

Interpreting Quantum Mechanics and Predictability in Terms of Facts About the Universe

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A potentially new interpretation of quantum mechanics posits the state of the universe as a consistent set of facts that are instantiated in the correlations among entangled objects. A fact (or event) occurs exactly when the number or density of future possibilities decreases, and a quantum superposition exists if and only if the facts of the universe are consistent with the superposition. The interpretation sheds light on both in-principle and real-world predictability of the universe.

It's time to kill Laplace's demon. And while we're at it, let's put to rest the age-old notion that, given enough processing time on a sufficiently large computer, it is possible, in principle, to fully predict the future. It's not.

A future event can be accurately predicted only if it necessarily follows from all preceding facts. Because quantum mechanics ("QM") is central to any analysis of predictability, I will briefly discuss the history and relevant features of QM to provide the foundation for a potentially new interpretation, in which QM is characterized in terms of consistent facts, the history of which is embedded in correlations between entangled objects. Next, based on this interpretation, I will analyze whether the universe is predictable in principle, and then the extent to which the ubiquity of chaotic amplifications of quantum events permits a milder version of predictability.

I. BRIEF HISTORY OF THE QUANTUM

Quantum mechanics was born in 1900 when Planck solved the Ultraviolet Catastrophe – the empirically false classical prediction that the spectral intensity of a blackbody diverges with decreasing wavelength – by discretizing electromagnetic radiation with energy $E = hc/\lambda$, where h is Planck's constant, c is the speed of light, and λ is wavelength. Given the known classical relationship between radiant energy and pressure ($E = pc$, where p is momentum), Planck's

Einstein confirmed through his explanation of the photoelectric effect that light was indeed absorbed as discrete particles, dubbed photons. Given that light was already known to exhibit interference effects describable by a wave equation, de Broglie surmised that matter might display similar interference effects and

be describable by a comparable wave equation, a fact that was empirically confirmed for electrons by Davisson and Germer in 1927.

In a monumental thought experiment² in 1927 [1], Heisenberg realized that any attempt to localize an object by illuminating it with light depended heavily on the light's wavelength: the wavelength had to be shorter than the dimensions of the object and its localization. However, the illumination would inherently impart momentum to the object, and the shorter the wavelength, the greater the potential impact. Given the tiny size of Planck's constant, the effect was expected to be inconsequential for anything but the smallest and lightest objects, implying an effective (if fuzzy) boundary between the "quantum" and "classical" worlds. Like any principle in physics, the so-called Heisenberg Uncertainty Principle ("HUP") cannot be proven; empirical verification is always subject to future falsification. (See footnote 4.) Amazingly, HUP waited nearly four decades for an experimental confirmation. [2].

Once it was shown that the interference patterns produced by particles could be explained by wave-like behavior, it was posited that a particle is fully specified, in time and one dimension, by a wave $\Psi(x,t)$ that is the superposition of a complete set of normalized plane waves $\varphi_k = c_k e^{i(kx - \omega t)}$, where c_k are complex amplitudes:

$$\Psi(x,t) = \sum c_k e^{i(kx - \omega t)} \quad (1)$$

The wave function is mathematically interpreted such that the probability of finding the particle between positions x_1 and x_2 at time t is:

$$P(x_1 < x < x_2, t) = \int_{x_1}^{x_2} \Psi^*(x,t) \Psi(x,t) dx$$

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¹ Momentum can also be written as $p = \hbar k$, where \hbar is the reduced Planck's constant and k is wave number.

² To be fair, he was trying to give a realistic interpretation of the commutator, derived via wave mechanics, in which $[\mathbf{X}, \mathbf{P}] = i\hbar$.

One problem with the above description is that, classically, an object is described in phase space – i.e., an object cannot be fully described at a given time without reference to both where it is (position) and where it’s going (velocity or momentum). However, the above equations make reference, at a given time, only to position; that is, in QM, objects are described in *configuration* space, not *phase* space. What happened to the dependency on momentum?

A related problem is the weirdness of describing an object as a wave, given that a wave represented by equation (1) has neither position nor momentum. Having said that, we can use $p = h/\lambda$ to *infer* a momentum.³ Then, equation (1) tells us that the state of an object is a summation over all momenta. Further, it’s a simple and fascinating mathematical fact that, via a process called Fourier transform, a position distribution can be created by an infinite sum of standing waves, called harmonics. If the harmonics themselves correspond to momenta, then just as the state of an object can be represented as a sum over all possible momenta, it can equivalently be represented as a sum over all possible positions. In other words, equation (1) inherently embeds both position and momentum information, thus validating the lack of dependency on *both* position and momentum.⁴

This still does not *explain* what happened to the dependency on both position and momentum, nor how to make predictions about the evolution of a system without both sets of information. A wave does not have a position (until it stops being a wave, I suppose). For example, a photon – i.e., the thing that is measured as a localized “blip” on a detector – is not a wave, so if the photon is completely describable as a wave, then its detection location is inherently indeterministic and

³ In fact, this assumption underlies the very foundation of wave mechanics. Note that for any $\varphi_k = c_k e^{i(kx - \omega t)}$, $\frac{\hbar \partial}{i \partial x} \varphi_k = \hbar k \varphi_k = p \varphi_k$ only if $p = \hbar k$. It was realized that the mathematical operation $-i\hbar \frac{\partial}{\partial x}$ could be defined as the momentum operator \hat{P} (in one dimension) and utilized on its own. If \hat{P} acts on eigenstate φ_k then it yields eigenvalue $p = \hbar k$ multiplied by φ_k , which is another way of saying that φ_k has a distinct momentum, while the superposition in equation (1) may not.

⁴ The fact that the quantum state of a system can be described in configuration space is often ignored in the typical textbook “proof” of HUP. In fact, given the assumptions that an object is fully described by a wave and that the momentum wave packet of a particular quantum state is equal to the Fourier transform of the position wave packet for the same state, HUP is a mathematical tautology that provides no further information. Experimental verification of HUP can render support for the assumptions of QM, not vice versa.

⁵ Clearly, I exclude a statistical or probabilistic “prediction,” which does not specify what an observer will actually witness.

cannot be predicted. Oddly, while the wave state is itself deterministic, it does not uniquely⁵ determine or predict what any observer will witness.

If our intuition is correct that predictability of an object depends on its description in phase space, but if the state of an object is entirely specified in configuration space, then the information necessary to predict the object simply does not exist. On this basis alone, many argue that the universe is fundamentally unpredictable.⁶

However, something important is missing from this analysis. The quantum wave state evolves linearly and deterministically, and the state of the world is presumably fully describable by a wave state. Yesterday, I connected a Geiger counter to my computer and programmed it so that if the decay of a radon atom was detected in a particular time interval, it would, tomorrow, order a pizza from QuantumPizza.com. But I heard the Geiger counter click yesterday, so I can (with reasonable confidence) predict a pizza delivery tomorrow, a prediction that the quantum wave state can *not* make without the relevant facts.⁷ I therefore argue that the happening of events and the universal consistency of those facts are actually central to an understanding of QM.

II. QUANTUM MECHANICS REINTERPRETED

In light of the above description, I propose a potentially novel interpretation or characterization of QM in terms of facts about the universe, as follows.

The state of the universe is a particular chronological⁸ set of facts (or events), and the relationships between objects in the universe comprise the information storing and instantiating those facts.

⁶ One important interpretation of QM maintains determinism simply by denying that a state is fully specified in configuration space. In 1952, David Bohm re-introduced the missing dependencies by positing that each object has an actual position given by initial conditions and an actual momentum that is determined at each point in space by a “guiding” equation: assuming that wave state Ψ is normalized, the momentum of an object at a position is simply the real value of the momentum operator acting on Ψ at that position. [3]. However, Bohmian mechanics is currently empirically indistinguishable from, and thus as unpredictable as, any other interpretation of QM because the purported definite position of each object is inherently unknowable.

⁷ Space limitations prevent a discussion of how various interpretations of QM address the apparent incompatibility between the completeness and linearity of the wave function and the observation that we never witness superpositions.

⁸ Relative to some object or observer. Rovelli, in his “Relational” interpretation, points to special relativity as motivating the conclusion that quantum states are relative among interacting systems. [4]. In the same vein, Brukner derived a no-go theorem for observer-independent facts. [5].

Those facts must be consistent throughout the entire universe.

A fact occurs exactly when the number (or density) of future possibilities decreases.⁹ The universe is subject to certain physical laws such that every fact limits future facts and is limited by prior facts. A fact does not necessarily require an “impact” or “interaction” as colloquially understood.¹⁰

A (quantum) superposition exists if and only if the facts of the universe are consistent with the superposition. For example, in the case of the classic double-slit interference experiment with the particle passing the slits at time T_0 , the particle is in a superposition of passing through both slits if and only if there is no fact about the particle’s location in one slit or another at time T_0 . If even a single photon, for example, correlated to the location of the particle in one slit or the other at time T_0 , scurries away at light speed, there is a fact about the location of the particle and it cannot be in a superposition at time T_0 . In the unlikely event that the experiment is set up so that the photon later gets uncorrelated such that no “which-path” information is ever available, then the particle, amazingly, cannot be in a superposition at time T_0 . Such a “delayed-choice quantum eraser experiment” (cf. [9]) demonstrates that whether an event occurs seems to depend on the *future* permanence of a correlating fact. In reality, the “window of opportunity” to prevent the decoherence of a superposition is extremely short, so we don’t generally need to wait long before we can officially declare the happening of an event.¹¹

Quantum uncertainty (e.g., in the form of the HUP) is simply one type of superposition, in which a spread of possible positions and a spread of possible momenta are related. For instance, if a particle is tightly localized at time T_0 , then the facts of the universe at that time are consistent with a wide spread of possible momenta – i.e., a superposition of many momenta exists at T_0 .

For simplicity, the following example proceeds classically – that is, I have assumed various classical

laws, such as Newton’s First Law, Conservation of Momentum, etc. – to show how facts limit future facts and are limited by prior facts. However, the actual laws governing the evolution of a system may be different or even unknown.

Imagine N objects ($\{O_1, \dots, O_N\}$), which need not be microscopic “particles,” distributed in position and velocity in three-dimensional space discretized into M possibilities per dimension. Each possible combination of position (X) and momentum (P) vectors for each and every object may be considered a single point in classical phase space, yielding a total of $M^{(6N)}$ such points/possibilities.¹² A fact (or event) is anything that reduces the number of such possibilities, so one example of a fact is an impact between two objects. Assume for simplicity that an impact between two objects is always repulsive and their masses are equal, so an impact just has the effect of swapping the objects’ velocities. Assume also that an impact occurs only when two objects are at the same location at the same time; we will neglect fields.

Let us choose one set of possibilities at time T_0 , specifically the set in which object O_1 has a particular position X_1 and three possible momenta P_{11}, P_{12}, P_{13} , and object O_2 has a particular position X_2 and three possible momenta P_{21}, P_{22}, P_{23} , as shown in Fig. 1 below. For the sake of demonstration, these values are chosen such that O_1 with P_{11} will, at time T_1 , reach the same location in space as O_2 with P_{21} ; also, O_1 with P_{12} will, at time T_2 (which may be different from T_1), reach the same location in space as O_2 with P_{23} ; but every other combination always results in non-coinciding future positions.

Assume there are no restrictions on the possible locations and momenta of other objects¹³, so let’s ignore those other combinations and simply write the nine points in phase space as $\{X_1, P_{11}, X_2, P_{21}\}, \{X_1, P_{11}, X_2, P_{22}\}$, etc.

⁹ Really I mean “new fact.” If event A necessarily implies event B, I don’t mean to suggest that event B does not occur, but rather that the state of the universe is not further specified or limited by event B, so for the sake of efficiency I’ll only consider events that reduce future possibilities.

¹⁰ Elitzur *et al.* unintentionally provide a great example as to how quantum mechanical events can occur without an “interaction.” [6]. Whether or not their suggested method disturbs a measured system’s internal quantum state, it undoubtedly produces facts that reduce the number of future possibilities.

¹¹ “The coherence vanishes as soon as a single quantum is lost to the environment.” [7]. Since there will always be those who claim that any process is in-principle reversible simply by acting on a system with the reverse Hamiltonian, we could limit ourselves to “any fact of which the news is already

propagating outward at the speed of light, so that the information can never, even in principle, be gathered together again in order to ‘uncause’ the fact.” [8]. If we indeed live in a universe with nonnegative curvature, as is currently believed, and given that almost the entire night sky is black, we can regard virtually every photon emitted into space as irretrievable.

¹² In reality, I doubt that spacetime is discretized, in which case there is no fundamental limit to the possibilities or number of events that reduce their density. Because the specification of facts requires information, the question of whether the universe can accommodate new information will be addressed in Footnote 16.

¹³ In other words, assume no entanglements with other objects, an exceptionally unlikely situation that significantly simplifies the analysis.

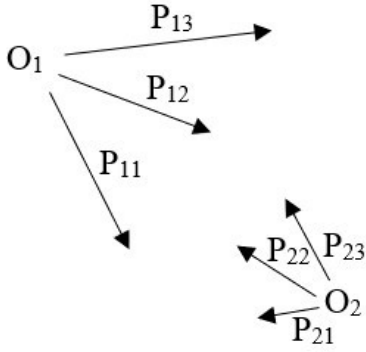


Fig. 1. Nine possibilities for two objects.

We now add the following fact about the universe: by time T_3 (which is after T_1 and T_2), O_1 and O_2 have interacted with each other but not with any other objects. That is, they reach the same location in space and then repel, thus swapping their momenta. Notice that this fact has the effect of reducing the number of possible combinations that can exist at T_3 . Specifically, only the two possibilities, $\{X_1, P_{11}, X_2, P_{21}\}$ and $\{X_1, P_{12}, X_2, P_{23}\}$ as they existed at time T_0 , can now exist at T_3 , at which time they have evolved, respectively, to $\{X_1', P_{21}, X_2', P_{11}\}$ and $\{X_1'', P_{23}, X_2'', P_{12}\}$.

This reduction in the number of combinations has two features. First, there are broad categories of individual momenta that simply cannot occur: specifically, at time T_3 , O_1 cannot have a phase that traces it back to (or is correlated to) the phase $\{X_1, P_{13}\}$ at time T_0 , just as O_2 cannot be traced back or correlated to the phase $\{X_2, P_{22}\}$ at T_0 , and no future measurement can contradict this.¹⁴ Second, while other broad categories of individual momenta may not be ruled out, there are now *correlations* between the possible momenta of the objects. For example, if an evolution of O_1 from state $\{X_1', P_{21}\}$ exists at some later time, then a corresponding evolution of O_2 from state $\{X_2', P_{11}\}$ must also exist.¹⁵ If a future fact rules out one, then it rules out both. Similarly, if an evolution of O_1 from state $\{X_1'', P_{23}\}$ exists at some later time, then a corresponding evolution of O_2 from state $\{X_2'', P_{12}\}$ must also exist. These two objects are now entangled, no matter the distance between them. Even spacelike separated measurement events on O_1 by Alice and O_2 by

Bob will be perfectly correlated, whether or not they know about the correlations.

Note that Alice's measurement of O_1 results in a correlation with O_1 . Instead of killing the correlation between O_1 and O_2 , the measurement by Alice will ultimately be correlated to O_2 via O_1 . That is, entanglement is additive, ubiquitous, and probably universal. If each impact between objects results in a new correlation between them, then, over time, most objects directly or indirectly become entangled with each other. If this interpretation is correct, then the universe creates new facts, reduces future possibilities, and correlates the possibilities of one system with those of another, *ad infinitum*, so that the possibilities for any one object depend, in some sense, on the possibilities of every other.¹⁶

Having explained this example, I want to consider the effects of facts on the universe in reducing the entire phase space of possibilities, and whether any interesting or large-scale pattern or structure emerges, such as in the *distributions* of object positions and/or momenta. I suspect that after enough events, some objects would start to appear fixed relative to other objects, and once most or all objects are entangled/correlated, they would begin to show a (potentially fuzzy) localization relative to each other. Is it possible that the remaining space of possibilities could demonstrate a degree of position fuzziness that varies approximately inversely with momentum fuzziness, *a la* HUP? Ultimately, could quantum uncertainty relationships, and perhaps even predictions consistent with a quantum wave state, emerge from a sufficiently large specification of facts?

Unlike other interpretations of QM, I assert the centrality of facts to the underlying ontology. They are not merely pesky "measurements" whose conflict with the linearity of the wave function needs to be explained away; rather, the occurrence of events may be fundamental to the very foundation of physics. This interpretation need not conflict with established doctrine. For example, the quantum wave state may, as in Bohmian mechanics, determine a momentum at each position, with quantum events acting to reduce the density of possibilities in *configuration* space. However, if, as suggested previously, the specification of sufficiently many facts results in a pattern in the remaining space of possibilities that converges to

¹⁴ I'm *not* asserting that an event after T_0 *retroactively* eliminates possibilities at T_0 . Rather, while at T_0 there were nine possibilities, there are only two at T_3 .

¹⁵ These evolutions are also correlated with an impact at T_1 , not T_2 , which suggests that time itself may be an emergent phenomenon when facts reduce phase space possibilities.

¹⁶ Rovelli's Relational interpretation specifically relies on the premises that the amount of information that can be extracted from a finite region of phase space is finite and that an interaction allows new information about a system to be

acquired. [10]. He resolves the apparent paradox by countering the gain of new information with the potential loss of old information. However, his first premise assumes the constancy of Planck's constant. A value of h that decreases with time – e.g., if h emerges from a reduction in the density of possibilities – could accommodate the continuous net creation of new facts in the universe. Further, others have noted that the information content of the universe could grow naturally with time (cf. [11]) or if the rate of expansion of the universe exceeds that of increase in entropy (cf. [12]).

quantum uncertainty, then QM may be shown to *emerge* from the underlying facts. Given the current lack of any *a priori* explanation for QM, as well as the FAPP impossibility of determining the quantum state of systems larger than a few particles, such a result would be deeply satisfying.

Further, the characterization of a superposition as the absence of a relevant fact may help to explain why we never observe superpositions: we cannot observe the lack of a fact. For example, the results of the classic double-slit interference experiment could be reinterpreted as a statement that there *is* a fact about the particle being localized in the region defined by the two slits but there is *not* a fact about its being localized in either one of the slits. This observation can be demonstrated by noting that when a particle passes through a single slit of width $2\Delta x$, the distribution of detection events on a distant screen appears as a Fraunhofer diffraction pattern having a certain width. However, if the single slit is regarded as two side-by-side slits each of width Δx , we can associate with each slit a wave producing its own Fraunhofer diffraction envelope having double the width. The interference between these two waves then produces an interference pattern that, incredibly enough, is identical to the single-slit Fraunhofer diffraction distribution of the entire slit. In other words, the observed fact that a particle traversed a slit turns out to imply a superposition with regard to where *within* the slit the particle traversed.

III. PREDICTABILITY

A. In-Principle Predictability

Predictability is always relative to an observer. If QM can be interpreted in terms of consistent facts about the universe, predictability might be couched colloquially in terms of this question: What has to be true so that I observe what I currently observe, and how do those facts limit what I might observe in the future?

Strictly speaking, an observer's ability to accurately predict a particular future fact requires that: a) it is the only possibility consistent with all previous facts; and b) the observer knows this. First consider the problem of knowledge. Because the record of facts is stored in relationships between entangled objects, and because there is no way to know if and how any two objects are entangled without already knowing the event that entangled them, no observer can know all prior facts. And while not all predictions require a knowledge

of all prior facts, there are certainly *some* predictions, with respect to a certain observer, that require knowledge of unknown and unknowable facts.

Next consider the problem of uniqueness of a future outcome. Even if an observer could know all relevant prior facts, there may still be different future possibilities (manifested as the future evolutions of current superpositions) that are consistent with the facts. For instance, if there is no fact about whether A is true, but there might be a fact about it tomorrow, and the particular future fact that the observer wants to predict hinges on the truth of A, then current knowledge of all relevant facts is still inadequate to make an accurate prediction.

Therefore, there are at least some predictions that cannot accurately be made, even in principle.¹⁷ For instance, the detection location of a particle in a particular single-slit diffraction experiment is unpredictable because it fails to satisfy a) above. The fact that there are some accurate predictions that can be made, in light of a) and b), does nothing to invalidate the inherent unpredictability of the universe. For instance, regarding the particle in the diffraction experiment, can we make an accurate prediction about whether, one second later, the particle will be detected at a location 300,000,000 meters away? The answer is yes: special relativity guarantees that the particle will *not* be detected there. Great, but this doesn't tell us much; the existence of physical laws that constrain future possibilities based on prior facts inevitably implies a very loose type of predictability that does nothing to elucidate whether the universe is truly in-principle predictable.

B. Predictability in the Real World

Perhaps I am being too strict. "Will the bullet from a rifle hit a certain target?" is a question about which a reasonably good prediction can be made, even though an extremely unlikely set of quantum events could cause the bullet to veer off course. Can't we just discount unlikely events and define predictability as "99% predictable"? No. When we ask questions that are actually relevant to real-world needs and experience, it turns out that unlikely quantum events, on sum, actually dominate the outcome.

Consider: "Will it rain on our wedding day?" "What will the stock market look like next month?" and "When and where will the next pandemic begin?" Answering these questions depends heavily on two features of the universe: chaos; and amplification of

¹⁷ And what of computability? To the extent the universe is subject to laws such that a particular initial state can evolve into some states (or one state, if deterministic) but not others, the universe inherently computes. Indeed, both the computational and storage capacities of the universe have been

estimated in [13]. However, a computable process must be deterministic (though a deterministic process is not necessarily computable). [14]. To the extent that nondeterministic events, constraining the future and constrained by the past, underlie the nature of the universe, it cannot be computable.

quantum events. Chaotic systems are, by their nature, extraordinarily sensitive to initial conditions. Moreover, all chaotic systems will, given enough time, become chaotic even as the level of precision in their initial conditions approaches infinity. Given that the actual precision in any physical system can never exceed that dictated by QM, it is a foregone conclusion that every chaotic system – weather, populations, markets – is inherently unpredictable.

Second, natural amplifications of quantum events happen constantly.¹⁸ While all quantum events create correlations, some events, merely by chance, set off a chain of events that ultimately correlate a significant outcome with the original quantum event. Imagine a system on the “tipping point” of evolving from one state to another. We can now posit a quantum event occurring at the right place and time that would nudge the system one way or another. Even without amplification, chaos theory instructs us that we don’t have to go back very far in a system’s timed evolution before tiny quantum fluctuations, which are ubiquitous at the particle level, control the outcome. When we include random amplifications of quantum events, the time before a system becomes chaotic is even less.

Consider the statement, "It rained yesterday." If that’s a fact about the universe, then for it to mean something, there must be correlates today (e.g., mud) that instantiate that fact. Those correlates themselves cause further correlations, day after day, year after year, so that in a billion years there is still a fact about whether it rained yesterday. That fact gets embedded in the history of the universe; just as it was limited by prior facts, it likewise limits future facts. Further, due to the combination of chaos and quantum amplification, it may very well be that that fact is the manifestation of the amplification of a single quantum event. If so, then the fact that it rained yesterday is a direct correlate of a quantum event!

If consistent facts, and correlations among objects instantiating those facts, represent the fundamental ontology of the universe, then the universe is guaranteed to be unpredictable in principle. Further, the chaotic amplification of quantum events guarantees that the vast majority of real-world events, even in the short term, are fundamentally unpredictable.

Laplace’s demon be damned.

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¹⁸ An engineered amplification, for example, is the audible “click” of a Geiger counter upon detecting the decay of a radioisotope.