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Highlights

- LKT test applied to asset bubbles detection using the dividend and price-earnings ratio from the NASDAQ Composite Index.
- We found an unexpected negative bubble in NASDAQ from Feb 1973 to Jun 1992 for both dividend and the price-earnings ratios.
- The findings imply that the stocks listed on the NASDAQ were undervalued with respect to their fundamental values.
- We also found a positive bubble ranging from December 1998 to July 2001 for the dividend-price price-earnings ratios.
- All findings situated around the period of the Dotcom bubble usually recorded by the media and previous academic studies.

Signalling the Dotcom Bubble: A Multiple Changes in Persistence Approach

Vitor Leone and Otavio Ribeiro de Medeiros

Abstract

This study investigates multiple changes in persistence in the dividend-price and price-earnings ratio of the NASDAQ composite index. Recent time series methods that are capable of signalling and dating asset price bubbles are employed, in particular the method developed by Leybourne et al. (2007). The method allows for breaks between periods in which the data are integrated of order zero I(0) and integrated of order one I(1). The results confirm the existence of the so-called Dotcom bubble with its start and end dates. Furthermore, an unexpected negative bubble was also identified, extending from the beginning of the 1970s to the beginning of the 1990s, suggesting that the NASDAQ stock prices were below their fundamental values as indicated by their dividend yields, finding not previously reported in the literature. As the tools used by regulators take considerable time to take effect, methods capable of picking up warnings signals of the start of a bubble could be very useful. We conjecture that the methodology can also be applied to study recent phenomena in real estate, commodity and foreign exchange markets.

Keywords: NASDAQ, Order of Integration, Persistence shifts, Rational Bubble, Unit roots.

JEL classifications: C10, C32, F15, G12, G14, G15

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1. Introduction

Starting from the second half of the 1990s, a rapid growth in the internet sector and related industries,[1] fuelled by the supply of new internet IPOs, resulted in the U.S. stock market experiencing a remarkable rise in its main indices. The internet-heavy NASDAQ Composite Index rose from 775.20 in January 1995 to 2,505.89 in January 1999 and more than doubled from this point to its peak of 5,048.62 on 10th of March 2000 (Scherbina, 2013).[2] Afterwards it declined to a low of 1,314.85 in August 2002. This growth and subsequent fall in stock prices has led leading economists, such as Shiller (2001) and Stiglitz (2003), to suggest the existence of a bubble in the US stock market. Indeed, Stiglitz has called the stock market bubble the seed of destruction of the US economy in the 1990s.

Academic research investigating the existence of bubbles in stock markets is quite extensive (e.g., Campbell and Shiller, 1987, 1988; Diba and Grossman, 1988; Froot and Obstfeld, 1991; Craine, 1993; Timmermann, 1995; Crowder and Wohar, 1998; Lamont, 1998; Thaler, 1999; Shiller, 2000b; Cooper et al., 2001; Ritter and Welch, 2002; Ofek and Richardson, 2002; Lamont and Thaler, 2003; Brunnermeier and Nagel, 2004; Sollis, 2006; Hong and Stein , 2007; Stiglitz, 2009; Gutierrez, 2011; and Griffin et al., 2011; Philips, Shi and Yu, 2012, among others)[3].

According to the present value theory of finance, fundamental or fair stock prices are determined by the sum of the present discounted values of expected future dividends. Specifically, stock prices are equal to the present value of rationally expected or optimally forecast future real dividends, discounted by a constant discount rate. This model is generally

[1] Macroeconomic conditions were favourable during the second half of the 1990s when the US economy experienced a remarkable performance in productivity growth. From 1995Q4 to 2000Q4 productivity growth per worker was 2.4% per year with a GDP growth rate of 3.5% per year, compared to only 1.4% per year from 1972Q2 to 1995Q4 (Cunado et al., 2005).
[2] The Standard and Poor's 500 (S&P 500) jumped from 330 in December 1990 to 1469 in December 1999.
[3] Most of these papers test the existence of stock market bubbles applied to annual data from the S&P 500.

used by economists and investment analysts as a tool to explain the behaviour of aggregate market indices (Shiller, 1981). In fact, since market uncertainties and frictions exist, and because the discount rate may vary, it is expected that actual prices fluctuate around fundamental values (LeRoy, 1989). The bubble terminology suggests a likely deviation of the stock price from the fundamental value, proxied by the present value of all its future cash flows, earning or dividends.

The underlying theory for testing rational bubbles in stock prices can be based on studies by Campbell, Lo and McKinlay (1997), Campbell and Shiller (1987), Craine (1993) and Koustas and Serletis (2005), to cite a few. For example, Campbell and Shiller (1987) when testing for cointegration between stock prices and dividends using annual data for the S&P 500 from 1871 to 1986, obtained persistent deviations of stock prices from the present-value model, and attributed these deviations to the existence of rational bubbles. Froot and Obstfeld (1991) and Craine (1993) reach the same conclusion, testing for a unit root in the price–dividend ratio using annual data of the S&P 500 for 1900–1988 and 1876–1988 respectively. However, Diba and Grossman (1988) find that stock prices do not contain explosive rational bubbles, when analysing data of S&P 500 index for 1871–1986.

The explanation for using dividend-price information is simple: dividend yields provide a measure of how stocks are valued vis-à-vis their fundamentals. Low dividend yields can be seen as an indication of overpriced stocks compared to their earning ability, represented by their dividends (or future dividends), and high dividend yields are seen as indication of underpriced stocks. Looking at the dividend yield time-series tells even more: incessantly decreasing dividend-price ratios may be held as an evidence of worsening overpricing, i.e. a bubble, because if prices are constantly rising, these rising expectations should at some point be realised as higher dividends. If price expectations keep rising, but higher dividends fail to materialise, the price rise is not due to fundamentals (i.e. earning ability). In other words, the price can be seen as a composite of fundamental value plus a rational bubble component, as described e.g. by Craine (1993): "rational bubbles satisfy an equilibrium pricing restriction implying that agents expect them to grow fast enough to earn the expected rate of return. The explosive growth causes the stock's price to diverge from its fundamental value".

Recent studies suggest that persistence shifts or systematic deviations in the relationship between stock prices and dividends can be related to the occurrence of rational bubbles in stock markets (Cunado et al. 2005, Sollis, 2006; Sanso-Navarro, 2009; Philips, Shi and Yu, 2012). Those studies apply a number of testing procedures intended to deal with processes showing changes in persistence in the order of integration of economic and financial time series[4]. The idea is to locate the point at which the construction of the dividend yield series changes to a unit root (or even explosive) series, using time-series methodology with slight modifications (regime shifts or changes in persistence).

Among these new testing procedures, and in connection to the present paper, we highlight the method developed by Leybourne, Kim and Taylor (2007), hereafter LKT. This method has the purpose of testing for and dating multiple changes in the order of integration of a time series between trend stationary I(0) and difference-stationary I(1) regimes, based on sequences of doubly-recursive implementations of regression-based unit root statistics.

Since a bubble sees stock prices deviate from their fundamental values, it can been argued that a non-stationary dividend-price ratio characterizes a bubble process. Therefore, if a dividend-price series suffers a change in persistence from I(0) to I(1), this can be considered as evidence of a bubble. Further, if the dividend-price ratio changes back to I(0), this would imply that the bubble has collapsed. Sollis (2006), using the S&P composite index and the

[4] Generically such procedures have been suggested for example by Kim (2000), Kim et al. (2002), Busetti and Taylor (2001) and Busetti and Taylor (2004), Leybourne, Kim and Taylor (2004) and Harvey et al. (2006) Shin et al (2010a, 2010b) as well as by Phillips et al. (2011).

centre for research in security prices (CRSP) data on dividend-price ratio of the New York Stock Exchange Index, tested for changes in persistence on the dividend-price ratio, using a test developed by Leybourne, Kim, Smith and Newbold (2003) and Kim et al. (2002). He found evidence of shifts from level-stationary to a difference-stationary process, supporting the hypothesis of a time-varying rational bubble. In fact, this test is a predecessor to the LKT approach as it tests for a single change in persistence, but without any date identification. As put forward by Leybourne et al. (2007), "in general, the tests for a single change in persistence will not be consistent against processes which display multiple changes in persistence. Where multiple changes in persistence occur these procedures also cannot be used in general to consistently partition the data into its separate I(0) and I(1) regimes".[5]

It should be noted that LKT's method was not developed specifically to test for asset price bubbles, and criticisms have been highlighted by academics in relation to studies applying a unit root test framework to characterise a bubble episode (Gürkaynak, 2008). Nevertheless, in early bubble studies using basic unit root tests (e.g. Diba and Grossman, 1988; Evans, 1991; Van Norden and Vigfusson, 1998; Hall et al., 1999) the main weaknesses were the inability to identify and date the changes in the order of integration in the period under investigation. As Gürkaynak (2008, p.179) puts it, "The unit root based tests have difficulty detecting collapsing bubbles because these behave more like stationary processes than like explosive processes as a result of periodic collapses involved". Therefore, by applying the LKT method to test for multiple changes in persistence in time series, we aim to address the issue highlighted by Gürkaynak. In other words, the capability of the method for identifying and dating regime changes allows the possibility that rational bubbles can collapse[6].

[5] As put forward by Bai (2000, p. 304), "because a myriad of political and economic factors may alter the data generating process, multiple changes may be a more accurate characterisation of economic time series".
[6] As the existence of bubbles entails changes in persistence in prices, it can be used to suggest the presence of bubbles. Other authors have used the LKT method to test multiple changes in persistence in non-bubble

Following the same idea of dating regime changes, a different approach to test for bubbles has recently been proposed by Phillips, Wu and Yu (2011), hereafter PWY, later refined by Phillips, Shi and Yu (2013, 2013a). The PWY method is a recursive test procedure which provides a mechanism for testing explosive behaviour, dating the origin and collapse of a bubble and presenting valid confidence intervals for explosive growth rates. The method involves the recursive implementation of a right-tail unit root test (I(1)>1) and a supreme test, together with a new limit theory for moderately explosive behaviour.

In this paper, in contrast to the majority of studies that have used annual S&P 500 data to detect rational bubbles without calendar date identification, we use the methodology developed by LKT to test for the Dotcom bubble using monthly data in the NASDAQ Composite Index, comparing our results with PWY's approach to test for the same bubble.

In other words, our aim by applying the LKT method for signalling asset prices bubbles is to add to the existing literature on time-series and unit root approaches for rational bubbles detection, and counterpoint this against the methodology developed by PYW to detect explosive behaviour, which also characterises a bubble[7]. We also expect to contribute to the debate regarding some of the criticisms attributed to unit root tests framework, by introducing methods capable of identifying and attributing calendar dates to the bubble episode under investigation, something traditional tests do not offer. The traditional tests in the unit root literature, the Dickey-Fuller (DF, 1979, 1981) test and its augmented version (ADF) have been shown to have severe limitations, especially for changes in persistence[8].

contexts. For example, Noriega and Ramos-Francia (2009) use the LKT procedure to test for change in persistence in the US inflation, suggesting that these changes were linked to changes in economic policy.

[7] Recently Homm and Breitung (2012) also produced a study covering different time-series methodologies to test for rational bubbles.

[8] Busetti and Taylor (2004) showed that the traditional ADF test is not consistent in the case of changes in persistence, as the test does not converge to minus infinity with sample size when applied to series containing persistence breaks caused by the I(1) part's dominance in test results.

The paper is organised as follows: Section 2 summarises the LKT and the PWY methods; Section 3 describes the data; Section 4 presents the empirical results and checks the robustness of the results; and Section 5 presents the concluding remarks.

2. Methodology

Leybourne et al. (2007)[9] develop a test for multiple changes in persistence, i.e. changes in the order of integration of a time series, which allows consistent estimation of the change dates and is also robust to the presence of multiple level breaks. The test is the only extant methodology which is consistent when multiple changes in persistence take place.

The data generation process (DGP) consists of the following Time-Varying (TV) AR(p):

$$y_t = d_t + u_t, \qquad u_t = \rho_i u_{t-1} + \sum_{j=1}^{k_j} \phi_{ij} \Delta u_{t-j} + \varepsilon_t, \qquad t = 1,\ldots,T \tag{1}$$

where y_t is the series being tested, $d_t = z_t' \beta$ is the deterministic kernel and ε_t is a martingale difference sequence. In Eq. (1), u_t is assumed to be a TV AR(p) process, rewritten such that $k_i = p_i - 1$, $i = 1,\ldots, m+1$, where m is the number of changes in persistence. The null hypothesis being tested is H_0: $y_t \sim (1)$ throughout the sample, and the alternative is H_1: y_t experiences one or more changes in persistence between I(1) and I(0) or vice-versa. Under H_1, ρ_i is subject to $m \geq 1$ unknown persistence changes, producing $m+1$ segments with change points given by $\tau_1<\tau_2<\ldots<\tau_{m-1}<\tau_m$. The procedure divides y_t, t = 1,…,T into separate I(0) and

I(1) regimes, and consistently estimates the change points. LKT define the fraction $\tau \in (\lambda,1)$,

[9] Early versions of the test for a single change in persistence were developed by Harvey et al. (2006) and Leybourne et al. (2006).

for a given λ in (0,1), and base their test H_0 vs. H_1 on the local GLS de-trended ADF unit root statistic (Elliot et al.,1996), that uses the sample observations between λT and τT, called DFG(λ,τ), obtained as the standard t-statistic associated with $\hat{\rho}_i$ in the fitted regression:

$$\Delta y_t^d = \hat{\rho}_i y_{t-1}^d + \sum_{j=1}^{k_i} \hat{b}_{i,j} \Delta y_{t-j}^d + \hat{\varepsilon}_t, \qquad t = \lambda T, \lambda T+1, \ldots, \tau T \tag{2}$$

with $y_t^d \equiv y_t - z_t'\hat{\beta}$ and $\hat{\beta}$ the OLS estimate of β in the regression of $y_{\lambda,T}$ on $z_{\lambda,T}$, where $y_{\lambda,T} \equiv (y_{\lambda T}, y_{\lambda T+1} - \bar{\alpha} y_{\lambda T}, \ldots, y_{\tau T} - \bar{\alpha} y_{\tau T-1})'$ and $z_{\lambda,T} \equiv (z_{\lambda T}, z_{\lambda T+1} - \bar{\alpha} z_{\lambda T}, \ldots, z_{\tau T} - \bar{\alpha} z_{\tau T-1})'$, with $\bar{\alpha} = 1 + \bar{c}/T$, and $\bar{c} < 0$. The test is based on doubly-recursive sequences of DF type unit root statistics $M \equiv \inf_{\lambda \in (0,1)} \inf_{\tau \in (\lambda,1)} DF_G(\lambda, \tau)$ with corresponding estimators given by $(\hat{\lambda}, \hat{\tau}) \equiv \arg\inf \lambda \in (0,1) \inf \tau \in (\lambda,1) \, \mathrm{DFG}(\lambda, \tau)$.

Application of the M test yields the start and end points (i.e. the interval $[\hat{\lambda}, \hat{\tau}]$) of the first I(0) regime over the whole sample. The presence of any further I(0) regimes are detected sequentially by applying the M statistic to each of the resulting subintervals $[0, \hat{\lambda}]$ and $[\hat{\tau}, 1]$. Continuing in this way all I(0) regimes, together with their start and end points, can be identified. The period between the end point of one I(0) regime and the start point of the next I(0) regime corresponds to an I(1) regime.

The Phillips et al. (2011) method allows for the detection of both the explosiveness in the bubble process and its start and end dates. The PWY method implements recursive right-tailed unit root tests. The tests were developed as follows.

Given a time series x_t (log stock price or log dividend), the augmented Dickey-Fuller (ADF) test for a unit root is applied against the alternative of an explosive root (right-tailed test). That is, equation (3) is estimated by least squares for a certain number of lags J.

$$x_t = \mu_x + \delta x_{t-1} + \sum_{j=1}^{J} \phi_j \Delta x_{t-j} + \varepsilon_{x,t}, \qquad \varepsilon_{x,t} \sim NID\left(0, \sigma_x^2\right) \tag{3}$$

Significance tests or an information criterion can be used to determine the lag parameter *J*. The unit root null hypothesis is H_0: $\delta = 1$ and the right-tailed alternative hypothesis is H_1: $\delta > 1$. Eq. (3) is estimated repeatedly in forward recursive regressions, using subsets of the sample data which are incremented by one observation at each run. If the first regression involves $\tau_0 = [nr_0]$ observations, for some fraction τ_0 of the total sample, where [ ] represents the integer part of the argument, subsequent regressions employ this originating data set supplemented by successive observations, giving a sample of size $\tau = [nr]$ for $r_0 < r < 1$. Denoting the corresponding t-statistic by ADF_r and hence ADF_1 corresponds to the full sample. Thus, under the null,

$$ADF_r \Rightarrow \frac{\int_0^r \tilde{W} dW}{\left(\int_0^r \tilde{W}^2\right)^{1/2}} \qquad \text{and} \qquad \sup_{r \in [r_0,1]} ADF_r \Rightarrow \sup_{r \in [r_0,1]} \frac{\int_0^r \tilde{W} dW}{\left(\int_0^r \tilde{W}^2\right)^{1/2}} \tag{4}$$

where W is the standard Brownian motion and $\tilde{W}(r) = W(r) - \int_0^1 W$ is demeaned Brownian motion. Comparing $\sup_r ADF_r$ with the right tailed critical values from $\sup_{r \in [r_0,1]} \int_0^r \tilde{W} dW / \left(\int_0^r \tilde{W}^2\right)^{1/2}$ provides a test for a unit root against explosiveness, which characterises a bubble. To locate the origin and the conclusion of exuberance, one can match the time series of the recursive test statistic ADF_r, with $r \in [r_0, 1]$, against the right tailed critical values of the asymptotic distribution of the standard Dickey-Fuller t-statistic. In particular, if r_e is the date of origin and r_f is the collapse date of explosive behaviour in the data, estimates of these dates are obtained as:

$$\hat{r}_e = \inf_{s \geq r_0} \{ s : ADF_s > cv_{\beta_n}^{adf}(s) \}, \quad \hat{r}_f = \inf_{s \geq \hat{r}_e} \{ s : ADF_s < cv_{\beta_n}^{adf}(s) \}, \tag{5}$$

where $cv_{\beta_n}^{adf}(s)$ is the right-tail critical value of ADF_s with a significance level of β_n.

3. The Data

The data utilised in this study consist of monthly observations of the NASDAQ dividend-price ratio and the NASDAQ composite price index, obtained from Thomson Reuters Datastream for the period spanning from January 1973 to December 2013, totalling 492 observations. The real NASDAQ composite price index is generated by deflating the nominal price index by the US CPI index, also collected from Datastream. The real NASDAQ composite dividend index is calculated based on the NASDAQ composite dividend-price ratio and the NASDAQ composite nominal price index, deflated by the US CPI index. The LKT tests are applied to the natural log of the series. Table 1 summarises the descriptive statistics for the raw data and their natural logarithms.

Table 1: Summary of descriptive statistics for the NASDAQ dividend-price ratio

	Real price index	**Real dividend index**	**d-p ratio**	**Log(d-p ratio)**
Mean	287.2654	287.9986	1.588659	0.205429
Median	207.0028	233.0149	1.180000	0.165514
Maximum	1175.556	987.9407	5.010000	1.611436
Minimum	46.75840	143.0622	0.170000	-1.771957
Std. Dev.	214.0360	152.4419	1.149710	0.737831
Skewness	1.033131	2.533315	1.078285	-0.135897
Kurtosis	3.946336	10.00359	3.015576	2.636398
Jarque-Bera	105.8824	1531.780	95.34622	4.224596
Probability	0.000000	0.000000	0.000000	0.120960
Observations	492	492	492	492

It can be seen that the real price and the real dividend indices are positively skewed, leptokurtic and not normally distributed. The raw data series of the dividend-price ratio has a positive skewness, meaning that most of the data are concentrated below the mean of 1.59. The distribution is also slightly platykurtic. The logarithm of the dividend price ratio is slightly skewed to the right, platykurtic and normally distributed.

4. Empirical Results

In this empirical application, following the suggestion in LKT, we set λ=1/T such that λT=1, making τ = 0.10. To determine the value of k_i, we use the Schwarz information criterion (Schwarz, 1978) which defines the appropriate lag length for values of k_i having a maximum value of 12, for every sample or sub-sample regression computed.

We report the results obtained for the version of the test with an intercept and a linear time trend, that is, since $d_t = z_t^{'}\beta$ is the deterministic components a linear trend implies $(z_t = [1,t]^{'} \; and \; \beta = [\beta_0, \beta_1]^{'}$[10]. The application of the test with no intercept and no time trend could not find any I(0) regime inside the series tested[11].

The M test is initially applied over the whole sample (January 1973–December 2013). An interior I(0) regime is detected between August 2001 and August 2009, as the calculated M statistic of −5.553 and the critical value from LKT for T=492 is −5.078 results in rejection of the unit root null hypothesis at the 1% level.

The test is then applied over January 1973–July 2001 (343 observations) and a second I(0) regime is found from July 1992 to November 1998, with the null rejected at the 10% level (M statistic = −4.756; critical value at 10% = −4.400). Applying the test over the remaining periods of January 1973−June 1992, December 1998−July 2001 and September

[10] For a constant only $(z_t = 1, and \beta = \beta_0)$

[11] These results are available from authors on request.

2009-December2013 results in non-rejection of the unit root null hypothesis, meaning that no further I(0) regime could be identified. The conclusion is that these periods are I(1) regimes. The results are summarised in Table 2.

Table 2: Results of the LKT test for the NASDAQ dividend-price ratio

Period Tested		Result		Regime	Duration (months)	$\hat{k}_i$	M-Statistic	Critical Value
Start	**End**	**Start**	**End**					
1973:01	**2013:12**	2001:08	2009:08	I(0)	12	12	-5.553	-5.078 (1%)
1973:01	**2001:07**	1992:07	1998:11	I(0)	78	2	-4.756	-4.400 (10%)
1973:01	**1992:06**	-	-	I(1)	234	10	-3.678	-4.422 (10%)
1998:12	**2001:07**	-	-	I(1)	32	0	-3.677	-5.459 (10%)
2009:09	**2013:12**	-	-	I(1)	52	0	-3.721	-4.970(10%)

Fig. 1 shows the trajectory of real NASDAQ price and dividend indices over the sample period, with the depiction of the I(1) regimes. For the first I(1) regime, from January 1973 to June 1992, stock prices are systematically below dividends. This unexpected result may suggest the occurrence of a negative asset price bubble during this period. A negative bubble, contrary to a positive one, would have market prices falling below their fundamental values.

Filardo (2011) argues that while this correspondence might be appealing, the driving force of positive and negative asset price bubbles are likely to be different. Positive price bubbles, be they in equities, housing, commodities, or other widely held assets, are expected to inflate progressively over time. The main driver is usually overconfidence that manifests itself in increased risk appetite (technically, less risk aversion) and optimistic expectations of future earnings. While the prices are misaligned with longer-term fundamentals, the root of big bubbles is generally linked to periods of history when innovations, real or financial, nurture an environment of excessive optimism about the future.

Negative asset price bubbles, in contrast, would seem to come in a wider variety of types. On the one hand, negative asset price bubbles can expand and burst in a way similar to positive asset price bubbles, as irrational beliefs permeate an economy; for negative asset price bubbles it would be irrational pessimism. One can envision this bubble process building slowly over time, as pessimism and risk aversion breed further pessimism and risk aversion. On the other hand, negative asset price bubbles could be initiated in a more dramatic fashion. In this case, a sudden negative overreaction to current events leads to a significant and immediate underpricing of risk.

Looking at the economic history of the US, the negative asset price views and results obtained from the LKT procedure may be supported by overreaction and pessimism linked to economic and political events between 1973-1992: 1973-1974 (OPEC, Watergate, inflation and recession); 1976-1978 (stagflation, budget deficits and trade deficits); 1980-1982 (high interest rates, the second OPEC crisis, recession and inflation); 1987 (stock market crash); 1990 (The Gulf war and the collapse of the Japanese stock market).

The second I(1) regime, of December 1998−July 2001, displays prices well above dividends. During these 32 months the NASDAQ composite index increased sharply, deviating considerably from dividends and consequently its expected fundamental value. Immediately after peaking, prices fell abruptly by June 2001 suggesting the collapse of the bubble. This second regime partially captured part of the Dotcom bubble of the late 1990s and, as already mentioned, positive bubbles can be caused by real or financial innovations. Cunado et al. (2005) argue that globalisation and the information technology revolution are behind the spectacular rise of the NASDAQ index, as most of its constituents were technology companies. The third I(1) regime seems to capture the recent financial crisis and will not be discussed in this paper.

Fig. 1: Results of the LKT test and the NASDAQ price and dividend indices

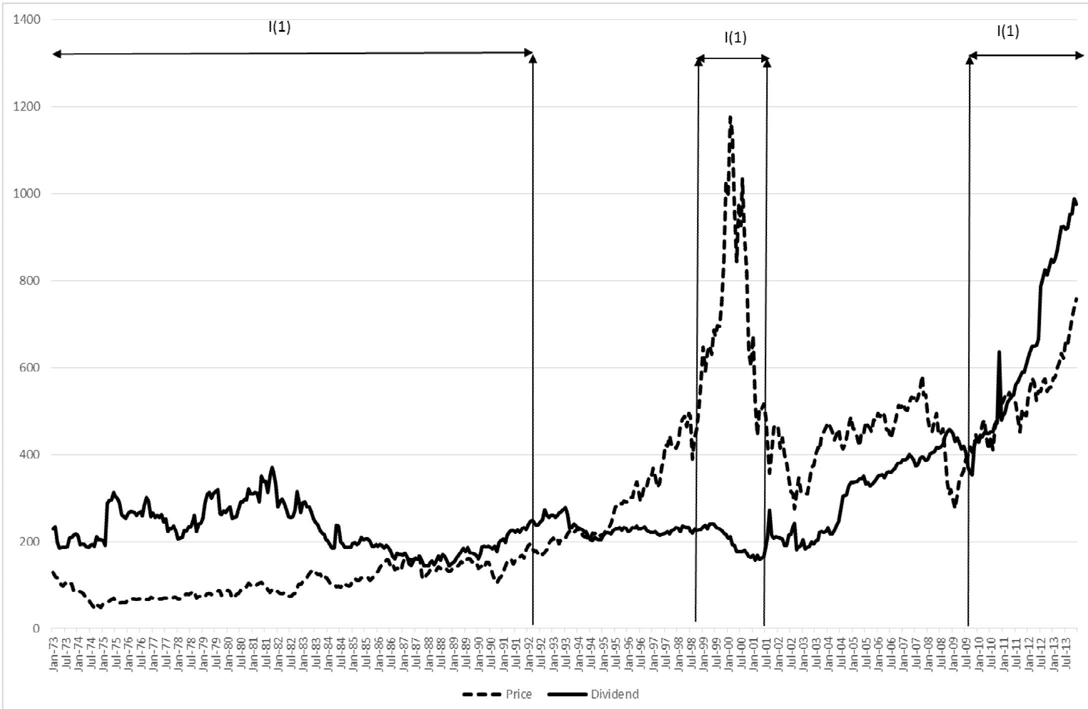
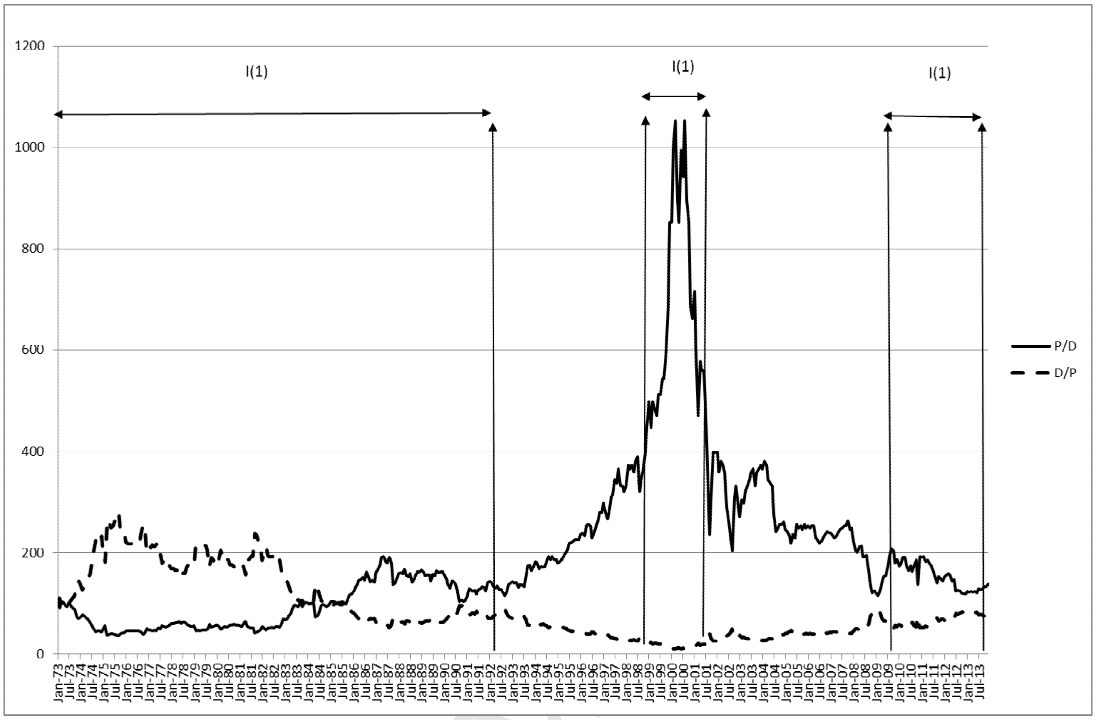


Fig. 2 presents the dividend-price ratio and its inverse, the price-dividend ratio, over the sample period showing the (1) regimes, with both series being normalised as 100 in 1973:01. It can also be seen from Fig. 2 that the dividend-price ratio is very high during the first I(1) (1973:01-1992:06) period and falls steadily from the end of this period throughout the first I(0) period, reaching a minimum during the second I(1) (1998:12-2001:07) period. The price-dividend path shows how stock prices were depressed with respect to dividends during the first I(1) period, while in the second I(1) period, prices rocketed with respect to dividends. Combining Figs. 1 and 2, it becomes clear that the dividend-price ratio falls from a high level in the first I(1) period to a minimum in the second I(1): by February 1993, stock prices start to rise, reaching a maximum by March 2000, making the dividend-price ratio fall to its minimum in the same month.

Fig. 2: The dividend-price ratio and the results of the LKT test



4.1 Robustness and the PWY Test

As the depressed prices captured for the first I(1) period, to our knowledge, have not being reported in the rational bubble tests literature, we check for the robustness of the LKT dividend yield results by applying the methodology to the NASDAQ price-earnings ratio.

The choice of a price-earnings ratio follows the same argument as the dividend-price ratio; that is, the idea of stock prices being determined by the sum of present discounted values of future earnings. The ratio is generated by dividing the current market price of a share by its latest earnings. A high P/E ratio might imply an overvalued market and a low P/E ratio and undervalued one. [12]

[12] Evidence of low P/E ratios generating abnormal returns is well documented in studies by Basu (1975, 1977, 1983); Keim (1988); Lakonishok et al. (1994); Gregory et al. (2001, 2003) and Anderson and Brooks (2006), to cite a few.

Monthly data on the NASDAQ price-earnings ratio was obtained from Datastream for the period January 1973 to December 2013, giving 492 observations. The results from the LKT test applied to the price earnings ratio are summarised on table 3 below:

Table 3: Results of the LKT test for the NASDAQ price-earnings ratio

Period Tested		**Result**						
Start	**End**	**Start**	**End**	**Regime**	**Duration (Months)**	$\hat{k}_i$	**M-Statistic**	**Critical Value**
Jan-73	**Dec-13**	**Oct-99**	**Sep-08**	I(0)	108	10	-7.512	-5.078(1%)
Jan-73	**Sep-99**	**Mar-73**	**Jan-78**	I(0)	59	9	-5.713	-5.169(1%)
Feb-78	**Sep-99**	-	-	I(1)	259	0	-4.008	-4.422(10%)
Oct-08	**Dec-13**	-	-	I(1)	63	4	-2.297	-4.486(10%)

As reported for the dividend-price ratio in Table 2, the LKT results for the price-earnings ratio suggest the existence of two I(1) periods. Although not exactly the same, the first non-stationary period for the price-earnings ratio appears to embrace a considerable part of the unexpected negative bubble identified in the dividend-price ratio. By spanning from February-1978 to September-1999 (259 months) it also picks up a reasonable partition of the Dotcom period that peaked by March 2000 (P/E ratio=68.3). The second period, as observed for the dividend-price ratio, also captures the recent financial crisis.

Fig.3 depicts the two I(1) periods for the decomposed price-earnings ratio. Prices are below earnings from February-1978 to December-1985. They are also below earnings from September 1989 to December 1998 but, for this second period, both price and earnings show an increasing pattern, suggesting that the increases in the index might be explained by prospective future positive earnings.

Fig. 3: The LKT results and the Decomposed NASDAQ Price-Earnings Ratio

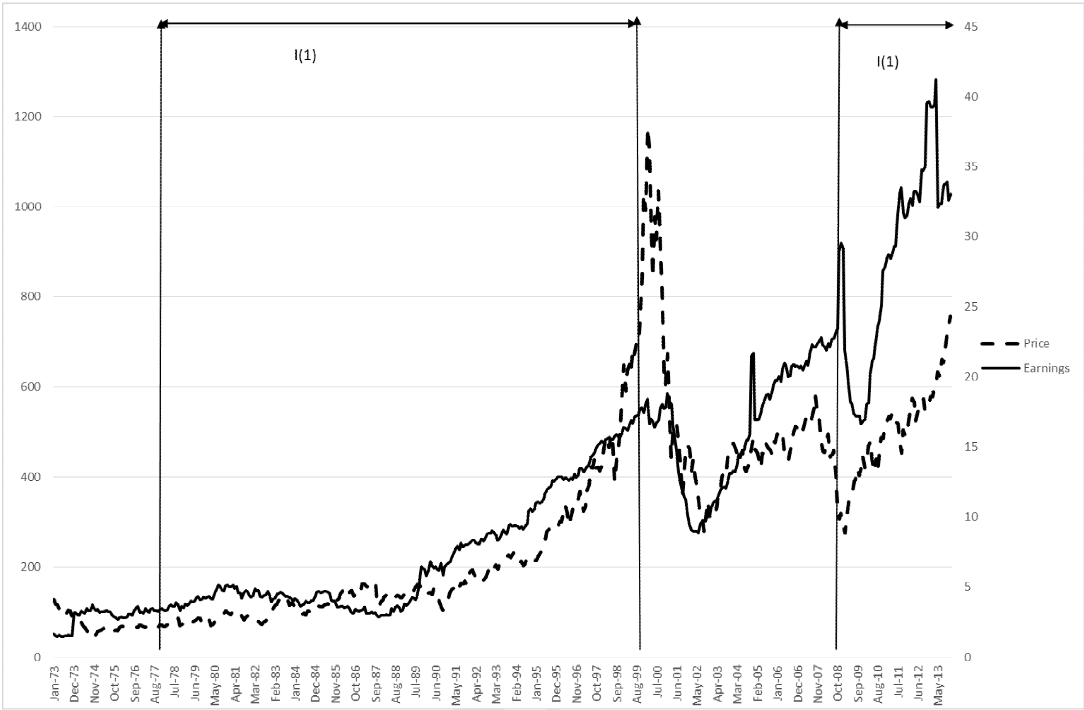


Fig. 4: Logs of Dividend and Price Earnings Ratios and Overlapping I(1) Periods

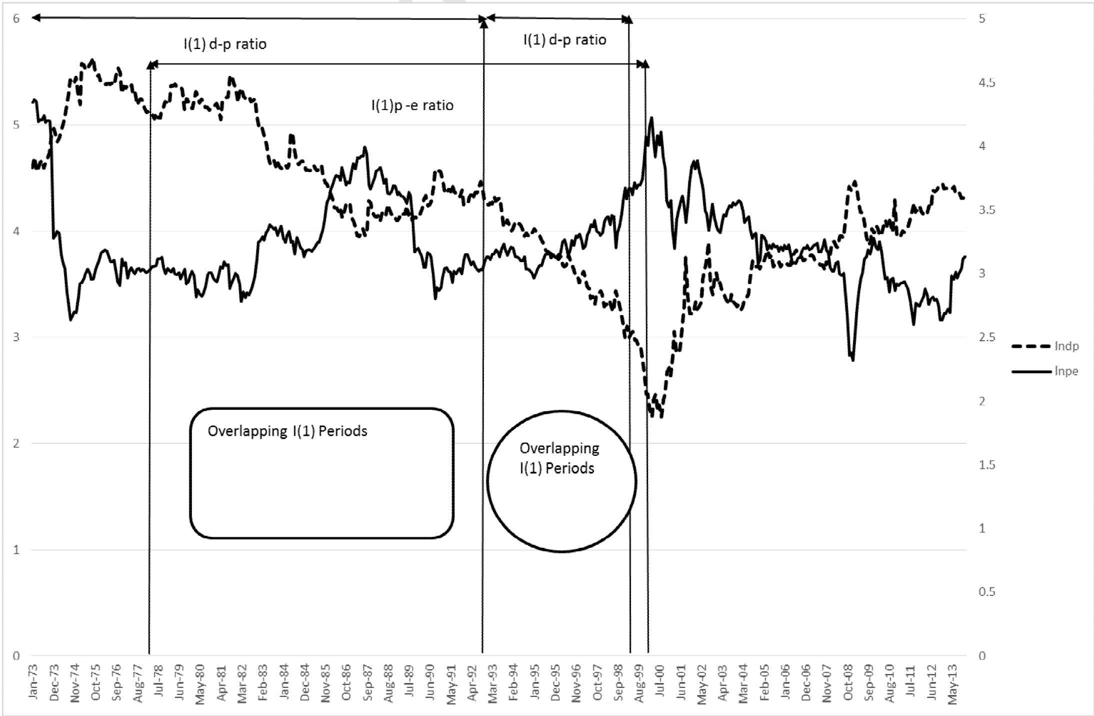


Fig. 4 depicts the logs of the dividend-price and price-earnings ratios respectively, with the overlapping I(1) periods identified for both series. At first glance it appears that during the suggested periods, where prices are below both dividends and earnings, the dividend-price ratio tends to deliver a higher yield than the price-earnings ratio. On the other hand, during periods of systematic price increases perhaps attained to an asset price bubble (likely to be fed by perspective positive future earnings), the price-earnings ratio tends to deliver higher yields. Nevertheless, the indication here is that both the dividend and the price earnings ratios suggest that during the 70s and 80s the NASDAQ was undervalued.

We also consider the PWY test for a bubble in NASDAQ, using the real monthly NASDAQ composite price index and the real monthly NASDAQ composite dividend index with a sample covering the period from February 1973 to June 2005, comprising 389 monthly observations. The $\sup_{r\in[r_0,1]} ADF_r$ test provides significant evidence of explosiveness in the price data at the 1% level, suggesting the presence of a bubble (price exuberance), but no evidence in the dividend data. Indeed, the dividend series is fairly stable and consequently non-explosive. Lintner (1956), almost 60 years ago, highlighted the idea that changes in dividend policies tend to be smooth, as companies tend to set long-run target dividends-to-earnings ratios according to the amount of positive net-present-value (NPV) projects they have available; and, as earnings increases are not always sustainable, the dividend policy is not changed until managers can see that new earnings levels are sustainable[13].

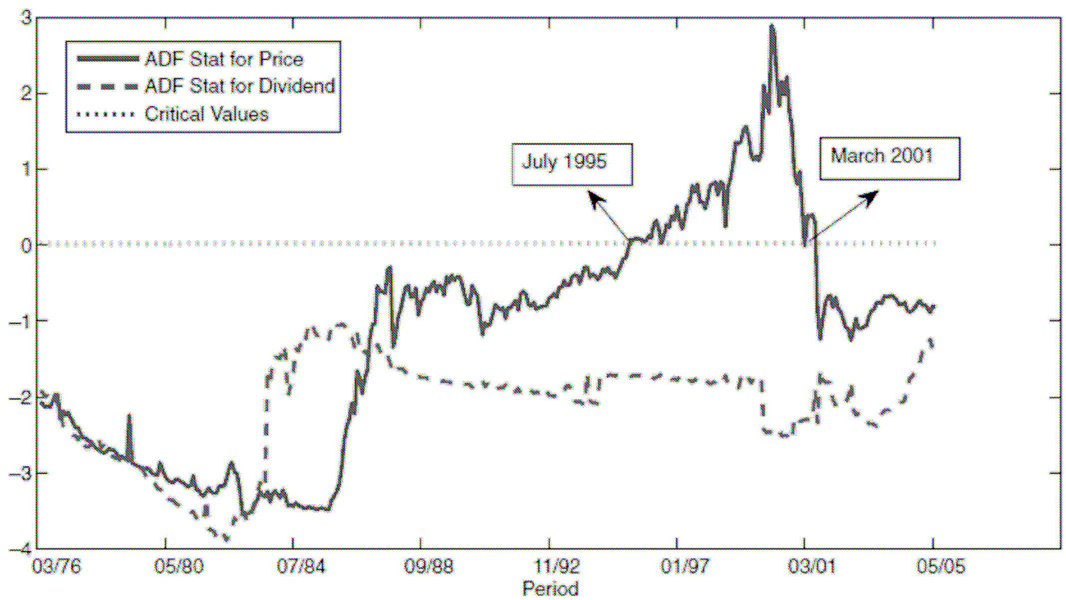
The stock price series is also found to be non-explosive for the initial sample, which suggests no evidence of a bubble in the initial data. This behaviour persists until June 1995. The test detects the presence of explosive behaviour in the data in July 1995 and the evidence of a bubble becomes stronger thereafter, reaching a peak in February 2000. The bubble lasts until February 2001, and by March 2001 they find evidence that the bubble has collapsed. In April 2001, the evidence of a bubble shows up again and persists until July 2001. In August 2001, no further evidence of a bubble is present in the data. Under the assumption of a

[13] A review of dividend policy theories and empirical evidence can be found in Al-Malkawi et al. (2010).

constant discount rate, PWY consider that the data show sufficient conditions for the presence of a bubble. Fig. 5 summarises PWY's ADF test results applied to NASDAQ.

It should be mentioned that a second test for explosive behaviour in the NASDAQ, by Phillips and Yu (2010),[14] using the same methodology of PWY but with a sample extending from January 1990 to January 2009, find different start and end dates for the NASDAQ bubble: June 1995 and November 2000, respectively.

Fig. 5: PWY's results for Logs of NASDAQ Price and Dividend April 1976 to June 2005.



Source: Phillips, Wu and Yu (2011)

Table 4 summarises the results of testing for bubbles in NASDAQ using LKT's test for change in persistence, versus PWY's tests for explosive behaviour. The following remarks are in order. First, the negative bubble found by LKT's method could not be found by PWY's method, since the latter is nor suitable to test for depressed prices, i.e. prices that

[14] Although Phillips and Yu's paper is dated as 2010, it is actually later than the PWY (2011) paper.

lie below their fundamental values as established by dividends or earnings[15]. PWY's method can only find explosive behaviour, which corresponds to positive bubbles.

Table 4. Evidence of bubbles by the LKT and the PWY tests

Method	Evidence	Start	End	Duration (months)
LKT (d-p ratio)	Negative bubble	1973:02	1992:06	238
LKT(d-p ratio)	Bubble	1998:12	2001:07	31
LKT (p-e ratio)	Some overlapping with Negative bubble and Dotcom bubble	1978:02	1999:09	259
PWY	Bubble	1995:07	2001:03	68
PWY	Bubble	1995:06	2000:11	65

Second, neither method is precise concerning the beginning and end of the bubble. The two tests using the PWY methodology cited in this paper found different dates for the beginning and ending of the Dotcom bubble (4th and 5th rows of Table 4). Using the LKT method, we also find different start and end dates for the bubble, when using different sample periods.

Third, it becomes apparent that the LKT's method is less sensitive to the beginning of a bubble, as it only captures the beginning of the bubble as December 1998, when prices have already gone some way from their fundamental values, as seen in Fig. 1. However, when the price-earnings ratio is used, the test clusters both the period where there is a suggestion of a negative bubble and a substantial part of the Dotcom period. This can also be seen by comparing this with the two PWY bubble starting points of July 1995 and June 1995, respectively. If we consider that on average, the media registers the Dotcom bubble as

[15] According to Shiller (2003, p. 91), a negative bubble occurs when price movements propel "further downward price movements, promoting word-of-mouth pessimism, until the market reaches an unsustainably low level". Other references to negative bubbles can be found, for example, in Blanchard and Watson (1982), Flood and Roderick (1990), Payne and Waters (2005), and Shiller (2000a).

starting in April 1997 and ending in June 2003, we see that the PWY method picks up the bubble 27 months earlier than it actually happened and the LKT method captures the bubble 20 months later than it actually occurred. Additionally, the LKT method takes longer to identify the end of the bubble in the case of the dividend price ratio but not for the price-earnings ratio (recording it in as July 2001 and September 1999 respectively), against PWY's bubble ending by March 2001 and November 2000, respectively. Therefore, both methods capture the end of the bubble much earlier than recorded by the financial media.

5. Conclusions

This study has investigated multiple changes in persistence of time series data on the dividend-price and price-earnings ratio for the NASDAQ composite index over the period 1973-2013, by applying a time-series method capable of signalling and dating asset price bubbles.

The method developed by Leybourne et al. (2007), the LKT test, is part of a new batch of procedures that address a key flaw in previous time-series and unit root approaches – their inability to detect and attribute calendar dates to changes in persistence on the stationary and non-stationary condition of a series under investigation. That is, by being able to find when a time-series changes its regime from I(0) to I(1), and vice-versa, the LKT approach allows us to conjecture the initiation and collapse of rational bubbles.

Our empirical results show unexpected depressed prices, signalling an undervalued market and a likely negative bubble in the NASDAQ, spanning February 1973 to June 1992 in the case of the dividend-price ratio series, partially confirmed by the price-earnings series, from February 1978 to December 1985. The depressed prices may not be seem as a surprise as it could be a result of overreaction and pessimism linked to economic and political events between 1970s-1990s: 1973-1974 (OPEC, Watergate, inflation and recession); 1976-1978 (stagflation, budget deficits and trade deficits); 1980-1982 (high interest rates, the second OPEC crisis, recession and inflation); 1987 (stock market crash); 1990 (The Gulf war and the collapse of the Japanese stock market).

The results find a positive bubble, from December 1998 to July 2001 for the dividend-price ratio, again partially confirmed by the price-earnings ratio that identified a much longer I(1) period spanning February 1978 to September 1999. Nevertheless, both periods identified by the LKT test are reasonably situated around the period when the Dotcom bubble is usually recorded, both by the media and academic studies. In fact, most of the financial media records the Dotcom bubble as starting in April 1997 and ending in June 2003.

As most studies on rational bubble detection generally use annual data on the S&P 500, we the compare our results from LKT's method to those recorded by Phillips et al.(2011), the PWY method, who provide strong evidence for a bubble in the NASDAQ Index at the end of the 1990s. The first PWY test, aimed at flagging and dating the NASDAQ bubble, finds the bubble starting in July 1995 and ending in March 2001 (Phillips et al., 2011)). The second PWY test, using a different sample, finds the bubble starting in June 1995 and ending in November 2011 (Phillips and Yu, 2010).

Our remarks, based on the findings of this paper, is that the LKT and the PWY methods are complementary, as far as positive bubbles are concerned, although they lack agreement with respect to the bubble start and end dates. The PWY method finds the bubble starting much earlier than the LKT method, but the latter is closer to the average media versions. With respect to the end of the bubble, the differences are much smaller between the findings of the two methods, although both methods find the bubble ending much earlier than what is reported by the media. It also should be mentioned that only the LKT test is capable of finding both positive negative bubbles while the PWY test, by definition, can only find positive bubbles.

The results also suggest that, although not fully precise, the possibility of finding changes in persistence or explosive behaviour, plus the capability of identifying calendar dates to those episodes, flag not just the likely use of such unit root methods to test for asset price bubbles, but also opens a new dimension for the analysis of economic and financial time series data. For example, because both methodologies appear to provide timely warning signals of exuberant prices, central banks could use these indicators to promote financial stability and achieve the objectives of macro-stability. Thus, as the tools used by regulators take considerable time to take effect, methods capable of picking up warnings signals of the start of a bubble could be very useful. Finally we conjecture that the LKT and the PWY methods can also be applied to study recent phenomena in real estate, commodity and foreign exchange markets.

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Table 1: Summary of descriptive statistics for the NASDAQ dividend-price ratio

	Real price index	Real dividend index	d-p ratio	Log(d-p ratio)
Mean	287.2654	287.9986	1.588659	0.205429
Median	207.0028	233.0149	1.180000	0.165514
Maximum	1175.556	987.9407	5.010000	1.611436
Minimum	46.75840	143.0622	0.170000	-1.771957
Std. Dev.	214.0360	152.4419	1.149710	0.737831
Skewness	1.033131	2.533315	1.078285	-0.135897
Kurtosis	3.946336	10.00359	3.015576	2.636398
Jarque-Bera	105.8824	1531.780	95.34622	4.224596
Probability	0.000000	0.000000	0.000000	0.120960
Observations	492	492	492	492

Table 2: Results of the LKT test for the NASDAQ dividend-price ratio

Period Tested		Result		Regime	Duration (months)	$\hat{k}_i$	M-Statistic	Critical Value
Start	End	Start	End					
1973:01	2013:12	2001:08	2009:08	I(0)	12	12	-5.553	-5.078 (1%)
1973:01	2001:07	1992:07	1998:11	I(0)	78	2	-4.756	-4.400 (10%)
1973:01	1992:06	-	-	I(1)	234	10	-3.678	-4.422 (10%)
1998:12	2001:07	-	-	I(1)	32	0	-3.677	-5.459 (10%)
2009:09	2013:12	-	-	I(1)	52	0	-3.721	-4.970(10%)

Table 3: Results of the LKT test for the NASDAQ price-earnings ratio

Period Tested		Result						
Start	End	Start	End	Regime	Duration (Months)	$\hat{k}_i$	M-Statistic	Critical Value
Jan-73	Dec-13	Oct-99	Sep-08	I(0)	108	10	-7.512	-5.078(1%)
Jan-73	Sep-99	Mar-73	Jan-78	I(0)	59	9	-5.713	-5.169(1%)
Feb-78	Sep-99	-	-	I(1)	259	0	-4.008	-4.422(10%)
Oct-08	Dec-13	-	-	I(1)	63	4	-2.297	-4.486(10%)

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