Erratum: "Resolution of a shock in hyperbolic systems modified by weak dispersion" [Chaos 15, 037103 (2005)]

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1. Formula (23) on p. 5 should be

$$m=1: \quad V_2=V_3=\tfrac{1}{3}(\beta_1+2\beta_3), \quad V_1=\beta_1.$$

2. On p. 13, where the problem of the decay of a step for the defocusing modified KdV [mKdV(d)] equation is considered, after formula (90), in the case (i), a wrong expression for the velocity/amplitude relationship for the mKdV(d) soliton was used. This led to an incorrect expression for the lead soliton amplitude in the mKdV(d) dispersive shock. The correct expression for the amplitude is derived and discussed below.

For the initial step (88) satisfying the condition of the case (i), $u_+^2-u_-^2>0$, the speeds of the mKdV(d) dispersive shock edges $s^{\pm}(u_+,u_-)$ are determined by expressions (89). Then the amplitude of the lead soliton a^+ is found from the kinematic condition $c_s(u^+,a^+)=s^+$ [cf. (62); here we use a different set of independent variables for the function c_s], where the relationship between the velocity and the amplitude for the mKdV(d) soliton moving against the background $\bar{u}$ is obtained by an elementary analysis of the traveling wave solution in the soliton limit and has the form $c_s(\bar{u},a_s)=-\bar{u}^2\pm 2\bar{u}a_s/3-a_s^2/6$. Different signs in this expression correspond to solitons of different polarity supported by the mKdV(d) equation. Now, after simple calculation one gets $a^+=2(|u_+|-|u_-|)$. For $u_+u_-\geqslant 0$ this coincides with the KdV result (31) [to make such a comparison in the case when $u^+>u^-\geqslant 0$, the KdV equation should be taken with the negative sign for the nonlinear term, which would result in the change $\Delta\to-\Delta$ in (31)]. The location of the mKdV(d) dispersive shock given by (89), however, is different from the respective KdV case; for the KdV equation with the same initial data one has $|s^+-s^-|=\frac{5}{3}(|u_+|-|u_-|)t$ rather than $\frac{5}{3}(u_+^2-u_-^2)t$.

Of course, these results could have been inferred from the already known periodic solution and the Riemann form of the modulation system for the mKdV(d) equation (see Ref. 1, for instance) but here they were obtained without invoking an integrable structure of the mKdV(d) equation, as an illustration of the effectiveness of the general transition relations for a dispersive shock.

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[1] A. M. Kamchatnov, A. Spire, and V. V. Konotop, "Dissipationless shock wave in a discrete nonlinear Schrödinger equation," J. Phys. A **37**, 5547–5568 (2004).

