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Imitation of optical coherence tomography images by wave Monte Carlo-based approach implemented with the Leontovich–Fock equation

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Abstract. We present a computational modeling approach for imitation of the time-domain optical coherence tomography (OCT) images of biotissues. The developed modeling technique is based on the implementation of the Leontovich–Fock equation into the wave Monte Carlo (MC) method. We discuss the benefits of the developed computational model in comparison to the conventional MC method based on the modeling of OCT images of a nevus. The developed model takes into account diffraction on bulk-absorbing microstructures and allows consideration of the influence of the amplitude–phase profile of the wave beam on the quality of the OCT images. The selection of optical parameters of modeling medium, used for simulation of optical radiation propagation in biotissues, is based on the results obtained experimentally by OCT. The developed computational model can be used for imitation of the light waves propagation both in time-domain and spectral-domain OCT approaches. © 2020 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.59.6.061626](https://doi.org/10.1117/1.OE.59.6.061626)]

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1 Introduction

The time-domain optical coherence tomography (OCT) is based on the interference of low-coherent optical radiation backscattered from medium inhomogeneities with the reference beam and is used extensively for structural and functional imaging of biological tissues.¹ The interpretation of the OCT images and quantitative assessment of the structural and optical parameters of turbid scattering biological tissues requires the development of realistic computational models for ultimate understanding of the formation of OCT images.

Typically, for modeling of the light propagation in biological tissues and tissue-like scattering media, including mimicking of OCT images, either a plane wave approximation^{2–5} or the conventional Monte Carlo (MC) method^{6–15} is used. In fact, both of these approaches have some disadvantages. The vector MC method takes into account the wave phenomena, such as diffraction and/or coherence, utilizing artificial methods developed outside of the original model of radiation transfer, as the individual photon trajectories are counted independently.

Modeling of light propagation within the random medium based on the Huygens–Fresnel principle allows considering diffraction but only for bulk inhomogeneities with sharp

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boundaries.^{16–21} The propagation of light within biological tissue is modeled typically by a set of particles also known as “photon packets” (Ref. 22). The Huygens–Fresnel principle approach has a high computational complexity since the wavefront is considered as a set of point sources, described by differential equation.²³

Using the Maxwell equations for the OCT simulations is suitable for modeling complex (profiled) laser beams.^{24–29} However, it is very time-consuming, even using parallel algorithms,^{30–32} and requires allocation of massive computer resources that significantly limits the spatial area of interest.

An approach based on a quasioptics approximation has been developed, where stochastic medium inhomogeneity was modeled by a set of phase screens with a random spatial distribution of the phase incursion.³³ A model of optical wave radiation propagation in a biotissue based on the Leontovich–Fock stochastic equation also referred to as a quasi-optics approximation was proposed.³⁴ The advantage of the model is that it provides an effective numerical implementation. Applying this approach for the simulation of OCT images of biological tissues requires a generalization of the model for the case of a partially coherent beam. The latter can be implemented as a set of statistical tests, where each optical wave with a given random phase distribution at the entrance to the medium propagates independently, and finally, their superposition is counted. By analogy with the conventional MC, this approach is known as the wave MC.³⁵

To imitate OCT signal formation by MC, it is necessary to calculate the propagation of the backward wave reflected from the boundaries of structural inhomogeneities localized within the medium. The combination of the wave MC method and the wave equation in the quasioptics approximation potentially makes it possible to simulate with reasonable computing resource requirements of the wave beams with a complex amplitude–phase profile and to take into account diffraction. In the current paper, utilizing the Leontovich–Fock stochastic parabolic equation, we introduce a new wave MC-based model for simulation of the time-domain OCT signals and 2D images.

2 Approach to Modeling of Biological Tissue Visualization Structure by the OCT Method

2.1 Model of Optical Radiation Propagation in Biological Tissue

Let us denote the electric field of an optical wave in the following form:

$$E(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{i\omega_0 t}, \quad (1)$$

where $\omega_0 = k_0 c$ is the carrier (central) frequency. Then, the corresponding Helmholtz equation for a randomly inhomogeneous medium is calculated as

$$\Delta\psi(\mathbf{r}, t) + k_0^2\psi(\mathbf{r}, t) = -k_0^2(\langle\epsilon\rangle + \delta\epsilon)\psi(\mathbf{r}, t), \quad (2)$$

where $\delta\epsilon$ is the dielectric permeability fluctuations ϵ and $\Delta \equiv \nabla^2$ is the Laplace operator. Further, time dependence further will be omitted.

Taken into account that $\psi(\mathbf{r})$ is presented as $\psi(\mathbf{r}) = U(\mathbf{r})e^{ik_0 z}$, Eq. (2) in a quasioptics approximation takes the form:

$$2ik_0 \frac{\partial}{\partial z} U(\mathbf{r}_\perp, z) + \Delta_\perp U(\mathbf{r}_\perp, z) = -k_0^2(\langle\epsilon\rangle + \delta\epsilon)U(\mathbf{r}_\perp, z). \quad (3)$$

Assuming that $\nabla\langle\epsilon\rangle = 0$, Eq. (3) can be simplified as

$$\frac{\partial}{\partial z} \tilde{U}(\mathbf{r}_\perp, z, t) = \frac{i}{4} \Delta_\perp \tilde{U} + i\tilde{n}(\mathbf{r}_\perp, z)\tilde{U}, \quad (4)$$

where $\tilde{n}(\mathbf{r}_\perp, z) \equiv k_0 L_R \tilde{n}_c$ is the normalized complex refractive index of a medium; $\tilde{n}_c = n + \delta\tilde{n}$ is the complex refractive index of the medium, where the real part represents the phase incursion, and the imaginary one characterizes absorption, $\delta\tilde{n}$ is the medium refractive index fluctuations; $L_R = k_0 R_0^2/2$ is the Rayleigh length equal to half the distance at which a collimated beam

with a Gaussian shape of the intensity distribution in the cross-section doubles in diameter; R_0 is the initial radius of the beam; $k_0 = 2\pi/\lambda_0$ is the wavenumber at the central wavelength λ_0 ; $r_\perp \rightarrow r_\perp/R_0$ is dimensionless transverse coordinate; $z \rightarrow z/L_R$ is the dimensionless coordinate along the beam propagation direction; ∇_\perp is the Nabla operator, which includes differentiation only over transverse coordinates; $\tilde{U}(\mathbf{r}_\perp, z, t) = \tilde{A}(\mathbf{r}_\perp, z, t)e^{i\tilde{\varphi}(\mathbf{r}_\perp, z, t)}$ is the slowly changing complex amplitude of the light field normalized to its maximum initial value; $\tilde{A}(\mathbf{r}_\perp, z, t)$ and $\tilde{\varphi}(\mathbf{r}_\perp, z, t)$ are, respectively, amplitude and phase. Hereinafter, the wave symbol denotes a random function.

Let us describe the phase $\tilde{\varphi}(t)$ by a random function with the Lorentzian shape of the spectrum fluctuations:³⁶

$$\langle \tilde{\varphi}(t - \tau)\tilde{\varphi}(t) \rangle = D_\varphi e^{-\tau/T_{\text{corr}}}, \tag{5}$$

where T_{corr} defines the temporal coherence of the optical wave radiation and determines the longitudinal resolution in OCT; and D_φ is the phase fluctuation dispersion.

The field amplitude is defined as follows:

$$\tilde{A}(\mathbf{r}_\perp, t) = A(\mathbf{r}_\perp, t) + \delta\tilde{A}(\mathbf{r}_\perp, t).$$

Similar to Eq. (5), the amplitude fluctuations $\delta\tilde{A}(t)$ are described as follows:

$$\langle \delta\tilde{A}(t - \tau)\delta\tilde{A}(t) \rangle = D_A e^{-\tau/T_{\text{corr}}},$$

where D_A is the amplitude fluctuation dispersion (usually, $D_A \ll 1$), $\langle \delta A(t) \rangle = 0$.

The phase $\tilde{\varphi}$ can be found from the Leontovich–Fock equation:

$$\partial_z \tilde{\varphi} = -\partial_z \text{Im}(\ln(\tilde{U})) = \tilde{n}(\mathbf{r}_\perp, z) + \frac{\nabla_\perp^2 |\tilde{A}|}{4|\tilde{A}|}. \tag{6}$$

The simulation of time-domain OCT images of biotissue was carried out based on Eqs. (4)–(6) for a set of realizations of random optical waves with given characteristics at the entrance to the medium. Then, the interference of reference wave and the superposition of the waves backreflected/scattered from structural inhomogeneities of the biological tissue during an observation time T are counted as follows:

$$I_{\text{oct}}(\mathbf{r}_\perp, L) = \int_T dt \langle \tilde{A}_{\text{sc}}(\mathbf{r}_\perp, t)\tilde{A}_0(\mathbf{r}_\perp, t - n_0 L/c) \text{Re}(e^{i(\tilde{\varphi}_{\text{sc}}(t) - \tilde{\varphi}(t - n_0 L/c))}) \rangle / T, \tag{7}$$

where \tilde{A}_{sc} and $\tilde{\varphi}_{\text{sc}}(t)$ are the amplitude and phase of the optical wave backscattered from the tissue; \tilde{A}_0 is the amplitude of the source light field; L is the difference of the interferometer arms lengths (a variable parameter that determines the position of the observed layer in the medium in the time-domain OCT); c is the speed of light; and n_0 is the refractive index of air.

2.2 Model of Randomly Inhomogeneous Medium

We use the following designations:

$$\langle \delta\tilde{n}(\mathbf{r})\delta\tilde{n}(\mathbf{r} - \delta\mathbf{r}) \rangle = C(\delta\mathbf{r}) = \int_{V_k} e^{-i\mathbf{k}\delta\mathbf{r}} C(\mathbf{k}) d\mathbf{k}.$$

where $\mathbf{r} = \mathbf{r}_\perp + z \cdot \vec{e}_z$, where \vec{e}_z is the Cartesian coordinate system unit vector and $C(\mathbf{k})$ is defined in the frame of the phenomenological model:³⁷

$$C(\mathbf{k}) = C_0 \frac{1}{(1 + l_c^2 |\mathbf{k}|^2)^{D_f/2}}. \tag{8}$$

The medium model parameters include the correlation radius l_c , the magnitude D_f of refractive index fluctuations, and the dispersion of refractive index fluctuations C_0 . Setting these parameters determines the type of biological tissue.^{38–43}

Here and following, we use the following notation for triple integrals:

$$\int_{V_k} f(\mathbf{k})d\mathbf{k} \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(k_x, k_y, k_z)dk_x dk_y dk_z.$$

The “volume” V is defined in a corresponding 3D real number space.

Let us establish a link between the statistical characteristics of the medium parameters and the parameters of the scattering indicatrix in the radiation transfer equation (RTE), which is a typical base of the conventional MC approach.

Let us designate the two-point statistical moment (average of the product of the optical wave amplitudes in two spatial points \mathbf{r}_1 and \mathbf{r}_2), i.e., to the second-order coherence function as follows:

$$\Gamma_2(\mathbf{R}, \delta\mathbf{r}) = \langle U(\mathbf{R} + \delta\mathbf{r})U^*(\mathbf{R} - \delta\mathbf{r}) \rangle_{\delta\epsilon}.$$

Here, $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$; $\delta\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)/2$. Differentiating Γ_2 on $\nabla_{\mathbf{R}}\nabla_{\delta\mathbf{r}}$, we get

$$\nabla_{\mathbf{R}}\nabla_{\delta\mathbf{r}}\Gamma_2 = \frac{1}{2}k_0^2 \langle \delta\epsilon(\mathbf{R} + \delta\mathbf{r})U(\mathbf{R} + \delta\mathbf{r})U^*(\mathbf{R} - \delta\mathbf{r}) - \delta\epsilon(\mathbf{R} - \delta\mathbf{r})U(\mathbf{R} + \delta\mathbf{r})U^*(\mathbf{R} - \delta\mathbf{r}) \rangle_{\delta\epsilon}. \quad (9)$$

Taking into account that $\delta\epsilon$ is the random isotropic homogeneous Gaussian distribution, and integrating Eq. (9) in parts, we obtain

$$i\nabla_{\mathbf{R}}\nabla_{\delta\mathbf{r}}\Gamma_2 = k_0^2 \langle (\delta\epsilon(\mathbf{R} - \delta\mathbf{r})\delta\epsilon(\mathbf{R} + \delta\mathbf{r})) \rangle \Gamma_2.$$

Presenting the function Γ_2 by Wigner function:

$$J(\mathbf{R}, \mathbf{k}) = \int_{V_r} \Gamma_2(\mathbf{R}, \delta\mathbf{r})e^{-i\mathbf{k}\delta\mathbf{r}}d\delta\mathbf{r},$$

which gives us the following:

$$\mathbf{k}\nabla_{\mathbf{R}}J(\mathbf{R}, \mathbf{k}) = \int_{V_k} \Theta(\mathbf{k} - \mathbf{k}')J(\mathbf{R}, \mathbf{k}')d\mathbf{k}', \quad (10)$$

where $\Theta(\mathbf{k} - \mathbf{k}') = k_0^2 \int_{V_r} e^{-i(\mathbf{k}-\mathbf{k}')\delta\mathbf{r}} \langle \delta\epsilon(\mathbf{R} - \delta\mathbf{r})\delta\epsilon(\mathbf{R} + \delta\mathbf{r}) \rangle d\delta\mathbf{r} = \frac{k_0^2}{4} C(\mathbf{k} - \mathbf{k}')$.

Further, assuming that the Wigner function J localized inside a sphere $|\mathbf{k}| = k_0$. Thus,

$$J(\mathbf{R}, \mathbf{k}) \approx \delta(|\mathbf{k}| - k_0)I_J(\mathbf{R}, \mathbf{k}), \quad (11)$$

where $I_J(\mathbf{R}, \mathbf{k})$ is the brightness function. By denoting $\mathbf{s} \equiv \mathbf{k}/k_0$, omit further the dependence of $I_J(\mathbf{R}, \mathbf{s}k_0)$ on k_0 , and multiplying Eq. (10) on $|\mathbf{k}|^2 d|\mathbf{k}|$ and integrating, we obtain

$$\mathbf{s}\nabla_{\mathbf{R}}I_J(\mathbf{R}, \mathbf{s}) = \frac{k_0^2}{4} \int_{S^2} \left(\int_0^{\infty} C(\mathbf{s}|\mathbf{k}| - \mathbf{s}'k_0)|\mathbf{k}|^2 d|\mathbf{k}| \right) I_J(\mathbf{R}, \mathbf{s}')ds'. \quad (12)$$

Comparing Eq. (12) with the RTE,⁴⁰

$$\mathbf{s}\nabla_{\mathbf{R}}I_J(\mathbf{R}, \mathbf{s}) = \frac{\mu_{sc}}{4\pi} \int_{S^2} p(\mathbf{s}, \mathbf{s}')I_J(\mathbf{R}, \mathbf{s}')ds', \quad (13)$$

we can conclude that

$$\frac{\mu_{sc}}{4\pi} p(\mathbf{s}, \mathbf{s}') = \frac{k_0^2}{4} \int_0^{\infty} C(\mathbf{s}|\mathbf{k}| - \mathbf{s}'k_0)|\mathbf{k}|^2 d|\mathbf{k}|, \quad (14)$$

where μ_{sc} is the scattering cross-section, $p(\mathbf{s}, \mathbf{s}')$ is the scattering phase function described by Heney–Greenstein formula:⁴⁴

$$p(\mathbf{s}, \mathbf{s}') = \frac{1 - g^2}{[1 + g^2 - g \cos(\theta)]^{3/2}}, \quad (15)$$

where $\mathbf{s} \cdot \mathbf{s}' = \cos(\theta)$ is the scalar product of unit the vectors, and g is the anisotropy scattering parameter. This function is often used in MC simulation of optical radiation propagation in tissue. From a theoretical point of view, the constraints of the Heney–Greenstein formula are associated with a discrepancy for anisotropic media, but, in practice, this approach reproduces the experimental data well enough.⁴⁵

Substituting Eq. (14) with Eq. (8):

$$p(\mathbf{s}, \mathbf{s}') = \frac{k_0^2 \pi}{\mu_{sc}} \int_0^\infty \frac{C_0}{(1 + (l_c k_0)^2 (|\mathbf{k}|^2 - 2 \cos(\theta) + 1))^{D_f/2}} |\mathbf{k}|^2 d|\mathbf{k}|. \quad (16)$$

Using Eqs. (14)–(16), μ_{sc} is expressed through C_0 , l_c , D_f , and g as

$$\mu_{sc} = \frac{k_0^2 \pi (1 + g^2 - g)^{3/2}}{1 - g^2} \int_0^\infty \frac{C_0}{[1 + (l_c k_0)^2 (|\mathbf{k}|^2 - 1)]^{D_f/2}} |\mathbf{k}|^2 d|\mathbf{k}|. \quad (17)$$

This equation allows us to link medium parameters for the conventional MC and wave MC simulation and comparative analysis.

The reflection coefficient (reflectivity) of the layer with thickness δz is determined as follows:

$$\tilde{R}(\mathbf{r}_\perp, z) = \frac{\partial_z \tilde{n} \delta z}{2\tilde{n}}.$$

Note that this parameter characterizes the relationship between the amplitudes of the forward propagated \tilde{A} and backscattered optical waves \tilde{A}_{sc} : $\tilde{A}_{sc} = \tilde{A} \sqrt{\tilde{R}}$.

3 Numerical Simulation

3.1 Numerical Simulation Algorithm

The splitting into physical processes approach was used for the numerical solution of Eqs. (4)–(6): at each step along the propagation direction, at first, the problem of free propagation is solved; then, the diffraction step alternates with the step of propagation through the phase screen describing the spatial distribution of the refractive index of the medium. Let us rewrite Eq. (4) in an operator form:

$$\frac{\partial}{\partial z} \tilde{U} = [\hat{\mathbf{D}} + \hat{\mathbf{T}}] \tilde{U},$$

where $\hat{\mathbf{D}} = \frac{i}{4} \nabla_\perp^2$ is the operator of the diffraction, $\hat{\mathbf{T}} = i\tilde{n}(\mathbf{r}_\perp, z)$ is the operator of passing the phase screen with propagation distance $h_z = \delta z$.

The optical wave transformation during the step δz along the propagation direction is represented as

$$\tilde{U}(z + \delta z) = e^{\delta z \hat{\mathbf{D}}/2} (e^{\delta z \hat{\mathbf{T}}} (e^{\delta z \hat{\mathbf{D}}/2} (e^{\delta z \hat{\mathbf{T}}/2} \tilde{U}(z))))). \quad (18)$$

The numerical scheme [Eq. (18)] is not completely conservative, and that can cause errors for strongly inhomogeneous media.⁴⁶ Therefore, a completely conservative numerical scheme had also been established.^{47,48} In the framework of model media studied, both numerical schemes gave matching results.

The size of the skin inhomogeneities is of micron scale, and the diameter of the initial beam was selected of the 1 mm order. The depth of penetration of the optical wave into the biological tissue can be up to several hundred microns. To avoid the influence of boundary effects and take

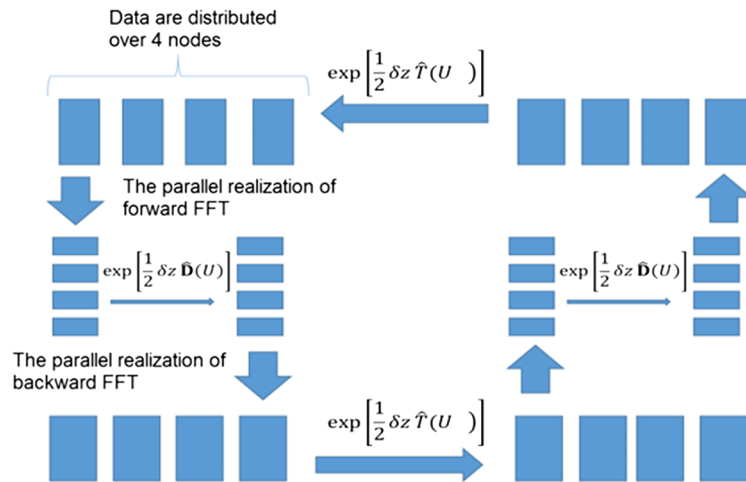


Fig. 1 Block diagram of calculation of an optical beam transformation for the step δz along the direction propagation, according to Eq. (18). The scheme corresponds to a case of four computational nodes.

into account the low-frequency spatial modes, which could appear during the wave propagation, the number of grid nodes was chosen as $4096 \times 4096 \times 1024$.

To speed up the calculations, we also used a slit-like beam transversal shape. The numerical scheme described in Ref. 49 was used to implement parallel computing. To do it, the spatial grid was decomposed according to one of the coordinates (along the coordinate of the narrow part of the slit beam) and, using the parallel version of the fast Fourier transform, the step of calculating the diffraction and the step of passing the phase screen through pointwise multiplication of arrays at each node were carried out. Figure 1 shows a block diagram of the computational algorithm.

The spatial distribution of the refractive index fluctuations was calculated, as shown in Ref. 37:

$$\delta \tilde{n} = \text{Re} \left[\hat{F}^{-1} \left(\text{Rand} \sqrt{C(\mathbf{k}) / (8\pi N_x N_y N_z)} \right) \right] + \text{Im} \left[\hat{F}^{-1} \left(\text{Rand} \sqrt{C(\mathbf{k}) / (8\pi N_x N_y N_z)} \right) \right]. \quad (19)$$

Here, \hat{F} describes the fast Fourier transform operator; Rand is the random number generated for normal distribution with zero mean and unit variance; and N_x , N_y , N_z are the voxel numbers along the corresponding axes of the Cartesian coordinate system.

3.2 Implementation Examples

To compare the simulation results of recording the characteristics of biological tissue with a wave and particle OCT methods, we used a skin model with nevus,^{4,5,12,50,51} which is used extensively in various modeling studies utilizing conventional MC. The relationship between the parameters of the microscopic model of a randomly inhomogeneous medium [Eq. (8)] and the scattering indicatrix used by the corpuscular MC was established according to Eq. (17) (see Fig. 2).

Figure 3 shows the results of the numerical simulation of biological tissue OCT visualization based on the skin with the nevus model presented in Fig. 2.

In the space area near coordinates (0.3, 0.5), a relatively bright structure is visible at the “basal layer–nevus–dermis” border, which is connected most likely with the optical wave focusing on the boundary between the granular and basal layers. The same structure is visible in the image obtained by the corpuscular MC method but with strong distortions compared to the wave method. The corpuscular MC simulation of a photon trajectory was conducted until the photon had been arrived at the plane of detector or absorbed. Dolganova et al.¹² used the total number of trajectories equal to 10^7 . Evidently, MC simulations had been conducted on a very rough mesh. It seems that this is the main reason of the artifacts mentioned already.

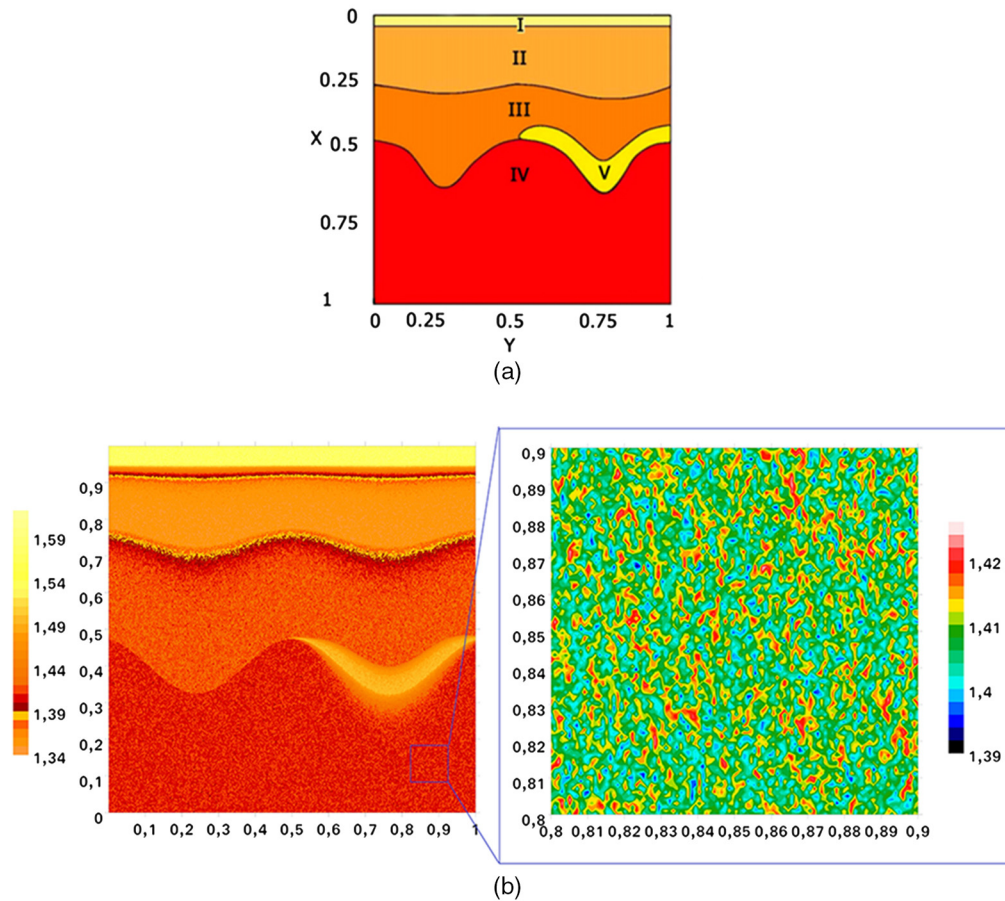


Fig. 2 A skin with a nevus model^{12,49,50} (scales X and Y are normalized to 1 mm): (a) spatial distribution of media parameters: area I (upper stratum corneum): $n = 1.54$, $\mu_{sc} = 35 \text{ mm}^{-1}$, $\mu_a = 0.02 \text{ mm}^{-1}$, $g = 0.9$; $C_0 = 0.047$, $l_c = 1.5 \mu\text{m}$, $D_f = 3.9$; area II (epidermis, granular layer): $n = 1.34$, $\mu_{sc} = 5 \text{ mm}^{-1}$, $\mu_a = 0.015 \text{ mm}^{-1}$, $g = 0.95$; $C_0 = 0.021$, $l_c = 3.2 \mu\text{m}$, $D_f = 3.5$; area III (epidermis, basal layer): $n = 1.4$, $\mu_{sc} = 12 \text{ mm}^{-1}$, $\mu_a = 0.02 \text{ mm}^{-1}$, $g = 0.85$; $C_0 = 0.034$, $l_c = 1.05 \mu\text{m}$, $D_f = 3.7$; area IV (dermis): $n = 1.39$, $\mu_{sc} = 12 \text{ mm}^{-1}$, $\mu_a = 0.1 \text{ mm}^{-1}$, $g = 0.9$; $C_0 = 0.033$, $l_c = 1.6 \mu\text{m}$, $D_f = 3.8$; area V (nevus): $n = 1.38$, $\mu_{sc} = 7 \text{ mm}^{-1}$, $\mu_a = 0.013 \text{ mm}^{-1}$, $g = 0.8$; $C_0 = 0.02$, $l_c = 0.8 \mu\text{m}$, $D_f = 3.6$. (b) An example of the spatial distribution of the $\text{Re}(\tilde{n}_c)$ implementation for the above parameter values (C_0 , D_f , and l_c). On the sidebar: an example of the implementation of the spatial distribution of the refractive index calculated following Eq. (19).

Figure 4 shows the results of the numerical simulation of biological tissue OCT visualization using Bessel optical wave beams, which are diffracted minimally, using the model of skin with the nevus.

Figure 5(c) shows the diffraction spreading of the optical wave beam.⁵² The quality of biological tissue OCT visualization using the Bessel optical wave beam with a transverse profile $J_0(20|r_{\perp}|)$ is lower than for the case of the Bessel optical wave beam with a transverse profile $J_0(6|r_{\perp}|)$.

Figure 5 shows an example of tissue OCT visualization for the case of the skin model shown in Fig. 2 in the absence of layer V (nevus).

The speckle noise was shown to increase under optical wave beam propagation in the tissue. It coincides, for example, with experimental results for a medium with suspensions of polystyrene microspheres.⁶ The speckle image distortion is enhanced for optical wave beams with a more complicated transverse profile. The ability to evaluate such effects is a fundamental advantage of the proposed approach. It can be suitable for the optimization of an experimental implementation of the OCT.⁵³

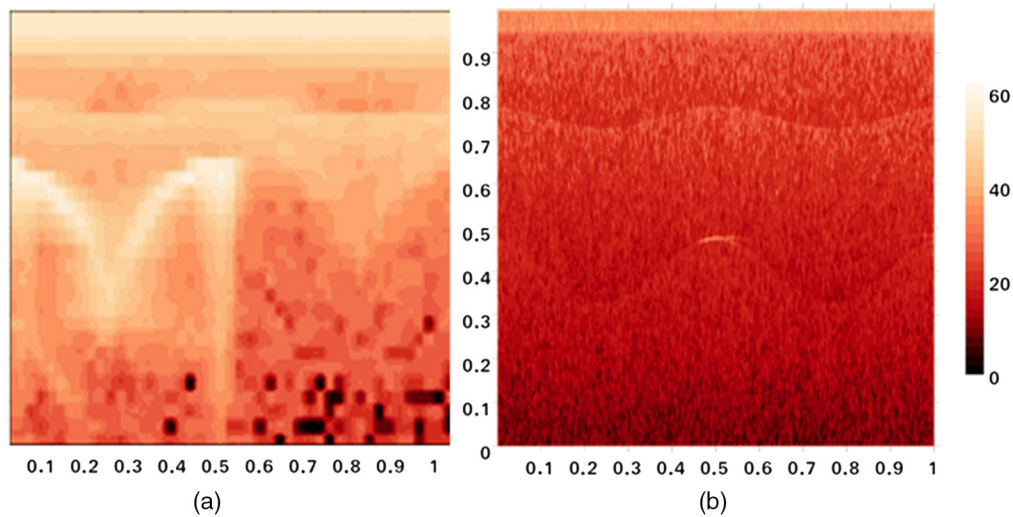


Fig. 3 The results of the numerical simulation of biological tissue OCT visualization based on the skin with the nevus model (Fig. 2; X and Y scales are normalized to 1 mm): (a) the implementation of the corpuscular MC method¹²; (b) the a of the MC wave method using a slit optical wave beam with a Gaussian profile with transverse dimensions of 1000 and 50 μm , the coherence length is 10 μm .

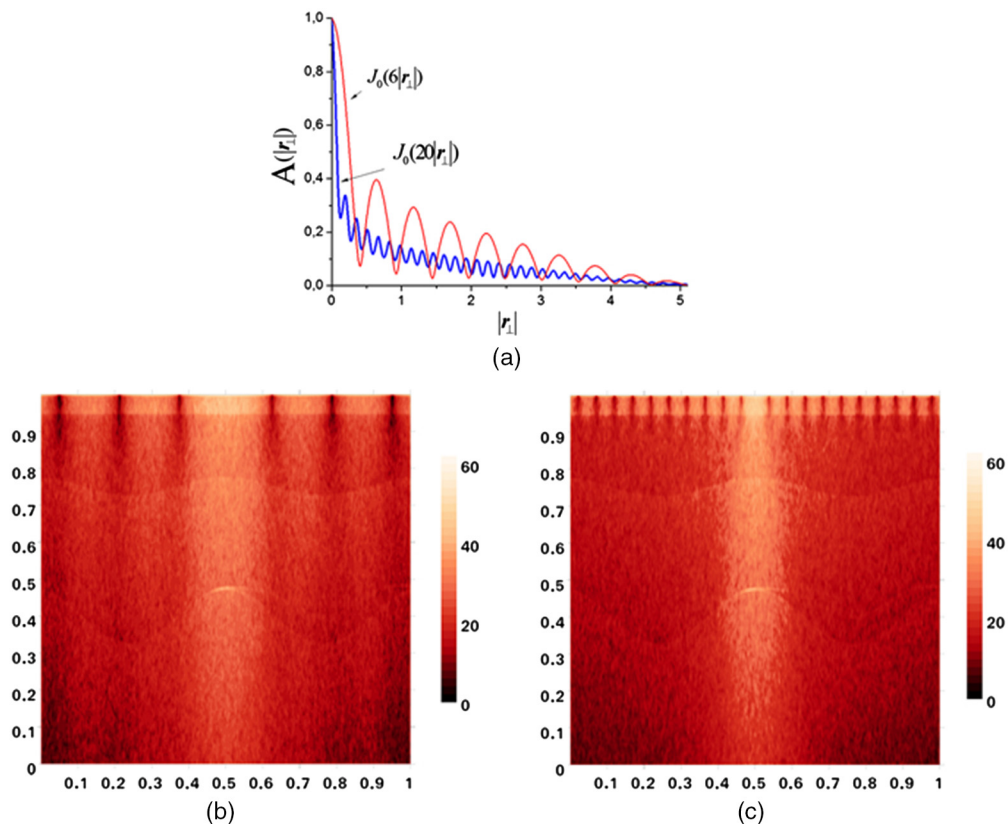


Fig. 4 The results of the numerical simulation of biological tissue OCT visualization based on the skin with the nevus model using Bessel optical wave beams with soft nullification on the edges of the mesh by super-Gaussian function (X and Y scales are normalized to 1 mm): (a) the transverse profiles of Bessel optical wave beam beams at the entrance to the medium; (b) the implementation of the MC wave method using the Bessel optical wave beam with a transverse profile $J_0(20|r_{\perp}|)$, the coherence length is 10 μm ; and (c) the implementation of the MC wave method using the Bessel optical wave beam with a transverse profile $J_0(6|r_{\perp}|)$, the coherence length is 10 μm .

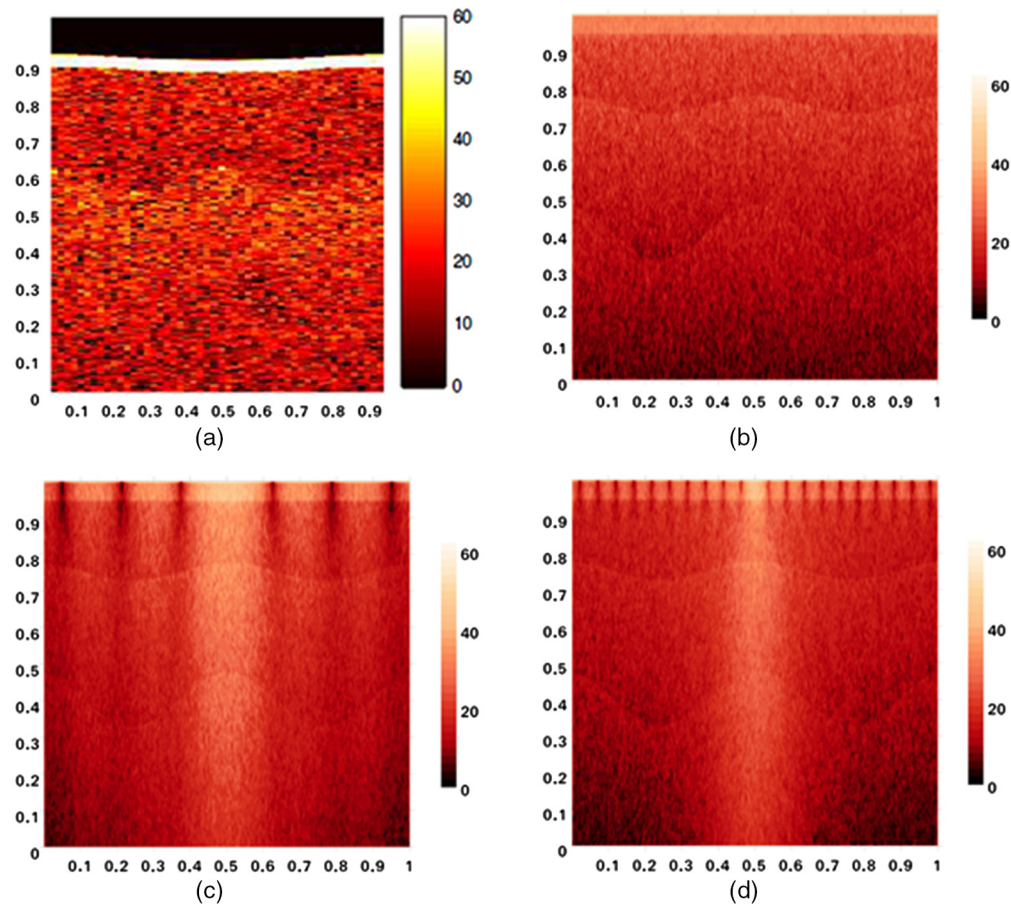


Fig. 5 The results of the numerical simulation of biological tissue OCT visualization based on the skin model without the nevus (see Fig. 2; X and Y scales are normalized to 1 mm): (a) the implementation of the corpuscular MC method, the coherence length is $10\ \mu\text{m}^{50}$; (b) the implementation of the MC wave method using a slit optical wave beam with a Gaussian profile with transverse dimensions of 1000 and $50\ \mu\text{m}$, the coherence length is $10\ \mu\text{m}$; (c) the implementation of the MC wave method using a Bessel optical wave beam with a transverse profile $J_0(20|\mathbf{r}_\perp|)$, the coherence length is $10\ \mu\text{m}$; and (d) the implementation of the MC wave method using a Bessel optical wave beam with a transverse profile $J_0(6|\mathbf{r}_\perp|)$, the coherence length is $10\ \mu\text{m}$.

4 Summary and Conclusion

We introduce a computational modeling approach specially developed for mimicking the time-domain OCT images of biotissues. The developed modeling technique is based on the implementation of the Leontovich–Fock equation into the wave MC method. The selection of optical parameters of modeling medium, used for simulation of optical radiation propagation in biotissues, is based on the results obtained experimentally by OCT. The developed computational modeling approach allows one to take into account diffraction effects on bulk, absorbing objects and the influence of the amplitude–phase profile of the wave beam on the quality of OCT imaging. The limitations of the approach are associated, first of all, with the conditions of applicability of the parabolic equation used. The accuracy of the latter is determined by the value of the numerical aperture, which should be much less than unity within the framework of the quasioptics approximation. Secondly, the presented results were obtained without multiple scattering. The speed of the numerical implementation essentially depends on the available computing resources and the optimization of the numerical scheme. Simulation of the time-domain OCT images presented in the current version of the MC model on a computer with 24 processors takes 3 to 4 h. The developed computational model can be used for imitation of the light waves propagation both in time-domain and spectral-domain OCT approaches.

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