Gettier Problems and Logical Properties of Justification

Václav Rajlich

Wayne State University, Detroit, MI 48202
rajlich@wayne.edu

Abstract. In the classical account of knowledge, S knows that P if and only if S believes that P, S is justified in believing that P, and P is true (JTB).. In 1963, Gettier presented two problems that casted doubt on this account. Since then, numerous authors proposed modifications or clarifications of JTB, however, these efforts have not produced a satisfactory solution. In this paper, the focus is on logical properties of justification. The Gettier problem Case II is expressed in sentential logic and Gettier Minimal Assumption (GMA) is introduced. It is shown that Gettier must have used GMA or some other assumption that entails GMA in his construction of Case II. Rejection of GMA solves Gettier problem Case II and it is a step towards a better understanding of the logical properties of justification and knowledge.

Keywords: Sentential logic, Gettier Minimal Assumption, Transmissibility of Evidence.

1 Introduction

In the classical account of knowledge, *S* knows that *P* if and only if *S* believes that *P*, *S* is justified in believing that *P*, and *P* is true. This account of knowledge is called "justified true belief", abbreviated *JTB*. In 1963, Gettier presented two problems that casted doubt on this account [1]. Since then, numerous authors proposed modifications or clarifications of *JTB*. However, these efforts have not produced a satisfactory solution [2]. In this paper, the focus is on logical properties of justification with aim to prove that they are the real cause of the Gettier problems.

The past approaches to Gettier problem often relied on intuition as the main tool [3, 4]. In this paper, intuition is augment by use of sentential (Boolean, propositional) logic [5]. Sentential logic is a classical and widely accessible formal framework. It offers simplicity and clarity that may be lacking in various non-classical logical systems proposed for epistemology. Logical calculator is used in the proofs of the sentential logic propositions of this paper [6].

2 JTB in Sentential Logic

For a belief *P*, the *truth value* is denoted as $V(P)$ and it is either *true* or *false*. Agent *S justifies* belief *P* by gathering supporting evidence and reasoning about it. The result is formalized as $J_S(P)$. If the justification supports belief *P* and meets a reasonable standard, the value of $J_S(P)$ is *true*. If the justification fails or is inadequate, then $J_S(P)$ is *false*. Then according to *JTB*, belief *P* of *S* qualifies as knowledge, denoted as $K_S(P)$, if both its justification $J_S(P)$ and truth value $V(P)$ are *true*. Formally, this is expressed in the following way:

(JTBSL) Agent *S* has a belief *P* and this belief is *knowledge* $K_S(P)$ if and only if $K_S(P) = J_S(P) \wedge V(P) = true$.

This definition of knowledge has several properties:

(Fallibilism) Justification and truth value are independent of each other and may produce disparate results. There can be unjustified beliefs that are *true* and there can be justified beliefs that turn out to be *false* [7].

(Distributive laws for truth values) It is widely accepted that truth values follow the distributive rules, i.e. $V(X \wedge Y) = V(X) \wedge V(Y)$, $V(X \vee Y) = V(X) \vee V(Y)$, $V(\neg X) = \neg V(X)$. This allows the following shorthand for easier readability: We drop predicate *V* from the formulas in the rest of the paper, i.e. *X* will mean both "proposition *X*" and "truth value *V(X)*." From the context, it will be always clear which meaning symbol *X* represents.

(Nontrivial properties of justification) In contrast to distributive laws for truth values, logical properties of justification have to be treated with caution. For example, it is intuitively clear that failed justification for proposition *A* does not imply that its negation is justified, i.e. $\neg J_S(A)$ does not imply $J_S(\neg A)$. In this paper, we investigate whether Gettier problem points to additional nontrivial properties of justification.

(JTB+) There have been numerous attempts to change *JTB* in order to solve Gettier problems and many of them added a new property to *JTB*. As long as these new accounts retain "justified and true" part of knowledge account, reasoning used in this paper applies to these modified accounts also.

The rest of the paper deals with the Gettier problem Case II (Gettier 1963). It also deals with a single agent *S* and as a notational convenience, $J_S(P)$ and $K_S(P)$ will be replaced by simplified notation *J(P)* and *K(P)*, i.e., subscript that identifies a specific agent will be dropped. Note that the meanings of *J(P)* and *K(P)* remain "agent *S* holds justified belief *P*" and "belief *P* of agent *S* is knowledge," respectively.

3 Gettier Problem Case II and Reconstructed Gettier Reasoning

Gettier presented the following example: Smith has credible evidence that Frank owns a Ford car. However, in reality, Frank does not own a Ford car and the evidence is misleading. Simultaneously, Smith has no evidence that Brown is in Barcelona and by coincidence, Brown actually is in Barcelona. According to Gettier, the proposition "Frank owns a Ford or Brown is in Barcelona" satisfies *JTB* account but intuitively, it is not knowledge. This contradiction is the *Gettier problem, Case II* [1]. Many authors over the past 50 years searched for a better account of knowledge that would avoid Gettier problem Case II, without finding a satisfactory solution [2].

Note that Gettier does not reveal details of his reasoning. In order to reconstruct it, Gettier problem is formulated in sentential logic. Let *F* denote proposition "Frank owns Ford" and *B* denote proposition "Brown is in Barcelona", then *J(F)* and *B* are *true* and *F* and *J(B)* are *false*. Gettier concluded that in this situation, $K(F \vee B)$ is *true.* This is formalized in the following way:

(G) $J(F) \wedge \neg F \wedge \neg J(B) \wedge B => J(F \vee B) \wedge (F \vee B)$

The literals of proposition *G* are *J(F), F, J(B), B,* and because of the nontrivial properties of justification, *J(F ∨B)* is also treated as a literal. Then *G* is not a tautology and it is true only under certain assumptions, called "patches."

In general, a *patch of proposition X* is a pro tion *P* that specifies the assumption under which *X* is true, i.e. *P => X* is a tautology. Note that patches have transitive property, i.e. if *P* is a patch of *Q* and *Q* is a patch of *R* then *P* is also a patch of *R.* An example of a patch of *G* is proposition *GMA*, defined in the next definition.

(GMA) $J(F) \wedge \neg J(B) => J(F \vee B)$

Lemma 1

GMA is a patch of *G*.

Proof

Lemma 1 is equivalent to the following proposition:

(S1) $(J(F) \wedge \neg J(B) => J(F \vee B)) =>$
$((J(F) \wedge \neg F \wedge \neg J(B) \wedge B) => (J(F \vee B) \wedge (F \vee B)))$

The formula *S1* is a tautology, per logic calculator.

For a more detailed proof, note that proposition *S1* is *true* for all combinations of values in which *J(F) = false* or *J(B) = true*. For all remaining combinations, *J(F) = true* and *J(B) = false*. Substituting these values into proposition *S1,* we get:

(S2) $J(F \vee B) => (\neg F \wedge B => J(F \vee B) \wedge (F \vee B)).$

Proposition *S2* is *true* for *J(F ∨B)= false.* For *J(F ∨B)= true,* we get:

(S3) $(\neg F \wedge B) => (F \vee B)$

Proposition *S3* is a tautology. This completes the proof of proposition *S1* and *Lemma 1.* □

Later, we will show that *GMA* plays a significant role in Gettier reasoning. Besides *GMA,* there are many additional patches of *G*. We can divide them into two broad groups: Patches that contain literal *J(F ∨B)* and patches that do not. The patches that do not contain *J(F ∨B)* have the following property:

Lemma 2

Let *Q* be a patch of *G* that does not contain literal *J(F ∨B).* Then
$Q => \neg(J(F) \wedge \neg F \wedge \neg J(B) \wedge B).$

Proof

If *Q* is a patch of *G,* then *Q => G*, i.e.

(S4) $Q \Rightarrow ((J(F) \wedge \neg F \wedge \neg J(B) \wedge B) \Rightarrow J(F \vee B) \wedge (F \vee B))$ is a tautology.

When substituting a specific truth value for a literal in a tautology, we still get a tautology. Let's substitute *false* for $J(F \vee B)$ in *S4* and we get

(S5) $Q \Rightarrow ((J(F) \wedge \neg F \wedge \neg J(B) \wedge B) \Rightarrow false)$

Lemma 2 follows from proposition *S5*. □

Lemma 2 states that patches that do not contain $J(F \vee B)$ must contradict the antecedent of *G*, i.e. must contradict $J(F) \wedge \neg F \wedge \neg J(B) \wedge B$. Those patches are contrived, and we conclude that they were not a part of Gettier's reasoning.

For patches that contain $J(F \vee B)$, *justification patches* consist of only justification literals while *mixed patches* contain also truth value literals. Mixed patches either contain redundant truth value literals (an example is patch $J(F \vee B) \wedge (\neg F \vee F)$,) or they impose relation between truth values and justifications and hence they violate fallibilism mentioned in Section 1 (an example is patch $B \Rightarrow J(F \vee B)$.) In either case, we conclude that mixed patches play no part in Gettier's reasoning.

In contrast, justification patches describe relations among the justifications and hence they comply with fallibilism. Among them, *minimal justification patches* play a special role:

(M) Minimal justification patch M of *X* is a justification patch of *X* so that for every justification patch *P* of *X*, $P \Rightarrow M$.

Definition *M* allows us to formulate the following theorem:

Theorem 1

GMA is a minimal justification patch of *G*.

Proof

We already know that *GMA* is a patch of *G* by *Lemma 1*. *GMA* is a justification patch because it consists of justification literals only. The only part that remains to be proven is the fact that *GMA* is a minimal justification patch of *G*.

Let *P* be a justification patch of *G*, then *P* consists of justification literals and $P \Rightarrow G$ is tautology, i.e.

(S6) $P \Rightarrow (J(F) \wedge \neg F \wedge \neg J(B) \wedge B \Rightarrow (F \vee B) \wedge J(F \vee B))$ is tautology.

After substituting $B = true$ and $F = false$ into *S6*, we obtain

(S7) $P \Rightarrow ((J(F) \wedge \neg J(B) \Rightarrow J(F \vee B))$.

Hint: Note that substituting $B = true$ and $F = false$ into *P* we obtain *P* again, because *P* consists of justification literals only and therefore does not contain literals *B* and *F*.

Proposition *S7* also must be tautology, because the result of a substitution of any logical values for literals into tautology *S4* is also a tautology. Therefore, for any justification patch *P*, *P => GMA* and this proves the Theorem 1. □

In summary, proposition *G* describes the Gettier problem Case II. However, *G* is not a tautology and it is true only under an assumption *P*. For every such assumption that is reasonable (i.e. does not just contradict antecedent of *G*) and fallibilist, *P => GMA => G*, as *Theorem 1* shows In other words, Gettier in the construction of his Case II **must have assumed** either *GMA*, or some other justification patch *P* that entails *GMA*. Hence *GMA* plays a key role and will be called *Gettier minimal assumption.*

Rejection of *GMA* and all propositions that entail *GMA* solves Gettier problem Case II, without need to reject or modify *JTB*. This rejection is a step towards understanding logical properties of justification and Gettier problem Case II can be understood as a counterexample against *GMA*, rather than a counterexample against *JTB*.

4 Further Justification Fallacies

This section lists several propositions that entail *GMA* and Gettier problem Case II is also a counterexample against them.

(EoJ) Extension of justification is the proposition $J(F) \Rightarrow J(F \lor B)$

Theorem 2 *EoJ => GMA*

Proof
EoJ implies $J(F) \land \neg J(B) \Rightarrow J(F \lor B)$, i.e. it implies *GMA*. □

(DoJ) Distribution of justification is the proposition $J(F) \lor J(B) \Rightarrow J(F \lor B)$

Theorem 3 *DoJ => EoJ*

Proof
DoJ implies $J(F) \Rightarrow J(F \lor B))$), i.e. it implies *EoJ*. □

One of the variants of epistemic closure is transmissibility of evidence [8]. While several papers already deal with its failure, additional proof is the next theorem:

(ToE) Transmissibility of evidence asserts for every proposition *P* and *Q*,
$J(Q) \land (Q \Rightarrow R) \Rightarrow J(R)$

Theorem 4 *ToE => EoJ*

Proof

Let *R = Q ⋁T*, then *J(Q) ∧ (Q => Q ⋁T) => J(Q ⋁ T)*, and *J(Q) => J(Q ⋁ T)*, which is equivalent to *EoJ*. □

5 Conclusion

Due to Gettier problems, many authors assumed that "justified true belief" *JTB* is an inadequate account of knowledge. However instead of that, the Gettier problem Case II can be understood to be a counterexample that invalidates certain simplistic assumptions about the logical Conclusion properties of justification. In this paper, several such assumptions are listed:

Gettier minimal assumption (GMA)
J(F) ⋀ ¬J(B) => J(F ⋁B)

Extension of justification (EoJ)
J(F) => J(F ⋁B)

Distribution of Justification (DoJ)
J(F) ⋁J(B) => J(F ⋁B)

Transmissibility of Evidence (ToE)
J(Q) ∧ (Q => R) => J(R)

Rejection of *GMA* and of assumptions that entail *it* (including *EoJ*, *DoJ*, *ToE*), solves Gettier problem Case II without modification of *JTB*. This rejection of *GMA* is a step towards a better understanding the nontrivial logical properties of justification and knowledge

References

[1] E. Gettier, " Is Justified True Belief Knowledge?," *Analysis*, pp. 121 - 123, 1963.

[2] A. Hazlett, "The maturation of the Gettier problem," *Philosophical Studies*, vol. 172, pp. 1-6, 2014.

[3] E. Sosa, "Experimental philosophy and philosophical intuition," *Philosophical Studies*, vol. 132, pp. 99-107, 2007.

[4] A. I. Goldman, "Philosophical intuitions: Their target, their source, and their epistemic status," *Grazer Philosophische Studien*, vol. 74, pp. 1-26, 2007.

[5] A. Hausman, H. Kahane, and P. Tidman, *Logic and philosophy: a modern introduction*: Cengage Learning, 2012.

[6] T. Tammet. *Logictools*. Available: http://logictools.org/index.html

[7] S. Hetherington, "The significance of fallibilism within Gettier's challenge: a case study," *Philosophia*, vol. 40, pp. 539-547, 2012.

[8] S. Luper, "Epistemic closure," in *The Stanford Encyclopedia of Philosophy*, E. N. Zalta, Ed., Fall 2012 ed, 2016.