Vertical Integration in Multi-commodity Energy Markets

Undergraduate Honors Research Thesis

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Abstract: In a simple energy-market structure consisting of natural gas and electricity suppliers, suppliers exercise market power by adjusting their production levels to influence the market price. This is done with the aim of maximizing their individual profits. This paper examines the impact of vertical integration, whereby firms supply both natural gas and electricity, on market efficiency. Market equilibria, which is the point at which no firm has incentive to deviate unilaterally, was modeled using a computational approach. Stylized profit-maximization problems were examined with vertically integrated and dis-integrated firms using a quantity-setting Nash-Cournot framework. Karush-Kuhn-Tucker (KKT) conditions are used to characterize optimal solutions to the firms' profit-maximization problems. Combining the KKT conditions of all of the firms allows us to compute market equilibria efficiently. The complementarity framework is used to examine the incentives of integrated and dis-integrated firms to exercise market [illegible] real-world energy systems are used to examine the impacts of potential mergers between natural gas and electric firms on market efficiency. This work can inform policy makers and regulators in determining how markets should be structured to ensure increased efficiency. Policy makers and regulators could employ a modeling technique, such as the one developed in this paper, to screen potential vertical mergers in energy and other market settings.

Contents

1. Introduction

Energy markets are often a focus of operations research. Different optimization problems can be used to model markets and predict their behavior. Insights from models like these can benefit decision making within companies and policy making from regulators. Throughout these models, however, the key component that tends to be missing is vertical integration. Vertical integration is when a firm owns multiple steps in the supply chain (Amadeo et al). In the context of energy markets, this can mean a firm owns both natural gas plants, which produce and sell natural gas, and electricity plants, which buy that natural gas as fuel to produce electricity. Being involved in multiple levels of the supply chain has the potential to increase a firm's capacity to exercise market power. A firm uses market power to manipulate the price of a market commodity, such as natural gas and electricity. By exploring the scarcely explored effects of vertical integration, this research has the potential to discover crucial information regarding the impacts on different players in the market.

An important concept when analyzing energy markets is social welfare, which is the sum of consumer welfare and producer welfare. Social welfare can be considered the total benefit to all parties. Consumer welfare is the difference between the price a consumer is theoretically willing to pay and the actual price. Mathematically, it is the area between the demand curve and the price, which is computed as the integral under the demand curve minus the area below the price. Producer welfare is the difference between the price a producer is theoretically willing to sell for and the actual price. Mathematically, it is the difference between the area above the supply curve and below the price, which is computed as the area below the price minus the integral of the supply curve. The value of producer welfare, consumer welfare, and overall social welfare can be calculated at a given market equilibrium. This is a key component of analyzing the effects of different manipulations of the market structure.

There are two aims of this research. The first aim is to explore how to represent vertical integration using complementarity models. Vertical integration has not been modeled using complementarity models in prior literature. This research works to develop a modeling framework that better represents vertical integration in energy markets. The second aim is to use the model to examine the impact of vertical integration on the market efficiency. Different levels of integration will be modeled, solved, and analyzed. Social welfare will be used as a metric, along with market prices and production quantities, to see which parties benefit and which are harmed by merger. This information can be crucial to outside regulators to ensure that regulations are made to benefit social welfare and protect consumers from excessive abuse of market power.

The remainder of the paper is structured as follows. Section 2 will discuss the methodology used in this research. Section 3 will discuss the Nash Cournot model developed. Section 4 will then present the results from an examination of merger. Finally, Section 5 section will discuss potential future directions for this research.

2. Methodology

This project has an iterative structure that involves 5 main steps. These steps are listed below.

1. Formulate Model
2. Program Model using GAMS
3. Check Model Accuracy
4. Manipulate Market Structure
5. Analyze Results

During the "Formulate Model" step, the desired market structure is formulated as a complementarity model. During the "Program Model in GAMS" step, the formulation is converted into KKT conditions so that the equilibrium can be solved by a nonlinear or mixed integer solver using the GAMS software package. These equations will then be coded into GAMS using the algebraic modeling language (AML). During the "Check Model Accuracy" step, the GAMS model will be solved and the solution analyzed to ensure that the model is behaving as expected. During the "Manipulate Market Structure" step, the formulation will be adjusted to reflect different market situations, such as varying levels of vertical integration. During the "Analyze Results" step, the solutions that are provided by GAMS during the previous step will be analyzed to find insights on the relationships between market structure and outcomes.

Energy markets are commonly represented using a complementarity model. A complementarity model is a method of modeling and solving optimization problems that involves converting the optimization problem into a set of Karush-Kuhn-Tucker conditions, or KKT conditions. KKT conditions are a series of necessary conditions that must be satisfied if a given solution is optimal. For the generic optimization problem

$$\min_{x} f(x) \tag{1}$$

$$h_i(x) = 0, \forall i = 1,2, \dots, m \tag{2}$$

$$g_j(x) \leq 0, \forall j = 1,2, \dots, n \tag{3}$$

where $f(x)$, $h_i(x)$, and $g_j(x)$ are continuously differentiable, an optimal solution, x^*, must satisfy the following KKT conditions

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i{}^* \nabla h_i(x^*) + \sum_{j=1}^{n} \mu_j{}^* \nabla g_j(x^*) = 0 \tag{4}$$

$$g_j(x^*)\mu_j{}^* = 0, \forall j = 1,2, \dots, m \quad (5)$$

$$\mu_j{}^* \geq 0, \forall j = 1,2, \dots, m \quad (6)$$

$$h_i(x^*) = 0, \forall i = 1,2, \dots, n \quad (7)$$

where $\lambda_i{}^*$ and $\mu_j{}^*$ are the Lagrangian multipliers of the optimization problem and x^* is regular. Condition (4) is the stationarity condition. Condition (5) is the complementary slackness condition. Condition (6) is the Lagrangian non-negativity condition. Condition (7) is the equality constraint.

If $f(x)$ is convex, $h_i(x)$ are linear, and $g_j(x) \leq 0$ are convex, then these KKT conditions are sufficient to determine that the solution x^* is globally optimal. Using these KKT conditions allows for multiple optimization problems to be solved simultaneously to reach an equilibrium.

3. Nash Cournot Model

The wholesale market price of natural gas and electricity are each determined by an inverse demand function. These functions are defined as

$$A^G - B^G r^G \quad (8)$$

$$A^E - B^E r^E \quad (9)$$

where A^G and B^G are the inverse demand parameters of the natural gas firm, A^E and B^E are the inverse demand parameters of the electricity firm, and r^G and r^E are the retail demand of natural gas and electricity respectively. r^G and r^E can be defined as

$$r^G = \sum_m (q_m{}^G) - \sum_n (\eta_n q_n{}^E) \quad (10)$$

$$r^E = \sum_n (q_n{}^E) \quad (11)$$

where $q_m{}^G$ and $q_n{}^E$ are the production quantities of natural gas firm m and electricity firm n respectively and η_n is the heat rate of firm n. A heat rate is the ratio between the thermal energy that goes in an electricity-producing mechanism and the electricity that comes out. For practical purposes, it is the conversion between gas fuel and electricity.

The firms are modeled as profit-maximizers. Profit per unit is defined as the wholesale market price per unit minus the production cost per unit. Multiplying the profit per unit by the units of production for a firm gives that firm's total profit. The profit of natural gas firm i is thus defined as

$$\left(A^G - B^G\left(\sum_m ({q_m}^G) - \sum_n (\eta_n {q_n}^E)\right) - {c_i}^G\right){q_i}^G \tag{12}$$

where ${c_i}^G$ is the production cost per unit of natural gas of firm i. The production of natural gas is subject to a non-negativity constraint and a production capacity constraint, which can be defined for firm i respectively as

$${q_i}^G \geq 0 \tag{13}$$

$${q_i}^G \leq {C_{P_i}}^G \tag{14}$$

where ${C_{P_i}}^G$ is the natural gas production capacity of firm i.

Electricity firms may use natural gas as a fuel, meaning that the production cost also includes the fuel cost, which is the wholesale price of natural gas multiplied by the heat rate of the electricity firm. With this fuel cost included, the profit of electricity firm j is defined as

$$\left(A^E - B^E\left(\sum_n ({q_n}^E)\right) - {c_j}^E + \left(A^G - B^G\left(\sum_m ({q_m}^G) - \sum_n (\eta_n {q_n}^E)\right)\right)\eta_j\right){q_j}^E \tag{15}$$

where ${c_j}^E$ is the non-fuel production cost per unit of electricity of firm j. The production of electricity is subject to a non-negativity constraint and a production capacity constraint, which can be defined for firm j respectively as

$${q_j}^E \geq 0 \tag{16}$$

$${q_j}^E \leq {C_{P_j}}^E \tag{17}$$

where ${C_{P_j}}^E$ is the electricity production capacity of firm j.

The integrated firms are simply modeled as the profit maximization of the summation of the profits from natural gas as defined in (12) and the profits from electricity as defined in (15), subject to the constraints of both markets defined in (13)-(14) and (16)-(17). Integrated firm k is thus formulated as

$$\begin{aligned}\max_{{q_k}^G, {q_k}^E} &\left(A^G - B^G\left(\sum_m ({q_m}^G) - \sum_n (\eta_n {q_n}^E)\right) - {c_k}^G\right){q_k}^G \\ &+ \left(A^E - B^E\left(\sum_n ({q_n}^E)\right) - {c_k}^E + \left(A^G - B^G\left(\sum_m ({q_m}^G) - \sum_n (\eta_n {q_n}^E)\right)\right)\eta_k\right){q_k}^E\end{aligned} \tag{18}$$

$$q_k{}^G \geq 0 \tag{19}$$

$$q_k{}^E \geq 0 \tag{20}$$

$$q_k{}^G \leq C_{P_k}{}^G \tag{21}$$

$$q_k{}^E \leq C_{P_k}{}^E \tag{22}$$

All three types of firms can be formulated as an integrated firm as shown in problem (18)-(22). If firm k is a natural gas only firm, it can be formulated as integrated firm simply by setting the production capacity of electricity for firm k, $C_{P_k}{}^E$, equal to zero. The combination of constraints (20) and (22) would require $q_k{}^E = 0$. Similarly, if firm k is an electricity only firm, it can be formulated as integrated firm simply by setting the production capacity of natural gas for firm k, $C_{P_k}{}^G$, equal to zero. The combination of constraints (19) and (21) would require $q_k{}^G = 0$. No changes need to be made to problem (18)-(22) if firm k is an integrated firm. Using complementarity principles, each optimization problem is converted to KKT Conditions shown below

$$-A^G + B^G\left(\sum_m (q_m{}^G) - \sum_n (\eta_n q_n{}^E)\right) + c_k{}^G + B^G q_k{}^G - B^G \eta_k q_k{}^E - \mu_{1_k} + \mu_{3_k} = 0 \tag{23}$$

$$B^G \eta_k q_k{}^G - A^E + B^E \sum_n (q_n{}^E) + c_k{}^E + \left(A^G - B^G\left(\sum_m (q_m{}^G) - \sum_n (\eta_n q_n{}^E)\right)\right)\eta_k + B^E q_k{}^E + B^G \eta_k{}^2 q_k{}^E - \mu_{2_k} + \mu_{4_k} = 0 \tag{24}$$

$$\mu_{1_k}(-q_k{}^G) = 0 \tag{25}$$

$$\mu_{2_k}(-q_k{}^E) = 0 \tag{26}$$

$$\mu_{3_k}\left(q_k{}^G - C_{P_k}{}^G\right) = 0 \tag{27}$$

$$\mu_{4_k}\left(q_k{}^E - C_{P_k}{}^E\right) = 0 \tag{28}$$

$$\mu_{1_k} \geq 0 \tag{29}$$

$$\mu_{2_k} \geq 0 \tag{30}$$

$$\mu_{3_k} \geq 0 \tag{31}$$

$$\mu_{4_k} \geq 0 \tag{32}$$

where conditions (23) and (24) are the stationarity conditions defined in equation (4), conditions (25)-(28) are the complementary slackness conditions defined in equation (5), and conditions (30)-(33) are the Lagrangian non-negativity constraints defined in equation (6). These sets of KKT Conditions for each firm are then solved simultaneously as a system of equations to find a market equilibrium.

4. Results

The Nash Cournot Model developed in the previous section was used to study merger. The parameters were set to the arbitrary values below.

$A^G = 30$

$B^G = 3$

${c_k}^G = 2, \forall k$

${C_{P_k}}^G = 20, \forall k$

$A^E = 30$

$B^E = 3$

${c_k}^E = 2, \forall k$

${C_{P_k}}^E = 20, \forall k$

$\eta_k = 0.05, \forall k$

For this study, a small model with two natural gas firms and two electricity firms was used to examine how merger impacts the market efficiency. *Case 1* is a scenario without any vertical integration. *Case 2* has one natural gas firm merged with one electricity firm to create a market with one integrated firm and two disintegrated firms (one of each commodity). *Case 3* has both natural gas firms each merged with an electricity firm to create a market with two integrated firms and no disintegrated firms. These four firms over these three cases are described in figure 1 below.

Key			
	Case 1: **No Integration**	**Case 2:** **1 Integrated**	**Case 3:** **2 Integrated**
f1	Gas firm	*Integrated firm*	*Integrated firm*
f2	Gas firm	Gas firm	*Integrated firm*
f3	Electric firm	None (absorbed by f1)	None (absorbed by f1)
f4	Electric firm	Electric firm	None (absorbed by f2)

Figure 1: Firm type key of merger case study.

These three cases were run using GAMS. Figure 2 below presents the key metrics of the equilibrium, including an examination of the firm profits, market clearing price, and welfare at the market equilibrium.

		Case 1: No Integration	Case 2: 1 Integrated	Case 3: 2 Integrated
Gas	Price	11.638	11.488	11.333
Elec	Price	11.736	11.897	12.052
Total Gas Production		6.426	6.426	6.522
Total Elec Production		6.088	6.088	5.982
Fuel Gas Demand		0.3044	0.3044	0.2991
Retail Gas Demand		6.1216	6.1216	6.2229
Profit	f1	30.962	*58.76*	*58.807*
	f2	30.962	30.009	*58.807*
	f3	27.866	N/A	N/A
	f4	27.866	28.897	N/A
Producer Welfare	Gas	61.924	61.411	60.866
	Elec	55.732	56.255	56.748
	Total	117.656	117.666	117.614
Consumer Welfare	Gas	127.427	128.003	128.593
	Elec	127.043	126.411	125.791
	Total	254.47	254.414	254.384
Social Welfare	Gas	189.351	189.414	189.459
	Elec	182.774	182.666	182.539
	Total	372.125	372.08	371.998
% Producer Welfare		31.6%	31.6%	31.6%
% Consumer Welfare		68.4%	68.4%	68.4%

Figure 2: Key metrics of case study examination of merger.

As seen by the metrics in Figure 2, changes caused by merger were negligible. According to these results, merger does not significantly impact the market equilibrium.

5. Future Work

Moving forward with this research, the stationarity conditions will be examined analytically. This should uncover how the incentives of each firm change as parameters change, as well as the differences between the incentives of natural gas firms, electric firms, and integrated firms. Better understanding the incentives of firms through the analytical analysis of stationarity conditions will help to predict the results of different market manipulations as well as explaining the results seen in case studies, such as the results from the previous section.

The parameters used in Section 4 were chosen arbitrarily and the differences in equilibrium due to merger were negligible. It is possible that non-arbitrary parameters may produce non-negligible differences in equilibrium. Moving forward, the Nash Cournot Model formulated earlier will be calibrated to produce a more realistic version of the model. Data from the United States Energy Information Administration will be examined to determine realistic market parameters and values, and then those values will be used to create a calibrated model.

The effects of merger can be studying using this new calibrated model. I hypothesize that an integrated firm with electricity production dependent on natural gas as a fuel will be incentivized to produce more natural gas in order to drive the prices of fuel down. Manipulation of the market such as this may have negative impacts on the welfare of the system.

Beyond the scope of this research, there is the potential for the model to be expanded for other uses. A user-friendly version of the model may be developed for policy makers and regulators. Such a model could assist in determining market structure and screening potential mergers. In addition, a more generalized model could be developed so that it may be adapted to accommodate other markets with a similar structure.

References

Amadeo, Kimberly. “Vertical Integration, Its Pros and Cons with Examples .” The Balance, Dotdash, 5 Nov. 2018, www.thebalance.com/what-is-vertical-integration-3305807. Accessed 7 May 2019