Different Geometries for Special Relativity

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Abstract

This paper introduces a different time-measuring convention for special relativity (SR), where a time interval t can be measured by dc, the distance traveled from an origin by the spherical wave-front of a light pulse c. Adoption of this convention leads to a Euclidean geometry for SR, different from the Euclidean geometry already proposed by Montanus. The present geometry is governed by the functions of the circle, rather than the hyperbola, and the spherical wave-front of a light pulse provides both a fourth set t of frame-dependent coordinate points and a parameter w for measuring intervals that are invariant between reference frames. Since sine values under the circle range from 1 to 0, rather than 1 to ∞, the new model does not allow, for a reference frame velocity ≈ c, any interval to have length ≈ ∞. Furthermore, the form of the new model excludes any notion of "travel" with respect to time.

Key words: time, conventionalism, Minkowski geometry, circular function geometry, time travel, infinite energy

1. INTRODUCTION

This paper argues that the established non-Euclidean geometry of Minkowski is not the exclusive or "necessary" means of modeling special relativity (SR). Given different conventions for space-time, SR can be satisfactorily modeled with Euclidean geometry, governed by the functions of the circle, rather than the hyperbola. Hans Montanus has already described one such Euclidean model, which he has called absolute Euclidean space-time (AEST).(1,2) This paper introduces another relativistic Euclidean geometry, herein termed circular function space-time (CFS).

The value of considering these alternative methods of modeling SR is not so much to advocate the use of one rather than another but to test the extent to which physicists are entitled to claim that the structure of the natural world "corresponds" to the particular properties of the geometry employed. Architects would not quarrel whether a perspective projection drawing of a building is more or less "true" than an orthographic one, as long as they agree that both representations accurately identify the interrelationships of the building's observable features. This also seems to be the relevant criterion in considering different space-time geometries. Where there is disagreement about what is represented, it would appear sensible to consider more fully the consistency of the means of representation. It will be argued below that one benefit of considering Euclidean space-time geometry is that it shows that conundrums concerning infinite quantities and "travel" relative to time are due to the properties of Minkowski geometry rather than those of nature.

2. THE CONVENTIONALIST DEFINITION OF TIME

Minkowski's geometry results from the approach Einstein adopted to define space and time. Einstein chose to define each solely by appealing to the established conventions physicists used to measure them.(3) The presence of rigid measuring sticks would indicate intervals of space, and, as for time, "… we understand by the 'time' of an event the reading (position of the hands) of that one of these clocks which is in the immediate vicinity (in space) of the event."(4)

This pragmatic approach to space and time is termed "conventionalist,"(5) and it allowed Einstein to bypass deeper philosophical consideration of these terms simply by postulating that in physics, spatial intervals are indicated by observations of rigid rods and temporal intervals are indicated by observing the movements of the hands of a pocket watch. Conventionally, rods are measured in meters and the displacements of hands about a circular clock face are measured in seconds of the arc. By focusing solely on the observation of instruments, it would appear that Einstein disposed of any need for a deeper understanding of how it is that these observations can satisfactorily identify the essential features of space and time. However, the most naive consideration of the behavior of a watch shows that Einstein's conventionalist definition of time is far from being either self-evident or self-sufficient.

Observation of the kind of pocket watch he describes above reveals the "position of the hands" to be cyclical and reversible about the same starting point. Yet the convention of Western science is to see time as a linearly asymmetric ordering, such as the t axis of Minkowski geometry that is formed by the graphic plotting of successive clock readings. Furthermore, while it is conventional to measure the displacement of clock hands in seconds, it is conceivable that if all watches had been constructed with a standard size face, then the displacements of all hands could have been measured in spatial units, such as millimeters. Had physicists created such a convention in the era when the pocket watch was being developed, then it is highly unlikely Minkowski would have developed a space-time geometry that employed two different units of measure, meter *and* second.

These considerations reveal Einstein's conventionalist definition of time to be an impoverished one, since its adoption is only justifiable by appeal to more fundamental inherited concepts of time. The first of these is that any record of the displacements of a clock hand shows its journey about the clock face to be one of constantly increasing length. The second is that the observer must acquire records of such events in order to acquire any usable notion of time. Since events appear as transient orderings, it is only the records of them that appear to form the "tangible" or "durable" physical orderings, which can be usefully employed in theoretical speculation.

Let us here consider record-keeping as a feature of the kind of mental activity that also includes the mapping and geometrical analysis of spatio-temporal events. The key characteristic of recording is that every registry of data is additive, never subtractive. Hence, a "later" record must always accumulate more data than an "earlier" one, in just the way that a given natural number set must by definition always include all its predecessor numbers among its elements. The Western convention for ordering a set of clock readings T requires that the readings $\{t_1, t_2, t_3, \ldots\}$ be arranged in the linear, asymmetric order of the set of counting numbers N in order to form a set of time coordinates. The conventions for this ordering of T date back at least as far as Aristotle ("Time is the number of motion in respect of before and after")[6] and were established independently of any later theories concerning irreversible processes and the like.

Hence it is not easy to divorce the raw observation of the movements of hands about a clock face from the recognition that records always show their journey to comprise a constantly accumulating number of regular displacements. An observable process that perfectly demonstrates such a constant asymmetric journey is the propagation of light from a point origin in space. Light propagation c is, of course, the third essential observable measure for Einstein's SR theory. Yet, rather than adopt intervals of light propagation to indicate time, Einstein and Minkowski chose to retain "seconds" of the arc for understandable practical reasons.

However, Minkowski recognized that the expansion of a light pulse provided a fundamental "natural" measure for his geometric manifold, which could be viewed either in terms of a "dynamic" temporal period, such as 1 s of light travel, or in terms of a "static" distance of 300 000 km of light travel.[7] The fact that these measures were completely interchangeable led Minkowski to claim that this geometric manifold was a unitary one, being the union of two sets of different interval measures, meter and second.[8] Hyperbolic geometry provided the most elegant means of uniting these different measures of the same relativistic interval, which preserved the distinctions between space and time in the form of a "time-like" region within the "light cone" and a "space-like" region outside of it.

Yet if it is true that the measures of light meter and light second are interchangeable, then in principle it should be possible to use only *one* of these measures in modeling the space-time continuum. This possibility is easily demonstrated by considering the mapping of nonrelativistic events such as those illustrated in Fig. 1, which is a scale map of two objects on a table, a stopwatch with hands C, and a ball A, rolling across the surface.

Observers create the map by plotting what they record to be the successive coincidences $\{C_1, A_1\}$, $\{C_2, A_2\}$, ... of the positions of the second hand about the clock face with the positions of the ball on the tabletop. Since the map plots all displacements to scale, it gives observers freedom of choice about whether

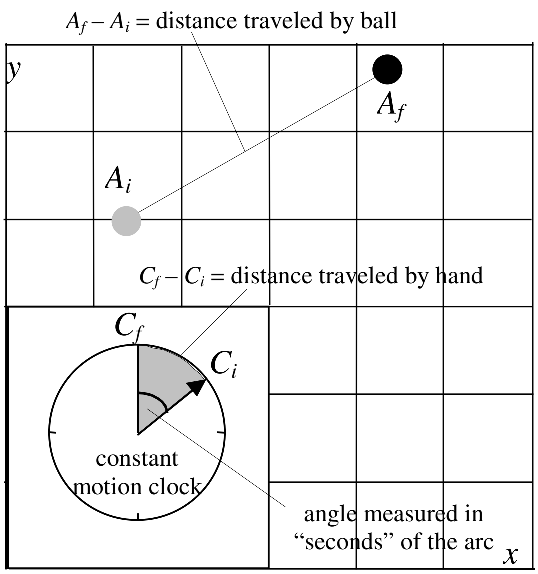


Figure 1. Velocity of a body A, measured as a ratio of distances traveled.

they wish to measure the intervals traveled by the hand of C as "seconds" or as "millimeters." While some observers could choose to characterize the intervals $\{C_1, A_1\}$ to $\{C_2, A_2\}$ over which these events are recorded as "periods of duration," that does not prevent other observers from electing to measure these same intervals in terms of the displacement they actually *observe*, as distinct from the "time elapse" they "sense."

To subscribers of this alternative convention, the map itself simply shows two spatial displacements dA and dC. Comparing the lengths of these two displacements provides a proper measure of their relative motion, without reference to seconds. Under this new convention, time is always measured in units of spatial displacement, and the velocity v_A of the moving ball can be redefined as the ratio of its displacement dA, from an initial position A_i to a final one A_f, relative to the displacement dC, over the interval i to f by the designated clock motion C:

$$v_a = (A_f - A_i) : (C_f - C_i) = \frac{dA}{dC}. \tag{1}$$

In situations involving SR, if light propagation is selected as the standard clock, then its properties as the natural constant of SR allow for the development of a further new rule for mapping events. Since SR postulates that nothing travels at a velocity greater than c, then we have the following:

$$\begin{aligned}&\text{For all } v,\ v_c \geq v.\\ &\text{Hence for } da > 0,\ dc > 0,\\ &\text{and where } dc = 0,\ \text{then } da = 0.\end{aligned} \tag{2}$$

This choice of convention then permits observers to substitute the time "measured in light seconds" axis ct of the Minkowski "light cone" with a "time measured in distances traveled by a light pulse" parameter w. As is illustrated in Fig. 2, rather than being a flat plane like the ct one of Minkowski space, w is the curved surface of a "light sphere" centered on the spatial origin of the light pulse o, with dc being the radius of w. Events occurring within this sphere may then be analyzed using circular function geometry, revealing a Euclidean analogue, CFS, of Minkowski's hyperbolic function space-time (HFS).

In common with AEST, CFS is what Montanus calls an "absolute" space-time in that it takes the observer's reference frame as the preferred one and takes the observer's time as the time parameter.[9] However, AEST retains the ct axis of Minkowski geometry as a general time parameter and introduces a fifth dimension, the proper times of objects, to act as their time coordinates.[10] In contrast, CFS is only four-dimensional, and, as will be discussed below, its time parameter is the spherical surface w, which may be seen to comprise an infinite set of unique points t_p that act as the time coordinates of objects.

3. CIRCULAR FUNCTION ANALYSIS OF SR

Figure 2 is a two-dimensional map of the relativistic relationship between an observer's reference frame S and a translating reference frame S'. The map is made by the same means as employed by its Minkowski analogue, where observers in the S frame of reference retrospectively plot data retrieved from a series of observational devices such as movie cameras, placed at regular distances throughout the S frame. The circle, centered on the origin o of the S coordinate system o, x, y, represents their observations of the wave-front w of a light flash c, expanding equidistantly from o. Since c is a natural constant, observers who retrieve data from detectors that are regularly spaced throughout their reference frame will obtain records of a regular sequence of light expansion intervals, dc_1, dc_2, dc_3, …, the unit length of which are determined by the actual spacing of their observational devices. S observers may then use any sequence of these "snapshot" records of the expanding wave-front s to relate the coincidences of all "point"

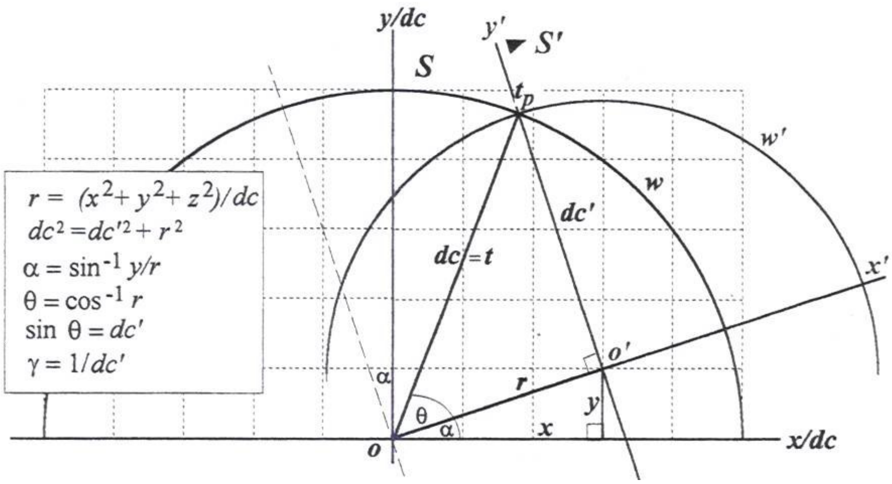


Figure 2. CFS modeling of translating system S', viewed from observers' rest-frame S.

particles observed at "point" places in S, with what they judge to be the simultaneous position of s.

The time coordinates of any reference frame may then be found as a unique pair of points t_p located on the surface w. The points t_p are found by following the trajectory of the given reference frame over the interval dc and plotting the axis that is perpendicular to the direction of the trajectory, which in the case of S' in Fig. 2 is the y' axis. S' is a translating, inertial frame of reference, separated from the x, y plane of S by a negligible point distance in the z direction. Over the interval dc, S' translates $0.63dc$ in a $+xy$ direction. Assume that the light flash c is triggered by the near coincidence of the origins of S and S'. Observers in S can then find two points where the y' axis meets the sphere w, and by mapping a straight line dc from their origin to one of these points t_p, they find a common coordinate for both their measurement of time and that of observers in S'.

Since the trajectory of S' is relatively diagonal to observers in S, they rotate their own y axis by the angle α to find the point t_p at which the wave-front c meets y' across a given translation interval $r = (x^2 + y^2 + z^2)^{1/2}$, where

$$\alpha = \cos^{-1} r. \tag{3}$$

The length of the line dc that connects the observer's origin o to t_p gives the observer's constant, frame-dependent, space-time interval t:

$$dc = t = 1. \tag{4}$$

The interval separating t_p from o' gives the S' frame's simultaneous measurement dc' of the observer's interval dc. The angle θ_r relating the juncture of dc and dc' is given by

$$\theta_r = \cos^{-1} r. \tag{5}$$

Following the first postulate of SR that the velocity of light propagation is the same for observers in all inertial reference frames, the judgment of the length of dc' will remain frame invariant, with dc' being another function of the circle, i.e.,

$$dc' = \sin \theta_r, \tag{6}$$

and

$$dc'^2 = dc^2 - (dx^2 + dy^2 + dz^2). \tag{7}$$

The length of dc' then provides the frame-invariant metric ds for CFS, analogous to that of the Minkowski (HFS) metric:

$$\begin{aligned} &\text{HFS: } ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \\ &\text{CFS: } ds^2 = dc^2 - (dx^2 + dy^2 + dz^2). \end{aligned} \tag{8}$$

As for the spatial separation between two points A and B, we have the following:

$$\text{HFS: } \Delta s_{AB}^2 = c^2(t_A - t_B)^2 - (r_A - r_B)^2 = c^2(\tau_A - \tau_B)^2, \tag{9}$$

$$\text{CFS: } \Delta s_{AB}^2 = (dc_A - dc_B)^2 - (r_A - r_B)^2 = (dc'_A - dc'_B)^2. \tag{10}$$

In Minkowski geometry there is an equivalence between ds, the proper time τ, and the velocity parameter $\beta = [1 - (v/c)^2]^{1/2}$, which is also found in CFS:

$$\text{HFS: } ds^2 = c^2 d\tau^2, \quad \text{CFS: } ds^2 = dc'^2. \tag{11}$$

From this it follows that, in CFS,

$$dc' = ds = \tau = \beta. \tag{12}$$

Figures 3a and 3b further compare CFS and HFS, showing that in CFS, the observer's plotting of successive t_p points on the axes of translating inertial frames reveals a straight line ot_p, related to the origin of the S coordinate system by the angle θ_r. These lines can be designated in CFS as "clock lines," since at any point Δr along the trajectory of a translating inertial frame, the frame's proper time $\tau = dc' = \sin\theta_r$ can be easily measured using the familiar rules of circular function geometry:

$$\cos^2\theta_r + \sin^2\theta_r = 1 = dc. \tag{13}$$

By combining rotation of axes with measurement of dc' under a clock line, observers can further deduce the Lorentz transformations of their coordinates to the simultaneous space of translating inertial frames, such as S' in Fig. 2. Observers in S use the frame-invariant length of dc' to determine the simultaneous space of S', which is another light sphere of relatively smaller dimension, shown in Fig. 2 by the gray circle centered on o'. Hence in CFS the Lorentz transformation of coordinates between reference frames is a displacement and shrinking of Euclidean spheres rather than a tilting and stretching of non-Euclidean light cones. In CFS the Lorentz transformation factor γ is given by

$$\gamma = \frac{1}{\sin\theta_r} = \frac{dc}{dc'}. \tag{14}$$

Returning to the example illustrated in Fig. 2, for $v = 0.63c$, $\theta_r = \cos^{-1} 0.63 = 51°$. Then $\gamma = 1/\sin 51° = 1.29$, which is exactly the same value found for a frame S' translating at $0.63c$ when using the standard SR equation for the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}. \tag{15}$$

Thus the functions of the circle — radius, sine and cosine — can be used to represent, respectively, t, τ, and v for the analysis of "inertial" reference frames as described in SR. CFS analysis of accelerating reference frames raises issues that will only be touched upon here.

As illustrated in Fig. 4, the presence of forces acting on a translating object will be indicated by the curvature of clock lines and the corresponding variations in the length of $dc' = \tau$ relative to the length of $ot_p = dc = t$. In the case of gravitational forces, this does not necessarily imply a curvature of the axes or deformation of the light sphere, especially not over the infinitesimal distances presumed in CFS. The only space-time "curvature" is that of clock lines within the (locally) Euclidean space. This is also characteristic of AEST, where Montanus argues that the notion of curved space-time is dispensable.[11]

4. COMPARISON OF MOMENTUM-ENERGY VECTORS IN CFS AND HFS

Figure 5b illustrates how in Minkowski geometry the relationships between relativistic mass, energy, and momentum, m, E, and p, are derived by dividing all vector lengths under the hyperbolic parameter s by the frame-invariant displacement $d\tau$.

The analogous operation in CFS is to divide all vector lengths under the circular parameter w by the displacement dc, as illustrated in Fig. 5a. Different energy-momentum relationships appear under the circle compared to the hyperbola. Since the radius $dc = t$ is the chosen reference frame interval, the ratio dc/dc indicates the total energy E_0 of objects relatively at rest to that frame. In Minkowski geometry the frame-invariant vector $m = d\tau$ is the chosen measure and may have values ranging from 1 to ∞. In CFS, since $dc = dt = E_0 = 1$, the ratio dc/dc' gives E_0', the relative *rest* energy in the accelerated frame, and dc/dr indicates the relativistic momentum p of the accelerated reference frame in terms of $E_0/p = 1/p$. Thus, in CFS vector space,

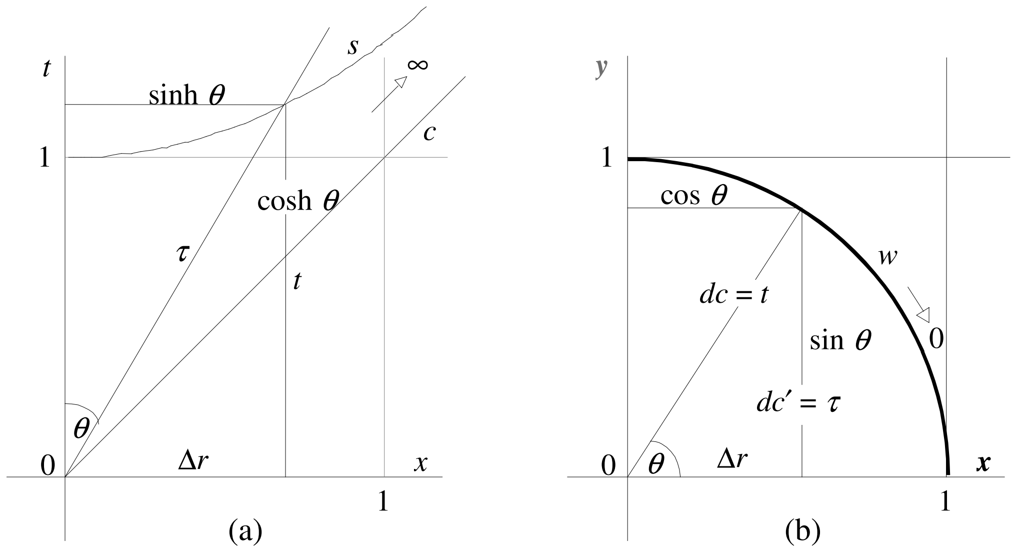


Figure 3. Comparison of Minkowski and CFS mapping of the translation $\Delta r = 0.55$.

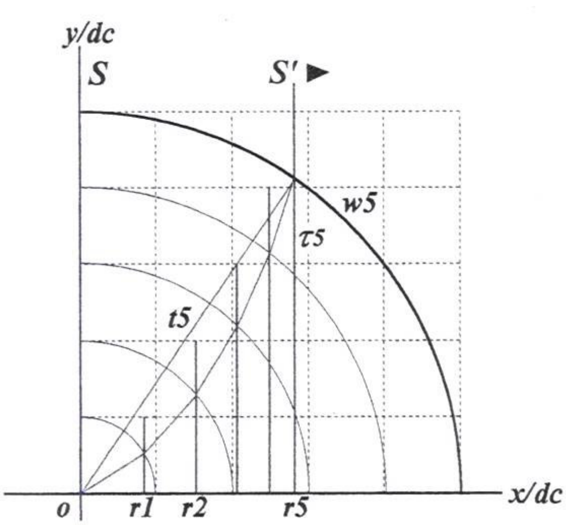


Figure 4. CFS mapping of accelerating reference frames.

$$\sin^2 dc + \cos^2 dc = dc^2 = (E_0')^2 + \left(\frac{E_0}{p}\right)^2 = 1^2, \quad (16)$$

$$E_0 = dc = dt = 1, \quad (17)$$

$$E_0' = dc' = d\tau = \frac{1}{\gamma} = \frac{E_0}{\gamma}, \quad (18)$$

$$m = \frac{E_0}{E_0'} = \frac{dc}{dc'} = \gamma E_0. \quad (19)$$

As comparison of Figs. 5a and 5b shows, in CFS, the displacement $dc' = \sin\theta$ diminishes from 1 at relative rest to 0 at the limit spatial displacement $dr = dc$, whereas in the Minkowski analogue, the directrix of the hyperbola, $d\tau = \cosh\theta$, grows from 1 to infinity at the limit velocity $v = c$. Hence in CFS comparison of the rest value ^{0} with the limit velocity value * for momentum-energy reveals the following set of relationships:

$$\frac{E_0}{p^0} = 0, \; E_0 = dc, \; p^0 = 0, \quad (20)$$

$$\frac{E_0}{p^*} = 1, \; E_0 = dc, \; p^* = dc, \quad (21)$$

$$\frac{E_0}{E_0'^0} = 1, \; E_0 = dc, \; E_0'^0 = dc, \quad (22)$$

$$\frac{E_0}{E_0'^*} = 0, \; E_0 = dc, \; E_0'^* = 0. \quad (23)$$

In one respect, (20)–(23) lead to the same conclusions derived from Minkowski momentum-energy vectors: For differing frame velocities, total momentum-energy remains constant in all frames, but measurements of momentum and energy vary from frame to frame. In common with HFS, CFS indicates that the

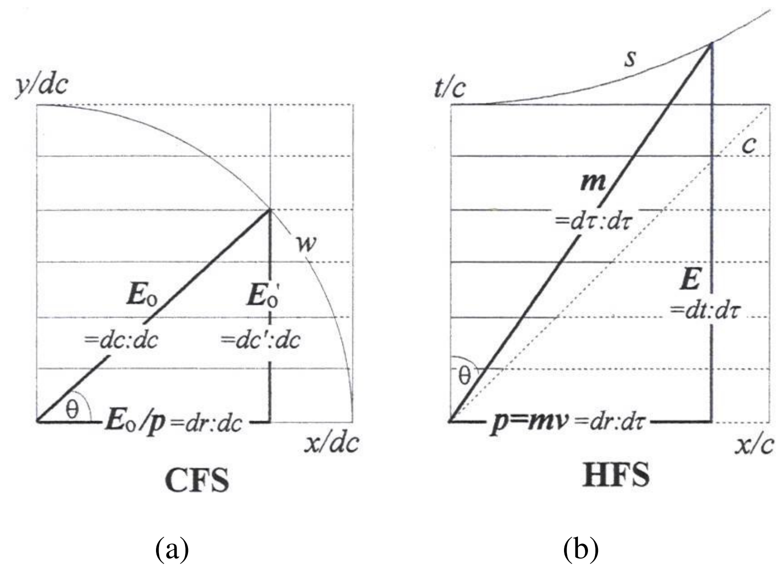


Figure 5. Comparison of momentum-energy 4-vectors in CFS and HFS.

observer's judgment of E_0' will be that it relatively diminishes from 1 to 0 at the limit, while E_0/p grows in proportion from 0 to 1 at the limit. However, (16)–(23) reveal a fundamental difference between CFS and HFS, which is that $1 = dc$, and dc is a *finite* physical measure, 2.998×10^8 lm. In CFS, combination of (20)–(23) indicates that in the limit case $v = c$,

$$\frac{p^*}{E_0} = \frac{E^*}{E_0} = dc = 1. \tag{24}$$

These are the correct values required to preserve the principle of equivalence between the two reference frames ^{0} and *. This is not something obviously evident from the mathematics of the Lorentz factor equation (15) and HFS, which indicate that $\gamma^* = \infty$ and $p^* = \infty$. In CFS, $\gamma^* = dc/dc'^* = 0$, so in order to derive the result $\gamma^* = \infty$ from CFS, one must make reference to the values of cosec $\theta = 1/\sin\theta$. However, it is evident from (6)–(13) above that cosec θ is *not* one of the functions under the circle through which CFS represents the fundamental measures of observer's time, proper time, and velocity. These differences between HFS and CFS appear due only to the peculiarities of the chosen mathematical form and therefore should not be regarded as fundamental. Furthermore, it should be noted that in its original Einsteinian form the Lorentz factor equation (15) is an algebraic expression for measures of velocity, *not* mass-energy. There is an important difference between the fact that there may be an infinity of differing lengths between 0 and 1 for sin θ under the circle and the fact that the value 0 or 1 also corresponds to a physically finite measure dc, the radius of the observer's light sphere. By considering the difference between the limit values of dc'^0 and dc'^* in terms of the factor γ, where $1/\gamma^0 = 1$ and $1/\gamma^* = 0$, it is found that

$$\frac{1}{\gamma^0} - \frac{1}{\gamma^*} = dc = 1 = E_0, \tag{25}$$

which leads to

$$\gamma^0 - \gamma^* = \frac{1}{dc} = \frac{1}{E_0}. \tag{26}$$

Multiplying both $1/dc$ and $1/E_0$ by dc gives

$$\frac{dc}{E_0} = 1 = \gamma^0 - \gamma^*. \tag{27}$$

The conclusion of these equations that both $\gamma^0 = 1$ and $\gamma^0 - \gamma^* = 1$ accord properly with the principle of relativity. The situation should never arise where observers in some rest-frame who measure the dimensions of physical processes in their frame as finite would encounter situations where other observers could measure the same processes as relatively infinite. The standard approach in relativity theory has been to exclude this possibility by arguing that the infinite dimensions of both the Lorentz factor equation and limit Minkowski "length" show it would require an impossible "infinite" energy to accelerate a particle to $v = c$. Unfortunately, the general theory can be used to hypothesize gravitational black holes as naturally occurring mechanisms that defy this alleged exclusion principle by accelerating particles to light velocity at its "event horizon"!

CFS then offers an alternative solution to this conflict by indicating that the proportions of γ^0, E^*, and p^* are all equal to $dc = 1$ and that the infinite dimensions of the Lorentz factor and Minkowski length are solely features of a chosen mathematical model, rather than the natural world. The fact that this world can also be modeled by analogous geometries such as AEST and CFS indicates that the scales of Minkowski geometry are not necessarily true of natural scales. This insight may be developed further by proportioning all the values of (27) solely in terms of natural constants.

Strictly speaking, there are no units for relativistic ratios such as E/m or c/E.[12] To find the numerical value of the ratio dc/E_0 requires finding a natural constant for E_0 in order to proportion it with the natural constant c. The obvious choice is the quantum of energy, Planck's constant h, essential to the photon hypothesis for light propagation c. In conventional units the ratio c/h appears as a momentum of 4.524×10^{41} kg/m^{-1}/s^{-1}. Division by the factor c gives this ratio in units of mass-energy, 1.509×10^{33} kg. The same number value is found in the relationship $1/E_0$ in (26), either by seeing it in terms of $1/h$ or by seeing it as the ratio between h and the measuring units of mass-energy, (1 kg/m^2/s^{-2})/h. Hence these ratios put a number, 1.509×10^{33}, to the difference $\gamma^0 - \gamma^*$, which will now be denoted as the limit Lorentz factor value γ^i.

This finite limit value γ^i can be used to give a consistent account of a hypothetical test object α, translating at $v = c$. To observers in any sublight reference frame, such an object is indistinguishable from a photon, because in this case $m_0/m_0' = dc/dc' = \gamma^* = 0$. Imagine that the frequency f of this photon is numerically equivalent to the factor γ^i; then its relativistic energy would be

$$\begin{aligned} E_\alpha &= hf_\alpha = h\gamma^i \\ &= (6.626\times10^{-34}\ \text{kg/m}^2/\text{s}^{-2})(1.509\times10^{33}) \\ &= 1\ \text{kg/m}^2/\text{s}^{-2}. \end{aligned} \tag{28}$$

The wavelength λ of this photon would be

$$\lambda_\alpha = \frac{c}{f} = \frac{c}{\gamma^i} = 1.987\times10^{-25}\ \text{m}. \tag{29}$$

Now consider that this same object α might be a particle with $v = c$. From (28) observers in sublight frames would deduce that the hypothesized rest energy of this object was (1 kg/m^2/s^{-2})/$\gamma^i = h$. Consider next how they would judge α's de Broglie and Compton wavelengths, λ_B and λ_C. For this velocity $E_\alpha = p_\alpha$, so E_α needs to be divided by the conversion factor c to obtain p_α in conventional units and by c^2 to obtain its relativistic mass m_α, giving

$$\lambda_{B\alpha} = \frac{h}{p_\alpha} = \frac{h}{E_\alpha / c} = \frac{h}{h/c} = 1.987\times10^{-25}\ \text{m}, \tag{30}$$

$$\begin{aligned} \lambda_{C\alpha} &= \frac{h}{m_\alpha c} = \frac{h}{(E_\alpha / c^2)c} = \frac{h}{(h/c^2)c} \\ &= 1.987\times10^{-25}\ \text{m}. \end{aligned} \tag{31}$$

Thus for any test object in this limit case there is no distinction between its properties as a particle and as a photon, since

$$\lambda_\alpha = \lambda_{B\alpha} = \lambda_{C\alpha} = 1.987\times10^{-25}\ \text{m}. \tag{32}$$

It is noteworthy that Montanus also derives the conclusion from the consideration of AEST that photons and other hypothetically massless particles should be considered to have a rest mass.[11] For its part, CFS indicates that SR and quantum theory appear to meet at a limit Lorentz factor γ^i. This should not be interpreted as an argument for the quantization of space-time at the scale of 10^{-25} m. It is only to say that for any particle α accelerated to $v_\alpha \approx c - 10^{-38}$ m/s, where $\gamma E_{0\alpha} = (1/h)E_{0\alpha}$, E_α will not rise for any further increase in v_α. In principle, this hypothesis could be the subject of an experimental test.

5. TIME TRAVEL

Time "travel" is a philosophically difficult idea encouraged by the fact that the construction of the Minkowski light cone gives the time axis a clear "direction" relative to spatial axes, and it partitions the continuum into "past" and "future." Since the construction of the light cone also sets c as the limit rate for all physical signals, efforts to consider faster than light (FTL) signals lead to paradox when it is found that some observers of the same events will judge FTL signals to be traveling "back" into the past.

By contrast, the construction of the light sphere does not permit any notion of time travel, and so it allows FTL signaling to be considered without paradox. Although measurements in CFS are made relative to four coordinates, x, y, z, and dc, dc is *not* a member of the mutually orthogonal axis set x, y, z. Since the temporal coordinates of CFS are found on the surfaces of asymmetrically expanding light spheres w, by definition, no light sphere will reverse direction relative to x, y, z. So although the "travel" of a reference frame may be reversible relative to x, y, z, it can never be so with respect to $dc = t$. Furthermore, the mapping of CFS starts at any chosen "now" and leads only to later events, so it does not divide space-time into past and future zones, as does Minkowski mapping.

An example serves to show that events that appear paradoxical when mapped in Minkowski geometry are not so when mapped in CFS. The example is also useful in demonstrating how CFS can be used to analyze transformations of coordinates between reference frames. The example follows a thought experiment summarized by Penrose,[(13)] which is illustrated in Fig. 6b.

The Minkowski space-time diagram there shows the point of view of observers at Q, who judge themselves to be between two reference frames P and R each translating away from Q at $v = 0.8c$ in opposite x directions. Observers in reference frame P send an FTL signal at $3c$ toward Q and R. Call the transmission of this signal event A, which from the Q viewpoint occurs at time $t = 0$, and at a place A, 0.5 m of light travel away on their $-x$ axis. P observers judge that their FTL signal arrives at event B_P, 2.1 lm distant, at a time $t_P = 0.7$ m. For observers in Q, B is an event in the R frame of reference, at which a mirror is located that reflects the FTL signal back to the P frame, where it arrives at an event C.

In Minkowski geometry, FTL signals lie in the space-like region outside the light cone, and the plotting of the coordinate transformations for translating frames like P and R requires Q observers to calculate the relative slope $S_r = \tan\theta_r$ of the t axes in P and R, which determines the relative folding and stretching of their Cartesian grid coordinates. From the Q perspective the sum of these transformations produces the paradox that the FTL signals travel relatively backward in time such that observers in all three frames must agree that event C precedes its cause, event B.

Penrose, in common with many others,[(14)] believes such paradoxes demonstrate the absurdity of FTL signaling and prove the validity of the original postulate that c marks the limit of any signal velocity. A more considered view might suggest that these paradoxes only prove that Minkowski geometry cannot model behavior at variance with relativity theory's original assumptions about c. CFS is also constructed on assumptions about the nature of light propagation, but these do not require a limit for signal velocity. CFS measures the motions of objects by mutual reference and c just happens to be the chosen reference constant in situations involving the Einstein principle of relativity. That allows these same events to be mapped without paradox in CFS.

In the CFS map shown in Fig. 6a, Q observers set event A as the starting point $t = 0$, for analysis. From the Q viewpoint, at this time, the position of A is at – = 0.5 m of light travel and the position of the R origin is at $+x = 0.424$ lm, separated from A by 0.924 lm. Q observers will determine that the distance Δr_R traveled by R before the signal of $v = 3c$ arrives from A at event B is

$$\begin{aligned}\Delta r_R &= \frac{\cos 36.9°(\Delta r_R + 0.924)}{3/1}\\ &= \frac{0.8(\Delta r_R + 0.924)}{3} = 0.336 \text{ lm.}\end{aligned} \tag{33}$$

Since Q observers judge P to be coincident at A at $t = 0$, then they must judge that the time taken for the FTL signal to cover the interval AB is

$$\Delta t_{AB} = \frac{\Delta r_R}{\cos 36.9°} = 0.42 \text{ lm.} \tag{34}$$

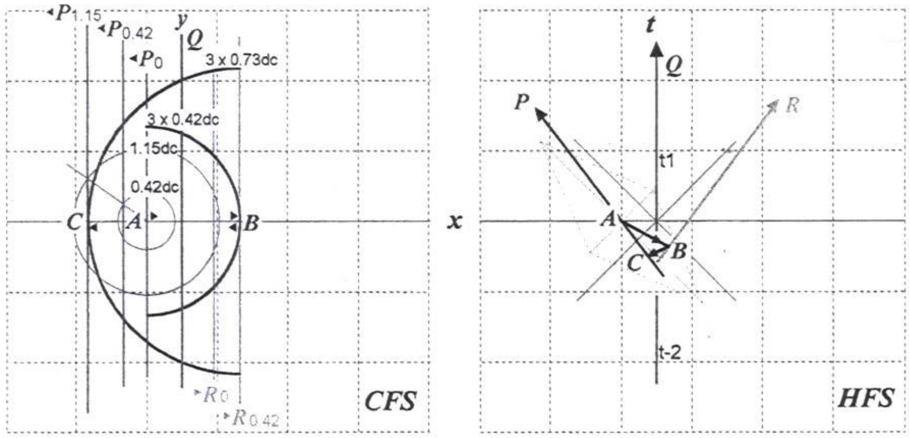
Hence the temporal separation Δt_{AB} between A and B is +0.42 lm, and the spatial separation AB_Q is

$$AB_Q = \frac{\Delta t_{AB}}{dc/3dc} = \frac{0.42}{1/3} = 1.26 \text{ lm.} \tag{35}$$

From the Q viewpoint, for observers in P, the intervals AB and Δt_{AB} will appear as

$$AB_P = \frac{AB_Q}{\sin 36.9°} = 2.1 \text{ lm,} \tag{36}$$

$$\Delta t_P = \frac{\Delta t_Q}{\sin 36.9°} = 0.7 \text{ lm.} \tag{37}$$



(a) (b)

Figure 6. Mapping FTL signaling from *A* to *B* to *C*, using CFS and HFS.

These are the same values initially specified for this thought experiment, and none of the time values are negative. Then, for the *Q* judgment of the same FTL signal reflected back from *B* to *C*,

$$\Delta r_{BC} = \frac{\cos 36.9°(\Delta r_R + 1.26)}{3/1} = 0.58 \text{ lm}, \quad (38)$$

$$\Delta t_{BC} = \frac{\Delta r}{\cos 36.9°} = 0.725 \text{ lm}, \quad (39)$$

$$BC_Q = \frac{\Delta t_{BC}}{dc/3dc} = \frac{0.725}{1/3} = 2.176 \text{ lm}, \quad (40)$$

and

$$\Delta t_{AB} + \Delta t_{BC} = 0.42 \text{ lm} + 0.725 \text{ lm} = 1.145 \text{ lm}. \quad (41)$$

From the *Q* viewpoint, for observers in *P*, the intervals Δr_{BC} and Δt_{BC} will appear as

$$BC_P = \frac{BC_Q}{\sin 36.9°} = 0.966 \text{ lm}, \quad (42)$$

$$\Delta t_P = \frac{\Delta t_Q}{\sin 36.9°} = 0.907 \text{ lm}. \quad (43)$$

Since all the time intervals here are positive, the paradox found when using Minkowski mapping disappears. This difference in results suggests that time travel may be an appropriate metaphor for describing the characteristics of world-lines in Minkowski geometry, but a wholly inappropriate description of temporal ordering in both space-time theory and nature.

6. CONCLUSION

CFS is a Euclidean geometry for SR that emerges from a philosophical critique of Einstein's conventionalist definition of time. Montanus, who has found another Euclidean analogue of Minkowski geometry, AEST, comments that his model shows Einstein got relativity mostly "right," while Minkowski got the foundations of space-time mainly wrong.[15] The conclusions of this paper are that space-time geometries are no more "right" than one another if they produce the same numerical results. When they do not, then physicists are obliged to attend more closely to the distinction between something being a property of a mathematical entity and it being a property of nature. It is also important that the geometry be easily usable. CFS is perhaps the simplest geometry to learn, since it maps the translations of systems in the way that they might actually be observed. However, the others each offer valuable insights into relativity, and CFS, only outlined in this paper, might not prove to be as useful in analyzing more complex situations involving gravitation and dynamics. One service CFS may perform is to clarify the concept of physical time, which Einstein did not get wrong, but neither did he get right.

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Résumé

Cet article présente une convention différente de la mesure du temps de la relativité spéciale où un intervalle de temps t pourrait être mesuré par dc, la distance voyagée d'une origine par le front d'onde sphérique d'une pulsation lumineuse c. L'adoption de cette convention mène à une géométrie euclidienne pour la relativité spéciale, qui est différente de celle de la géométrie euclidienne déjà proposée par Montanus. La géométrie actuelle est régie par les fonctions du cercle, plutôt que par l'hyperbole, et le front d'onde sphérique d'une pulsation lumineuse fournit soit un quatrième ensemble t des points des coordonnées et un paramètre w pour mesurer les intervalles qui sont invariables entre les référentiels. Puisque les valeurs de sinus sous le cercle s'étendent de 1 à 0, plutôt que de 1 à ∞, le nouveau modèle ne permet pas dans le cas d'un objet voyageant à la vitesse ≈ c, que toute intervalle ait une longueur ≈ ∞. En outre, la forme du nouveau modèle exclut la notion du 'voyage' relativement au temps.

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