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Finite-time Average Consensus in a Byzantine Environment Using Stochastic Set-Valued Observers

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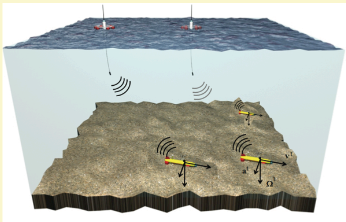
Objectives:

- Compute the average of the initial states in finite-time
- Guarantee fault detection and bounds on the maximum possible deviation from an attack
- Incorporate the transmissions stochastic information in the fault detection mechanism.

1. Motivation

Nodes need to distributedly agree on a common value:

- Smart Grids when deciding the needed power
- Robot swarms to find rendezvous points
- Social Networks making a pool about a subject.



In all these examples a single node can drift the entire network to any desired value!

2. Problem Statement

The objective is to compute the average of some quantity of interest. If faulty, any node i must detect the fault using only local information communicated from their neighbors.

Network Model

$G = (V, E)$
$E \subseteq V \times V$

System Model

$$S^i : \begin{cases} x(k+1) = \left(A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell(k) A_\ell\right) x(k) + B(k)u(k) \\ y^i(k) = C^i(k)x(k) \end{cases}$$

n_Δ number of uncertainties
Each S^i is a Linear Parameter-Varying (LPV) system
$\Delta_\ell(k)$ are scalar uncertainties

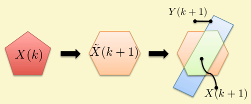
Byzantine Consensus Problem:
Either detect non-zero signals $u(k)$ using $y(k)$ without the knowledge of $B(k)$ or compute the final consensus value.

3. Proposed Solution

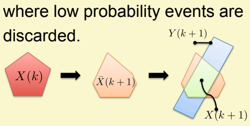
Each node runs a Stochastic Set-Valued Observer (SSVO).
The estimates are intersected as to produce less conservative polytopes.

1) Compute the next set-valued estimates

Example of a SVO update for horizon 1.



Example of a SSVO update where low probability events are discarded.



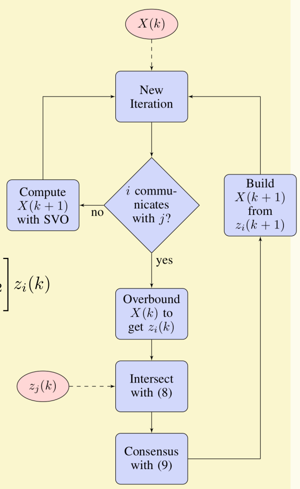
2) Overbound using a hyper-parallelepiped to transmit estimates to neighbors

3) Intersect estimates by performing

$$z_i(k) = z_j(k) = \max(z_i(k), z_j(k))$$

4) Perform a consensus update on the estimates interval

$$z_i(k+1) = \left[\left(\frac{1}{2}(e_i - e_j)(e_j - e_i)^\intercal + I_{n_x}\right) \otimes I_2\right] z_i(k)$$



Flowchart of the proposed algorithm

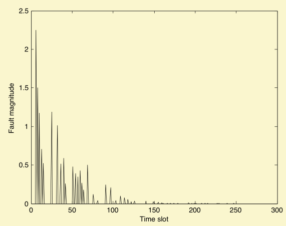
Remark:

Fault detection consists in checking if the result of the intersection in step 3) is the empty set.

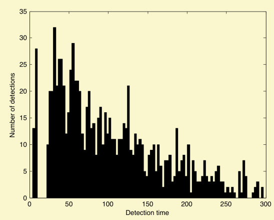
[1] Silvestre, D.; Rosa, P.; Cunha, R.; Hespanha, J.P.; Silvestre, C., "Gossip average consensus in a Byzantine environment using stochastic Set-Valued Observers," *IEEE 52nd Annual Conference on Decision and Control (CDC), pp.4373,4378, 10-13 Dec. 2013*

4. Results

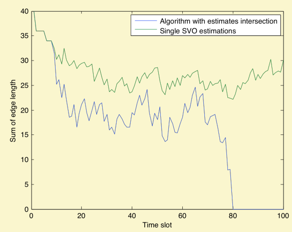
- The magnitude of an attacker fault signal is bounded.



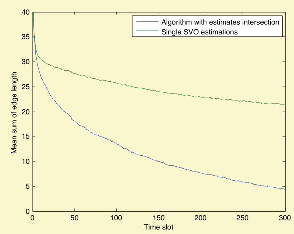
- Faults are more likely to be detected as time increases.



- Finite-time consensus is achieved.



- Less conservative set-valued estimates.



Main result:

The algorithm can either detect and intruder or compute the average consensus in finite-time.

[2] Silvestre, D.; Rosa, P.; Hespanha, J.P.; Silvestre, C., "Finite-time Average Consensus in a Byzantine Environment Using Set-Valued Observers," American Control Conference (ACC), *pp.3023,3028, 4-6 Jun. 2014*

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