University of Dundee

Income stratification and the measurement of interdistributional inequality between multiple groups

Allanson, Paul

Publication date:
2014

Citation for published version (APA):
Allanson, P. (2014). Income stratification and the measurement of interdistributional inequality between multiple groups. (Dundee Discussion Papers in Economics; No. 281). Dundee: Economic Studies, University of Dundee.

Dundee Discussion Papers in Economics

Income stratification and the measurement of interdistributional inequality between multiple groups

Paul Allanson

Department of
Economic Studies
University of Dundee
Dundee
DD1 4HN

Working Paper
No. 281
October 2014

ISSN: 1473-236X

Income stratification and the measurement of interdistributional inequality between multiple groups

Paul Allanson[1]

Economic Studies, University of Dundee, United Kingdom

Abstract

This paper proposes a new class of stratification indices that measure interdistributional inequality between multiple groups. The class is based on a conceptualisation of stratification as a process that results in a hierarchical ordering of groups and therefore seeks to capture not only the extent to which groups form well-defined strata in the income distribution but also the scale of the resultant differences in income standards between them, where these two factors play the same role as identification and alienation respectively in the measurement of polarisation. The properties of the class as a whole are investigated as well as those of selected members of it: the first two integer members may be interpreted as measuring the overall incidence and depth of stratification, while higher-order members are directly sensitive to the severity of stratification between groups. An illustrative application provides an empirical analysis of global income stratification by regions in 1993.

JEL codes: D31, D63
Key words: income stratification; interdistributional inequality; multiple groups

[1] Corresponding author. Economics Studies, University of Dundee, Perth Road, Dundee DD1 4HN, UK. Tel: +44 01382 384377 Fax: +44 01382 384691. Email: p.f.allanson@dundee.ac.uk.

1. Introduction

The concept of stratification is deeply embedded within sociology, most notably in relation to the analysis of social class, but has only been of relatively recent concern within the economics literature. Thus Yitzhaki and Lerman (1991) in their seminal article quote a definition by the sociologist Lasswell (1965, p.10): "In its general meaning a stratum is a horizontal layer, usually thought of as between, above or below other such layers or strata. Stratification is the process of forming observable layers, or the state of being comprised of layers." Key to this definition is the idea that stratification, unlike segregation, implies a hierarchical ordering of groups according to some metric where in many economic settings it will be possible to quantify the scale of the resultant differences in outcomes between groups. For example, occupational segregation in a labour market context will only lead to stratification in the earnings distribution if one group is crowded into lower paid occupations, with the resultant scale of economic disadvantage due to employment discrimination depending not only on the degree of segregation but also on the size of occupational pay differentials. Conversely, direct wage discrimination may not lead to significant stratification if groups are distributed equally among higher and lower paid occupations. In this paper we propose a class of stratification measures that depend in general on both the extent to which groups form well-defined layers or strata in the distribution of some economic outcome and the scale of between-group differences in those outcomes, since both are necessary consequences of the process of stratification. For expositional purposes we refer to "income stratification" though the measures are equally applicable to consumption, wealth or earnings.

The measurement of stratification from our perspective requires a comparison of inequality between two or more distributions, rather than the conventional focus of inequality analysis on the dispersion of outcomes within one distribution, where this may be expected to generate additional insights into the relative economic position of different groups. For example, examining gender pay differentials over the whole of the wage distribution rather than just in terms of the average wage gap can shed more light on the nature of the disadvantage faced by women in the labour market (e.g. Jenkins, 1994; van Kerm 2013). Similarly, the comparison of income distributions by race in South Africa (e.g. Allanson and Atkins, 2005; Gradin, 2012) or by country in the world (e.g. Milanovic and Yitzhaki, 2002; Lakner and Milanovic, 2013) may be informative about the legacy of apartheid and the impact of globalisation respectively.

A small but distinct literature on 'interdistributional inequality' has sought to develop graphical tools to facilitate such comparisons of distributions along with summary measures of economic distance between them (see Deutsch and Silber (1999) for an overview). One major limitation of this literature is that it is very largely restricted to the comparison of one distribution with another.[2] Thus, whereas the standard tools for the decomposition of inequality by population group yield measures of between-group inequality that are applicable to two or more groups, all but one of the summary measures of economic distance or group disadvantage reviewed in Yalonetsky (2012) are only applicable to pairwise comparisons. The sole exception is the class of 'ethical distance functions' in which an equally distributed equivalent (EDE) income standard[3] is first computed for each distribution and then the two income standards are compared (Shorrocks, 1982). The main contribution of this paper is to propose a class of indices that is both applicable to multiple groups and more informative about the nature of interdistributional inequality than ethical distance functions.

---

[2] See, for example, Bishop et al. (2010). Andreoli and Zoli (2012) provides a recent exception but does not proceed to define any corresponding multilateral summary indices.

[3] An income standard summarizes an entire distribution as a single 'representative income' level and should ideally satisfy various axioms including linear homogeneity and sub-group consistency (see Foster et al., 2013).

The indices are built up by aggregating the economic distance between each distinct pair of distributions in the population of interest to yield a class of measures that are defined as weighted averages with weights that depend solely on group frequencies. These measures are given in general as increasing functions of both the extent to which groups constitute distinct strata in the income distribution and between-group differences in income standards, where these two factors play the same role as identification and alienation respectively in the measurement of polarisation (Esteban and Ray, 1994; Duclos et al., 2004). An important difference is that pairwise identification in our approach is equal to the difference in the odds that the income of a randomly chosen member of the richer group is more rather than less than that of a randomly selected member of the poorer group, rather than being a function of relative frequencies within income classes or at particular levels of income. We therefore refer to our proposed class of measures as stratification rather than polarisation indices, although the two sets of measures do exhibit similar properties in many respects.

Alienation between each pair of groups is in turn defined as a power function of the absolute difference in income standards between them, where the choice of power determines the 'disadvantage' sensitivity of the measure. Analogously to the interpretation of Foster-Greer-Thorbecke (FGT) poverty measures (Foster et al., 1984), the first two integer members of the class may be interpreted as measuring the overall incidence and depth of stratification, while higher-order members are directly sensitive to the severity of stratification between groups in the population. More specifically, the first member may be interpreted as the population weighted mean difference in the odds that the income of a randomly chosen member of a richer group is more rather than less than that of a randomly selected member of a poorer group. The second further takes into account the depth of stratification and can be represented graphically on a generalised Lorenz curve diagram given that it is simply equal to the absolute Yitzhaki and Lerman (1991) between-group Gini index if the income standard is specified as the arithmetic mean. Higher-order members measure alienation as convex functions of pairwise income standard gaps and are therefore directly sensitive to the distribution of disadvantage among pairs of groups.

The paper is organised as follows. The next section introduces some basic notation and definitions employed in the paper. Section 3 focuses on the choice of a suitable measure of stratification in the two group case for which aggregation is not an issue. Section 4 extends the analysis to more than two groups, obtaining the new class of multilateral stratification indices by aggregation of the pairwise indices. Consideration is given to the properties of both the class as a whole and specific members of it. Section 5 compares the new class of indices with the Yitzhaki (1994) overlapping measures and briefly considers the properties of two sets of related measures based on alternative normalisations of the alienation function. Section 6 provides an empirical illustration based on the Milanovic and Yitzhaki (2002) analysis of world inequality in 1993 by regions. The final section summarises the contribution and offers some suggestions for further research.

2. Notation and definitions

We consider a population divided into $K \geq 2$ mutually exclusive and exhaustive groups that are ordered by some income standard from the poorest to the richest group. Let Y_k, $F_k(Y_k)$, $f_k(Y_k)$, n_k, p_k, μ_k, s_k and θ_k represent respectively the income variable, cumulative distribution function, probability density function, population, population share, expected value, income share and income standard of group k *(k=1,...K)*. The overall population $Y_u = Y_1 \cup Y_2 \ldots \cup Y_K$ is the union of all groups with size $N = \sum_k n_k$, distribution function $F_u(Y_u) = \sum_k p_k F_k(Y_k)$ and expected value $\mu_u = \sum_k p_k \mu_k$. All incomes are assumed to be positive to allow for the general definition of income standards. The (fractional) ranking of group k incomes in the group l and overall income distributions are given as $F_l(Y_k)$ and

$F_u(Y_k)$ respectively, with corresponding mean ranks $\bar{F}_{kl}$ and $\bar{F}_{ku}$. Assuming continuity of the income distributions, $\mathrm{P}(Y_k > Y_l) = \bar{F}_{kl}$ denotes the probability that the income of a random member of group *k* is more than that of a random member of group *l*, where this is known as the probability of transvariation (Gini, 1916, 1959) if groups are ordered by the arithmetic mean of income with *k* and *l* denoting the poorer and richer groups respectively.[4] If two or more groups have identical income standards then they are ranked such that $\mathrm{P}(Y_k > Y_l) < 0.5 < \mathrm{P}(Y_l > Y_k)$ for all relevant pairwise comparisons, where this secondary criterion for ranking distributions will generate a transitive ordering if the probability relationship between the sub-set of groups exhibits mutual rank transitivity (see De Baets et al., 2010).[5] Finally if the two distributions cannot be ranked on the basis of either criteria (e.g. if the two income distributions are identical) then the various indices to be considered below are invariant to the ordering of the groups, which is therefore chosen arbitrarily.

Following Mookherjee and Shorrocks (1982), the conventional group-wise decomposition of the Gini index $G = 2\,\mathrm{cov}\big(Y_u, F_u(Y_u)\big)/\mu_u$ may be written as $G = G_W + G_B + R$ where $G_W = \sum_k p_k s_k G_k$ with $G_{kk} = 2\,\mathrm{cov}\big(Y_k, F_k(Y_k)\big)/\mu_k$ denoting the Gini index of group *k*; $G_B = 0.5\sum_k \sum_l p_k p_l \left|\mu_l - \mu_k\right|/\mu_u$; and the residual *R* is interpreted as an 'interaction effect'. The alternative decomposition of Yitzhaki and Lerman (1991) yields the identity $G = G_w + G_b$ where $G_w = \sum_k s_k G_{ku}$ with $G_{ku} = 2\,\mathrm{cov}\big(Y_k, F_u(Y_k)\big)/\mu_k$; and $G_b = 2\sum_k p_k\left(\mu_k - \mu_u\right)\left(\bar{F}_{ku} - 0.5\right)/\mu_u$. Following Yitzhaki and Lerman (1991), Yitzhaki

---

[4] Continuity is assumed for notational convenience, implying that the probability of a randomly chosen member of group *k* having the same income as a randomly selected member of group *l* will have measure zero. The treatment of ties is discussed below in footnote 8.

[5] The need for the transitivity condition arises iff there are more than two groups with the same income standard given that $\mathrm{P}(Y_l > Y_k) > 0.5$ and $\mathrm{P}(Y_m > Y_l) > 0.5$ does not necessarily imply $\mathrm{P}(Y_m > Y_k) > 0.5$. The empirical significance of the issue is likely to be limited but the condition can always be checked should the need arise. Note that $\bar{F}_{ku} = \bar{F}_{lu}$ does not imply $\mathrm{P}(Y_k > Y_l) = 0.5$ so ranking in ascending order of average ranks in the overall distribution may not be sufficient to order groups that are distinguishable on a pairwise basis.

(1994) sets out to measure stratification in terms of the relationship between the within-group measures G_{kk} and G_{ku}, whereas the class of indices defined in this paper build on links that have recently been established between the between-group indices G_B and G_b. We discuss the construction of the new class of indices in the following two sections and compare them with the Yitzhaki (1994) overlapping measures in Section 5.

3. The choice of stratification measure in the two group case

This section proposes a class of pairwise stratification measures that are defined as the product of an identification index and an alienation function. We first consider the choice and properties of the identification index, which is a modified version of the Gastwirth (1975) index of earnings differentials, before proceeding to the specification of the class of stratification indices.

Gastwirth (1975) considers the problem of comparing male and female earning distributions in the light of the observation that men earn more than women on average, proposing a measure of earnings differentials *TPROB* that is equal to twice the 'probability that a randomly chosen woman earns at least as much as a randomly selected man' (p.32). In our notation, the index is defined by Gastwirth as $TPROB = 2\int_0^\infty \left[1 - F_1(Y_1)\right] f_2(Y_2)\ \delta Y_2$ with groups 1 and 2 referring to women and men respectively. *TPROB* provides a unit free measure that will take a minimum value of zero if the highest paid women earns less than the lowest paid man and an "ideal" value of one when the two earnings distributions are identical. Gastwirth (1975, p.33) argues that *TPROB* "will detect any advancement of women relative to men"[6] and is therefore superior to measures such as the Theil (1971) overlap measure $OVL = \int_0^\infty \min\left\{ f_1(Y_1), f_2(Y_2)\right\} \partial y$,[7] which is only sensitive to movements

---

[6] This is not strictly accurate. See below for further discussion.

[7] Following Deutsch and Silber (1997), Anderson et al. (2009) consider *OVL* as a polarization measure.

across the income level(s) at which the density functions intersect, and the ratio of medians which is open to similar criticism.

We consider here the complementary index $I_{12} = 1 - TPROB$ as a measure of the degree of identification of the two groups in a binary setting. Given continuity of the income distributions, $I_{12} = 1 - 2\,\mathrm{P}(Y_1 > Y_2) = \left(1 - \mathrm{P}(Y_1 > Y_2)\right) - \mathrm{P}(Y_1 > Y_2) = \mathrm{P}(Y_2 > Y_1) - \mathrm{P}(Y_1 > Y_2)$ so the index may be interpreted as the difference in the odds that a randomly chosen man will receive more rather than less than a randomly selected woman.[8] It follows immediately that the index is symmetric since $I_{12} = -I_{21}$. I_{12} will take its maximum value of 1 if the two groups are fully identified in the sense that group membership can be unequivocally determined from an individual's position in the income distribution: no man will earn less than any woman if there is complete segregation of the groups into separate layers in the income distribution so not only are all men among the highest earners but also all the highest earners are men. Conversely I_{12} will equal zero if the two distributions are identical such that knowledge of an individuals' position in the income distribution is entirely uninformative of their group identity: a randomly chosen man is equally likely to earn more rather than less than a randomly selected woman if the two groups are indistinguishable.

I_{12} has appeared in the recent literature both as a measure of discrimination (Le Breton *et al.*, 2008, 2012) and of the loss of between-group inequality due to overlapping (Monti and Santoro, 2011). Le Breton *et al.* (2008) obtain the result that I_{12} is equal to the first-order discrimination index $\Delta_1 = 2\int_0^1 \left(q - \Phi^1(q)\right) dq$ where $\Phi^1(q) = F_2\left(F_1^{-1}(q)\right)$ is interpreted as a first-order discrimination curve and defined as the proportion of men with incomes no more than the female income quantile associated with the poorest proportion *q* of

---

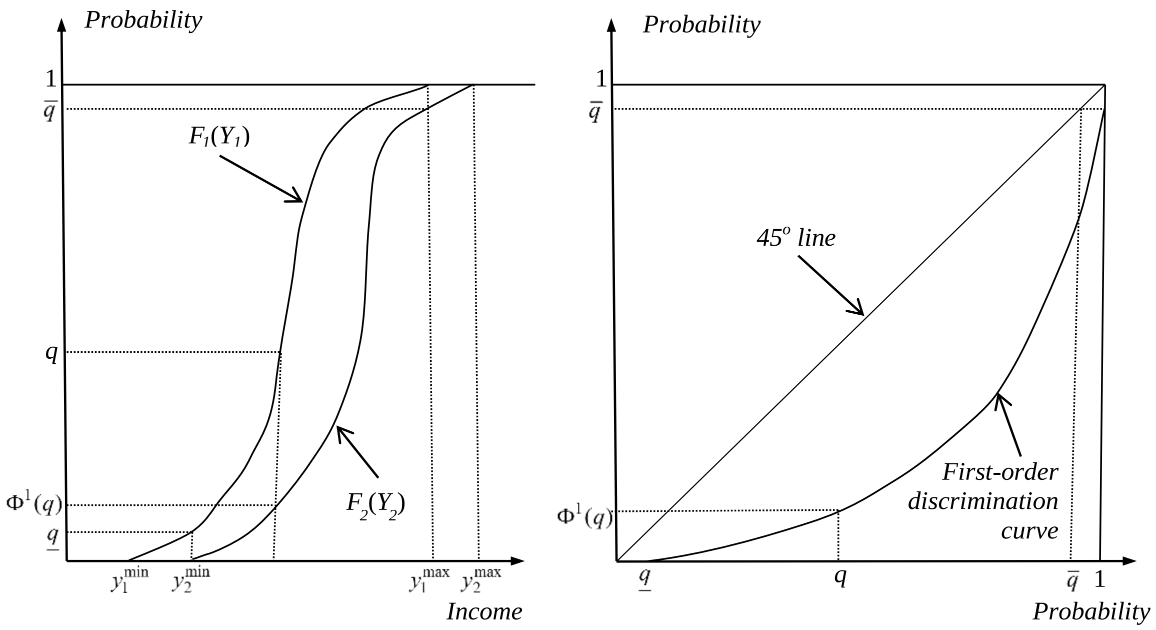
[8] Note that if $\mathrm{P}(Y_1 = Y_2) \neq 0$ then the definition of the identification indices may be extended to give $I_{12} = 1 - 2[\mathrm{P}(Y_1 > Y_2) + 0.5\,\mathrm{P}(Y_1 = Y_2)]$ and $I_{21} = 1 - 2[\mathrm{P}(Y_2 > Y_1) + 0.5\,\mathrm{P}(Y_1 = Y_2)]$ such that $I_{12} = \mathrm{P}(Y_2 > Y_1) - \mathrm{P}(Y_1 > Y_2) = -I_{21}$ as before, with this treatment also providing a feasible solution to the problem of ties in empirical work.

women: for example, if $\Phi^1(0.5) = 0.2$ then only 20% of men have incomes no more than the female median income. Figure 1 illustrates $\Phi^1(q)$ which is an 'interdistributional Lorenz curve of the first type' (Butler and McDonald, 1987) and takes the form of an increasing function that will coincide with the 45º line only if the two distributions are identical. Δ_1 is defined as twice the area between this diagonal and $\Phi^1(q)$ with portions of $\Phi^1(q)$ below the diagonal counting positively to the measure while those above it contribute negatively. Thus Δ_1 provides a measure of 'net' discrimination, with $\Phi^1(q)$ only lying everywhere below the diagonal if male incomes first-order stochastically dominate female incomes, and may equal zero even if the two distributions are not identical (see Yalonetsky, 2012). Δ_1 is responsive to changes in individual male and female incomes over the common support of the two distributions but not, for example, to progressive income transfers between women receiving less than the lowest male income $y_2^{\min}$ or between men with more than the highest female income $y_1^{\max}$.

Monti and Santoro (2011) show that the ratio of the alternative between-group Gini indices G_b and G_B is equal to $I_{12} = G_b / G_B = 1 - 2\,\mathrm{P}(Y_1 > Y_2)$ in the two group case, and follow Milanovic and Yitzhaki (2002, p.161) in interpreting the index in terms of the loss of between-group inequality due to overlapping. In particular, I_{12} will take its maximum value of one when the two groups occupy exclusive income ranges such that there is perfect stratification in the sense of Lasswell (1965). It is also shown that the minimum value of I_{12} is not zero since G_b, unlike G_B, can be negative when mean incomes by group are negatively correlated with mean ranks.

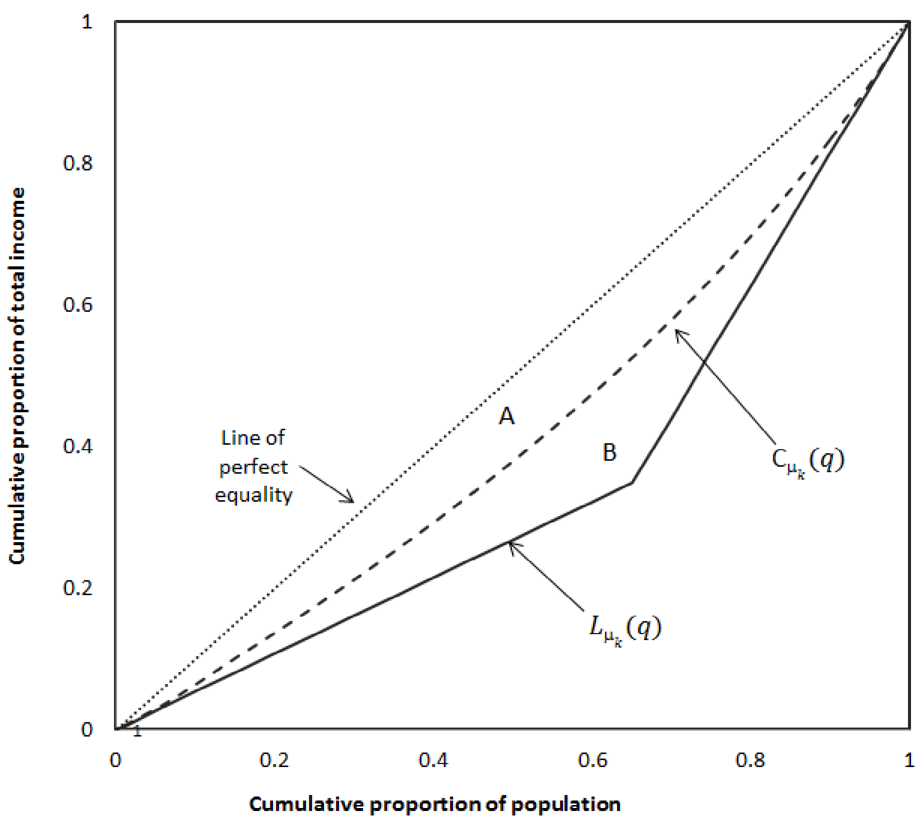
Monti and Santoro (2011) proceed to give a graphical interpretation of I_{12} similar to that in Le Breton *et al.* (2008, 2012) but the link with between-group Gini indices suggests an alternative representation in terms of Lorenz and concentration curves that will prove useful in the multilateral case. Specifically, if we consider the smoothed distribution Y_{μ_k} obtained

Figure 1 Construction of first-order discrimination curve



Source: Adapted from Le Breton et al. (2012)

Figure 2 Representation of I_{12} based on Lorenz and concentration curves



by assigning to each individual in the population the mean income of the group to which they belong then $F_{\mu_k}(Y_{\mu_k}) = p_1F_1(\mu_1) + p_2F_2(\mu_2)$, $G_B = 2\operatorname{cov}\left(Y_{\mu_k}, F_{\mu_k}(Y_{\mu_k})\right)\big/\mu_u$ is the Gini index of smoothed income and $G_b = 2\operatorname{cov}\left(Y_{\mu_k}, F_u(Y_u)\right)\big/\mu_u$ is the concentration index of smoothed income with ranks based on the original income distribution. Hence I_{12} is in general equal to the ratio of the concentration index to the Gini index for the smoothed distribution. Figure 2 plots the proportion of smoothed income received by the first 100q per cent of people when ranked from poorest to richest in the smoothed and original distributions, $L_{\mu_k}(q)$ and $C_{\mu_k}(q)$ respectively, with I_{12} equal to the ratio of area A to area (A+B) if $C_{\mu_k}(q)$ lies everywhere below the line of equality.

Given the identification index I_{12}, an obvious candidate for a stratification index in the binary case is the absolute between-group inequality measure $\mu_u G_b = \mu_u G_B I_{12} = (\mu_2 - \mu_1) I_{12}$ where, following Esteban and Ray (1994), $(\mu_2 - \mu_1) \geq 0$ may be interpreted as a measure of alienation between groups. In practice, we employ a generalised alienation function $A_{12}(\upsilon, \alpha)$ that yields the following class of pairwise stratification indices:

$$S_{12}(\upsilon,\alpha) = A_{12}(\upsilon,\alpha)\; I_{12} = \left(\theta_2(\alpha) - \theta_1(\alpha)\right)^{\upsilon} \left(\mathrm{P}(Y_2 > Y_1) - \mathrm{P}(Y_1 > Y_2)\right);\ \upsilon \geq 0;\ 0 \leq \alpha \leq 1 \qquad (1)$$

$$\text{where}\quad \theta_j(\alpha) = \mu_j^{\alpha} = \begin{cases} \left(\dfrac{\sum_i^{n_j} y_{ij}^{\alpha}}{n_j}\right)^{1/\alpha} & \text{if } 0 < \alpha \leq 1 \\ \left(\prod_i^{n_j} y_{ij}\right)^{1/n_j} & \text{if } \alpha = 0 \end{cases} \;;\ j = 1, 2, \qquad (1a)$$

Thus alienation $A_{12}(\upsilon, \alpha)$ is given as a power function of the difference in income standards between the groups and will be non-negative by definition since $\theta_2(\alpha) - \theta_1(\alpha) \geq 0$ by construction. Income standards $\theta_j(\alpha)$ are defined as generalised or $\alpha-$order means and may therefore be interpreted, following Blackorby et al. (1981), as social welfare measures with α being the Atkinson (1970) inequality aversion parameter. The parameter υ may be interpreted as an indicator of "group disadvantage" aversion in that a society in which the gap in income standards between the two groups is twice as large will have 2^{υ} times the level of stratification for any given degree of pairwise identification. Alternatively, υ is the elasticity of stratification with respect to the income standard gap, so that a 1% increase in the gap leads to a υ% increase in between-group stratification ceteris paribus. In general there seems no reason to believe that aversion to individual income inequality and to group disadvantage will be the same so α and υ are treated as independent parameters.

The parametric class of measures $S_{12}(\upsilon,\alpha)$ gives analysts and policymakers an instrument to evaluate stratification with varying sensitivity to distributional issues depending on social preferences. In particular, $(\mu_2-\mu_1)I_{12}=\mu_u G_b=\mu_u G_B-2(\mu_2-\mu_1)\mathrm{P}(Y_1\geq Y_2)$ is obtained as a special case when $\alpha=\upsilon=1$, where the final term may be interpreted as a measure of the loss of absolute between-group inequality due to overlapping. Additional flexibility can be gained through the normalisation of the alienation function, which is considered in Section 5 following the generalisation of the stratification index to allow for multiple groups.

4. Generalisation to the case of two or more groups

This section considers the generalisation of the pairwise measures $S_{12}(\upsilon,\alpha)$ to provide a class of multilateral stratification indices that are applicable to two or more groups. The key to our approach is to build up the multilateral indices by aggregating the pairwise indices over each distinct pair of distributions so as to yield an overall index that is a weighted average of the pairwise indices. The pairwise index provides an attractive building block for this purpose as the contribution of each pair of groups to the value of the overall measure can be interpreted in a straightforward manner given the inherent symmetry of $S_{12}(\upsilon,\alpha)$. Following the definition of the new class of multilateral stratification indices, we explore the properties of both the class as a whole and specific members of it.

4.1 Definition of the class of indices

Let $S_{kl}(\upsilon,\alpha) = A_{kl}(\upsilon,\alpha)I_{kl}$ be the pairwise index for two groups $l > k$ such that $\left(\mu_l^\alpha - \mu_k^\alpha\right) \geq 0$ by definition. We propose the following class of multilateral indices:

$$\begin{aligned} S(\upsilon,\alpha) &= \sum_k \sum_{l>k} p_{kl} S_{kl}(\upsilon,\alpha) = \sum_k \sum_{l>k} A_{kl}(\upsilon,\alpha) I_{kl}; \qquad \upsilon \geq 0;\ \ 0 \leq \alpha \leq 1 \\ &= \sum_k \sum_{l>k} p_{kl} \left(\mu_l^\alpha - \mu_k^\alpha\right)^\upsilon \left(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l)\right) \qquad (2) \\ &= \sum_k \sum_{l\neq k} \left(\frac{p_k}{p_k + p_l}\right) p_{kl} \left|\mu_l^\alpha - \mu_k^\alpha\right|^\upsilon \mathrm{sgn}(l-k)\left(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l)\right); \end{aligned}$$

where $$p_{kl} = p_{lk} = \left(\frac{2 p_k p_l}{\sum_k \sum_{l\neq k} p_k p_l}\right) = \left(\frac{2 p_k p_l}{1 - \sum_j p_j^2}\right); \qquad (2a)$$

$$\mathrm{sgn}(l-k) = \begin{cases} 1 & if\ l-k > 0 \\ -1 & l-k < 0 \end{cases}. \qquad (2b)$$

Thus $S(\upsilon,\alpha)$ is a population weighted average of the pairwise indices $S_{kl}(\upsilon,\alpha)$ with non-negative weights p_{kl} that are defined to sum to one over the set of distinct pairs of groups, i.e. for all $l > k$. Hence, p_{kl} may be interpreted as the probability that two individuals randomly selected with replacement from the population will be members of groups *k* and *l* conditional on them not being members of the same group. The third line of (1) follows since $I_{kl} = -I_{lk}$ and $\left(\mu_l^\alpha - \mu_k^\alpha\right) = -\left(\mu_k^\alpha - \mu_l^\alpha\right)$, with the use of the sign function $\mathrm{sgn}(l-k)$ allowing for the possibility that $\left(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l)\right)$ may be less than zero even if $l > k$, and makes use of the convention that the relative contribution of each group to the pairwise weight p_{kl} is in proportion to group sizes.

4.2 General properties of the class of indices

For two or more groups the properties of $S(\upsilon,\alpha)$ are as follows:

(I) Normalisation

The index $S(\upsilon,\alpha)$ is normalised to take a value of zero if for each pair of groups then either $I_{kl} = 0$ or $A_{kl}(\upsilon,\alpha) = 0$ or $A_{kl}(\upsilon,\alpha) = I_{kl} = 0$. Thus stratification will only be non-zero if there is at least one pair of groups that is both identified to some extent and consists of groups with non-identical income standards. Conversely, as with the binary index, zero values do not necessarily imply that all distributions are identical.

The index will be unit free with a maximum value of one if the disadvantage aversion parameter υ is set equal zero. Otherwise $S(\upsilon,\alpha)$ is unbounded from either above or below with the same units as $A_{kl}(\upsilon,\alpha)$. To see that negative values of $S(\upsilon,\alpha)$ are possible with multiple groups, consider a population that consists of three groups *k*, *l*, and *m* with population shares of 2/5, 1/5 and 2/5 respectively and incomes Y_k={5, 5, 5, 5, 5, 5}; Y_l={3, 9, 9} and Y_m={4, 4, 4, 4, 13, 13} measured in dollars. Hence $\mu_k < \mu_l < \mu_m$, with $\mu_k = 5$, $\mu_l = 7$ and $\mu_m = 9$; and $I_{kl} = 1/3$, $I_{lm} = 1/9$ and $I_{km} = -1/3$ since $\mathrm{P}(Y_k > Y_l) = 1/3$, $\mathrm{P}(Y_l > Y_m) = 4/9$ and $\mathrm{P}(Y_k > Y_m) = 2/3$. For the first three integer members of the class with $\alpha = 1$, we obtain from (2) that $S(0,1) = -1/18$, $S(1,1) = -4/9$ dollars and $S(2,1) = -20/9$ dollars squared.

(II) Invariance axioms

a. Symmetry: $S(\upsilon,\alpha)$ is unaffected by the permutation of groups

b. Population replication: $S(\upsilon,\alpha)$ is invariant to the replication of the population within each group while holding the population shares of the groups constant.

c. Income measurement invariance: The pairwise identification indices I_{kl} are invariant to affine transformations of individual welfare levels,[9] while the alienation functions $A_{kl}(\upsilon,\alpha)$ are homogeneous of degree υ in the difference in income standards $\left(\mu_l^\alpha - \mu_k^\alpha\right) \geq 0$. Hence $S(\upsilon,\alpha)$ is invariant to affine transformations of individual welfare levels if $\upsilon = 0$ and translation invariant otherwise, with these properties extending to invariance in individual incomes if additionally $\alpha = 1$. Alternative normalisations of $A_{kl}(\upsilon,\alpha)$, to be discussed in the

[9] Note that the I_{kl} 's are not in general invariant to order-preserving transformations of individual welfare levels because such transformations can have an effect on identification through the ordering of groups by income standards.

next section, yield stratification measures that are invariant to scalar and affine transformations with $\upsilon \neq 0$.

d. Continuity: Assuming continuous income distribution functions than $S(\upsilon,\alpha)$ will be continuous for all $\upsilon \neq 0$. In the case of $S(0,\alpha)$ a small change in an individual income that leads to a change in the ordering of groups by income standards may give rise to a discontinuous change in the value of index, where this property is similar to the discontinuity of the FGT poverty measure P(0) at the poverty line when a small change in income takes an individual either above or below the line. In all other cases the transition will be smooth because pairwise stratification tends to zero as the difference in income standards between the pair of groups tends to zero.

(III) Dominance axioms: The dominance properties of the index may be characterised in terms of identification and alienation axioms, as in Esteban and Ray (1994) and Duclos et al. (2004). For the sake of generality the discussion is couched in terms of the distribution of welfare and income standards, rather than of income and mean incomes: the two approaches coincide for the sub-class of indices with $\alpha = 1$, i.e. $S(\upsilon,1)$.

a. Identification: We define the identification axiom with reference to a population consisting of two or more groups with symmetric, unimodal welfare densities with compact supports $f_k(Y_k^\alpha)$ and corresponding income standards μ_k^α.[10] The need to define the population to which the axiom applies reflects the absence of simple stratification dominance properties that might apply to all populations, unlike the corresponding dominance axioms - such as the Pigou-Dalton transfer principle - in inequality analysis. In particular, identification is inherently a

[10] Symmetry implies that $f_k(\mu_k^\alpha - w) = f_k(\mu_k^\alpha + w)$ for all $w \in [0, \mu_k^\alpha]$ and unimodality that $f_k(Y_k^\alpha)$ is non-decreasing on $[0, \mu_k^\alpha]$.

characteristic of groups so the impact on identification of any particular change in individual welfare levels will inevitably depend on the configuration of groups in the population (see Esteban and Ray (1994) for further discussion).

Given this setting, a symmetric, income standard-preserving "squeeze" in the welfare distribution of one group, as shown in Figure 3a, cannot reduce identification and hence stratification. As in the measurement of polarisation (cf. Esteban and Ray, 1994), it is the identification axiom that distinguishes stratification from inequality, since a reduction in within-group variation holding between-group differences constant will lead to a fall in inequality according to the Pigou-Dalton transfer principle.

More specifically, we follow Duclos et al. (2004) in defining a λ-squeeze of the density $f_k(Y_k^{\alpha})$ as:

$$f_k^{\lambda}(Y_k^{\alpha}) = \frac{1}{\lambda} f_k\left(\frac{Y_k^{\alpha} - (1-\lambda)\mu_k^{\alpha}}{\lambda}\right); \quad 0 < \lambda < 1 \tag{3}$$

where $f_k^{\lambda}(Y_k^{\alpha})$ is also symmetric and has the same income standard as $f_k(Y_k^{\alpha})$ but is second-order stochastically dominant. To see that $S(\upsilon,\alpha)$ cannot fall due to a λ-squeeze note that the contribution of group *k* to overall stratification can be written from (2) as:

$$S_k(\upsilon,\alpha) = \sum_{l \neq k}\left(\frac{p_k}{p_k + p_l}\right) p_{kl} \left|\mu_l^{\alpha} - \mu_k^{\alpha}\right|^{\upsilon} \operatorname{sgn}(l-k) I_{kl}; \tag{4}$$

which will not fall if the degree of identification of group *k* does not fall with respect to either poorer or richer groups, i.e. the pairwise indices I_{kl} do not fall if $l > k$ and do not rise if $k > l$.

Figure 3a. Income-standard preserving "squeeze" of group *k* welfare distribution

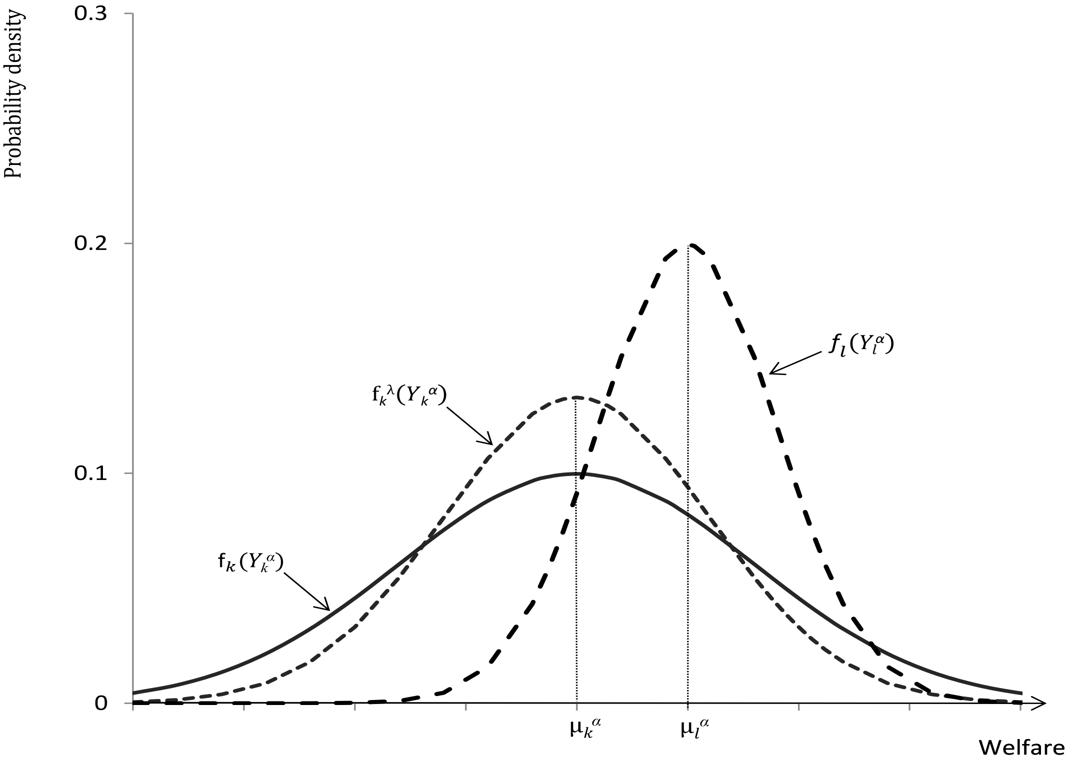
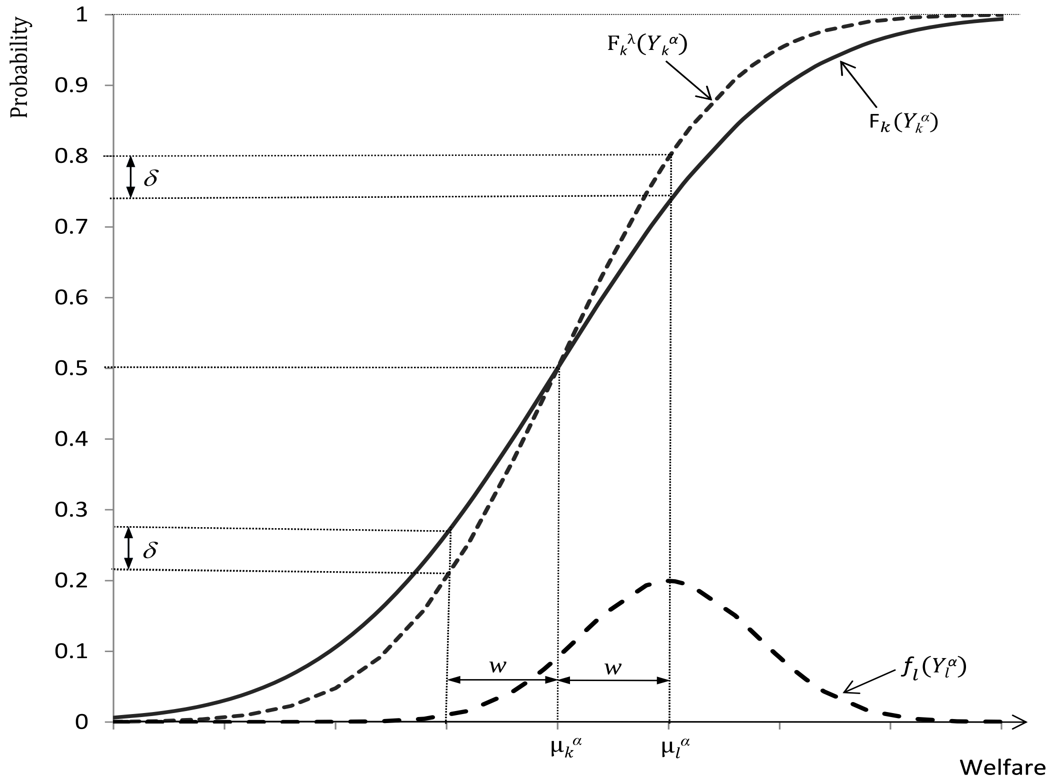


Figure 3b. Graphical proof of identification axiom



We demonstrate that the condition will hold in the former case in which $(\mu_l^\alpha - \mu_k^\alpha) \geq 0$,[11] with extension to the latter case immediate given the symmetry assumptions. The proof is illustrated in Figure 3b which shows the original and squeezed group *k* distribution functions $F_k(Y_k^\alpha)$ and $F_k^\lambda(Y_k^\alpha)$ respectively. Note that $F_k^\lambda(Y_k^\alpha) > F_k(Y_k^\alpha)$ if $Y_k^\alpha > \mu_k^\alpha$ and vice versa, with the absolute difference between the two curves being symmetric about the income standard given the symmetry of the welfare distributions. For $l > k$, we need to show that $I_{kl}^\lambda - I_{kl} = 2\int_0^\infty [F_k^\lambda(Y_k^\alpha) - F_k(Y_k^\alpha)] f_l(Y_l^\alpha)\ \delta Y_l^\alpha \geq 0$, where $I_{kl}^\lambda - I_{kl}$ is a weighted sum of the differences at each welfare level with weights given by the group *l* welfare density $f_l\left(Y_l^\alpha\right)$. Consider first the limiting case $\mu_l^\alpha = \mu_k^\alpha$ in which the weights $f_l\left(Y_l^\alpha\right)$ will also be symmetric about the common income standard such that $I_{kl}^\lambda - I_{kl} = 0$ by construction, with both $I_{kl}^\lambda = 0$ and $I_{kl} = 0$. For $\mu_l^\alpha > \mu_k^\alpha$, the mode of $f_l\left(Y_l^\alpha\right)$ will lie to the right of μ_k^α, as shown in the diagram, and we can proceed to sign $I_{kl}^\lambda - I_{kl}$ as follows. First note that $f_l\left(Y_l^\alpha\right)$ is strictly increasing over the range $\mu_k^\alpha \pm (\mu_l^\alpha - \mu_k^\alpha)$ so $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for any pair of points $\mu_k^\alpha \pm w$ with $0 < w \leq (\mu_l^\alpha - \mu_k^\alpha)$. Moreover $f_l\left(Y_l^\alpha\right)$ is symmetric about the mode at μ_l^α so it will also be the case that $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for any pair of points $\mu_k^\alpha \pm w$ for which $w > (\mu_l^\alpha - \mu_k^\alpha)$. Hence we can conclude that $I_{kl}^\lambda - I_{kl} > 0$ since $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for all possible *w*.

Finally, it should be recognised that the characterisation of identification is substantially different in the measurement of stratification and polarisation despite the superficial similarities. In particular, Duclos et al. (2004) apply the

[11] Le Breton et al. (2008) seek to establish an analogous relationship between second-order stochastic dominance and second-order discrimination (i.e. identification in our terminology) but only manage to show that it will hold if the density of the reference function $f_l(Y_l^\alpha)$ is decreasing over the entire support of $F_k(Y_k^\alpha)$, implying that the group *l* distribution must be positively skewed with mode of zero.

"squeeze" operator to so-called "basic densities" that would be fully identified in our framework even before the application of the operator because they are assumed to have disjoint supports. More generally, the identification function (4) for any group k depends in our approach on the extent to which group membership can be determined from individuals' ranks within the income distribution rather than on the density $f_k(Y_k^{\alpha})$ at any given welfare level. This difference fundamentally distinguishes the measurement of income stratification between a set of exogenously classified groups from that of income polarisation whether with or without predetermined groups.

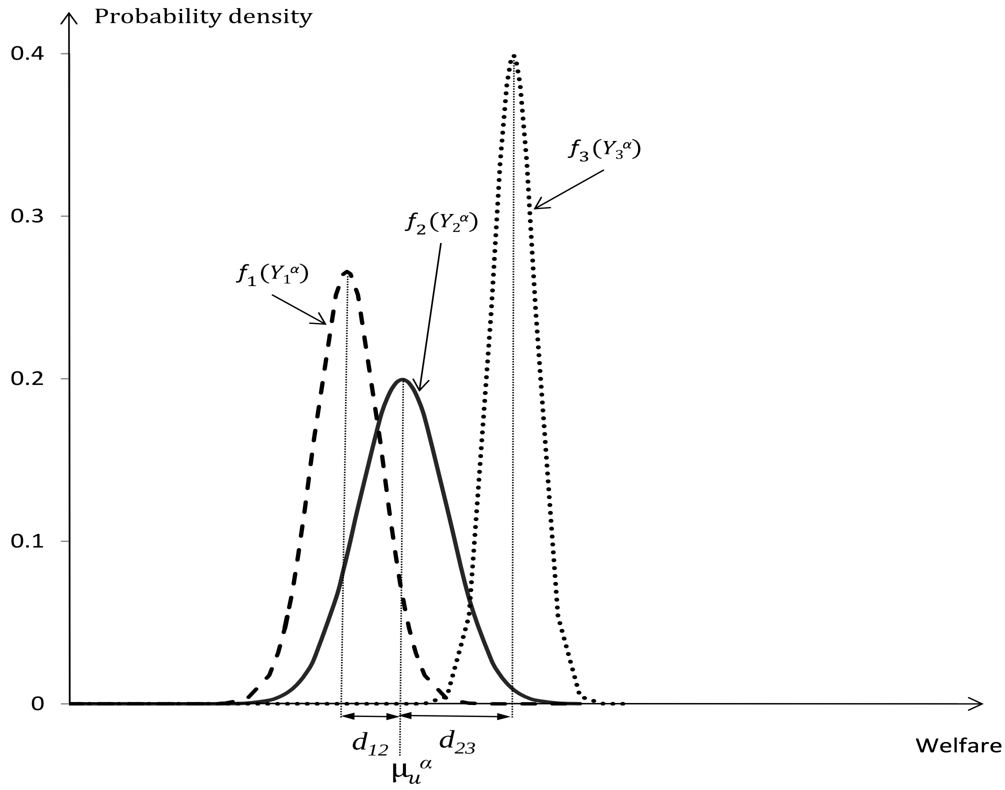
b. Alienation: The specification of the alienation axiom is more straightforward as it will apply to any population consisting of two or more groups. Given the invariance properties of the pairwise identification indices I_{kl} and the homogeneity in welfare levels of the alienation functions $A_{kl}(\upsilon,\alpha)$, an identification-preserving scalar expansion of all welfare differences about the overall population income standard μ_u^{α} will unambiguously increase alienation. Thus we define a global or γ−spread of population welfare levels Y_u^{α} as:

$$Y_u^{\alpha}(\gamma) = \mu_u^{\alpha} + \gamma\left(Y_u^{\alpha} - \mu_u^{\alpha}\right); \quad \gamma > 1 \qquad (5)$$

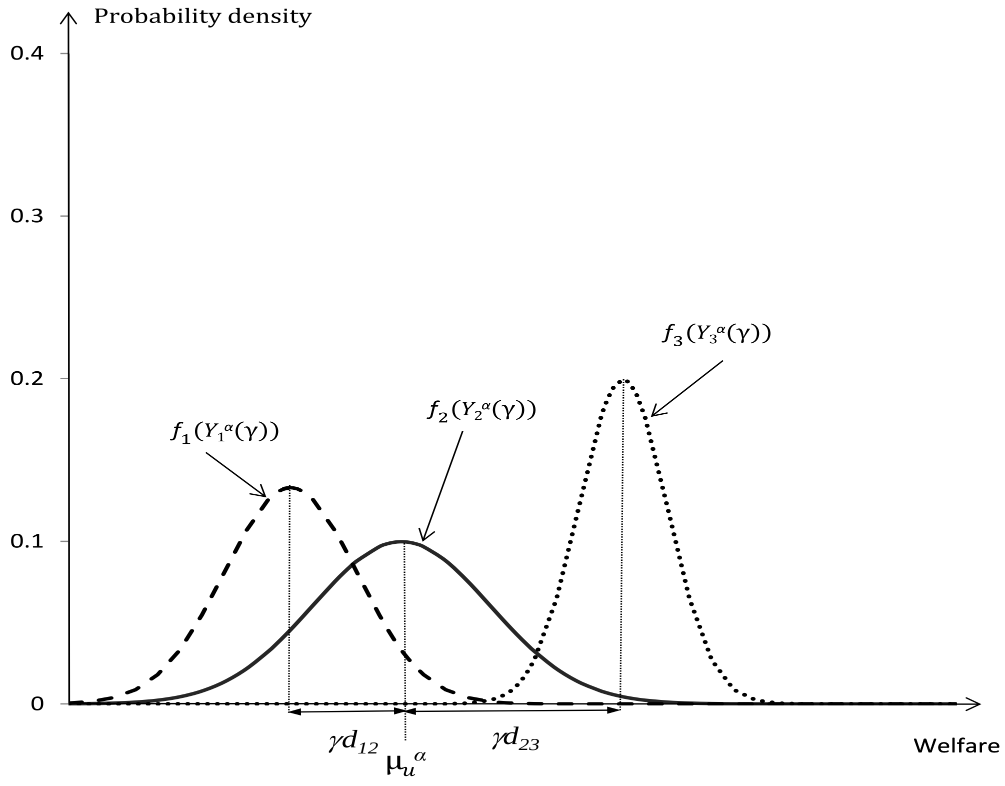
where $Y_u^{\alpha}(\gamma)$ will have the same population income standard μ_u^{α} as Y_u^{α} but give rise to higher levels of alienation and hence stratification if $\gamma > 1$. Figure 4 illustrates the idea of a γ−spread about the population mean μ_u^{α}, clearly capturing the idea that stratification is an increasing function of between-group differences in income standards. The parallel with the polarisation literature is again clear but, in the spirit of FGT poverty measures, we additionally allow for different degrees of group disadvantage aversion through the choice of the parameter υ.

Figure 4. Illustration of alienation axiom

(A) Original welfare levels



(B) γ-spread welfare levels



(IV) Decomposability and population composition: The index $S(\upsilon,\alpha)$ is a weighted average of the pairwise stratification indices $S_{kl}(\upsilon,\alpha)$, which provide estimates of the contribution of each distinct pair of groups to overall stratification. Furthermore, the pairwise indices may be meaningfully aggregated, given symmetry, to yield unique estimates $S_k(\upsilon,\alpha)$ of the contribution of each group to overall stratification using (4).

Overall stratification will unambiguously rise if stratification between any pair of groups increases holding population shares constant. Nevertheless it is important to remember that stratification is a property of groups and therefore not independent of the partition of the population into groups.[12] For example, splitting one group into two or more sub-groups that each possesses the same income distribution as the parent group will lead to a fall in the population average level of stratification $S(\upsilon,\alpha)$ given that $S_{kl}(\upsilon,\alpha)=0$ by definition for all pairs of sub-groups. By extension $S(\upsilon,\alpha)$ is not invariant to the replication of the population by the replication of groups.

For any given set of K groups with income distributions $F_k(Y_k)$, stratification will be maximised if the population is equally divided between the two groups with the largest pairwise index $S_{kl}(\upsilon,\alpha)$. In the case of $S(0,\alpha)$ this will be the pair of groups that exhibits the highest degree of differentiation from each other into separate layers as measured by the pairwise identification indices I_{kl}, where this pair may usually be expected to consist of the richest and poorest groups in the population although this need not always be the case. For $\upsilon \neq 0$, stratification will also depend on the degree of alienation and it will more likely be the case that $S_{kl}(\upsilon,\alpha)$ will be maximised with the population equally split between the richest and poorest groups, given that these two groups must exhibit the greatest alienation as measured by the pairwise alienation

[12] We have previously noted a link between the measurement of stratification and between-group inequality: subgroup consistency in the measurement of inequality requires overall inequality, not the within-group and between-group components, to be invariant to the partition.

functions $A_{kl}(\upsilon,\alpha)$. The parallel with the measurement of polarisation is once again obvious (cf. Estaban and Ray, 1994, p.837)

4.3 Properties of $S(0,\alpha)$

The first member of the class, $S(0,\alpha)$, may be re-written from (1) as:

$$S(0,\alpha)=\sum_k\sum_{l>k}p_{kl}I_{kl}=\sum_k\sum_{l>k}p_{kl}\left(P(Y_l>Y_k)-\mathrm{P}(Y_k>Y_l)\right)=1-2\sum_k\sum_{l>k}p_{kl}\,\mathrm{P}(Y_k>Y_l); \qquad (6)$$

where $S(0,\alpha)$ is the population-weighted average level of pairwise identification since $\sum_k\sum_{l>k}p_{kl}=1$ by definition. More explicitly, $S(0,\alpha)$ measures the difference in the odds that the income of a randomly chosen member of a richer group is more rather than less than that of a randomly selected member of a poorer group, where groups are first ordered by income standards μ_k^{α} and then by pairwise comparison of ranks in the case of tied groups. Alternatively, the index is equal to one less twice the population weighted average probability that a randomly chosen member of a poorer group receives more than a randomly selected member of a richer group.

Thus $S(0,\alpha)$ may be interpreted as a headcount or incidence measure of stratification that captures the extent to which individuals' positions within the income distribution are determined by group membership: if group membership is entirely uninformative as a predictor of relative rank then $S(0,\alpha)=0$, whereas if membership of a particular group restricts individuals to a single interval or range of ranks exclusively occupied by members of their own group then $S(0,\alpha)=1$. With only two groups, the reduction in headcount stratification caused by a unit increase in individual welfare levels would be greatest for members of the poorer group with incomes equal to the modal welfare level in the richer group. With more than two groups, the issue is more complicated as there is a need to consider which group to target as well as to identify which members of the targeted group to support, where this will depend for intermediate groups on the net change in identification due to unit changes in individual welfare levels. Nevertheless it is readily apparent that increasing the welfare of the poorest group, let alone the welfare of the poorest members of that group, will not necessarily have the most impact on headcount stratification: indeed $S(0,\alpha)$ is invariant to changes in the incomes of individuals in the poorest group who receive less than the lowest income level in any other group so long as these changes do not increase any of their incomes above that level.

Zhou (2012) has independently proposed a stratification measure S_{ZHOU} that is identical to $S(0,\alpha)$ except that the groups are ordered by $\bar{F}_{ku}$ alone on the assumption of no ties between groups. Zhou defends his choice of measure on the grounds that it is invariant to all rank-preserving transformations of income but this is achieved by conflating the determination of the hierarchical ordering of groups with the measurement of the degree of identification between them given that $\bar{F}_{ku} = \sum_l p_l \bar{F}_{kl}$.[13] In our view these are independent steps with income standards providing a more compelling primary criterion for the establishment of the relative economic standing of groups. Nevertheless S_{ZHOU} will prove useful in applications in which only an ordinal measure of wellbeing is available such that is not possible to calculate representative welfare levels as a basis for ordering groups.

The main virtue of the headcount index is that it is easy to understand as it only depends on the extent to which groups form more or less distinct strata and not on the associated differences in income standards. But stratification is more than identification and alienation must also be taken into account in order to obtain an index that fully captures the richness of the concept.

---

[13] Zhou further conjectures that $S_{ZHOU} \geq 0$, which would imply that the index is bounded within the unit interval. However the following variant on the main text example shows this not to be the case: let the population shares of the three groups *k*, *l*, and *m* be 4/14, 3/14 and 7/14 with incomes $Y_k=\{5, 5, 5, 5\}$; $Y_l=\{3, 9, 9\}$ and $Y_m=\{4, 4, 4, 4, 13, 13, 13\}$ then $\bar{F}_{lu} < \bar{F}_{ku} < \bar{F}_{mu}$ since $\bar{F}_{ku} = 7\tfrac{1}{2}$, $\bar{F}_{lu} = 7\tfrac{1}{3}$ and $\bar{F}_{mu} = 7\tfrac{4}{7}$. Hence $I_{lk} = -7/21$, $I_{lm} = 5/21$ and $I_{km} = -3/21$, since $\mathrm{P}(Y_l > Y_k) = 2/3$, $\mathrm{P}(Y_l > Y_m) = 8/21$ and $\mathrm{P}(Y_k > Y_m) = 4/7$, to give $S_{ZHOU} = -3/61$.

4.4 Properties of $S(1,\alpha)$

The second member of the class, $S(1,\alpha)$, may be re-written from (1) as:

$$\begin{aligned} S(1,\alpha) &= \sum_k \sum_{l>k} p_{kl} A_{kl}(1,\alpha) I_{kl} = \sum_k \sum_{l>k} p_{kl} \left(\mu_l^\alpha - \mu_k^\alpha\right)\left(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l)\right) \\ &= S(0,\alpha)\bar{D}(\alpha) + \mathrm{cov}\left(A_{kl}(1,\alpha), I_{kl}\right) \end{aligned} \tag{7}$$

where $\bar{D}(\alpha) = \sum_k \sum_{l>k} p_{kl}\left(\mu_l^\alpha - \mu_k^\alpha\right)$ is the population mean income standard gap and $\mathrm{cov}\left(A_{kl}(1,\alpha), I_{kl}\right) = \sum_k \sum_{l>k} p_{kl}\left(\left(\mu_l^\alpha - \mu_k^\alpha\right) - \bar{D}(\alpha)\right)\left(I_{kl} - S(0,\alpha)\right)$ is the population covariance between pairwise income standard gaps and identification indices.

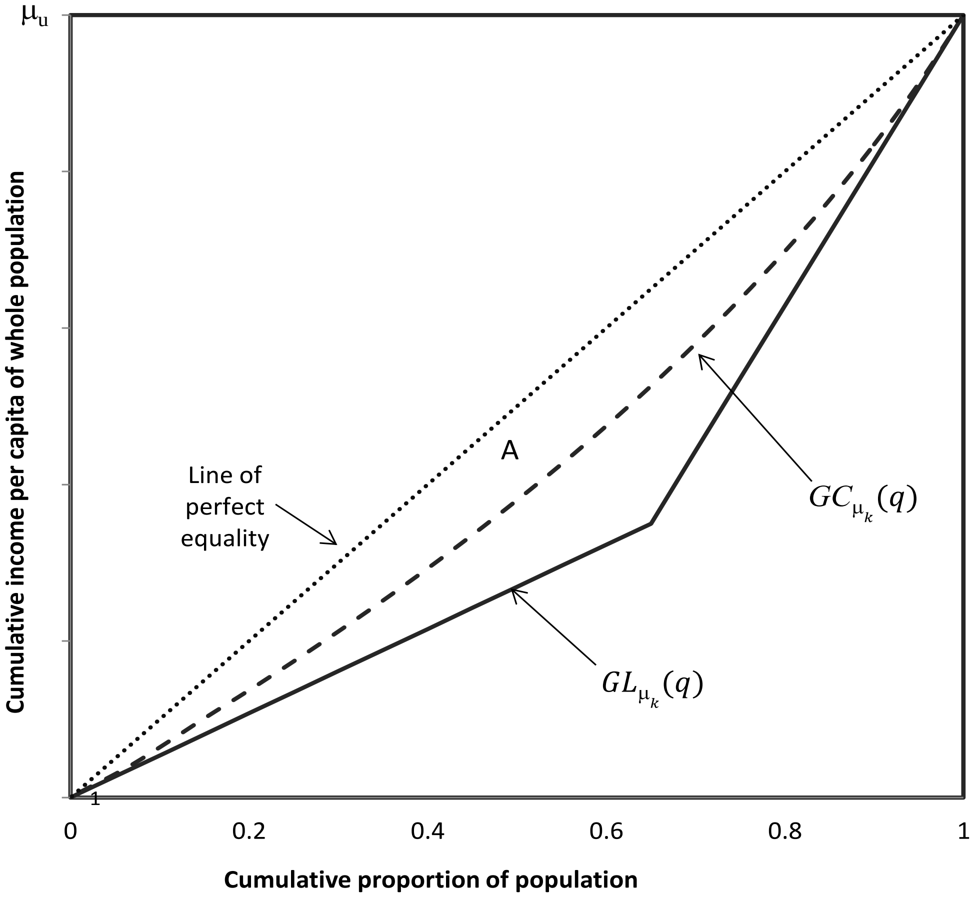
$S(1,\alpha)$ is again interpretable as a population weighted average but the contribution that any particular pair of groups makes to the value of the overall index now depends not only on the pairwise identification index I_{kl} but also on the (absolute) difference in income standards between them. For example, the lack of overlap between a rich and a poor group will count more towards the ‘stratification gap’ as measured by $S(1,\alpha)$ than the same lack between two moderately well-off groups: in the limit, two groups with identical income standards will not figure at all however large the difference in odds that a randomly chosen member of one group will be better off than a randomly selected member of the other group.

$S(1,\alpha)$ therefore reflects not only the incidence but also the depth of stratification, differentiating between pairs of groups on the basis of the size of the income standard or disadvantage gap between them. More specifically, the last line of (7) shows that $S(1,\alpha)$ is equal to the product of the mean levels of identification $S(0,\alpha)$ and alienation $\bar{D}(\alpha)$, plus the covariance between pairwise alienation and identification which will typically be positive. With only two groups, the reduction in the stratification gap caused by a unit increase in individual welfare levels would again be greatest for members of the poorer group with incomes equal to the modal welfare level in the richer group. And, more generally, it will also be the case that increasing the welfare of members of the poorest group may not necessarily have the most impact on the stratification gap given that alienation is a linear function of the income standard gap. For the specific index $S(1,1)$, it may be noted that the minimum cost of eliminating alienation through a policy of group-specific lump sum transfers will be equal to $\sum_{k \neq K}(\mu_K - \mu_k)n_k$ if transfers were perfectly targeted, i.e. the sum over all but the richest group of the product of the mean income gap with the richest group and group size.

Allanson (2014) has recently generalised the Monti and Santoro (2011) result on the relationship between G_b and G_B to more than two groups, implicitly identifying $S(1,1)$ in the process as $\mu_u G_b = 2\,\mathrm{cov}(Y_{\mu_k}, F_u(Y_u))$ where Y_{μ_k} is the smoothed distribution defined in the previous section. This correspondence suggests a graphical interpretation of $S(1,1)$ along the lines of that provided for I_{12} in Section 3 but based on the generalised Lorenz curve. Figure 5 plots the cumulative mean smoothed income (i.e. cumulated smoothed income divided by the total population) of the first 100q per cent of people when ranked from poorest to richest in the smoothed and original distributions, $GL_{\mu_k}(q)$ and $GC_{\mu_k}(q)$ respectively, with $S(1,1)$ simply equal to twice the area A if $GC_{\mu_k}(q)$ lies everywhere below the line of equality. More generally, $S(1,\alpha)$ is simply the generalised concentration index of the smoothed distribution $Y_{\mu_k^\alpha}$ obtained by assigning to each individual in the population the income standard μ_k^α of the group to which they belong, i.e. $S(1,\alpha) = \mu_u^\alpha G_b^\alpha = 2\,\mathrm{cov}(Y_{\mu_k^\alpha}, F_u(Y_u))$.

The sub-class of indices $S(1,\alpha)$ thus provides a direct link between the measurement of stratification and between-group inequality and has the further advantage of a simple graphical representation using a familiar tool from stochastic dominance analysis. Unlike $S(0,\alpha)$, the concept of stratification implied by $S(1,\alpha)$ requires the joint presence of identification and alienation. However $S(1,\alpha)$ is not directly related to the distribution of disadvantage among groups, as pairwise alienation is simply given as the size of the income standard gap, which may not be an appropriate assumption in all cases.

Figure 5. Representation of $S(1,1)$ using generalised Lorenz and concentration curves



4.6 Properties of $S(\upsilon,\alpha)$ with $\upsilon>1$

All indices $S(\upsilon,\alpha)$ with $\upsilon>1$ have alienation functions that are convex functions of pairwise income standard gaps and are therefore directly sensitive to the distribution of disadvantage among pairs of groups. For example, consider a population consisting of three equal sized groups with $I_{12}=I_{23}$, i.e. the middle group is equally identified with respect to the two other groups. It then follows from Jensen's inequality that stratification will be minimised if $\mu_2^\alpha-\mu_1^\alpha=\mu_3^\alpha-\mu_2^\alpha$, i.e. the income standard of the middle group is also equidistant between those of the two other groups. By implication, stratification will be higher in this population the closer the income standard of the middle group to that of either the richest or the poorest group, holding all other things constant.

Thus $S(\upsilon,\alpha)$ reflects not only the incidence and depth but also the severity of stratification if $\upsilon > 1$. In particular, if $\upsilon = 2$ then the alienation function is equal to the squared income standard gap and one pair of groups with income standards twice as far apart as another pair will contribute four times as much to the stratification index holding all other factors equal.[14] Higher values of υ imply greater disadvantage aversion: in the limit as $\upsilon \to \infty$ then the value of the index will be dominated by the pairwise stratification between the richest and the poorest groups, with the poorest group − though not necessarily the poorest members of it − providing the most cost-effective target for an anti-stratification support policy.

5. Discussion.

The class of stratification indices $S(\upsilon,\alpha) = \sum_k \sum_{l>k} p_{kl} A_{kl}(\upsilon,\alpha) I_{kl}$ builds on links between the alternative definitions of the between-group Gini with each pairwise identification index I_{kl} equal to the ratio of G_b to G_B in the sub-population consisting of groups *k* and *l* only, and $S(1,1)$ identical to $\mu_u G_b$ in the overall population. In contrast, Yitzhaki (1994) builds on Yitzhaki and Lerman (1991) to provide an alternative approach to the measurement of stratification based on the relationship between the alternative within-group Gini measures G_{kk} and G_{ku}. This section compares the relative merits of the two sets of measures and also

---

[14] The index in this case may be written as:

$$S(2,\alpha) = \sum_k \sum_{l>k} p_{kl} A_{kl}(2,\alpha) I_{kl} = \sum_k \sum_{l>k} p_{kl} \left(\mu_l^\alpha - \mu_k^\alpha\right)^2 I_{kl}$$
$$= S(0,\alpha)\left(\text{var}\left(A_{kl}(1,\alpha)\right) + \bar{D}(\alpha)^2\right) + \text{cov}\left(A_{kl}(1,\alpha), I_{kl}\right)\bar{D}(\alpha) + \text{cov}\left(A_{kl}(2,\alpha), I_{kl}\right)$$

where $\text{var}\left(A_{kl}(1,\alpha)\right) = \text{var}\left(\mu_l^\alpha - \mu_k^\alpha\right) = \sum_k \sum_{l>k} p_{kl}\left(\left(\mu_l^\alpha - \mu_k^\alpha\right) - \bar{D}(\alpha)\right)^2$ is the variance of the income standard gap, $\text{cov}\left(A_{kl}(2,\alpha), I_{kl}\right) = \sum_k \sum_{l>k} p_{kl}\left(\left(\mu_l^\alpha - \mu_k^\alpha\right) - \bar{D}(\alpha)\right)^2 \left(I_{kl} - S(0,\alpha)\right)$ is the (unstandardized) co-skewness between the income standard gap and the identification index, and the last equality holds since $\left(\mu_l^\alpha - \mu_k^\alpha\right)^2 = \left(\left(\mu_l^\alpha - \mu_k^\alpha\right) - \bar{D}(\alpha)\right)^2 + 2\left(\mu_l^\alpha - \mu_k^\alpha\right)\bar{D}(\alpha) - \bar{D}(\alpha)^2$.

briefly considers the properties of two simple variants on $S(\upsilon,\alpha)$ based on normalised versions of the alienation functions $A_{kl}(\upsilon,\alpha)$.

5.1. Comparison with the Yitzhaki (1994) overlapping measures

Yitzhaki (1994) shows that the within-group component G_w from the exact decomposition of the Gini index $G = G_w + G_b$ can be written as $G_w = \sum_k s_k G_{ku} = \sum_k s_k G_{kk} O_k$ where O_k denotes the 'overlapping' index of group *k* with the entire population. In turn $O_k = \sum_l p_l O_{lk}$ where the pairwise index $O_{lk} = \text{cov}(Y_k, F_l(Y_k)) / \text{cov}(Y_k, F_k(Y_k))$ lies in the open interval [0, 2] and is an increasing function of the fraction of group *l* that is located in the income range of group *k*. In other words, each O_{lk} measures the degree of 'overlapping' of group *l* by group *k*, where 'overlapping' is interpreted as non-stratification in the sense of Lasswell (1965), and O_k is given as the population-weighted average of these indices. The properties of the pairwise indices are fully expounded in Yitzhaki (1994) so discussion here is limited to the observation that the asymmetry of the pairwise indices, O_{lk} and O_{kl}, gives rise to problems of both interpretation and aggregation.

With regard to interpretation, knowledge of the values of both O_{lk} and O_{kl} is required to understand the relationship between the income distributions of the two groups. In particular, a zero value of O_{lk} may arise either if the two distributions have no common support in which case O_{kl} equals zero as well, or if all the incomes in group *l* are concentrated at a point in the income distribution of group *k* in which case $O_{kl} > 0$ with O_{kl} taking the maximum value of 2 when all group *l* incomes are equal to the group *k* mean income. The first of these two cases corresponds in our framework to a state of perfect identification in which the unit-free index $S_{kl}(0,\alpha)$ would equal one; while the limit of the second case implies zero stratification according to all $S_{kl}(\upsilon,1)$ measures due to the complete absence of both identification and mean income-gap alienation. Hence individual pairwise overlapping indices cannot really be interpreted as measuring pairwise stratification, and even in combination can only be thus interpreted with care in the absence of clearly defined identification and alienation functions. In contrast, pairwise stratification in our framework is based on explicit identification and alienation axioms that give rise to symmetric functions that are readily interpretable.

The asymmetry of the pairwise indices also prevents aggregation to the population level, with the groupwise indices O_k capturing the extent of overlapping of the overall population distribution $F_u(Y_u)$ by the group distribution $F_k(Y_k)$, not the overall degree of overlapping *per se*. In contrast our framework allows for the construction of both group $S_k(\upsilon,\alpha)$ and overall population $S(\upsilon,\alpha)$ indices, where the contribution of any pairwise index $S_{kl}(\upsilon,\alpha)$ to the value of each can be interpreted in a straightforward manner. This is of considerable practical importance because it is therefore possible to judge not only whether the overall level of stratification is higher in one population compared to another but also to estimate the contribution of individual groups to observed levels of overall stratification with the further potential to identify the characteristics or factors that contribute to stratification.

5.2 Normalisations

We have already noted that the framework provides analysts and policymakers with the flexibility to evaluate stratification with varying sensitivity to distributional issues through the parameterisation of the alienation function. Additionally, it is possible to normalise these functions so that alienation is not measured in absolute terms but in relative or standardised terms. We consider two variants on $S(\upsilon,\alpha)$ that normalise the absolute income standard gap $\left(\mu_l^\alpha-\mu_k^\alpha\right)$ with respect to the group l income standard μ_l^α and the population average income standard gap $\bar{\mathrm{D}}(\alpha)$, with both reducing to the unnormalised incidence measure $S(\upsilon,\alpha)$ if $\upsilon=0$.

5.2.1 Relative stratification measures

Treating the income standard of the richer group as the relevant "poverty line" in each pairwise comparison, it is possible to define a class of relative stratification gap measures:

$$\tilde{S}(\upsilon,\alpha)=\sum_{k}\sum_{l>k} p_{kl}\tilde{A}_{kl}(\upsilon,\alpha)I_{kl} \;=\sum_{k}\sum_{l>k} p_{kl}\left(\frac{\mu_l^{\alpha}-\mu_k^{\alpha}}{\mu_l^{\alpha}}\right)^{\upsilon}\left(\mathrm{P}(Y_l>Y_k)-\mathrm{P}(Y_k>Y_l)\right); \qquad (9)$$

where $\tilde{A}_{kl}(\upsilon,\alpha)$ captures the relative rather than the absolute disadvantage faced by the poorer group. Both $\tilde{A}_{kl}(\upsilon,\alpha)$ and $A_{kl}(\upsilon,\alpha)$ may be interpreted as measures of the magnitude of relative deprivation inasmuch as they capture "the extent of the difference between the desired situation and that of the person desiring it" (Runciman, 1966, p.10), but the income standard gap is expressed in proportional terms in the case of $\tilde{A}_{kl}(\upsilon,\alpha)$. This results in a stratification index $\tilde{S}(\upsilon,\alpha)$ that is more sensitive to absolute differences in income standards between the poorest groups in the income distribution since the same income standard gap between two groups will contribute more to $\tilde{S}(\upsilon,\alpha)$ the lower is the income standard of the richer of the two groups with the level of identification held constant. For example, consider again the example of a population consisting of three equal sized groups with the middle group equally identified with respect to the two other groups then stratification will be maximised if $\mu_2^{\alpha}=\sqrt{\mu_1^{\alpha}\mu_3^{\alpha}}<\left(\mu_1^{\alpha}+\mu_3^{\alpha}\right)/2$, i.e. the income standard of the middle group is closer to the bottom than the top group. Thus, in contrast to $S(1,\alpha)$, $\tilde{S}(1,\alpha)$ is not invariant to the distribution of absolute income standard gaps among pairs of groups all other things equal, nor is it symmetric to the distribution of income standard gaps about the overall mean unlike higher-order $S(\upsilon,\alpha)$ measures.

By definition, $\tilde{S}(\upsilon,\alpha)$ are unit free measures that are invariant to the scalar transformation of welfare levels and bounded from above by unity. These properties may prove particularly useful for the comparison of (relative) stratification gaps either over time or across countries. $\tilde{S}(\upsilon,\alpha)$ will be a decreasing function of the disadvantage aversion parameter υ as with FGT poverty measures.

5.2.2 Standardised stratification measures

It may also be useful to normalise each income standard gap by the population-weighted mean income standard gap, to yield a class of standardised stratification gap measures:

$$\widehat{S}(\upsilon,\alpha)=\sum_{k}\sum_{l>k}p_{kl}\widehat{A}_{kl}(\upsilon,\alpha)I_{kl}=\sum_{k}\sum_{l>k}p_{kl}\left(\frac{\mu_l^{\alpha}-\mu_k^{\alpha}}{\bar{D}(\alpha)}\right)^{\upsilon}\left(\mathrm{P}\left(Y_l>Y_k\right)-\mathrm{P}\left(Y_k>Y_l\right)\right); \qquad (10)$$

where the standardised alienation function $\widehat{A}_{kl}(\upsilon,\alpha)$ is unit free.

In particular if $\upsilon=1$ then the normalised index may be written as:

$$\widehat{S}(1,\alpha)=\sum_{k}\sum_{l>k}p_{kl}\widehat{A}_{kl}(1,\alpha)I_{kl}=\sum_{k}\sum_{l>k}\frac{p_k p_l\left(\mu_l^{\alpha}-\mu_k^{\alpha}\right)}{\sum_{k}\sum_{l>k}p_k p_l\left(\mu_l^{\alpha}-\mu_k^{\alpha}\right)}I_{kl}\equiv\sum_{k}\sum_{l>k}w_{kl}I_{kl}; \qquad (11)$$

where the weights w_{kl} are non-negative and sum to unity. Thus $\widehat{S}(1,\alpha)$ may be interpreted as a weighted average identification index like $S(0,\alpha)$ but with pairwise weights equal to shares in the total income standard gap $N\bar{D}(\alpha)$. It is easily shown that $\widehat{S}(1,\alpha)$ is equal to $S(0,\alpha)$ plus $\mathrm{cov}(\widehat{A}_{kl}(1,\alpha),I_{kl})$ where the covariance between the standardised alienation function and pairwise identification indices may be expected to be positive. Like $S(0,\alpha)$, $\widehat{S}(1,\alpha)$ is invariant to affine transformations of income but also to the replication of population by the replication of groups. Allanson (2014) has previously identified $\widehat{S}(1,\alpha)$ as the ratio of G_b to G_B, given that the denominator in the weights function is simply equal to $0.5\mu_u G_B$, with Heller and Yitzhaki (2006) interpreting this ratio as a measure the 'quality of identification' achieved in the classification of individual groups by means of some continuous characteristic.

By extension, $\widehat{S}(\upsilon,\alpha)$ may in general be interpreted as a class of weighted identification indices where the form of the weighting functions $w_{kl}(\upsilon,\alpha)\equiv p_{kl}\widehat{A}_{kl}(\upsilon,\alpha)$

resembles those employed in the definition of the class of general additive decomposable (GAD) inequality indices (Foster and Shneyerov, 1999) but specified in terms of mean normalised differences in income standards between pairs of groups, rather than mean normalised income standards of individual groups, since the concern is to measure the distance between distributions. However it should be noted that the resultant weights only sum to one if either $\upsilon = 0$ or $\upsilon = 1$, as is also the case with GAD inequality indices.

6. Empirical illustration.

By way of illustration, this section follows Allanson (2014) in further elaborating the empirical analysis presented in Milanovic and Yitzhaki (2002) of world inequality by regions in 1993.[15] The top panel in Table 1 presents estimates from their Tables 4 and 7 of population shares, p_k; mean incomes, μ_k; and mean rankings in the income distributions of each region, $\bar{F}_{kl} = P(Y_k > Y_l)$, and the world $\bar{F}_{ku} = P(Y_k > Y_u)$. This shows that Africa was the poorest region in per capita terms followed by Asia; Eastern Europe and the Former Soviet Union (EFSU); Latin America and the Caribbean (LAC); and Western Europe/North America/Oceania (WENAO). However the mean rank of Africans in the Asian income distribution was 0.515, implying that an African chosen at random was likely to have been better off than a randomly chosen Asian, and the mean rank of Africans in the world distribution was also higher than that of Asians. Mean ranks for all other pairs of regions are consistent with the ordering of mean incomes.

The remaining panels show the constituent elements of the stratification indices as identified in the last line of (2), with the stratification indices themselves given in Table 2. Note that the population weights $p_{kl} p_k / (p_k + p_l)$ reflect the relative frequencies of distinct regional pairs and do not sum across columns to give the population shares p_k. The pattern of pairwise signed identification indices $\text{sgn}(l-k) I_{kl}$ and absolute mean income gaps

[15] These regions are referred to as 'continents' in Milanovic and Yitzhaki (2002) though the correspondence is not exact.

$\left|\mu_l - \mu_k\right|$ reveals that the regions of the world are broadly divided into three broad layers or strata – with Africa and Asia at the bottom, EFSU and LAC in the middle and WENAO on its own at the top of the world income distribution – where the degree of both identification and alienation between regions in the same layer was much lower than that between regions in different strata. Indeed, there was virtually no stratification of the African and Asian distributions in the bottom stratum nor of the EFSU and LAC distributions in the middle layer, with pairwise identifications indices close to zero and mean income gaps less than \$1000. In contrast, the WENAO income distribution was highly stratified from those of every other region, with the relevant pairwise identification indices ranging between 0.656 and 0.902 and all mean income gaps greater than twice the mean world income level of \$3000. All other pairwise measures were intermediate with the population-weighted mean identification index and mean income gap equal to 0.518 and \$4007 respectively.

The top panel of Table 2 reports the headcount index $S(0,1)$, which is equal to the population-weighted mean identification index reported in Table 1. Thus the difference in the odds that the income of a randomly chosen member of a richer region was more rather than less than that of a randomly selected member of a poorer region was equal to 0.518. It follows immediately that the population-weighted mean probability of transvariation was equal to 0.241. The pairwise decomposition shows that the overall level of identification was mainly driven by the existence of the largely separate WENAO stratum at the top of the world income distribution, with the Asia/WENAO pair alone contribute nearly half of the total value of $S(0,1)$[16] as a result of the populousness of the two regions and the low degree of overlap between their income distributions. At the other extreme, the EFSU/LAC and Africa/Asia pairs made a negligible contribution to the total due to the lack of pairwise identification of their income distributions, with the negative value of the latter arising because the probability of transvariation between the two regions, i.e. $P\left(Y_{Africa} > Y_{Asia}\right)$, was greater than 0.5.

[16] Since (0.0475+0.1981)/0.518=0.474.

Table 1. Constituent elements of income stratification calculations

	Pop^n share (%)	Mean income ($PPP)	Mean rank in income distribution of:					
Column (l) *Row (k)*			Africa	Asia	EFSU	LAC	WENAO	World
Africa	10.0	1310.0	0.500	0.515	0.275	0.261	0.049	0.407
Asia	59.5	1594.6	0.485	0.500	0.265	0.247	0.064	0.397
EFSU	7.8	2780.9	0.725	0.735	0.500	0.483	0.136	0.609
LAC	8.4	3639.8	0.739	0.753	0.517	0.500	0.172	0.629
WENAO	14.3	10012.4	0.951	0.936	0.864	0.828	0.500	0.861
World	**100.0**	**3031.8**						**0.500**
			Population weights: $p_{kl} p_k / (p_k + p_l)$					Sum
Africa			~	0.029	0.015	0.015	0.020	0.078
Asia			0.169	~	0.136	0.146	0.227	0.678
EFSU			0.011	0.018	~	0.010	0.013	0.053
LAC			0.013	0.021	0.011	~	0.015	0.060
WENAO			0.028	0.055	0.024	0.025	~	0.131
World								**1.000**
			Signed pairwise identification indices: $\text{sgn}(l-k)I_{kl}$					Weighted mean
Africa			~	-0.030	0.450	0.478	0.902	0.393
Asia			-0.030	~	0.470	0.506	0.872	0.488
EFSU			0.450	0.470	~	0.034	0.728	0.443
LAC			0.478	0.506	0.034	~	0.656	0.448
WENAO			0.902	0.872	0.728	0.656	~	0.811
World								**0.518**
			Absolute mean income gaps: $\lvert\mu_l - \mu_k\rvert$					Weighted mean
Africa			~	284.6	1470.9	2329.8	8702.4	3021.7
Asia			284.6	~	1186.3	2045.2	8417.8	3567.8
EFSU			1470.9	1186.3	~	858.9	7231.5	2678.7
LAC			2329.8	2045.2	858.9	~	6372.6	2957.0
WENAO			8702.4	8417.8	7231.5	6372.6	~	7871.9
World								**4007.4**

Notes: Top panel. Source: Milanovic and Yitzhaki (2002) Tables 4 and 7 - see also Table 1 for the list of countries in each region (EFSU – Eastern Europe and Former Soviet Union; LAC – Latin America and Caribbean; WENAO – Western Europe, North America and Oceania). Other panels. Author's own calculations.

Table 2. Income stratification between regions of the world

	Africa	Asia	EFSU	LAC	WENAO	Sum	*Share*
Headcount stratification							
Africa	~	-0.0009	0.0066	0.0073	0.0177	0.031	*5.9%*
Asia	-0.0051	~	0.0639	0.0738	0.1981	0.331	*63.9%*
EFSU	0.0051	0.0084	~	0.0004	0.0095	0.023	*4.5%*
LAC	0.0061	0.0105	0.0004	~	0.0097	0.027	*5.2%*
WENAO	0.0252	0.0475	0.0174	0.0165	~	0.107	*20.6%*
$S(0,1)$						**0.518**	
Stratification gap $PPP							
Africa	~	-0.2	9.7	17.0	153.8	180.2	*5.8%*
Asia	-1.4	~	75.8	150.8	1667.7	1892.9	*60.9%*
EFSU	7.5	9.9	~	0.3	68.6	86.4	*2.8%*
LAC	14.3	21.4	0.3	~	62.0	98.0	*3.2%*
WENAO	218.9	400.2	125.7	104.9	~	849.7	*27.3%*
$S(1,1)$						**3107.2**	
Relative stratification gap							
Africa	~	-0.0002	0.0035	0.0047	0.0154	0.023	6.3%
Asia	-0.0009	~	0.0273	0.0414	0.1666	0.234	63.0%
EFSU	0.0027	0.0036	~	0.0001	0.0069	0.013	3.6%
LAC	0.0039	0.0059	0.0001	~	0.0062	0.016	4.3%
WENAO	0.0219	0.0400	0.0126	0.0105	~	0.085	22.8%
$\tilde{S}(1,1)$						**0.372**	
Standardised stratification gap							
Africa	~	-0.0001	0.0024	0.0042	0.0384	0.045	5.8%
Asia	-0.0004	~	0.0189	0.0376	0.4161	0.472	60.9%
EFSU	0.0019	0.0025	~	0.0001	0.0171	0.022	2.8%
LAC	0.0036	0.0053	0.0001	~	0.0155	0.024	3.2%
WENAO	0.0546	0.0999	0.0314	0.0262	~	0.212	27.3%
$\hat{S}(1,1)$						**0.775**	
Squared stratification gap ($PPP/1000)^2							
Africa	~	-0.0001	0.0142	0.0395	1.3385	1.392	*5.9%*
Asia	-0.0004	~	0.0900	0.3085	14.0381	14.436	*61.0%*
EFSU	0.0110	0.0118	~	0.0003	0.4964	0.519	*2.2%*
LAC	0.0332	0.0437	0.0003	~	0.3951	0.472	*2.0%*
WENAO	1.9053	3.3684	0.9089	0.6688	~	6.851	*28.9%*
$S(2,1)$						**23.671**	

Source: Author's own calculations.

The second panel reports the stratification gap index $S(1,1)$ which also reflects the depth of stratification and may loosely be interpreted as a measure of the perceived average difference in mean incomes between regions based on individuals' actual positions in the world income distribution, where this would only equal the actual average mean income gap if all regional income distributions were fully identified such that the probability of transvariation was zero. Thus the stratification gap of $3107 may be compared to the mean income gap $\bar{D}(1)$ of $4007 reported in Table 1, with the difference reflecting the odds that a randomly chosen individual in a poorer region had a higher income than a randomly selected individual in a richer region. Alternatively, following Milanovic and Yitzhaki (2002), the difference of $900 represents the loss of absolute between-group inequality due to the overlapping of regional income distributions since $\bar{D}(1) = \mu_u G_B$ and $S(1,1) = \mu_u G_b$. In comparison to $S(0,1)$, WENAO accounts for an even larger share of the total value of the index as a result of the above-average mean income differences between WENAO and every other region in the world. In contrast, the shares of the "middle income" regions, EFSU and LAC, fall particularly sharply as a result of their intermediate position in the world income distribution and correspondingly lower mean income gaps compared to other regions.

The next two panels present results on the alternative normalisations of $S(1,1)$. The relative stratification gap index $\tilde{S}(1,1)$ was 0.372, which may be interpreted as the perceived average relative difference in mean incomes between regions based on individuals' actual positions in the world income distribution. $\tilde{S}(1,1)$ is less than the headcount index $S(0,1)$ by construction, since the pairwise mean income gaps reported in Table 1 are all strictly positive. The pairwise decomposition shows increases in the relative contributions of all regions except WENAO compared to those for $S(1,1)$, reflecting the greater sensitivity of relative stratification gap indices to income standard gaps between pairs of groups at the bottom of the income distribution. In other words, $\tilde{S}(1,1)$ gives more weight than $S(1,1)$ to differences in mean incomes between the poorer regions of the world in the calculation of the overall measure of the stratification gap.

The standardised stratification gap index $\hat{S}(1,1)$ was 0.775 and, like $S(0,1)$, may be interpreted as a weighted average identification index but with total income gap rather than population weights. Given that $\hat{S}(1,1) = S(0,1) + \text{cov}(\hat{A}_{kl}(1,\alpha), I_{kl})$, the larger value of $\hat{S}(1,1)$ reflects the positive correlation between pairwise mean income gaps and identification indices, i.e. region pairs that formed more clearly defined regional strata in their combined income distribution also tended to have had larger differences in mean incomes. The value of the index may also be identified, following Allanson (2014), as the ratio of G_b to G_B, with 0.775=3107/4007. The pairwise decomposition is identical to $S(1,1)$ but differs from that given in Allanson (2014) who splits pairwise contributions equally between regions rather than by population shares.

The final panel reports the squared stratification gap $S(2,1)$ which was 23.6 million dollars squared. The squared measure puts greater weight on larger mean income gaps compared to $S(1,1)$ leading, as expected, to increases in the relative contributions of the regions at the top and bottom of the world income distribution – WENAO, Africa and Asia – at the expense of those in the middle – EFSU and LAC. Higher-order indices (i.e. with $\upsilon > 2$) would place increasingly greater weight on the relative contributions of the regions at the top and bottom of the world income distribution, with the pairwise stratification between the poorest and richest regions dominating the value of the index in the limit.

Overall the various stratification indices all portray a broadly similar picture of the pattern of stratification given that the correlation coefficient between the pairwise identification indices and mean income gaps was equal to 0.88. We have argued that stratification necessarily results in both pairwise identification and alienation so this positive correlation is to be expected but the strength of the association will likely differ depending on the specific nature of the process under consideration. In any case, reporting a range of indices serves to provide a fuller characterisation of the nature of stratification given that each individual measure has a clear and distinct interpretation in terms of the outcomes of the process. Recalling that a ceteris paribus increase in within-group inequality will (typically) reduce stratification, the combination in some poorer Asian countries, most notably China and India, of high per capita growth rates and the emergence of prosperous middle classes may be expected to have reduced overall levels of both alienation and identification between regions in more recent years.[17]

7. Conclusion

This paper offers a new class of indices that is based on a conceptualisation of stratification as a process that results in a hierarchical ordering of groups and therefore seeks to capture not only the extent to which groups form well-defined layers or strata in the income distribution but also the scale of the resultant differences in income standards between them, where these two factors play the same role as identification and alienation respectively in the measurement of polarisation (Esteban and Ray, 1994; Duclos et al., 2004). One important difference is that pairwise identification in our approach is equal to the difference in the odds that the income of a randomly chosen member of the richer group is more rather than less than that of a randomly selected member of the poorer group, rather than being a function of relative frequencies within income classes or at particular levels of income. Moreover, alienation between pairs of groups is defined as a power function of the absolute difference in income standards between them, providing a parametric class of measures that may be used by analysts and policymakers to evaluate the impact of differing degrees of inequality and disadvantage aversion on stratification.

[17] See Milanovic (2012) and Lakner and Milanovic (2013) for further discussion and evidence on trends in between-country inequality.

The main theoretical advantage of the proposed class of indices over existing measures of interdistributional inequality is that the indices are applicable to multiple groups and yet provide more information than ethical distance functions. The dominance properties of the indices are similar to those of the Duclos et al. (2004) polarisation measures. First the identification axiom distinguishes stratification from inequality since an income standard-preserving "squeeze" in the welfare distribution of one group cannot reduce identification under certain specified conditions whereas it will lead to a fall in inequality according to the Pigou-Dalton transfer principle. More straightforwardly, an identification-preserving scalar expansion of all welfare differences about the overall population income standard will unambiguously increase alienation and hence stratification. Finally stratification will typically be maximised if the population is equally divided between the richest and poorest groups. However it is important to recognise that stratification is not the same as polarisation due to the different characterisations of identification employed in the two sets of measures, with an axiomatic derivation of the proposed class of stratification measures remaining a topic for further research. The link between the stratification gap measure and the generalised Lorenz curve further suggests that it may be possible to establish welfare foundations for at least some members of the new class of indices.

The other major attraction of the proposed class of measures is their ease of interpretation and practical utility. In particular, the headcount or incidence measure gives the odds that the income of a randomly chosen member of a richer group is more rather than less than that of a randomly selected member of a poorer group, while the stratification gap also reflects the depth of stratification and index and may be interpreted as a measure of the perceived average difference in income standards between groups based on individuals' actual positions in the world income distribution. Each index is a population-weighted average of pairwise indices so it is possible to estimate the contribution of individual groups to observed levels of overall stratification with the further potential to identify the characteristics or factors that contribute to stratification. Reporting a range of measures rather than just one enables a fuller characterisation of the nature of stratification as shown by the illustrative study of global stratification in this paper. Estimation and inference procedures remain an issue for future work, with the Frick et al. (2006) estimator of the Yitzhaki and Lerman (1991) between-group Gini index providing a possible starting point. Given suitable procedures, it would be of interest to examine changes in global stratification over time as well as consider applications to a range of other socioeconomic phenomena such as the racial wage hierarchy in South Africa and gender pay differentials in earnings.

References

Allanson, P. (2014) Income stratification and between-group inequality, Economics Letters, 124, 227–230.

Allanson, P. and Atkins, J., (2005) The Evolution of the Racial Wage Hierarchy in Post-Apartheid South Africa, Journal of Development Studies, 41(6), 1023-50.

Anderson G. J., Ge, Y. and Leo, T. W. (2009) Distributional Overlap: Simple, Multivariate, Parametric and Non-Parametric Tests for Alienation, Convergence and General Distributional Difference Issues. Econometric Reviews, 29, 3, 247-275.

Andreoli, F. and Zoli, C. (2012) On the Measurement of Dissimilarity and Related Orders. ECINEQ, Society for the Study of Economic Inequality Working paper 2012–274.

Atkinson, A. B. (1970) On the measurement of inequality, J. Econ. Theory 2, 244–263.

Blackorby, C., Donaldson, D. and Auersperg, M. (1981) A new procedure for the measurement of inequality within and among population subgroups. Canadian Journal of Economics 14, 665–685

Bishop, J. A., Chow, K. V. and Zeager, L. A. (2010) Visualizing and Testing Convergence Between Two Income Distributions. Journal of Income Distribution 19, 1, 2-19.

Butler, R. J. and McDonald, J. B. (1987) Interdistributional Income Inequality. Journal of Business and Economic Statistics 5, 13-18.

De Baets, B., De Meyer, H. and De Loof, K. (2010) On the cycle-transitivity of the mutual rank probability relation of a poset, Fuzzy Sets and Systems 161, 2695–2708.

Deutsch, J. and Silber, J. (1997) Gini's "Transvariazione" and the measurement of distance between distributions. Empirical Economics 22, 547–554.

Deutsch, J. and Silber, J. (1999) Inequality Decomposition by Population Subgroup and the Analysis of Interdistributional Inequality, in J. Silber (Ed.), Handbook of Income Inequality Measurement (Dordrecht: Kluwer) pp. 363-397.

Duclos, J-Y, Esteban, J. and Ray, D. (2004) Polarization: Concepts, Measurement, Estimation. Econometrica 72, 1737–1772.

Esteban, J. and Ray, D. (1994) On the Measurement of Polarization. Econometrica, 62, 819–852.

Foster, J., Greer, J. and Thorbecke, E. (1984): A class of decomposable poverty measures. Econometrica, 52, 761–776.

Foster, J., Seth, S., Lokshin, M., and Sajaia, Z. (2013). A Unified Approach to Measuring Poverty and Inequality: Theory and Practice. Washington, DC: World Bank. doi: 10.1596/978-0-8213-8461-9

Foster, J.E. and Shneyerov, A.A. (1999) A general class of additively decomposable inequality measures. Economic Theory 14, 89–111.

Frick, R. J., Goebel, J., Schechtman, E., Wagner, G. G. and Yitzhaki S. (2006) Using Analysis of Gini (ANOGI) for Detecting Whether Two Sub-Samples Represent the Same Universe: The German Socio-Economic Panel Study (SOEP) Experience, Sociological Methods and Research, 34(4), 427–68.

Gastwirth, J.L. (1975) Statistical measures of earnings differentials. The American Statistician 29 (1), 32-35.

Gini, C. (1916) Il Concetto di "Transvariazione" e le sue prime applicazioni," Studi di Economia, Finanza e Statistica, editi del Giornali degli Economisti e Revista de Statistica, Reprinted in Gini (1959).

Gini, C. (1959) Memorie de Metadologia Statistica: Volume Secondo – Transvariazione. Rome: Libreria Goliardica.

Gradin, C. (2012) Race, Poverty and Deprivation in South Africa, Journal of African Economies, 22, 2, 187–238.

Heller, J. and Yitzhaki, S. (2006) Assigning fossil specimens to a given recent classification when the distribution of character variation is not normal, Systematics and Biodiversity, 4, 2, 161-172.

Jenkins, S.P. (1994) Earnings discrimination measurement: a distributional approach. Journal of Econometrics 61, 81–102

Lakner, C, and Milanovic, B. (2013) Global Income Distribution From the Fall of the Berlin Wall to the Great Recession, World Bank Policy Research Working Paper 6719.

Lasswell, T. E. (1965) Class and stratum. Houghton Mifflin, Boston.

Le Breton M., Michelangeli A. and Peluso, E. (2008) Wage Discrimination Measurement: In Defense of a Simple but Informative Statistical Tool. Università Commerciale Luigi Bocconi, Centre for Research on the Public Sector, Working paper 112.

Le Breton M., Michelangeli A. and Peluso, E. (2012) A stochastic dominance approach to the measurement of discrimination. Journal of Economic Theory 147, 1342–1350

Milanovic, B. (2012) Global Income Inequality by the Numbers: in History and Now — An Overview. World Bank Policy Research Working Paper 6259.

Milanovic, B. and Yitzhaki, S. (2002) Decomposing World income distribution: does the World have a middle class? Review of Income and Wealth 48 (2), 155–178.

Monti, M. and Santoro, A. (2011) Stratification and between-group inequality: A new interpretation. Review of Income and Wealth 57 (3), 412–427.

Mookherjee, D. and Shorrocks, A.F. (1982) A Decomposition Analysis of the Trend in UK Income Inequality. The Economic Journal 92 (368), 886–902.

Runciman, W.G. (1966) Relative Deprivation and Social Justice. Routledge, London

Shorrocks, A.F. (1982) On the Distance between Income Distributions. Econometrica, 50, 1337–1339.

van Kerm, P. (2013) Generalized measures of wage differentials. Empirical Economics, 45, 465–482.

Yalonetsky, G. (2012) Measuring Group Disadvantage with Inter-distributional Inequality Indices: A Critical Review and Some Amendments to Existing Indices, Economics: The Open-Access, Open-Assessment E-Journal, 6, 2012–9.

Yitzhaki, S. (1994) Economic distance and overlapping of distributions. Journal of Econometrics 61 (1), 147-159.

Yitzhaki, S. and Lerman, R. (1991) Income stratification and income inequality. Review of Income and Wealth 37 (3), 313-329.

Zhou, X. (2012) A Nonparametric Index of Stratification. Sociological Methodology, 42, 365-389.