant NORAD SPACETRACK propagation models.

2.5.1 NORAD TLE

A typical TLE set can be obtained from the celestrak website (Kelso 2006). For example, a TLE for a GPS BII-09 (PRN 15) acquired on 4th October 2006 (Day 277) at 10:50 UTC is shown in Table 2.2.

'1 20830U 90088A 06275.19442019 .00000034 00000-0 10000-3 0 9887' '2 20830 54.7397 262.1253 0097535 156.4284 204.0294 2.00565857117480'

Table 2.2 A typical NORAD TLE element set.

Kelso (1998a) interpreted in detail the variable names of each data element contained in the two-line element set. Instead of using generic characters to describe them, here, an example of a two-line element data set is presented in Table 2.2. Each line of a NORAD two-line element set consists 69-character lines of data which is used together with NORAD SGP4/SDP4 orbital propagation model to determine the position and velocity of the respective satellite. The first line of this element set is interpreted as follows:

1	First line number of the element data set
20830	Satellite number
U	Unclassified data, publicly available
90	International Designator (Last two digits of launch year, i.e. this
	satellite is launched in year 1990)
088	International Designator (Launch number of the year)
А	International Designator (Piece of the launch)
06	Epoch Year (Last two digits of year)
275.19442019	Epoch (Day of the year and fractional portion of the day)
.00000034	First Time Derivative of the Mean Motion
00000-0	Second Time Derivative of Mean Motion (starting from decimal
	point)
10000-3	BSTAR drag term (starting from decimal point)

0	Ephemeris type
988	Element number
7	Checksum (Modulo 10), other than number, minus signs = 1,
	letters, blanks, period or plus sign $= 0$

Epoch Year and epoch corresponds to 2th of Octotber, 2006 and the fraction part corresponds to the hour, minute and second of the UTC time of that day. The second line of the NORAD element set as presented by Table 2.2 is interpreted as follows,

2	second line number of the element data set
20830	satellite number
54.7397	Satellite inclination(degrees)
262.1253	Right Ascension of the Ascending Node (degrees)
0097535	Eccentricity (starting from decimal point)
156.4284	Argument of Perigee (degrees)
204.0294	Mean Anomaly (degrees)
2.00565857	Mean Motion (revolutions per day)
11748	Revolution number at epoch (revolution)
0	Checksum (Modulo 10)

However, the element sets maintained by NORAD are "mean" values obtained by removing periodic variations by certain specific methods and in a particular way. In addition to the 6 Keplerian elements the NORAD TLE includes the first and second time derivatives of the mean motion, the reference time of the epoch, and a drag related parameter. It is possible to always reconstruct these periodic variations from the base element sets. In order to obtain good predictions, these periodic variations must necessarily be reconstructed in exactly the same way they were removed by NORAD and by employing methods that are compatible with the methods employed to remove them in the first place. Hence, employing the NORAD element sets with a different prediction model, no matter how representative and accurate it may be, will result in erroneous predictions. The NORAD element sets must be used with one of the prediction models described in the SPACETRACK Report No. 3 by Hoots and Roehrich (1980) in order to retain maximum prediction accuracy. This is, undoubtedly, an issue that has to be addressed carefully, particularly if NORAD TLEs are employed for interoperable navigation satellite orbit predictions as these element sets cannot be employed with just any prediction model.

2.5.2 NORAD SPACETRACK Propagation Models

For navigation applications it is also important to recognise that all space vehicles are classified by NORAD as near-Earth (orbital period less than 225 minutes) or deep-space (orbital period greater than or equal to 225 minutes) (Hoots and Roehrich 1980). Depending on the period, the NORAD element sets are automatically generated with the near-Earth or deep-space models and the associated prediction methods. Most navigation satellites fulfill the criteria of a deep-space object, and thus this type of model is more appropriate for them. Thus, almost exclusively, the navigation satellites should be dealt with by employing the prediction models for the deep-space objects.

Although five models for prediction of satellite position and velocity are available for use with the NORAD TLE sets, the first and second of these are specifically for near Earth satellites. The first one of these, SGP, was developed by Hilton and Kuhlman (1966) based on a simplification of the work of Kozai (1959) for its gravitational model and a linear model for the drag effect on mean motion while the second model, SGP4, employs the extensive analytical theory of Lane and Cranford (1969), which uses the solution of Brouwer (1959) for its gravitational model and a power density function for its atmospheric model. The SGP8 model for near-Earth satellites is obtained by simplification of an extensive analytical theory of Hoots (1980), which uses the same gravitational and atmospheric models as Lane and Cranford but integrates the differential equations by a different method. The models SDP4 and

SDP8 are extensions of SGP4 and SGP8 for deep-space satellites. The deep-space equations for the SDP4 model were developed by Hujsak (1979). The deep-space equations model the gravitational effects of the Moon and Sun, which contribute mainly to the secular rates and long periodics, as well as certain sectoral and tesseral Earth harmonics, which are of particular importance for half-day and one-day period orbits, an issue which was discussed in the previous section. The deep-space effects are modelled in SDP8 with the same equations used in SDP4. Further details of the evolution of the models may be found in Hoots et al. (2004).

One method for real-time prediction of the position and velocity of a satellite or any Earth-orbiting object, is by using the NORAD TLE set. Every NORAD TLE set can be used with a NORAD orbital model algorithm (namely SGP, SGP4, SDP4, SGP8 and SGP8) to predict and determine the position and velocity of the corresponding satellite. Each satellite has its own TLE set, which is generated by NORAD and likely to change on an as-needed basis rather than according to an established timetable (Kelso 1998b). The method of orbit propagation models used with suitable modification in this research is SDP4, which is discussed in Appendix A. The suitable modification of this method is presented in section 4.4.

2.6 POSITION DETERMINATION FUNDAMENTAL

The position of a point can be measured relative to some known positions. A simple way to show and, hence, prove this is by an illustration of a two-dimensional case as shown in Figure 2.2.



Figure 2.2 Two-dimensional user position and three satellites positions.

The above figure shows that three known satellites are required at that particular instance. If a single satellite is used, the trace of a point with constant distance to the fixed point is a circle in two-dimensional space. Two satellites will have two circles, which intersect at two points. With three satellites S1, S2 and S3, the user position U can be uniquely determined from three distances x1, x2 and x3, as presented in Figure 2.2. This method of determining the relative positions of the receiver using the geometry of triangles is known as trilateration. The trilateration method forms the basis of satellite position determination.

The same logic applies in three-dimensional space; the user will require four satellites to determine his/her position. This does not, however, account for the offset and bias errors; if these are included, a further (fifth) satellite is required. Practically, four

satellites are sufficient; since one of the solutions will be near to the surface of the Earth and the other one is somewhere in space approximately twice the orbital radius of the satellites further away from Earth. Normally, the user is near to the surface of the Earth, which means the user position can be determined uniquely with four satellites, after including offset and bias errors.

2.6.1 Satellite Signals Flow through a Receiver



Figure 2.3 A simplified flowchart of signals flow through a receiver.

Figure 2.3 shows a simplified flowchart of satellite signals flow through a receiver. Firstly, the input signals are detected and digitised. Signal acquisition follows, whereby the necessary parameters, that is, the beginning of the code period and the carrier frequency of the input signals, are obtained. These parameters will then be passed to the signal tracking process. Once signal tracking is achieved, then the

navigation data can be obtained, each message subframe is then matched, whereby the ephemeris data are acquired, the parity checked and pseudoranges of each satellite and user receiver are determined. In turn, the satellites' positions and the user receiver position are calculated. The user receiver position can then be adjusted in the user's desired coordinate frame.



2.6.2 Signal Tracking

Figure 2.4 Carrier and code tracking loops [adapted from Tsui 2000, pg. 174].

ADC	Analog-to-Digital Converter
MA	Moving Average filter
sqrt	square root process
Σ	summation
e/d select	selecting outputs to their proper connectors

lpf	low pass filter
osc	oscillator
arctan	arctangent comparator
90°	90° phase shift

A GPS signal is a bi-phase coded signal (Tsui 2000, pg. 173) and likewise, generically, all GNSS signals are bi-phase coded signals. Due to the continuous relative motions of the corresponding satellite and receiver, the Doppler effect alters both carrier and code frequencies. In order to track the GNSS signal, the code information must be separated from the carrier signal. As shown in Figure 2.4, these two-phase-locked loops are needed to track the GNSS signal; the code loop tracking the GNSS code signal information, and the carrier loop tracking the carrier frequency. These two loops are coupled together.

The code loop generates three outputs, i.e. an early code, a prompt code and a late code. The prompt code is fed into a multiplier, which strips off the GNSS code signal from the bi-phase coded signal and hence produces an output of a continuous wave (cw) without the GNSS code signal. This cw signal with phase transition caused only by the navigation data is applied as the input to the carrier loop. Similarly, the cw with the carrier frequency produced from the carrier loop is fed into a multiplier which strips off the GNSS code signal. The output from the bi-phase coded signal and produces the GNSS code signal. The output from this multiplier, which is a signal with only a GNSS code and no carrier frequency, becomes the input to the code loop.

This process of signal tracking is then achieved and the navigation data can be obtained.

2.6.3 Code-based Measurement

The code-based pseudorange measurement is a measurement of the distance between the navigation satellite and the user receiver measuring the time taken by the signal to travel from the satellite to the receiver. It uses the leading edge of TLM word in the GNSS signal message as the reference point for initial alignment purpose.

A satellite receiver generates an internal replica of the signal similar to the GNSS signal that it receives. However, as the signal reaches the receiver, the internally generated replica of the signal has advanced and the two do not properly line up with each other. The delay in the signal received in relation to the internal replica is the time required for the signal to reach the receiver. The distance between the satellite and the receiver is estimated by multiplying this delay by the speed of propagation.Later, the position of the satellite can be obtained from the ephemeris and, hence, the user position can be determined.

2.6.4 Carrier Phase Measurement (Real Time Kinematics)

The carrier phase measurement, also referred to as Real Time Kinematics (RTK) is a generic term to describe the technique using the carrier phase of the GNSS signal instead of the navigation data encoded on the signal (as in code pseudorange measurement) where a single, local reference station broadcast the PNT corrections in real time to a stationary or a roving receiver. The system is known as Carrier Phase Enhancement (CPGPS) when referring to GPS. This technique is widely used in map-making, land-surveying, hydrographic surveying and atmospheric modelling

The range measurement accuracy of the carrier phase is about 1% of one bit-width. A GPS civilian coarse-acquisition (C/A) code signal transmits a bit for every 0.98 microsecond, which gives the receiver and accuracy about 0.01 microsecond or

roughly 3 metres in distance. For the military-only P(Y) signal which is 10 time faster than its civilian counterpart, the receiver range measurement accuracy is about 30 cm.

Unlike the code-based measurement, the carrier phase was not initially designed to measure the distance between the satellite and the receiver. The cycles of the carrier phase do not contain any reference point for alignment, hence the difficulty in properly aligning the signal received and the internal replica of the signal. This problem is known as integer ambiguity. However, a complex statistical method can reduce this problem as presented in Appendix E, although it cannot eliminate the problem altogether.

2.7 MEASUREMENT OF PSEUDORANGE

The navigational satellite receiver depends on accurate range measurements in order to determine the precise position of the user. Ideally, the range between a navigational satellite and a user receiver is simply as follows,

$$d_{i} = \sqrt{(x_{sv} - x_{u})^{2} + (y_{sv} - y_{u})^{2} + (z_{sv} - z_{u})^{2}}$$
(2.37)

where d_i is the geometric distance between the i^{th} navigational satellite and the user receiver; (x_{sv}, y_{sv}, z_{sv}) is the known three-dimensional satellite position (in meters); and (x_u, y_u, z_u) is the unknown three dimensional user position (in meters).

However, due to the delay in the timing measurement of the propagation, receiver clock offset from the satellite time (the receiver clock is generally not synchronised with GPS system time (Brown and Hwang 1997; and Kline 1997)) and bias (due to time drift); extra terms have to be incorporated to these errors. Hence, the term is pseudorange rather than range because of the erroneous range measurement, which is expressed as:

$$d_{i} = \sqrt{(x_{sv} - x_{u})^{2} + (y_{sv} - y_{u})^{2} + (z_{sv} - z_{u})^{2} + cb_{ut}}$$
(2.38)

where c is the speed of light, b_{ut} is the user clock offset and bias, and the remaining parameters are as defined as in the previous equation. However, the clock errors are not the only errors which affect the accuracy of the satellite-user range. There are several other errors, which can affect the performance of a navigational satellite receiver. This will be explained further in the next chapter.

Dilution of Precision (DOP) describes the imprecision in measuring the user position. DOP is an estimate through a least squares adjustment procedure, the receiver position and the clock offset are not computed accurately enough. DOP is a function of satellite geometry only. If there are a lot of visible satellites and the user is spoilt for choices, then, the lowest DOP value should be as small as possible to obtain the greatest user position accuracy

2.7.1 Measurement Errors in Code-based Measurement

Measurement errors in code-based measurement can be due to several factors, i.e. clock biases (satellite and receiver), intentional dithering of the clock by the respective GNSS services (selective availability, anti-spoofing and possibly selective deniability), error in the calculated ephemeris (satellite orbital position error), signal propagation errors (differences in propagation speed through vacuum, ionosphere and troposphere), code multipath error and random code measurement (thermal) noise.

2.7.2 Measurement Errors in Carrier Phase Measurement

Measurement errors in carrier phase measurement are due to all the factors mentioned in section 2.7.1 plus integer ambiguity. Instead of code multipath error and random code measurement (thermal) noise, carrier phase measurement suffers from similar forms of error called carrier multipath error and random carrier measurement (thermal) noise respectively.

CHAPTER 3

PSEUDORANGE ERROR MODELLING FOR POSITION ESTIMATION

Previous chapters have discussed, navigation satellites and their augmentation systems (chapter one) and the real-time orbit prediction models of these satellites and the pseudorange measurement (chapter two). Thus, the basis for satellite-based position estimation and measurement has been laid out.

This chapter will continue from where the work has left off so far. Knowing the orbital propagation models used and simulated for positioning prediction, the next step is to convert the signals received from the satellites into useable information, in this case the unknown user position. Simultaneously, measurement errors and biases must be modelled and estimated to get an accurate estimation of the position of the satellite from the measurement. By acquiring this processed information, therefore, the unknown user position can be estimated.

This chapter is devoted to pseudorange error modelling for position estimation. It is arranged in a sequence, starting from the pseudorange error model; modelling and simulation of pseudorange errors; and finally estimation and prediction of pseudorange.

3.1 PSEUDORANGE ERROR MODELS

The satellite-based positioning of the user is modelled by code pseudorange and full phase carrier measurement. The measurement of code pseudorange $\tilde{\rho}$ by a user receiver can be accurately modelled as Brown and Hwang (1997), Farrel and Barth (1999), and Farrel and Givargis (2000) show, as follows,

$$\widetilde{\rho} = \left((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2 \right)^{0.5} + c\Delta t_r(t) + MP(t) + \eta(t) + c\Delta t_{sv}(t) + SA(t) + E(t) + c\Delta t_{ion}(t) + c\Delta t_{ir}(t)$$
(3.1)

where

 $\hat{x}_{sv}, \hat{y}_{sv}, \hat{z}_{sv}$ satellite estimate positions in x, y, z components

 x_u , y_u , z_u user positions in x, y, z in components

c speed of light

- $\Delta t_r(t)$ receiver clock bias
- $\Delta t_{sv}(t)$ satellite clock bias
- $\Delta SA(t)$ selective availability error
- E(t) error in the calculated ephemeris
- $\Delta t_{ion}(t)$ dispersive ionospheric error
- $\Delta t_{ir}(t)$ nondispersive tropospheric error
- $\Delta MP(t)$ code multipath error
- $\eta(t)$ random code measurement noise

Meanwhile, the measurement of the full carrier phase $\tilde{\phi}$ can be accurately modelled as,

$$(\tilde{\phi} + N) \cdot \lambda = ((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2)^{0.5} + c\Delta t_r(t) + mp(t) + \zeta(t) + c\Delta t_{sv}(t) + SA(t) + E(t) - c\Delta t_{ion}(t) + c\Delta t_{tr}(t)$$
(3.2)

where

 $\lambda = c/f$ wavelength corresponding to the carrier frequency *f*;

CHAPTER 3: PSEUDORANGE ERROR MODELLING FOR SATELLITE-BASED POSITION ESTIMATION

- mp(t) phase multipath error;
- $\zeta(t)$ random phase measurement noise;
- *N* constant integer phase ambiguity (the whole number between the satellite and the receiver at initial time measurement).

The interesting fact about the carrier signal is that the phase multipath mp(t) and phase random measurement noise $\zeta(t)$ are approximately 100 times smaller than the corresponding errors in code pseudorange, ie code multipath MP(t) and random code measurement noise $\eta(t)$ (Farrel and Barth 1999). This is the reason why carrier phase measurement is used for sub-metre accuracy for precision positioning. However, this equation is rendered useless if the constant integer phase ambiguity N is not properly estimated.

Also, a significant difference between the measurements of code pseudorange and carrier phase is the ionosphere effect, where the code signal transmission delays while the phase signal transmission advances. The effect on code and phase measurements have the same amount but opposite sign. Therefore, in equation 3.1 (measurement of the code pseudorange), the ionospheric term is positive but in equation 3.2 (measurement of the carrier phase), the ionospheric term is negative.

If a precisely surveyed location (x_0, y_0, z_0) of a reference receiver is known, this information can be used to estimate the reference-to-satellite range (Farrel and Givargis 2000)

$$\hat{R}_{0} = \left(\left(\hat{x}_{sv} - x_{0} \right)^{2} + \left(\hat{y}_{sv} - y_{0} \right)^{2} + \left(\hat{z}_{sv} - z_{0} \right)^{2} \right)^{0.5}$$
(3.3)

Hence, the basic range space differential correction (for each satellite) is determined by differencing the estimated and measured reference-to-satellite ranges (Farrel and Givargis 2000)

$$\hat{\Delta}_{DGPS} = \hat{R}_0 - \tilde{\rho} \tag{3.4}$$

$$= -(c\Delta t_{0}(t) + c\Delta t_{sv}(t) + SA(t) + E(t) + c\Delta t_{a}(t) + MP(t) + \eta(t))$$
(3.5)

CHAPTER 3: PSEUDORANGE ERROR MODELLING FOR SATELLITE-BASED POSITION ESTIMATION

where Δt_0 represents the bias in the reference receiver clock and $c\Delta t_a$ is a total of combined errors of ionospheric and tropospheric propagation delays.

The broadcast corrections should be corrected to remove the reference receiver and satellite clock errors. Therefore, the broadcast corrections will take the form (Farrel and Givargis 2000)

$$\hat{\Delta}_{GPS} = \hat{R}_0 + c\Delta t_0(t) + c\Delta t_{sv}(t) - \tilde{\rho}$$
(3.6)

$$= -(c\,\partial t_0(t) + c\,\partial t_{sv}(t) + SA(t) + E(t) + c\Delta t_a(t) + MP(t) + \eta(t))$$

$$(3.7)$$

where $c \delta t_0$ and $c \delta t_{sv}$ represent the residual reference receiver and satellite clock errors.

To summarise, the error terms in both equations above can be classified into two categories, as presented in Table 3.1.

Common Mode Errors	Receiver Dependant Errors
satellite clock bias	receiver clock bias
selective availability error	code and phase multipath errors
error in the calculated ephemeris	random measurement noise
dispersive ionospheric delay	constant integer phase ambiguity
nondispersive tropospheric delay	

 Table 3.1 Pseudorange error classification.

Further scrutinising Table 3.1, the clock biases for the receiver and the satellite, which belong to different categories use the same model but different values for certain parameters. The clock bias errors, receiver dependent errors and common mode errors are deliberated in the following sections.

Also, an important note to be discussed here is that all error models used might not be the best models, which are normally computationally complex, but most of them are easy and simple algorithms available from various literatures.

These two categories of pseudorange errors will be elaborated on further in the next three sections.
3.2 CLOCK BIAS ERRORS

This section will concentrate on receiver and satellite clock bias errors. Although satellite clock bias is a common mode error and receiver clock bias is a receiver dependent error, both errors have the same error model and, thus, it is imperative to deal with them in one section.

In this research, the model of the clock bias error is based on the one derived by Brown and Hwang (1997). The two-state clock model can be represented as in Figure 3.1.



Figure 3.1 General two-static model describing clock errors.

In continuous form, the clock errors can be modelled as follows

$$\begin{bmatrix} \dot{x}_{cb} \\ \dot{x}_{cd} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{cb} \\ x_{cd} \end{bmatrix} + \begin{bmatrix} u_{cb} \\ u_{cd} \end{bmatrix}$$
(3.8)

where

- x_{cb} clock bias state
- x_{cd} clock drift state
- u_{cb} white noise driving function for x_{cb}
- u_{cd} white noise driving function for x_{cd}

From the continuous form equation, the State Transition Matrix (STM) can be derived as

$$\boldsymbol{\varphi} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \tag{3.9}$$

where Δt is the interval between each step.

The discrete state of the process at time t_k is described as

$$\mathbf{x}_{k} = \begin{bmatrix} x_{cb,k} \\ x_{cd,k} \end{bmatrix}$$
(3.10)

where $x_{cb,k}$ and $x_{cd,k}$ are the respective discrete clock bias and drift state errors. $x_{cb,k}$ will contribute to the pseudorange error model. The discrete noise can be defined as follows

$$\mathbf{w}_{k} = \begin{bmatrix} w_{cb,k} \\ w_{cd,k} \end{bmatrix}$$
(3.11)

where $w_{cb,k}$ and $w_{cd,k}$ are the respective discrete white noise for $x_{cb,k}$ and $x_{cd,k}$. In this work, a normally distributed random number is used for this discrete white noise driving function. Therefore, in discrete form, the clock error model can be described as

$$\mathbf{x}_{k} = \boldsymbol{\varphi} \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{3.12}$$

The covariance matrix associated with \mathbf{w}_k is denoted as \mathbf{Q}_k . The relationship between \mathbf{w}_k is described as \mathbf{Q}_k as follows

$$E\left[\mathbf{w}_{k}\mathbf{w}_{i}^{T}\right] = \begin{cases} \mathbf{Q}_{k}, i = k\\ 0, i \neq k \end{cases}$$
(3.13)

Hence, Brown and Hwang (1997) define \mathbf{Q}_k as

$$\mathbf{Q}_{k} = \begin{bmatrix} S_{f} \Delta t + \frac{S_{g} \Delta t^{3}}{3} & \frac{S_{g} \Delta t^{2}}{2} \\ \frac{S_{g} \Delta t^{2}}{2} & S_{g} \Delta t \end{bmatrix}$$
(3.14)

where S_f and S_g are the respective spectral amplitudes associated with the white noise driving functions u_{cb} and u_{cd} Both are defined as,

$$S_f = 2h_0$$

$$S_g = 8\pi^2 h_{-2}$$

The parameters of h_0 and h_{-2} will be defined later, after the alternative \mathbf{Q}_k covariance error model is formulated. Alternatively, \mathbf{Q}_k can be defined as

$$\mathbf{Q}_{k} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}$$
(3.15)

where

$$q_{11} = \frac{h_0}{2}\Delta t + 2h_{-1}\Delta t^2 + \frac{2}{3}\pi^2 h_{-2}\Delta t^3$$
(3.16)

$$q_{12} = q_{21} = h_{-1}\Delta t + \pi^2 h_{-2}\Delta t^2$$
(3.17)

$$q_{22} = \frac{h_0}{2\Delta t} + 4h_{-1} + \frac{8}{3}\pi^2 h_{-2}\Delta t$$
(3.18)

The parameters of h_0 , h_{-1} and h_{-2} are typical Allan variance parameters. The values for these parameters are shown in Table 3.2.

Timing Standard	h ₀	h.1	h.2	
Crystal	$2 \cdot 10^{-19}$	$7 \cdot 10^{-21}$	$2 \cdot 10^{-20}$	
Ovenised Crystal	$8 \cdot 10^{-20}$	$2 \cdot 10^{-21}$	$4 \cdot 10^{-23}$	
Rubidium	$2 \cdot 10^{-20}$	$7 \cdot 10^{-24}$	$4 \cdot 10^{-29}$	

 Table 3.2 Typical Allan variance parameters for various timing standards (Brown and Hwang 1997).

The values of h_0 , h_{-1} and h_{-2} determined the type of clock bias and drift errors. For the user receiver clock, the inexpensive crystal and ovenised crystal are used, while the satellite clock uses the more accurate Rubidium and Caesium.

Langley (1993) notes that the linear combinations of between-receivers (or betweensatellites) differences could generate new observables with significantly reduced errors. The illustration can be referred to Figure 3.2.



Figure 3.2 Linear combinations of satellites and receivers (Langley 1993).

The single difference of two receivers tracking the same satellite is able to eliminate the satellite clock bias. Hence, the full carrier phase measurement (equation 3.2) becomes

$$\Delta(\widetilde{\phi} + N) \cdot \lambda = \Delta((\widehat{x}_{sv} - x_u)^2 + (\widehat{y}_{sv} - y_u)^2 + (\widehat{z}_{sv} - z_u)^2)^{0.5} + \Delta c \Delta t_r(t) + \Delta mp(t) + \Delta \zeta(t) + \Delta SA(t) + \Delta E(t) - \Delta c \Delta t_{ion}(t) + \Delta c \Delta t_{tr}(t)$$
(3.19)

The single difference of one receiver tracking two satellites is able to eliminate the receiver clock bias. Hence, equation 3.2 becomes

$$\Delta(\widetilde{\phi} + N) \cdot \lambda = \Delta((\widehat{x}_{sv} - x_u)^2 + (\widehat{y}_{sv} - y_u)^2 + (\widehat{z}_{sv} - z_u)^2)^{0.5} + \Delta mp(t)$$

+
$$\Delta \zeta(t) + \Delta c \Delta t_{sv}(t) + \Delta SA(t) + \Delta E(t) - \Delta c \Delta t_{ion}(t) + \Delta c \Delta t_{ir}(t)$$
(3.20)

The double difference of either equation 3.19 or 3.20 will eliminate both satellite and receiver clock biases. Hence, the equation now becomes

$$\nabla \Delta \left(\widetilde{\phi} + N \right) \cdot \lambda = \nabla \Delta \left(\left(\hat{x}_{sv} - x_u \right)^2 + \left(\hat{y}_{sv} - y_u \right)^2 + \left(\hat{z}_{sv} - z_u \right)^2 \right)^{0.5} + \nabla \Delta m p(t)$$

$$+ \nabla \Delta \zeta(t) + \nabla \Delta S A(t) + \nabla \Delta E(t) - \nabla \Delta c \Delta t_{ion}(t) + \nabla \Delta c \Delta t_{tr}(t)$$

$$(3.21)$$

3.3 RECEIVER DEPENDENT ERRORS

This section will concentrate on receiver dependent errors, which have been listed in Table 3.1 apart from the receiver clock bias, i.e. multipath error (for code pseudorange model) and phase multipath error (for full carrier phase pseudorange model), signal strength and noise (random measurement noise) and constant integer bias.

3.3.1 Multipath Error Model

Multipath modelling is not easy but an assumption can be made, i.e. multipath being a more or less slowly varying bias (Wolf 2000). Multipath can be modelled as Gauss-Markov processes or white noise. These processes have an exponential autocorrelation function with variance, σ^2 and time constant, $\frac{1}{\beta}$ (Brown and Hwang 1997; and Rankin 1994).

$$\mathbf{R}(\tau) = \sigma^2 e^{-\beta|t|} \tag{3.22}$$

The Gauss-Markov terms are modelled by

$$\mathbf{x}_{k+1} = e^{-\beta^* \Delta t} \mathbf{x}_k + \mathbf{w}_k \tag{3.23}$$

where \mathbf{x}_k is the parameter being simulated, \mathbf{w}_k is the Gaussian white noise, and ΔT is the sample time. The standard deviation, σ , and time constant, $\frac{1}{\beta}$, are listed in Table 3.3 for the Gauss-Markov multipath noise terms (Rankin 1994).

Error Parameter	Std. Dev. (meters)	Time (sec)
C/A standard	5.0	600
C/A narrow	0.25	600
Р	1.0	600
L1	0.048	600

 Table 3.3 Parameters for Gauss-Markov multipath error source.

3.3.2 Signal Strength and Noise

Signal strength and noise, otherwise known as the receiver random noise measurement, v_{rer} , is the accuracy with which the code or carrier can be tracked (Rankin 1994). Table 3.4 lists the approximations of measurement noise parameters.

Error Parameter	Std. Dev. (meters)
C/A standard	3.0
C/A narrow	0.1
Р	03
L1 carrier	0.0019

 Table 3.4 Measurement noise parameters.

3.3.3 Constant Integer Phase Ambiguity

Constant integer phase ambiguity is normally shortened to integer ambiguity. Integer ambiguity is a problem for carrier phase measurement and does not affect code pseudorange. Unlike the code pseudorange signals, which are intentionally encoded so that they can be aligned easily with the internally generated copy of the same signal within the user receiver, each carrier phase cycle is identical to one another. There is no way to identify them, thus making it extremely difficult to properly align the signals with the signal copy generated internally. As a result of this 'improper' alignment of the carrier phase signal, integer ambiguity error is introduced in multiples of 20 cm.

To resolve this problem, a complex statistical method is presented in appendix E.2 ambiguity resolution.

3.4 COMMON MODE ERRORS

This section will concentrate on common mode errors, which have been listed in Table 3.1, apart from the satellite clock bias, namely nondispersive ionospheric error model, dispersive tropospheric error model, selective availability and errors in the calculated ephemeris.

3.4.1 Ionospheric Delay

Satellite navigation radio signals traveling through the ionosphere are dispersive. This affects the code and phase measurements in the opposite sense. The ionospheric effects are dependent on carrier frequency and can be sufficiently modelled to first order as

$$c\Delta t_{ion}(t) = \frac{40.3}{f^2} TEC \tag{3.24}$$

where TEC is the total electron content along the signal path and, f is the satellite carrier frequency.

Wolf (2000) uses a good approximation of the nominal *TEC* distribution using a Chapman Profile as shown in Figure 3.3.



Figure 3.3 Chapman Profile of the ionosphere (Wolf 2000).

Normally a navigation satellite orbiting altitude is between 15,000 km and 35,000 km from Earth, i.e. GLONASS orbits the Earth at an altitude of 19,100 km, GPS orbiting altitude is approximately 20,200 kilometres and Galileo orbital altitude is approximately 23,222 km.

From Figure 3.3, *TEC* can be obtained by integrating the area enclaved by the altitude of the satellite. The area of the graph can be divided into several smaller areas, and each of these areas can be calculated.

$$\log(F(x)) = mx + c \tag{3.25}$$

$$F(x) = 10^{mx+c} (3.26)$$

$$TEC = \text{Area enclaved} = \int_{x0}^{x1} F(x) \cdot dx = \int_{x0}^{x1} 10^{mx+c} \cdot dx = \frac{10^{mx+c}}{m \cdot \ln(10)} \Big|_{x0}^{x1}$$
(3.27)

Wolf (2000) also notes that "The error of the model has been assumed to be 50%. This value is added to the observation variance", which will be included into the overall ionospheric error model.

A simpler model as shown by Rankin (1994) using Gauss-Markov process is listed in Table 3.5.

Error Parameter	Std. Dev. (meters)	Time (sec)
Ionosphere	5.0	1800

 Table 3.5 Parameter for Gauss-Markov ionosphere error source.

3.4.2 Improved Modelling of the Ionosphere

Improved modelling of the ionosphere, and thus better prediction of the *TEC*, is by using the established models, either the two widely used empirical models (International Reference Ionosphere (IRI) model or NeQuick model), the broadcast model (Klobuchar model) or GPS data driven models (Global Ionospheric Maps (GIMs)) (Orús, Hernández-Pajares, Juan, Sanz. and García Fernández 2002).

The International Reference Ionosphere (IRI) is an international project sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI) (International Reference Ionosphere 2009 and Coïsson, Radicella and Nava 2002). These organisations formed a Working Group in the late sixties to produce an empirical standard model of the ionosphere, based on all available data sources. Several steadily improved editions of the model have been released. For given location, time and date, IRI provides monthly averages of the electron density, electron temperature, ion temperature, and ion composition in the altitude range from 50 km to 2000 km as well as *TEC* to the same altitude. The IRI-2001 and IRI-2007 source codes and software are available on the website (International Reference Ionosphere 2009).

The NeQuick model is a quick-run model based on the DGR profiler concept (Di Giovanni and Radicella 1990). It is derived from models developed under the European Commission, COST 238 and COST 231 and developed at Abdus Salam International Centre for Theoretical Physics (ARPL-ICTP) of Trieste, Italy and University of Graz, Austria (Coïsson, Radicella and Nava 2002). The output is the vertical or slant profile of electron density and the corresponding total electron content to any given height up to 20,000 km (Hochegger, Nava, Radicella and Leitinger 2000; Radicella and Leitinger, 2001).

Abdullah, Awang-Mat, Mohd-Zain, Abdullah and Nik-Zulkifli (2007) conclude that the ionospheric models (IRI2001 and NeQuick) are suitable in predicting the value of *TEC* during normal quiet day, but during the occurrences of a phenomenon called the Travelling Ionospheric Disturbance, or TID for short (IPS Radio and Space Services 2008), which is a 'wavelike' motion in the ionosphere that can cause the focusing and defocusing of radio waves due to solar activity, the value of *TEC* differs greatly (with other methods used for measuring the *TEC*).

The Klobuchar model is an ionospheric broadcast model for single-frequency user as described by Klobuchar (1987). Although two frequency and even three frequency receivers have became more widely available, the Klobuchar model is included here due to its simplicity in terms of its computational requirement and the Klobuchar algorithm itself. Klobuchar (1987) concluded that 50% rms ionospheric error reduction can be obtained with this algorithm.

GPS data driven models include the Global Ionospheric Maps (GIMs) and Real-time US-Total Electron Content (Liu, Skone, Gao and Komjathy 2005). GIMS was provided and developed by the Center for Orbit Determination in Europe (CODE) while Real-time US-Total Electron Content was evolved through a collaboration between the Space Weather Prediction Center (SWPC), the National Geodetic Survey

(NGS), the National Geophysical Data Center (NGDC), and the Global Systems Division (GSD) National Geophysical Data Center 2009).

GIMs are generated at 2-hour intervals and 13 snapshots are available to users each day. The GIMs are usually provided in two formats: IONosphere Map Exchange format (IONEX) (Schaer and Gurtner 1998). and Bernese ION format. The map file in IONEX format can be directly employed at user locations to estimate *TEC* values for a given satellite-receiver line-of-sight interpolation method, while the latter format is specifically for users of Bernese software.

Real-time US-Total Electron Content products include maps of vertical TEC over continental US, estimate uncertainties, recent trends based on the past 10 days of TEC information and ASCII data files for both vertical and slant TEC in near real time (Space Weather Prediction Center 2009). This technique is driven by data from ground-based Global Positioning System (GPS) dual frequency receivers. The primary data stream comes from the Maritime and Nationwide Differential GPS (M/NDGPS) real time network of stations operated by the US Coast Guard (USCG), and is provided to SWPC by the NGS continuously operating reference stations (CORS) network. Secondary data streams are provided by the GPS/Met network (meteorological application of GPS data) and the Real Time IGS (International GNSS Service) network. Currently, there are about 80 CORS, 30 GPS/Met, and 15 IGS stations ingested into the model. This number has been gradually increasing and will be augmented by Federal Aviation Administration/Wide Area Augmentation System (FAA/WAAS) data in the future (National Geophysical Data Center 2009). The ionospheric products are computed by a Kalman-based data assimilation algorithm called "MAGIC" (Spencer, Robertson and Mader 2004).

Orus et. al. (2002) also conclude that the best performance amongst the three types of models on a global scale is using GPS data driven models, which present an error of 24% of the rms with respect to TOPEX TEC, instead of the 41% of error of IRI climatological model and an 54% of error using the GPS broadcast model.

Further derivation of the improved modelling of the ionosphere is mentioned in detail in Appendix B.

3.4.3 Tropospheric Delay

Unlike the ionosphere, GNSS signals travelling through the tropospheric region are not dispersive. Hence, the error model used is much simpler compared to the ionospheric error model. Navigation satellites do not transmit any ephemeris data regarding tropospheric correction.

Tsui (2000) presents a fairly simple tropospheric delay in meters as shown below:

$$c\Delta t_{trop}(t) = \frac{2.47}{\sin\xi + 0.0121}$$
(3.28)

where ξ is the elevation angle between the user receiver and the satellite.

Wolf (2000) utilises the Saastamionen tropospheric model which is presented as follows:

$$c\Delta t_{trop}(t) = \frac{0.002277}{\cos\left(\frac{\pi}{2} - \xi\right)} \cdot \left[P + \left(\frac{1225}{T} + 0.05\right) \cdot e - \tan\left(\frac{\pi}{2} - \xi\right) \right]$$
(3.29)

where the notations used in this model is similar to the preceding model. Other variables defined as the atmospheric pressure P, temperature T and partial pressure of water vapour e. A 20% residual error is added into the model.

A simpler Gauss-Markov process model as shown by Rankin (1994) is listed in Table 3.6.

Error Parameter	Std. Dev. (meters)	Time (sec)
Troposphere	2.0	3600

Table 3.6: Parameter for Gauss-Markov troposphere error source.

A more complete, yet complex model presented by Penna, Dodson and Chen (2001) is included in Appendix C.

3.4.4 Selective Availability

The purpose of selective availability is to degrade the performance of the GPS. The signal degradation is achieved by dithering the satellite clock frequency and providing only a coarse description of the satellite ephemeris (Tsui 2000).

On May 1, 2000, President Clinton (Office of the Press Secretary 2000) decreed:

"My March 1996 Presidential Decision Directive included in the goals for GPS to: "encourage acceptance and integration of GPS into peaceful civil, commercial and scientific applications worldwide; and to encourage private sector investment in and use of U.S. GPS technologies and services." To meet these goals, I committed the U.S. to discontinuing the use of SA by 2006 with an annual assessment of its continued use beginning this year...The decision to discontinue selective availability is the latest measure in an ongoing effort to make GPS more responsive to civil and commercial users worldwide...This increase in accuracy will allow new GPS applications to emerge and continue to enhance the lives of people around the world."

President Clinton's decree effectively rendered selective availability obsolete, especially emerging technology such as anti-jamming techniques, selective deniability and modernisation of GPS become available. As decreed, in the year 2006, selective availability was switched off completely.

Selective availability can be accurately modelled by the following Table 3.7 (Rankin 1994):

Error Parameter	Std. Dev. (meters)	Time (sec)			
Selective Availability	30.0	180			
Table 3.7 Parameters for Gauss-Markov selective availability error source.					

If selective availability is included (or switched on), the total correlated noise will be dominated by selective availability.

To widen the coverage and increase position accuracy of GPS, Differential GPS (DGPS) are commonplace. Now, nearly all commercial GPS units available on the market offer DGPS data inputs, which provide better positional accuracy.

3.4.5 Errors in Ephemeris

Errors in calculated ephemeris (satellite orbital position error) represent some error contribution towards pseudorange error modelling. An example of an error source is from calculating eccentric anomaly *E* from mean anomaly *M* and eccentricity *e*. $E = M + e \cdot \sin E$ (3.30)

This equation is normally evaluated in an iterative nested loop which computationally contributes a minor, but significant error in the calculated ephemeris.

In this work, errors in calculated ephemeris are considered as non-existent since the simulation does not use ephemeris to calculate the satellite position, yet these errors are worth mentioning.

This error can still be modelled as presented by Rankin (1994), which is shown in the following Table 3.8.

Error Parameter	Std. Dev. (meters)	Time (sec)
Error in Ephemeris	3.0	1800

 Table 3.8 Parameters for Gauss-Markov ephemeris error source.

3.5 SIMULATION OF PSEUDORANGE ERRORS

All the required pseudorange error models have been defined in previous sections. In fact, the total pseudorange errors have been formulated in equations 3.4, 3.5, 3.6 and 3.7. Receiver and satellite clock biases simulation is shown as in the following Figure 3.4:



Figure 3.4 Graph of receiver and satellite clock bias errors.

Selective availability and multipaths are shown as in the following Figures 3.5 and 3.6:



Figure 3.5 Graph of selective availability and multipath errors.



Selective Availability and Multipath Errors

Figure 3.6 Graph of selective availability and multipath errors (zoomed-in).

Ionospheric delay models are shown as in the following Figure 3.7:



Figure 3.7 Graph of ionospheric models based on Chapman Profile and Gauss-Markov.

Several tropospheric delay models are presented in the following Figure 3.8:



Figure 3.8 Graph of troposheric delay models against elevation angle.



Figure 3.9 Pseudorange errors without selective availability.



Figure 3.10 Pseudorange errors with selective availability.

89

The pseudorange errors with and without selective availability are based on equation 3.5. These can be represented as in Figures 3.9 and 3.10. The magnitudes of pseudorange errors with selective availability are higher than those with selective availability switched off.

3.6 ESTIMATION AND PREDICTION OF PSEUDORANGE

Tsui (2000, pg. 202-209) explains in detail on finding pseudorange. In short, the pseudorange can be measured only in a relative way because there is no absolute time reference, the only time reference being the sampling frequency, while the clock bias of the receiver is an unknown quantity. The sampling frequency is 5 Mhz, hence the interval between individual data is 200 ns. The beginning point of subframe 1 of each satellite under observation is used as a reference point and is transmitted at the same time except for the clock correction terms of each satellite. As an example, assume that there are four satellites under observation, which have the relative times (*diff of dat* – the relative transit time, compared to the first satellite) 0, 35935, 47222 and -15232 (each is in unit of 200 ns). The pseudorange ρ can be found using

$$\rho = c \cdot (const + diff \, of \, dat + finetime) \tag{3.31}$$

where *c* is the speed of light (c = 299792458 m/s),

finetime is not included in this example (*finetime* = 0). *finetime* is used to obtain time resolution better than 200 ns.

const is an arbitrary selected constant to make pseudoranges positive.

For this example, let us take const = 75 milliseconds. Hence the four relative pseudoranges can be calculated as

$$\rho = 299792458 \times (75 \times 10^{-3})$$
 seconds
 $\rho = 299792458 \times (75 \times 10^{-3} + 35935 \times 200 \times 10^{-9})$ seconds

$$\rho = 299792458 \times \left(75 \times 10^{-3} + 4722 \times 200 \times 10^{-9}\right) \text{ seconds}$$

$$\rho = 299792458 \times \left(75 \times 10^{-3} - 15232 \times 200 \times 10^{-9}\right) \text{ seconds}$$

In this research work, the estimation and prediction of a given navigational satellite position and velocity are obtained using an algorithm based on NORAD SPACETRACK REPORT NO. 3. The original C++ program was written by Michael F. Henry (Henry 2005) and has subsequently been translated and ported into MATLAB with suitable modification as part of this work. The MATLAB program has been verified by comparing the output with TrakStar version 2.65 written by Dr. T.S. Kelso (Kelso 2000) and the results for the MATLAB program with the same set of inputs matched exactly with TrakStar.

If a precisely surveyed location of an observer is known, then the reference-to-satellite range (equation 3.3) can be calculated. This value will then become the pseudorange by adding the clock bias of the receiver.

CHAPTER 4

DIFFERENTIAL CORRECTION ESTIMATION FOR INTEROPERABILITY

In the preceding chapter, the dynamic model of the satellite position was developed. Thus by using suitable and proper error models, the position of the unknown user can be estimated.

Following on from the previous chapter, it can be assumed that the position of the user is now already known, although it may not be precise enough. Hence, the differential corrections are introduced with the purpose of enhancing the precision of the user position. Another issue, which arises, is the interoperability of the different navigation satellite systems currently in use and how signals arising from them can be utilised to work together or interoperate at either the system or the signal levels. Finally, the feasibility of the solution for the interoperability problem is envisaged by using adaptive Kalman filter.

4.1 STANDARD DIFFERENTIAL SATELLITE NAVIGATION REFERENCE STATION ALGORITHMS

Differential satellite navigation reference station has the same basis as Differential GPS (DGPS) but it is a more generic term, which is used for any type of navigation satellite systems. A differential satellite navigation reference station has its location precisely surveyed and this becomes the local reference. For a reference station to emit a

broadcast correction signal, it is essential to remove the reference receiver and satellite clock errors. This has been described by equations 3.6 and 3.7.

If the position of an stationary observer or a roving vehicle is unknown, its position can be found by calculating its pseudorange and carrier phase measurements. Simultaneously, the reference station receives navigation satellite signals and estimates pseudorange and carrier phase measurements before transmitting the broadcast corrections to another receiver (stationary or roving vehicle). Since the position of the reference station is accurately known, the station calculates the timing of the travel time of the navigation satellite signals and compares it with the actual travel time. The difference is transmitted as differential correction, sans receiver and satellite clock errors. The differential correction prior to transmitting is called broadcast ephemeris correction, which is then sends to other receivers to correct the user receivers' own position. Effectively, this improved the user position accuracy. The reference station transmits a correction signal transmission using radio broadcast, typically in UHF or VHF band.

4.2 KALMAN FILTER OF THE DIFFERENTIAL CORRECTION

Kalman filter is an optimal recursive data processing algorithm which requires the estimated state from previous time step and the current measurement to compute the estimate for the current state. It does not require any history of estimates or measurements.

Kalman filter can be used for differential correction. Farrel and Givargis (1999 and 2000) have reviewed several existing algorithms and also proposed two new algorithms for DGPS reference station design, all of which are based on Kalman filtering

methodology. The propagation model for implementing the Kalman filter is assumed to be of the form:

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{\omega}(k) \qquad \mathbf{y}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{v}(k)$$
(4.1)

The first (existing) algorithm calculates reference station correction by passing the basic correction (equation 3.6) through a three state Kalman filter with $\mathbf{x} = \begin{bmatrix} c & v & a \end{bmatrix}^T$ where c is the filtered correction (i.e. pseudorange error), v is the rate of change of the correction and a is the acceleration of the correction. The state-space model is parameterised by:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
(4.2)

The second (existing) algorithm is based on a four state filter $\mathbf{x} = \begin{bmatrix} c & v & a & e \end{bmatrix}^T$ where c, v, and a are defined as in the previous algorithm and e is the difference between the rates of change of the code and carrier phase correction. The state-space model is parameterised by:

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & T & \frac{T^2}{2} & 0\\ 0 & 1 & T & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(4.3)

and:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}$$
(4.4)

Farrel and Givargis (2000) noted that the first algorithm (equations 4.1-4.2) filter is suboptimal since it neglects the time correlation in the multipath errors which have been modeled as measurement noise while the second algorithm (equations 4.3-4.4) does not model the code multipath as a separate state, instead including the code multipath in the

term. Therefore, the term has significant time correlation violating the standard Kalman filter assumptions. Farrel and Givargis (2000) developed and designed two algorithms for reference station, namely Single and Two Frequency Reference Station algorithms.

Single Frequency Reference Station algorithm has the following properties:

$$\mathbf{x} = \begin{bmatrix} r_c & \dot{r}_c & MP & I_a & N \end{bmatrix}^T \tag{4.5}$$

where the first three state variables are the range correction and its first two derivatives, excluding ionospheric effect, *MP* is the code multipath which is to be removed, I_a is the ionospheric effects and *N* is the carrier integer ambiguity. The measurement matrices **H**₁ and **H**₂ for the two observable variables z_1 and z_2 are defined as follows,

$$z_1(t) = \mathbf{H}_1 \mathbf{x}(t), \qquad \mathbf{H}_1 = \begin{bmatrix} -1 & 0 & 0 & -1 & -\frac{f_2}{f_1} & 0 \end{bmatrix}$$
 (4.6)

$$z_2(t) = \mathbf{H}_2 \mathbf{x}(t), \qquad \mathbf{H}_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & \frac{1}{\lambda_2} & -1 \end{bmatrix}$$
 (4.7)

The output matrices \mathbf{L}_1 and \mathbf{L}_2 for the range correction $y_1(t)$ and range rate correction $y_2(t)$ is presented as follows,

$$y_{1}(t) = \mathbf{L}_{1}\mathbf{x}(t), \qquad \mathbf{L}_{1} = \begin{bmatrix} -1 & 0 & 0 & 0 & -\frac{f_{2}}{f_{1}} & 0 \end{bmatrix}$$
(4.8)
$$y_{2}(t) = \mathbf{L}_{2}\mathbf{x}(t), \qquad \mathbf{L}_{2} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
(4.9)

Hence, the state-space model for the single frequency system is

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_a} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_M} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + \mathbf{\omega}(t)$$
(4.10)

where τ_a , τ_M and τ_i are the correlation times of the acceleration, multipath and ionospheric respectively.

The Two Frequency Reference Station algorithm has the following properties:

$$\mathbf{x} = \begin{bmatrix} r_c & \dot{r}_c & MP_1 & MP_2 & I_a & N_1 & N_2 \end{bmatrix}^T$$
(4.11)

where all the states are similar to the one defined in equation 4.5, with some variant states, MP_1 , MP_2 , N_1 and N_2 where the number of the subscript of the states refers to the first and second carrier frequency. Apart from the measurement matrices \mathbf{H}_1 and \mathbf{H}_2 , and the two observable variables z_1 and z_2 defined is equations 4.6 and 4.7, the additional measurement matrices \mathbf{H}_3 and \mathbf{H}_4 for the extra two observable variables z_3 and z_4 are expressed as follows,

$$z_3(t) = \mathbf{H}_3 \mathbf{x}(t), \qquad \mathbf{H}_3 = \begin{bmatrix} -1 & 0 & 0 & 0 & -1 & -\frac{f_1}{f_2} & 0 & 0 \end{bmatrix}$$
 (4.12)

$$z_4(t) = \mathbf{H}_4 \mathbf{x}(t), \qquad \mathbf{H}_4 = \begin{bmatrix} -\frac{1}{\lambda_2} & 0 & 0 & 0 & \frac{1}{\lambda_1} & 0 & -1 \end{bmatrix}$$
 (4.13)

Now, the output matrices \mathbf{L}_1 and \mathbf{L}_2 for the range correction $y_1(t)$ and range rate correction $y_2(t)$ is presented as follows,

$$y_1(t) = \mathbf{L}_1 \mathbf{x}(t), \qquad \mathbf{L}_1(t) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & -\frac{f_2}{f_1} & 0 & 0 \end{bmatrix}$$
 (4.14)

$$y_2(t) = \mathbf{L}_2 \mathbf{x}(t), \qquad \mathbf{L}_2(t) = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
 (4.15)

Hence, the state-space model the two frequency system is

where τ_a , τ_M and τ_i are the correlation times of the acceleration, multipath and ionospheric respectively.

These two algorithms proposed by Farrel and Givargis (2000) and the two other basic existing algorithms mentioned much earlier will be analysed and subsequently serve as a basis for the new two and three frequency reference station algorithms which are potentially expected to be able to deal with an interoperable system of navigation satellites.

4.3 INTRODUCING INTEROPERABILITY

Interoperability is an idea of using signals from several systems in order to enhance the quality of ephemeris data acquired where there might be significant disturbances and also to provide improved global coverage at the user receiver level. Prasad and Ruggieri (2005), using GALILEO, explain the concept of interoperability by defining it into three grades: *coexistence* or compatibility (i.e. absence of interoperability), *alternative use*, and *combined use* (full interoperability). *Coexistence* means that one system will not degrade the services of another system. *Alternative use* means that there is integration at the user receiver level between systems; the user can use the same receiver for several systems or even use several systems to have new or similar services with enhanced performance. *Combined use* means there is full integration at the system level between two or more systems. Prasad and Ruggieri (2005) continue that interoperability with other satellite navigation systems, interoperability with terrestrial navigation systems and interoperability with non-navigation systems.

In this research, one proposed method of implementing interoperability is to convert navigation systems ephemeris into a unified standard format. The conversion process is shown in the following Figure 4.1.



Figure 4.1 Navigation satellite orbital determination and ephemeris conversion processes.

Since our main focus is on interoperability, an extension of the NORAD TLE set is proposed so both GLONASS, GPS and possibly Galileo satellite ephemerides can be computed, thus facilitating the use of any of these navigation satellites for position and velocity estimation. (The space segment of Galileo is intended to consist of a total of 30 satellites in mean Earth circular orbits configured as a Walker constellation, i.e. distributed over three orbital planes at an altitude of 23,616 km, with an inclination of 56°.) Our extension is simply to append the residual acceleration vectors, \ddot{x}_{res} , \ddot{y}_{res} , \ddot{z}_{res} , to the standard NORAD TLE after the last element, the ballistic drag term, B^* .

The residuals are assumed not to include any gravitational or atmospheric drag effects. With the extended TLE one may compute the user's correct position, employing data from any navigation satellite or any DGPS ground station. Even if one does not intend to use the extended TLE it provides a basis for interoperable computations of the satellite and user positions.

4.4 EPHEMERIS CONVERSION FOR INTEROPERABILITY

A typical TLE set can be obtained from the celestrak website (Kelso 2006). For an example, a TLE for a GPS BII-09 (PRN 15) acquired on 4th October 2006 at 10:50 UTC is shown in Table 2.2 as follows,

```
'1 20830U 90088A 06275.19442019 .00000034 00000-0 10000-3 0 9887'
'2 20830 54.7397 262.1253 0097535 156.4284 204.0294 2.00565857117480'
```

 Table 2.2 (revisit) A typical NORAD TLE element set.

Using the NORAD SPACETRACK algorithms written in MATLAB code during the course of this research, the outputs of TLE in Table 2.2, over a period of 1440 minutes or, put more simply, one day, are obtained and shown in Table 4.1. The positions and velocities of any satellites listed in celestrak.com (Kelso 2006) can be adjusted to predict the respective satellite positions by employing the "*TimeAdjust*" parameter.

The "*TimeAdjust*" parameter is estimated by employing the equation,

TimeAdjust = ((Julian date at 0hr. UTC) – Epoch time of the satellite) \times 24 hours \times 60 mins.

CHAPTER 4: DIFFERENTIAL CORRECTION ESTIMATION FOR
INTEROPERABILITY

T in	X	Y	Ζ	Xdot	Ydot	Zdot
(mins)	(km)	(km)	(km)	(km/s)	(km/s)	(km/s)
0	-3671.68	-26544.4	0.46745	2.198751	-0.28789	3.134585
360	3218.612	26122.26	-559.604	-2.24698	0.225574	-3.18997
720	-3408.83	-26574.5	387.685	2.207771	-0.2207	3.134051
1080	2949.769	26144.71	-953.509	-2.25528	0.155846	-3.18789
1440	-3144.88	-26596.5	774.9553	2.21614	-0.1534	3.132528

Table 4.1 Real-time prediction (SDP4) of GPS BII-09 starting at epoch time (time=0).

The "*TimeAdjust*" parameter provides the bias for the time variable and should be added to the "*Times*" parameter in the iterative process of the algorithm. It is important that this parameter is estimated accurately as it influences the secular contributions to the elemental values. From this example, it was found that some calculated values of the SDP4 algorithm can be used to generate the GPS ephemeris, provided the "*TimeAdjust*" parameter can be estimated accurately. For example, the reference time at 4th October 2006, 0:00 UTC can be estimated as follows:

Julian date of the reference time = 2454012.5 days

Epoch of the satellite (GPS BII-09 (PRN 15)) = 2454010.69442019 days

Hence, *TimeAdjust* will be 2600.034926310182 minutes.

The outputs, for a recurring duration of 360 minutes, since 4th October 2006 at 0:00 UTC are as in the Table 4.2.

<i>T</i> in	X	Y	Ζ	Xdot	Ydot	Zdot
(mins	(km)	(km)	(km)	(km/s)	(km/s)	(km/s)
0	-8183.74	19947.05	-15354.2	-1.9575	-2.49021	-2.25341
360	7862.894	-20559.7	15029.67	1.955353	2.444961	2.258546
720	-8417.41	19638.86	-15630.2	-1.93637	-2.5421	-2.21223
1080	8096.158	-20256.9	15306.58	1.935484	2.497396	2.219023
1440	-8648.46	19324.37	-15901.2	-1.91466	-2.59315	-2.17033

Table 4.2 Real-time prediction of satellite GPS BII-09 starting at

4th October 2006, 0:00 UTC.

From this example, it was found that, some calculated values of the SDP4 algorithm could be used to generate the GPS ephemeris. The process is briefly illustrated here and

to begin with, the elements of the GPS ephemeris are classified into several categories, as shown in Table 4.3.

Group Name	GPS Ephemeris Elements
SDP4 calculated values	Inclination (i_0)
	Right Ascension (Ω_0)
	Argument of Perigee (ω_0)
Derived elements	Time of Ephemeris (<i>Toe</i>)
	Eccentricity (<i>e</i>)
	Mean Anomaly (M_0)
	Semi-Major Axis (a)
Amplitudes of sine & cosine harmonics	of orbital radius (C_{rs}, C_{rc});
correction (Short Period) terms	of inclination (C_{is}, C_{ic});
	of argument of latitude $(C_{us}, C_{uc});$
Secular terms	Mean Motion Difference (<i>dn</i>)
(These components are zero at Epoch)	Rate of Right Ascension ($\dot{\Omega}$)
	Rate of Inclination (\dot{i})

 Table 4.3 Classification of GPS ephemeris.

From Table 4.3, it is seen that GPS ephemeris elements can be classified into 3 categories, i.e. direct SDP4 calculated values, derived elements and time-varying or secular elements. Direct SDP4 calculated values are taken directly from SDP4 program and all ephemeris of the time-varying elements category are equated to zero as the computation is assumed to be at Epoch after adjustment employing the "*TimeAdjust*" parameter. As for derived elements, this requires some degree of mathematical manipulation to acquire these values.

Starting with the time of ephemeris, *Toe*, let us take the reference date to be the same as the previous example (4^{th} October 2006, 0:00 UTC = 2454012.5 days = 212026680000 seconds). The most significant aspect of the computation of the GPS ephemeris is to recognise that the argument of the latitude is defined differently in the GPS ephemeris navigation message and in the SDP4 procedure. This would enable one to establish the relationships between the short period correction terms in SDP4 and the sine and cosine

correction terms in the GPS ephemeris navigation message. The results of our computation are illustrated in Table 4.4 for the satellite (GPS BII-09 (PRN 15)) starting on 4th October 2006 at 0:00 UTC.

	SDP4 computation of GPS ephemeris (for different epochs in mins)					
GPS Ephemeris Element	0	360	720	1080	1440	
SDP4 calculated values						
Inclination (<i>i0</i>)	0.95504	0.95504	0.95503	0.95503	0.95503	
Right Ascension (Ω_0)	4.57366	4.57348	4.57330	4.57311	4.57293	
Argument of Perigee (ω_0)	2.72954	2.72962	2.72969	2.72977	2.72984	
Derived elements						
Time of Ephemeris (Toe) (in secs)	129600	151200	172800	194400	216000	
Eccentricity (E)	9.83965E-03	9.83965E-03	9.83968E-03	9.83974E-03	9.83982E-03	
Mean Anomaly (M0)	1.20535	4.35589	1.22324	4.37378	1.24114	
Semi-Major Axis (a)	26560.15695	26560.15695	26560.15695	26560.15695	26560.15695	
Cosine correction of orbital radius (Crc)	2.76164E-01	2.76164E-01	2.76164E-01	2.76164E-01	2.76164E-01	
Sine correction of orbital radius (Crs)	1.18348E-02	1.18352E-02	1.18356E-02	1.18359E-02	1.18362E-02	
Cosine correction of inclination (Cic)	2.20556E-05	2.20556E-05	2.20556E-05	2.20556E-05	2.20556E-05	
Sine correction of inclination (<i>Cis</i>)	9.45177E-07	9.45208E-07	9.45236E-07	9.45262E-07	9.45284E-07	
Cos correction of arg. of latitude (<i>Cuc</i>)	2.03159E-01	-1.09566E+03	1.20403E-01	9.56505E+01	8.56407E-02	
Sine correction of arg. of latitude (<i>Cus</i>)	-1.07323E-02	-3.15232E+00	-1.07607E-02	-3.15402E+00	-1.08033E-02	
Time-varying elements						
Mean Motion Difference (<i>dn</i>)	0	0	0	0	0	
Rate of Right Ascension (Ωdot)	0	0	0	0	0	
Rate of Inclination (<i>idot</i>)	0	0	0	0	0	

Table 4.4 SDP4 computation of approximated GPS ephemeris elements (for GPS BII-09, PRN 15) starting on 4th October 2006 AT 0:00 UTC.

09, PKN 15) starting on 4 October 2000 AT 0.00 UTC.

[The values have been truncated for clarity and presentation purposes]

Applying these approximated GPS ephemeris values into a standard GPS calculation, the result of the computation can be shown as in Table 4.5. In the same table, the different outputs between both models (direct SDP4 outputs and SDP4-based GPS ephemeris computation) are compared. The comparisons indicate a very close match. In fact, the calculation errors are in the order of micrometers (10^{-6}).

	Time (mins)	0	360	720	1080	1440
SDP4 Model outputs	X (km)	-8183.7398	7862.8943	-8417.4140	8096.1577	-8648.4647
	Y(km)	19947.0451	-20559.6598	19638.8581	-20256.9300	19324.3650
	Z (km)	-15354.1707	15029.6682	-15630.2318	15306.5809	-15901.2327
SDP4-based GPS ephemeris computation	X (km)	-8183.7398	7862.8943	-8417.4140	8096.1577	-8648.4647
	Y (km)	19947.0451	-20559.6598	19638.8581	-20256.9300	19324.3650
	Z (km)	-15354.1707	15029.6682	-15630.2318	15306.5809	-15901.2327
Positional Differences	ΔX (km)	4.3401E-09	-3.4397E-09	3.7599E-09	-3.0095E-09	-2.6102E-09
	ΔY (km)	2.7976E-09	2.1028E-09	8.0036E-10	7.0213E-10	-3.0996E-09
	ΔZ (km)	4.3001E-09	-3.6998E-09	1.7008E-09	2.0991E-09	3.0013E-10

Table 4.5 Comparison between direct SDP4 outputs, SDP4-based GPS ephemeris

 computation for satellite GPS BII-09 (PRN 15) and the differences.

[The values have been truncated for clarity and presentation purposes]

Further improvements are currently being made by a proper inclusion of the secular corrections to increase the accuracy of the computed ephemeris. These clearly show that interoperable differential navigation satellite reference stations have the potential to convert different types of ephemerides to cater for various end user receivers. This implies that a GPS user is potentially able to receive an ephemeris from a GLONASS satellite for instance, after the interoperable stations process the message conversion.

4.5 Ensuring Consistency of Error Models for Interoperability

One of the major requirements for ensuring interoperability is to ensure consistency of the error models. While the GPS error models can be derived from the NORAD deep-space equations (refer appendix A), it is apparent that the GLONASS error model (refer section 2.4) and the NORAD deep-space model are not consistent with each other. On the other hand, the GLONASS model is strikingly simple and so the choice is between simplicity and consistency. In order to enhance consistency, it was decided to include a

drag model and the effects of the gravitation of the Moon and Sun as enunciated in the deep-space equations, into the GLONASS model. The influence of atmospheric drag results in one of the most significant perturbations to a satellite in low Earth orbit, especially below 400-600 km but does not generally influence navigation satellites.

Taking into account the rotation of the upper atmosphere with the Earth, the drag acceleration on a satellite is:

$$d\mathbf{F}_{drag} = F_r \mathbf{e}_r + F_s \mathbf{e}_s + F_n \mathbf{e}_n = -\frac{1}{2} C_D \rho A v_{rel} \mathbf{v}_{rel}$$
(4.17)

where C_D is the drag coefficient, A is the projected satellite area, ρ is the atmospheric density which is assumed to satisfy a power-law function,

$$\mathbf{v}_{rel} = \mathbf{v} - \omega_{ua} \mathbf{e}_z \times r \mathbf{e}_r \,, \tag{4.18}$$

and ω_{ua} is the rotational angular velocity of the Earth's upper atmosphere, which is assumed to be fixed. (C_D is the drag coefficient, which depends to a very large extent on the shape and surface of the satellite. For a sphere it is less than 2.2 and for a cylinder it is about 3. The drag coefficient, C_D , is not as trivial to evaluate as it may seem. Since atmospheric density is very low at the altitudes of satellite orbits, even low Earth orbits, the ordinary continuum-flow theory of conventional aerodynamics does not apply and the appropriate regime is that of free-molecule flow). The last element in the NORAD TLE, the ballistic drag term, B^* , is employed for the computation of the drag coefficient C_D according to the formula:

$$C_D = \frac{4mB^*}{\rho_0 A} \tag{4.19}$$

where ρ_0 is the reference value of the atmospheric density, A is the average crosssectional or projected area of the satellite of mass m. At an altitude of 450 km, the approximate density is given by:

$$\rho = 1.585 \times 10^{-12} \text{ kg m}^{-3} \tag{4.20}$$

The deep-space power law model for the density variations with the altitude is adopted in this thesis and is given by:

$$\rho = \rho_0 \left[((q_0 - s)/(r - s))^4 \right]$$
(4.21)

where r is the radial distance from the centre of the Earth and q_0 and s are altitude parameters defining the model. Assuming a circular orbit and neglecting terms of the order of $(\omega_{ua}/n)^2$ it can be shown that,

$$v_{rel} = na(1 - (\omega_{ua}/n)\cos i), F_r = 0,$$
 (4.22)

$$F_{s}\mathbf{e}_{s} + F_{n}\mathbf{e}_{n} = -\frac{1}{2}C_{D}\rho Aa^{2}n((1-(\omega_{ua}/n)\cos i)\mathbf{e}_{s} + (\omega_{ua}/n)\sin i\cos\theta \mathbf{e}_{n})$$
(4.23)

With these models incorporated into the GLONASS error model, the residuals, \ddot{x}_{res} , \ddot{y}_{res} , \ddot{z}_{res} , are not to be interpreted in the usual sense but must be assumed not to include any gravitational or atmospheric drag effects. When this is done the computations of the errors cannot be carried out by employing the standard GLONASS model but by following the methods outlined here.

4.6 ADAPTIVE KALMAN FILTER

The discrete Kalman filter, (outlined for example by Brown and Huang 1997) is the basis for developing the adaptive Kalman filter algorithm. Consider a linear discrete time model representing the error correction states of a generic differential satellite navigation system given by:

$$\mathbf{x}_{k} = \mathbf{\Phi}_{k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{4.24a}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k, \qquad (4.24b)$$

where \mathbf{x}_k is a $(n \times 1)$ state vector, $\mathbf{\Phi}_{k-1}$ is a $(n \times n)$ transition matrix, \mathbf{z}_k is a $(m \times 1)$ measurement vector and \mathbf{H}_k is a $(m \times n)$ state to measurement distribution matrix.

Variables \mathbf{w}_k and \mathbf{v}_k are uncorrelated Gaussian white noise sequences with zero means:

$$E\{\mathbf{w}_k\} = E\{\mathbf{v}_k\} = 0 \tag{4.25}$$

and covariance matrices defined by:

$$E\{\mathbf{w}_{k}\mathbf{v}_{i}^{T}\}=0 \text{ and } E\{\mathbf{w}_{k}\mathbf{w}_{i}^{T}\}=0, \qquad (4.26a)$$

$$E\left\{\mathbf{w}_{k}\mathbf{v}_{i}^{T}\right\}=0 \text{ for } i\neq k, \qquad (4.26b)$$

and

$$E\{\mathbf{w}_{k}\mathbf{w}_{k}^{T}\}=\mathbf{Q}_{k}, E\{\mathbf{v}_{k}\mathbf{v}_{k}^{T}\}=\mathbf{R}_{k}$$
(4.26c)

where $E\{\cdot\}$ is the expectation operator. The parameters, \mathbf{Q}_k and \mathbf{R}_k are the covariance matrices of the process noise sequence, \mathbf{w}_k and the measurement noise sequence, \mathbf{v}_k respectively.

The state and covariance prediction equations defining the Kalman filter (KF) are:

$$\hat{\mathbf{x}}_{k} = \mathbf{\Phi}_{k-1} \hat{\mathbf{x}}_{k-1} \tag{4.27a}$$

$$\hat{\mathbf{P}}_{k}^{-} = \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T} + \mathbf{Q}_{k-1}$$
(4.27b)

where $\hat{\mathbf{x}}_k$ is the state vector predicted from the corrected state vector $\hat{\mathbf{x}}_{k-1}$ estimated at the end of the previous epoch, $\hat{\mathbf{P}}_k^-$ is the corresponding predicted state covariance matrix and \mathbf{P}_{k-1} is the corresponding predicted state covariance matrix at the end of the previous epoch. The measurement correction or update equations defining the KF are:

$$\mathbf{K}_{k} = \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T} \left(\mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T} + \mathbf{R}_{k} \right)^{-1}$$
(4.28a)

$$\hat{\mathbf{x}}_{k} = \hat{\mathbf{x}}_{k}^{-} + \mathbf{K}_{k} \left(\mathbf{z}_{k} - \mathbf{H}_{k} \hat{\mathbf{x}}_{k}^{-} \right)$$
(4.28b)

$$\hat{\mathbf{P}}_{k} = \left(\mathbf{I} - \mathbf{K}_{k}\mathbf{H}_{k}\right)\hat{\mathbf{P}}_{k}^{-} \tag{4.28c}$$

where \mathbf{K}_{k} is the optimal Kalman gain, which defines the correction that must be added to the predicted state vector in order to obtain the estimate. The correction is a function of the innovation sequence, \mathbf{v}_{k} expressed by:

$$\mathbf{v}_{k} = \left(\mathbf{z}_{k} - \mathbf{H}_{k}\hat{\mathbf{x}}_{k}^{-}\right). \tag{4.29}$$

The innovation sequence is a white Gaussian noise sequence with zero mean when the filter is optimal. Moreover the observation error and state estimation error are orthogonal to each other. The innovation sequence is different from the residual which is defined as:

$$\mathbf{r}_{k} = \left(\mathbf{z}_{k} - \mathbf{H}_{k}\hat{\mathbf{x}}_{k}\right) = \mathbf{v}_{k} + \mathbf{H}_{k}\left(\mathbf{x}_{k} - \hat{\mathbf{x}}_{k}\right).$$
(4.30)

Thus employing equation 4.30 one could express the measurement noise \mathbf{v}_k as a linear combination of two independent components, the residual, \mathbf{r}_k and the optimal error in the estimate. Eliminating the measurements the innovation sequence may be expressed as:

$$\mathbf{v}_{k} = \mathbf{H}_{k} \left(\mathbf{x}_{k} - \hat{\mathbf{x}}_{k}^{-} \right) + \mathbf{v}_{k}$$
(4.31)

and the covariance of the innovation is,

$$E\left\{\mathbf{v}_{k}\mathbf{v}_{k}^{T}\right\} = \mathbf{H}_{k}\hat{\mathbf{P}}_{k}^{-}\mathbf{H}_{k}^{T} + \mathbf{R}_{k}.$$
(4.32)

Assuming that the models are linear but with predicted states and measurements corrupted by some additive Gaussian noise with known variance of the type described in equations 4.24, then it is known that the KF converges to the steady state regardless of the initial conditions. The adaptive KF therefore assumes that the magnitudes of the covariance matrices of the additive Gaussian noises are unknown and seeks to estimate the noise covariance matrices \mathbf{Q}_k and \mathbf{R}_k pertaining respectively to the process and the measurement noise models. The adaptive KF is thus a method of self-tuning for adapting the covariance matrices, \mathbf{Q}_k and \mathbf{R}_k , of the process and measurement noise model sequences. It is achieved by making the statistics of the KF innovation sequences consistent with their theoretical covariances. This principle was established by Mehra (1972) and can be employed to tune both \mathbf{Q}_k and \mathbf{R}_k . An estimate of the covariance of the innovation is obtained by averaging the previous innovation sequence over a window length N:

$$\mathbf{C}_{\upsilon}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \mathbf{v}_{k} \mathbf{v}_{k}^{T}$$
(4.33)
and the covariance of the measurement noise sequence may be updated in principle by employing the relation:

$$\hat{\mathbf{R}}_{k} = \mathbf{C}_{\nu}^{k,N} - \mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T}$$
(4.34)

Assuming a fixed window length, the covariance matrix may be recursively updated by employing the recursive relation:

$$\mathbf{C}_{\upsilon}^{k+1,N} = \mathbf{C}_{\upsilon}^{k,N} + \frac{\left(\mathbf{v}_{k+1}\mathbf{v}_{k+1}^{T} - \mathbf{v}_{k-N+1}\mathbf{v}_{k-N+1}^{T}\right)}{N}$$
(4.35)

One could also directly estimate \mathbf{R}_k from the measurement residual. In this case it has been shown by Mohamed and Schwarz (1999) that one has:

$$\hat{\mathbf{R}}_{k} = \mathbf{C}_{r}^{k,N} + \mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T}$$
(4.36)

where,

$$\mathbf{C}_{r}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \mathbf{r}_{j} \mathbf{r}_{j}^{T} .$$
(4.37)

The covariance of the process noise satisfies the equation (rearrange from equation 4.27b and substitute for equation 4.28c):

$$\mathbf{Q}_{k-1} = \hat{\mathbf{P}}_{k}^{-} - \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T}$$

$$\mathbf{Q}_{k-1} = \mathbf{K}_{k} \mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-} + \hat{\mathbf{P}}_{k} - \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T}$$

$$(4.38)$$

Recognising that the state estimate is an optimal estimate and considering the covariance of the state correction:

$$\mathbf{C}_{\Delta x}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} (\hat{\mathbf{x}}_{k} - \hat{\mathbf{x}}_{k}^{-}) (\hat{\mathbf{x}}_{k} - \hat{\mathbf{x}}_{k}^{-})^{T}$$

$$\mathbf{C}_{\Delta x}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta \mathbf{x} \Delta \mathbf{x}^{T} , \qquad (4.39)$$

where

$$\Delta \mathbf{x} = \left(\mathbf{x}_{k} - \hat{\mathbf{x}}_{k}^{-}\right) - \left(\mathbf{x}_{k} - \hat{\mathbf{x}}_{k}\right). \tag{4.40}$$

CHAPTER 4: DIFFERENTIAL CORRECTION ESTIMATION FOR INTEROPERABILITY

Calculating the covariance of $\Delta \mathbf{x}$ and recognising that the corrected error is orthogonal to the predicted error, and substitutes for $\hat{\mathbf{P}}_{k}^{-} - \hat{\mathbf{P}}_{k} = \mathbf{K}_{k}\mathbf{H}_{k}\hat{\mathbf{P}}_{k}^{-}$ (rearrange from equation 4.28c), it may be expressed as:

$$\mathbf{C}_{\Delta x}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta \mathbf{x} \Delta \mathbf{x}^{T} \approx \hat{\mathbf{P}}_{k}^{-} - \hat{\mathbf{P}}_{k} = \mathbf{K}_{k} \mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-}.$$
(4.41)

The covariance of the state correction, which is linearly related to the innovation, may also be expressed as:

$$\mathbf{C}_{\Delta x}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta \mathbf{x} \Delta \mathbf{x}^{T} \approx \mathbf{K}_{k} \mathbf{C}_{\upsilon}^{k,N} \mathbf{K}_{k}^{T}.$$
(4.42)

This relationship between the covariance matrices suggests that the update of \mathbf{R}_k could be done by employing the covariance of the residual while the update of \mathbf{Q}_k could be done by employing the covariance of the state correction. Hence the equation for updating the covariance of the process noise may be expressed in principle as:

$$\mathbf{Q}_{k-1} = \hat{\mathbf{Q}}_{k-1} \equiv \mathbf{C}_{\Delta x}^{k,N} + \hat{\mathbf{P}}_{k} - \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T}.$$
(4.43a)

In some references (see for example Myers and Tapley, 1976; Blanchet, Frankignoul and Cane, 1997) an unbiased estimator is employed for the covariance of the state correction and equation 4.43a is expressed as:

$$\mathbf{Q}_{k-1} = \frac{N}{N-1} \mathbf{C}_{\Delta x}^{k,N} + \hat{\mathbf{P}}_{k} - \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T}.$$
(4.43b)

4.7 FILTERING FOR INTEROPERABILITY

Standard Kalman filter is designed for linear problems and is not suitable for nonlinearity. Therefore, suitable types of filter design must be employed for, one; processing ephemerides from a variety of satellites and two; the processed ephemerides output must be filtered with the orbital errors.

CHAPTER 4: DIFFERENTIAL CORRECTION ESTIMATION FOR INTEROPERABILITY

However, most dynamic models employed for purposes of estimation or filtering of pseudorange errors or orbit ephemeris errors are generally not linear. To extend and overcome the limitations of linear models, a number of approaches such as the EKF have been proposed in the literature for nonlinear estimations using a variety of approaches. Unlike the KF, the EKF may diverge, if the consecutive linearisations are not a good approximation of the linear model over the entire uncertainty domain. Yet the EKF provides a simple and practical approach to dealing with essential nonlinear dynamics. The model takes the form:

$$\mathbf{x}_{k} = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1}$$
(4.44)

$$\mathbf{z}_{k} = \mathbf{h}_{k} \left(\mathbf{x}_{k} \right) + \mathbf{v}_{k} \,. \tag{4.45}$$

Given the Jacobians:

$$\boldsymbol{\Phi}_{k-1} = \nabla \mathbf{f}_{k-1} \left(\hat{\mathbf{x}}_{k-1} \right) \Big|_{k-1}, \tag{4.46}$$

and

$$\mathbf{H}_{k} = \nabla \mathbf{h}_{k} \left(\hat{\mathbf{x}}_{k}^{-} \right) \Big|_{k}, \qquad (4.47)$$

the state prediction equation defining the EKF is:

$$\hat{\mathbf{x}}_{k}^{-} = \mathbf{f}_{k-1} \left(\hat{\mathbf{x}}_{k-1} \right) \tag{4.48}$$

while the covariance prediction equation is:

$$\hat{\mathbf{P}}_{k}^{-} = \mathbf{\Phi}_{k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k-1}^{T} + \mathbf{Q}_{k-1}.$$
(4.49)

The measurement correction equations defining the EKF are:

$$\mathbf{K}_{k} = \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T} \left(\mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-} \mathbf{H}_{k}^{T} + \mathbf{R}_{k} \right)^{-1}$$
(4.50)

$$\hat{\mathbf{x}}_{k} = \hat{\mathbf{x}}_{k}^{-} + \mathbf{K}_{k} \left[\mathbf{z}_{k} - \mathbf{h}_{k} \left(\hat{\mathbf{x}}_{k}^{-} \right) \right]$$
(4.51)

$$\hat{\mathbf{P}}_{k} = \left(\mathbf{I} - \mathbf{K}_{k}\mathbf{H}_{k}\right)\hat{\mathbf{P}}_{k}^{-}.$$
(4.52)

Equations 4.49, 4.50 and 4.52 are identical to equations 4.27b, 4.28a and 4.28c respectively.

For the purpose of interoperability, the EKF approach was adopted to estimate pseudorange errors using an adaptive approach. The methods of adapting the parameter

matrices, \mathbf{Q}_k and \mathbf{R}_k , defined earlier for the case of the linear discrete model may be employed.

4.8 UNSCENTED KALMAN FILTERING

The Unscented Kalman Filter, or UKF in short, gets its name from the unscented transformation, which is a method of calculating the mean and covariance of a random variable undergoing nonlinear transformation $\mathbf{y} = \mathbf{f}(\mathbf{w})$. Although it is a derivative-free approach, it does not really address the divergence problem. In essence the method constructs a set of *sigma vectors* and propagates them through the same nonlinear function. The mean and covariance of the transformed vector are approximated as a weighted sum of the transformed *sigma vectors* and their covariance matrices.

Consider a random variable \mathbf{w} with dimension L which undergoes the nonlinear transformation, $\mathbf{y} = \mathbf{f}(\mathbf{w})$. The initial conditions are that \mathbf{w} has a mean $\overline{\mathbf{w}}$ and a covariance \mathbf{P}_{ww} . To calculate the statistics of \mathbf{y} , a matrix χ of 2L+1 sigma vectors is formed. Sigma vector points are calculated according to the following conditions:

$$\boldsymbol{\chi}_0 = \overline{\mathbf{w}} \tag{4.53a}$$

$$\boldsymbol{\chi}_{i} = \overline{\mathbf{w}} + \left(\sqrt{(L+\lambda)\mathbf{P}_{ww}}\right)_{i}, \qquad i = 1, 2, \dots, L, \qquad (4.53b)$$

$$\boldsymbol{\chi}_{i} = \overline{\mathbf{w}} - \left(\sqrt{(L+\lambda)\mathbf{P}_{ww}}\right)_{i}, \qquad i = L+1, L+2, \dots, 2L, \qquad (4.53c)$$

where

 $\lambda = \alpha^2 (L + \kappa) - L$ is a scaling parameter, α is a scaling parameter between 0 and 1 and κ is a secondary scaling parameter. $(\sqrt{(L + \lambda)} \mathbf{P}_{ww})_i$ is the *i*th column of the matrix square root. This matrix square root can be obtained by Cholesky factorisation. The weights associated with the sigma vectors are calculated from the following:

$$W_0^{(m)} = \lambda / (L + \lambda) \tag{4.54a}$$

CHAPTER 4: DIFFERENTIAL CORRECTION ESTIMATION FOR INTEROPERABILITY

$$W_0^{(c)} = \left(\lambda/(L+\lambda)\right) + 1 - \alpha^2 + \beta \tag{4.54b}$$

$$W_i^{(m)} = W_i^{(c)} = 1/2(L + \lambda), \qquad i = 1, 2, ..., 2L,$$
(4.54c)

where β is chosen as 2 for Gaussian distributed variables. The mean, covariance and cross-covariance of **y** calculated using the UT are given by:

$$\mathbf{y}_{i} = \mathbf{f}(\mathbf{\chi}_{i})$$
 $i = 1, 2, ..., 2L,$ (4.55a)

$$\overline{\mathbf{y}} \approx \sum_{i=0}^{2L} W_i^{(m)} \mathbf{y}_i \qquad i = 1, 2, \dots, 2L, \qquad (4.55b)$$

$$\mathbf{P}_{yy} \approx \sum_{i=0}^{2L} W_i^{(c)} (\mathbf{y}_i - \overline{\mathbf{y}}) (\mathbf{y}_i - \overline{\mathbf{y}})^T \qquad i = 1, 2, \dots, 2L, \qquad (4.55c)$$

$$\mathbf{P}_{xy} \approx \sum_{i=0}^{2L} W_i^{(c)} (\boldsymbol{\chi}_i - \overline{\boldsymbol{\chi}}) (\mathbf{y}_i - \overline{\mathbf{y}})^T \qquad i = 1, 2, \dots, 2L, \qquad (4.55d)$$

where $\overline{\chi}$ is the mean of the sigma points vector χ , $W_i^{(m)}$ and $W_i^{(c)}$ are the set of weights defined in a manner so approximations of the mean and covariance are accurate up to third order for Gaussian inputs for all nonlinearities, and to at least second order for non-Gaussian inputs. The sigma points in the sigma vectors are updated using the nonlinear model equations without any linearisation.

Given a general discrete nonlinear dynamic system in the form:

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k, \qquad \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k$$
(4.56)

where $\mathbf{x}_k \in \mathbf{R}^n$ is the state vector, $\mathbf{u}_k \in \mathbf{R}^r$ is the known input vector, $\mathbf{y}_k \in \mathbf{R}^m$ is the output vector at time k. \mathbf{w}_k and \mathbf{v}_k are, respectively, the disturbance or process noise and sensor noise vectors, which are assumed to Gaussian white noise with zero mean. Furthermore \mathbf{Q}_k and \mathbf{R}_k are assumed to be the covariance matrices of the process noise sequence, \mathbf{w}_k and the measurement noise sequence, \mathbf{v}_k respectively. The unscented transformations (UT) of the states are denoted as:

$$\mathbf{f}^{UT} = \mathbf{f}^{UT} \left(\mathbf{x}_k, \mathbf{u}_k \right), \qquad \mathbf{h}^{UT} = \mathbf{h}^{UT} \left(\mathbf{x}_k \right)$$
(4.57)

while the transformed covariance matrices and cross-covariance are respectively denoted as:

$$\mathbf{P}_{k}^{f} = \mathbf{P}_{k}^{f} \left(\hat{\mathbf{x}}_{k}, \mathbf{u}_{k} \right), \qquad \mathbf{P}_{k}^{h} = \mathbf{P}_{k}^{h} \left(\hat{\mathbf{x}}_{k} \right)$$
(4.58a)

and

$$\mathbf{P}_{k}^{fh} = \mathbf{P}_{k}^{fh} \left(\hat{\mathbf{x}}_{k}, \mathbf{u}_{k} \right). \tag{4.58b}$$

Equations 4.57 and 4.58a unscented transformation can be visualised as dimensions in set topological form as illustrated in Figure 4.2. This illustration also shows that UT estimates better and more accurate posterior mean and covariance to the second order, while EKF accuracy is up to the first order.



Figure 4.2 An example of filtering a Gaussian prior propagated through an highly nonlinear function (Yuan 2004).

The UKF estimator can then be expressed in a compact form. The state time-update equation, the predicted covariance, the Kalman gain, the state estimate and the corrected covariance are respectively given by:

$$\hat{\mathbf{x}}_{k}^{-} = \mathbf{f}_{k-1}^{UT} (\mathbf{x}_{k-1})$$
(4.59a)

$$\hat{\mathbf{P}}_{k}^{-} = \mathbf{P}_{k-1}^{f} + \mathbf{Q}_{k-1}$$
(4.59b)

$$\mathbf{K}_{k} = \hat{\mathbf{P}}_{k}^{fh-} \left(\hat{\mathbf{P}}_{k}^{h-} + \mathbf{R}_{k} \right)^{-1}$$
(4.59c)

$$\hat{\mathbf{x}}_{k} = \hat{\mathbf{x}}_{k}^{-} + \mathbf{K}_{k} \left[\mathbf{z}_{k} - \mathbf{h}_{k}^{UT} \left(\hat{\mathbf{x}}_{k}^{-} \right) \right]$$
(4.59d)

$$\hat{\mathbf{P}}_{k} = \hat{\mathbf{P}}_{k}^{-} - \mathbf{K}_{k} \left(\hat{\mathbf{P}}_{k}^{h-} + \mathbf{R}_{k} \right)^{-1} \mathbf{K}_{k}^{T}.$$
(4.59e)

Thus higher order nonlinear models capturing significant aspects of the dynamics may be employed to ensure that the KF algorithm can be implemented to effectively estimate the states in practice.

For these purposes, the UKF approach was adopted to estimate orbit parameters using an adaptive approach. The methods of adapting the parameter matrices, \mathbf{Q}_k and \mathbf{R}_k , defined earlier for the case of the linear discrete model may be employed.

CHAPTER 5

ADAPTIVE ORBIT ESTIMATION FOR INTEROPERABILITY

The navigation problems associated with terminal aircraft guidance refers to position determination of an individual vehicle with respect to some point local to the environment, as is the case with aircraft landing systems.

In this chapter, the main consideration is the issue of corrections to the orbiting satellites ephemeris. One of the major requirements that must be met in order to establish generic interoperable systems is to employ independent and yet consistent error models to ensure that the ephemerides employed by the different systems can be easily converted from one to another. In fact there is a need to use a standard ephemeris to identify a satellite in an orbit. Currently different satellite navigation systems, such as GPS, GLONASS, and Galileo, use different methods for orbit estimation, correction and prediction. Moreover the error dynamics models used are extremely complex (see for example Hoots et. al. 2004). Thus the aim is to develop an interoperable orbit estimation method that bears a direct straightforward relationship to the various methods currently in use. The method of modelling the nonlinear propagation dynamics was chosen after considering a number of methods, rotating Cartesian coordinate dynamics and the Kustanheimo-Stiefel four-parameter method. This work also explores the application of various adaptive Kalman filters, including the UKF, to the orbit estimation problem.

This chapter presents results for a relative navigation filter that achieves CAT3-level precision from a customised navigation satellite receiver's data message and the SDP4/SDP8 algorithms to establish the measured data and a precise, robust Unscented

Kalman Filter (UKF) using a suitable nonlinear propagation method. The results indicate the method is particularly suitable for estimating the orbit ephemeris of navigation satellites facilitating interoperable differential GNSS operation.

5.1 THE ORBIT ESTIMATION PROBLEM

This research work has extensively studied the problem of modelling of orbit mechanics for purposes of applying the UKF methodology. Firstly, the application of the UKF to the family of methods established by NORAD for the computation of the orbit position and velocity from the two-line elements (SGP4, SDP4 etc.) was investigated. These are essentially a transformation of the two-line element data, albeit a dynamic one, and are not suitable for applying the unscented transformation (UT), which is the basis for the UKF. Secondly, the orbit dynamics were modelled by applying the Lagrange planetary equations. Here again it was found that the UT could not be successfully applied because of inherently nonlinear transformations such as Kepler's equation and inverse trigonometric functions coupled with the modulus function. The transformed covariance matrices were generally unrealistic and not positive definite because of the presence of singularities and it was generally not possible to apply the UKF approach. Thirdly, the regularisation approach involving the Kustanheimo-Stiefel four-parameter method was considered. This transformation not only involves a quaternion like representation of the orbit parameters but also a transformation of the time variable (see for example Stiefel and Scheifele 1971). The presence of this latter transformation made it difficult to apply the unscented transformation without linearising the transformation of the independent variable. The final method considered was based on using Cartesian coordinates in a rotating frame of reference, which is discussed further below. In all these cases the measurements were assumed to be provided by the pseudorange and the Cartesian coordinates of the satellite's position.

The most commonly employed model in navigation theory is based on the Lagrange planetary equations for the Keplerian orbital elements which is the basis for a variety of satellite error models; see for example Filipski and Varatharajoo (2006). However these equations, that are patently nonlinear, may not provide the best parameterisation of the orbit for purposes of orbit estimation.

5.2 Orbit Dynamic Propagation Models

Several orbit dynamic propagation models have been extensively studied for the purpose of interoperable orbit estimations. These few candidate models are the orbit modelling in the rotating Cartesian coordinate dynamics (Earth-Centered, Earth-Fixed coordinate) known as the Euler-Hill frame, Hill-Clohessy-Wiltshire equations and the Lagrange Planetary equations.

The HCW equation is valid for circular and near-circular orbits for two close-orbiting satellites. The HCW equations are considered, but not used in this work due to the fact that it is an orbit dynamic propagation about a linearised circular orbit. These equations are mentioned and derived extensively because of the similarity used in this work, and can be considered as an extension to the HCW equations.

The Lagrange planetary equations of motion (LPE) are considered for orbit dynamics propagation in this course of research. However, during this research, several trials applying Unscented Kalman Filter (UKF) onto LPE have been made, but were unsuccessful. The problem lies in the fact that LPE can not be approximated by second order, which UKF assumes any measurement can be approximated to second or third order. Unfortunately, approximating LPE to second or third order estimation does not yield any useful results.

Finally, after comparing the Lagrange Planetary Equations in terms of the Keplerian elements and the Cartesian coordinate formulations in a rotating frame, the latter formulation was preferred.

Further details and derivations of these orbit dynamic propagation models are discussed in Appendix D.

5.3 UKF-BASED ORBIT ESTIMATION

In the case of the classical linear KF, which is not only an optimal filter but also an asymptotically stable filter, the filter estimates can be expected to follow the measurements closely even when the states of the process or plant model are unstable. However, in the above orbit model, it is not possible to apply the linear KF and, for this reason, the UKF is chosen. This is demonstrated in Figures 5.9 and discuss in the last section of this chapter.

The UKF is based on approximating the probability distribution function rather than on approximating a nonlinear function as in the case of EKF. The state distributions are approximated by a Gaussian probability density, which is represented by a set of deterministically chosen sample points. The nonlinear filtering using the Gaussian representation of the posterior probability density via a set of deterministically chosen sample points is the basis for the UKF. Thus, it is based on statistical linearisation of the state dynamics rather than analytical linearisation (as in the EKF). The statistical linearisation is performed by employing linear regression using a set of regression (sample) points. The mean and covariance at the sigma points represent the true mean and covariance of the Gaussian density. When transformed to the nonlinear systems, they represent the true mean and covariance accurately only to the second order of the nonlinearity. Thus this can be a severe limitation of the UKF unless the nonlinearities are limited to the first and second order in the process model. One of the difficulties that one encounters repeatedly while using the UKF algorithm is the fact the matrix \mathbf{P}_{ww} in equations 4.53a and 4.53b is not positive definite. Consequently one needs to choose \mathbf{Q}_k and \mathbf{R}_k in equations 4.59 to be sufficiently positive definite so as to prevent \mathbf{P}_{ww} from becoming negative definite. This imposes an undue and unrealistic constraint on the nature of the noise sequences, which would no longer represent the true statistics of the process and sensor noise vectors.

To avoid the problem in computing the square root of \mathbf{P}_{ww} , which is not positive definite, this research employs the method of singular value decomposition (SVD) and then replaces the singular values by their absolute values. This is a perfectly valid alternative in computing the sigma points and there is then no need to choose \mathbf{Q}_k and \mathbf{R}_k in equations 4.59 to be sufficiently positive definite so as to prevent \mathbf{P}_{ww} from becoming negative definite. This modification of the UKF algorithm results in a remarkable improvement in the performance of the UKF.

5.4 ADAPTIVE UKF-BASED ORBIT ESTIMATION

In order to employ the UKF when precise statistics of the process and measurement noise vectors are not available, the adaptive filter method proposed by Song, Qi and Han (2006) is used to estimate the orbit parameters. The covariance matrixes of measurement residuals are recursively updated in the UKF. The measurement and state noise covariance matrices, in the case of the UKF, may be expressed as:

$$\hat{\mathbf{R}}_{k} \equiv \mathbf{C}_{\nu}^{k,N} - \hat{\mathbf{P}}_{k}^{h-}, \tag{5.1a}$$

$$\hat{\mathbf{Q}}_{k-1} \equiv \mathbf{C}_{\Delta x}^{k,N} + \hat{\mathbf{P}}_{k} - \mathbf{P}_{k-1}^{f}$$
(5.1b)

which are analogous to equation 4.34 and the right hand side of equation 4.43. Correspondingly, following equation 4.36 we may express the measurement noise covariance as,

$$\hat{\mathbf{R}}_{k} = \mathbf{C}_{r}^{k,N} + \hat{\mathbf{P}}_{k}^{h}$$
(5.2)

which involves the further computation of $\hat{\mathbf{P}}_{k}^{h}$, by applying the unscented nonlinear transformation, $\mathbf{h}^{UT}(\hat{\mathbf{x}}_{k})$ to the state estimate, $\hat{\mathbf{x}}_{k}$. The measurement noise covariance may be updated in principle by employing the equation 5.1a.

The nonlinear relationships between the covariance matrices also suggests that the update of \mathbf{R}_k could be done by employing the covariance of the residual equation 5.2 while the update of \mathbf{Q}_k could be done by employing the covariance of the state correction equation 5.1b. However, the simultaneous adaptation of both \mathbf{Q}_k and \mathbf{R}_k is not considered robust, as discussed by Blanchet, Frankignoul and Cane, 1997. For this reason we restrict our attention to \mathbf{Q}_k adaptation as it is the process statistics that are really unknown. Furthermore it was observed that the magnitudes of the filter gains were relatively small and for this reason equation 5.1b was approximated as:

$$\hat{\mathbf{Q}}_{k-1} \approx \mathbf{C}_{\Delta x}^{k,N}. \tag{5.3}$$

5.5 SIMULATIONS AND VALIDATION

Differential Global Position System (DGPS) is a mode of operation of the Global Positioning System (GPS) satellite based positioning system that employs a reference station at a known location to calculate and broadcast corrections that could be applied to the pseudorange by users in the vicinity of the receiver station. This approach is known to increase positional accuracy. In the literature, several algorithms have been developed that are designed to remove the effects of the so-called common mode errors in all receivers in the vicinity of the reference station. These algorithms are based on the

concepts of optimal filtering in general and on the Extended Kalman Filter (EKF) in particular, developed along the lines suggested by Farrell and Givargis (2000) and Farrell, Givargis and Barth (2000).

In an aircraft landing system, not only does the pilot need to know his accurate position, but also the reference station, which require a preliminary estimate of his position. In this case, the IDGPS (Inverted DGPS) would be more suitable than DGPS. In IDGPS, a vehicle sends its GPS position information, usually in NMEA format, to the reference station and the differential correction is made at the reference station, not at the GPS receiver in the vehicle. However, in contrast to a standard IGPS system, which does not require an RTCM transmission to the vehicle, the pilot requires an update on his position from the reference station. Thus this situation can be handled provided the aircraft itself is treated as a roving virtual reference centre. The objective in using multiple reference stations in a network for GPS corrections is to model and correct for distance-dependent errors that reduce the accuracy of conventional Real Time Kinematic (RTK) or DGPS positions in proportion to the distance from a rover to its nearest reference station. It is well known that the most significant sources of error affecting precise GPS positioning are the ionosphere, troposphere and satellite orbits. The influence of the ionospheric error on different frequencies in the L-band used by satellite navigation systems is well understood. The ionosphere, which is subject to rapid and localised disturbances, is the main restriction on the station density in a reference network. The troposphere and orbit errors have an equal effect on all ranging signals used by current satellite-based global navigation systems. The aim of a reference network is to model and estimate these error sources and provide this network correction information to the roving vehicle so that they may derive positions with a higher degree of accuracy than with conventional RTK.

In an earlier paper, Vepa and Zhahir (2008) discuss the development of two and three frequency reference station algorithms that may be employed with any navigation satellite. The motivation behind the design of the algorithms has been the need for

reference station algorithms that can deal with an interoperable system of navigation satellites to obtain high accuracy positioning information local to the roving vehicle. In order to achieve interoperability we provided for additional satellite orbit corrections that will ensure the consistency of satellite orbit predictions. To account for the fact that we are now dealing with a variety of satellites, we made no assumptions of the error covariance matrices and adopted an adaptive filter based on the Method of Maximum Likelihood Estimation (MMLE), a technique applied to the EKF by Mehra (1970). However, corrections of the orbiting satellite's ephemeris are assumed to be independent of the other common mode errors and were not considered.



Figure 5.1a GLONASS satellite position prediction normalised to orbit radius.



Figure 5.1b GLONASS satellite normalised velocity prediction.

Equations 2.30-2.33 can be numerically integrated and compared with the position and velocity data for a typical GLONASS navigation satellite independently generated from the NORAD TLE data set from the celestrak website (2008), by using the SDP4 method, (Hoots et. al. 2004) with the position normalised to a mean altitude of 25,490 km, and the velocity to the mean circular velocity of 3.9545 km/sec. These position and velocity responses are shown in Figures 5.1a-5.1d respectively.



Figure5.1cGLONASSsatellitenormalised position prediction error.



Figure 5.1d GLONASS satellite normalised velocity prediction error.

These results are obtained by using the GLONASS error model which only includes C_{20} term and not the C_{22} term in the equations 2.30-2.33. The results indicate that the simulated response follows the measured position and velocity data quite accurately. However, looking very closely at the figures, one may observe that the simulated responses drift very slowly away from the measurements due to the presence of secular terms, thus establishing the need for filtering. While the drift can be eliminated by using

the Lagrange planetary equations (LPEs), which is equivalent to including both the C_{20} term and the C_{22} term in the equations 2.30-2.33, the order of the errors are typical of all the simulation methods with the exception of the regularisation approach involving the Kustanheimo-Stiefel four-parameter method.



Figure 5.2a GLONASS satellite normalised position prediction error obtained by using the Lagrange equations.



Figure 5.2b GLONASS satellite normalised velocity prediction error obtained by using the Lagrange equations.

The position and velocity errors obtained by using the LPEs are shown in Figures 5.2a and 5.2b. It is also observed that the simulations correctly predict the harmonic responses, which are absent in the responses obtained from the Hill-Clohessy-Wiltshire type linearised equations of motion.

In Figures 5.3 and 5.4 the state estimates for the position and velocity errors and the error in the measurement estimate are shown for the same satellite as in Figures 5.1. The propagation model used was also the same as the one used to obtain Figures 5.1.





Figure 5.3a GLONASS satellite UKFbased normalised position estimate error.

Figure 5.3b GLONASS satellite UKFbased normalised velocity estimate error.

The measurement vector consists of six independent simulations of the position and velocity as well actual measurements of the pseudorange. The maximum predicted error in the pseudorange is thus less than 10m relative to the data generated for the GLONASS satellite. It is clear that the estimates tend to follow the states of the plant model and the measured position and velocity data. Moreover the observed drift rates in the simulations are reduced. However there is a need for some caution in applying the UKF due to its limitations.



Figure 5.4 GLONASS satellite UKF-based pseudorange estimate error.

In Figures 5.5 and 5.6 the state estimates for the position and velocity errors and the error in the measurement estimate are shown for the same satellite as in Figures 5.3 and 5.4, where the estimates are now obtained by the modified UKF.



Figure 5.5a GLONASS satellite modified UKF-based normalised position estimate error.





The maximum predicted error in the pseudorange is now reduced to less than 1mm relative to the data generated for the GLONASS satellite. Moreover it is clear that the

estimated error is considerably more uniform in Figure 5.7 than it is in Figure 5.5, where it is quite visibly sinusoid and biased. Thus, with the use of the proposed modification in place it is possible to substantially improve the performance of the UKF, because it facilitates the use of the most appropriate approximations for the noise statistics.



Figure 5.6 GLONASS satellite modified UKF-based pseudorange estimate error.

We also observe from Figure 5.6 that the magnitude of the measurement error is still biased. This is to be expected as we are only seeking to estimate the orbital errors, which contribute exclusively to the errors in the satellite's ephemeris.

The results of applying the adaptation scheme, with the additional modification in computing the square root of the covariance matrices by employing SVD as discussed in the preceding section, are illustrated in Figures 5.7 and 5.8. These results clearly demonstrate the usefulness of the adaptive modified UKF. The results also indicate that the bias and drift in the estimate produced by the adaptive UKF, as it approaches steady state, are of the same order as the modified UKF. Moreover it takes at least an hour to approach steady state.



Figure 5.7a GLONASS satellite adaptive UKF-based normalised position estimate error.



adaptive UKF-based normalised velocity estimate error.



Figure 5.8 GLONASS satellite adaptive UKF-based pseudorange estimate error.

It is observed that the UKF is tracking the true orbit over the entire time frame. The performance of the UKF is generally better than either the conventional KF or the EKF. The main reason for the better performance of the UKF is that the UT approximates the mean and the covariance to third order, which is better than linearisation. Furthermore

the modified UKF facilitates the use of arbitrary realistic models of the process and measurement noise statistics and thus gives a very good estimate of a navigation satellite's pseudorange.





Figure5.9aGLONASSsatellitenormalisedpositionpredictioncomparison using UKF and EKF filters.

Figure 5.9b GLONASS satellite normalised velocity prediction comparison using UKF and EKF filters.

As mentioned before in section 5.3, it is not possible to apply the linear KF to the selected orbit model using the Cartesian coordinate formulations in a rotating frame and, for this reason, the UKF is chosen. This orbit estimation problem is patently nonlinear. The result of using the traditional Extended Kalman filter (EKF) is included and compared with UKF in Figures 5.9. The high error estimate accuracy obtained with the UKF was not surprising as the estimation was based on simulated measurements (for repeatability of results) which were corrupted by predictable Gaussian noise. The UKF is particularly well suited for orbit estimation as the nature of the leading

CHAPTER 5: ADAPTIVE ORBIT ESTIMATION FOR INTEROPERABILITY

nonlinearities is primarily quadratic (with a relatively smaller cubic contribution) although the noise may not always be perfectly Gaussian.

In most orbit predictions, there is little *a priori* information about the state and measurement noise inputs. For this reason, adaptive filtering is appropriate as it allows for the interoperable operation of the orbit estimator, as it permits one to switch from one satellite model to another. Thus, the adaptive UKF serves to generate pseudorange corrections in an interoperable differential GNSS application. Moreover, the performance of the adaptive UKF is almost as accurate as the modified UKF.

Although the standard UKF was initially a promising alternative feature of the orbital dynamics, which has led to the belief that the standard UKF must be employed with appropriate restrictions on the noise covariance statistics to facilitate the calculation of the sigma points, it nonetheless has a number of shortcomings, in particular, being not positive definite. In order to address some of these, a modified approach to the UKF is proposed. The proposed modified UKF uses singular value decomposition rather than Cholesky decomposition to estimate the sigma points. Moreover, the singular values are replaced by their absolute values in the decomposition. Thus, this work presents the results of the application of the modified approach to the UKF to orbit estimation to demonstrate its superiority over the standard approach.

Precise, adaptive UKFs and a suitable nonlinear propagation method are used to estimate the orbit ephemeris and the differential position and the navigation filter mixing errors. The presented results indicate the method is particularly suitable for estimating the orbit ephemeris of navigation satellites and the differential position and navigation filter mixing errors, thus facilitating interoperable DGNSS operation for aircraft landing.

CHAPTER 6

ENHANCED ACCURACY ALGORITHMS

In previous chapters, the emphasis of the work is based on code pseudorange measurement. Differential GNSS using code-correlating techniques allow the unknown user position to be determined at accuracies of up to 2-3 metres, if the user receiver is static and 5-10 metres if the user receiver is in motion. Enhanced accuracy of positioning at the centimetre or even millimetre scale can be achieved by using differential carrier phase measurement. By making the filter algorithms adaptive, any GNSS observable can be applied interchangeably by the base reference station, hence making it interoperable with enhanced positional accuracy.

The introduction of carrier phase measurements has led to the development of a new breed of satellite navigation receivers, which are able to combine carrier phase measurement and code based measurement of the pseudorange. In chapter seven, the application of such receivers to the problem of inertial navigation will be examined. For that reason, some of the processes that have been implemented in the modern satellite navigation receivers are explained here, so the receiver outputs can be characterised appropriately.

6.1 INTRODUCTION

When first initiated, positions determination algorithms in GPS and GLONASS were designed by using binary code sequences modulated onto the respective carrier but did not account for carrier phase measurement (Forsell 1997). Later, the carrier phase

measurements were included to obtain centimetre scale accuracy. Integration of carrier phase measurements is viable for high accuracy positioning, which, however, brings in the problem of integer phase ambiguity cycles determination. This problem is due to the fact that the user receiver cannot directly measure the number of carrier cycles between the receiver and the corresponding satellite, but the change in the number of carrier cycles is measurable. The straightforward solution to this problem is limited to static user receivers or to limited spatial corrections between a roving user and the base reference station.

Revisiting equation 3.2, the carrier-phase observable is given as:

$$\begin{aligned} & (\widetilde{\phi} + N) \cdot \lambda = \left((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2 \right)^{0.5} + c\Delta t_r(t) + mp(t) \\ & + \zeta(t) + c\Delta t_{sv}(t) + SA(t) + E(t) - c\Delta t_{ion}(t) + c\Delta t_{tr}(t) \end{aligned}$$
(3.2)

Now, since selective availability has been switched off completely by decree of President Clinton (Office of the Press Secretary 2000) starting year 2006, the SA(t) term can be dropped. If all the common mode errors and the receiver clock bias can be collected into a single term; say $e_{cm}(t)$, and taking the actual pseudorange as $\rho = \left((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2 \right)^{0.5}$, then equation 3.2 becomes: $(\tilde{\phi} + N) \cdot \lambda = \rho + e_{cm}(t) + mp(t) + \zeta(t)$ (6.1)

6.2 SOLUTION TO INTEGER PHASE AMBIGUITY

There are two ways to solve the problem of integer phase ambiguity, either by eliminating the constant integer ambiguity by differencing the carrier-phase measurement across each time epoch or estimating the constant integer ambiguity. The former uses the Doppler carrier-phase processing method and the latter, the integer ambiguity resolution method. Carrier Phase Differential works on the principle of the Doppler phenomenon. To avoid the need for differential implementation time

differencing of the phase is done thus getting rid of the integer ambiguity and facilitating the direct estimation of velocity (further details are found in Farrell and Barth (1999)). Meanwhile, the integer ambiguity resolution method has two stages; the first stage is the initial estimate of ambiguity, which will be used as an initialisation for the second stage, the integer search algorithm, which determines the value of the integer ambiguity.

Forsell (1997) compares two methods for real-time ambiguity resolution. The first, wide-laning is a method using frequency differences between two suitably spaced carriers (Forsell 1995), and secondly, tone-ranging, which uses modulation signals on one carrier (Hatch 1996).

Hatch (2000) categorises ambiguity resolution techniques into two types: Geometry Independent, which are insensitive to tropospheric refraction, have a greater degree of freedom and require simple verification; and Geometry Dependent, which are the total opposite to the description of Geometry Independent. For Geometry Independent, the most common technique is ambiguity resolution in measurement space, which uses smoothed code for wide-lane ambiguity resolution, followed by estimating wide-lane resolved values to step to narrow-lane. Two Geometry Dependent techniques are ambiguity resolution in position space, which utilises Counselman's ambiguity function, and ambiguity resolution in ambiguity space, which searches for minimum residuals as a function of ambiguity combinations.

Further details and discussion are included in Appendix E.

6.3 CARRIER PHASE SMOOTHING APPLIED TO DGPS: The Hatch Filter

The carrier-smoothed code processing is based on the concept that estimating the bias in the integrated carrier phase measurement is essential in order to convert it into an absolute measurement of range. Although the carrier phase can be very accurately measured, the integrated carrier phase information cannot be directly mixed with the pseudorange since there is a phase ambiguity between the receiver and the satellite, which is equal to an integral multiple of two times pi. However, the change in the pseudorange between observations at different points of time (epochs) approximately equals the change in the integrated carrier phase. The change in the integrated carrier phase can, though, be determined with far more accuracy than the change in pseudorange. Carrier-smoothed code processing uses the carrier phase information to correct the code phase tracking loop to reduce multipath and receiver noise on the pseudoranges. Navigation equipment with a high precision requirement (e.g. aircraft autopilot for aircraft landing) and satellite navigation reference stations for differential correction (e.g. LAAS) are two examples. The smoothing of pseudorange observations using carrier phase observations has been elaborated by Hatch (1982), who introduced a recursive algorithm known as the Hatch filter. The Hatch filter is a simple onedimensional filter that uses the carrier-phase measurement to recursively update the pseudorange.

The multipath error, $v_{\rho mk}$ may be modelled as a first order Gauss-Markov process and hence can be considered to be the output of the process defined by,

 $v_{\rho m k} = a_m v_{\rho m (k-1)} + v_{m p i}.$

where a_m is the measured acceleration vector and v_{mpi} is a white noise process, representing the multipath component of the noise in the *i*th code pseudorange measurement.

In most modern receivers, the Hatch filter is a single-frequency carrier-smoothing filter, which can be modelled as a mixing filter of code-based and carrier phase measurements. If a total of L_{ϕ} measurements are made between two successive code measurements, the Hatch filter output at epoch k can be expressed in terms of the filter's output at epoch k-1 in the form:

$$\hat{\rho}_{h(k)} = \frac{L_{\phi} - 1}{L_{\phi}} \left(\hat{\rho}_{h(k-1)} + \phi_{m(k)} - \phi_{m(k-1)} \right) + \frac{1}{L_{\phi}} \rho_{mc(i)}$$
(6.2)

$$L_{\phi} = \frac{\tau_{\phi}}{T\phi} \tag{6.3}$$

where $\phi_{m(k)}$ and $\phi_{m(k-1)}$ are the measured carrier phase at epochs k and k-1 respectively, $\rho_{mc(i)}$ is the measured i^{th} code pseudorange, τ_{ϕ} is the filter time constant and T_{ϕ} is the sampling interval.

After smoothing, the noise of the smoothing code measurement can be described by Hwang and McGraw (1998) as follows:

$$\sigma_{\hat{\rho}_{h(k)}} \approx \sqrt{\sigma_{\phi}^2 + \frac{1}{2L_{\phi}}\sigma_{\rho_{mc(i)}}^2}$$
(6.4)

where σ_{ϕ} and $\sigma_{\rho_{mc(i)}}$ are the standard deviations for the carrier phase measurement noise and the code-based measurement noise respectively.

6.4 MODELLING THE HATCH FILTER

In this work, it is assumed that the code-based measurement is used to initialise N, due to the integer cycle ambiguity of the carrier phase, which causes a very poor initial position. Consequently, it is also assumed that the ambiguity error in the measured carrier phase is initially estimated, corrected and eliminated within the receiver. This assumption will enable the carrier phase ambiguity to be assumed as a carrier phase

noise. Thus the effect of the Hatch filter is only to reduce the noise and the Hatch filter may be equivalently modelled as:

$$\begin{bmatrix} \hat{v}_{k+1}^{\phi} \\ \hat{m}_{k+1} \\ \hat{v}_{k+1} \end{bmatrix} = \frac{1}{L_{\phi}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{\phi} a_{m} & 0 \\ -(L_{\phi} - 1) & a_{m} & (L_{\phi} - 1) \end{bmatrix} \begin{bmatrix} \hat{v}_{k}^{\phi} \\ \hat{m}_{k} \\ \hat{v}_{k} \end{bmatrix} + \frac{1}{L_{\phi}} \begin{bmatrix} L_{\phi} & 0 & 0 \\ 0 & L_{\phi} & 0 \\ (L_{\phi} - 1) & 1 & 1 \end{bmatrix} \begin{bmatrix} v^{\phi} \\ v_{mp} \\ v_{pc} \end{bmatrix}$$
(6.5)

where L_{ϕ} and a_m have been described in previous equations, the first state \hat{v}_k^{ϕ} is the current pseudorange error due to ambiguity, the second state \hat{m}_k is the current pseudorange error due to the multipath component and the third state \hat{v}_k is the current complete pseudorange error state representing the Hatch filter and their white noise processes are corresponding to v^{ϕ} , v_{mp} and v_{pc} .

$$\rho_{me} = \rho + \hat{v}_k + v_{po} \tag{6.6}$$

where ρ_{me} is the estimate of the code pseudorange may be expressed in terms of the actual magnitude of the pseudorange vector ρ and $v_{\rho 0}$ is a zero-mean white noise process, representing the receiver noise. In the above filter, the additional assumption is made that the white noise processes v^{ϕ} , v_{mp} and v_{pc} are stationary. This is due to the fact that these white noise processes are modelled as first order zero-mean Gauss-Markov processes and these processes are invariant after a translation of time.

6.5 SINGLE AND MULTI-FREQUENCY FILTERS

Tropospheric and ionospheric delays are significant, requiring accurate evaluation to implement a stable Hatch filter. The ionosphere advances the phases and delays the codes on a carrier signals in equal magnitude. Although tropospheric delays are independent of frequency, the magnitude of the ionospheric delay is inversely proportional to the square of the carrier frequency. The relationship is used to form an ionospheric independent observable using dual and triple frequency phase and code observations. The ionospheric error at a single frequency could be estimated adaptively

using a method outlined by Kim, Walter and Powell (2007). As the ionospheric error is dependent on the carrier frequency, several multi-frequency methods have also been presented for eliminating the ionospheric errors.

Recent developments in satellite navigation include GPS modernisation and the development of the European GALILEO system, which have led to the development of new algorithms. Following the GPS modernisation scheme, a third GPS frequency, L5 centred at 1176.45 MHz is being transmitted from Block IIF satellites, the first of which was launched in 2005 (Fontana et. al. 2001). Using the three frequency observations, a number of linear combinations are possible with characteristics such as longer wavelength, long ionospheric delay, less measurement noise and retention of the integer property of phase ambiguities.

Revisiting equation 3.24:

$$c\Delta t_{ion}(t) = \frac{40.3}{f^2} TEC$$
(3.24)

which is a generalisation for both single and dual frequency receivers.

Enhancements in accuracy can be made by using three carrier frequency receivers as proposed by Forssell, Martin-Neira and Harris (1997) called the Wavelength-Gap-Bridging method as illustrated in Figure 6.1.



Figure 6.1 The Wavelength-Gap-Bridging method (Forssell et. al. 1997).

The same technique as in appendix E.1 is applied but using two stages, where the first stage estimates the super wide-lane ambiguity integer. The output of the first stage then becomes the input for the second stage, which is to estimate the wide-lane ambiguity integer. Further accuracy of integer ambiguity resolution can be done via an integer search as explained in appendix E.2

6.6 MULTIPATH ESTIMATION

Two issues associated with the Hatch filter are multipath and ionospheric error induced divergence of the Hatch filter. Several modifications of the Hatch filter have been proposed to mitigate the effect of multipath and ionospheric error. Ray, Cannon and Fenton (2001) have proposed a multi-antenna method for mitigation of multipath effect. The process flowchart can be illustrated as in Figure 6.2.



Figure 6.2 The process flowchart for multipath mitigation.

The interest of this research is the description of the Kalman filter as presented in the flowchart, which eventually leads to the estimation of code and carrier phase multipath errors. Details of the multipath mitigation process can be retrieved from the text.

Ray et al (2001) identify that code-range, carrier phase and signal-to-noise ratio (SNR) measurements are all affected by multipath. These three measurements can be parameterised and developed as state variables for the Kalman filters. The five unknown parameters or the state variables are given as follows,

α		Reflection coefficien t	
α'		Correlation ratio	
γ_0	=	Reflected signal relative phase at the reference antenna	(6.7)
$ heta_0$		Reflected signal elevation	
$arphi_0$		Reflected signal azimuth	

The single difference code and carrier phase measurements are free from atmospheric delay errors, satellite orbital errors and satellite clock errors. These can be expressed respectively as follows:

$$\Delta P_{0i} = \Delta \rho_{0i} + c\Delta t_{0i} + \Delta \varepsilon_{p,0i} + \Delta \varepsilon_{Mp,0i}$$
(6.8)

$$\Delta \Phi_{0i} = \Delta \rho_{0i} + c \Delta t_{0i} + \Delta N_{0i} \lambda + \Delta \varepsilon_{\varphi,0i} + \Delta \varepsilon_{M\varphi,0i}$$
(6.9)

where ΔP is the measured code pseudorange single difference between antennae in unit metres,

 $\Delta\Phi$ is the measured carrier phase single difference between antennae in unit metres,

 $\Delta \rho$ is the range difference due to spatial separation between antennae in unit metres,

 $c\Delta t$ is the receiver clock bias difference in unit metres,

 ΔN is the integer ambiguity difference in unit cycles,

 $\Delta \varepsilon_p$ is the receiver code noise difference in unit metres,

 $\Delta \varepsilon_{\omega}$ is the receiver carrier phase noise difference in unit metres,

 $\Delta \varepsilon_{Mp}$ is the code pseudorange multipath error difference in unit metres and

 $\Delta \varepsilon_{M\varphi}$ is the carrier phase multipath error difference in unit metres.

If the receivers are driven by a common clock and their code range and carrier phase measurements corrected for the antennae's spatial separations, then the single difference code range and carrier phase measurements are reduced to:

$$\Delta P_{0i} = \Delta \varepsilon_{p,0i} + \Delta \varepsilon_{Mp,0i} \tag{6.10}$$

$$\Delta \Phi_{0i} = \Delta N_{0i} \lambda + \Delta \varepsilon_{\varphi,0i} + \Delta \varepsilon_{M\varphi,0i} \tag{6.11}$$

Then, the integer cycles can be easily removed, as the carrier phase multipath and noise combined are smaller in comparison to the carrier wavelength. Hence, equation 6.11 is further reduced and becomes:

$$\Delta \Phi_{0i} = \Delta \varepsilon_{\varphi,0i} + \Delta \varepsilon_{M\varphi,0i} \tag{6.12}$$

Omitting receiver code and carrier phase noises yield:

$$\Delta \tau_{0i} = \Delta P_{0i} = \Delta \varepsilon_{Mp,0i} \tag{6.13}$$

$$\Delta \Psi_{0i} = \Delta \Phi_{0i} = \Delta \varepsilon_{M\varphi,0i} \tag{6.14}$$

where $\Delta \tau_{0i}$ is the difference in code multipath error and

 $\Delta \Psi_{0i}$ is the difference in carrier phase multipath error.

Both differences in multipath errors (code and carrier phase) are at two closely spaced antennae and are assuming the reflected signal strength or the reflection coefficient is the same for both antennae. The difference in code multipath error, $\Delta \tau_{0i}$, can also be expressed as:

$$\Delta \tau_{0i} = \left(\frac{T(1-\alpha^2)(\cos\gamma_0 - \cos\gamma_i)}{1+\alpha\cos\gamma_0 + \alpha\cos\gamma_i + \alpha^2\cos\gamma_0\cos\gamma_i}\right)$$
(6.15)

where

T is the chip width in unit metres and

 γ_i is the reflected signal relative phase at antenna *i*.

Likewise, the difference in carrier phase multipath error $\Delta \Psi_{0i}$ can be expressed as:

$$\Delta \Psi_{0i} = \arctan\left(\frac{\alpha \alpha' \sin \gamma_0 - \alpha \alpha' \sin \gamma_i + \alpha^2 \alpha'^2 \sin(\gamma_0 - \gamma_i)}{1 + \alpha \alpha' \cos \gamma_0 - \alpha \alpha' \cos \gamma_i + \alpha^2 \alpha'^2 \cos(\gamma_0 - \gamma_i)}\right)$$
(6.16)

The SNR, R_{0i} , between two closely spaced antennae, assuming the noise power spectral density for both antenna is the same, is given as:

$$R_{0i} = \frac{P_i / N_i}{P_0 / N_0} = \frac{\left(1 + \alpha^2 \alpha'^2 + 2\alpha^2 \alpha'^2 \cos \gamma_i\right)}{\left(1 + \alpha^2 \alpha'^2 + 2\alpha^2 \alpha'^2 \cos \gamma_0\right)}$$
(6.17)

Let us further assume the bandwidth of the two receivers to be the same and that C_0 and C_i are the carriers to the noise power spectral densities of a satellite signal; the SNR now becomes:

$$R_{0i} = \frac{P_i}{P_0} = 10^{\binom{C_i - C_0}{10}}$$
(6.18)

In *m* closely spaced antennae, if one of the antennae in the (m-1) antenna pairs is common, then the number of measurements would be (m-1) single difference code range measurements, (m-1) single difference carrier phase measurements and (m-1) SNRs. Hence, the measurement matrix becomes:

$$z = \begin{bmatrix} \Delta \tau_{0,1} & \cdots & \Delta \tau_{0,m-1} & \Delta \Psi_{0,1} & \cdots & \Delta \Psi_{0,m-1} & \Delta R_{0,1} & \cdots & \Delta R_{0,m-1} \end{bmatrix}^T$$
(6.19)

The relationship between the state variables and the measurements is contained in the design matrix. The relationship is nonlinear, therefore the partial derivatives with respect to the unknown parameters must be computed. The resulting design matrix is:

$$\begin{bmatrix} \frac{\delta(\tau_{0,1})}{\delta a} & \frac{\delta(\tau_{0,1})}{\delta a'} & \frac{\delta(\tau_{0,1})}{\delta \gamma_{0}} & \frac{\delta(\tau_{0,1})}{\delta \theta_{0}} & \frac{\delta(\tau_{0,1})}{\delta \rho_{0}} \\ \frac{\vdots}{\delta(\tau_{0,m-1})} & \frac{\vdots}{\delta a'} & \frac{\vdots}{\delta(\tau_{0,m-1})} & \frac{\delta(\tau_{0,m-1})}{\delta \gamma_{0}} & \frac{\delta(\tau_{0,m-1})}{\delta \theta_{0}} & \frac{\delta(\tau_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(\Psi_{0,1})}{\delta a} & \frac{\delta(\Psi_{0,1})}{\delta a'} & \frac{\delta(\Psi_{0,1})}{\delta \gamma_{0}} & \frac{\delta(\Psi_{0,1})}{\delta \theta_{0}} & \frac{\delta(\Psi_{0,1})}{\delta \rho_{0}} \\ \frac{\delta(\Psi_{0,m-1})}{\delta a} & \frac{\delta(\Psi_{0,m-1})}{\delta a'} & \frac{\delta(\Psi_{0,m-1})}{\delta \gamma_{0}} & \frac{\delta(\Psi_{0,m-1})}{\delta \theta_{0}} & \frac{\delta(\Psi_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,1})}{\delta a} & \frac{\delta(R_{0,1})}{\delta a'} & \frac{\delta(R_{0,1})}{\delta \gamma_{0}} & \frac{\delta(R_{0,1})}{\delta \theta_{0}} & \frac{\delta(R_{0,1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta a} & \frac{\delta(R_{0,m-1})}{\delta a'} & \frac{\delta(R_{0,m-1})}{\delta \gamma_{0}} & \frac{\delta(R_{0,m-1})}{\delta \theta_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} & \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\ \frac{\delta(R_{0,m-1})}{\delta \rho_{0}} \\$$

The state variables in the Kalman filter are described as simple first-order Gauss-Markov processes. The correlation time was selected to be about 1 minute and an appropriate process noise was selected for each of the state variables.

This Kalman filter is used to estimate the unknown parameters for a particular satellite. These filter estimates refer to composite multipath, since the parameters based on the measurements are affected by multipath from all sources in the environment. After the parameters are estimated, multipath errors for code, $\hat{\tau}_c$, and carrier phase, $\Delta \Psi$, can be estimated using the following equations:

$$\hat{\tau}_c = \frac{\alpha T (1 - \alpha') \cos \gamma}{1 + \alpha \cos \gamma} \tag{6.21}$$

$$\Delta \Psi = \arctan\left(\frac{R(\tau_c - \tau_1)\alpha \sin \gamma}{R(\tau_c) + R(\tau_c - \tau_1)\alpha \cos \gamma}\right) = \arctan\left(\frac{\alpha \alpha' \sin \gamma}{1 + \alpha \alpha' \cos \gamma}\right)$$
(6.22)

6.7 WAAS CORRECTION ALGORITHMS

One way to enhance the accuracy of the carrier phase measurements is by applying the WAAS correction algorithm (Kim et. al. 2007). Presenting equation E.3 (refer Appendix E):

$$(\Delta \widetilde{\phi} + \Delta N) \cdot \lambda = \Delta r + \Delta (c \Delta t_r(t)) + \Delta (mp(t)) + \Delta (\zeta(t)) + \Delta (c \Delta t_{sv}(t)) + \Delta (E(t))$$

- $\Delta (c \Delta t_{ion}(t)) + \Delta (c \Delta t_{tr}(t))$ (E.3)

Dropping the frontal Δ sign, assuming that there is no cycle slip and collecting carrier phase multipath, error in the calculate ephemeris and random measurement noise terms into a single term ε , the equation can now be expressed as:

$$\widetilde{\phi} \cdot \lambda = r + c\Delta t_r(t) + c\Delta t_{sv}(t) - c\Delta t_{ion}(t) + c\Delta t_{tr}(t) + \varepsilon$$
(6.23)

which differs slightly from equation 6.1. A single difference of carrier phase measurements from a satellite k at two successive epochs, t and 0, is given as follows: $(\phi_t^k - \phi_0^k) \cdot \lambda = r_t^k - r_0^k + \Delta(c\Delta t_r(t)) + \Delta(c\Delta t_{sv}(t)) - \Delta(c\Delta t_{ion}(t)) + \Delta(c\Delta t_{tr}(t)) + \Delta \varepsilon_t^k$ (6.24)

Applying WAAS satellite clock-ephemeris error corrections and tropospheric error correction, linearising the terms with respect to a reference position, with a short base line, equation 6.24 becomes:

$$y_{t,0}^{k} = r_{t}^{k} - r_{0}^{k} - \left(\hat{r}_{R,t}^{k} - \hat{r}_{R,0}^{k}\right) + \Delta(c\Delta t_{r}(t)) - \Delta(c\Delta t_{ion}(t)) + \Delta\widetilde{\varepsilon}_{t}^{k}$$

$$= -1_{t}^{k} \cdot \delta \overline{x}_{t,0} + \Delta(c\Delta t_{r}(t)) + b_{REF,t}^{k} - \Delta(c\Delta t_{ion}(t)) + \Delta\widetilde{\varepsilon}_{t}^{k}$$
(6.25)

where \hat{r}_{R}^{k} is the computed distance between the satellite k and a reference position using the broadcast ephemeris,
1_t^k is a line of sight vector to the satellite k at time t,

 $\delta \bar{x}_{t,0}$ is the relative position of a receiver from the position at time 0,

 $b_{\text{REF},t}$ is an error caused from the imperfect knowledge of reference position at time t,

 $\Delta \tilde{\varepsilon}_t^k$ includes residual correction errors and higher order modelling errors due to linearisation in addition to $\Delta \varepsilon_t^k$.

6.8 ADAPTIVE FILTERING FOR INTEROPERABILITY

The concept of adaptive filtering for interoperability is implemented by a switching mechanism. Assuming the satellite navigation system currently in used is GPS an in order to switch from GPS to another system, say, GLONASS, a transition period for the switchover is required, so that there will be an overlap between the two systems. This uses a soft-switching technique and, therefore, is not an abrupt change, with a gradual switching and graceful changeover to another system. The complete system itself can either run the entire three GNSS in parallel or provide a lead-time in between two systems before the switching is committed.

The switching mechanism involves switching between two GNSS systems (inter-system switching), and not between individual satellites. Even though switching between individual GPS and Galileo satellites (intra-system switching) is possible due to the use of the same channel access method, there is a need to test the integrity and accuracy of the data repeatedly. Hence, this work only considers the inter-system switching mechanism.

As stated above, both systems must be in operation for some time. While adaptive filtering systems are suitable for such soft-switching, it is important to establish a sequence of initialisation algorithms which will facilitate the soft-switching. The data for this soft-switching algorithm will depend largely on the differences in performance

of the GPS, GLONASS, Galileo and other such GNSS systems. However, it is possible in principle to establish soft-switching algorithms based on adaptive UKF filters.

Hence, adaptive UKF provides the basic framework for the design of such interoperable systems.

6.9 INTEGRATION AND SYSTEM DESIGN

In order that pseudorange errors (or carrier-phase measurement errors) and orbital errors can be broadcast and applied by a rover, computation and consolidation of errors must be done. In this section, approaches to composite error estimation are presented. Later, the switching mechanism for the interoperable system is presented. Finally, the complete architecture of the system is laid out.

6.9.1 Approaches to Composite Error Estimation

In this work, two approaches have been devised for the composite error estimation. The first approach is the parallel approach (see Figure 6.3) and the second approach is the serial approach (see Figure 6.4).

In the parallel approach, an adaptive extended Kalman filter (AEKF) used for estimating pseudorange errors with an initially fixed Q_k (process noise) covariance matrix is set up in parallel with an adaptive unscented Kalman filter (AUKF) used for estimating orbital errors with an initially fixed R_k (measurement noise) covariance matrix. The outputs of the AEKF are the estimated pseudorange errors and R_k covariance matrix while the outputs of the AUKF are the estimated orbital errors and Q_k covariance matrix. All these four outputs will be added into the mixing Kalman filter, which will process and produce the composite error estimation as the output. In the serial approach, an AEKF is connected in series with an AUKF. The AEKF has pseudorange errors and an initially fixed Q_k covariance matrix as its input and produces an estimated pseudorange error and an asymptotically adapted R_k covariance matrix. These two outputs along with orbital errors will be the input to the AUKF. The composite error estimation is the output for the whole process.



Figure 6.3 Parallel approach to composite error estimation.



Figure 6.4 Serial approach to composite error estimation.

Similarly, carrier-phase measurement errors are used in place of code pseudorange errors. Generally it is assumed that the process and measurement covariance matrices

are unknown. This particularly true if one is switching between GLONASS, GPS and other satellite systems. The need for extended Kalman filtering arises as the equations are generally nonlinear and are linearised prior to applying the filtering algorithm. In this work, the equations were concurrently linearised during the derivation of the error equations.

6.9.2 Interoperable System: The Switching Mechanism

As described in section 6.8, the concept of adaptive filtering for interoperability is implemented by a switching mechanism. In this subsection, the switching mechanism is presented as in Figure 6.5.



Figure 6.5 A switching mechanism for interoperability.

Each of the navigation satellite receivers has its own adaptive UKF, which estimates the error covariance matrix and other statistical data of each system, taking into

account the availability of the signals and the least magnitude of error estimates by each adaptive UKF, and compares in the Compare block. The Compare block chooses which system is optimal and activates the switch to connect to the corresponding system. The default system in use is GPS.

For the switching to occur, a minimum of two systems must be available and operational at all times to ensure smooth interoperability. The switching mechanism is readily extensible by adding Compass/Beidou receivers and/or future GNSS receiver blocks in the above diagram.

6.9.3 The Complete System Architecture



The full operation of the system can be illustrated as in Figure 6.6.

Figure 6.6 Complete system architecture.

The switching mechanism and the composite error estimation subsystems have been elaborated on subsections 6.9.1 and 6.9.2. The output of the latter is combined with the broadcast correction signal of the local differential satellite navigation reference

station, and these are fed into the smoothing filter. The smoothing filter processes several coupled states in parallel and estimates the user position.

6.10 RECOMMENDATIONS ON IMPLEMENTATION

These algorithms are intended for implementation in a variety of applications for unmanned aerial vehicles, airborne survey and gravimetry, and remote sensing by direct geo-referencing of aerial imagery. It is expected that the enhanced accuracy algorithms would facilitate the development of aircraft landing systems based exclusively on satellite navigation receivers. The enhanced accuracy algorithms may be employed with the UKF to the mixing problem, so as to facilitate the mixing of measurements made by either a GNSS or a DGNSS and a variety of low-cost or high-precision INS sensors, as illustrated in the next chapter.

CHAPTER 7

ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY

Mixing navigation information can result in improved navigation positional accuracy. Furthermore, applying an unscented Kalman filter as an adaptive mixing filter will enable navigation observables to be used as measurements for estimating the pseudorange. This customised, interoperable pseudorange is 'mixed' with INS information produced by adaptive mixing filters for simultaneous interoperability, which is the main theme of this chapter.

7.1 PRINCIPLES OF THE MIXING FILTER

The basic idea of a mixing filter is a system, which accepts two or more inputs and gives improved output accuracy without compromising the overall system performance. Consider two different methods of measuring the same physical attribute concurrently, for example, using two different satellite navigation sensors to measure the instantaneous position or velocity of a particular vehicle. Although having two satellite navigation sensors can be considered as redundant, there are some benefits associated with this implementation, which will be explained as follows.

Assuming that one of the measurements is corrupted or one of the satellite navigation systems is down, by continuously observing the observables of both systems, there is a possibility of determining which of the systems is misbehaving. Even the faulty system can be identified with inertial navigation system (INS) information. This benefit is called integrity monitoring. However, if only one system is being used, there is no way

to monitor the system integrity and, hence, to determine which one of the systems is misbehaving.

In case one of the signals becomes temporarily unavailable, the estimation of that attribute can still be made possible by the other signal. This benefit is called availability, which continually provides the user with necessary navigational information without disruption, even during the downtime of one of the systems.

A well known example of mixing information is an INS vertical channel mixed with barometric height. This method has been proven to improve vertical accuracy of the vehicle. A widely used implementation of mixing filters is the complementary filter. An example of a complementary filter can be best illustrated as in Figure 7.1.



Figure 7.1 Schematic form of a complementary filter.

In the above figure, System 1 produces an output of θ_1 , which has low noise content and a fast response, but it is subjected to drift rate. System 2 produces an output of θ_2 , which has high noise content, but has a good long-term accuracy and bounded error estimation. The output of this complementary filter θ_0 , in respect of weighted gain *K* can be formulated as follows:

$$\theta_0 = \theta_1 + K \int (\theta_2 - \theta_0) \cdot dt \tag{7.1}$$

In this research, the inputs to the mixing filter are obtained from a generic satellite navigation system and INS. The majority of the satellite navigation systems currently in use are GPS based, although there are also other types of satellite navigation systems like GLONASS in use. Galileo and COMPASS, when operational, are also considered to be in this category. However, for the sake of generalisation, in this text, the term "satellite navigation" will be used to refer to any generic global satellite navigation system. The integrated satellite navigation-INS implementation based on the mixing filter concept is known to improve navigational performance.

In the next section, the mixing of satellite navigation-INS outputs using adaptive filtering for interoperability will be further elaborated.

7.2 ADAPTIVE SATELLITE NAVIGATION-INS MIXING FILTERS

With the availability of additional measurements, a host of Kalman filter based fusion algorithms have been developed (see Adam, Rivlin and Rotstein (1999) for an example) to compensate for misalignment and IMU errors (Waldmann, 2007). The Kalman filter is itself a two-stage process involving both state propagation and error correction. Kalman filter-based approaches have been proposed to integrate imaging vision sensors to provide for multi-sensor inertial navigation and alignment (see for example Hafskjold, Jalving, Hagen and Gade (2000), Roumeliotis, Johnson and Montgomery (2002) and Wang, Garratt, Lambert, Wang, Han, and Sinclair (2008)). One popular approach is to combine measurements made by GPS receiver with the traditional strapped down navigation system measurements (Eck and Geering (2000), Vik and Fossen (2001), Wagner and Wienecke (2003)). When no rate gyro measurements are made and it is still possible to make other measurements using satellite navigation aids such as GPS, which can provide estimates of the pseudorange or of carrier smoothed pseudorange and the carrier phase differentials, the algorithms for the computation of

the navigation position and orientation can be greatly simplified. While errors may still be classified as coning errors that arise because finite rotations do not commute, sculling errors that are due to incorrect thrust velocity computation as coordinate frames rotate between data samples, and as scrolling errors arising from velocity and position updates occurring at distinctly different rates, the relative contributions of these error sources to the total navigation error can be significantly different when a sensor fusion approach is adopted.

GPS aided INS development has progressed in two distinct directions. In the first case, there have been substantial efforts to develop high fidelity navigation systems for attitude and position estimation. These include high accuracy systems for both geomatic and navigation applications (see for example Mohamed (1999), Grejner-Brzezinska and Wang (1998), Qin, Zhang, Zhang and Xu (2006), Liu, Tian and Huang (2001) and Farrell, Givargis and Barth (2000)). These systems recommend the use of either highly sophisticated angular rate sensors or carrier phase and differential carrier phase measurement systems to achieve the improved accuracy. For navigation applications Rios and White (2000), Bye, Hartmann and Killen (1998) and Salychev, Voronov, Cannon, Nayak and Lachapelle (2000) have considered the development of low cost GPS aided inertial navigation systems. Nordlund (2000), Wan, E.A., and van der Merwe (2001) have recommended the use of nonlinear estimation algorithms as a matter of course.

In order for interoperable satellite navigation-INS mixing filters to be presented, an overview of INS is introduced, followed by strapdown INS, gyro-free strapdown INS, high precision INS, process modelling with gyro-free acceleration measurements, process modelling with carrier phase measurements, customised satellite navigation measurement modelling, and finally the formulation of the satellite navigation-INS mixing filter.

7.2.1 Overview of INS

An Inertial Navigation System (INS) is a standard navigation aid that uses a computer and accelerometers to continuously compute the position, velocity, acceleration and orientation of a vehicle in motion whether as a stand-alone or with external aiding systems.

Two main types of INS, according to Farrel and Barth (1999), are the mechanisedplatform approach and the strapdown approach. Farrel and Barth (1999) define the mechanised-platform approach as driving a set of actuators to maintain the alignment of the platform with the coordinate axes of a desired navigation coordinate system independent of the motion of the vehicle relative to the navigation frame. Farrel and Barth (1999) go on to define the strapdown approach, which mounts the instrument platform directly on the vehicle chassis and transforms the inertial measurements to the navigation frame computationally.

High accuracy integrated navigation systems based on carrier-phase satellite navigation systems such as the Global Positioning System (GPS) and Inertial Navigation System (INS) are under development for a variety of applications in unmanned aerial vehicles (UAVs), airborne survey and gravimetry and remote sensing by direct geo-referencing of aerial imagery (Farrell and Barth, 1999; Farrell, Givargis and Barth, 2000; Yang, Farrell and Tan, 2001a, 2001b). With the availability of several fully operational satellite navigation systems, it has been recognised that an optimal combination of one or more satellite navigation systems with inertial navigation has a number of advantages over stand-alone inertial or satellite navigation. Each satellite contributes its high accuracy and stability over time, enabling continuous monitoring of inertial sensor errors. Implementation of closed-loop INS errors within a certain boundary, leading to increased estimation accuracy. Thus the satellite navigation aiding information is used to reduce the estimate errors in the INS state and to continuously calibrate the inertial sensors. This

results in improved INS accuracy. On the other hand, INS contributes immunity from satellite outages. During periods when signals from some or all of the satellites become unavailable, the INS continues to provide vehicle state information. The INS also provides for continuous attitude monitoring, and the reduction of the carrier phase ambiguity search volume/time. Using a carrier phase-based and calibrated satellite navigation system, high to medium accuracy inertial system, attitude accuracy in the range of 10-30 arcsec can be achieved in principle [Grejner-Brzezinska and Wang, 1998]. Therefore, the integrated approach has been shown to result in improved reliability, latency, bandwidth, and update rate improvements relative to the satellite navigation only approach.

7.2.2 Strapdown INS

The strapdown INS is preferable compare to the mechanised-platform due to smaller size, less expensive, requires less power. The strapdown system eliminates the need of gimbals. Furthermore, the strapdown has a higher update rate of about 2000 Hz, in comparison to the gimballed system that normally has update rates of 50-60 Hz. This higher rate is required to keep the maximum angular measurement within a practical range for real rate gyroscopes. Nowadays, strapdown systems are mass produced due to cheap, fast and reliable digital computers and are more practical to use compared to the gimbals.

7.2.3 Gyro-free Strapdown INS

The concept of gyro-free measurement of angular acceleration using linear accelerometers was proposed by Schuler, Grammatikos and Fegley (1967) more than forty years ago. Subsequently, Padgoanker, Krieger and King (1975) and Mital and King (1979) considered the computation of rigid body rotations from measurements of linear acceleration obtained from body fixed linear accelerometers. Moreover, it was felt that to obtain stable outputs of rotational motion a minimum of nine

accelerometers are necessary. However Chen, Lee and DeBra (1994) were able to show that six accelerometers are quite adequate to measurement rigid body rotations. Since their work, a few alternate schemes using nine accelerometers have emerged, such as the one proposed by Wang, Ding and Zhao (2003). In most of these proposals the six accelerometer unit was considered as an independent sensor but was not fully integrated into a strapped down navigation system.

Figure 7.2 shows a six linear accelerometers configuration for a Gyro-free strapdown INS.



Figure 7.2 The GYROCUBE: A sensor for inertial measurements; the directions of the arrows indicates the direction of sensitivity of the accelerometers.

7.2.4 High Precision INS

Although there have been several studies of the integration of satellite navigations systems with inertial navigation systems (see for example Wang, Lachapelle and Cannon, 2004), most of these have been restricted to low cost solutions. With the low cost solutions, it is practically impossible to obtain accurate estimates of the attitude. Most of the low cost solutions use a complement of solid state accelerometers and do not use the more expensive rate or even attitude gyros required for precise attitude estimation. Accurate estimation of the attitude will require an independent measurement of the attitude or even the attitude and angular velocity vector.

Compared to a low cost solution, the development of a high precision integrated system would involve fibre-optic gyro-based angular velocity measurements (Bye, Hartmann and Killen, 1998) and multiple-antenna based attitude measurements.

In addition to the pseudorange measurement, a carrier phase measurement is usually provided in many modern satellite navigation receivers. Two types of measurement are available from a typical satellite navigation system (Hatch 1982). The relative phase between the received reconstructed carrier phase and the receiver clock phase at a particular epoch may be measured. This measurement is a fine measurement of pseudorange in terms of the non-integer number of cycles with the integer number or whole cycles deleted. Another form of carrier related measurement that is more common is obtained by integrating the rate of change of relative phase over a specific time interval as determined by the receiver clock. To complement the angular velocity and/or attitude measurements either of the carrier phase measurements are used in several high precision satellite navigation applications to recursively smooth and improve code-based range measurements via the use of an embedded filter and an embedded fast ambiguity resolution method.

The well known observable for satellite aided attitude determination is the difference in the carrier phase between a master antenna and slave antenna located at two different positions on an aircraft. If the integer ambiguity can be resolved, the carrier phase difference measurement is the only addition measurement required for attitude determination. The phase difference can be shown to be a function of the pseudorange difference and the differential ambiguity. The bias and noise in this measurement could be eliminated by taking the difference of two independent measurements of the phase difference, a process known as double differencing. The double difference is a linear combination of four phase measurements obtained from two different satellite navigation spacecraft at the two antennae of the receiver. Satellite navigation receivers have been built to provide such measurements directly but are more expensive than the simpler code phase receivers. Provided such measurements are available, the accuracy of the translation and angular velocities could be substantially improved. This aspect involves the measurement of attitude using multiple GPS antenna as well as attitude estimation, which has been discussed by Vepa (2010) and is included for high precision and Doppler-aided high precision applications.

7.2.5 Process Modelling with Gyro-free Acceleration Measurements

The basic navigation equations have been derived by Farrell and Barth (1999). These are summarised here for completeness:

$$\begin{bmatrix} \dot{V}_{N} \\ \dot{V}_{E} \\ \dot{V}_{D} \end{bmatrix} = \begin{bmatrix} A_{N} \\ A_{E} \\ A_{D} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 2\omega_{s}\sin\lambda + \frac{V_{E}}{R_{P} + h}\tan\lambda \end{bmatrix} V_{N} + \begin{bmatrix} 2\omega_{s}\cos\lambda + \frac{V_{E}}{R_{P} + h} \end{bmatrix} V_{D} \\ - \frac{V_{N}^{2}}{R_{M} + h} - \begin{bmatrix} 2\omega_{s}\cos\lambda + \frac{V_{E}}{R_{P} + h} \end{bmatrix} V_{E} \end{bmatrix},$$

$$\dot{\lambda} = \frac{V_N}{R_M + h}, \qquad \dot{\phi} = \frac{V_E \sec \lambda}{R_P + h}, \qquad \dot{h} = -V_D, \qquad (7.2)$$

where V_N , V_E and V_D are the north, east and down velocities in the local tangent plane, with reference to a local geodetic frame often referred to as the navigation frame (*n*-frame) or north-east-down frame. The last three equations relate these velocities to the rate of change of the geodetic latitude (λ), the rate of change of longitude (φ) and the altitude (h). A_N , A_E and A_D are the north, east, down components of the measured acceleration in the *n*-frame which must be compensated for by adding the acceleration due to gravity g, in down direction, ω_s is the angular velocity of the Earth, R_M and R_P are the radii of curvature in the meridian and prime vertical at a given latitude. Unit vectors in the *n*-frame are related to the unit vectors in the Earth-centred inertial frame according to the relations:

$$\begin{bmatrix} i_G \\ j_G \\ k_G \end{bmatrix} = \mathbf{D}_{n,I} \begin{bmatrix} i_I \\ j_I \\ k_I \end{bmatrix}, \ \mathbf{D}_{n,I} \equiv \begin{bmatrix} -\sin\lambda\cos(\varphi + \Xi) & -\sin\lambda\sin(\varphi + \Xi) & \cos\lambda \\ -\sin(\varphi + \Xi) & \cos(\varphi + \Xi) & 0 \\ -\cos\lambda\cos(\varphi + \Xi) & -\cos\lambda\sin(\varphi + \Xi) & -\sin\lambda \end{bmatrix}$$
(7.3)

where Ξ is the hour angle of the vernal equinox. The vector of the north, east, down components of the measured acceleration in the *n*-frame is related to the body components of the measured acceleration, by the transformation:

$$\mathbf{A}_{NED} = \mathbf{D}_{n,b} \mathbf{A}_{body},\tag{7.4}$$

where the transformation of the measured body acceleration components to the north, east, down components in the *n*-frame $\mathbf{D}_{n,b}$, satisfies the differential equation:

$$\dot{\mathbf{D}}_{n,b} + \mathbf{\Omega}_G \mathbf{D}_{n,b} = \mathbf{D}_{n,b} \mathbf{\Omega}_b.$$
(7.5)

In equation 7.5 the matrix Ω_G is obtained from the components of the angular velocity vector of the local geodetic frame or *n* frame. The angular velocity vector of the local geodetic frame or *n* frame may be expressed in terms of the Earth angular velocity in the local geodetic frame ω_s as:

$$\boldsymbol{\omega}_{G} = \boldsymbol{\omega}_{s} + \begin{bmatrix} \dot{\varphi} \cos \lambda \\ -\dot{\lambda} \\ -\dot{\varphi} \sin \lambda \end{bmatrix} \text{ with } \boldsymbol{\omega}_{s} = \boldsymbol{\omega}_{s} \begin{bmatrix} \cos \lambda \\ 0 \\ -\sin \lambda \end{bmatrix}.$$
(7.6)

Given a vector, $\boldsymbol{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$, $\boldsymbol{\omega}_{\times}$ is defined by the relation:

$$\mathbf{\omega}_{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}.$$
(7.7)

Then Ω_G is defined as $\Omega_G = \omega_{G\times}$. Similarly Ω_b is defined as $\Omega_b = \omega_{b\times}$ where ω_b is the body angular velocity in the body fixed frame.

In principle, the scalar acceleration measurements may be expressed as:

$$a_{\mathbf{i}} = \mathbf{z}^{i} \cdot \left[\ddot{\mathbf{R}}_{\mathbf{I}} - \mathbf{r}^{i} \times \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}^{i} \right], \ i = 1, 2, 3....6$$
(7.8)

where \mathbf{z}^{i} , is the direction of sensitivity of the *i*th accelerometer, \mathbf{r}^{i} is the position vector of the accelerometer location in the body fixed frame, $\boldsymbol{\omega} = \boldsymbol{\omega}_{b}$ and $\ddot{\mathbf{R}}_{I}$ is the

inertial acceleration of the origin of the body frame. With six accelerometers it is, in principle, possible to express:

$$\begin{bmatrix} -\mathbf{z}^{i} \cdot \mathbf{r}_{x}^{i} & \mathbf{z}^{i} \begin{bmatrix} \dot{\mathbf{\omega}} \\ \dot{\mathbf{R}}_{I} \end{bmatrix} = a_{i} - \mathbf{z}^{i} \cdot \begin{bmatrix} \mathbf{\omega} \times \mathbf{\omega} \times \mathbf{r}^{i} \end{bmatrix} = a_{i} + f_{i} (\mathbf{z}^{i}, \mathbf{r}^{i}, \mathbf{\omega}),$$
(7.9)

where

$$f_i(\mathbf{z}^i, \mathbf{r}^i, \boldsymbol{\omega}) \equiv -\mathbf{z}^i \cdot [\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}^i].$$
(7.10)

It follows that:

$$f_{i}(\mathbf{z}^{i},\mathbf{r}^{i},\boldsymbol{\omega}) = z_{1}^{i}((\omega_{3}^{2}+\omega_{2}^{2})r_{1}^{i}-\omega_{1}\omega_{2}r_{2}^{i}-\omega_{1}\omega_{3}r_{3}^{i}) + z_{2}^{i}(-\omega_{2}\omega_{1}r_{1}^{i}+(\omega_{1}^{2}+\omega_{3}^{2})r_{2}^{i}-\omega_{2}\omega_{3}r_{3}^{i}) + z_{3}^{i}(-\omega_{3}\omega_{1}r_{1}^{i}-\omega_{3}\omega_{2}r_{2}^{i}+(\omega_{2}^{2}+\omega_{1}^{2})r_{3}^{i}).$$
(7.11)

Defining the vectors \mathbf{d}_i as:

$$\begin{bmatrix} \mathbf{d}_{i} \end{bmatrix} = \begin{bmatrix} -\mathbf{z}^{i} \cdot \mathbf{r}_{x}^{i} & \mathbf{z}^{i} \end{bmatrix} = \begin{bmatrix} z_{3}^{i} r_{2}^{i} - z_{2}^{i} r_{3}^{i} & z_{1}^{i} r_{3}^{i} - z_{3}^{i} r_{1}^{i} & z_{2}^{i} r_{1}^{i} - z_{1}^{i} r_{2}^{i} & z_{1}^{i} & z_{2}^{i} & z_{3}^{i} \end{bmatrix}$$
(7.12)

equation 7.9 may be expressed as:

$$\mathbf{D}\begin{bmatrix} \dot{\boldsymbol{\omega}}^T & \ddot{\mathbf{R}}_1^T \end{bmatrix}^T = \mathbf{A} + \mathbf{F},$$
(7.13)
where $\mathbf{D} = \begin{bmatrix} \mathbf{d}_1^T & \mathbf{d}_2^T & \mathbf{d}_3^T & \mathbf{d}_4^T & \mathbf{d}_5^T & \mathbf{d}_6^T \end{bmatrix}^T, \ \mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^T,$
and $\mathbf{F} = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \end{bmatrix}^T.$

Equation 7.13 may be expressed as:

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A} + \mathbf{F}), \ \ddot{\mathbf{R}}_{\mathbf{I}} = \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A} + \mathbf{F}).$$
(7.14)

At this stage it is important to recognise that the definition of the functions $f_i(\mathbf{z}^i, \mathbf{r}^i, \boldsymbol{\omega})$, must be modified after considering that measurements of acceleration must be compensated by adding the local acceleration due to gravity. Furthermore, the definition of the acceleration of gravity generally includes the centripetal acceleration due to the Earth's induced rotation rate vector, $\boldsymbol{\omega}_s$. For this reason, one defines:

$$\Delta f_i(\mathbf{z}^i, \mathbf{r}^i, \boldsymbol{\omega}) \equiv \mathbf{z}^i \cdot \left(\left[\boldsymbol{\omega}_s \times \boldsymbol{\omega}_s \times \mathbf{r}^i \right] - \left[\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}^i \right] \right), \tag{7.15}$$

and the equation 7.14 may now be expressed as:

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A}_m + \mathbf{G} + \Delta \mathbf{F} + \mathbf{b} + \mathbf{n}), \tag{7.16}$$

and the body components of the measured acceleration are:

$$\mathbf{A}_{body} = \begin{bmatrix} [\mathbf{0}_{3\times3} & \mathbf{I}_{3\times3}] \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A}_m + \Delta \mathbf{F} + \mathbf{b} + \mathbf{n}), \tag{7.17}$$

where **G** is the gravitational component of the acceleration in the body frame: $\Delta \mathbf{F} = \begin{bmatrix} \Delta f_1 & \Delta f_2 & \Delta f_3 & \Delta f_4 & \Delta f_5 & \Delta f_6 \end{bmatrix}^T$, **b** is a measurement bias vector and **n** is a measurement noise vector. It is possible to choose the location \mathbf{r}^i , and the direction of the measurements \mathbf{z}^i , such that, in equation 7.16:

$$\begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \mathbf{D}^{-1} \mathbf{G} = \mathbf{0}.$$
(7.18)

Hence it follows that:

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A}_m + \Delta \mathbf{F} + \mathbf{b} + \mathbf{n}), \tag{7.19}$$

and:

$$\mathbf{A}_{NED} = \mathbf{D}_{n,b} \begin{bmatrix} [\mathbf{0}_{3\times3} & \mathbf{I}_{3\times3}] \end{bmatrix} \mathbf{D}^{-1} (\mathbf{A}_m + \Delta \mathbf{F} + \mathbf{b} + \mathbf{n}), \qquad \mathbf{D}_{n,b} = \mathbf{D}_{n,l} \mathbf{D}_{b,l}^{-1}.$$
(7.20)

When the accelerometers are located on the faces of a rectangular cuboid, as shown in Figure 7.2, the vectors \mathbf{z}^i and \mathbf{r}^i may be expressed as:

$$\begin{bmatrix} \mathbf{z}^{1} \\ \mathbf{z}^{2} \\ \mathbf{z}^{3} \\ \mathbf{z}^{4} \\ \mathbf{z}^{5} \\ \mathbf{z}^{6} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix},$$

$$(7.21)$$

$$\begin{bmatrix} \mathbf{r}^{1} & \mathbf{r}^{2} & \mathbf{r}^{3} & \mathbf{r}^{4} & \mathbf{r}^{5} & \mathbf{r}^{6} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -r_{1}^{3} & r_{1}^{3} & 0 & 0 \\ 0 & -r_{2}^{2} & 0 & 0 & r_{2}^{2} & 0 \\ -r_{3}^{1} & 0 & 0 & 0 & 0 & r_{3}^{1} \end{bmatrix}.$$

$$(7.22)$$

Equation 7.19 may now be integrated, in principle to obtain the body angular velocity vector, $\boldsymbol{\omega} = \boldsymbol{\omega}_{b}$. The attitude quaternion is then computed from the equations:

$$\dot{\mathbf{q}} = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}) \mathbf{q}, \ \boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}, \ \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix},$$
(7.23)

where the quaternion is subject to the constraint $\mathbf{q}^T \cdot \mathbf{q} = \mathbf{1}$. Once the solution for the quaternion is known, the transformation from the inertial to the body fixed frame $\mathbf{D}_{b,I}$ is computed from:

$$\mathbf{D}_{b,I}(\mathbf{q}) = \begin{bmatrix} q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & q_4^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}.$$
(7.24)

and its inverse is obtained using the same equation by changing the sign of q_4 . The required transformation $\mathbf{D}_{n,b}$ may then be computed without matrix inversion from $\mathbf{D}_{n,l}\mathbf{D}_{b,l}^{-1}$, the transformations from the inertial to the *n*-frame and the inverse transformation from the inertial to the body fixed frame. Alternately $\mathbf{D}_{n,b}$ may be computed directly from the associated quaternion, representing the relative attitude of the navigation from relative to the body frame.

7.2.6 Process Modelling with Carrier Phase Measurements

The process modelling with carrier phase measurement follows the same models as presented by equations 7.2 – 7.8. The departure of both process modelling starts off following equation 7.8. Assuming that all accelerometers are co-located and with three independent accelerometer measurements it is, in principle, possible to express: $\mathbf{z}^{i}\ddot{\mathbf{R}}_{I} = a_{i} + [\mathbf{z}^{i} \cdot \mathbf{r}_{\times}]\dot{\boldsymbol{\omega}} - \mathbf{z}^{i} \cdot [\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}] = a_{i} + [\mathbf{z}^{i} \cdot \mathbf{r}_{\times}]\dot{\boldsymbol{\omega}} + f_{i}(\mathbf{z}^{i}, \mathbf{r}, \boldsymbol{\omega}),$ (7.25) where $\mathbf{r} = \mathbf{r}^{i}$ $f_{i}(\mathbf{z}^{i}, \mathbf{r}, \boldsymbol{\omega}) \equiv -\mathbf{z}^{i} \cdot [\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}].$ (7.26)

Equation 7.26 is similar to equation 7.10, with slight differences in notation and equation 7.26 is a more general form to equation 7.10. It follows that:

$$f_{i}(\mathbf{z}^{i},\mathbf{r}^{i},\boldsymbol{\omega}) = z_{1}^{i}((\omega_{3}^{2}+\omega_{2}^{2})r_{1}-\omega_{1}\omega_{2}r_{2}-\omega_{1}\omega_{3}r_{3}) + z_{2}^{i}(-\omega_{2}\omega_{1}r_{1}+(\omega_{1}^{2}+\omega_{3}^{2})r_{2}-\omega_{2}\omega_{3}r_{3}) + z_{3}^{i}(-\omega_{3}\omega_{1}r_{1}-\omega_{3}\omega_{2}r_{2}+(\omega_{2}^{2}+\omega_{1}^{2})r_{3}).$$
(7.11)

Defining the vectors \mathbf{d}_i as:

$$\begin{bmatrix} \mathbf{d}_i \end{bmatrix} = \begin{bmatrix} -\mathbf{z}^i \cdot \mathbf{r}_x^i & \mathbf{z}^i \end{bmatrix} = \begin{bmatrix} z_3^i r_2^i - z_2^i r_3^i & z_1^i r_3^i - z_3^i r_1^i & z_2^i r_1^i - z_1^i r_2^i & z_1^i & z_2^i & z_3^i \end{bmatrix}$$
(7.12)

$$\begin{bmatrix} \mathbf{d}_i \end{bmatrix} = \begin{bmatrix} \mathbf{z}^i \end{bmatrix} = \begin{bmatrix} z_1^i & z_2^i & z_3^i \end{bmatrix}$$
(7.27)

equation 7.25 may be expressed as:

$$\mathbf{D\ddot{R}}_{I} = \mathbf{A} + \mathbf{D}[\mathbf{r}_{x}]\dot{\boldsymbol{\omega}} + \mathbf{DF}, \qquad (7.28)$$

where $\mathbf{D} = \begin{bmatrix} \mathbf{d}_1^T & \mathbf{d}_2^T & \mathbf{d}_3^T \end{bmatrix}^T$, $\mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^T$, and $\mathbf{F} = \begin{bmatrix} (\omega_3^2 + \omega_2^2)r_1 - \omega_1\omega_2r_2 - \omega_1\omega_3r_3 \\ -\omega_2\omega_1r_1 + (\omega_1^2 + \omega_3^2)r_2 - \omega_2\omega_3r_3 \\ -\omega_3\omega_1r_1 - \omega_3\omega_2r_2 + (\omega_2^2 + \omega_1^2)r_3 \end{bmatrix}$.

Equation 7.28 may be expressed as:

$$\ddot{\mathbf{R}}_{\mathbf{I}} = \mathbf{D}^{-1}\mathbf{A} + [\mathbf{r}_{\times}]\dot{\boldsymbol{\omega}} + \mathbf{F}.$$
(7.29)

At this stage it is important to recognise that the definition of the function vector **F** must be modified considering that measurements of acceleration must be compensated by adding the local acceleration due to gravity. Furthermore, the definition of the acceleration of gravity generally includes the centripetal acceleration due the Earth's rotation rate vector, $\boldsymbol{\omega}_{s}$. For this reason, one defines:

$$\Delta \mathbf{F} = ([\boldsymbol{\omega}_s \times \boldsymbol{\omega}_s \times \mathbf{r}] - [\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r}]). \tag{7.30}$$

Equation 7.30 is similar to equation 7.15, with slight differences in notation and equation 7.30 is a more general form to equation 7.15. The equation 7.29 gives the body components of the acceleration as:

$$\mathbf{A}_{body}^{T} = \mathbf{D}^{-1} \left(\mathbf{A}_{m} + \mathbf{G} + \mathbf{b}_{1} + \mathbf{n}_{1} \right) + \Delta \mathbf{F}, \qquad (7.31)$$

where **G** is the gravitational component of the acceleration in the body frame, \mathbf{A}_m is the actual measured acceleration vector obtained from a triad of pendulous accelerometers, \mathbf{b}_1 is a measurement bias and drift vector and \mathbf{n}_1 is a measurement white noise vector. The north, east and down accelerations are:

$$\mathbf{A}_{NED}^{T} = \mathbf{D}_{n,b} \left[\mathbf{D}^{-1} \left(\mathbf{A}_{m} + \mathbf{G} + \mathbf{b}_{1} + \mathbf{n}_{1} \right) + \Delta \mathbf{F} \right], \ \mathbf{D}_{n,b} = \mathbf{D}_{n,I} \mathbf{D}_{b,I}^{-1}.$$
(7.32)

The north, east and down accelerations may be expressed in terms of the measured north, east and down components of the acceleration and north, east and down components of the gravity vector as:

$$\mathbf{A}_{NED}^{T} = \mathbf{A}_{NED} + \mathbf{G}_{NED},\tag{7.33}$$

where

$$\mathbf{A}_{NED} = \mathbf{D}_{n,b} \Big[\mathbf{D}^{-1} \big(\mathbf{A}_m + \mathbf{b}_1 + \mathbf{n}_1 \big) + \Delta \mathbf{F} \Big],$$
(7.34a)

and

$$\mathbf{G}_{NED} = \mathbf{D}_{n,b} \begin{bmatrix} \mathbf{D}^{-1} \mathbf{G} \end{bmatrix} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T.$$
(7.34b)

The WGS-84 model of the local acceleration due to gravity is defined as,

$$g = g_0 \left(s^2 \left(1 - \frac{2}{a} \left(1 + f + m - 2fs^2 \right) h + \frac{3}{a^2} h^2 \right), \ s = \sin \phi,$$

 $g_0(s^2) = \gamma_a (1 + 0.0052790414s^2 + 0.0000232718s^4 + 0.0000001262s^6 + 0.0000000007s^8)$ where $\gamma_a = 9.7803267715 \ m/sec^2$ is equatorial acceleration due to gravity, which is corrected for latitude variations and altitude h, variations $m = \omega_s^2 a^2 b/GM_0$ which is evaluated as, m = 0.00344978600308, from estimates of GM_0 which is the product of the universal gravitation constant and the Earth mass, and ω_s which is the Earth's sidereal rate, ϕ is the common or geodetic latitude is the angle between the equatorial plane and a line that is normal to the reference ellipsoid. Depending on the flattening, f, it may be slightly different from the geocentric (geographic) latitude, which is the angle between the equatorial plane and a line from the centre of the ellipsoid. The Earth flattening factor, f is defined as,

$$f = \frac{a-b}{a}.$$

It may also be obtained from the eccentricity *e* and is related to it by,

$$e^{2} = f(2-f) = 1-(1-f)^{2}.$$

The WGS-84 parameters used for Earth's semi-major axis a, semi-minor axis b and f are respectively given by, a = 6378137 m, b = 6356752.3142 m and f = 298.257223563. However since the same gravity model in used in the simulation and measurement, it cancels out and the results are quite independent of the model.

The drift and bias vectors are assumed to be first order Gauss-Markov processes given by:

$$\dot{\mathbf{b}}_1 = \mathbf{b}_2 + \mathbf{n}_2 , \ \mathbf{b}_2 = \mathbf{n}_3$$
 (7.35)

where \mathbf{n}_2 and \mathbf{n}_3 , are a white noise vector driving the processes.

The body angular velocity vector, $\boldsymbol{\omega} = \boldsymbol{\omega}_b$, is assumed to be measured by a triad of fibre optic laser gyros. Thus the measure angular velocity vector is assumed to be related to the body angular vector:

$$\mathbf{L}\boldsymbol{\omega}_b = \boldsymbol{\omega}_m + \mathbf{b}_3 + \mathbf{n}_4 \tag{7.36}$$

where **L** is the matrix of the three directions of sensitivity of the fibre-optic laser gyros, $\boldsymbol{\omega}_m$ the actual measured angular velocities, \mathbf{b}_2 is a measurement bias and drift vector and \mathbf{n}_3 is a measurement white noise vector. Following Savage (1998a) and Savage (1998b) the bias and drift vector is assumed to be a first order Gauss-Markov process given by:

$$\mathbf{b}_3 = \mathbf{b}_4 + \mathbf{n}_5, \ \mathbf{b}_4 = \mathbf{n}_6 \tag{7.37}$$

where \mathbf{n}_5 and \mathbf{n}_6 , are a white noise vector driving the processes.

It is assumed that there is no need to scale either the acceleration or angular velocity measurements as the sensors are assumed to be calibrated. Thus, no provision is made for scaling the measurements. Furthermore, when the three accelerometer measurement axes and fibre-optic gyro measurement axes coincide with the body axes, it can be assumed that nominally, $\mathbf{D} = \mathbf{L} = \mathbf{I}_{3\times 3}$.

Similarly, as in subsection 7.2.5, Process Modelling with Carrier Phase Measurements, the attitude quaternion is then computed from the equations:

$$\dot{\mathbf{q}} = \frac{1}{2} \boldsymbol{\Omega}(\boldsymbol{\omega}) \mathbf{q}, \ \boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}, \ \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix},$$
(7.23)

where the quaternion components are subject to the constraint $q_1^2 + q_2^2 + q_3^2 + q_4^2 = \mathbf{1}$. Once the solution for the quaternion is known, the transformation from the inertial to the body fixed frame $\mathbf{D}_{b,l}$ is computed from:

$$\mathbf{D}_{b,I}(\mathbf{q}) = \begin{bmatrix} q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & q_4^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}.$$
(7.24)

and its inverse is obtained from the same equation by changing the sign of q_4 . The required transformation $\mathbf{D}_{n,b}$ may then be computed without matrix inversion from $\mathbf{D}_{n,l}\mathbf{D}_{b,l}^{-1}$, the transformations from the inertial to the *n*-frame and the inverse transformation from the inertial to the body fixed frame. Alternatively, $\mathbf{D}_{n,b}$ may be computed directly from the associated quaternion, representing the attitude of the navigation relative to the body frame.

7.2.7 Customised Satellite Navigation Measurement Modelling

The satellite-based position estimation and measurement has been sufficiently modelled and elaborated in full detail in chapter three. Re-visiting equation 3.1 as follows, the measured code pseudorange, $\tilde{\rho}$ is given:

$$\widetilde{\rho} = \left((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2 \right)^{0.5} + c\Delta t_r(t) + MP(t) + \eta(t) + c\Delta t_{sv}(t) + SA(t) + E(t) + c\Delta t_{ion}(t) + c\Delta t_{tr}(t)$$
(3.1)

The actual magnitude of the pseudorange vector ρ , can be expressed as:

$$\rho = \left((\hat{x}_{sv} - x_u)^2 + (\hat{y}_{sv} - y_u)^2 + (\hat{z}_{sv} - z_u)^2 \right)^{0.5}$$
(7.38)

Thus the estimate of the pseudorange $\hat{\rho}$ may be expressed in terms of the actual magnitude of the pseudorange vector ρ , as:

$$\hat{\rho} = \tilde{\rho} - c\Delta t_r(t) - MP(t) - c\Delta t_{sv}(t) - SA(t) - E(t) - c\Delta t_{ion}(t) - c\Delta t_{tr}(t)$$
(7.39a)

$$\hat{\rho} = \rho + \eta(t) \tag{7.39b}$$

The actual pseudorange vector is related to the geodetic latitude λ , geocentric latitude λ_s , longitude ϕ and altitude *h*, by the relations:

$$\boldsymbol{\rho} = \begin{bmatrix} r_s \cos \lambda_s \cos \phi + h \cos \lambda \cos \phi \\ r_s \cos \lambda_s \sin \phi + h \cos \lambda \sin \phi \\ r_s \sin \lambda_s + h \sin \lambda \end{bmatrix}$$
(7.40)

where ρ is the Earth-centered, Earth-fixed position vector of the aircraft, r_s the radius at a surface point of the flattened Earth ellipsoid and λ_s are defined in terms of the flattening *f* and the equatorial radius R_e as:

$$\lambda_s = \arctan\left((1-f)^2 \tan \lambda\right) \tag{7.41}$$

and

$$r_{s} = \sqrt{R_{e}^{2} / \left(1 + \left(1 / (1 - f)^{2} - 1\right) \sin^{2} \lambda_{s}\right)}.$$
(7.42)

The change in attitude of an aircraft over a period of time can be observed by comparing the current measured phase differential with the initial phase differential measured at some initial reference time. Thus, this difference in the measured phase differential could be expressed as:

$$\Delta \phi_m = \frac{2\pi}{\lambda} \left(\mathbf{d} \cdot \left(\mathbf{r}_B - \mathbf{r}_0 \right) \right) \tag{7.43}$$

where \mathbf{r}_B is the navigation satellite's sight line vector at the current time and \mathbf{r}_0 is the navigation satellite's sight line vector at the initial reference time. The navigation satellite's sight line vector \mathbf{r}_B could be expressed in terms of the satellite's body coordinates. However, since the body attitude may be defined in terms of the quaternion, the transformation relating the estimate of current sight line vector $\hat{\mathbf{r}}$ in the inertial coordinates to the current sight line vector \mathbf{r}_B in body coordinates may be expressed in terms of the quaternion, the quaternion components. Hence:

$$\mathbf{r}_{B} = \mathbf{D}_{b,I}(\mathbf{q})\hat{\mathbf{r}} . \tag{7.44}$$

An estimate of current sight line vector $\hat{\mathbf{r}}$ in the orbiting coordinates can generally be obtained by an independent Kalman filter or by employing an algorithm such as NORAD's SDP4, SDP8 or SGP4 methods (Hoots et. al. 2004). It therefore follows that the difference in the measured phase differential could be expressed as:

$$\Delta \phi_m = \frac{2\pi}{\lambda} \left(\mathbf{d} \cdot \left(\mathbf{D}_{b,I} \left(\mathbf{q} \right) - \mathbf{I} \right) \hat{\mathbf{r}} \right) + \frac{2\pi}{\lambda} \left(\mathbf{d} \cdot \left(\hat{\mathbf{r}} - \mathbf{r_0} \right) \right), \tag{7.45}$$

and using the constraint on the components of the quaternion, $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$, we may write, $\mathbf{D}_{b,I}(\mathbf{q}) - \mathbf{I}$ as:

$$\mathbf{D}_{b,I}(\mathbf{q}) - \mathbf{I} \equiv \Delta \mathbf{D}_{b,I}(\mathbf{q}) = -2 \begin{bmatrix} \left(q_2^2 + q_3^2\right) & -\left(q_1q_2 + q_3q_4\right) & -\left(q_1q_3 - q_2q_4\right) \\ -\left(q_1q_2 - q_3q_4\right) & \left(q_1^2 + q_3^2\right) & -\left(q_2q_3 + q_1q_4\right) \\ -\left(q_1q_3 + q_2q_4\right) & -\left(q_2q_3 - q_1q_4\right) & \left(q_1^2 + q_2^2\right) \end{bmatrix}$$
(7.46)

which is a homogeneous quadratic function of the components of the quaternion. Thus, a discrete measurement of the error in the difference of the phase differentials due to changes in the attitude can be expressed as:

$$z_{m} \equiv \Delta \phi_{m} - \frac{2\pi}{\lambda} \left(\mathbf{d} \cdot \left(\hat{\mathbf{r}} - \mathbf{r}_{0} \right) \right) = \frac{2\pi}{\lambda} \left(\mathbf{d} \cdot \Delta \mathbf{D}_{b,I} \left(\mathbf{q}(k) \right) \hat{\mathbf{r}} \right) + v_{\phi}$$
(7.47)

where v_{ϕ} is an additive Gaussian random variable representing a white noise or deltacorrelated stochastic process. For three-axis measurement of the attitude one would require three independent measurements, which may be expressed as:

$$z_{mi} \equiv \Delta \phi_{mi} - \frac{2\pi}{\lambda} \left(\mathbf{d}_i \cdot (\hat{\mathbf{r}} - \mathbf{r_0}) \right) = \frac{2\pi}{\lambda} \left(\mathbf{d}_i \cdot \Delta \mathbf{D}_{b,l} \left(\mathbf{q}(k) \right) \hat{\mathbf{r}} \right) + v_{\phi i}, \ i = 1, 2, 3.$$
(7.48)

To complement these pseudorange measurements we assume that we also have independent measurements of the altitude and east geodetic longitude. This is necessary as the altitude and longitude kinematics have been included in the process model. Measurements of the altitude may be obtained from a radar altimeter while there are a variety of ways to obtain the east geodetic longitude. Alternatively the longitude kinematics may be deleted from the process model.

7.2.8 Formulation of the Satellite Navigation-INS Mixing Filter

Consider a random variable \mathbf{w} with dimension L which is going through the nonlinear transformation, $\mathbf{y} = \mathbf{f}(\mathbf{w})$. The initial conditions are that \mathbf{w} has a mean $\overline{\mathbf{w}}$ and a covariance \mathbf{P}_{ww} . To calculate the statistics of \mathbf{y} , a matrix $\boldsymbol{\chi}$ of 2L+1 sigma vectors is formed. We have chosen to use the scaled unscented transformation proposed by Julier (2002), as this transformation gives one the added flexibility of scaling the sigma points to ensure that the covariance matrices are always positive definite.

Given a general discrete nonlinear dynamic system in the form:

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k, \ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k$$
(7.49)

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\mathbf{u}_k \in \mathbb{R}^r$ is the known input vector, $\mathbf{y}_k \in \mathbb{R}^m$ is the output vector at time *k*. \mathbf{w}_k and \mathbf{v}_k are, respectively, the disturbance or process noise and sensor noise vectors, which are assumed to Gaussian white noise with zero mean.

Furthermore \mathbf{Q}_k and \mathbf{R}_k are assumed to be the covariance matrices of the process noise sequence, \mathbf{w}_k and the measurement noise sequence, \mathbf{v}_k respectively. The unscented transformations of the states are denoted as:

$$\mathbf{f}_{k}^{UT} = \mathbf{f}_{k}^{UT} \left(\mathbf{x}_{k}, \mathbf{u}_{k} \right), \ \mathbf{h}_{k}^{UT} = \mathbf{h}_{k}^{UT} \left(\mathbf{x}_{k} \right)$$
(7.50)

while the transformed covariance matrices and cross-covariance are respectively denoted as:

$$\mathbf{P}_{k}^{ff} = \mathbf{P}_{k}^{ff}\left(\hat{\mathbf{x}}_{k}, \mathbf{u}_{k}\right), \ \mathbf{P}_{k}^{hh-} = \mathbf{P}_{k}^{hh}\left(\hat{\mathbf{x}}_{k}^{-}\right)$$
(7.51a)

and

$$\mathbf{P}_{k}^{xh-} = \mathbf{P}_{k}^{xh-} \left(\hat{\mathbf{x}}_{k}^{-}, \mathbf{u}_{k} \right).$$
(7.51b)

The UKF estimator can then be expressed in a compact form. The state time-update equation, the propagated covariance, the Kalman gain, the state estimate and the updated covariance are respectively given by:

$$\hat{\mathbf{x}}_{k}^{-} = \mathbf{f}_{k-1}^{UT} \left(\hat{\mathbf{x}}_{k-1} \right) \tag{7.52a}$$

$$\hat{\mathbf{P}}_{k}^{-} = \mathbf{P}_{k-1}^{ff} + \mathbf{Q}_{k-1}$$
(7.52b)

$$\mathbf{K}_{k} = \hat{\mathbf{P}}_{k}^{xh-} \left(\hat{\mathbf{P}}_{k}^{hh-} + \mathbf{R}_{k} \right)^{-1}$$
(7.52c)

$$\hat{\mathbf{x}}_{k} = \hat{\mathbf{x}}_{k}^{-} + \mathbf{K}_{k} \left[\mathbf{z}_{k} - \mathbf{h}_{k}^{UT} \left(\hat{\mathbf{x}}_{k}^{-} \right) \right]$$
(7.52d)

$$\hat{\mathbf{P}}_{\mathbf{k}} = \hat{\mathbf{P}}_{k}^{-} - \mathbf{K}_{k} \left(\hat{\mathbf{P}}_{k}^{hh-} + \mathbf{R}_{k} \right)^{-1} \mathbf{K}_{k}^{T}.$$
(7.52e)

Equations 7.52a-e are in the same form as the traditional Kalman filter and the extended Kalman filter. Thus higher order nonlinear models capturing significant aspects of the dynamics may be employed to ensure that the Kalman filter algorithm can be implemented to effectively estimate the states in practice. Hence, the set of equations 4.56-4.59 and the set of equations 7.49-7.52 are identical.

In order to employ the UKF when precise statistics of the process and measurement noise vectors are not available, the adaptive filter method proposed by Song, Qi and Han (2006) is used to estimate the orbit parameters. The covariance matrixes of measurement residuals are recursively updated in the UKF. The measurement noise covariance matrices, in the case of the UKF, may be expressed as:

$$\hat{\mathbf{R}}_{k} = \mathbf{C}_{r}^{k,N} + \hat{\mathbf{P}}_{k}^{hh}$$
(7.53)

where $\mathbf{C}_{r}^{k,N}$ is defined in terms of the sample size N and the residual \mathbf{r}_{k} as:

$$\mathbf{C}_{r}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \mathbf{r}_{j} \mathbf{r}_{j}^{T}, \ \mathbf{r}_{k} = \left(\mathbf{z}_{k} - \mathbf{H}_{k} \hat{\mathbf{x}}_{k}\right) = \mathbf{v}_{k} + \mathbf{H}_{k} \left(\mathbf{x}_{k} - \hat{\mathbf{x}}_{k}\right).$$
(7.54)

Equation 7.53 involves the further computation of $\hat{\mathbf{P}}_{k}^{hh}$, by applying the unscented nonlinear transformation, $\mathbf{h}_{k}^{UT}(\hat{\mathbf{x}}_{k})$ to the state estimate, $\hat{\mathbf{x}}_{k}$. The measurement noise covariance may be updated in principle by employing equation 7.53. The nonlinear relationship between the covariance matrices also suggests that the update of \mathbf{R}_{k} could be done by employing the covariance of the residual.

In the application considered in this work, the adaptation of \mathbf{Q}_k is implemented, as it is the process statistics that is often unknown or may be considered to vary. It was observed that the magnitudes of the filter gains were relatively small and for this reason the exact expression for an estimate of \mathbf{Q}_k :

$$\hat{\mathbf{Q}}_{k-1} \equiv \mathbf{C}_{\Delta x}^{k,N} + \hat{\mathbf{P}}_{k} - \mathbf{P}_{k-1}^{ff}$$
(7.55a)

was approximated as:

$$\hat{\mathbf{Q}}_{k-1} \approx \mathbf{C}_{\Delta x}^{k,N} \tag{7.55b}$$

where $\mathbf{C}_{\Delta x}^{k,N}$ is defined as:

$$\mathbf{C}_{\Delta x}^{k,N} = \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta \mathbf{x} \Delta \mathbf{x}^{T} \approx \hat{\mathbf{P}}_{k}^{-} - \hat{\mathbf{P}}_{k} = \mathbf{K}_{k} \mathbf{H}_{k} \hat{\mathbf{P}}_{k}^{-}, \qquad (7.56)$$

and

$$\Delta \mathbf{x} = \left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right) - \left(\mathbf{x}_k - \hat{\mathbf{x}}_k\right). \tag{7.57}$$

7.3 APPLICATION OF THE ADAPTIVE UKF TO SATELLITE NAVIGATION-INS MIXING FILTERS

The process model for applying the adaptive UKF is given by equations 7.2, 7.19 and 7.23. For low-cost solution, \mathbf{A}_{NED} vector in equation 7.2 is given by equations 7.20, 7.24 and 7.3, while for high precision and Doppler-aided high precision mechanisation, the \mathbf{A}_{NED} vector in equations 7.2 is given by equations 7.32-7.34, 7.24 and 7.3. In using equation 7.23 with the UKF it is important to ensure that the constraint which the quaternion must satisfy is met by the estimates. This is ensured by repeated application of the method proposed by Vepa (2010) where the quaternion normalisation is considered as a nonlinear transformation and performed by applying the unscented transformation sequentially. The measurement model for the low-cost solution is given by equations 7.39 to 7.42, while the measurement model for high precision and Doppler-aided high precision mechanisation is given by equations 6.5 and 6.6 and their carrier phase measurement model is given by equations 7.46 to 7.48.

In this work, three types of filters are tested, namely the low cost satellite navigation-INS mixing filter, high precision satellite navigation-INS mixing filter and the Doppleraided high precision satellite navigation-INS mixing filter. Each of the filters' output simulation is compared with standard UKF estimation and adaptive UKF estimation. This choice of filters selected, is dependent on the nature and availability of auxiliary sensors and hardware. There are several other possibilities but only these three were considered for purposes of comparison. In the low cost case, no gyroscopic sensors were used, while in the Doppler-aided case it was assumed that velocity measurements are available with the kind of accuracy associated with Doppler measurements.

To test the filters' performance, rather than subject it to realistic accelerations over an extended period of time, the system is subjected to intense accelerations and sustained rotations over a short time frame. The initial altitude of the vehicle was assumed to be

10,000 metres while the initial location was assumed to be above London Heathrow. The exact \mathbf{Q}_k adaptation algorithm-based UKF was initially compared with the non-adaptive (standard) UKF and it was found that it outperforms the standard UKF in every department as presented in the following subsections. Further results are now included to provide supporting evidence for the conclusions.

7.3.1 Low Cost Satellite Navigation-INS Mixing Filter

It must be recognised at the outset that the process error covariance is relatively quite large as the accelerometers being used are generally low cost MEMS type sensors. The implication of the use of these accelerometers, which are characterised by a relatively large standard deviation in the measured acceleration, is that the pseudorange measurement error correction, due to the availability of the additional accelerometer measurements, is expected to be relatively small in comparison with the total user equivalent range error. The real issue is that that the navigation mixing filter is able to deal with the large uncertainties associated with low grade accelerometers. Bearing this in mind, the UKF is first implemented as a mixing filter to facilitate GPS-INS integration and these results are discussed in the first instance. Furthermore, as no measurements of attitudes are deemed to be available, the estimates of the attitude quaternion are not expected to be unique or consistent. Yet the associated direction cosine matrix is expected to be uniquely estimated.



Figure 7.3a Estimates of the latitude, longitude and altitude (LLA) compared with the corresponding simulations.

The rotation rate of the vehicle is assumed to consist of two components oscillating at different frequencies. In the first instance, the north, east and down velocity equations and the angular velocity equations were each subjected to three independent slowly varying biases and the corresponding 19 states of the filter were estimated by applying the UKF algorithm. Figure 7.3a shows the estimates of the latitude, longitude and attitude over a typical epoch of 60 seconds (= 3×10^4 time steps) and compared with the corresponding simulations. The time step for implementing the estimator was chosen as, $\Delta t = 0.002$ seconds. Figure 7.3b presents the corresponding velocities in the north, east and down directions, while while Figures 7.3c and 7.3d shows the estimated horizontal and vertical velocities compare with the corresponding simulations respectively. Figure 7.3e presents the angular velocities that the system is subjected to. The user position error is depicted in Figure 7.3f and the pseudorange measurement estimate error is illustrated in Figure 7.3g.



Figure 7.3b Estimates of the north, east and down (NED) velocities compared with the corresponding simulations.



Figure 7.3c Estimates of the horizontal velocities compared with the corresponding simulations.

CHAPTER 7: ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY



Figure 7.3d Estimates of the vertical velocities compared with the corresponding simulations.



Figure 7.3e Estimates of the body angular velocities, in body coordinates, compared with the corresponding simulations.



Figure 7.3f Estimate of the user position error.



Figure 7.3g Estimate of the pseudorange measurement error.

Although this estimate is relatively very small, as expected, the estimated velocities in the north, east and down directions shown in Figure 7.3b and one of the angular velocity components shown in Figure 7.3e are apparently drifting away from the simulated values. To remedy the situation the biases introduced into some of the north, east and down velocity equations, as well as into some of the angular velocity equations, are also assumed to drift slowly at slowly varying rates. While these modifications improved the performance of the filter marginally, the north velocity and the yaw angular rate components continued to drift at an unacceptably fast rate. However, changing the process noise covariance matrix did have a dramatic effect on the drift rate and the performance of the filter, which improved significantly.

For this reason, at this stage and in subsequent subsections, it was decided to update the process covariance matrix adaptively.



Figure 7.4a Comparison of the latitude, longitude and altitude (LLA) obtained by simulation, standard UKF estimation and adaptive UKF estimation.



Figure 7.4b Comparison of the north, east and down (NED) translational velocities obtained by simulation, standard and adaptive UKF estimation.



Figure 7.4c Comparison of the horizontal velocities obtained by simulation, standard and adaptive UKF.
CHAPTER 7: ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY



Figure 7.4d Comparison of the vertical velocities obtained by simulation, standard and adaptive UKF.



Figure 7.4e Comparison of the body angular velocities, in body coordinates, obtained by simulation, standard and adaptive UKF.



Figure 7.4f Comparison of user position errors obtained by simulation, standard and adaptive UKF.



Figure 7.4g Comparison of pseudorange measurement errors obtained by simulation, standard and adaptive UKF.

The results of applying the adaptation of \mathbf{Q}_k exactly (equation 7.55a) are shown in Figures 7.4 over an epoch of 60 seconds (= 3×10^4 time steps with the time step for implementing the estimator was chosen as, $\Delta t = 0.002$ seconds). It is clear from these figures that the exact \mathbf{Q}_k adaptation algorithm outperforms the standard UKF in every department. Similar results were obtained by applying the approximate \mathbf{Q}_k adaptation algorithm (equation 7.55b).

The rapid variations in the estimated angular velocities in Figure 7.4e are due to the estimator attempting to follow the rapid rotations of the body. It must be noted that the tests that were carried out have been done with exceptionally high translational and angular velocities. In reality a low-cost satellite-aided inertial navigation system would never be subjected to such extremes. The user position and pseudorange estimate error remains within the bounds shown in Figure 7.4f and 7.4g.

7.3.2 High Precision Satellite Navigation-INS Mixing Filter

It must be recognised at the outset that the process error covariance is relatively quite low as both the accelerometers and rate gyros being used are high precision type sensors. The implication of the use of these accelerometers, which are characterised by a relatively low standard deviation in the measured acceleration, is that the pseudorange measurement error correction, due to the availability of the additional accelerometer measurements, is expected to be relatively of the same order in comparison with the total user equivalent range error. The real issue is that the navigation mixing filter is not only able to deal with the uncertainties associated with the sensors, but also be able to estimate the user position to a desired level of accuracy. Bearing this in mind, the UKF is first implemented as a mixing filter to facilitate GPS-INS integration, and these results are discussed in the first instance. Furthermore, although no measurements of attitudes are deemed to be available, the estimates of the attitude quaternion are expected to be unique or consistent, due to the presence of the rate gyro measurements, and the associated direction cosine matrix is expected to be uniquely estimated.

The simulations as shown figures 7.5a, 7.5b, 7.5c, 7.5d and 7.5e and it is observed that it outperforms the standard UKF in every department. The comparison is made over a typical epoch of the first 4 seconds (= 2×10^4 time steps) as the UKF filters converge to a steady state well before the end of this time frame and the filter's response is compared with the corresponding simulations. The time step for implementing the estimator was chosen as, $\Delta t = 0.0002$ seconds. The number of visible satellites is assumed to be 3.



Figure 7.5a Comparison of simulated and UKF estimated navigation positions (LLA
latitude, longitude and attitude) over 20000 time steps (equivalent to a time frame of 4 seconds).

In the first instance the north, east and down velocity equations and the angular velocity equations were each subjected to three independent slowly varying biases and the corresponding 31 states of the filter were estimated by applying the UKF algorithms.



Figure 7.5b Comparison of simulated and UKF estimated navigation velocities (north east and down) over 20000 time steps (equivalent to a time frame of 4





Figure 7.5c Comparison of simulated and UKF estimated body attitude quaternion components over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.5d Errors in the UKF estimated pseudorange for three satellites over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.5e Errors in the UKF estimated three-axis user position components over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.6a Comparison of simulated and adaptive UKF estimated navigation positions (LLA - latitude, longitude and attitude) over 20000 time steps (equivalent to a time frame of 4 seconds).

In Figure 7.6a, just the adaptive UKF estimates of the latitude, longitude and attitude are compared with the corresponding simulations. In Figure 7.6b are presented the corresponding velocities in the north, east and down directions. It should be noted that in the standard UKF and adaptive UKF comparisons, the simulated responses are slightly different due to differences in the disturbances. However they are of the same orders of magnitude thus facilitating the comparison of errors. It may be observed that the errors in the horizontal velocity components (north and east) are relative high. This is due to the fact that there is no information in the measurements that will help separate the components of velocity in the horizontal plane. For this reason the case with additional three axis Doppler measurements is considered in the next subsection. Figures 7.6c and 7.6d compares the estimated and simulated components of the velocity in the horizontal and vertical planes which shows almost an insignificant error in these two components. Thus the difficulty is in resolving the velocity in the horizontal plane into its north and east component. Figure 7.6e present the

corresponding body attitude quaternion components. The pseudorange measurement estimate error is illustrated in Figure 7.6f. The user position error is depicted in Figure 7.6g.



Figure. 7.6b Comparison of simulated and adaptive UKF estimated navigation velocities (north, east and down) over 20000 time steps (equivalent to a time frame of

4 seconds).



Figure 7.6c Comparison of simulated and adaptive UKF estimated navigation horizontal velocities over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.6d Comparison of simulated and adaptive UKF estimated navigation vertical velocities over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.6e Comparison of simulated and adaptive UKF estimated body attitude quaternion components over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.6f Errors in the adaptive UKF estimated pseudorange for three satellites over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.6g Errors in the adaptive UKF estimated three-axis user position components over 20000 time steps (equivalent to a time frame of 4 seconds).

CHAPTER 7: ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY

The pseudorange estimate error in the Figure 7.6f and the user position errors in Figure 7.6g remain within certain limits.

7.3.3 Doppler-Aided High Precision Satellite Navigation-INS Mixing Filter

Although all the estimated errors are relatively very small as expected, only the estimated north and east velocity components shown in figure 7.6b differed slightly from the simulated components. To remedy the situation additional Doppler aided measurements of the velocities were assumed to be available. These additional measurements improved the performance of the filter.



Figure 7.7a Comparison of simulated and adaptive Doppler-aided UKF estimated navigation velocities (north, east and down) over 20000 time steps (equivalent to a time frame of 4 seconds).



Figure 7.7b Comparison of simulated and adaptive Doppler-aided UKF estimated body attitude quaternion components over 20000 time steps (equivalent to a time frame of 4 seconds)

The results of applying the adaptation of \mathbf{Q}_k exactly (equation 7.55a) on the velocity and quaternion components are shown in Figures 7.7a and 7.7b over an epoch of the first 4 seconds (= 2×10^4 time steps). All the other estimate errors behave quite similarly to those shown in figure 7.6a-7.6g and are not shown. Moreover the pseudorange estimate and the user position estimate errors remain within the bounds shown in Figures 7.6f and 7.6g and are not shown. It is observed that the accuracy of the estimate of components of the quaternion is maintained in spite of considerable variations in their magnitude. Generally it was observed that the most inaccurate component of the quaternion was in fact the one with the lowest magnitude. This error is due to the fact that both the simulated quaternion and the estimated quaternion are being forced to satisfy the normalisation constraint $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$ exactly. Consequently the errors in the major components of the quaternion cause a significant error in the component with the lowest magnitude.

CHAPTER 7: ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY

In practice it may be essential to trade-off the error in the component with the lowest magnitude by allowing a small normalisation error.

It is observed that the performance of the adaptive UKF based estimations has improved when the addition Doppler measurements were made available to the filter, particularly in resolving the velocity ion the horizontal plane to its north and east components. It must be noted that the tests that were carried out have been done with exceptionally high translational and angular velocities. In reality a satellite aided inertial navigation system would be subjected to such extremes only on certain rare occasions.

Finally to demonstrate the efficacy adaptive UKF estimator over a relatively long period of the time, the filter is run over a 30 seconds time frame and the results over the last 4 seconds compared with the simulations in Figures 7.8a-7.8g.



Figure 7.8a Comparison of simulated and adaptive UKF estimated navigation positions (LLA - latitude, longitude and attitude in metres) over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8b Comparison of simulated and adaptive UKF estimated navigation velocities (north, east and down) over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8c Comparison of simulated and adaptive UKF estimated navigation horizontal velocities over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8d. Comparison of simulated and adaptive UKF estimated navigation vertical velocities over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8e Comparison of simulated and adaptive UKF estimated body attitude quaternion components over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8f Comparison of errors in the UKF and adaptive UKF estimated pseudoranges for three satellites over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).



Figure 7.8g Comparison of errors in the UKF and adaptive UKF estimated three-axis user position components over the last 20000 time steps (equivalent to the last 4 seconds in a 30 seconds time frame).

It is particularly interesting to observe that the adaptive UKF estimator is able to predict the north, east and down velocity components without any assistance from the Doppler measurements. Moreover it is able to predict the body attitude accurately over the entire time frame, even though the aircraft seems to have been reduced to a state of sustained rotations at the end of the 30 second time frame and the body attitude components are continuously changing.

7.4 INTEROPERABLE SATELLITE NAVIGATION-INS Mixing Filters

In this thesis, the feasibility of implementing an adaptive unscented Kalman filter-based mixing filter, which is used to develop a low-cost satellite-aided inertial navigation system and a high accuracy satellite-aided inertial navigation system, with and without Doppler-aided techniques have been demonstrated.

For the low-cost solution, the measurements were assumed to be made by six low-cost ADXL203, type two, axis accelerometers and a low-cost altimeter. While the estimates of the pseudorange using a standard UKF were of acceptable accuracy, it was also found the estimates of the north, east and down velocities and the body axis angular velocities did not converge over a long time frame. For this reason the adaptive UKF algorithm was used with the process covariance matrix updated recursively. When the adaptive UKF algorithm was used there was a dramatic 200% minimum reduction in the errors in the estimated north, east and down velocities, and a 60% minimum reduction in the errors in the estimated body angular velocities at the end of the time frame. Moreover, when additional measurements of the true airspeed and vertical airspeed were available the estimated velocities were seen to converge to the simulated values.

For high precision applications, the acceleration and angular velocity measurements were assumed to be made by three high accuracy accelerometers and three fibre-optic ring-laser rate gyros. As with to the low-cost solution, the estimates of the pseudorange using a standard UKF were of acceptable accuracy, but it was also found the estimates based on adaptive UKF algorithm provided extremely accurate estimates of the positioning variables and reasonably accurate estimates of the body quaternion components. Moreover, when additional Doppler-aided measurements of the velocity components were available, the estimated velocities were seen to converge to the simulated values even more rapidly.

CHAPTER 7: ADAPTIVE MIXING FILTERS FOR INTEROPERABILITY

The methodology may be developed as a stand-alone system or employed in conjunction with a traditional strapped down inertial navigation system for purposes of initial alignment. Moreover, the feasibility of employing adaptive mixing facilitates the possibility of using the system in an interoperable fashion with satellite navigation measurements.

It is important to note that any generic satellite navigation system observables can be used as a measurement of pseudoranges for the satellite navigation-INS filter. This is due to the adaptive nature of the UKF, which will adapt according to the error covariance matrices; therefore the adaptive mixing filter presented here is interoperable.

CHAPTER 8

CONCLUSIONS

In this chapter, a summary of findings is presented, followed by the contributions and achievements. Finally, recommendations for future works are presented.

8.1 SUMMARY OF FINDINGS

With the availability of several fully operational satellite navigation systems, it has been recognised that an optimal combination of the output of one or more satellite navigation systems with the output of an inertial navigation system has a number of advantages over a stand-alone inertial or satellite navigation system. The use of adaptation facilitates interoperable mixing of the outputs of any satellite navigation system with the output of an inertial navigation system. Similar advantages could be gained by the application of the UKF-based estimation methods to the various components of the differential GNSS (DGNSS) reference station error estimation algorithms. Furthermore, the application of adaptive UKF-based estimation could, in principle, facilitate interoperability.

8.2 THESIS CONTRIBUTION AND ACHIEVEMENTS

The main contributions of this research work so far are as follows:

- Implementing the NORAD SPACETRACK SDP4 and SGP4 algorithms with suitable modifications so that they can be employed for navigation satellites in MATLAB code. If a precisely surveyed location of an observer is known, then the reference-to-satellite range of that observer to the corresponding satellite can be calculated. By adding the receiver clock biases to the reference-to-satellite range, the pseudoranges of the user receiver with its respective satellites can be simulated.
- Proposed and validated a method of converting NORAD TLE for GPS operational satellites into GPS 16 element ephemeris.
- Further improvements are made by a proper inclusion of the secular corrections to increase the accuracy of the ephemeris. These clearly show that an interoperable differential navigation satellite reference station has the potential to convert different types of ephemerides to cater for various end user receivers. This implies that a GPS user is potentially able to receive an ephemeris from a GLONASS satellite, for instance, after the interoperable stations process the message conversion at the system level.
- Although the standard UKF was initially a promising alternative, features of the orbital dynamics led to the conclusion that the standard UKF must be employed with appropriate restrictions on the noise covariance statistics, to facilitate the calculation of the sigma points. The superiority of the UKF over the EKF is clearly and unambiguously demonstrated in Chapter 5, where it was possible to implement both methods as it was possible to linearise the governing dynamical equations. This finding is in conformance with the examples that have been

CHAPTER 8: CONCLUSIONS

demonstrated in the literature by Julier and Uhlmann (2004). The fundamental advantage of the UKF is that it is a derivative free method. To address some of the shortcomings of the standard UKF, a modified approach to the UKF is proposed. The proposed modified UKF uses singular value decomposition (SVD) rather than Cholesky decomposition to estimate the sigma points. Moreover, the singular values are replaced by their absolute values in the decomposition. Thus, this work presents the results of the application of the modified approach to the UKF to orbit estimation to demonstrate its superiority over the standard approach. This modification of the UKF algorithm resulted in a remarkable improvement in the performance of the UKF.

The main achievements of this research work so far are as follows:

- The results indicate the method used is particularly suitable for estimating the orbit ephemeris of navigation satellites and the differential position and navigation filter mixing errors, thus facilitating interoperable DGNSS operation for aircraft landing.
- This thesis has demonstrated the feasibility of implementing an adaptive unscented Kalman filter-based mixing filter which is used to develop a low-cost satellite-aided inertial navigation system or/and a high-accuracy satellite-aided inertial navigation system with and without Doppler-aided techniques. In both cases, the output performance of the adaptive unscented Kalman filter exceeded that of the standard, non-adaptive equivalent for the same test case.

8.3 RECOMMENDATIONS FOR FUTURE WORK

Several recommendations for future work are suggested below:

- Apart from the notably successful implementation of adaptive unscented Kalman filters used in this research, the direction of this work could be progressed further by applying several potential filters for interoperable algorithms. The filters proposed for further consideration are the particle, polynomial and high-gain filters.
- This thesis concentrates mainly on designing and simulating models of the interoperable algorithms on MATLAB. Further testing of the interoperable algorithms using a suitable test bed is necessary for verification of the system.
- Testing using real-time data acquired from a standard GNSS receiver, both stationary and roving vehicle, is highly desirable.
- Further research is needed to fine-tune the interoperable algorithms and thus, facilitated interoperable DGNSS operation for safe aircraft landing.

APPENDIX

Details of certain models and derivations are given and discussed in the appendices.

APPENDIX A: THE SDP4 MODEL

The SDP4 model is one of the propagation models of NORAD element sets based on Hoots and Roehrich (1980). The main SDP4 algorithm is listed here in detail.

The NORAD mean element sets can be used for prediction with SDP4. The original mean motion (n_0'') and semi-major axis (a_0'') are first recovered from the input elements by the equations:

$$a_1 = \left(\frac{k_e}{n_0}\right)^{\frac{2}{3}} \tag{A.1}$$

$$\delta_{1} = \frac{3}{2} \frac{k_{2}}{a_{1}^{2}} \frac{\left(3\cos^{2} i_{0} - 1\right)}{\left(1 - e_{0}^{2}\right)^{\frac{3}{2}}}$$
(A.2)

$$a_0 = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$
(A.3)

$$\delta_0 = \frac{3}{2} \frac{k_2}{a_0^2} \frac{\left(3\cos^2 i_0 - 1\right)}{\left(1 - e_0^2\right)^{\frac{3}{2}}} \tag{A.4}$$

$$n_0'' = \frac{n_0}{1 + \delta_0}$$
(A.5)

$$a_0'' = \frac{a_0}{1 - \delta_0}$$
(A.6)

For perigee between 98 kilometers and 156 kilometers, the value of the constant *s* used in SDP4 is changed to:

$$s^* = a_0'' (1 - e_0) - s + a_E$$
(A.7)

For perigee below 98 kilometers, the value of s is changed to:

$$s^* = 20/XKMPER + a_E \tag{A.8}$$

If the value of s is changed, then the value of $(q_0 - s)^4$ must be replaced by:

$$(q_0 - s^*)^4 = \left[\left[(q_0 - s)^4 \right]^{\frac{1}{4}} + s - s^* \right]^4$$
(A.9)

Then, calculate the constants (using the appropriate values of s and $(q_0 - s)^4$):

$$\theta = \cos i_0 \tag{A.10}$$

$$\xi = \frac{1}{a_0'' - s} \tag{A.11}$$

$$\beta_0 = \left(1 - e_0^2\right)^{\frac{1}{2}}$$
(A.12)

$$\eta = a_0'' e_0 \xi \tag{A.13}$$

$$C_{2} = (q_{0} - s)^{4} \xi^{4} n_{0}'' (1 - \eta^{2})^{-\frac{7}{2}} \left[a_{0}'' \left(1 + \frac{3}{2} \eta^{2} + 4e_{0} \eta + e_{0} \eta^{3} \right) + \frac{3}{2} \frac{k_{2} \xi}{(1 - \eta^{2})} \left(-\frac{1}{2} + \frac{3}{2} \theta^{2} \right) (8 + 24\eta^{2} + 3\eta^{4}) \right]$$
(A.14)

$$C_{1} = B'C_{2}$$
(A.15)

$$C_{4} = 2n_{0}''(q_{0} - s)^{4}\xi^{4}a_{0}''\beta_{0}^{2}(1 - \eta^{2})^{-\frac{7}{2}} \left(\left[2\eta(1 + e_{0}\eta) + \frac{1}{2}e_{0} + \frac{1}{2}\eta^{3} \right] - \frac{2k_{2}\xi}{a_{0}''(1 - \eta^{2})} \times \left[3\left(1 - 3\theta^{2}\left(1 + \frac{3}{2}\eta^{2} - 2e_{0}\eta - \frac{1}{2}e_{0}\eta^{3}\right) + \frac{3}{4}\left(1 - \theta^{2}\right)\left(2\eta^{2} - e_{0}\eta - e_{0}\eta^{3}\right)\cos 2\omega_{0} \right] \right)$$
(A.16)

$$\dot{M} = \left[1 + \frac{3k_2(-1+3\theta^2)}{2a_0''^2\beta_0^3} + \frac{3k_2^2(13-78\theta^2+137\theta^4)}{16a_0''^4\beta_0^7}\right]n_0''$$
(A.17)

$$\dot{\omega} = \left[-\frac{3k_2(-1+5\theta^2)}{2a_0''^2\beta_0^4} + \frac{3k_2^2(7-114\theta^2+395\theta^4)}{16a_0''^4\beta_0^8} + \frac{5k_4(3-36\theta^2+49\theta^4)}{4a_0''^4\beta_0^8} \right] n_0''$$
(A.18)

$$\dot{\Omega}_1 = -\frac{3k_2\theta}{a_0'^2\beta_0^4}n_0'' \tag{A.19}$$

$$\dot{\Omega} = \dot{\Omega}_{1} + \left[\frac{3k_{2}^{2}\left(4\theta - 19\theta^{3}\right)}{2a_{0}^{\prime\prime}\beta_{0}^{8}} + \frac{5k_{4}\theta\left(3 - 7\theta^{2}\right)}{2a_{0}^{\prime\prime}\beta_{0}^{8}}\right]n_{0}^{\prime\prime}$$
(A.20)

At this point SDP4 calls the initialisation of DEEP routine, which calculates all initialised quantities needed for the deep-space perturbations.

The secular effects of gravity are included by:

$$M_{DF} = M_0 + \dot{M} (t - t_0)$$
(A.21)

$$\omega_{DF} = \omega_0 + \dot{\omega} (t - t_0) \tag{A.22}$$

$$\Omega_{DF} = \Omega_0 + \dot{\Omega} (t - t_0) \tag{A.23}$$

where $(t - t_0)$ is time since epoch. The secular effect of drag on longitude of ascending node is included by:

$$\Omega = \Omega_{DF} - \frac{21}{2} \frac{n_0'' k_2 \theta}{a_0''^2 \beta_0^2} C_1 (t - t_0)^2$$
(A.24)

Next, SDP4 calls the secular section of DEEP, which adds the deep-space secular effects and long-period resonance effects to the six classical orbital elements. The secular effects of drag are included in the remaining elements by:

$$a = a_{DS} \left[1 - C_1 (t - t_0) \right]^2 \tag{A.25}$$

$$e = e_{DS} - B^* C_4 (t - t_0)$$
 (A.26)

$$IL = M_{DS} + \omega_{DS} + \Omega_{DS} + n_0'' \left[\frac{3}{2} C_1 (t - t_0)^2 \right]$$
(A.27)

where a_{DS} , e_{DS} , M_{DS} , ω_{DS} , and Ω_{DS} , are the values of n_0 , e_0 , M_{DF} , ω_{DF} , and Ω after deep-space secular and resonance perturbations have been applied.

Here SDP4 calls the periodics section of DEEP, which adds the deep-space lunar and solar periodics to the orbital elements. From this point on, it will be assumed that n, e,

APPENDIX

I, ω , Ω , and M are the mean motion, eccentricity, inclination, argument of perigee, longitude of ascending node, and mean anomaly after lunar-solar periodics have been added.

Add the long-period periodic terms:

$$a_{xN} = e\cos\omega \tag{A.28}$$

$$\beta = \sqrt{\left(1 - e^2\right)} \tag{A.29}$$

$$IL_{L} = \frac{A_{3,0} \sin i_{0}}{8k_{2}a \left|\beta^{2}} \left(e \cos \omega\right) \left(\frac{3+5\theta}{1+\theta}\right)$$
(A.30)

$$a_{yNL} = \frac{A_{3,0} \sin i_0}{4k_2 a \beta^2}$$
(A.31)

$$IL_T = IL + IL_L \tag{A.32}$$

$$a_{yN} = e\sin\omega + a_{yNL} \tag{A.33}$$

Solve Kepler's equation for $(E + \omega)$ by defining:

$$U = IL_T - \Omega \tag{A.34}$$

and using the iteration equation:

$$(E+\omega)_{i+1} = (E+\omega)_i + \Delta(E+\omega)_i$$
(A.35)

with

$$\Delta (E+\omega)_i = \frac{U-a_{yN}\cos(E+\omega)_i + a_{xN}\sin(E+\omega)_i - (E+\omega)_i}{-a_{yN}\sin(E+\omega)_i + a_{xN}\sin(E+\omega)_i + 1}$$
(A.36)

and

$$\left(E+\omega\right)_{1}=U\tag{A.37}$$

The following equations are used to calculate preliminary quantities needed for shortperiod periodics:

$$e\cos E = a_{xN}\cos(E+\omega) + a_{yN}\sin(E+\omega)$$
(A.38)

$$e\sin E = a_{xN}\sin(E+\omega) - a_{yN}\cos(E+\omega)$$
(A.39)

$$e_L = \left(a_{xN}^2 + a_{yN}^2\right)^{\frac{1}{2}}$$
(A.40)

$$p_L = \left(1 - e_L^2\right) \tag{A.41}$$

$$r = a(1 - e\cos E) \tag{A.42}$$

$$\dot{r} = k_e \frac{\sqrt{a}}{r} e \sin E \tag{A.43}$$

$$r\dot{f} = k_e \frac{\sqrt{p_L}}{r}$$
(A.44)

$$\cos u = \frac{a}{r} \left[\cos(E + \omega) - a_{xN} + \frac{a_{yN}(e\sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$
(A.45)

$$\sin u = \frac{a}{r} \left[\sin(E + \omega) - a_{yN} + \frac{a_{xN}(e \sin E)}{1 + \sqrt{1 - e_L^2}} \right]$$
(A.46)

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right) \tag{A.47}$$

$$\Delta r = \frac{k_2}{2p_L} \left(1 - \theta^2\right) \cos 2u \tag{A.48}$$

$$\Delta u = -\frac{k_2}{4p_L^2} (7\theta^2 - 1) \sin 2u$$
 (A.49)

$$\Delta\Omega = \frac{3k_2\theta}{2p_L^2}\sin 2u \tag{A.50}$$

$$\Delta i = \frac{3k_2\theta}{2p_L^2} \sin i_0 \cos 2u \tag{A.51}$$

$$\Delta \dot{r} = -\frac{k_2 n}{p_L} \left(1 - \theta^2\right) \sin 2u \tag{A.52}$$

$$\Delta r \dot{f} = \frac{k_2 n}{p_L} \left[\left(1 - \theta^2 \right) \cos 2u - \frac{3}{2} \left(1 - 3\theta^2 \right) \right]$$
(A.53)

The short-period periodics are added to give the osculating quantities:

$$r_{k} = r \left[1 - \frac{3}{2} k_{2} \frac{\sqrt{1 - e_{L}^{2}}}{p_{L}^{2}} \left(3\theta^{2} - 1 \right) \right] + \Delta r$$
(A.54)

$$u_k = u + \Delta u \tag{A.55}$$

$\Omega_k = \Omega + \Delta \Omega$	(A.56)
$i_k = I + \Delta i$	(A.57)
$\dot{r}_k = \dot{r} + \Delta \dot{r}$	(A.58)

$$r\dot{f}_k = r\dot{f} + \Delta r\dot{f} \tag{A.59}$$

Then unit orientation vectors are calculated by:

$$U = M\sin u_k + N\cos u_k \tag{A.60}$$

$$V = M\cos u_k - N\sin u_k \tag{A.61}$$

where

$$M = \begin{cases} M_x = -\sin\Omega_k \cos i_k \\ M_y = \cos\Omega_k \cos i_k \\ M_z = \sin i_k \end{cases}$$
(A.62)

$$N = \begin{cases} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{cases}$$
(A.63)

Then position and velocity are given by:

$$\mathbf{r} = r_k \mathbf{U} \tag{A.64}$$

and

$$\dot{\mathbf{r}} = \dot{r}_k \mathbf{U} + \left(r\dot{f}\right)_k \mathbf{V} \tag{A.65}$$

APPENDIX B: IONOSPHERIC MODELS

A better method of predicting the TEC, rather than using the logarithmic scale of the Chapman Profile, which varies with the altitude without considering variations in other parameters, is by using either of the two widely used empirical models (International Reference Ionosphere (IRI) model or NeQuick model), the broadcast model (Klobuchar model) or GPS data driven models (Global Ionospheric Maps (GIMs)) to predict and hence estimate the value of TEC. (Orús, Hernández-Pajares, Juan, Sanz. and García Fernández 2002).

B.1 Empirical Models

IRI and NeQuick models are two widely used empirical models. For IRI models, International Reference Ionosphere (2009) provides the resources (source code, online computation etc.) for IRI-2001 model and the latest IRI-2007 model. Here, only the NeQuick 2 (Nava, Coïsson and Radicella 2008) is presented.

The basic inputs of the NeQuick model are position, time and solar flux (or solar number); the output is the electron concentration at the given location and time.

The NeQuick 2 analytical formulation

Before describing the NeQuick 2 in detail, recall that an Epstein layer (Rawer 1982) is expressed by:

$$N_{Epstein}(h; h \max, N \max, B) = \frac{4N \max}{\left(1 + \exp\left(\frac{h - h \max}{B}\right)\right)^2} \exp\left(\frac{h - h \max}{B}\right)$$
(B.1)

where $N \max$ is the layer peak electron density, $h \max$ is the layer peak height and B is the layer thickness parameter.

The bottomside formulation

Using the expressions $NmE = 0.124(foE)^2$, $NmF1 = 0.124(foF1)^2$, $NmF2 = 0.124(foF2)^2$ for *E*, *F*1 and *F*2 layer peak electron densities (in $10^{11}m^{-3}$), respectively, hmE, hmF1 and hmF2 for the *E*, *F*1 and *F*2 layer peak heights (in km), respectively, and *BE*, *B*1 and *B*2 for the *E*, *F*1 and *F*2 layer thickness parameters (in km), respectively the bottomside of the NeQuick 2 can be expressed as a sum of semi-Epstein layers as follows:

$$N_{bot}(h) = N_E(h) + N_{F1}(h) + N_{F2}(h)$$
(B.2)

where

$$N_{E}(h) = \frac{4Nm^{*}E}{\left(1 + \exp\left(\frac{h - hmE}{BE}\xi(h)\right)\right)^{2}} \times \exp\left(\frac{h - hmE}{BE}\xi(h)\right)$$
(B.3)

$$N_{F1}(h) = \frac{4Nm^*F1}{\left(1 + \exp\left(\frac{h - hmF1}{B1}\xi(h)\right)\right)^2} \times \exp\left(\frac{h - hmF1}{B1}\xi(h)\right)$$
(B.4)

$$N_{F2}(h) = \frac{4NmF2}{\left(1 + \exp\left(\frac{h - hmF2}{B2}\right)\right)^2} \times \exp\left(\frac{h - hmF2}{B2}\right)$$
(B.5)

with

$$Nm^*E = NmE - N_{F1}(hmE) - N_{F2}(hmE)$$
 (B.6)

$$Nm^*F1 = NmF1 - N_E(hmE) - N_{F2}(hmF1)$$
 (B.7)

and

$$\xi(h) = \exp\left(\frac{10}{1+1|h-hmF2|}\right) \tag{B.8}$$

is a function that ensures a "fadeout" of the *E* and *F*1 layers in the vicinity of the *F*2 layer peak in order to avoid secondary maxima around hmF2. In accordance with the behaviour of the *F*1 layer, expressions (B.6) and (B.7) can be slightly modified. The thickness parameters take different values for the bottomside and for

the topside of each layer (BE_{bot} and BE_{top} for the *E* layer, $B1_{bot}$ and $B1_{top}$ for the *F*1 layer, $B2_{bot}$ and $B2_{top}$ for the *F*2 layer).

The topside formulation

The model topside is represented by a semi-Epstein layer with a height thickness parameter H:

$$N(h) = \frac{4NmF2}{(1 + \exp(z))^2} \exp(z)$$
(B.9)

with

$$z = \frac{h - hmF2}{H} \tag{B.10}$$

$$H = H_0 \left[1 + \frac{rg(h - hmF2)}{rH_0 + g(h - hmF2)} \right]$$
(B.11)

where the constant parameters

$$r = 100$$
 (B.12)

$$g = 0.125$$
 (B.13)

are used to control the increase of H.

Parameter modelling of peak heights

The heights in km of the E, F1 and F2 layer maximum densities are given by:

$$hmE = 120$$
 (B.14)

$$hmF1 = \frac{hmE + hmF2}{2} \tag{B.15}$$

$$hmF2 = \frac{1490MF}{M + \Delta M} - 176 \tag{B.16}$$

where

$$\Delta M = \begin{cases} \frac{0.253}{foF2} - 0.012 \\ -0.012 \\ 0.0196M^2 + 1 \end{cases}$$
(B.17)
(B.17)
(B.17)

$$MF = M \sqrt{\frac{0.0196M^2 + 1}{1.2967M^2 - 1}}$$
(B.18)

and

$$M = M(3000)F2 (B.19)$$

Parameter modelling of thickness

The semi-thickness parameters BE_{bot} and BE_{top} (for the *E* layer), $B1_{bot}$ and $B1_{top}$ (for the *F*1 layer) and $B2_{bot}$ and *H* for the *F*2 layer) are given in km and expressed by the following relations:

$$BE_{bot} = 5 \tag{B.20}$$

$$BE_{top} = \max(0.5(hmF1 - hmE),7)$$
 (B.21)

$$B1_{hot} = 0.5(hmF1 - hmE)$$
 (B.22)

$$B1_{top} = 0.3(hmF2 - hmF1)$$
(B.23)

$$B2_{bot} = \frac{0.385NmF2}{(dN/dh)_{max}}$$
(B.24)

$$H = kB2_{bot} \left[1 + \frac{rg(h - hmF2)}{rkB2_{bot} + g(h - hmF2)} \right]$$
(B.25)

Expression (B.24) depends on the value of the maximum of the electron density derivative with respect to height. The maximum is computed from foF2 and M(3000)F2 values, using the empirical relation (Mosert de Gonzales and Radicella 1990) given as:

$$\ln\left(\left(\frac{dN}{dh}\right)_{\max}\right) = -3.467 + 1.714\ln(foF2) + 2.02\ln(M(3000)F2)$$
(B.26)

where dN/dh is in $(10^9 m^{-3} km^{-1})$ and foF2 in (MHz). Expression (B.25) is the same as (B.11) with $H_0 = kB2_{bot}$. The parameter k, which appears in equation (B.25) is given by Coïsson, Radicella, Leitinger and Nava (2006):

$$k = 3.22 - 0.0538 \, foF2 - 0.00664 hmF2 + 0.113 \frac{hmF2}{B2_{bot}} + 0.00257R12 \tag{B.27}$$

where hmF2 (km), foF2 (MHz), are the F2 layer peak parameters, $B2_{bot}$ (km) the thickness of the F2 bottomside and R12 the smoothed sunspot number. As inferred from the experimental data analysis, the restriction $k \ge 1$ is applied to the model.

Parameter modelling of critical frequency and propagation factor

Taking into account the fact that the NeQuick model has been designed mostly for trans-ionospheric propagation applications, the representation of the lower part of the ionosphere has been kept as simple as possible. The Titheridge model for *foE* (Leitinger, Titheridge, Kirchengast and Rothleitner 1995; Titheridge 1996) has been adopted. It is based on the seasonal relationship between the solar zenith angle χ and *foE* given as:

$$(foE)^2 = (a_e \sqrt{F107})^2 (\cos \chi_{eff})^{0.6}$$
 (B.28)

where a_e is the seasonal term represented in Table B.1, F107 is the 10.7 cm solar radio noise flux and χ_{eff} is the solar zenith angle:

a _e	Month North	Month South
1.131	1,2,11,12	5,6,7,8
1.112	3,4,9,10	3,4,9,10
1.093	5,6,7,8	1,2,11,12

Table B.1 Seasonal term to compute foE in the Titheridge's model (equation B.28)for the northern and southern hemisphere (Nava, Coïsson and Radicella, 2008).

$$\chi_{eff} = \chi \quad when \quad \chi \le 86.23^{\circ} \tag{B.29}$$

$$\chi_{eff} = 90^{\circ} - 0.24^{\circ} \exp(20^{\circ} - 0.2\chi) \quad when \quad \chi > 86.23^{\circ}$$
(B.30)

where equation B.29 is used during daytime and equation B.30 during night time. An exponential day-night transition is used to ensure the continuity of foE and its first derivative at the solar terminator.

Following Leitinger, Zhang and Radicella (2005), foF1 is related to foE by:

$$foF1 = \begin{cases} 1.4 \, foE & \text{if } foE \ge 2\\ 0 & \text{if } foE < 2\\ 0.85 * 1.4 \, foE & \text{if } 1.4 \, foE > 0.85 \, foF2 \end{cases}$$
(B.31)

B.2 Broadcast Model

The Klobuchar model is an ionospheric broadcast model for single-frequency user as described in Klobuchar (1987). The notations used for the Klobuchar model are the user approximate geodetic latitude (Φ_u ,), longitude (λ_u), elevation angle (*E*), and azimuth (*A*) to the particular GPS satellite for which you wish to calculate the ionospheric time delay. The coefficients α_n and β_n are transmitted as part of the satellite message. All angles are in units of semi-circle and time is in seconds.

The algorithms are as follows:

1. Calculate the Earth-centred angle:

$$\psi = \frac{0.0137}{E + 0.11} - 0.022 \quad (semicircles) \tag{B.32}$$

2. Compute the sub-ionospheric latitude:

$$\Phi_I = \Phi_u + \psi \cos A \tag{B.33}$$

If $\Phi_I > +0.416$, then $\Phi_I = +0.416$. If $\Phi_I < -0.416$, then $\Phi_I = -0.416$.

3.	Compute the sub-ionospheric longitude:	
λ_I	$=\lambda_{u}+\frac{\psi\sin A}{\cos\Phi_{I}}$	(B.34)
4.	Find the geomagnetic latitude:	
Φ,	$_{n} = \Phi_{I} + 0.064 \cos(\lambda_{I} - 1.617)$	(B.35)
5.	Find the local time:	
<i>t</i> =	$4.32 \times 10^4 \lambda_I + GPS \ time \ (sec)$	(B.36)

If t > 86400, then t = t - 86400. If t < 0, add 86400.

`2

6. Compute the slant factor:

$$F = 1.0 + 16.0 \times (0.53 - E)^{3}$$
(B.37)

7. Compute the ionospheric time delay:

$$T_{IONO} = F \times \left[5 \times 10^{-9} + \sum_{n=0}^{3} \alpha_n \Phi_m^n \times \left(1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right]$$
(B.38)

where

$$x = \frac{2\pi(t - 50400)}{\sum_{n=0}^{3} \beta_n \Phi_m^n}$$

Note: T_{IONO} is referred to the L_1 frequency. If the user requires the ionospheric timedelay correction on the L_2 , the correction term must be multiplied by a constant of 1.65.

B.3 GPS Data Driven Models

Global Ionospheric Maps (GIMs) and Real-time US-Total Electron Content are two products of GPS data driven models. For the latter, the Space Weather Prediction Center (2009) provides Vertical and Slant TEC over the Continental US (CONUS) in near real-time. Here, only the GIMs using the IONEX format (Schaer and Gurtner (1998)) is presented.
Three different procedures to compute the TEC E as a function of *geocentric latitude* β , *longitude* λ and *universal time t*, when TEC maps $E_i = E(T_i), i = 1, 2, ..., n$ at disposal:

1. Simply take the nearest TEC map $E_i = E(T_i)$ at epoch T_i :

$$E(\beta,\lambda,t) = E_i(\beta,\lambda) \tag{B.39}$$

where $|t - T_i| = \min$.

2. Interpolate between two consecutive TEC $E_i = E(T_i)$ maps and $E_{i+1} = E(T_{i+1})$:

$$E(\beta,\lambda,t) = \frac{T_{i+1} - t}{T_{i+1} - T} E_i(\beta,\lambda) + \frac{t - T_i}{T_{i+1} - T_i} E_{i+1}(\beta,\lambda)$$
(B.40)

where $T_i \leq t \leq T_{i+1}$.

3. Interpolate between consecutive rotated TEC maps:

$$E(\beta,\lambda,t) = \frac{T_{i+1} - t}{T_{i+1} - T} E_i(\beta,\lambda_i') + \frac{t - T_i}{T_{i+1} - T_i} E_{i+1}(\beta,\lambda_{i+1}')$$
(B.41)

where $T_i \leq t \leq T_{i+1}$ and $\lambda'_i = \lambda + (t - T_i)$. The TEC maps are rotated by $(t - T_i)$ around the Z-axis in order to compensate to a great extent the strong correlation between the ionosphere and the Sun's position. Note that method (B.39) can be refined accordingly by taking the nearest *rotated* map: $E(\beta, \lambda, t) = E_i(\beta, \lambda')$.

APPENDIX C: EGNOS TROPOSPHERIC CORRECTION MODEL

Another tropospheric error model under consideration is that proposed by Penna, Dodson and Chen (2001) for EGNOS tropospheric correction model. The total tropospheric delay for a receiver-to-satellite range at elevation angle α using:

$$c\Delta t_{trop}(t) = (d_{dry} + d_{wet}) \times MF(\alpha)$$
(C.1)

where d_{dry} is the zenith 'dry' (hydrostatic) delay,

 d_{wet} is the zenith 'wet' delay, and

 $MF(\alpha)$ is the mapping function to 'map' the zenith total delay to the appropriate receiver-to-satellite elevation angle.

$$d_{dry} = z_{dry} \left[1 - \frac{\beta H}{T} \right]^{\frac{s}{R_d \beta}}$$
(C.2)

$$d_{wet} = z_{wet} \left[1 - \frac{\beta H}{T} \right]^{\frac{(\lambda+1)g}{R_d \beta} - 1}$$
(C.3)

g is the gravitational attraction at the surface of Earth (m/s^2)

H is the height of the receiver above mean sea level (m),

T is the temperature at mean sea level (K),

 β is the temperature lapse rate (K/m),

 $R_d = 287.054 \,\text{J/kg/K}$,

k is the water vapour lapse rate (dimensionless),

 z_{dry} is the zenith `dry' delay at mean sea level,

 z_{wet} is the zenith `wet' delay at mean sea level.

$$z_{dry} = \frac{10^{-6} k_1 R_d P}{g_m}$$
(C.4)

$$z_{wet} = \frac{10^{-6} k_2 R_d}{g_m (\lambda + 1) - \beta R_d} \times \frac{e}{T}$$
(C.5)

where $k_1 = 77.604 \text{ K/mbar}$,

P is the pressure at mean sea level (mbar),

 $g_m = 9.784 \,\mathrm{m/s}$,

 $k_2 = 382000 \,\mathrm{K}^2/\mathrm{mbar}$,

e is the water vapour pressure at mean sea level (mbar).

The average values and seasonal variations for the five meteorological parameters are given in Table C.1. Using the values detailed in Table C.1, each meteorological parameter value (ξ) may then be computed using:

$$\xi(\phi, D) = \xi_0(\phi) - \Delta\xi(\phi) \times \cos\left[\frac{2\pi(D - D_{\min})}{365.25}\right]$$
(C.6)

where u is the receiver's latitude,

D is the day-of-year (starting with 1 January),

 $D_{\min} = 28$ for northern latitudes,

 $D_{\min} = 211$ for southern latitudes,

 ξ_0 and $\Delta\xi$ are the average and seasonal variation respectively for the particular parameter at the receiver's latitude.

Average					
Latitude (°)	P_0 (mbar)	$T_0(\mathbf{K})$	e_0 (mbar)	$\beta_0 (\text{K/m})$	λ_{O}
≤15	1013.25	299.65	26.31	6.30E-03	2.77
30	1017.25	294.15	21.79	6.05E-03	3.15
45	1015.75	283.15	11.66	5.58E-03	2.57
60	1011.75	272.15	6.78	5.39E-03	1.81
≥75	1013.00	263.65	4.11	4.53E-03	1.55
Seasonal Variation					
Latitude (°)	ΔP (mbar)	$\Delta T(\mathbf{K})$	Δe (mbar)	$\Delta\beta$ (K/m)	$\Delta\lambda$
≤15	0.00	0.00	0.00	0.00e-00	0.00
30	-3.75	7.00	8.85	0.25e-03	0.33
45	-2.25	11.00	7.24	0.32e-03	0.46
60	-1.75	15.00	5.36	0.81e-03	0.74
≥75	-0.50	14.50	3.39	0.62e-03	0.30

Table C.1 Average and seasonal variation values of the five meteorological parametersused by the EGNOS model (Penna et al 2001).

The mapping function $(MF(\alpha))$ is expressed as:

$$MF(\alpha) = \frac{1.001}{\sqrt{0.002001 + \sin^2 \alpha}}$$
(C.7)

The mapping function is not valid for elevation angles of less than 5 degrees (RTCA 1999).

APPENDIX D: ORBIT DYNAMIC PROPAGATION MODELS

Several orbit dynamic propagation models have been extensively studied for the purpose of interoperable orbit estimations. These few candidate models are the orbit modelling in the rotating Cartesian coordinate dynamics (Earth-Centered, Earth-Fixed coordinate) known as the Euler-Hill frame, Hill-Clohessy-Wiltshire equations and the Lagrange Planetary equations.

D.1 Perturbation Forces: The Euler-Hill Frame

Orbital dynamics has been classically expressed in terms of Cartesian position and velocity coordinates in inertial and in rotating coordinate frames. The Euler-Hill frame, also known as the Clohessy-Wiltshire frame, is a rotating Cartesian frame orbiting with the satellite around the Earth, as shown in Figure D.1.



Figure D.1 Relative motion rotating Euler-Hill reference frame (Kasdin and Gurfil 2004).

This coordinate frame, denoted by \Re , is defined by unit vector \hat{x} , \hat{y} , \hat{z} ; with the satellite at its origin. \mathbf{r}_1 is the relative position of *Satellite 1* from Earth, \mathbf{r}_2 is the relative position of *Satellite 2* from Earth and \mathbf{r} is the relative position of *Satellite 2* from *Satellite 1*. The angle between \mathbf{r}_1 and \mathbf{r} is denoted by α . The origin is set at the reference point (either satellite, but in this illustration, just *Satellite 1*) on the circular

reference orbit plane, the positive \hat{x} -axis ($\hat{\mathbf{X}}$) directed radially outwards along the extrapolation of the local radius vector, the positive \hat{y} -axis ($\hat{\mathbf{Y}}$) pointed along the direction of motion and the \hat{z} -axis ($\hat{\mathbf{Z}}$) completes the final orthogonal axis, which are normal to the reference orbit plane. This coordinate frame, as mentioned earlier, rotates with mean motion of $n = \sqrt{\frac{\mu}{a^3}}$ where μ is the standard gravitational constant. The slanted thick circular line is the orbit for *Satellite 2*.

D.2 Hill-Clohessy-Wiltshire Equations

The Hill-Clohessy-Wiltshire (HCW) is a method that can be used for linear orbit theory, specifically in analysing equations of motion between two satellites, which orbit near to each other. Figure D.1 is useful in order to explain how these equations can be derived.

Firstly, the equation of motion for *Satellite 1* is defined:

$$\ddot{\mathbf{r}}_1 = -\frac{\mu \mathbf{r}_1}{r_1^3} \tag{D.1}$$

Next, the equation of motion for *Satellite 2* is defined. Assuming a rendezvous is required, thrusting is included. Other forces such as drag for lower orbiting satellites or solar radiation pressure for higher orbiting satellites is added. Hence, the equation of motion for *Satellite 2* becomes:

$$\ddot{\mathbf{r}}_2 = -\frac{\mu \mathbf{r}_2}{r_2^3} + \mathbf{F}$$
(D.2)

The relative range vector, \mathbf{r} from Satellite 1 to Satellite 2 can be found as follows:

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 \tag{D.3a}$$

Differentiating yields:

$$\dot{\mathbf{r}} = \dot{\mathbf{r}}_2 - \dot{\mathbf{r}}_1 \tag{D.3b}$$

Differentiating once again gives:

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_2 - \ddot{\mathbf{r}}_1 \tag{D.3c}$$

Substituting the two-body equations of motion, (D.1) and (D.2) into (D.3c) gives:

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}_2}{r_2^3} + \mathbf{F} + \frac{\mu \mathbf{r}_1}{r_1^3} \tag{D.4}$$

Re-arranging equation (D.3a) for Satellite 2:

 $\mathbf{r}_2 = \mathbf{r}_1 + \mathbf{r}$

The position vector of *Satellite 2* can be obtained by dividing the position of *Satellite 2*, \mathbf{r}_2 with its magnitude cubed, r_2^3 . Also, the cosine law is applied to the magnitude of the vector of the *Satellite 2* cubed, on the denominator of the right-hand side of the equation. Hence:

$$\frac{\mathbf{r}_2}{r_2^3} = \frac{\mathbf{r}_1 + \mathbf{r}}{(r_1^2 - 2\mathbf{r}_1 \cdot \mathbf{r} + r^2)^{3/2}}$$

Assuming that the magnitude of the relative vector, r^2 is smaller than r_1^2 , $(r_1^2 >>> r^2 \Rightarrow r^2 \approx 0)$ reduces the above equation to :

$$\frac{\mathbf{r}_2}{r_2^3} = \frac{\mathbf{r}_1 + \mathbf{r}}{(r_1^2 - 2\mathbf{r}_1 \cdot \mathbf{r})^{3/2}}$$

Factoring out the r_1^2 term yields:

$$\frac{\mathbf{r}_2}{r_2^3} = \frac{\mathbf{r}_1 + \mathbf{r}}{r_1^3} \frac{1}{\left(1 - \frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1^2}\right)^{3/2}}$$

Simplify the above equation by using binomial series on the dot-product terms. The binomial series has the form:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots$$

where x is replaced by the dot-product term of the denominator. Hence:

$$\frac{\mathbf{r}_{2}}{r_{2}^{3}} = \frac{\mathbf{r}_{1} + \mathbf{r}}{r_{1}^{3}} \left\{ 1 - \frac{3}{2} \left(\frac{2\mathbf{r}_{1} \cdot \mathbf{r}}{r_{1}^{2}} \right) + \dots \right\}$$

Substituting this result into equation (D.4):

$$\ddot{\mathbf{r}} = -\mu \left\{ \frac{\mathbf{r}_1 + \mathbf{r}}{r_1^3} \left\{ 1 - \frac{3}{2} \left(\frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1^2} \right) + \ldots \right\} \right\} + \mathbf{F} + \frac{\mu \mathbf{r}_1}{r_1^3}$$

Expanding and removing terms of opposite signs and keeping first-order terms while omitting higher-order terms yields:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r_1^3} \left\{ -\frac{3\mathbf{r}_1}{2} \left(\frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1^2} \right) + \mathbf{r} - \frac{3\mathbf{r}}{2} \left(\frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1^2} \right) \right\} + \mathbf{F}$$

Assume the third term in the bracket, $\frac{3\mathbf{r}}{2}\left(\frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1^2}\right)$, is small enough and can be

dropped; hence the equation is further reduced to:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r_1^3} \left\{ -\frac{3\mathbf{r}_1}{2r_1} \left(\frac{2\mathbf{r}_1 \cdot \mathbf{r}}{r_1} \right) + \mathbf{r} \right\} + \mathbf{F}$$

Introducing $\frac{\mathbf{r}_1}{r} = \hat{\mathbf{X}}$ and $\hat{\mathbf{X}} \cdot \mathbf{r} = x$ into the equation, which brings:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r_1^3} \left\{ -3x\hat{\mathbf{X}} + \mathbf{r} \right\} + \mathbf{F}$$
(D.5)

This equation expresses the relative acceleration of *Satellite 2* with respect of *Satellite 1* in an *inertial* frame, which is adequate for a fixed, non-rotating time-invariant frame. However, since the coordinate system of *Satellite 1* is rotating and changes with time, an extension to the above equation with rotation transformation for acceleration is required. The relationship between the inertial acceleration vector, \mathbf{a}_{hor} and the rotating acceleration vector, \mathbf{a}_{Rot} is expressed generically as follows:

$$\mathbf{a}_{Iner} = \mathbf{a}_{Rot} + 2(\mathbf{\Omega}_{R/I} \times \mathbf{v}_{Rot}) - \dot{\mathbf{\Omega}}_{R/I} \times \mathbf{r}_{Rot} - \mathbf{\Omega}_{R/I} \times (\mathbf{\Omega}_{R/I} \times \mathbf{r}_{Rot}) + \mathbf{a}_{Origin}$$
(D.6)

where $\Omega_{R/I}$ and $\dot{\Omega}_{R/I}$ represent the angular velocity and the rate of angular velocity of the Euler-Hill reference frame with respect to the inertial frame respectively. The second term on the right-hand side of equation (D.6), $2(\Omega_{R/I} \times \mathbf{v}_{Rot})$, is *Coriolis acceleration*, followed by the *tangential acceleration*, $\dot{\Omega}_{R/I} \times \mathbf{r}_{Rot}$, which is equal to zero for a circular orbit, and the fourth term, $\Omega_{R/I} \times (\Omega_{R/I} \times \mathbf{r}_{Rot})$, represents the *centrifugal acceleration*. The final term, \mathbf{a}_{Origin} , is the origin acceleration, which, in this case, simply equals zero since *Satellite 1*'s coordinate frame is not accelerating. By shifting \mathbf{a}_{Rot} to the left-hand side and \mathbf{a}_{Iner} to the right-hand side of the expression, the relative acceleration vector equation becomes:

$$\mathbf{a}_{Rot} = \mathbf{a}_{Iner} - 2(\mathbf{\Omega}_{R/I} \times \mathbf{v}_{Rot}) - \dot{\mathbf{\Omega}}_{R/I} \times \mathbf{r}_{Rot} - \mathbf{\Omega}_{R/I} \times (\mathbf{\Omega}_{R/I} \times \mathbf{r}_{Rot})$$
(D.7)

Writing equation (D.7) so as to comply with this section convention, it will then becomes:

$$\ddot{\mathbf{r}}_{Rot} = \ddot{\mathbf{r}}_{Iner} - 2(\boldsymbol{\omega}_{R/I} \times \dot{\mathbf{r}}_{Rot}) - \dot{\boldsymbol{\omega}}_{R/I} \times \mathbf{r}_{Rot} - \boldsymbol{\omega}_{R/I} \times (\boldsymbol{\omega}_{R/I} \times \mathbf{r}_{Rot})$$
(D.8)

Assuming the orbit of *Satellite 1* is circular, then the angular rate is equal to the mean motion of *Satellite 1*, $n = \sqrt{\frac{\mu}{a^3}} \Rightarrow n \equiv \omega = \sqrt{\frac{\mu}{\mathbf{r}_1^3}}$. Generally, the angular rate is defined

as follows: $\omega = \sqrt{\frac{\mu p}{\mathbf{r}_1^4}}$, for any type of orbits. In xyz Euler-Hill frame reference components:

$$\boldsymbol{\omega} = \begin{bmatrix} 0\\0\\\omega \end{bmatrix} \text{ and } \mathbf{r}_{Rot} = \begin{bmatrix} x\\y\\z \end{bmatrix}$$

Differentiating twice for the rotating position vector yields:

$$\dot{\mathbf{r}}_{Rot} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \text{ and } \ddot{\mathbf{r}}_{Rot} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}$$

Now, expanding the second, third and forth terms of equation (D.8) in Euler-Hill reference frame respectively:

$$\boldsymbol{\omega}_{R/I} \times \dot{\mathbf{r}}_{Rot} = \begin{bmatrix} \hat{\mathbf{X}} & \hat{\mathbf{Y}} & \hat{\mathbf{Z}} \\ 0 & 0 & \omega \\ \dot{x} & \dot{y} & \dot{z} \end{bmatrix} = -\omega \dot{y} \hat{\mathbf{X}} + \omega \dot{x} \hat{\mathbf{Y}}$$

$$\dot{\boldsymbol{\omega}}_{R/I} \times \mathbf{r}_{Rot} = \begin{bmatrix} \hat{\mathbf{X}} & \hat{\mathbf{Y}} & \hat{\mathbf{Z}} \\ 0 & 0 & \dot{\boldsymbol{\omega}} \\ x & y & z \end{bmatrix} = -\dot{\omega}y\hat{\mathbf{X}} + \dot{\omega}x\hat{\mathbf{Y}}$$
$$\boldsymbol{\omega}_{R/I} \times (\boldsymbol{\omega}_{R/I} \times \mathbf{r}_{Rot}) = \boldsymbol{\omega}_{R/I} \times \begin{bmatrix} \hat{\mathbf{X}} & \hat{\mathbf{Y}} & \hat{\mathbf{Z}} \\ 0 & 0 & \boldsymbol{\omega} \\ x & y & z \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}} & \hat{\mathbf{Y}} & \hat{\mathbf{Z}} \\ 0 & 0 & \boldsymbol{\omega} \\ -\boldsymbol{\omega}y & \boldsymbol{\omega}x & 0 \end{bmatrix} = -\boldsymbol{\omega}^2 x\hat{\mathbf{X}} + \boldsymbol{\omega}^2 y\hat{\mathbf{Y}}$$

Collecting these three terms into equation (D.8) and substituting the first term of equation (D.8) $\ddot{\mathbf{r}}_{lner}$ with equation (D.5) as $\ddot{\mathbf{r}}_{lner} = \mathbf{r}$, hence equation (D.8) becomes:

$$\ddot{\mathbf{r}}_{Rot} = \frac{\mu}{r_1^3} \left\{ -3x\hat{\mathbf{X}} + \mathbf{r} \right\} + \mathbf{F} - 2(-\omega \dot{y}\hat{\mathbf{X}} + \omega \dot{x}\hat{\mathbf{Y}}) - (-\dot{\omega}y\hat{\mathbf{X}} + \dot{\omega}x\hat{\mathbf{Y}}) - (-\omega^2 x\hat{\mathbf{X}} + \omega^2 y\hat{\mathbf{Y}})$$
(D.9)

as $\mathbf{r} = \mathbf{r}_{Rot} = x\hat{\mathbf{X}} + y\hat{\mathbf{Y}} + z\hat{\mathbf{Z}}$ and assuming the orbit of *Satellite 1* is circular, $\omega = \sqrt{\frac{\mu}{\mathbf{r}_1^3}} \Rightarrow \omega^2 = \frac{\mu}{\mathbf{r}_1^3}$ and $\dot{\omega} = 0$, the expression now reduces to: $\ddot{\mathbf{r}}_{Rot} = -\omega^2 \{x\hat{\mathbf{X}} + y\hat{\mathbf{Y}} + z\hat{\mathbf{Z}} - 3x\hat{\mathbf{X}}\} + \mathbf{F} + 2\omega \dot{y}\hat{\mathbf{X}} - 2\omega \dot{x}\hat{\mathbf{Y}} + \omega^2 x\hat{\mathbf{X}} + \omega^2 y\hat{\mathbf{Y}}$ (D.10) Separating each vector component and introducing $\mathbf{F} = \begin{bmatrix} f_x \\ f_y \\ f \end{bmatrix}$ in component forms;

this will result in the HCW equations:

$$\begin{vmatrix} \ddot{x} - 2\omega \dot{y} - 3\omega^2 x = f_x \\ \ddot{y} + 2\omega^2 \dot{x} = f_y \\ \ddot{z} + \omega^2 z = f_z \end{vmatrix}$$
 (D.11a-c)

D.3 The Lagrange Planetary Equations

The Lagrange planetary equations of motion (LPE) came from a variation of parameters (VOP) because the orbital elements, which are constant in a two-body equation (unperturbed system), are now changing (due to perturbation). Knowing the

APPENDIX

solution for the unperturbed system is the key to solving the perturbed system (a three- or more-body equation).

In a two-body system, the six Keplerian orbital elements plus the time (as described earlier in section 2.2) are constant. Altering the two-body system with a relatively small perturbation force in comparison to either of the two-body forces, the new equations of motion will comprise the original two-body systems' equation of motion and the time-varying change of the osculating elements. In other words, the six Keplerian orbital elements, which were constants, are now varied with respect to time – this is the gist of VOP. If \mathbf{c} is the matrix form of the Keplerian orbital elements, then the time-varying elements can be describes as follows:

$$\frac{d\mathbf{c}}{dt} = f(\mathbf{c}, t) \tag{D.12}$$

The derivation of Lagrange's VOP (please see Vallado 2007 for derivation) gives rise the LPE, which can either be in the Lagrangian form of VOP as follows:

$$\frac{da}{dt} = \frac{2}{na} \frac{\partial R}{\partial M_0}$$

$$\frac{de}{dt} = \frac{1 - e^2}{na^2 e} \frac{\partial R}{\partial M_0} - \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial R}{\partial \omega}$$

$$\frac{di}{dt} = \frac{1}{na^2 \sqrt{1 - e^2}} \frac{\partial R}{\sin(i)} \left(\cos(i) \frac{\partial R}{\partial \omega} - \frac{\partial R}{\partial \Omega} \right)$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial R}{\partial e} - \frac{\cot(i)}{na^2 \sqrt{1 - e^2}} \frac{\partial R}{\partial i}$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sqrt{1 - e^2}} \frac{\partial R}{\sin(i)} \frac{\partial R}{\partial i}$$

$$\frac{dM_0}{dt} = -\frac{1 - e^2}{na^2 e} \frac{\partial R}{\partial e} - \frac{2}{na} \frac{\partial R}{\partial a}$$
(D.13a-f)

which accounts for conservative forces only. This form of LPE is regularly used in the analysis of non-Keplerian orbits due to the fact that most perturbation forces are conservative. The other derivation of VOP produces the Gaussian form, which can handle conservative and non-conservative forces. The equations are as follows:

$$\begin{aligned} \frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left\{ e \cdot \sin(v)F_{\chi} + \frac{p}{r}F_{\gamma} \right\} \\ \frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na} \left\{ \sin(v)F_{\chi} + \left(\cos(v) + \frac{e + \cos(v)}{1 + e \cdot \cos(v)} \right)F_{\gamma} \right\} \\ \frac{di}{dt} &= \frac{r \cdot \cos(u)}{na^2\sqrt{1-e^2}}F_Z \\ \frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{nae} - \left\{ \cos(v)F_{\chi} + \sin(v)\left(1 + \frac{r}{p}F_{\gamma}\right) \right\} - \frac{r \cdot \cot(i)\sin(u)}{h}F_Z \\ \frac{d\Omega}{dt} &= \frac{r \cdot \sin(u)}{na^2\sqrt{1-e^2}\sin(i)}F_Z \\ \frac{dM_0}{dt} &= \frac{1}{na^2e} \left\{ (p \cdot \cos(v) - 2er)F_{\chi} + (p + r)\sin(v)F_{\gamma} \right\} \end{aligned}$$
(D.14a-f)

where F_X , F_Y and, F_Z are the perturbing resultant force components resolved in the xyz Euler-Hill rotating reference frame.

D.4 Orbit Modeling in Earth-Centred Cartesian Coordinates

In a rotating reference frame, a dynamic model for the acceleration is obtained by including the effects of normal central force field and the primary disturbance effect due to the Earth's equatorial bulge and flattening at the poles. The Earth's equatorial bulge and flattening at the poles is a result of the Earth's oblateness and is represented by two coefficients, C_{2i} , where i = 0 and 2.

A length scale and a time normalisation defined by:

$$r_s = \left(\mu/\Omega_n^2\right)^{1/3}, \ \tau = \Omega_n t$$

are introduced, where Ω_n is the angular velocity of the rotating frame. The equations of motion are then expressed in terms of non-dimensional Cartesian coordinates, \tilde{x} , \tilde{y} , \tilde{z} , as:

$$d\tilde{x}/d\tau = \tilde{x}', \ d\tilde{y}/d\tau = \tilde{y}', \ d\tilde{z}/d\tau = \tilde{z}', \tag{D.15a}$$

$$\frac{d\tilde{x}'}{d\tau} = -\frac{\mu_n}{\tilde{r}^3}\tilde{x} + \mu_n\frac{\partial\tilde{U}_2}{\partial\tilde{x}} + \tilde{x} + 2\tilde{y}' + \tilde{x}''_{res}, \qquad (D.15b)$$

$$\frac{d\tilde{y}'}{d\tau} = -\frac{\mu_n}{\tilde{r}^3}\tilde{y} + \mu_n\frac{\partial\tilde{U}_2}{\partial\tilde{y}} + \tilde{y} - 2\tilde{x}' + \tilde{y}''_{res},\tag{D.15c}$$

$$\frac{d\tilde{z}'}{d\tau} = -\frac{\mu_n}{\tilde{r}^3}\tilde{z} + \mu_n\frac{\partial U_2}{\partial\tilde{z}} + \tilde{z}''_{res},$$
(D.15d)

where, $\mu_n = \mu / \Omega_n^2 r_s^3 = 1$, $\tilde{r} = \sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}$, μ is the gravity parameter, and \tilde{x}''_{res} , \tilde{y}''_{res} , \tilde{z}''_{res} are the residual accelerations mainly due to the gravitational effects of the Moon and Sun. These are generally modelled as the sum of biases and periodic terms including secondary harmonics.

The Earth's gravitational perturbation potential can be expressed as:

$$U_{2} = \frac{\mu C_{20}}{r^{3}} \left(1 - \frac{3}{2} \cos^{2} \delta \right) + \frac{3\mu C_{22}}{r^{3}} \cos^{2} \delta \cos 2\lambda$$
(D.16)

where *r* is the distance from the body's centre of mass, δ is the latitude measured from the equatorial plane, and λ is the longitude measured from the long end of the body (about 15° west longitude in the case of the Earth). In Earth-fixed Cartesian coordinates, with the *x*-*y* plane in the Earth's equatorial plane, the potential may be approximately expressed as:

$$U_{2} = -\mu C_{20} \frac{\left(r^{2} - 3z^{2}\right)}{2r^{5}} + 3\mu C_{22} \frac{\left(x^{2} - y^{2}\right)}{r^{5}},$$
 (D.17)
where $r = \sqrt{x^{2} + y^{2} + z^{2}}.$

In terms of the normalised Earth-fixed $(\tilde{x}_e, \tilde{y}_e, \tilde{z}_e)$ and rotating $(\tilde{x}, \tilde{y}, \tilde{z})$ coordinates, assuming that the reference *x*-*y* plane is inclined to the Earth's equatorial plane by a fixed angle, the gradients of the non-dimensional Earth's gravitational perturbation potential, U_2 , in rotating coordinates are:

$$\frac{\partial \widetilde{U}_2}{\partial \widetilde{x}} = -\frac{\widetilde{C}_{20}\widetilde{x}}{\widetilde{r}^5} + \frac{5\widetilde{C}_{20}\widetilde{x}}{2\widetilde{r}^7} \left(\widetilde{r}^2 - 3\overline{z}_e^2\right) + \frac{3\widetilde{C}_{20}\overline{x}_e}{\widetilde{r}^5} \sin i \cos \Omega_r \tau + \frac{6\widetilde{C}_{22}\overline{x}}{\widetilde{r}^5} - \frac{15\widetilde{C}_{22}\widetilde{x}}{\widetilde{r}^7} \left(\overline{x}_e^2 - \widetilde{y}_e^2\right),$$

$$\begin{aligned} &(\text{D.18a})\\ \frac{\partial \tilde{U}_2}{\partial \tilde{y}} = -\frac{\tilde{C}_{20}\tilde{y}}{\tilde{r}^5} + \frac{5\tilde{C}_{20}\tilde{y}}{2\tilde{r}^7} \left(\tilde{r}^2 - 3\bar{z}_e^2\right) - \frac{3\tilde{C}_{20}\bar{x}_e}{\tilde{r}^5} \sin i \sin \Omega_r \tau + \frac{6\tilde{C}_{22}\bar{y}}{\tilde{r}^5} - \frac{15\tilde{C}_{22}\tilde{y}}{\tilde{r}^7} \left(\bar{x}_e^2 - \tilde{y}_e^2\right), \end{aligned}$$

$$(\text{D.18b}) \\ \frac{\partial \tilde{U}_2}{\partial \tilde{z}} = -\frac{\tilde{C}_{20}\tilde{z}}{\tilde{r}^5} + \frac{3\tilde{C}_{20}\bar{z}_e}{\tilde{r}^5} \cos i + \frac{5\tilde{C}_{20}\tilde{z}}{2\tilde{r}^7} \left(\tilde{r}^2 - 3\bar{z}_e^2\right) - \frac{6\tilde{C}_{22}\bar{x}_e}{\tilde{r}^5} \sin i - \frac{15\tilde{C}_{22}\tilde{z}}{\tilde{r}^7} \left(\bar{x}_e^2 - \tilde{y}_e^2\right), \end{aligned}$$

$$(\text{D.18b}) \\ (\text{D.18b}) \\ (\text{D.18c}) \end{aligned}$$

where,

$$\begin{bmatrix} \tilde{x}_{e} \\ \tilde{y}_{e} \\ \tilde{z}_{e} \end{bmatrix} = \begin{bmatrix} \cos \Omega_{r} \tau & -\sin \Omega_{r} \tau & 0 \\ \sin \Omega_{r} \tau & \cos \Omega_{r} \tau & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \mathbf{T}_{\Omega} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix},$$

$$\begin{bmatrix} \bar{x}_{e} \\ \bar{y}_{e} \\ \bar{z}_{e} \end{bmatrix} = \begin{bmatrix} \cos i & 0 & -\sin i \\ 0 & 1 & 0 \\ \sin i & 0 & \cos i \end{bmatrix} \begin{bmatrix} \tilde{x}_{e} \\ \tilde{y}_{e} \\ \tilde{z}_{e} \end{bmatrix} = \mathbf{T}_{i} \begin{bmatrix} \tilde{x}_{e} \\ \tilde{y}_{e} \\ \tilde{z}_{e} \end{bmatrix},$$

$$\bar{x} = \bar{x}_{e} \cos i \cos \Omega_{r} \tau + \bar{y}_{e} \sin \Omega_{r} \tau,$$

$$\bar{y} = -\bar{x}_{e} \cos i \sin \Omega_{r} \tau + \bar{y}_{e} \cos \Omega_{r} \tau,$$

 Ω_r is the relative angular velocity of the satellite to the Earth fixed frame, *i* is the inclination orbit to the Earth's equatorial plane and $\tilde{C}_{2i} = C_{2i}/r_s^2$. The oblateness coefficients, C_{2i} , are also related to the principal moments of inertia of the Earth and could be expressed in terms of alternate relationships to the zonal harmonic coefficients, $J_2 = 1.082616 \times 10^{-3}$, $J_3 = -2.53881 \times 10^{-6}$, $J_4 = -1.65597 \times 10^{-6}$ and to $J_{21} = 0$, $J_{22} = 1.86 \times 10^{-6}$, $J_{31} = 2.1061 \times 10^{-6}$. The orbit is defined by equations D.15 and D.18.

APPENDIX E: AMBIGUITY ESTIMATION AND RESOLUTION

There are two ways to solve the integer phase ambiguity, either by eliminating the constant integer ambiguity by differencing the carrier-phase measurement across each time epoch, or by estimating the constant integer ambiguity. The former uses the Doppler carrier-phase processing method and the latter, the integer ambiguity resolution method. The integer ambiguity resolution method has two stages; the first stage is the initial estimate of ambiguity, which will be used as an initialisation for the second stage. The second stage is the integer search algorithm, which determines the value of the integer ambiguity.

Forsell (1997) compares two methods for real-time ambiguity resolution. Firstly, widelaning is a method using frequency differences between two suitably spaced carriers (Forsell 1995) and secondly, tone-ranging, which uses modulation signals on one carrier (Hatch 1996).

Hatch (2000) categorises ambiguity resolution into Geometry Independent, which is insensitive to tropospheric refraction, has greater degree of freedom and simple verification; and Geometry Dependent, which is totally opposite to the description of Geometry Independent. For Geometry Independent, the technique used is ambiguity resolution in measurement space, which uses smoothed code for wide-lane ambiguity resolution, then wide-lane resolved value to step to narrow-lane. There exist two techniques for Geometry Dependent, ambiguity resolution in position space, which utilises Counselman's ambiguity function and ambiguity resolution in ambiguity space, which searches for minimum residuals as a function of ambiguity combinations.

E.1 Ambiguity Estimation

An accurate estimation of carrier-phase integer ambiguities will give a reliable and precise relative positioning using differential GPS. By measuring the relative phase shift of the carrier frequency used, the positioning accuracy can reach up to centimetre scale, and even to millimetre scale. This technique of measurement can eliminate clock and atmospheric errors when applied in differential mode of operation. If the user carrier phase measurement (as in equation 3.2) is corrected with the reference base station, the differential phase can be written as follows:

$$\Delta \widetilde{\phi} \cdot \lambda = \left(\widetilde{\phi} - \widetilde{\phi}_0\right) \cdot \lambda \tag{E.1a}$$

$$=\mathbf{h}(\mathbf{x}-\mathbf{x}_{0})-(N-N_{0})\lambda+(mp-mp_{0})+(\xi-\xi_{0})$$
(E.1b)

where $\tilde{\phi}_0$ is the phase measurement of the reference base station, **h** is the vector between the antenna and the satellites, $(\mathbf{x} - \mathbf{x}_0)$ is the linearised position, and all other remaining terms are the same as before. Similarly, the differential correction of equation 3.2 becomes:

$$\begin{aligned} \left(\Delta \widetilde{\phi} + \Delta N\right) \cdot \lambda &= \left(\left(\hat{x}_{sv} - x_{u}\right)^{2} + \left(\hat{y}_{sv} - y_{u}\right)^{2} + \left(\hat{z}_{sv} - z_{u}\right)^{2}\right)^{0.5} \\ &- \left(\left(\hat{x}_{sv} - x_{0}\right)^{2} + \left(\hat{y}_{sv} - y_{0}\right)^{2} + \left(\hat{z}_{sv} - z_{0}\right)^{2}\right)^{0.5} + c\Delta t_{r}(t) - c\Delta t_{r,0}(t) \\ &+ mp(t) - mp_{0}(t) + \zeta(t) - \zeta_{0}(t) + c\Delta t_{sv}(t) - c\Delta t_{sv,0}(t) + E(t) - E_{0}(t) \\ &- c\Delta t_{ion}(t) - \left(-c\Delta t_{ion,0}(t)\right) + c\Delta t_{tr}(t) - c\Delta t_{tr,0}(t) \end{aligned}$$
(E.2)

using Δ to indicate differences of an expression and taking:

 $\Delta r = \left(\left(\hat{x}_{sv} - x_u \right)^2 + \left(\hat{y}_{sv} - y_u \right)^2 + \left(\hat{z}_{sv} - z_u \right)^2 \right)^{0.5} - \left(\left(\hat{x}_{sv} - x_0 \right)^2 + \left(\hat{y}_{sv} - y_0 \right)^2 + \left(\hat{z}_{sv} - z_0 \right)^2 \right)^{0.5}$ equation E.2 becomes:

$$(\Delta \widetilde{\phi} + \Delta N) \cdot \lambda = \Delta r + \Delta (c \Delta t_r(t)) + \Delta (mp(t)) + \Delta (\zeta(t)) + \Delta (c \Delta t_{sv}(t)) + \Delta (E(t))$$

- $\Delta (c \Delta t_{ion}(t)) + \Delta (c \Delta t_{tr}(t))$ (E.3)

Apart from ionospheric delay error, all other common-mode errors have been eliminated through differential mode of operation. Hence, equation E.3 becomes:

$$\left(\Delta\widetilde{\phi} + \Delta N\right) \cdot \lambda = \Delta r - \Delta(c\Delta t_{ion}(t)) + \Delta(mp(t)) + \Delta(\zeta(t))$$
(E.4)

Dividing both sides by λ , the equation becomes:

$$\left(\Delta\widetilde{\phi} + \Delta N\right) = \frac{\Delta r}{\lambda} - \frac{\Delta(c\Delta t_{ion}(t))}{\lambda} + \frac{\Delta(mp(t))}{\lambda} + \frac{\Delta(\zeta(t))}{\lambda}$$
(E.5)

now, substitute equation 3.25 for the ionospheric delay, $c\Delta t_{ion}(t)$ into equation E.5:

$$\left(\Delta\tilde{\phi} + \Delta N\right) = \frac{\Delta r}{\lambda} - \frac{\Delta\left(\frac{40.3}{f^2} \cdot TEC\right)}{\lambda} + \frac{\Delta(mp(t))}{\lambda} + \frac{\Delta(\zeta(t))}{\lambda}$$
(E.6)

Assuming two carrier frequencies, f_1 and f_2 , where $f_1 > f_2$, the difference in frequency, f_{12} is given by:

$$f_{12} = f_1 - f_2 = c \cdot \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{c}{\lambda_{12}}$$
(E.7)

where c is the velocity of propagation.

Hence, the wide-lane wavelength is defined as follows:

$$\lambda_w = \lambda_{12} = \frac{c}{f_1 - f_2} \tag{E.8}$$

In contrast, a similar argument applied for a narrow-lane wavelength is defined as the velocity of propagation divided by the frequency summation of the two suitably spaced carriers, which is given by:

$$\lambda_n = \frac{c}{f_1 + f_2} \tag{E.9}$$

The phase measurements for equation E.6 for a single satellite at carrier frequencies f_1 and f_2 , can be represented as:

$$\left(\Delta \widetilde{\phi}_{1} + \Delta N_{1}\right) = \frac{\Delta r}{\lambda_{1}} - \frac{\Delta \left(\frac{40.3}{f^{2}} \cdot TEC\right)}{\lambda_{1}} + \frac{\Delta (mp(t))}{\lambda_{1}} + \frac{\Delta (\zeta(t))}{\lambda_{1}}$$
(E.10a)

$$\left(\Delta \widetilde{\phi}_{2} + \Delta N_{2}\right) = \frac{\Delta r}{\lambda_{2}} - \frac{\Delta \left(\frac{40.3}{f^{2}} \cdot TEC\right)}{\lambda_{2}} + \frac{\Delta (mp(t))}{\lambda_{2}} + \frac{\Delta (\zeta(t))}{\lambda_{2}}$$
(E.10b)

substituting equations E.10 with $\lambda_1 = \frac{c}{f_1}$ and $\lambda_2 = \frac{c}{f_2}$,

$$\left(\Delta\widetilde{\phi}_{1}+\Delta N_{1}\right)=\frac{f_{1}}{c}\Delta r-\frac{f_{1}}{c}\Delta\left(\frac{40.3}{f_{1}^{2}}\cdot TEC\right)+\frac{f_{1}}{c}\Delta\left(mp_{1}(t)\right)+\frac{f_{1}}{c}\Delta\left(\zeta_{1}(t)\right)$$
(E.11a)

$$\left(\Delta\tilde{\phi}_{2} + \Delta N_{2}\right) = \frac{f_{2}}{c}\Delta r - \frac{f_{2}}{c}\Delta\left(\frac{40.3}{f_{2}^{2}} \cdot TEC\right) + \frac{f_{2}}{c}\Delta(mp_{2}(t)) + \frac{f_{2}}{c}\Delta(\zeta_{2}(t))$$
(E.11b)

A summation of equations E.11 yields:

$$\left(\Delta \widetilde{\phi}_{1} + \Delta \widetilde{\phi}_{2}\right) + \left(\Delta N_{1} + \Delta N_{2}\right) = \left(\frac{f_{1}}{c} + \frac{f_{2}}{c}\right) \Delta r - \Delta \left(\frac{40.3 \cdot TEC}{f_{1}f_{2}}\right) \cdot \left(\frac{f_{2} + f_{1}}{c}\right) + \left(\frac{f_{1}}{c}\right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t))\right) + \left(\frac{f_{2}}{c}\right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t))\right)$$

$$(E.12a)$$

While the difference of equations E.11 yields:

$$\left(\Delta \widetilde{\phi}_{1} - \Delta \widetilde{\phi}_{2}\right) + \left(\Delta N_{1} - \Delta N_{2}\right) = \left(\frac{f_{1}}{c} - \frac{f_{2}}{c}\right) \Delta r - \Delta \left(\frac{40.3 \cdot TEC}{f_{1}f_{2}}\right) \cdot \left(\frac{f_{2} - f_{1}}{c}\right) + \left(\frac{f_{1}}{c}\right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t))\right) - \left(\frac{f_{2}}{c}\right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t))\right)$$
(E.12b)

Replacing terms in equations E.12 with, respectively, the wide-lane equation (E.8) and the narrow-lane equation (E.9) gives:

$$\left(\Delta \widetilde{\phi}_{1} + \Delta \widetilde{\phi}_{2} \right) + \left(\Delta N_{1} + \Delta N_{2} \right) = \left(\frac{1}{\lambda_{n}} \right) \cdot \Delta r - \left(\frac{1}{\lambda_{n}} \right) \cdot \Delta \left(\frac{40.3 \cdot TEC}{f_{1}f_{2}} \right)$$

$$+ \left(\frac{1}{\lambda_{1}} \right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t)) \right) + \left(\frac{1}{\lambda_{2}} \right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t)) \right)$$

$$\left(\Delta \widetilde{\phi}_{1} - \Delta \widetilde{\phi}_{2} \right) + \left(\Delta N_{1} - \Delta N_{2} \right) = \left(\frac{1}{\lambda_{w}} \right) \cdot \Delta r + \left(\frac{1}{\lambda_{w}} \right) \cdot \Delta \left(\frac{40.3 \cdot TEC}{f_{1}f_{2}} \right)$$

$$+ \left(\frac{1}{\lambda_{1}} \right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t)) \right) - \left(\frac{1}{\lambda_{2}} \right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t)) \right)$$

$$(E.13a)$$

$$(E.13b)$$

Re-arranging equations E.13 respectively can be written as:

$$\left(\Delta \widetilde{\phi}_{1} + \Delta \widetilde{\phi}_{2} \right) \cdot \lambda_{n} = \Delta r - \Delta \left(\frac{40.3 \cdot TEC}{f_{1} f_{2}} \right) - \left(\Delta N_{1} + \Delta N_{2} \right) \cdot \lambda_{n}$$

$$+ \left(\frac{\lambda_{n}}{\lambda_{1}} \right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t)) \right) + \left(\frac{\lambda_{n}}{\lambda_{2}} \right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t)) \right)$$

$$(E.14a)$$

$$\left(\Delta \widetilde{\phi}_{1} - \Delta \widetilde{\phi}_{2} \right) \cdot \lambda_{w} = \Delta r + \Delta \left(\frac{40.3 \cdot TEC}{f_{1} f_{2}} \right) - \left(\Delta N_{1} - \Delta N_{2} \right) \cdot \lambda_{w}$$

$$+ \left(\frac{\lambda_{w}}{\lambda_{1}} \right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t)) \right) - \left(\frac{\lambda_{w}}{\lambda_{2}} \right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t)) \right)$$

$$(E.14b)$$

Similarly, the code measurements for equation E.6 for a single satellite measuring at carrier frequencies f_1 and f_2 , can be represented as:

$$\frac{\Delta \tilde{\rho}_{1}}{\lambda_{1}} = \frac{\Delta r}{\lambda_{1}} + \frac{\Delta \left(\frac{40.3}{f_{1}^{2}} \cdot TEC\right)}{\lambda_{1}} + \frac{\Delta (MP_{1}(t))}{\lambda_{1}} + \frac{\Delta (\eta_{1}(t))}{\lambda_{1}} + \frac{\Delta (\eta_{1}(t))}{\lambda_{1}}$$
(E.15a)

$$\frac{\Delta\tilde{\rho}_2}{\lambda_2} = \frac{\Delta r}{\lambda_2} + \frac{\Delta\left(\frac{40.3}{f_2^2} \cdot TEC\right)}{\lambda_2} + \frac{\Delta(MP_2(t))}{\lambda_2} + \frac{\Delta(\eta_2(t))}{\lambda_2}$$
(E.15b)

A summation of equations E.15 yields:

$$\left(\frac{\Delta\tilde{\rho}_{1}}{\lambda_{1}} + \frac{\Delta\tilde{\rho}_{2}}{\lambda_{2}}\right) = \left(\frac{1}{\lambda_{1}} + \frac{1}{\lambda_{2}}\right)\Delta r + \Delta\left(\frac{40.3}{f_{2}^{2}} \cdot TEC\right) \cdot \left(\frac{f_{2} + f_{1}}{c}\right) + \left(\frac{1}{\lambda_{1}}\right)\left(\Delta(MP_{1}(t)) + \Delta(\eta_{1}(t))\right) + \left(\frac{1}{\lambda_{2}}\right)\left(\Delta(MP_{2}(t)) + \Delta(\eta_{2}(t))\right)$$

$$(E.16a)$$

While the difference of equations E.14 yields:

$$\left(\frac{\Delta\tilde{\rho}_{1}}{\lambda_{1}} - \frac{\Delta\tilde{\rho}_{2}}{\lambda_{2}}\right) = \left(\frac{1}{\lambda_{1}} - \frac{1}{\lambda_{2}}\right)\Delta r + \Delta\left(\frac{40.3}{f_{2}^{2}} \cdot TEC\right) \cdot \left(\frac{f_{2} - f_{1}}{c}\right) + \left(\frac{1}{\lambda_{1}}\right)\left(\Delta(MP_{1}(t)) + \Delta(\eta_{1}(t))\right) - \left(\frac{1}{\lambda_{2}}\right)\left(\Delta(MP_{2}(t)) + \Delta(\eta_{2}(t))\right) + \Delta(\eta_{2}(t))\right)$$
(E.16b)

Re-arranging equations E.16 respectively:

$$\left(\frac{\Delta\tilde{\rho}_{1}}{\lambda_{1}} + \frac{\Delta\tilde{\rho}_{2}}{\lambda_{2}}\right) \cdot \lambda_{n} = \Delta r + \Delta \left(\frac{40.3}{f_{2}^{2}} \cdot TEC\right) + \left(\frac{\lambda_{n}}{\lambda_{1}}\right) (\Delta(MP_{1}(t)) + \Delta(\eta_{1}(t))) + \left(\frac{\lambda_{n}}{\lambda_{2}}\right) (\Delta(MP_{2}(t)) + \Delta(\eta_{2}(t)))$$
(E.17a)

$$\left(\frac{\Delta\tilde{\rho}_{1}}{\lambda_{1}} + \frac{\Delta\tilde{\rho}_{2}}{\lambda_{2}}\right) \cdot \lambda_{w} = \Delta r - \Delta \left(\frac{40.3}{f_{2}^{2}} \cdot TEC\right) + \left(\frac{\lambda_{w}}{\lambda_{1}}\right) (\Delta(MP_{1}(t)) + \Delta(\eta_{1}(t))) - \left(\frac{\lambda_{w}}{\lambda_{2}}\right) (\Delta(MP_{2}(t)) + \Delta(\eta_{2}(t)))$$
(E.17b)

Revisiting equation E.14b:

$$\left(\Delta \widetilde{\phi}_{1} - \Delta \widetilde{\phi}_{2} \right) \cdot \lambda_{w} = \Delta r + \Delta \left(\frac{40.3 \cdot TEC}{f_{1} f_{2}} \right) - \left(\Delta N_{1} - \Delta N_{2} \right) \cdot \lambda_{w}$$

$$+ \left(\frac{\lambda_{w}}{\lambda_{1}} \right) \left(\Delta (mp_{1}(t)) + \Delta (\zeta_{1}(t)) \right) - \left(\frac{\lambda_{w}}{\lambda_{2}} \right) \left(\Delta (mp_{2}(t)) + \Delta (\zeta_{2}(t)) \right)$$

$$(E.14b)$$

The term $(\Delta N_1 - \Delta N_2)$ in equation E.14b can be further reduced to $\Delta N_1 - \Delta N_2 = (N_1 - N_0) - (N_2 - N_0) = N_1 - N_2.$

The right-hand sides of equation E.17a and equation E.14b are comparable. Hence, the difference between the two equations will result in:

$$\left(\frac{\Delta\tilde{\rho}_{1}}{\lambda_{1}} + \frac{\Delta\tilde{\rho}_{2}}{\lambda_{2}}\right) \cdot \lambda_{n} - \left(\Delta\tilde{\phi}_{1} - \Delta\tilde{\phi}_{2}\right) \cdot \lambda_{w} = (N_{1} - N_{2}) \cdot \lambda_{w} \\
+ \left(\frac{\lambda_{n}}{\lambda_{1}}\right) \left(\Delta(MP_{1}(t)) + \Delta(\eta_{1}(t))\right) + \left(\frac{\lambda_{n}}{\lambda_{2}}\right) \left(\Delta(MP_{2}(t)) + \Delta(\eta_{2}(t))\right) \\
+ \left(\frac{\lambda_{w}}{\lambda_{1}}\right) \left(\Delta(mp_{1}(t)) + \Delta(\zeta_{1}(t))\right) - \left(\frac{\lambda_{w}}{\lambda_{2}}\right) \left(\Delta(mp_{2}(t)) + \Delta(\zeta_{2}(t))\right)$$
(E.18)

From equation E.18, the estimation of the wide-lane integer, $(N_1 - N_2)$ can be done. The correct integer phase ambiguity is expected to be within three integers of the estimate. For an accurate integer ambiguity resolution, an integer search is required.

E.2 Ambiguity Resolution

After the initial estimate of the ambiguity, the next stage is the integer search for the correct ambiguity. A stepwise algorithm commences with ambiguity convergence of the combination with the longest wavelength, low ionosphere content and low noise. Once resolved, the ambiguities in combinations with shorter wavelengths may be estimated with greater reliability. Such an algorithm has been proposed for three-

frequency relative positioning (Hatch 1996). The concept of stepwise ambiguity resolution has facilitated simultaneous code and carrier update (CCU). Hwang (1991) introduced the concept of carrier phase riding (CPR), as one is able to update the integer ambiguity provided it is initial known, given incremental measurements or rate of change of the relative carrier phase. Teunissen (1994) has also presented a method for ambiguity resolution based on transforming and reparameterising the integer ambiguity. Forsell, Martin-Neira and Harris (1997) have proposed a method that uses the measurements at all three carrier frequencies. Henderson, Raquet and Maybeck (2002) have presented a multi-filter approach to ambiguity resolution.

This work uses the technique presented by Yang, Hatch and Sharpe (2002) based upon the concept of residual sensitivity matrix proposed by Hatch and Sharpe (2001), which relates the search integer ambiguity set to each carrier phase residual directly. The technique uses the singular value decomposition of the residual sensitivity matrix to find the minimum search space. This technique not only improves the calculation efficiency and ambiguity resolution time, but also improves the reliability. The search space is minimised by selecting only those combinations of possible ambiguity values which are consistent with the satellite geometry and the measurement residuals.

Equation E.1b terms can be collected and re-written as:

$$\Delta \phi \cdot \lambda = \begin{bmatrix} \mathbf{h} & \lambda \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \Delta N \end{bmatrix} + n_{\phi}$$
(E.19)

where $\Delta \phi$ is the differential carrier phase, **h** is the vector between the receiver antenna and the satellites, $\Delta \mathbf{x}$ is the linearised position, ΔN is the differential integer ambiguity and n_{ϕ} is the total differential phase noise, the sum of the multipath error $(mp - mp_0)$ and the carrier phase noise $(\xi - \xi_0)$. The resolution of integer ambiguity N can be accomplished by solving the integer ambiguity with special search and hypothesis testing techniques and validating the result to ensure the integer ambiguity solution is unique and correct. Equation E.19 can be re-written as:

$$(\Delta\phi + \Delta N)\lambda = \mathbf{h}\mathbf{x} + n_{\phi} \tag{E.20}$$

For n number of satellites in view, all the measurements can be written in array format as:

$$(\Delta \Phi + \Delta \mathbf{N})\lambda = \mathbf{H}\mathbf{x} + \mathbf{n}_{\phi} \tag{E.21}$$

where $\Delta \Phi = [\Delta \phi_1, \Delta \phi_2, \dots, \Delta \phi_n]^T$ is the differential carrier phase measurement vector formed by each satellite, $\Delta \mathbf{N} = [\Delta N_1, \Delta N_2, \dots, \Delta N_n]^T$ is the differential integer ambiguity vector formed by each satellite, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_n]^T$ is the measurement vector matrix from user to satellites with \mathbf{h}_i being the \mathbf{h} of the *i*th satellite and $\mathbf{n}_{\varphi} = [n_{\phi_1}, n_{\phi_2}, \dots, n_{\phi_n}]^T$ is the carrier phase measurement noise vector formed by each satellite.

The calculated initial ambiguity \hat{N}_0 can be estimated by rounding off using either the pseudorange or carrier phase smoothed pseudorange. Assuming the search width of each satellite as δN , the total candidate set number is δN^{n-1} with *n* being the number of satellites used. As an example, if $\delta N = 4$ and n = 7, the total number of search is $4^6 = 4096$ candidate set. For each candidate set, the real set is:

$$\hat{\mathbf{x}} = \left[\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\right]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \left(\Delta \boldsymbol{\Phi} + \hat{\mathbf{N}}_0 + \partial \mathbf{N}\right) \boldsymbol{\lambda}$$
(E.22)

where $\mathbf{R} = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{bmatrix}$ is the measurement covariance matrix formed by the

differential carrier phase noise, σ_i is the standard deviation of satellite *i* differential carrier phase noise, $\partial \mathbf{N} = [\partial N_1, \partial N_2, \dots, \partial N_n]^T$ is the integer ambiguity vector formed from search width ∂N for each satellite.

The residual sensitivity matrix, S, for a weighted least square can be described as in Hatch and Sharpe (2001):

$\mathbf{S} = \mathbf{I} - \left[\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}\right]^{-1} \mathbf{H}^T \mathbf{R}^{-1}$

where I is an identity matrix. The S matrix has the following properties,

- i. Symmetric
- ii. Zero sum of each row and column
- iii. Positive semidefinite
- iv. Equal idempotent: $\mathbf{S} = \mathbf{S}^2 = \mathbf{S}^3 = \dots$
- v. Rank equal to n-k: $rank(\mathbf{S}) = n-k$ (with k = 4 for single differential GPS and k = 3 for double differential GPS)
- vi. SVD calculation efficiency of **S** matrix: for SVD of $\mathbf{S} = \mathbf{U}\mathbf{X}\mathbf{V}^T$, one of the solution of **V** is equal to the eigenvector of **S**. The eigenvalue of **S** is either 1 or 0, and its eigenvectors are all real.

The calculated phase range residual vector is:

$$\Delta_{\Phi} = \left(\Delta \Phi + \hat{\mathbf{N}}_{0} + \delta \mathbf{N}\right) \lambda - \mathbf{H} \hat{\mathbf{x}}$$
(E.24)
$$= \left(\mathbf{I} - \mathbf{H} \left[\mathbf{H}^{T} \mathbf{R}^{-1} \mathbf{H}\right]^{-1} \mathbf{H}^{T} \mathbf{R}^{-1} \right) \left(\Delta \Phi + \hat{\mathbf{N}}_{0} + \delta \mathbf{N}\right) \lambda$$
(E.25)
$$= \mathbf{S} \left(\Delta \Phi + \hat{\mathbf{N}}_{0} + \delta \mathbf{N}\right) \lambda$$
(E.25)

The estimated phase standard deviation for candidate set \hat{N} is:

$$\sigma_{\Phi|\hat{\mathbf{N}}} = \sqrt{\frac{\Delta_{\Phi}^{T} \Delta_{\Phi}}{n-k}} \tag{E.26}$$

(E.23)

with k being the real state number of **x** (k = 4 for single differential GPS and k = 3 for double differential GPS). The ambiguity search target is to find the unique and correct candidate set with smallest $\sigma_{\Phi|\hat{N}}$. Since Δ_{Φ} is a vector, minimising $\sigma_{\Phi|\hat{N}}$ is equal to minimise the absolute value of each term of Δ_{Φ} .

The initial phase range residual vector can be defined as:

$$\Delta_{\Phi_0} = \mathbf{S} \left(\Delta \Phi + \hat{\mathbf{N}}_0 \right) \lambda \tag{E.27}$$

minimising the absolute value of Δ_{ϕ} in equation E.25 is equal to estimate ∂N that

$$\Delta_{\Phi_0} + \mathbf{S}\delta\mathbf{N}\lambda = 0 \Longrightarrow \mathbf{S}\delta\mathbf{N} = -\Delta_{\Phi_0}\frac{1}{\lambda} = -\mathbf{r}_0$$
(E.28)

with \mathbf{r}_0 being the initial phase range residual vector in unit of cycle.

Since **S** is not full rank, **S** can be re-written using Singular Value Decomposition (SVD) as:

$$\mathbf{S} = \mathbf{U}\mathbf{X}\mathbf{V}^{T} \tag{E.29}$$

where

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \end{bmatrix}$$
(E.30)

with

$$\mathbf{u}_i^T \mathbf{u}_i = 1, \quad \mathbf{u}_i^T \mathbf{u}_j = 0 \ (i \neq j)$$

with \mathbf{u}_i are orthogonal vectors and \mathbf{U} having full rank n:

$$\mathbf{X} = \begin{bmatrix} s_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \mathbf{0}_{(n-k)\times k} \\ \mathbf{0} & \cdots & s_{n-k} \\ & \mathbf{0}_{k\times(n-k)} & & \mathbf{0}_{k\times k} \end{bmatrix}$$
(E.31)

with $\mathbf{s}_1 = \cdots = \mathbf{s}_{n-k} = 1$ for matrix **S**; and

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix}$$
(E.32)

with

$$\mathbf{v}_i^T \mathbf{v}_i = 1 \qquad \mathbf{v}_i^T \mathbf{v}_j = 0 \ (i \neq j)$$

with \mathbf{v}_i are orthogonal vectors and \mathbf{V} having full rank n.

Now, estimate **S**, so that $\mathbf{S}\delta\mathbf{N} = -\mathbf{r}_0$, which leads to:

$$\mathbf{U}\mathbf{X}\mathbf{V}^{\mathrm{T}}\boldsymbol{\delta}\mathbf{N} = -\mathbf{r}_{0}$$

$$\Leftrightarrow \mathbf{X}\mathbf{V}^{\mathrm{T}}\boldsymbol{\delta}\mathbf{N} = -\mathbf{U}'\mathbf{r}_{0} = \mathbf{r}_{1}$$

$$\Leftrightarrow \begin{bmatrix} s_{1}\mathbf{v}_{1}^{\mathrm{T}} \\ \vdots \\ s_{n-k}\mathbf{v}_{n-k}^{\mathrm{T}} \\ \mathbf{0}_{k\times n} \end{bmatrix} \mathbf{\delta}\mathbf{N} = \mathbf{r}_{11}$$
(E.33)

Equation E.33 can be re-written as:

$$\begin{bmatrix} \mathbf{A}_{1} & \mathbf{A}_{2} \\ \mathbf{0}_{k\times(n-k)} & \mathbf{0}_{k\times k} \end{bmatrix} \begin{bmatrix} \mathbf{N}_{1f} \\ \mathbf{N}_{2f} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_{11} \\ \mathbf{r}_{12} \end{bmatrix}$$
(E.34)
$$\Leftrightarrow \mathbf{A}_{1} \mathbf{N}_{1f} + \mathbf{A}_{2} \mathbf{N}_{2f} = \mathbf{r}_{11}$$

$$\Leftrightarrow \mathbf{N}_{1f} = \mathbf{A}_{1}^{-1} (\mathbf{r}_{11} - \mathbf{A}_{2} \mathbf{N}_{2f})$$
(E.35)
$$\Leftrightarrow \mathbf{N}_{1} = round (\mathbf{A}_{1}^{-1} (\mathbf{r}_{11} - \mathbf{A}_{2} \mathbf{N}_{2f}))$$

$$\Leftrightarrow \mathbf{N}_{1} = round (\mathbf{C} - \mathbf{D} \mathbf{N}_{2})$$
(E.36)
with $\mathbf{r}_{12} = 0$, $\mathbf{C} = \mathbf{A}_{1}^{-1} \mathbf{r}_{11}$, and $\mathbf{D} = \mathbf{A}_{1}^{-1} \mathbf{A}_{2}$.

Equation E.36 relates two integer ambiguity subsets, which will reduce the search space from searching around n satellites for both N_1 and N_2 to searching around k satellites for N_2 only.

After calculating the integers N_1 and N_2 , substituting N_1 and N_2 into equation E.34 gives:

$$\mathbf{r} = \mathbf{r}_{11} - \left(\mathbf{A}_1 \mathbf{N}_1 + \mathbf{A}_2 \mathbf{N}_2\right) \tag{E.37}$$

which is the residual corresponding to integer set \mathbf{N}_1 and \mathbf{N}_2 .

Further calculation for improvement of this technique is referred to by Yang, Hatch and Sharpe (2002). Instead of using SVD as introduced in equation E.29, a better way to implement the space search reduction for S is by manipulating the properties of S.

APPENDIX F: KALMAN FILTER VARIANTS

This appendix presents the standard Kalman filter algorithms and its two commonly known variants, namely the extended Kalman filter and unscented Kalman filter.

F.1 Standard Kalman Filter

The standard Kalman filter (KF) is a linear optimal recursive predictor-corrector. A discrete–time Kalman filter as presented by Grewal and Andrews (2001) is presented here.

System dynamic model:

$$\mathbf{x}_{k} = \mathbf{\Phi}_{k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{F.1}$$

with $\mathbf{w}_k \sim N(0, \mathbf{Q}_k)$.

Measurement model:

$$\mathbf{z}_{k} = \mathbf{H}_{k}\mathbf{x}_{k} + \mathbf{v}_{k} \tag{F.2}$$

with $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$.

Initial conditions:

$$\mathbf{E}\langle \mathbf{x}_0 \rangle = \hat{\mathbf{x}}_0 \tag{F.3}$$

$$\mathbf{E} \left\langle \widetilde{\mathbf{x}}_{0} \widetilde{\mathbf{x}}_{0}^{T} \right\rangle = \mathbf{P}_{0} \tag{F.4}$$

Independence assumption:

$$\mathbf{E} \left\langle \mathbf{w}_{k} \mathbf{v}_{j}^{T} \right\rangle = 0 \qquad \text{for all values of } k \text{ and } j. \qquad (F.5)$$

State estimation extrapolation:

$$\hat{\mathbf{x}}_{k}(-) = \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_{k-1}(+) \tag{F.6}$$

Error covariance extrapolation:

$$\mathbf{P}_{k}\left(-\right) = \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1}\left(+\right)\mathbf{\Phi}_{k-1}^{T} + \mathbf{Q}_{k-1}$$
(F.7)

State estimate observational update:

$$\hat{\mathbf{x}}_{k}(+) = \hat{\mathbf{x}}_{k}(-) + \overline{\mathbf{K}}_{k}[\mathbf{z}_{k} - \mathbf{H}_{k}\hat{\mathbf{x}}(-)]$$
(F.8)

Error covariance update:

$$\mathbf{P}_{k}(+) = \left[\mathbf{I} - \overline{\mathbf{K}}_{k} \mathbf{H}_{k}\right] \mathbf{P}_{k}(-) \tag{F.9}$$

Kalman gain matrix:

$$\overline{\mathbf{K}}_{k} = \mathbf{P}_{k} \left(-\right) \mathbf{H}_{k}^{T} \left[\mathbf{H}_{k} \mathbf{P}_{k} \left(-\right) \mathbf{H}_{k}^{T} + \mathbf{R}_{k} \right]^{-1}$$
(F.10)

The basic steps of computational procedure for the discrete-time Kalman estimator:

- 1. Compute $\mathbf{P}_{k}(-)$ using $\mathbf{P}_{k-1}(+)$, $\mathbf{\Phi}_{k-1}$, and \mathbf{Q}_{k-1} .
- 2. Compute $\overline{\mathbf{K}}_k$ using $\mathbf{P}_k(-)$ (computed in step 1), \mathbf{H}_k and \mathbf{R}_k .
- 3. Compute $\mathbf{P}_k(+)$ using $\overline{\mathbf{K}}_k$ (computed in step 2) and $\mathbf{P}_k(-)$ (from step 1).
- 4. Compute successive values of $\hat{\mathbf{x}}_k(+)$ recursively using the computed values of $\overline{\mathbf{K}}_k$ (from step 3), given the initial estimate $\hat{\mathbf{x}}_0$ and the input data \mathbf{z}_k .

F.2 Extended Kalman Filter

The extended Kalman filter (EKF) is a nonlinear optimal recursive predictorcorrector variant to the linear Kalman filter. A discrete–time EKF as presented by Grewal and Andrews (2001) is presented here:

Nonlinear dynamic model:

$$\mathbf{x}_{k} = f_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1}$$
(F.11)

with $\mathbf{w}_k \sim N(0, \mathbf{Q}_k)$.

Measurement model:

$$\mathbf{z}_{k} = h_{k}(\mathbf{x}_{k}) + \mathbf{v}_{k} \tag{F.12}$$

with $\mathbf{v}_k \sim N(0, \mathbf{R}_k)$.

Nonlinear implementation equations:

1. Computing the predicted state estimate:

$$\hat{\mathbf{x}}_{k}(-) = f_{k-1}(\hat{\mathbf{x}}_{k-1}(+))$$
(F.13)
2. Computing the predicted measurement:

$$\mathbf{z}_{k} = h_{k}(\hat{\mathbf{x}}_{k}(-)) \tag{F.14}$$

Linear approximation equations:

$$\Phi_{k-1}^{[1]} \approx \frac{\partial f_k}{\partial x}\Big|_{x=\hat{x}_k(-)}$$
(F.15)

Conditioning the predicted estimate on the measurement:

$$\hat{\mathbf{x}}_{k}(+) = \hat{\mathbf{x}}_{k}(-) + \overline{\mathbf{K}}_{k}(\mathbf{z}_{k} - \hat{\mathbf{z}}_{k})$$
(F.16)

$$\mathbf{H}_{k-1}^{[1]} \approx \frac{\partial h_k}{\partial x} \bigg|_{x=\hat{x}_k(-)}$$
(F.17)

The next three equations are similar to equations F.7, F.9 and F.10.

Computing the *a priori* covariance matrix:

$$\mathbf{P}_{k}(-) = \mathbf{\Phi}_{k-1}^{[1]} \mathbf{P}_{k-1}(+) \mathbf{\Phi}_{k-1}^{[1]T} + \mathbf{Q}_{k-1}$$
(F.18)

Computing the Kalman gain:

$$\overline{\mathbf{K}}_{k} = \mathbf{P}_{k} \left(-\right) \mathbf{H}_{k}^{[1]T} \left[\mathbf{H}_{k}^{[1]} \mathbf{P}_{k} \left(-\right) \mathbf{H}_{k}^{[1]T} + \mathbf{R}_{k} \right]^{-1}$$
(F.19)

Computing the *a posteriori* covariance matrix:

$$\mathbf{P}_{k}(+) = \left\{ \mathbf{I} - \overline{\mathbf{K}}_{k} \mathbf{H}_{k}^{[1]} \right\} \mathbf{P}_{k}(-)$$
(F.20)

F.3 Unscented Kalman Filter

Julier and Uhlmann (2004) propose and develop the unscented Kalman filter (UKF), while realising the limitations of EKF, which has been shown to be difficult to implement, difficult to tune, and only reliable for systems that are almost linear on the time scale of the updates. Julier and Uhlmann (2004) summarise the general formulation of the unscented Kalman filter, which uses the unscented transformation described as follows.

Firstly, the set of sigma points is created by applying a sigma point selection algorithm. Then, the transformed set is given by instantiating each point through the process model:

$$\hat{\mathbf{x}}_{a,n}^{(i)} = \mathbf{f} \left[\mathbf{x}_{a,n}^{(i)}, \ \mathbf{u}_n \right]$$
(F.21)

The predicted mean is computed as:

$$\hat{\boldsymbol{\mu}}_{a,n} = \sum_{i=0}^{p} \mathbf{W}^{(i)} \hat{\mathbf{x}}_{a,n}^{(i)}$$
(F.22)

The predicted covariance is computed as:

$$\hat{\mathbf{K}}_{a,n} = \sum_{i=0}^{p} \mathbf{W}^{(i)} \{ \hat{\mathbf{x}}_{a,n}^{(i)} - \boldsymbol{\mu}_{a,n} \} \{ \hat{\mathbf{x}}_{a,n}^{(i)} - \boldsymbol{\mu}_{a,n} \}^{T}$$
(F.23)

Instantiate each of the prediction points through the observation model:

$$\hat{\mathbf{y}}_{n}^{(i)} = \mathbf{g} \left[\mathbf{x}_{a,n}^{(i)}, \quad \mathbf{u}_{n} \right]$$
(F.24)

The predicted observation is calculated by:

$$\hat{\mathbf{y}}_n = \sum_{i=0}^p \mathbf{W}^{(i)} \hat{\mathbf{y}}_n^{(i)}$$
(F.25)

The innovation covariance is:

$$\hat{\mathbf{S}}_{n} = \sum_{i=0}^{p} \mathbf{W}^{(i)} \{ \hat{\mathbf{y}}_{n}^{(i)} - \hat{\mathbf{y}}_{n} \} \{ \hat{\mathbf{y}}_{n}^{(i)} - \hat{\mathbf{y}}_{n} \}^{t}$$
(F.26)

The cross covariance matrix is determined by:

$$\hat{\mathbf{K}}_{n}^{xy} = \sum_{i=0}^{p} \mathbf{W}^{(i)} \{ \hat{\mathbf{x}}_{n}^{(i)} - \hat{\boldsymbol{\mu}}_{n} \} \{ \hat{\mathbf{x}}_{n}^{(i)} - \hat{\boldsymbol{\mu}}_{n} \}^{T}$$
(F.27)

Finally, the update can be performed using the normal Kalman filter equations:

$$\boldsymbol{\mu}_n = \hat{\boldsymbol{\mu}}_n + \mathbf{W}_n \boldsymbol{\nu}_n \tag{F.28a}$$

$$\mathbf{K}_{n} = \hat{\mathbf{K}}_{n} - \mathbf{W}_{n} \hat{\mathbf{S}}_{n} \mathbf{W}_{n}^{T}$$
(F.28b)

$$\boldsymbol{\nu}_n = \mathbf{y}_n - \hat{\mathbf{y}}_n \tag{F.28c}$$

$$\mathbf{W}_n = \hat{\mathbf{K}}_n^{xy} \mathbf{S}_n^{-1} \tag{F.28d}$$

APPENDIX G: THE EARTH'S GRAVITATIONAL POTTENTIAL

The full gravitational potential function is expressed as follows:

$$U(r,\phi,\lambda) = \frac{\mu}{r} \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R}{r}\right)^{n} P_{nm}(\sin\phi) \left[C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\right] \right]$$
(G.1)

where:

- r Distance from geo center to object
- ϕ Geocentric latitude of object
- λ Longitude of object
- *R* Equatorial radius magnitude of Earth.

When the order *n* equals 0 (m = 0), the coefficients are referred to as **zonal** harmonics. When n = m, the coefficients are referred to as **sectorial**. **Sectorial** harmonics account for the gravitational field variation in longitude. The coefficients are referred to as **tesseral** harmonics when $m \neq n \neq 0$.

The gravitational potential field originating from a satellite's planetary host is the source of the main external force affecting an orbiting satellite. The Earth is not uniformly spherical, as it is an oblate spheroid, nor is the mass distribution homogeneous and uniform. The associated non-spherical gravitational field may be expressed in terms of Legendre and associated Legendre polynomials, the distance r of the satellite from the Earth's centre, the longitude L of the satellite, measured from the Greenwich meridian and positive eastward, the latitude l of the satellite taken to be positive towards north and the equatorial Earth radius, $R_e = 6378.16$ km as (Deutsch 1963),

$$U = \frac{\mu}{r} - \frac{\mu}{r} \left[\sum_{n=2}^{\infty} \left(\frac{R_e}{r} \right)^n J_n \times P_n(\sin l) - \sum_{n=2}^{\infty} \sum_{m=1}^{\infty} \left(\frac{R_e}{r} \right)^n J_{nm} \times P_{nm}(\sin l) \cos(m(L - L_{nm})) \right]$$
(G.2)

where,

$$P_n(x) = \frac{1}{2^n!} \times \frac{d^n}{dx^n} (x^2 - 1)^n, \ P_{nm}(x) = (1 - x^2)^{\frac{m}{2}} \times \frac{d^m}{dx^m} P_n(x).$$

 J_n , J_{nm} and L_{mn} are constants characterising the Earths mass distribution. J_n are the **zonal** harmonics related to the Earth's oblateness and the later constants are associated with the **tesseral harmonics** related to the ellipticity of the equator which results in a 150m difference between the Earth's major and minor axes.

The principal perturbation to orbital elements, in the GPS orbit is due to the Earth's flattening given by J_2 in the expression for the total potential energy possessed by a satellite by virtue of the Earth's gravitational field. The effect of the J_2 perturbation can be computed from the Lagrange planetary equations. For the Earth, $J_2 = 1.08284 \times 10^{-3}$, $J_3 = -2.56 \times 10^{-6}$, $J_4 = -1.58 \times 10^{-6}$ and J_2 is at least about 1000 times larger, in magnitude, than all other J_n , for n > 4. For the Earth, $J_{21} = 0$, $J_{22} = 1.86 \times 10^{-6}$, $J_{31} = 2.1061 \times 10^{-6}$ and all other J_{nm} are at most equal to 1.0×10^{-6} .

The Earth's gravitational potential may also be expressed as,

$$U = \frac{\mu}{r} - \frac{\mu}{r} \left(\frac{R_e}{r}\right)^2 \left[J_2 \left(1 - \frac{3}{2} \cos^2 \delta \right) - 3J_{22} \cos^2 \delta \cos(2(L - L_{22})) \right]$$
(G.3)

with $L_{22} = -15^{\circ}$. As a geostationary satellite orbits the Earth along the equator, the latitude, δ , from the equatorial plane is always zero and since $R_e/r \ll 1$, the Earth's gravitational potential may be simplified.

REFERENCES

1. Abdullah, M., Awang-Mat, D.A., Mohd-Zain, A.F., Abdullah, S. and Nik-Zulkifli, S.S. (2007). Analysis of ionospheric models during ionospheric disturbances. **In**: *Proceedings of Asia-Pacific Conference on Applied Electromagnetics 2007*, Melaka, Malaysia. Dec. 4-6, 2007, pp. 1-4.

2. Adam, A., Rivlin, E., and Rotstein, H. (1999). Fusion of fixation and odometry for vehicle navigation. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, **29** (6), pp. 593-603.

3. Alkan, R.M., Karaman, H. and Sahin, M. (2005). GPS, GALILEO and GLONASS satellite navigation systems & GPS modernization. **In**: *Proceedings of 2nd International Conference on Recent Advances in Space Technologies, 2005. RAST 2005*, June 9-11, 2005, pp. 390- 394,

4. Blanchet, I., Frankignoul C. and Cane, M. A., (1997). A Comparison of adaptive Kalman filters for a tropical Pacific Ocean model. *Monthly Weather Review, American Meteorological Society*, **125** (1), pp. 40-58.

5. Brouwer, D. (1959). Solution of the problem of artificial satellite theory without drag. *Astronomical Journal*, **64**, pp. 378-397.

6. Brown, R. B. and Hwang, P. Y. C. (1997). *Introduction to Random Signals and Applied Kalman Filtering*. 3rd edition. John Wiley and Sons Inc. (0471128392).

7. Bye, C.T., Hartmann, G.L., Killen, A. (1998). Development of a FOG-based GPS/INS.
In: *IEEE Position Location and Navigation Symposium*, April 20-23, 1998, Palm Springs, CA, USA, pp. 264-271.

8. Chen, J., Lee, S. and DeBra, D. (1994). Gyroscope free strapdown inertial measurement unit by six linear accelerometers. *Journal of Guidance, Control and Dynamics*. **17** (2), pp. 286-290.

9. Coïsson, P., Radicella, S.M., Leitinger, R. and Nava, B. (2006). Topside density in IRI and NeQuick: features and limitations. *Advances in Space Research.* **37**, pp. 937-942.

10. Coïsson, P., Radicella, S.M., and Nava, B. (2002). Comparison of experimental topside electron concentration profiles with IRI and NEQuick models. *Annals of Geophysics*. **41** February, pp.111-116.

11. Deutsch, R. (1963). Orbital Dynamics of Space Vehicles, Chapter 8. Prentice Hall.

12. Di Giovanni, G. and Radicella, S.M. (1990). An analytical model of the electron density profile in the ionosphere. *Advances in Space Research*. **10** (11), pp. 27-30.

13. Easton, R.L. (1980). The navigation technology program. *Institute of Navigation*, Vol. I, Washington D.C., pp. 15-20.

14. Eck, C. and Geering, H.P. (2000). Error dynamics of model based INS/GPS navigation for an autonomously flying helicopter. **In**: *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Denver, CO, USA, August 2000, pp. 1-9.

15. Farrel, J. and Barth, M. (1999). *The Global Positioning System and Inertial Navigation: Theory and Practice*. New York: McGraw-Hill. (007022045X).

16. Farrel, J. and Givargis, T. (1999). Experimental differential GPS reference station evaluation. **In**: *Proceeding of American Control Conference*, San Diego, California, June 1999, pp. 3645–3649.

17. Farrel, J. and Givargis, T. (2000). Differential GPS reference station algorithm – design and analysis. *IEEE Transactions on Control Systems Technology*. **8** (3) May, pp. 519-531.

18. Farrell, J., Givargis, T. and Barth, M. (2000). Real-time differential carrier phase GPSaided INS. *IEEE Transactions on Control Systems Technology*. **8** (4), pp. 709-721.

19. Filipski, M.N. and Varatharajoo, R. (2006). Satellite orbit estimation using earth magnetic field measurements. *International Journal of Engineering and Technology*. **3** (2), pp. 263-271.

20. Fontana, R.D., Cheung, W. and Stansell, T. (2001). The modernised L2 civil signal. *GPS World*, **12**(9), pp. 28-34.

21. Forssell, B. (1995). Signal design for GNSS. In: *The 4th International Conference on Differential Satellite Navigation Systems, Proceedings of DSNS 95.* Bergen, Norway, April 1995.

22. Forssell, B. (1997). Comparison of wide-laning and tone-ranging. *Electronics Letters*.33 (18) 29 August, pp.1525-1526.

23. Forssell B., Martin-Neira M. and Harris R.A. (1997). Carrier phase ambiguity resolution in GNSS-2. **In**: *10th International Technical Meeting of the Satellite Division of the Institute of Navigation, Proceedings of the ION-GPS 1997*, Kansas City Convention Center, Kansas City, Missouri, USA, Sept 16-19, 1997, pp.1727-1736.

24. Fortescue, P., Stark, J. and Swinerd, G. (2003). *Spacecraft Systems Engineering*. 3rd Edition. West Sussex, England: John Wiley & Sons Ltd. (0470851023).

25. Grewal, M.S. and Andrews, A.P. (2001). *Kalman filtering: theory and practice using MATLAB*[®]. 2nd edition. New York, USA : John Wiley & Sons, Inc.(0471392545)

26. Grejner-Brzezinska, D.A. and Wang, J. (1998). Gravity modeling for high-accuracy GPS/INS integration. *Navigation*. **45** (3), pp.209-220.

27. Hafskjold, B.H., Jalving, B., Hagen, P.E., and Gade, K. (2000). Integrated camerabased navigation. *Journal of Navigation*. **52** (2), pp. 237-243.

28. Hatch, R. (1982). The synergism of GPS code and carrier measurements. In: *Proceedings of the Third International Geodetic Symposium on Satellite Doppler Positioning*, Vol. II. Las Cruces, New Mexico, USA. Magnavox Technical Paper MX-TM 3353-82. pp. 1213–1232.

29. Hatch, R.R. (1996, May). The promise of a third frequency. *GPS World*. **7**(5) May, pp. 55-58.

30. Hatch, R.R. (2000). GPS carrier-phase ambiguity resolution. **In**: *IMA "Hot Topics" Workshop: Mathematical Challenges in Global Positioning (GPS)*. August 16-18, 2000. Retrieved: Tuesday, 2 June 2009. Website: http://silver.ima.umn.edu/talks/workshops/8-16-18.2000/ Downloaded from: http://silver.ima.umn.edu/talks/workshops/8-16-18.2000/hatch/hatch.pdf : Institute of Mathematics and Its Applications, University of Minnesota, USA.

31. Hatch, R. and Sharpe, T. (2001). A computationally efficient ambiguity resolution technique. **In**: *14th International Technical Meeting of the Satellite Division of the Institute of Navigation, ION GPS-2001*, Salt Lake City, Utah, USA, September 11-14, 2001.

32. Hein, G. (2006). GNSS interoperability: achieving a global system of systems or "Does Everything Have to Be the Same?" **In**: *Working Papers, Inside GNSS, Gibbons Media & Research LLC*. January/February 2006 – Premiere Issue. [Online]. Available at: http://www.insidegnss.com/auto/0106_Working_Papers_IGM.pdf (Accessed: 18 December 2008)

33. Henderson, P., Raquet, J. and Maybeck, P. (2002). A multiple filter approach for precise kinematic DGPS positioning and carrier-phase ambiguity resolution. *Journal of The Institute of Navigation*, **49**, pp. 149–160.

34. Henry, M.F. (2005). NORAD C++ SGP4/SDP4 implementations (SxP4Test.zip).Retrieved/Downloaded:Tuesday,30May2006.Website:http://www.zeptomoby.com/satellites/index.htm.Downloadedfrom:http://www.zeptomoby.com/satellites/index.htm.Downloadedfrom:

35. Hilton, C.G. and Kuhlman, J.R. (1966). *Mathematical Models for the Space Defence Center*. Philco-Ford Corp., Publ. U-3871, Newport Beach, California, November, pp. 17-28.

36. Hochegger, G., Nava, B., Radicella, S.M. and Leitinger, R. (2000). A family of ionospheric models for different uses. *Physic and Chemistry of the Earth, Part C: Solar, Terrestrial & Planetary Science*. **25** (4), pp. 307-310.

37. Hoots, F.R. (1980). A short, efficient analytical satellite theory. *AIAA Paper* No. 80-1659, August 1980.
38. Hoots, F.R. and Roehrich, R.L. (1980). Models for propagation of NORAD element sets. *SPACETRACK Report No. 3*. U.S. Air Force Aerospace Defense Command, Colorado Springs, Colorado, December 1980. (Package Compiled by Kelso, T.S., 31 December 1988).

39. Hoots, F.R., Schumacher Jr., P.W. and Glover, R.A. (2004). History of analytical orbit modelling in the U.S. space surveillance system. *Journal of Guidance, Control and Dynamics*. **27** (2), March-April, pp. 174-185.

40. Hujsak, R.S. (1979). A restricted four body solution for resonating satellites without drag. *Project SPACETRACK Rept.* 1, U.S. Air Force Aerospace Defence Command, Colorado Springs, Colorado, November 1979.

41. Hwang, P.Y.C. (1991). Kinematic GPS for differential positioning: resolving integer ambiguity on the fly. *Journal of the Institute of Navigation*, **38** (1), Spring-1991.

42. Hwang, P.Y., McGraw, G. and Bader, J. (1998). Enhanced differential GPS carriersmoothed code processing using dual frequency measurements. **In**: *11th International Technical Meeting of the Satellite Division of the Institute of Navigation, Proceedings of ION GPS-98*, September 15-18, 1998, pp. 461–470.

43. ICAO. (1999). *Manual on Required Navigation Performance (RNP)*. Doc. 9613, June 1999.

44. Inside GNSS News. (2008). China to launch 10 compass satellites over next two years. In: *Inside GNSS News, Gibbons Media & Research LLC*. [Online]. Dated: December 8, 2008. Available at: <u>http://www.insidegnss.com/node/924</u> (Accessed: Thursday, 18 December 2008) 45. Institute of Electrical and Electronics Engineers. (1990). *IEEE Standard Computer Dictionary: A Compilation of IEEE Standard Computer Glossaries*. New York, USA.

46. International Committee on Global Navigation Satellite Systems. (2007). Available at: http://www.unoosa.org/oosa/en/SAP/gnss/icg.html (Accessed: Friday, 18 December 2008)

47. International Reference Ionosphere. (2009). IRI Home. Available at: http://iri.gsfc.nasa.gov/ (Accessed: Friday, 18 September 2009)

48. IPS Radio and Space Services. (2008). Glossary of solar terrestrial terms. *The Australian Space Weather Agency, Australian Government*. [Online]. Available at: <u>http://www.ips.gov.au/Educational/1/2/1/T</u> (Accessed: 30 September 2009)

49. Julier, S.J. (2002). The scaled unscented transformation. In: *Proceedings of the American Control Conference*, **6**, pp. 4555 – 4559.

50. Julier, S.J. and Uhlmann, J.K. (2004). Unscented filtering and nonlinear estimation. *Proceedings of the IEEE*, **92** (3) March 2004, pp. 401-422.

51. Kasdin, N.J. and Gurfil, P. (2004). Hamiltonian modelling of relative motion. *Annals of the New York Academy of Science*. **Vol. 1017**, pp. 138-157. [Online]. Available at: http://www3.interscience.wiley.com/cgi-bin/fulltext/118765778/PDFSTART (Accessed: Tuesday, 27 January 2009)

52. Kelso, T.S. (2000). *TrakStar Version 2.65 (trakstar26a.zip)*. Retrieved/Downloaded: Thursday, 03 August 2006 Website: <u>http://www.celestrak.com/software/tskelso-sw.asp</u> Downloaded from: <u>http://www.celestrak.com/software/satellite/trakstar26a.zip</u>

53. Kelso, T.S. (1998a). Frequently Asked Questions: Two-Line Element Set Format. *Satellite Times*. **4** (3), January 1998, pp. 52-54. Retrieved: Tuesday, 11 May 2010. Website: http://celestrak.com/columns/v04n03/

54. Kelso, T.S. (1998b). More frequently asked questions. *Satellite Times*. **4** (5), March 1998, pp. 76-77. Retrieved: Tuesday, 3 October 2006. Website: http://www.celestrak.com/columns/v04n05/

55. Kelso, T.S. (2006). *NORAD two-line element sets current data*. Last update: October 4, 2006, Retrieved: http://www.celestrak.com/NORAD/elements/ (Accessed: Wednesday, 04 October 2006).

56. Kim, E. Walter, T. and Powell, J. D.(2007). Adaptive carrier smoothing using code and carrier divergence. **In**: *Proceedings of ION NTM 2007*. Catamaran Resort Hotel, San Diego, California, January 22-24, 2007.

57. Kline, P.A. (1997). Atomic clock augmentation for receivers using the Global *Positioning System*. PhD Thesis. Virginia Polytechnic Institute and State University, Blacksburg, Virginia, February 1997.

58. Klobuchar, J.A. (1987). Ionospheric time-delay algorithm for single-frequency GPS users. *IEEE Transactions on Aerospace and Electronic Systems*. **AES-23** (3) May 1987, pp.325-331.

59. Kozai, Y. (1959). The motion of a close earth satellite. *Astronomical Journal*. **64** (1274), pp. 367-377.

60. Lane, M.H. and Cranford, K.H. (1969). An improved analytical drag theory for the artificial satellite problem. *AIAA Paper* 69-925, August 1969.

61. Langley, R. (1993). The GPS observables, GPS World, April, p. 52.

62. Lasiter E. M., and Parkinson, B. W. (1977). The operational status of NAVSTAR GPS. *Journal of Navigation*, **30**.

63. Leitinger, R., Titheridge, J.E., Kirchengast, G. and Rothleitner, W. (1995). A "simple" global empirical model for the F layer of the ionosphere. Wissenschaftliche Berichte 1/1995, IMG, University of Graz.

64. Leitinger, R., Zhang, M.L. and Radicella, S.M. (2005). An improved bottomside for the ionospheric electron density model NeQuick. *Annals of Geophysics*. **48** (3), pp. 525-534.

65. Liu, L., Tian, Z., and Huang, S-J. (2001). An algorithm for integrating GPS/INS attitude determination system, **In**: *Proceedings of Radar*, 2001 CIE International Conference on Radar, 2001, pp. 167 – 170.

66. Liu, Z. Skone, S., Gao, Y. and Komjathy, A. (2005). Ionospheric modeling using GPS data. *Journal GPS Solutions*. **9** (1), April, pp. 63-66.

67. Massatt, P. and Brady, W. (2002). Optimizing Performance Through Constellation Management. *Crosslink*. **3** (2) pp. 17-21. [Online]. Available at: http://www.aero.org/publications/crosslink/summer2002/03.html (Accessed: Tuesday, 11 May 2010)

68. Mehra, R. K., (1970). On the identification of variances and adaptive Kalman filtering. *IEEE Transactions on Automatic Control*, **AC-15** (2), pp. 175-184.

69. Mehra, R. K., (1972). Approaches to adaptive filtering. *IEEE Transactions on Automatic Control.* **17** (5), pp. 693-698.

70. Mital, N.K. and King, A.I. (1979). Computation of rigid-body rotation in three dimensional space from body-fixed linear acceleration measurement. *Journal of Applied Mechanics, Transactions of the American Society of Mechanical Engineers*, **46** (12), pp. 925-930.

71. Mohamed, A.H. (1999). Optimizing the estimation procedure in INS/GPS integration for kinematic applications. Ph.D. Dissertation, University of Calgary, Department of Geomatics Engineering, Calgary, Alberta.

72. Mohamed, A.H. and Schwarz, K. P., (1999). Adaptive Kalman filtering for INS/GPS," *Journal of Geodesy*, **73**, pp. 193–203.

73. Mosert de Gonzales, M. and Radicella, S.M. (1990). On a characteristic point at the base of F2 layer in the ionosphere. *Advances in Space Research*. **10** (11), pp. 17-25.

74. Myers, K.A. and Tapley, B.D., (1976). Adaptive sequential estimation with unknown noise statistics. *IEEE Transactions on Automatic Control*, **21**, pp. 520-523

75. National Executive Commitee for Space-Based Positioning, Navigation and Timing(PNT).(2010).ImageLibrary.[Online].Availableat:http://pnt.gov/public/images/constellation.gif (Accessed: Monday, 29 March 2010)

76. National Geophysical Data Center (2009). US Total Electron Content (Vertical and
Slant)-ProductArchive[Online].Availableat:http://www.ngdc.noaa.gov/stp/IONO/USTEC/ (Accessed: Thursday, 1 October 2009)

77. Nava, B., Coïsson, P. and Radicella, S.M. (2008). A new version of the NeQuick ionosphere electron density model. *Journal of Atmospheric and Solar-Terrestrial Physics*. **70**, 2008. pp. 1856-1862.

78. Nordlund, P.-J. (2000). Recursive state estimation of nonlinear systems with applications to integrated navigation, *Report LiTH-ISY-R-2321*, Linköping University, Sweden. (Content reachable in file 2321.pdf can be retrieved by anonymous ftp at ftp://control.isy.liu.se/).

79. Office of the Press Secretary. THE WHITE HOUSE. (May 1, 2000). *Statement By The President Regarding The United States' Decision To Stop Degrading Global Positioning System Accuracy*. Retrieved: Wednesday, 25 July 2007. from Removal Of GPS Selective Availability (SA), National Geodetic Survey, National Oceanic & Atmospheric Administration, U.S. Department of Commerce. Website: http://www.ngs.noaa.gov/FGCS/info/sans_SA/docs/statement.html

80. Orús, R., Hernández-Pajares, M., Juan, J.M., Sanz, J. and García Fernández, M. (2002). Performance of different TEC models to provide GPS ionospheric corrections. *Journal of Atmospheric and Solar-Terrestrial Physic.* **64**, pp. 2055-2062.

81. Padgoanker, A.J., Krieger, K.W. and King, A.I. (1975). Measurement of angular acceleration of a rigid body using linear accelerometers. *Journal of Applied Mechanics, Transactions of the American Society of Mechanical Engineers*, **42** (9), pp. 552-556.

82. Penna N.T., Dodson, A.H. and Chen, W. (2001). Assessment of EGNOS tropospheric correction model. *Journal of Navigation*, **54** (01) Jan., pp. 37-55.

83. Prasad, R. and Ruggieri, M. (2005). *Applied Satellite Navigation: Using GPS, GALILEO, and Augmentation Systems*. Boston, USA: Artech House.

84. Pridmore, J. and Rumens, D.J. (1989). Interoperability-how do we know when we have achieved it? *Command, Control, Communications and Management Information Systems, 1989, Third International Conference on*, May 2-4, 1989, pp. 192-205.

85. Qin, L., Zhang, W., Zhang, H. and Xu, W. (2006). Attitude measurement system based on micro-silicon accelerometer array, *Chaos, Solitons and Fractals*, **29**, pp. 141–147.

86. Radicella, S.M. and Leitinger, R. (2001). The evolution of the DGR approach to model electron density profiles. *Advances in Space Research*. **27** (1), pp. 35-40.

87. Rankin, J. (1994). GPS and differential GPS: an error model for sensor simulation. **In**: *IEEE Position Location and Navigation Symposium*. Las Vegas, NV, Apr 11-15, 1994, pp. 260-266.

88. Rawer, K. (1982). Replacement of the present sub-peak plasma density profile by a unique expression. *Advances in Space Research.* **2** (10), pp. 183-190.

89. Ray, J.K., Cannon M.E. and Fenton P. (2001). GPS code and carrier multipath mitigation using a multiantenna system, *IEEE Transactions on Aerospace and Electronic Systems*. **37** (1), pp. 183-195.

90. Rios, J.A. and White, E. (2000). Low cost solid state GPS/INS package. In: *Proceedings of the Institute of Navigation Conference, Institute of Navigation*, USA, September 2000.

91. Roumeliotis, S.I., Johnson, A.E. and Montgomery, J.F. (2002). Augmenting inertial navigation with image-based motion estimation. **In**: *Proceedings of the IEEE International Conference on Robotics and Automation*, Washington, DC, USA, pp. 4326-4333.

92. RTCA (1999). Minimum operational performance standards for Global Positioning System/Wide Area Augmentation System airborne equipment. RTCA DO-229B, Issued 10 June 1999.

93. Salychev, O.S., Voronov, V.V., Cannon, M. E., Nayak, R. and Lachapelle, G. (2000). Low cost INS/GPS integration: concepts and testing, In: *ION NTM 2000*, Anaheim, CA, January 26-28, 2000, pp. 98-105.

94. Savage, P. (1998a). Strapdown inertial navigation integration algorithm design, Part 1: attitude algorithms. *Journal of Guidance, Control and Dynamics*. **21** (1), pp. 19-28.

95. Savage, P. (1998b). Strapdown inertial navigation integration algorithm design, Part 2: velocity and position algorithms. *Journal of Guidance, Control and Dynamics*. **21** (3), pp. 208-221.

96. Schaer, S. and Gurtner, W. (1998). *IONEX: The IONosphere Map EXchange Format Version 1*, Astronomical Institute, University of Berne, Switzerland. [Online]. Available at: <u>ftp://igscb.jpl.nasa.gov/igscb/data/format/ionex1.ps</u> (Accessed: Wednesday, 30 September 2009)

97. Schuler, A.R., Grammatikos, A. and Fegley, K.A. (1967). Measuring rotational motion with linear accelerometers. *IEEE Transactions on Aerospace and Electronic Systems*. **AES-3** (3), pp. 465-471.

98. Song, Q., Qi, J. and Han, J. (2006). An adaptive UKF algorithm and its application in mobile robot control. **In**: *ROBIO '06. IEEE International Conference on Robotics and Biomimetics*, Kunming, China, pp. 1117-1122.

99. Space Weather Prediction Center (2009). Real-time US-Total Electron Content: Vertical and Slant. [Online]. Available at: <u>http://www.swpc.noaa.gov/ustec/index.html</u> (Accessed: Wednesday, 30 September 2009)

100. Spencer, P.S.J., Robertson, D.S. and Mader, G.L. (2004). Ionospheric data Assimilation methods for geodetic applications. **In**: *Proceedings of IEEE PLANS 2004*, Monterey, California, April 26-29, 2004, pp. 510-517.

101. Stiefel, E.L., and Scheifele, G. (1971). *Linear and Regular Celestial Mechanics* Springer Verlag, Berlin.

102. Teunissen, P. J. G., (1994). A new method for fast carrier phase ambiguity estimation. **In**: *IEEE Position Location and Navigation Symposium*, 1994, Las Vegas, Nevada. April 11-15, 1994, pp.562-573.

103. The International Committee on Global Navigation Satellite Systems (December 2007). *The International Committee on Global Navigation Satellite Systems (ICG): A forum to discuss Global Navigation Satellite Systems (GNSS) to benefit people around the world*. Retrieved: Thursday, 18 December 2008. from International Committee on Global Navigation Satellite Systems, United Nations Office for Outer Space Affairs. Website: http://www.unoosa.org/pdf/publications/icg_book01E.pdf

104. Titheridge, J.E. (1996). Re-modelling the ionosphere E region. Kleinheubacher Berichte 39, pp. 687-696.

105. Tsui, J.B.Y. (2000). Fundamental of Global Positioning System receivers: a software approach. New York: John Wiley & Sons, Inc.

106. Vallado, D.A. (2007) *Fundamental of Astrodynamics and Application*. (3rd Ed.).California & New York: Microcosm Press & Springer.

107. Vepa, R. (2010). Spacecraft large attitude estimation using a navigation sensor. *Royal Institute of Navigation's The Journal of Navigation*. **63** (01), Jan. 2010, pp. 89-104.

108. Vepa R. and Zhahir, A. (2008). Adaptive extended Kalman filter for DGPS and carrier-phase DGPS applications, Poster/paper presented at European Navigation Conference, ENC-GNSS-2008, held in Toulouse, France, 23-25 April, 2008.

109. Vik, B., and Fossen, T.I. (2001). A nonlinear observer for GPS and INS integration.
In: *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, Florida, USA, pp. 2956-2961.

110. Waldmann, J. (2007). Feedforward INS aiding: an investigation of maneuvers for in-flight alignment. SBA Controle & Automação. 18 (4), Natal Oct./Dec. 2007. Retrieved: Thursday, 26 February 2009. Downloaded from: www.scielo.br/pdf/ca/v18n4/a06v18n4.pdf ISSN 0103-1759.

111. Wagner, J.F. and Wienecke T. (2003). Satellite and inertial navigation - conventional and new fusion approaches. *Control Engineering Practice*. **11**, pp. 543-550.

112. Wan, E.A., and van der Merwe, R. (2001). The unscented Kalman filter. **In**: *Kalman Filtering and Neural Networks*, Chapter 7, Wiley Publishing, Ed. S. Haykin.

113. Wang, C., Lachapelle G. and Cannon, M. E. (2004). Development of an integrated low-cost GPS/rate gyro system for attitude determination. *The Journal of Navigation*. **57**, pp. 85–101.

114. Wang, J., Garratt, M., Lambert, A., Wang, J.J., Han, S., and Sinclair, D. (2008) Integration of GPS/INS/vision sensors to navigate unmanned aerial vehicles. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences.* Vol. **XXXVII**. Part B1. Beijing, pp. 963-970.

115. Wang, Q., Ding, M. and Zhao, P. (2003). A new scheme of non-gyro inertial measurement unit for estimating angular velocity. **In**: *Proceedings of the 29th Annual Conference of the IEEE Industrial Electronics Society, IECON '03.* **2**, pp. 1564-1567.

116. Wolf, R. (2000). *Satellite Orbit and Ephemeris Determination using Inter Satellite Links*. PhD Thesis. Faculty for Civil Engineer and Surveying, University of the German Federal Armed Forces Munich, Germany. (Universität der Bundeswehr München, Fakultät für Bauingenieur- und Vermessungswesen).

117. Yang, Y., Farrell, J.A. and Tan H.-S. (2001a). Carrier phase differential GPS-aided INS based vehicle control: experiment results. **In**: *Institute of Navigation, NTM 2001*, Long Beach, Jan. 22-24, 2001.

118. Yang, Y., Farrell, J.A. and Tan H.-S. (2001b). GPS-aided INS based control state calculation for AHS. In: *IEEE 2001 American Control Conference*, June 2001.

119. Yang, Y., Sharpe, R.T. and Hatch R.R. (2002). A fast ambiguity resolution technique for RTK embedded within a GPS receiver. **In**: *15th International Technical Meeting of the Satellite Division of The Institute of Navigation, Proceeding of ION GPS 2002*, Portland, Oregon; USA, September 24-27, 2002.

120. Yuan, D. (2004). The Unscented Series. **In**: UCSC VISUAL COMPUTING LABORATORY – Group Meetings. November 17, 2004. Retrieved: Wednesday, 12 May 2010. Website: http://users.soe.ucsc.edu/~taoswap/events.htm Downloaded from:

REFERENCES

http://www.soe.ucsc.edu/%7Etaoswap/GroupMeeting/UnscentedSeries_DanYuan_2004_11 _17.ppt : Jack Baskin School of Engineering, UC Santa Cruz, USA.

PUBLICATIONS

CONFERENCES

1. **A. Zhahir** and R. Vepa, "A Comparative Study of Various Ephemerides Employed for Real Time Navigational Satellite Orbit Prediction", *3rd National Real-Time Technology And Applications Symposium*, UPM, Serdang, Malaysia, 5th-6th December 2006

2. R.Vepa and **A. Zhahir**, "Adaptive Extended Kalman Filter for DGPS and Carrier-Phase DGPS Applications", *European Navigation Conference (ENC-GNSS 08)*, Pierre Baudis Convention Center, 11, Esplanade Compans Caffarelli, Toulouse, Midi-Pyrénées, France, April 23 - 25, 2008

3. R.Vepa and **A. Zhahir**, "Adaptive Kalman Filters for Orbit Estimation of Navigation Satellites for DGPS Applications", *NAV08/ILA37 The Navigation Conference & Exhibition: Navigation and Location: We are here!*, Church House, Westminster, London, United Kingdom, 28–30 October 2008

JOURNAL

1. R. Vepa and **A. Zhahir**, "High-Precision Kinematic Satellite and Doppler Aided Inertial Navigation System", submitted to *Journal of Navigation* on 24 September 2009. Submitted and accepted for publications, will appear in *Journal of Navigation* (Vol. 63 No. 4).