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Aircraft Performance Model Calibration and Validation for General Aviation Safety Analysis

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Performance models facilitate a wide range of safety analyses in aviation. In an ideal scenario, the performance models would show inherently good agreement with the true performance of the aircraft. However, in reality, this is rarely the case, either owing to underlying simplifications or due to the limited fidelity of applicable tools or data. In such cases, calibration is required to fine-tune the behavior of the performance models. For point-mass steady-state performance models, challenges arise due to the fact that there is no obvious, unique metric or flight condition at which to assess the accuracy of the model predictions, and since a large number of model parameters may potentially influence model accuracy. This work presents a two-level approach to aircraft performance model calibration. The first level consists of using manufacturer-developed performance manuals for calibration, while the second level provides additional refinement when flight data is available. The performance models considered in this work consist of aerodynamic and propulsion models (performance curves) that are capable of predicting the non-dimensional lift, drag, thrust, and torque at any given point in time. The framework is demonstrated on two representative General Aviation aircraft. The demonstrated approach results in models that can predict critical energy-based safety metrics with improved accuracy for use in retrospective safety analysis.

I. Introduction and Motivation

The development of models is ubiquitous in a broad range of performance studies and safety analyses. The ability to accurately model complex phenomena contributes greatly towards enhancing the understanding of their performance of vehicles in real-world scenarios. One of the main advantages of using performance models in safety analysis is the ability to use recorded data to predict unrecorded quantities of interest. For example, using recorded flight data from a digital flight data recorder (DFDR) [1] in conjunction with aircraft performance model can help predict non-dimensional lift, drag, thrust, etc. These quantities along with flight data can be used for various retrospective safety analyses such as evaluation of energy-based safety metrics [2, 3], monitoring of aircraft performance [4], detecting anomalous flights in a large set of flight data records [5–8], estimating the weight of the aircraft, identifying performance safety envelopes [9–12], etc. The quality of insights and results obtained from these retrospective analyses depends on the availability of well-calibrated performance models for predicting the vehicle's performance.

General Aviation (GA), according to the Federal Aviation Administration (FAA) is "*that portion of civil aviation that does not include scheduled or unscheduled air carriers or commercial space operations*". Despite the improving safety record of aviation operations, GA aircraft have traditionally had higher accident and incident rates [13]. The availability of additional data in the form of performance model predictions is a valuable asset for incident and accident investigation, retrospective analyses, flight training etc.

The problem being addressed in this paper is the calibration and validation of point-performance steady state models of small GA fixed-wing aircraft (such as the Cessna 172). These models are available in the form of variations of

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important non-dimensional quantities relevant for aircraft performance, such as lift curve, drag polar, propeller polar, etc. A detailed implementation of such models using empirical relations and assumptions can be found in Harrison et al. [14] and Min et al. [15]. In an ideal scenario, the performance models would show inherently good agreement with the true performance of the aircraft. However, in reality, this is almost never the case, either owing to underlying simplifications or assumptions or due to the limited fidelity of available or applicable analysis tools. In the case of point-performance steady-state performance models, challenges also arise due to the fact that there is no obvious, unique metric or flight condition at which to assess the accuracy of the model predictions, and since a large number of model parameters may potentially influence model accuracy. The availability of accurate calibration and validation data can limit the applicability of developed models. All the limitations mentioned necessitate the calibration of developed empirical models in a systematic fashion in order to deploy them for performance and safety analyses. Lack of data is often a major hindrance in retrospective safety analysis for GA applications [16]. Performance models aim to alleviate that problem to some extent by enabling estimation of quantities of interest that are not directly recorded in GA flight data recorders.

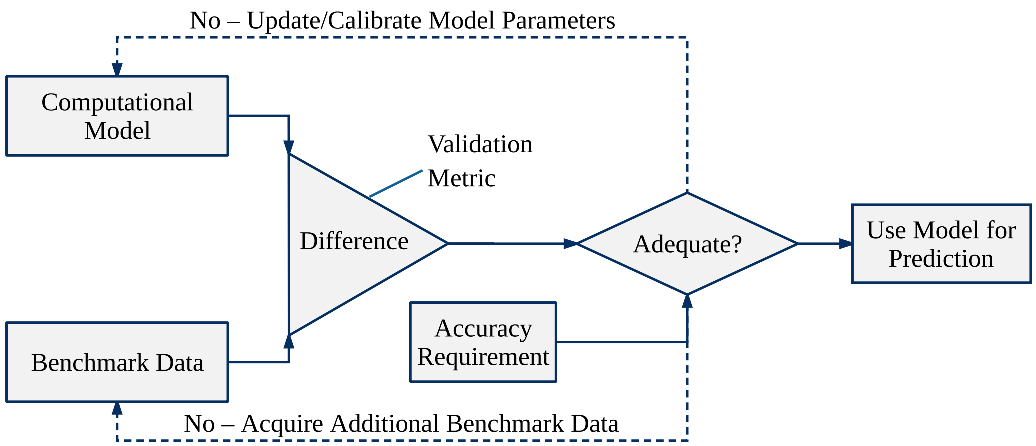


Fig. 1 Calibration of computational models and use for prediction (recreated based on Oberkampf and Barone [17])

Model calibration is defined as "*the process of adjusting numerical or physical modeling parameters in the computational model for the purpose of improving agreement with experimental data*" [18]. The overall process of calibrating computational models has been outlined by Oberkampf et al. [17, 19] and has been reproduced in Figure 1. The process consists of comparing the predictions from computational models with some benchmark data (usually publicly available). The comparison is made in terms of certain critical output parameters called validation metrics. If there is sufficient agreement between the computational model and benchmark data, the model is used for prediction. If not, the parameters of the model are updated or calibrated until the adequacy requirements are met. Wherever possible, additional benchmark data is acquired. In the type of calibration undertaken here, the form of the model is assumed to be fixed and the parameters are varied to achieve accuracy of model predictions. In such cases, the values of context-specific inputs and variables are unknown or uncertain and the observations or benchmark data are used to tune these parameters. There exist other types of model calibration frameworks such as system identification [20] or Bayesian Calibration [21] in which the model form is not assumed to be fixed. These frameworks are not considered here as they require more detailed input-output data which is typically not available for GA performance and safety studies.

The errors or uncertainties in the performance models can stem from various sources. There are two main types of uncertainty [22]: aleatory uncertainty due to inherent variation (irreducible) and epistemic uncertainty due to lack of knowledge (reducible). Even when dealing with performance models for the same GA aircraft, there can still be several reasons due to which the performance actually observed can differ from predicted performance. These errors arise due to the cumulative effect of factors not included in the model and random effects on the estimate. In the context of the present models, some of the important factors that can cause a difference in actual versus predicted performance are:

- Variations in aircraft gross weight between flights and the fact that gross weight is most often unknown (epistemic)
- Degradation in performance of aircraft systems (such as engine etc.) with age (epistemic) or additional drag due to unclean surfaces, insect accretions, etc.

- Changes or modifications made to aircraft that affect its aerodynamic behavior (epistemic)
- Variation in aircraft model (C172S versus C172R - epistemic)
- Model Inadequacy (epistemic)
- Unknown/Inaccurate model parameters (epistemic)
- Piloting skill (epistemic)
- Noise in recorded data (aleatory)
- Environmental conditions (aleatory)

Due to all these uncertainties, it is essential to calibrate the aircraft performance models prior to using them for safety analysis. In consideration of the above observations, the overarching research objective of this paper is identified as follows:

To demonstrate a systematic two-level framework for calibration and validation of General Aviation point-performance models for aerodynamics and propulsion to enable improved predictions of energy-based safety metrics

The rest of the paper is organized as follows: Section II describes the overall setup of the model calibration framework, Section III contains the implementation of the two-level calibration process utilized in this work to two representative GA aircraft, Section IV describes the application of calibrated models to an enhance the insights obtained during a safety analysis scenario, and Section V contains conclusions drawn from this work and potential avenues for future work.

II. Model Calibration Framework

While *performance model* can mean something different depending on the application, within the work described in this paper, the term is used in a specific context. An aircraft performance model consists of two individual disciplinary models: aerodynamics and propulsion. The aerodynamics model contains variations of non-dimensional lift and drag with the airplane angle-of-attack. The propulsion model consists of variations of non-dimensional thrust with vehicle speed and propeller revolutions per minute (RPM) as well as engine power lapse with altitude and temperature.

Within these general specifications, there exist numerous options for both aerodynamic and propulsive models, differing in terms of their underlying assumptions and fidelity. While the methods or rules for appropriate model selection in a broad sense is beyond the scope of this work, it is important to note the limitations of calibration as it pertains to model selection. Generally speaking a performance model is selected upon careful consideration of the domain of interest, which may include considerations such as the operating regime, vehicle configuration, and model application intent. From these considerations a performance model that captures the relevant physics, or other behavior of interest, should be selected that affords the most reasonable level of fidelity given the cost of model implementation. Once a model is chosen model calibration allows for the tailoring of a general model of behavior to be more representative of the particular system of interest. However, if the selected model lacks the capability to adequately describe some key aspect of the problem under consideration then the final results of the calibration process described herein will be modest at best.

Figure 2 outlines the schematic of the two-level process followed in this work for model calibration. Many of the decisions made in GA safety analysis depend upon the availability of data. The previous empirical models developed for GA operations ([14, 15]) utilized data from the public domain due to lack of flight data. These models were independently calibrated against available benchmarks. However, rarely are such performance models used in isolation. Sometimes, metrics that are not estimable using a single model alone can be estimated using predictions from a combination of models [3, 23]. While this could add uncertainty to the prediction, it opens up new avenues to test, calibrate, and fine-tune model parameters. Similarly, in the event of availability of some amount of flight data, a method needs to be available to utilize that information appropriately to improve model predictions. Therefore, two levels of calibration-data availability are defined, (i) *publicly available data* and (ii) *flight data*, and make up the corresponding two levels of the developed model calibration framework.

Three external inputs are required for the entire calibration framework and have been described in this section. The levels of the framework are elaborated in the next section on implementation. It is noted that recorded flight data might not always be available and therefore Level-II calibration is only possible when such data is available. In cases where required flight data is unavailable the calibration process is terminated after Level-I and the model(s) obtained after Level-I are used for safety analysis.

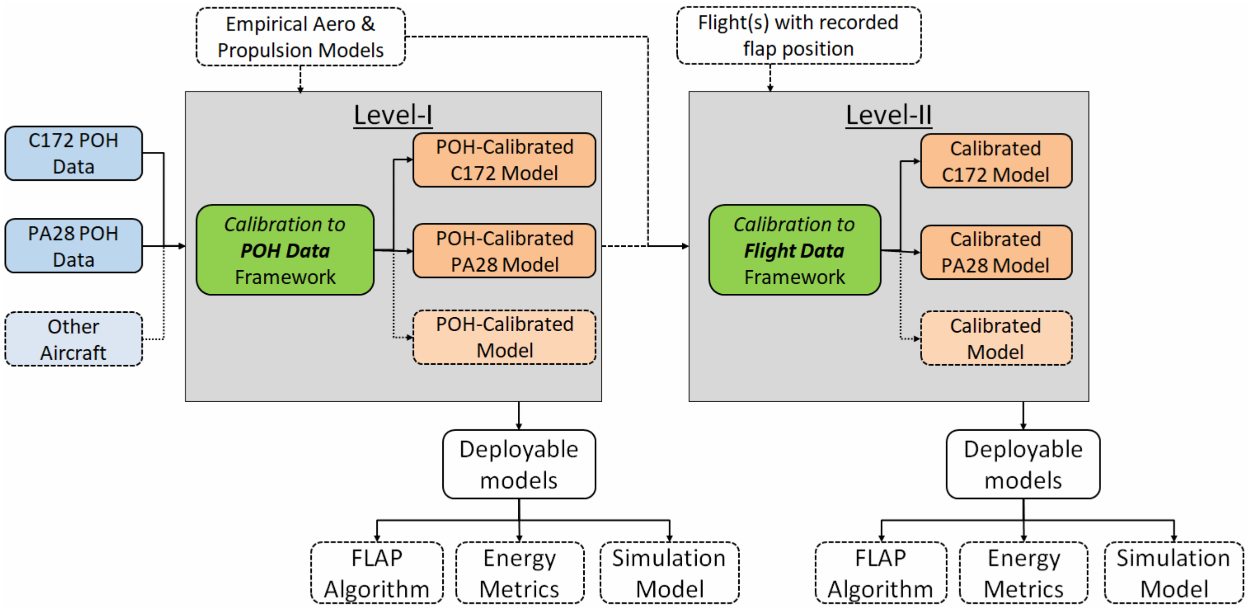


Fig. 2 Schematic of two-level calibration process

A. Aircraft Data

In order to begin the process, basic data about the geometry of the aircraft along with publicly available performance data for calibration and validation is required. Ideally, flight data or flight test data should be used, but this may not be readily available for the aircraft in question. An alternative source for this information is documentation published by the manufacturer of the aircraft such as the Pilot Operating Handbook (POH). The POH is a document developed by the airplane manufacturer and approved by the FAA which lists important information regarding the design, operations, and limitations of the aircraft, as well as its performance characteristics. Although the performance tables listed in the POH are idealized capabilities that a brand new aircraft can theoretically attain under ideal conditions, it is nevertheless a very valuable source of information for calibrating models. As a POH is also readily available for most aircraft, it would enable a calibration process that is easily repeatable. The data from the performance tables in the POH is collected in *comma-separated values* (CSV) format and compared against predicted performance of aircraft using models. Details on the data used in each level of the calibration process are provided in the subsequent section.

B. Empirical Performance Models

Figure 3 shows the flow of information in the individual discipline models used in this work. The initial development of the models can be found in Harrison et al. [14] and Min et al [15]. Given a set of inputs and environmental conditions, it is of interest to predict the aerodynamic and propulsive performance of a GA aircraft in terms of certain non-dimensional coefficients. While these (or similar) models of aerodynamics or propulsion may be validated against published data, it is advantageous to calibrate them together as an aircraft performance model due to the inherent uncertainties associated with using such models on actual flight data. One of the reasons for this is that model input data includes factors such as geometry, operational parameters, and ambient conditions, and can come from a range of sources that may not always be measurable or easily available. The current set of aerodynamic models consist of four lift-curves and drag polars (one for each of four flap settings - 0, 10, 20, and 30 deg. - as is typical on a GA aircraft). On the other hand, the propulsion model consists of a torque and thrust curve. A brief description of the models follows.

1. Propulsion

Among currently operated GA vehicles, the most common means of propulsion is a propeller driven by an internal combustion engine [24]. Following this trend, the propulsion model is similarly divided into individual models of internal combustion engine performance and propeller performance, which combined yield a propulsive model that is appropriate for a large subset of the GA fleet. Many approaches for engine modeling exist within the literature, and a

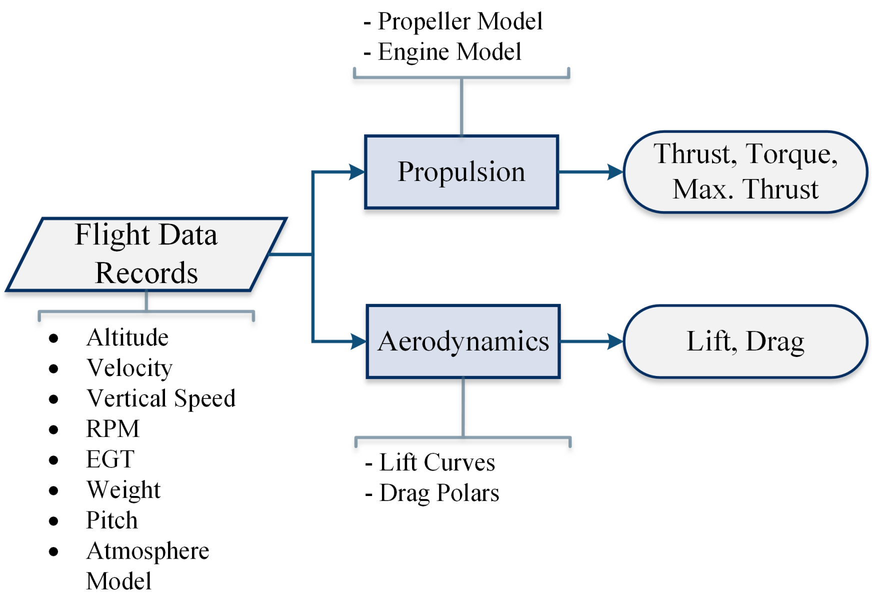


Fig. 3 Information flow in aircraft performance models

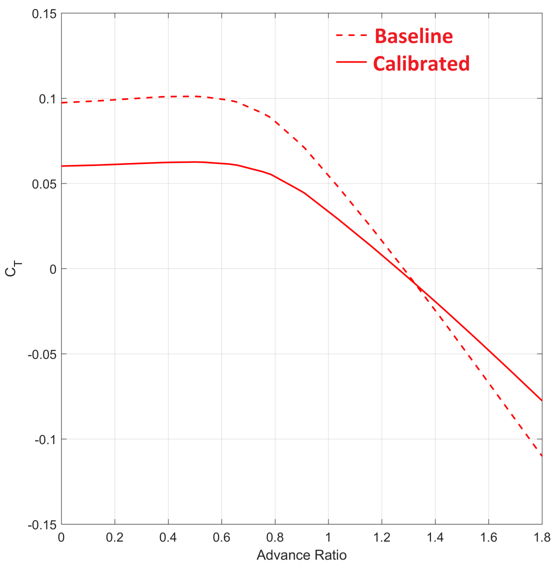
short survey and implementation of some notable methods can be found in previous work by the authors [14]. In that work, it was seen that a detailed modeling of the engine's cycle produces accurate results, though a high computational cost is incurred. This expense can, however, be avoided while still maintaining a reasonable level of accuracy by using published relationships for engine power lapse as functions of local atmospheric density ratio, such as that given by Gudmundsson [25]. As computational time and cost were important considerations for the model calibration framework, this latter approach for engine modeling was chosen.

Alongside the engine model, a model of propeller performance was implemented which predicts the efficiency characteristics of a given propeller geometry. As an input, the model requires propeller geometry in the form of blade chord lengths and pitch angles at several intervals along the span of the propeller blade. This geometric data is combined with airfoil aerodynamic data computed using XFOIL [26], an open-source 2-D aerodynamic prediction program. To generate the 3-D performance of the full propeller, the geometric and 2-D aerodynamic properties are then used with a code based on blade-element momentum theory [27–29]. When matched with the output of the engine model, this propeller model yields a prediction of the propulsive performance of the desired GA aircraft over a wide range of operating conditions. An example of the typical trends for the baseline model used within this work are shown in Figure 4(a) and 4(b).

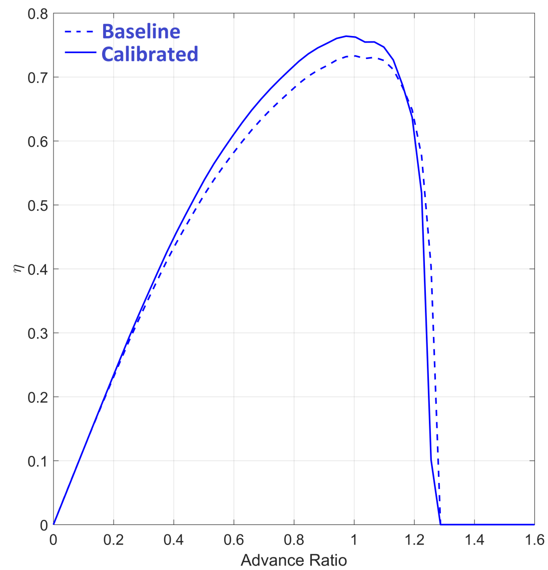
2. Aerodynamics

A variety of aerodynamic modeling methods for a fixed-wing aircraft exists ranging from first order approximations to high-fidelity computational fluid dynamics (CFD) tools. Among the existing aerodynamic performance prediction methods, a theoretical physics-based modeling method is the most appropriate method for developing an aerodynamic model for a fixed-wing GA aircraft because it only requires a minimal set of information that is publicly available and provided by authoritative and reliable sources. Min et al. reviewed, implemented, and comparatively evaluated some of the most well-known physics-based modeling methods for aerodynamic performance prediction as applied to a representative GA aircraft [15]. The inputs for the aerodynamic model developed by Min et al. are aircraft geometric information from the POH [30], publicly available empirical data [31], and typical operating conditions [30].

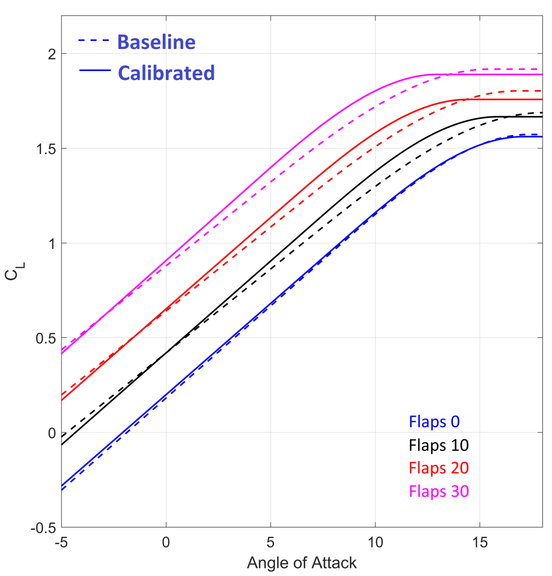
The outputs provided by the developed model are lift curves and drag polars for clean and flap-deployed configurations. A representation of the baseline aerodynamic model is shown in Figure 4(c) and 4(d). Although this aerodynamic model is validated against some reference data published in literature, the main limitation of this modeling method is that it is not always possible to acquire reliable reference data for model validation. Thus, it is necessary to develop a standard model calibration process that can be undertaken with publicly available data in order to add more reliability, flexibility, and repeatability to the suggested aerodynamic modeling method.



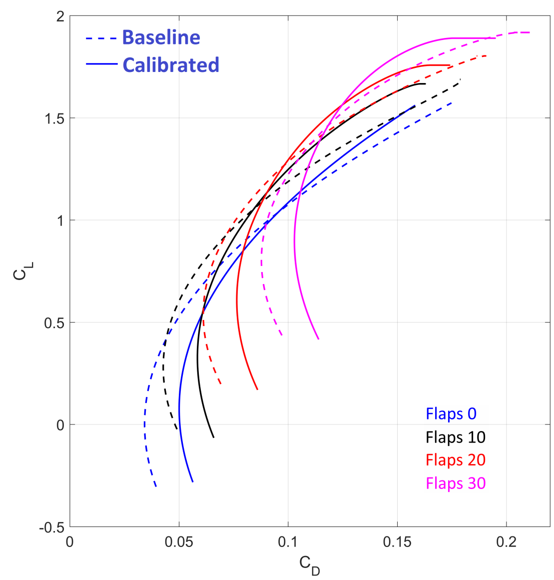
(a) Thrust curve of propulsion model.



(b) Propeller efficiency of propulsion model.



(c) Lift curves for four flap settings.



(d) Drag polars for four flap settings.

Fig. 4 Sample performance model with baseline and calibrated curves.

3. Parameterization

The empirical models available are in the form of curves that can be queried at the flight conditions of interest. In order to calibrate these models, it is necessary to parameterize the curves using appropriate factors to provide flexibility to the calibration process. In the calibration process, these factors are varied to modify the nature of the curves as desired. These *calibration factors* correspond to certain physical characteristics of the aircraft such as lift-curve slope for airfoil, wing, and aircraft, the lift and drag coefficient increments for different flap settings, scaling of intercepts for different curves, sectional airfoil characteristics of the propeller, slope shift of the propeller pitch, scaling of propeller chord, etc. The advantage of parameterizing in this manner is that it allows for an easier setup for optimization and allows modifying individual curves without affecting others.

A total of thirty calibration factors are included in the calibration process and a detailed list of each factor, its description, and range of possible values is included in appendix V. For each calibration factor, judgment is used in determining whether its effect on performance curves should be modeled as being of an additive or multiplicative nature. Similarly, the upper and lower limits are chosen so as to always satisfy physical constraints of the problem.

C. Recorded Flight Data

When available, recorded flight data from actual flights can be used to undertake Level-II of the calibration framework. The flight data obtained from the DFDR are a multivariate time series, whose lengths typically vary between records due to varying duration of flight. The number of parameters varies around 20 (in GA operations) and these parameters are recorded at a specific frequency (e.g., once per 1 second interval). The data consist of parameters related to the state, attitude, basic engine information, environmental conditions, Global Positioning System information, and others. While typical GA aircraft flight data record may not contain a rich set of parameters, the flight record used for calibration is expected to contain a rich set including parameters such as the altitude, true airspeed, vertical speed, engine RPM, outside air temperature, weight of the aircraft (or initial weight and fuel flow rate), position of the flaps, and others. In the current work, data recorded on a Cessna 172 and Piper Archer aircraft equipped with a Garmin G1000 system are used. At least one flight data record is required for performing calibration with the requirement of additional flight data records for validating the calibrated model.

III. Implementation and Results

Each of the two levels of the calibration framework outlined in Figure 2 follows the same stencil with different algorithms, benchmark data, etc. used for each. Figure 5 illustrates the common stencil used in both levels of the calibration process. The model(s) obtained from the process may be used directly for prediction or analysis or passed on to the next level in the framework. In order to modify the performance models in a systematic manner during calibration, they are parameterized using certain variables called calibration factors. A full list of calibration factors along with the performance model they modify and the level during which they are modified is provided in appendix V. The values of the calibration factors are collected in a single vector henceforth called the "calibration vector". Each calibration vector results in the generation of a unique set of performance model curves which can then be validated or tested. In each level, there are four important factors that need to be determined prior to calibration:

1) **Benchmark Data**: The data (either publicly available or collected flight data) used as the *truth* value in the calibration process
2) **Discrepancy Metrics**: The measure of disagreement between the assumed *truth* value and the model prediction, typically expressed as % root mean squared (RMS) error
3) **Calibration Factors**: The factors that are varied during the calibration process to optimize/minimize the discrepancy metric
4) **Optimization Strategy**: The setup (single vs multi-objective) and algorithm used for the optimization problem involved in calibrating the model

The subsequent sections describe the choices made for each of these four factors and the rationale behind these choices for Level-I and Level-II calibration. Figure 5 also illustrates how these factors fit into the overall calibration framework stencil for each level.

A. Level-I Calibration

1. Benchmark Data

As described previously, empirical models for aerodynamics and propulsion can be obtained independently and used in making predictions. However, due to the uncertainties noted, there might be context-specific information that precludes their efficient use in safety analysis. Therefore, it is preferable to simultaneously calibrate both aerodynamics and propulsion models. The benchmark data typically used for this level is publicly available information from manuals such as the POH. The POH lists important information regarding the design, operations, and limitations of the aircraft, as well as its performance characteristics. Although the POH performance tables are for a brand new aircraft under ideal conditions, they are nevertheless valuable for calibrating performance models. The POH contains tables that list the aircraft performance at various operating conditions and phases of flight. While some variation in the format of data presented is expected between different handbooks, most have some common performance tables, such as those

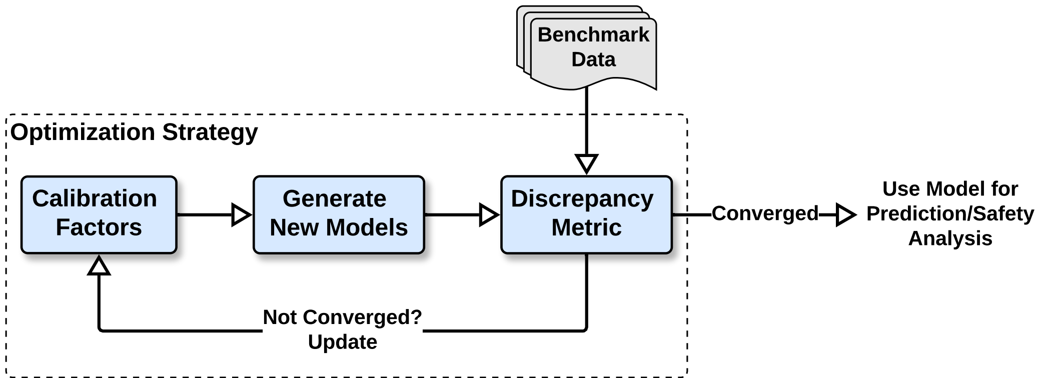


Fig. 5 Information flow in calibration of aircraft performance models for both levels of calibration

related to rate of climb, cruise performance, stall speeds, etc. In addition, the POH typically also identifies best glide speeds, speed for best rate of climb, etc. All of these conditions can be used to calibrate the performance models. For the purpose of this work, the conditions specified in Appendix V have been utilized. The aircraft under consideration are the Cessna 172 and the Piper Archer.

2. Discrepancy Metric

The performance tables/condition in some cases have multiple data points for comparison (such as cruise, rates of climb) and in other cases have a single scalar value to compare (glide angle, best glide speed). Therefore, the RMS error metric is computed for conditions having multiple calibration points so that a single value of error metric is obtained for each performance condition. Each phase of flight outlined in Table 1 will result in a single scalar error which will be combined to yield a vector.

Table 1 Various conditions from POH used in model calibration

Calibration Factors Utilized From	Phase of Flight/ Condition	Discrepancy Metric
Engine, Propeller	Cruise RPM Imbalance	$\frac{RPM_{pred} - RPM_{POH}}{RPM_{POH}}$
Aerodynamics, Propeller	Cruise Thrust-Drag Imbalance	$\frac{T-D}{T}$
Aerodynamics	Glide Angle	$\frac{FPA_{pred} - FPA_{POH}}{FPA_{POH}}$
Aerodynamics	Best Glide Speed	$\frac{Vbg_{pred} - Vbg_{POH}}{Vbg_{POH}}$

3. Calibration Factors

A full list of calibration factors is provided in Appendix V. The POH for the Cessna 172 (and many other similar aircraft) typically have very limited performance data for flapped configurations. Therefore, the calibration factors which modify the curves in these conditions cannot be tuned in Level-I calibration due to lack of suitable benchmark data for these conditions. The third column in the appendix V indicates the calibration factors used in Level-I calibration with a check mark.

4. Optimization Strategy

While the perfect model would ideally simultaneously minimize the error for all POH conditions, due to the epistemic uncertainty in the models it is expected that different phases of flight will have different errors. One of the most likely reasons for this is the fact that the models themselves have different fidelity in different operating conditions. For example, a drag polar from empirical build-up methods for clean configuration is expected to be more accurate than that for a configuration with flaps deployed, due to the inherent additional flow complexity (such as non-linearity, flow separation, sensitivity to flap geometry, etc.). Another possible reason is that there might be more calibration data for some conditions than others, leading to better predictions for certain phases of flight.

Since each phase of flight or POH condition will yield a different error metric, the calibration will be a multi-objective problem. A multi-objective optimization algorithm is therefore used for calibrating the performance models to the POH performance tables which can then enable choosing models as suitable points on a pareto front. Similar applications have been used in other contexts such as aircraft subsystem architecture optimization [32]. The well known algorithm NSGA-II is utilized for multi-objective optimization [33]. The implementation of NSGA-II within MATLAB is utilized to find the Pareto front of calibration vectors. Any model from this front is Pareto-optimal and may be chosen for the purposes of the application here. It is noted that the calibration up to the end of Level-I and generation of Pareto-optimal models is implemented without the use of a single flight data record. This is inherently valuable as detailed recording capability may not be readily available for all GA aircraft, even though performance predictions and safety assessments are nonetheless desired. In such cases, models calibrated using Level-I of the framework will prove to be useful.

5. Results

Figures 6 and 7 shows the distribution of error metrics outlined earlier for points on the obtained Pareto front using optimization for each of the two aircraft. As is evident from the figure, due to the limitations of model structure, there is no single point that minimizes all errors. The points depicted in the figure represent a Pareto-optimal set of models from which a specific model can be selected. In such situations, it is difficult to make a choice for the appropriate model and in the absence of further information, multiple models may be used to make predictions.

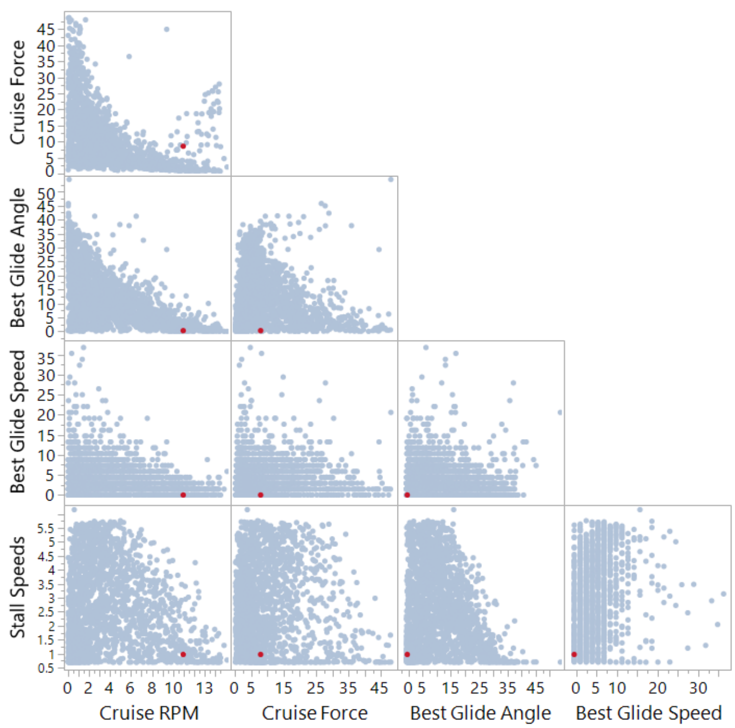


Fig. 6 Scatter of error metrics for Pareto-optimal Cessna 172 models obtained after Level-I calibration

For a chosen model, such as the ones highlighted in red in the figure, some amount of additional information is required for basic validation. This is usually in the form of flight data records with some parameters recorded. Since the quantities being estimated using the performance models are not recorded, even in the higher end of GA flight data recorders such as Garmin G1000, a suitable metric needs to be chosen for validation. The specific total energy rate of the aircraft, an energy-based metric used frequently in performance studies [3], can be evaluated from recorded flight data and also independently evaluated using performance models. Therefore, the value of this metric is evaluated at each point in a validation flight using the data and the models. The error obtained at each point is then visualized and

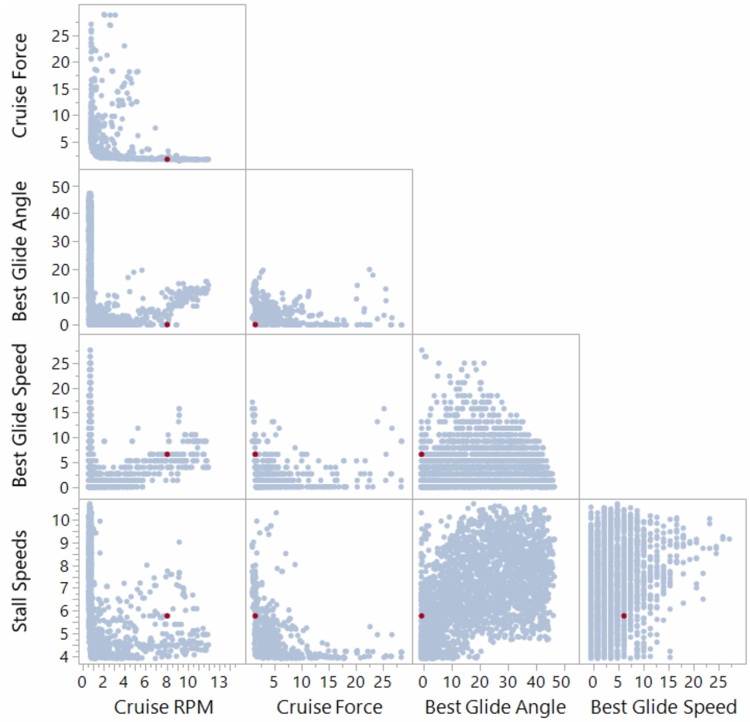


Fig. 7 Scatter of error metrics for Pareto-optimal Piper Archer models obtained after Level-I calibration

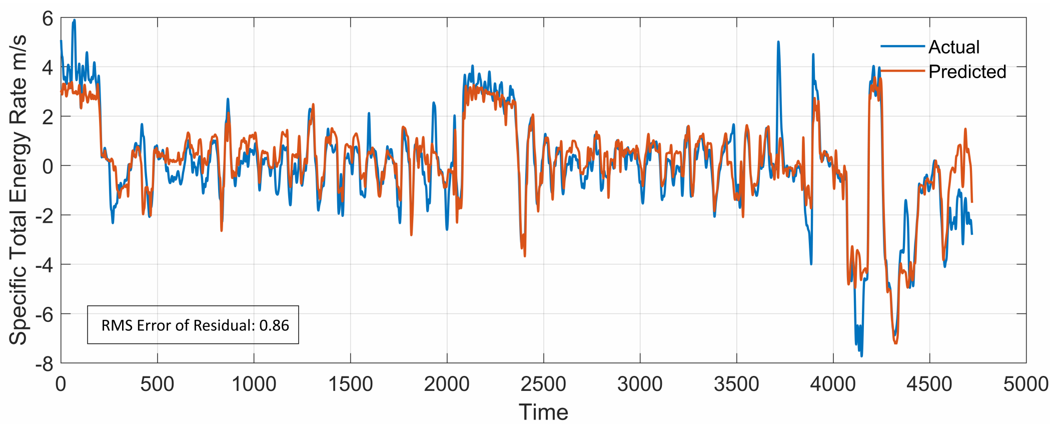


Fig. 8 Variation of specific total energy rate metric: actual and predicted (using level-I calibration) for Cessna 172

can be aggregated over the entire flight data record in the form of the root mean square error. Figures 8 and 9 show the variation of this metric for two flight data records (one each from the Cessna 172 and Piper Archer). As is evident from the figures, there is a good overall agreement between the actual value of the metric as measured in the flight data and the predicted value obtained using the calibrated performance models. The main advantage of using Level-I calibrated models is that when additional flight data is available that is not as rich in the set of recorded parameters, the calibrated models can be used to provide predictions for energy metrics of interest to be used in retrospective safety analysis. Similarly, in the event that sensitive flight data records cannot be shared between entities for various reasons, calibrated models that are validated on such flight records can be transferred between different users for retrospective and predictive safety analysis.

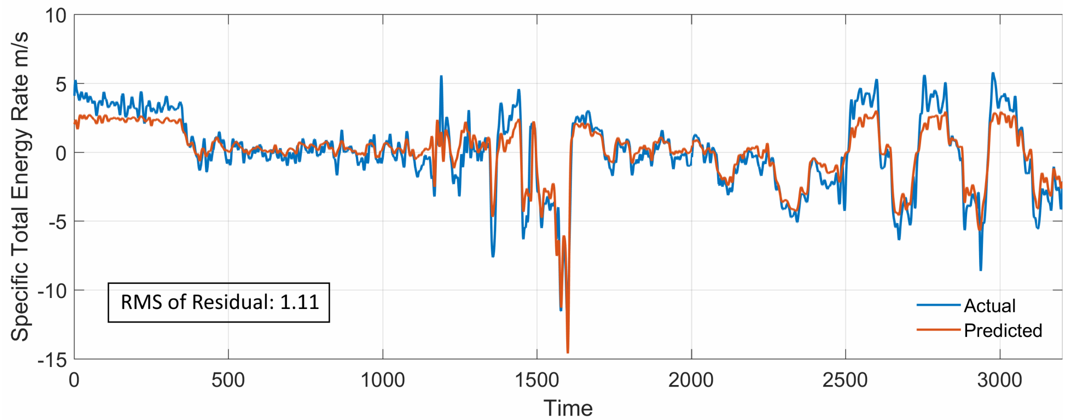


Fig. 9 Variation of specific total energy rate metric: actual and predicted (using level-I calibration) for Piper Archer

B. Level-II Calibration

In the Level-I calibration, only a limited number of flight conditions are available. This does not allow the calibration of all factors, but only those for which the POH provides directly relevant performance data. Most notably, POH performance data pertaining to aerodynamically dirty configurations is sparse, and thus calibration factors associated with the aerodynamic characteristics for such configurations cannot be tuned in the calibration process. Despite calibration to POH, models are expected to suffer from uncertainties. For example, Longmuir et al. [34] have demonstrated that something as simple as unwashed exteriors on commercial aircraft can increase drag up to 1%. GA aircraft are known to have many more deformities, non-flush sitting doors, protuberances, etc. that can cause significant increases in drag. Similarly, higher surface roughness can lead to degraded aerodynamic performance. Therefore, the second level of the model calibration framework seeks to improve model predictions using a limited amount of actual flight data.

1. Benchmark Data

The benchmark data for Level-II calibration come from a limited number of flights (from project partners) with certain key additional parameters recorded. This data may be obtained from a G1000-type system. An important piece of information assumed to be available is the position of the flaps at each point during the flight. This type of flight data is not required for all flights on which the models will be deployed, but only for a handful of flights whose data records will be used for model calibration. If the system cannot directly record flap position, then flap activity for such flights may also be manually logged. It should be specifically noted that flight data for flapped configurations allows the development of the Level-II calibration method.

2. Discrepancy Metric

In the case of Level-II calibration, a metric to measure the discrepancy is sought which can be independently evaluated from the calibration flight data and aircraft performance models. From previous work on energy-based metrics [2, 3], various metrics which are useful for retrospective safety analysis were identified. Additionally, the data required to compute each metric was also outlined. It is noted here that the specific total energy rate of the aircraft is a metric that can be evaluated using flight data as well as basic aircraft performance models. This metric is widely used in performance analysis [35–37], flight training [38], cockpit displays [39], etc. Because of these characteristics and the fact that this metric can be evaluated using available data, it is utilized for calibration of the performance models. For reference, it defined as:

$$STER = \underbrace{\frac{(T-D)\,V}{W}}_{\text{From performance models}} = \underbrace{\dot{h} + \frac{V \times \dot{V}}{g}}_{\text{From flight data}} \tag{1}$$

The discrepancy of this metric at each point in the calibration flight can be evaluated and aggregated over the duration of the entire flight using the RMS error. Therefore, for Level-II calibration, a single scalar metric is obtained for use in optimization.

3. Calibration Factors

Unlike Level-1 calibration using POH data, where the calibration factors are restricted to those for which test conditions exist in the manual, Level-II calibration can use the entire set of calibration factors (assuming that the flaps have been used at least once in the calibration flights). Therefore, all thirty calibration factors can be varied within their range in the Level-II calibration. While this increases the dimensionality of the space being explored, it enables sweeping a larger possible spread of performance curves.

4. Optimization Strategy

Unlike Level-I calibration, the Level-II calibration setup uses only a single discrepancy metric. This metric is calculated at each point in the flight data record(s) available for calibration. Therefore the root mean square (RMS) of this discrepancy over the entire flight is calculated and used. In case multiple flights are available for calibration, the RMS over all the flights can be used. Since this is an indirect calibration process, an evolutionary optimization algorithm is chosen. Additionally, as the calibration flight will only have limited flight conditions at which to test the discrepancy, there may be multiple local minima. Therefore, a genetic algorithm is used to ensure that the best possible calibration model is obtained.

Regarding implementation, the initial starting point of the algorithm can be chosen randomly within the ranges of calibration factors or using information from prior efforts. In particular, the pareto-optimal set of calibration vectors obtained from Level-I calibration can be used to "warm start" optimization in Level-II calibration. This may yield faster algorithm convergence and provide better results. The implementation of genetic algorithm in MATLAB is utilized for this purpose*.

5. Results

One of the advantages of Level-I (POH) calibration is that it does not require any flight data records. The resulting calibrated aircraft performance models obtained from this level are shown to have good predictive capability. Therefore, for Level-II calibration, it is useful to start the optimization from the POH-calibrated models obtained from Level-I calibration. This can accelerate the convergence of the calibration and produce models that have improved performance over the models from Level-I calibration. Starting from a Level-I calibrated model that performs well on the validation flight data record is advantageous, as it ensures that the predictive capability of the model is already good. An important difference noted previously is that the quantities being predicted by each model are not individually recorded in the flight data. Rather an energy metric that uses the difference between predictions from the two models is used. Therefore, simultaneously varying both models that are already providing good predictions might cause the optimization to move away from the minimum. Therefore, in Level-II calibration approach, the aerodynamics and propulsion models are varied one at a time and the optimization is performed in an iterative manner until convergence. The calibration factors are appropriately separated into aerodynamics and propulsion specific factors and inactive factors are held to their Level-I optima or optima from the previous iteration.

The results obtained from Level-II calibration consist of a single performance model rather than a Pareto-optimal set as was the case with Level-I calibration. Therefore, the single model obtained for each aircraft can be used to predict the previously identified energy metric for the validation flight data records. Figures 10 and 11 show the data traces of the STER metric for the same two validation flights shown earlier in Level-I calibration results. As is evident from the figures, Level-II calibration improves the agreement between actual and predicted values of the metrics for both the aircraft.

The overall effect of the calibration process on the performance model itself can be observed in Figure 4. It is observed from comparison between the baseline and calibrated curves that the fundamental shape of the curves is maintained through the calibration process. The changes to the models consist primarily of translations and local scaling of the underlying models. Yet, the improvements observed in the various metrics of interest suggest that while the modifications to the model were straightforward, their impact on the final outcome of the models can be quite pronounced.

IV. Application of Calibrated Models to Safety Analysis

The performance models thus calibrated can be utilized in a number of ways for safety analysis. For some analyses, these calibrated models are indispensable, while for others they increase the value and insights obtained. In general,

*https://www.mathworks.com/discovery/genetic-algorithm.html

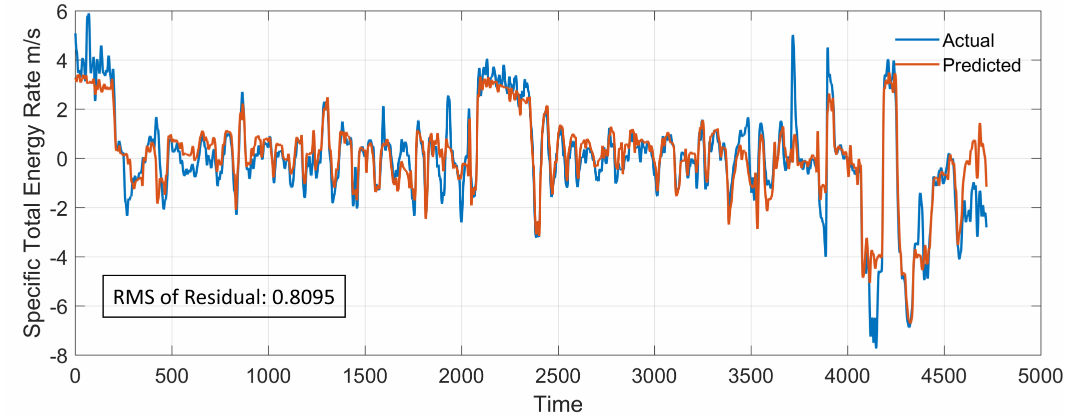


Fig. 10 Variation of specific total energy rate metric: actual and predicted (using level-II calibration) for Cessna 172

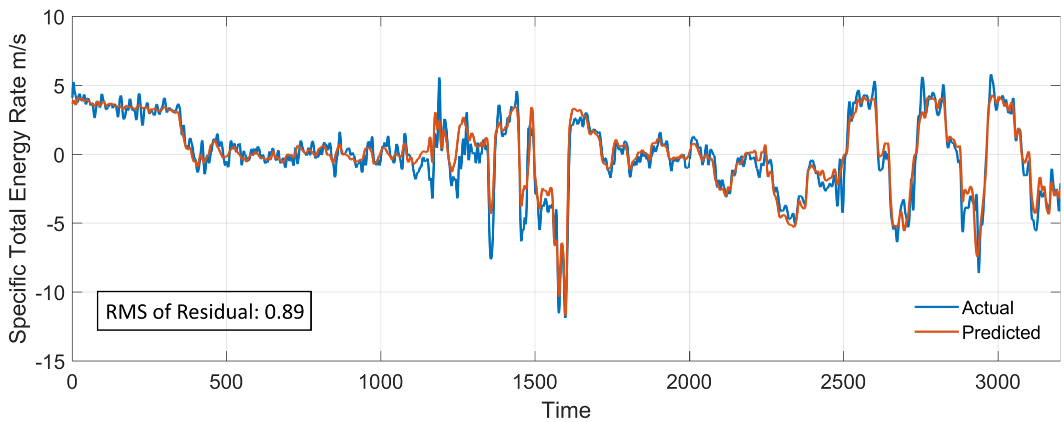


Fig. 11 Variation of specific total energy rate metric: actual and predicted (using level-II calibration) for Piper Archer

lack of information is sometimes a major hurdle in performing accident analyses, and methods to obtain richer data from flight operations are sought [40]. In this section, some methods of utilizing the predictions of performance models are identified and an application is demonstrated for illustrative purposes.

It is identified by the authors in previous work [2, 3] that various energy-based metrics are useful for retrospective safety analysis, particularly in critical phases of flight such as take-off and approach and landing. Energy management and energy state awareness are critical for the characterization, detection, and prevention of safety-critical conditions. Auguste has identified in their work that there is a lack of understanding and recognition of low energy states by pilots of GA aircraft [41]. A number of energy metrics require an accurate aircraft point-performance model to evaluate them. An example of a critical energy metric is presented earlier in equation 1 and used in the level-II calibration as well. The availability of calibrated performance models increases the applicability of energy metrics and enables estimation of energy metrics of interest despite uncertainty present due to possible unknown take-off weight of the aircraft.

In addition to metrics, estimation of the configuration of the aircraft (positions of flaps, etc.) is another factor important in energy management during approach and landing. For example, Louisy [42] tested a PA-28-161 Piper Warrior aircraft to develop an understanding of the effects of altitude and flap configuration on the ability of the aircraft to change its energy state. This is critical for developing an energy management system which is applicable to GA aircraft that can alert the pilot in situations of low energy conditions and recommends to the pilot the appropriate corrective action to restore conditions to a safe energy state.

In GA aircraft equipped with state-of-the-art data acquisition systems (e.g., Garmin G1000) may still lack the flap position among one of the logged parameters. Yet, flap activity is often a factor of interest for appropriate energy management as shown by Louisy [42]. Therefore, the ability to infer the flap position from flight data records (if not directly logged) with reasonable accuracy is desirable. Flap position may be estimated using point performance models in conjunction with energy metrics. Using calibrated performance models, the accuracy of this estimation capability can be further enhanced. For example, in Figures 12 and 13, the flap position is inferred using performance models obtained from Level-I and Level-II calibration for the Cessna 172 and Piper Archer aircraft respectively.

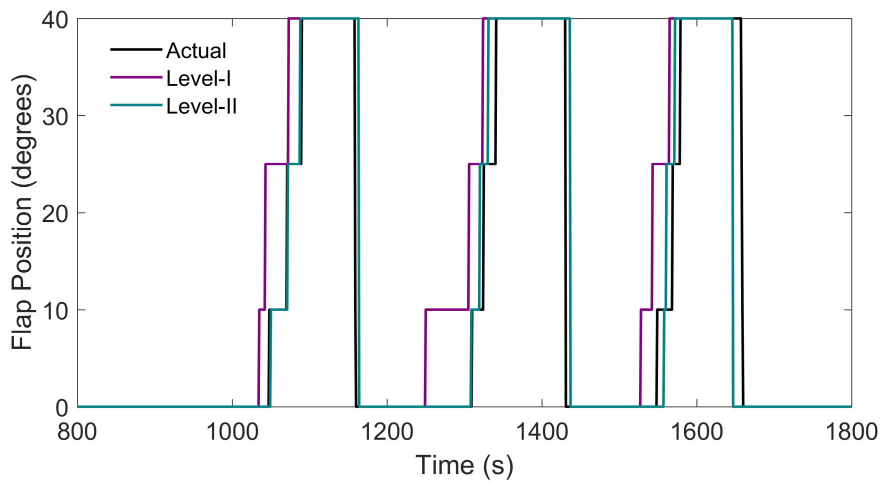


Fig. 12 Comparison of actual and predicted flap position using calibrated models for Cessna 172 aircraft

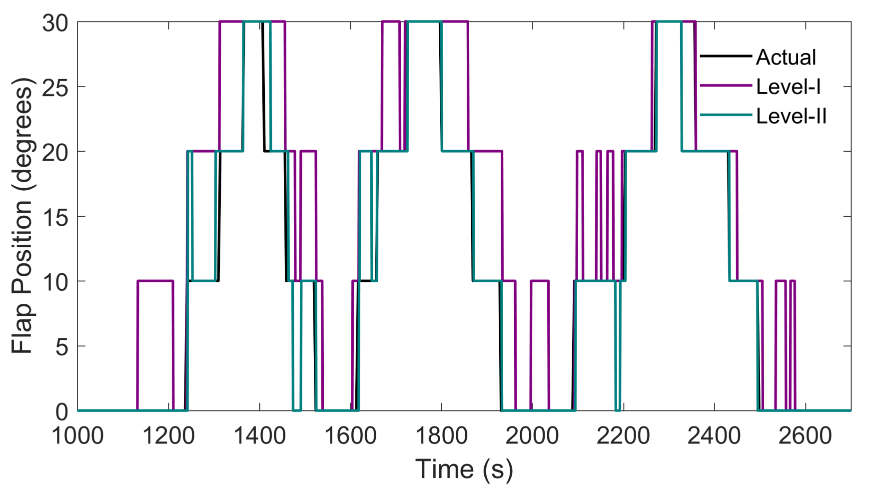


Fig. 13 Comparison of actual and predicted flap position using calibrated models for Piper Archer aircraft

Figure 12 shows the actual and predicted flap position using Level-I and Level-II calibrated models for the Cessna 172 aircraft. The actual flap activity comprised three complete flap cycles performed at three different airspeed. In each flap cycle, the aircraft starts in clean configuration, following which flaps are progressively deployed (detent by detent) and then progressively retracted, finishing in clean configuration once again. The predicted position using both Level-I and Level-II calibrated models is highly accurate in regions where the actual flap position is zero. For sections with non-zero flap positions, Level-I calibrated models do a reasonable job with some offset errors whereas Level-II calibrated models perform well and adhere to the actual variation of the flap position trace. Similarly, in Figure 13 the actual and predicted flap position using Level-I and Level-II calibrated models for the Piper Archer aircraft is shown. For this flight, the flap cycle is slightly more complicated than the Cessna aircraft and the Level-I model results in a few false positives (flap predicted non-zero when it is actually zero). The Level-II model performs much better for this aircraft as well indicating the advantage offered by calibration of models in flap activity prediction. Nevertheless, it is evident from both the figures that there is a good overall agreement between the actual and predicted flap position using both levels of calibrated models. As expected, the predictions made using Level-II calibrated models have a much closer match of flap activity than those made with Level-I calibrated models. Making continuous improvements in the model predictions thus provides improved estimates for the flap position for retrospective safety analysis as well as better in-flight energy management. This application represents one of ways in which calibrated performance models may be utilized for safety analysis of GA operations.

V. Conclusion

In this paper, a novel two-level framework for calibrating performance models of GA aircraft is demonstrated. The main motivation behind the development of the framework is the ability to have a single consistent method to calibrate performance models of multiple GA aircraft starting from the same generic baseline model. Since different types of data are available depending on the context, the calibration framework works on multiple levels in order to utilize the data in the best way possible. The improvement in model predictions for both levels of the framework are measured in terms of its ability to predict a frequently used energy metric for safety analysis - the specific total energy rate.

The first level of the framework provides the ability to obtain models with improved accuracy in STER prediction without the need for any actual flight data during the calibration process. The STER metric residual RMS for the calibrated Cessna 172 aircraft over an entire validation flight is 0.86 whereas for the calibrated Piper Archer aircraft is 1.11. For both these validation flights, the trace of the predicted metrics follows the actual trace very closely in all phases of the flight (Figures 8 and 9). The second level of the framework provides the ability to obtain models with improved predictions over the level-I models by utilizing actual annotated flight data records during calibration. The STER metric residual RMS for the level-II calibrated Cessna 172 over the entire validation flight is 0.81 whereas for the level-II calibrated Piper Archer aircraft is 0.89. Similar to level-I calibration, the traces of the metrics (Figures 10 and 11) show improved agreement with the actual trace of the metric. The framework developed in this work is widely applicable and is demonstrated on two common and representative single-engine naturally-aspirated General Aviation aircraft. The demonstrated approach results in an easily-repeatable process that can be used to calibrate models for a variety of similar aircraft in order to perform retrospective and predictive safety analyses. Lack of data is often a major hindrance in retrospective safety analysis for GA applications. This work alleviates that problem to some extent by enabling estimation of quantities of interest that are not directly recorded in GA flight data recorders.

An example application of the calibrated models for safety analysis is the automatic identification of flap position using kinematic flight data and calibrated performance models. Using the calibrated models (both level-I and level-II) developed in this work a good agreement between actual and predicted flap position is obtained for the demonstrated flight data records from both representative aircraft.

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Appendix: Description of Calibration Factors

Table 2 Description of Calibration Factors in Models

Model	Factor Description	Level-I Calibration	Level-II Calibration
Engine	Engine de-rate factor	✓	✓
Propeller	Vertical shift of airfoil sectional lift curve	✓	✓
Propeller	Scaling of airfoil sectional lift curve slope	✓	✓
Propeller	Scaling of airfoil sectional minimum drag	✓	✓
Propeller	Scaling of airfoil sectional quadratic parameters	✓	✓
Propeller	Scaling of airfoil sectional lift at min. drag	✓	✓
Propeller	Slope shift of propeller pitch	✓	✓
Propeller	Translational shift of propeller pitch	✓	✓
Propeller	Scaling of propeller chord	✓	✓
Aero.	Scaling of maximum lift coefficient for flap 0	✓	✓
Aero.	Scaling of maximum lift coefficient for flap 10	✓	✓
Aero.	Scaling of maximum lift coefficient for flap 20	✓	✓
Aero.	Scaling of maximum lift coefficient for flap 30	✓	✓
Aero.	Scaling of parasite drag factor for clean configuration	✓	✓
Aero.	Scaling of induced drag factor for clean configuration	✓	✓
Aero.	Drag polar shifting factor ($C_{D,minL}$) for clean configuration	✓	✓
Aero.	Scaling of parasite drag increment factor for flap 10 setting	– –	✓
Aero.	Scaling of induced drag increment factor for flap 10 setting	– –	✓
Aero.	Scaling of parasite drag increment factor for flap 20 setting	– –	✓
Aero.	Scaling of induced drag increment factor for flap 20 setting	– –	✓
Aero.	Scaling of parasite drag increment factor for flap 30 setting	– –	✓
Aero.	Scaling of induced drag increment factor for flap 30 setting	– –	✓
Aero.	Scaling of lift-curve slope for flap 0 setting	✓	✓
Aero.	Scaling of lift-curve slope for flap 10 setting	✓	✓
Aero.	Scaling of lift-curve slope for flap 20 setting	✓	✓
Aero.	Scaling of lift-curve slope for flap 30 setting	✓	✓
Aero.	Scaling of lift-curve intercept for flap 0 setting	✓	✓
Aero.	Scaling of lift-curve intercept for flap 10 setting	✓	✓
Aero.	Scaling of lift-curve intercept for flap 20 setting	✓	✓
Aero.	Scaling of lift-curve intercept for flap 30 setting	✓	✓

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