Finite-time Convergence Policies in State-dependent Social Networks

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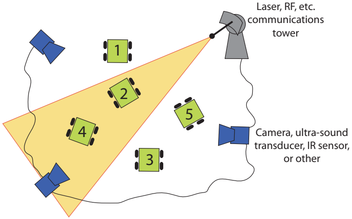
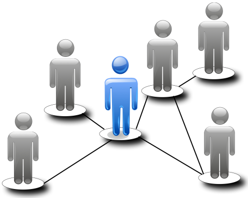
Outline

Motivation

- Sensor Networks - Transmission at variable communication radius can be modeled by the proposed approach.
- Robot Coordination - Fleet of robots wishes to have consensus on direction/speed or rendezvous point.
- Advertising in social networks - Identifying what are the key opinions in the final decision.



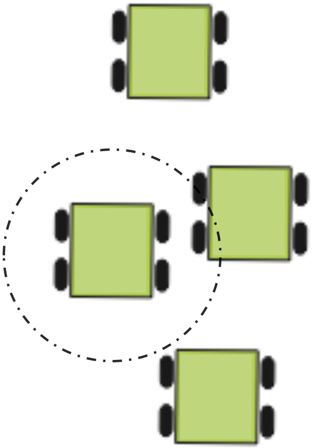


State-dependent Social Network

- A group of n people has an objective opinion about a subject.
- People seek those that share similar opinions.
- The number of social links is limited.
- Main issue: study the convergence rates of different interaction dynamics based on the previous observations.

Motivating Example

- Consider a set of vehicles with variable power communication antennae.
- To save resources the number of communicating vehicles should be reduced.
- The selected power influences the number of possible neighbors.
- Communication is based on proximity.



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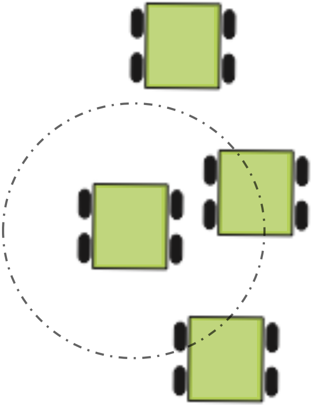

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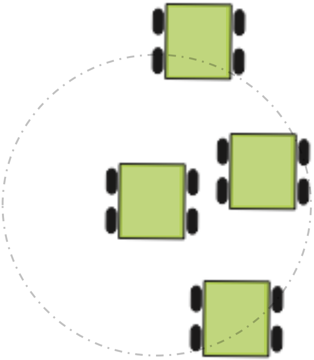


Problem Statement

- Take n nodes, where each node i updates its opinion as

$$x_i(k+1) = \alpha_k \min_{j \in N_i(k)} x_j(k) + (1-\alpha_k) \max_{j \in N_i(k)} x_j(k)$$

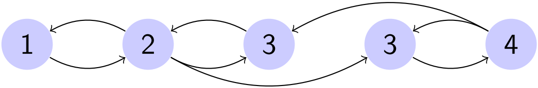
- Four different rules for selecting $N_i(k)$ are presented.

Convergence of State-dependent Social Networks Problem

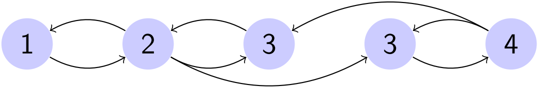
Does the n nodes converge to the same opinion asymptotically or in finite-time? If so, can we provide the convergence rate?

Neighbor Selection Rules (1/3)

- Standard Network - Each agent i picks η nodes with higher opinion and η with a smaller one.

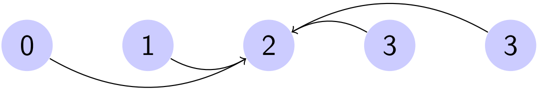


- Distinct Value - excludes neighbors with equal opinion.



Neighbor Selection Rules (2/3)

- Standard Network for $\eta = 2$.



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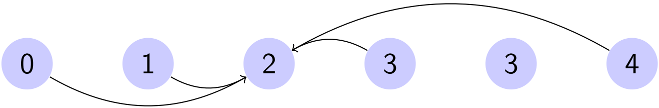
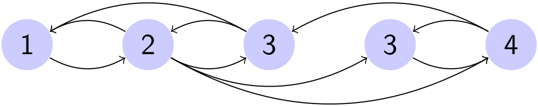
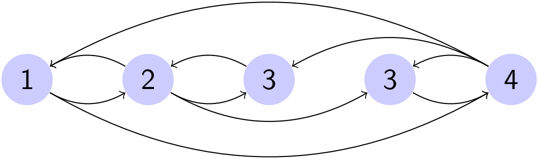


Neighbor Selection Rules (3/3)

- Distinct Neighbor - selects extra nodes with higher opinion if there are not enough nodes with smaller opinion, and vice-versa.



- Circular Value - selects nodes with extreme opinions (minimum and maximum) if there is not enough neighbors.



Convergence Results (1/2)

- Standard Network
  - If $\eta \geq n-1$, the network has finite-time convergence.
  - If $\eta < n-1$, the network converges asymptotically.
- For constant parameters, we have asymptotically exponential convergence governed by the second largest eigenvalue of

$$A_{ij} := \begin{cases} \alpha, \text{ if } j = \max(1, i-\eta) \\ 1-\alpha, \text{ if } j = \min(n, i+\eta) \\ 0, \text{ otherwise} \end{cases} .$$

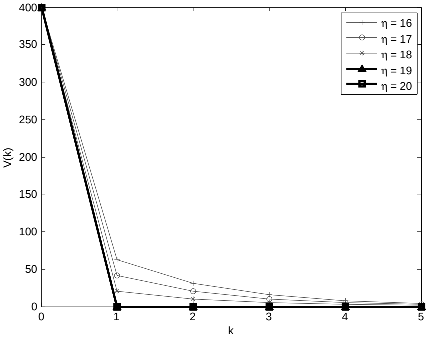
Convergence Results (2/2)

- Distinct Value network
  - If $\eta \geq \frac{n}{2}$, the network has finite-time convergence in $\lceil \log_2 n \rceil$ steps.
  - If $\eta < \frac{n}{2}$, the network converges asymptotically.
- Distinct Neighbor network has finite-time convergence in $\lceil \frac{n-(2\eta+1)}{2\eta} \rceil + 1$ steps for any $\eta \geq 1$.
- Circular Value network has finite-time convergence in $\lceil \frac{n-(2\eta+1)}{2\eta-1} \rceil + 1$ steps for any $\eta \geq 1$.

Simulation Results (1/2)

Setup: 20-node network with initial states $x_i(0) = i^2$.

- The Standard Network requires the complete network to have finite-time convergence.
- Distinct Value needs half the connections.
- Distinct Neighbor converges in finite-time for every value of η.
- An interesting remark is that for higher values of η the gain in convergence speed is diminished.



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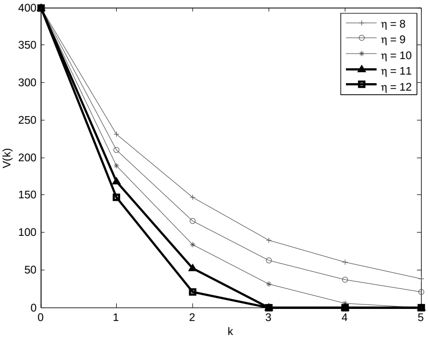

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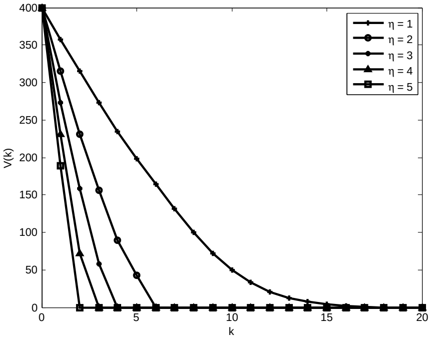
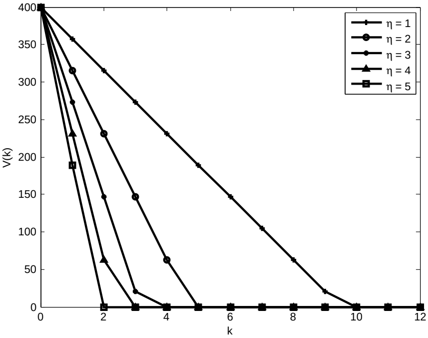


Simulation Results (2/2)

- The convergence for the Circular Value is similar to the Distinct Neighbor.
- Setup: 50-node network, $\eta = 1$ and initial states $x_i(0) = i^2$.
- Figure depicts that both the Standard Network and Distinct Value are very slow in convergence.
- Convergence is very similar for Distinct Neighbor and Circular Value. Circular Value was a comparison as it lacks physical meaning.



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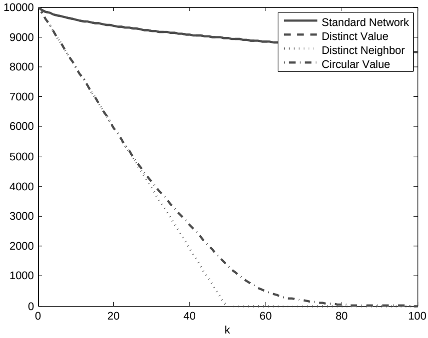

Concluding Remarks

Contributions:

- We model a social network as a distributed algorithm where the network is state-dependent with a fixed parameter of maximum number of connections;
- Considering only nodes with distinct opinion is shown to reduced the number of required neighbors to half the nodes in the network to obtain finite-time convergence;
- Finally, two strategies are investigated — one where nodes with extreme opinions contact with each other and another where agents require a fixed number of neighbors — and proved to converge in finite time, even when only communicating with 2 other nodes.

The end

- Thank you for your time.

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