# THE DYNAMIC STOCHASTIC LINEAR PROGRAMMING MODEL FOR MANAGEMENT IN THE CONSUMPTION OF FUEL IN FLEX

*Dn. Paulo César Chagas Rodrigues*
*Instituto Federal de Educação, Ciência e Tecnologia de São Paulo - Brazil*
*E-mail: paulo.rodrigues@ifsp.edu.br*

**Abstract**

From the amount invested in fuel prices, the rate of road that owners of vehicles powered by combustion need to run within the city and on the roads in the region, began a study in order to allow a better definition as to the cost of supply of vehicle with Ethanol or gasoline. And to analyze the equation that the government, through Petrobras announces in the media, because it is mentioned that to obtain the lowest cost when the supply should divide the value of a liter of Ethanol Gasoline for that case stay above 0.70 is the ideal fuel to petrol and vice versa.

**Keywords:** Dynamic stochastic, linear programming, consumption of fuel, flex car, Ethanol, Gasoline.

## 1    INTRODUCTION

With the price of ethanol and gasoline having a small difference, the users of motor vehicles are in doubt whether to fill up with ethanol or gasoline.

On this assumption it was decided to do a search value of ethanol and gasoline in the region of Avaré/SP, in order to develop a linear programming model in which to obtain the lowest cost when supplying the vehicle. This research is not measuring how many miles per gallon the car will do according to the volume of ethanol and gasoline in the tank.

This research is exploratory and will use the quantitative method, because if you want to look at gas stations in the region the charge and record the values that have variation.

This research will feed the price linear programming model, so you can have a brief idea of what volume of ethanol and gasoline, and so put position can know what really gives me the best option.

Lawrence and Pasternack (2001) discuss the mathematical programming is the branch of science dealing with the management of optimization problems, in which we want to maximize a function (eg as the return of profit, expected, or efficiency) or minimize a function (such as cost, time or distance), usually in a restricted environment.

According to Goldberg (2000), the need to represent the context of reality simply gave rise to the concept of modeling, which is defined as a process of finding a well-structured view of reality, models are simplified representations of reality, to certain situations and approaches.

Modeling is a method of quantitative research of an applied nature, refers to research on models of causal relationships between control variables and performance variables which can be developed, analyzed or tested (BERTRAND; FRANSOO, 2002).

Mathematical modeling is intended to structure and solve quantitative models that can be expressed mathematically and that within the field of Operations Research, Mathematical Programming are called. In practice, the Mathematical Programming supports decision making in managing large systems, especially those related to the treatment of quantified variables (GOLDBERG, 2000).

Ko, Mehnen and Tiwari (2010) add that a mathematical structure allows us to treat and represent uncertainties in the self-perception of uncertainty, imprecision, partial truth, and lack of information.

Allen and Schuster (2004) add that mathematical programming is often the tool used for modeling strategies in conditions of uncertainty. These models are static in nature, assuming that the probabilities of occurrence are known beforehand and that all decisions are taken at the same time.

To represent particular system in formulating a problem with models used is normally a set of equations or mathematical expressions. There, every decision is taken to be quantifiable associated with a variable of the model is called the decision variable whose value should be determined by the model. After formulating the

problem it created an objective function, which is formed by the decision variables and express the measure of effectiveness sought. Furthermore, from the actual limitations exist, it creates the model constraints are represented mathematically by equation and inequalities (Goldberg, 2000).

Bertrand and Fransoo(2002) explain that modeling can be classified as axiomatic and empirical research:

a) Research indicates the axiomatic process of obtaining resolutions of the model defined; and

b) The empirical research indicates that to obtain the results of the empirical process.

## 2  DYNAMIC STOCHASTIC PROGRAMMING MODEL

According to Lawrence and Pasternack (2001), Dynamic programming can be defined as models that can be thought of as problems of multiple stages in which a set of decisions is made "following."

According to García-Dastugue (2003), the mechanics of the optimization algorithm is based on dynamic programming. Lambert, Emmelhainz and Gardner (1996) discuss the dynamic programming is used to solve sequential decision problems in which dynamic programming starts solving the problem for the last stage and working backwards to solve the problem for previous stages of each one time.

To Kivinen (2007), the dynamic programming approach is applied in a solution to the problems that can be decomposed into a series of different stages (e.g., year 1, 2, 3 or projects A, B, C, etc.). at each step, the decision maker is a certain situation or state, which describes the amount of some resource that can be used to this and all subsequent phases. The challenge to the decision maker is making a series of decisions that are ideal for the whole process.

Grondin (1998) discuss the components of a dynamic programming that must be identified for solving multistage decision may include the following among many others:

a)  A variable stage;

b)  A (set of) state variable;

c)  A (set of) decision variable;

d)  The return (or cost) function (or table) for each stage;

e) An optimal value function to give the best return for the values of the stage and state variables;

f) A set of boundary conditions for the last step;

g) The stopping rule based on a value of resources (set of) total available in the first phase; and

h) The recursion-relation of a procedure for determining the value of the function, value, ideal for any stage and state.

### 2.1 Definitions

In this model the objective function is to minimize the cost of supply when the car with petrol and ethanol, for both variables were defined Ethanol and Gasoline and unit of measure was in liters. The constants that will enable the implementation of the objective function are viable cost (Y1), desired ethanol consumption (Y2), gasoline consumption desired (Y3), tank capacity of ethanol (Y4), petrol tank capacity (Y5), joint capacity (Y6) and fuel costs (Y7).

### 2.2 Decision variables

The decision variables are used in this model ethanol (X11) and petrol (X21), because these two variables will be displayed volume in liters of each fuel in the tank to be completed and allow to obtain the lowest cost when the supply.

These variables will be influenced by the price of fuel, as the fuel station, city, state or even region of the country, as prices can have a range of up to 100%.

This variation will exercise some influence on the volume of fuel to be put in the tank, because as the price change between ethanol and gasoline may inform the model to place different amounts.

### 2.3 Constraints

This set of constants will assist in defining the volume in liters of fuel supply to the vehicle, i.e. the amount of ethanol and gasoline to be put, as the tank capacity. Note that implicitly assume the non-negativity of the decision variables and will be supplemented with 1 or 0, which can be seen in Table 1.

The following features will equations that will compose the constants: Equation 1 shows the restriction value, which is the multiplication number of liters of

ethanol and cost of fuel, which must be less than or equal to the cost of viable * cost of fuel *  number of liters of gasoline.

$$Y17 * X1 <= Y1 * Y27 * X12 \qquad (1)$$

Equation 2 is the sum of the multiplication of the number of liters of ethanol * Tank capacity ethanol and multiplying the quantity of gasoline does * gas tank capacity, which is a restriction on the ability of ethanol as the fuel tank support. This restriction has the constant value 1 Y17 and Y27 has constant value 0.

$$Y14 * X11 + Y24 * X12 <= Y4 \qquad (2)$$

Equation 3 is the sum of the multiplication of the number of liters of ethanol * Tank capacity ethanol and multiplying the quantity of gasoline does * gas tank capacity, which is a restriction on the ability of ethanol as the fuel tank support . This restriction has the constant value 0 Y17 and Y27 constant has the value 1.

$$Y15 * X11 + Y25 * X12 <= Y5 \qquad (3)$$

Equation 4 represents the combined capacity of the two fuel tank which should be less than or equal to the capacity of the tank and it means the sum of multiplications of constant joint capacity * variable quantity of gallons of ethanol and constant joint capacity * variable number of gallons of gasoline.

$$Y16 * X11 + Y26 * X12 <= Y6 \qquad (4)$$

Equations 5 and 6 are not to force the negativity of the variables related to the volume in liters of ethanol and gasoline to be supplied in the car.

$$X11 >= 0 \qquad (5)$$

$$X21 >= 0 \qquad (6)$$

Equations 7 and 8, are responsible for creating Kilometer a delimitation between the minimum and maximum that the vehicle can make from the formulation amount of each fuel to be used during fueling of the car. In which the summation is carried out multiplications between the capacity of the tank with ethanol and the value in Km / l and that the vehicle is in Gasoline Tank capacity and the value in km / l that the vehicle is.

$$X11 * Y2 + X12 * Y3 >= Y4 * Y2 \qquad (7)$$

$$X11 * Y2 + X12 * Y3 <= Y5 * Y3 \qquad (8)$$

### 2.4 Objective function

The objective function of the model includes two contrasting components. The first measuring the volume in liters of gasoline and ethanol and second cost would be about the supply of fuel. From these variables and constants will be the calculation of the total cost when the supply of fuel in certain gas station.

The objective function of Equation 7 is to minimize the cost of supply, allowing it to obtain a financial economics, when supply and have a lead time between a supply and another constant.

$$\text{Minimizar } (Y17 * X11 + Y27 * X21) \qquad (7)$$

## 3   ILLUSTRATIVE EXAMPLE

Based on Table 1, is presented an application example and model validation: In Cost-effective (Y1) as the value is one way of forcing the supply with both fuels, the value of 0.6667 signifies the division between the values of Km / L when the vehicle makes Ethanol and Gasoline, i.e. division between 10 and 15 liters.

In Ethanol Consumption Desired (Y2) values are 1 and 0 respectively for the X11 and X21 column to inform you that should only be put Ethanol.

Gasoline Consumption In Desired (Y3) values are 0 and 1 respectively for the X11 and X21 column to inform you that should only be put Gasoline.

Already in Ethanol Tank capacity (Y4) to how many liters of Ethanol can be put in the fuel tank and should have the values 1 and 0.

To Gasoline Tank Capacity (Y5) to how many gallons of Gasoline may be placed in the fuel tank and must have values 0 and 1.

Combined capacity (Y6) will permit the validation of how many of each fuel tank and will fill the value is one for both columns.

The Cost of fuel (Y7) be the price recorded in the fuel pump which will assist in formulation of the cost of supply.

Table 1: sample run of the model

| | Ethanol | Gasoline | | |
|---|---|---|---|---|
| | X11 | X21 | | |
| Liters | 24 | 21 | | |
| | | | | |
| Cost-effective (Y1) | 1 | 1 | 0,6667 | |
| Ethanol Consumption desired (Y2) | 1 | 0 | 10 | Km/L |
| Gasoline Consumption desired (Y3) | 0 | 1 | 15 | Km/L |
| Tank capacity Ethanol (Y4) | 1 | 0 | 45 | L |
| Gasoline Tank Capacity (Y5) | 0 | 1 | 45 | L |
| Combined capacity (Y6) | 1 | 1 | 45 | L |
| Cost of fuel (Y7) | 1,570 | 2,690 | | R$ |
| | | | | |
| Objective function: | 94,18 | | | |
| | | | | |
| Constraints: | | | | |
| | 37,671 | <= | 37,671 | |
| | 24 | <= | 45 | |
| | 21 | <= | 45 | |
| | 45,000001 | <= | 45 | |
| | 24 | >= | 0 | |
| | 21 | >= | 0 | |
| | 554 | >= | 450 | |
| | 554 | <= | 675 | |

## 4   APPLICATION TO THE FUEL FLEX CAR

The application of the proposed model to a real world problem requires additional assumptions, complete specification of risk factors "fuel value", how many Kilometers per liter the vehicle wheel and the inclusion of the total cost of refueling in the objective function.

These data helped in the implementation of a mathematical model, which was implemented in the spreadsheet Microsoft Excel version 2010, with the implementation of Solver application during its execution sought to observe the variation amount of fuel as the price of fuel varied according to the fuel station.

In a second step we could see how many kilometers the vehicle could run as the amount of each fuel, which allows observing the cost / benefit that the driver may have with this solution. Mileage values as the vehicle can achieve with the mixture is

an estimate, it would have to observe the condition of the vehicle, location where this is heading, among others.

The Appendices A, B and C are respectively presented reports response, and sensitivity limits, so as to enable the validation of the model.

## 5    FINAL THOUGHTS

Challenges for modeling and solving mathematical problems, taking into account the dynamics of the decisions about the uncertainties are many and complex. Since the size of the problem increases exponentially with the number of steps, computational tractability, as the first obstacle on the modeling.

With the incorporation of data from fuel costs, mileage per gallon of each fuel and the fuel tank capacity, the objective function seeks to minimize the cost of supplying and allowing the vehicle can run as much as possible, between the pre-established by the vehicle manufacturer, because as is well known brand and each model may have very specific details.

The application of this model in practice has been going through empirical studies, conducted over several trips, in which the author tries to observe the value of fuel, depending on the region or city where it passes and observing the behavior of the vehicle during its operation. Also being done tests and observations, as the condition of the local road in good repair or bumpy, cities with slow or fast traffic, use of air conditioning, among other factors.

The development of this model is part of a project to allow a little more rational consumption of fossil fuel and Ethanol, allowing the reduction of the emission of toxic gases, engine durability, reduced spending on fuel, among others. This project seeks to establish a balance between successful academic innovation and attention to business requirements. From the use of an application widely used in business and personal life, the model can be used and tested, but for that you should have a minimum of knowledge about the use of the spreadsheet in Microsoft Excel and in the case of Plugin Solver, adding a practical dimension to the contributions described in this article.

**REFERENCES**

ALLEN S. J.; SCHUSTER, E. W. (2004) Controlling the risk for a agricultural harvest. **Manufacturing & Service Opertions Management**, v. 6, n. 3, p 225-236.

BERTRAND, J. W. M.; FRANSOO, J. C. (2002) Modeling and Simulation: Operations management research methodologies using quantitative modeling. **International Journal of Operations & Production Management**, v. 22, n. 02, p. 241-264.

GARCÍA-DASTUGUE, S. J. (2003) **Dynamic time-based postponement**: Conceptual development and empirical test. Ph.D, Ohio: Ohio State University.

GOLDBERG, D. E. (2000) **Genetic algorithms in search**, Optimization and Machine Leaning. Addison Wesley Limited.

GRONDIN, F. (1998) Improvements of the dynamic programming algorithm for tree bucking. **Wood and Fiber Science**, v. 30, n. 1, p. 91-104.

KIVINEN, V. P. (2007) **Design and testing of stand-specific bucking instructions for use on modern cut-to-length harvesters**. Dissertação (Department of Forest Resource Management), Helsinki: University Helsinki.

KO, M.; TIWARI, A.; MEHNEN, J. (2010) A review of soft computing applications in supply chain management. **Applied Soft Computing**. V. 10, p. 661-674.

LAMBERT, D. M.; EMMELHAINZ, M. A.; GARDNER, J. T. (1996) Developing and Implementing Supply Chain Partnerships. **The International Journal of Logistics Management**, v. 7, n. 2, p. 1-18.

LAWRENCE, J. A.; PASTERNACK, B. A. (2001) **Applied management science**: modeling, spreadsheet analysis, and communication for decision making. 2 ed. New Jersey: Wiley.

## Appendices' A

**Microsoft Excel 14.0 Relatório de Respostas**
**Planilha: [IJM&P1_I.xls]Plan2**
**Relatório Criado: 23/08/2012 16:05:03**
**Resultado: O Solver encontrou uma solução. Todas as Restrições e condições de adequação foram satisfeitas.**
**Mecanismo do Solver**
   Mecanismo: LP Simplex
   Tempo da Solução: 0,047 Segundos.
   Iterações: 5 Subproblemas: 0
**Opções do Solver**
   Tempo Máx. Ilimitado, Iterações Ilimitado, Precision 0,000001
   Subproblemas Máx. Ilimitado, Soluç. Máx. Núm. Inteiro Ilimitado, Tolerância de Número Inteiro 1%, Assumir Não Negativo

Célula do Objetivo (Mín.)

| Célula | Nome | Valor Original | Valor Final |
|---|---|---|---|
| $B$14 | função objetivo: X11 | 93,91 | 94,18 |

Células Variáveis

| Célula | Nome | Valor Original | Valor Final | Número Inteiro |
|---|---|---|---|---|
| $B$4 | Litros (X1) X11 | 24 | 24 | Conting. |
| $C$4 | Litros (X1) X12 | 21 | 21 | Conting. |

Restrições

| Célula | Nome | Valor da Célula | Fórmula | Status | Margem de Atraso |
|---|---|---|---|---|---|
| $B$17 | Y17 * X1 <= Y1 * Y27 * X12 X11 | 37,671 | $B$17<=$D$17 | Associação | 0 |
| $B$18 | Y14 * X11 + Y24 * X12 <= Y4 X11 | 24 | $B$18<=$D$18 | Não-associação | 21,00594648 |
| $B$19 | Y15 * X11 + Y25 * X12 <= Y5 X11 | 21 | $B$19<=$D$19 | Não-associação | 23,99405352 |
| $B$20 | Y16 * X11 + Y26 * X12 <= Y6 X11 | 45 | $B$20=$D$20 | Associação | 0 |
| $B$21 | X11 >= 0 X11 | 24 | $B$21>=$E$21 | Não-associação | 24 |
| $B$22 | X21 >= 0 X11 | 21 | $B$22>=$D$22 | Não-associação | 21 |
| $B$23 | X11 * Y2 + X12 * Y3 >= Y4 * Y2 X11 | 555 | $B$23>=$D$23 | Não-associação | 105 |
| $B$24 | X11 * Y2 + X12 * Y3 <= Y5 * Y3 X11 | 555 | $B$24<=$D$24 | Não-associação | 119,9702676 |

## Appendices' B

**Microsoft Excel 14.0 Relatório de Sensibilidade**
**Planilha: [IJM&P1_I.xls]Plan2**
**Relatório Criado: 23/08/2012 16:05:04**

Células Variáveis

| Célula | Nome | Final Valor | Reduzido Custo | Objetivo Coeficiente | Permitido Aumentar | Permitido Reduzir |
|---|---|---|---|---|---|---|
| $B$4 | Litros (X1) X11 | 23,99405352 | 0 | 1,57 | 1,12 | 1E+30 |
| $C$4 | Litros (X1) X12 | 21,00594648 | 0 | 2,69 | 1E+30 | 1,12 |

Restrições

| Célula | Nome | Final Valor | Sombra Preço | Restrição Lateral R.H. | Permitido Aumentar | Permitido Reduzir |
|---|---|---|---|---|---|---|
| $B$17 | Y17 * X1 <= Y1 * Y27 * X12 X11 | 37,67066402 | -0,333002973 | 0 | 70,65 | 80,7 |
| $B$18 | Y14 * X11 + Y24 * X12 <= Y4 X11 | 23,99405352 | 0 | 45 | 1E+30 | 21,00594648 |
| $B$19 | Y15 * X11 + Y25 * X12 <= Y5 X11 | 21,00594648 | 0 | 45 | 1E+30 | 23,99405352 |
| $B$20 | Y16 * X11 + Y26 * X12 <= Y6 X11 | 45 | 2,092814668 | 45 | 9,726797911 | 8,515468059 |
| $B$21 | X11 >= 0 X11 | 23,99405352 | 0 | 0 | 23,99405352 | 1E+30 |
| $B$22 | X21 >= 0 X11 | 21,00594648 | 0 | 0 | 21,00594648 | 1E+30 |
| $B$23 | X11 * Y2 + X12 * Y3 >= Y4 * Y2 X11 | 555,0297324 | 0 | 450 | 105,0297324 | 1E+30 |
| $B$24 | X11 * Y2 + X12 * Y3 <= Y5 * Y3 X11 | 555,0297324 | 0 | 675 | 1E+30 | 119,9702676 |

## Appendices' C

Microsoft Excel 14.0 Relatório de Limites
Planilha: [IJM&P1_I.xls]Plan2
Relatório Criado: 23/08/2012 16:05:04

| Célula | Objetivo Nome | Valor |
|---|---|---|
| $B$14 | função ob | 94,18 |

| Célula | Variável Nome | Valor | Inferior Limite | Objetivo Resultado | Superior Limite | Objetivo Resultado |
|---|---|---|---|---|---|---|
| $B$4 | Litros (X1) | 24 | 24 | 94 | 24 | 94 |
| $C$4 | Litros (X1) | 21 | 21 | 94 | 21 | 94 |