To Be Fair

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1. Introduction

In this paper I present a theory of what it is to be fair. I take my cue from Broome's well known 1990 account of fairness. Broome's basic thesis is that fairness is *the proportional satisfaction of claims*, and with this I am in at least partial agreement. But neither Broome nor anyone else (so far as I know) has laid down a theory of precisely what one must do in order to be fair. The theory offered here does just this.[1]

Fairness is at issue in what I call *situations*, occurrences where an *allocating agent* has an amount of a *good* and allocates it amongst a set of *receiving agents*, each of whom has a *claim* to the good in question.[2] The *outcome* of a situation is the allocation of goods produced by the allocating agent (which can be probabilistic). To illustrate:

<u>Situation 0:</u>

Jones possesses eight apples and has promised three apples to Smith, three to Brown, and two to Clark. So (we may suppose) Smith and Brown have a claim to three apples each, whilst Clark has a claim to two apples. She gives five apples to Smith and two to Clark (leaving her with remainder 1). So we have:

Allocating agent:	Jones
Good:	Apples (Divisible)
Amount of good possessed:	8
Receiving agents:	Smith, Brown, and Clark.
Claims:	Smith(3 apples), Brown(3 apples), Clark(2 apples)
Outcome:	Smith(5 apples), Brown(0 apples), Clark(2 apples)[3]

[1] The theory concerns what has been called by Stone (2007) *allocative* justice rather than *distributive* justice, *viz.* the issue of how an agent should allocate a particular good amongst a set of people with claims when 'charged with the task [of doing so] in a direct manner, by giving the good to one or more recipients rather than employing some indirect distributive process, like a market'. (2007: 277) In what follows I assume that the amount of good possessed by an allocating agent is fixed and that it is not possible for recipients to share any amount of a good (so I assume, for example, that it is not possible to fully satisfy two claims to one unit of a good with a single unit of the good). So the theory is of limited scope. It does not, for example, tell an agent what to do in Taurek-style cases (Taurek 1977).

[2] How do claims arise? This is an interesting question, but it is not one that I consider here, and the theory I present is silent about this – it merely applies to claims when they do arise.

[3] This outcome is, of course, intuitively unfair (and my theory will say that it is). But this is just an illustrative example for which I picked an arbitrary outcome.

We are only interested in particular features of situations. We are not interested in: Who the allocating agent is; why she has the good; what the good is; who the receiving agents are; how their claims arise. But we are interested in: Whether the good is divisible or indivisible; how much good is possessed by the allocating agent; how many receiving agents there are and what claims they have; which outcome occurs. So, below I represent situations in a minimal way by listing just these latter features. For example, Situation 0 is represented as:

<u>Situation 0:</u>

Good:	8 (Divisible)
Claims:	A(3), B(3), C(2)
Outcome:	A(5), B(0), C(2)

Jones might instead have allocated the good on the basis of a lottery, e.g. by giving each receiving agent a 1/3 chance of receiving all 8 applies. This outcome is represented as:

Outcome:	A(8), B(0), C(0) with probability 1/3
	A(0), B(8), C(0) with probability 1/3
	A(0), B(0), C(8) with probability 1/3

The basic principle that the theory to be presented is supposed to capture is:

The Fairness Claim (FC): In order to be fair an allocating agent [AA] must (i) do as much as she can to satisfy the claim of each receiving agent [RA] to as great a degree as possible whilst ensuring that (ii) each claim is treated equally.

FC is still not a precise statement of what it is to be fair (if it were, we wouldn't need a theory of fairness at all). The theory to be presented is supposed to correctly capture, elaborate, and make clear the content of FC. I turn now to that theory.

2. The Theory of Fairness

What does FC tell AA to do in a given situation? This depends on particular features of the situation. The easy cases are those situations involving divisible goods. There are just two kinds of case: those situations in which AA has enough good to go round and those in which AA does not. In the former kind of case it is straightforward that FC tells AA to satisfy each claim in full. In the latter kind of case it is straightforward that FC tells AA to use *all* of the good she possesses and assign to each RA a quantity of good that is proportional to their claim. (That she must use *all* of the good is demanded by part (i) of FC, and that she divides it proportionately is demanded by part (ii)). The general method for proportionately assigning goods in these latter kinds of situation is P(roportionality):

Method P:
(a.) Divide the total amount of good possessed by the total amount owed. Move to step Pb. (b.) Multiply the figure obtained in step Pa by what is owed to each RA to obtain a number for each RA. Move to step Pc. (c.) Give each RA an amount of good corresponding to the number obtained for them in step Pb.

Moreover, it is also clear that FC tells AA to follow method P in those situations involving indivisible goods that are such that the result of following method P results only in integers at step Pb.

Examples:

<u>Situation 1</u>

Good: 60 (Divisible)

Claims: A(20), B(20), C(20)

AA can satisfy all claims in full, so FC tells her to do the following:

Produce outcome: A(20), B(20), C(20)

<u>Situation 2:</u>

Good: 4 (Divisible)

Claims: A(2), B(4), C(8)

AA cannot satisfy all claims and the good is divisible, so FC tells her to follow method P and do the following:

Produce outcome: A(8/14), B(1+2/14) C(2+4/14)

<u>Situation 3:</u>

Good: 6 (indivisible)[4]

Claims: A(12), B(24)

Here, method P divides the good perfectly at step Pb. So FC tells her to do following:

Produce outcome: A(2), B(4)

[4] NB To be clear, when I say that the good is indivisible, I mean that each *unit* of the good is indivisible.

The situations left to consider are more interesting. They involve indivisible goods such that applying method P to them results in numbers with a fractional component at step Pb. In order to deal with these situations, first ask: what do the figures produced at step Pb represent? The answer is that they represent the *proportional claims* of the RAs – i.e. the amount of good that each RA has a claim to *given the amount of good available*. So in these situations, if method P assigns a number with a whole number component to any RA at step Pb, RA should be assigned an amount of good that is at least equal to that whole number in the final outcome. For example, consider:

<u>Situation 4:</u>

Good: 8 (indivisible)

Claims: A(5), B(4)

Pb tells us that the proportional claims are A(4+4/9) , B(3+5/9) – so both numbers have a whole number component and a fractional component. AA has enough good available to give at least 4 units of the good to A, and 3 to B. Given they each have a proportional claim to at least that much, AA should assign at least that much to them in the final outcome. This leaves AA with remainder 1, and the question remains: what should she do with the remainder?

In order to answer this question consider situations involving indivisible goods where applying method P results only in numbers with fractional components at step Pb, e.g.:

<u>Situation 5:</u>

Good: 2 (indivisible)

Claims: A(9), B(8), C(3)

Here the proportional claims are A(18/20), B(16/20), C(6/20). Now ask: what does FC demand of AA in situations of this kind? Here the answer is not so clear. But I have an attractive suggestion. AA cannot proportionately satisfy each claim. So instead I suggest she must assign a *probability* of being at least partially satisfied to each claim that is proportional to its strength and as *high* as possible. That she assigns a proportional probability of partial satisfaction to each claim is, I suggest, what is demanded by part (ii) of FC and that the probability is as high as possible is, I suggest, what is demanded by part (i). Thus the outcome that is produced should be one such that in no other possible outcome (in which the claims are treated equally) does any claim receive a higher chance of partial satisfaction. But what precise method should AA use to assign the probabilities in order to achieve this result?

Note first that each fractional proportional claim in situations of this kind is not a claim to that fraction *of the total good possessed by AA*, but rather to *1 unit of the good* *possessed by AA*. So, for example, in situation 5, A has a proportional claim to *18/20 of 1 unit of the available good*, not to *18/20 of the total good available*. Now, the good is indivisible, so 1 unit of the good is the smallest possible amount that any RA can receive. So, if the good is to be allocated at all, it is inevitable that some RA will receive more than they have a proportional claim to. But, once an RA has been assigned 1 unit of the available good, as they have already received more than they have a proportional claim to, they should not then be assigned *more* of the available good. If there is any remaining good it should be used to (at least partially) satisfy one of the other remaining claims. So we need to restrict our attention to those outcomes in which no RA receives more than 1 unit of the available good. So we can give an initial statement of the method required as L(ottery):

Method L:

In situations involving indivisible goods where applying method P results only in numbers with fractional components at step Pb:

- Assign to each RA a probability of receiving 1 unit of the available good that is equal to their proportional claim.

However, L is only a partial statement of the correct method. It tells AA which probabilities she needs to assign to each *RA*. But AA needs to know more than this. She needs to know which probabilities to assign to each *possible allocation of the good*. In situations where AA possesses only 1 unit of good, it is easy to see how to assign probabilities to the possible allocations that gives each RA the correct proportional chance of receiving 1 unit of the good. In such situations, for every RA, there is only a single allocation in which they receive 1 unit of the good, and AA simply has to assign to that outcome a probability equal to their proportional claim. For example, consider:

<u>Situation 6:</u>

Good: 1 (indivisible)

Claims: A(10), B(8), C(2), D(20)

Here the proportional claims are A(10/40), B(8/40), C(2/40), and D(20/40). There are just four allocations in which no RA receives more than 1 unit of good. And each RA receives 1 unit of the good in only one allocation. So AA simply has to assign a probability to each allocation that is equal to the proportional claim of the RA that receives the good in that allocation. That is, AA simply has to do the following:

Produce outcome: A(1), B(0), C(0), D(0) with probability 10/40

A(0), B(1), C(0), D(0) with probability 8/40

A(0), B(0), C(1), D(0) with probability 2/40

A(0), B(0), C(0), D(1) with probability 20/40

But in situations where AA possesses more than 1 unit of good things are not so simple. In such situations there will be multiple allocations in which each RA receives 1 unit of the available good. Situation 5 above is such a situation. There are three allocations in which no RA receives more than 1 unit of good. So AA must do the following:

Produce outcome: A(1), B(1), C(0) with probability x

A(1), B(0), C(1) with probability y

A(0), B(1), C(1) with probability z

But what values should AA give to x, y and z? Note first that A receives 1 unit of good in the first and second allocations, B receives 1 unit in the first and third allocations, and C receives 1 unit in the second and third allocations. So, AA must assign probabilities to these outcomes in such a way that the following equations are jointly satisfied:

$x+y=18/20$

$x+z=16/20$

$y+z=6/20$

In other words, the situation determines a system of linear equations that AA must solve in order to be fair. In this case the system has a unique solution, *viz*. $x=14/20$, $y=4/20$, and $z=2/20$. So, according to L2, AA must do the following:

Produce outcome: A(1), B(1), C(0) with probability 14/20

A(1), B(0), C(1) with probability 4/20

A(0), B(1), C(1) with probability 2/20

Clearly, the above way of proceeding generalises. All situations of this kind will determine a system of linear equations the solution of which will result in an outcome such that in no other possible outcome (in which the claims are treated equally) does any claim receive a higher chance of partial satisfaction. The number of equations in the system generated will be equal to the number of RAs, and the number of variables equal to the number of allocations. The only possible coefficients in any equation generated by situations can be 0 and 1, and there can never be more RAs than there are allocations. These two facts are enough (due to the Rouché-Capelli theorem) to guarantee that every system of linear equations will have at least one solution. But there need not be a *unique* solution. In general, if a situation determines a set of linear equations with more variables than equations, there will in fact be infinitely many solutions.

Consider:

Situation 7:

Good: 2 (indivisible)

Claims: A(5), B(9), C(10), D(6)

The proportional claims here are A(10/30), B(18/30), C(20/30), D(12/30). There are 6 allocations to consider, so to be fair AA must do the following:

Produce outcome: A(1), B(1), C(0), D(0) with probability x_1

A(1), B(0), C(1), D(0) with probability x_2

A(1), B(0), C(0), D(1) with probability x_3

A(0), B(1), C(1), D(0) with probability x_4

A(0), B(1), C(0), D(1) with probability x_5

A(0), B(0), C(1), D(1) with probability x_6

This determines the following system of linear equations:

$x_1+x_2+x_3=10/30$

$x_1+x_4+x_5=18/30$

$x_2+x_4+x_6=20/30$

$x_3+x_5+x_6=12/30$

Here there are four equations and six variables (and so two free parameters), and so this system has infinitely many solutions, e.g.:

1. $x_1=1/30$, $x_2=2/30$, $x_3=7/30$, $x_4=15/30$, $x_5=2/20$, $x_6=3/30$

2. $x_1=5/30$, $x_2=4/30$, $x_3=1/30$, $x_4=9/30$, $x_5=4/20$, $x_6=7/30$

….

But this matters not. Each solution assigns probabilities to the allocations in a way that gives each RA the correct overall probability of receiving 1 unit of the good. So all this means is that, in some situations, there are infinitely many ways to be fair. AA must just pick one.

I return now to the question posed above about what AA should do with the remainder good in situations where the good divides proportionately leaving such a remainder. It is now easy to see what should be done. We were considering situation 4 in which the proportional claims are A(4+4/9), B(3+5/9). A gets allocated 4 units of the good and B 3 units directly, leaving remainder 1. A has a remaining proportional claim to 4/9 of 1 unit of the good, and B 5/9 to 1 unit. So AA should simply run a lottery that gives A a 5/9 chance of getting the remaining unit, and B a 4/9 chance of getting it. In other words, AA should do the following:

Produce outcome: A(5), B(3) with probability 4/9

A(4), B(4) with probability 5/9

Of course, in more complicated situations, where there is more than 1 unit of good remaining, AA will have to allocate the remaining good by solving systems of linear equations in order to assign probabilities as before. But how this is to be done should now be clear.

Drawing all of the above together, then, we get the completed theory of what it is to be fair:

Theory of Fairness

An allocating agent is fair iff she follows the 1-2-3 method, viz:

1. If possible satisfy all claims in full and stop. Otherwise, move to step 2:

2. Satisfy all claims proportionally and to as great a degree as possible with the available good. If there is any remainder, move to step 3. That is:

(a.) Divide the total amount of good possessed by the total amount owed to obtain a number *g*. Move to step 2b.
(b.) Multiply *g* by what is owed to each receiving agent $R_1, R_2, \ldots R_n$ to obtain a number $r_1, r_2, \ldots r_n$ for each of them. Move to step 2c.
(c.) Assign each R_n a quantity of good corresponding to the whole number component of their number r_n. Move to step 2d.
(d.) If $r_1, r_2, \ldots r_n$ have no fractional component, stop. Otherwise, move to step 3.

3. Give all remaining proportional claims a chance of being satisfied that is proportional to their strength and as high as possible. That is:

(a.) Find all possible allocations in which all of the remaining good is allocated but no *R* receives more than 1 unit. Move to step 3b.
(b.) Label the allocations $x_1, x_2, \ldots x_n$. Form an equation for each R_n consisting of the addition of every allocation $x_1, x_2, \ldots x_n$ in which R_n receives 1 of the remainder good on the left-hand side, and the fractional component of r_n on the right-hand side. The result will be a system of linear equations. Move to step 3c.
(c.) Solve the system of linear equations formed in step 3b and produce a lottery in which each allocation $x_1 \ldots x_n$ is given a probability of occurring equal to the number given to it in the solution. (If there is more than one solution, then pick one of them.) Move to step 3d.
(d.) Produce whatever allocation the lottery decides and stop.

3. Relation to Broome's View

Broome would disagree with my theory on a number of grounds that stem from the basic principle that underlies Broome's view, *viz*:

- In order to be fair an allocating agent must satisfy all claims proportionately. (Broome 1990: 95)

Firstly, Broome's principle implies that so long as one satisfies claims proportionately it does not matter (so far as fairness is concerned) how much of the available good one uses in doing so. I reject this.[5] I think that in order to be fair to those who have claims one must do all one can to satisfy their claims and (where the sum of all claims is greater than or equal to the amount of good possessed) I think this entails using all of the good that one has available in some way or other. This is why the basic principle underlying my theory is not Broome's, but FC.

Secondly, Broome's principle implies that any deviation from proportional satisfaction is, strictly speaking, a deviation from fairness. So, on Broome's view, holding a lottery in situations involving indivisible goods where AA does not have enough to go round necessarily results in an allocation that deviates from fairness (1990: 97). I reject this too. Whilst it is true that the allocations that result from a lottery in such situations are such that some claims will be satisfied to a greater degree than others, in my view this is not a deviation from fairness, because each RA is treated equally; each is directly allocated as much of the good as possible given her proportional claim, and is assigned a proportional chance of receiving one unit of the remainder.

The second disagreement leads to a third.[6] In addition to requiring proportional satisfaction (due to fairness), Broome maintains that claims also require satisfaction *simpliciter* (1990: 95-96). So Broome also advocates the use of lotteries in those situations where the requirement to satisfy claims *simpliciter* is more important, all things considered, than the requirement to satisfy them proportionately. However, in such circumstances, Broome maintains, there is a need to minimise unfairness – i.e. to produce an allocation that is as close to one of perfect proportional satisfaction as possible. To see how this leads to disagreement, consider a situation in which AA possesses 1999 units of an indivisible good, one RA has a claim to 1000 units, and one-thousand RAs have a claim to 1 unit:

<u>Situation 8:</u>

Good:	1999
Claims:	A(1000), B_1(1), B_2(1), B_3(1)… B_{1000}(1)

[5] I do so for the reasons given by Hooker (2005: 340-41). See also Saunders (2010: 44-47).
[6] I thank an anonymous referee for pressing this point.

The proportional claims here are A(999+½), B_n(1999/2000). According to my theory, AA should do the following:

Produce outcome:	A(999), 1000xBs(1)	with probability ½
	A(1000), 999xBs(1), 1xB(0)	with probability ½ [7]

In each allocation only one RA fails to have her claim fully satisfied. But, in the first allocation (where the agent is A) the shortfall is only 0.1%, whereas in each of the others (where the agent is one of the Bs) the shortfall is 100%. So, the first allocation is closer to one of perfect proportional satisfaction than any of the others. So, on Broome's view, the first allocation is preferred to each of the others, and so is the one that should be produced with a probability close to or equal to 1.

I think the outcome recommended by my theory in this case (and those like it) is preferable to the one Broome would recommend for a reason that should now be clear. To weight the lottery in the way described is to treat the fractional component of A's proportional claim as being less important than those of the Bs. It is thus to treat the claims of the Bs preferentially to A's claim, and it is thus to treat the claims unequally, and thus unfairly.[8]

[7] There are 1000 allocations in which all but one B gets 1 unit of the good, each of which is assigned a probability of 1/2000 by AA. For brevity I have combined these allocations into a single line here. But note well: each individual B still has a 1999/2000 chance of receiving 1 unit of the good.

[8] I thank Stefano Predelli, Neil Sinclair, Harold Noonan, and Joe Noble for comments and helpful discussion in preparing this paper.

References

Broome, J. 1990. Fairness. *Proceedings of the Aristotelian Society* 91: 87-101.

Hooker, B. 2005. Fairness. *Ethical Theory and Moral Practice* 8: 329-352.

Saunders, B. 2010. Fairness Between Competing Claims *Res Publica* 16: 41-55.

Stone, P. 2007. Why Lotteries Are Just. *The Journal of Political Philosophy* 15: 276-295.

Taurek J.M. 1977. Should the numbers count? *Philosophy and Public Affairs* 6: 293-316.

Abstract:

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