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November 5, 1992

Dr. Albert B. Harvey
Program Director
Lightwave Technology
National Science Foundation
1800 G Street, N.W.
Washington, D.C. 20550

Dear Dr. Harvey:

Enclosed is the first-year progress report on my Research Initiation Award, along with an associated publication. I trust the report will meet with your satisfaction. I am, of course, at your service to elaborate or answer any questions about the research.

I would also like to note that I received the SBIR Phase II proposal you sent me, and that I will finish reviewing it very soon.

I remain quite grateful to the National Science Foundation for this support.

Sincerely,

Dr. Mary Ann Ingram
Assistant Professor
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mingram@ee.gatech.edu

Enclosures: Paper - 2 copies

Research Initiation Award Progress Report
For 1991-1992

Prepared by
Dr. Mary Ann Ingram
Georgia Institute of Technology

1 General

The objective of my RIA proposal was to determine the effect of polarization mode dispersion (PMD) on coherent receivers. We have made steady progress toward this goal in the first year. The larger goal of my research program has been to analyze transmission quality in the presence of new optical devices that may induce noise, dispersion or nonlinearities into the transmission link. In this context, the RIA proposal was motivated by the erbium-doped fiber amplifier (EDFA), because only in the very long optical links made possible by the EDFA, does PMD accumulate to appreciable levels. The RIA has also facilitated other research pertaining to the larger goal. This report describes the RIA-supported research, along with a brief description of my other projects in order to show the general good health of my research program.

The RIA has enabled me to change research areas from general problems in estimation, detection and signal processing to the applied area of optical communications. My research program is gaining momentum; the original graduate students, two of which are funded by the RIA, are producing meaningful results. We have presented our first conference papers this year and have published our first journal article in optical communications. The RIA-funded trip to the Optical Fiber Communications (OFC) Conference was very important for stimulating ideas and gaining contacts with other researchers and potential sponsors. We have begun contract development based on our first publications. I have also recently gained three more graduate students. The work with these new students represents new directions in optical communications and continued momentum.

In the next year, the priorities of the overall research program will be contract development and publication. The contract development is necessary because at least one of the two RIA-supported students and one of the new students will require support after this coming year. The RIA support continues to be very important as we build a reputation and court sponsors.

2 RIA Topic: Effects of PMD on Coherent Receivers

There are several different ways that coherent receivers can handle the polarization matching problem. Following the proposal, we started with what is probably the most popular choice, polarization diversity. We picked the simplest modulation scheme: amplitude shift keying (ASK). We studied the noncoherent demodulation scheme of two-branch phase diversity and we considered both square-law detection and envelope detection. Our efforts resulted in a paper that was presented at the SPIE Multigigabit Communications Conference, held in Boston in September. This paper is included with this report. We intend to improve on this paper and submit it for journal publication.

The conference paper is summarized as follows. The receiver is assumed to be subjected to transmitter and receiver polarization misalignment relative to the principal states of the optical fiber, phase noise, polarization mode dispersion and shot noise. We found that polarization and phase diversity receivers that use square law detection are not affected by PMD. To study the case of envelope detection, the recent numerical techniques of Beaulieu (see refs. 10 and 11 in the paper) were applied to compute the probability of bit error (BER). To our knowledge, no other authors have computed the BER for optical polarization and phase diversity receivers. Lengthy computational times limited our initial study to very low values of SNR. We found that the BER depends on the power distribution among the four branches of the receiver. In the absence of PMD, the power distribution depends on the polarization misalignment and the absolute phase of the carrier. The worst case was observed to be when the misalignment and phase are such that all of the received power is concentrated in one branch of the receiver. When PMD is present, the fiber output is depolarized and hence all of the power cannot be concentrated into one branch. Thus PMD actually lowers the BER relative to the worst case without PMD.

In the course of this study, we discovered that the optimal detection threshold for a polarization and phase diversity receiver strongly depends on the power distribution among the branches of the receiver. We are currently preparing a journal article on this subject. Fortunately, we were able to address more realistic SNR's in this case. An example of our results is

that if the optimal threshold is selected assuming an equal power distribution among the branches, but if the power distribution changes due to changes in the state-of polarization or the absolute phase of the light coming out of the fiber, then a four-branch receiver can suffer a sensitivity penalty of as much as 6 dB and a six branch receiver can suffer about 3 dB. We plan to give recommendations in the paper on how to select the threshold to minimize the sensitivity penalty. We think this paper will be of strong interest to researchers of coherent optical receivers and plan to submit it to either Electronics Letters or Journal of Lightwave Technology, depending on its finished length.

After the threshold study is complete and we make improvements on the conference paper, we will address PMD effects on diversity receivers that use other modulation techniques: DPSK and FSK. Following that, we will begin studying how PMD affects polarization controllers. I already have an undergraduate student creating some software that we will need for the polarization controller part of this effort. The final topic that we proposed is how PMD affects polarization demodulation. This part of the study is not expected to take very long.

3 Other Research in Optical Communications

This section describes ongoing research projects that are either evolved from the RIA topic or are not related to the RIA topic. I have included this section to show that my overall research program is sufficiently diversified and forward-looking.

A spin-off of the RIA effort has been to determine the best way to twist and helically wind a fiber (such as when creating an optical cable) so as to minimize PMD. I am pursuing funding from AT&T to complete the helical wind study.

A natural application of the helical wind study is to fiber-optic guided missiles (FOG-M). We wrote a white paper to the Army on coherent transmission for the FOG-M. They liked the white paper enough to invite us for a discussion in mid November.

The EDFA has also motivated another area of investigation. It seems

quite likely that EDFA's will be omnipresent in future optical networks. The statistical model of a network will therefore include noise sources distributed among switches, control nodes, multiplexers, demultiplexers, and other devices yet to be imagined. Several questions arise: 1) Will the statistics of the EDFA noise arriving at a terminating node be significantly different from that of a simple cascade of amplifiers in a single link?, 2) Will the noise have an effect on the *operation* of the switches, demultiplexers, etc. within the network, and 3) Will any devices actually suppress the EDFA noise? Until these questions occurred to me this year, I had confined my work to single transmission links. Now I am learning all about optical networks, from switching devices to network management schemes, along with two of my graduate students working in this area. These efforts are also leading to collaborative efforts with other faculty at Georgia Tech who build optoelectronic devices. For example, one of my students is analyzing the effect of EDFA noise on an optical serial-to-parallel converter that uses second harmonic generation, while the device is being built and tested by the other faculty member.

The other effort in optical communications involves post-detection processing of wavelength division multiplexed (WDM) digital signals to suppress crosstalk. The student doing most of this work is a former colleague from my radar signal processing days at the Georgia Tech Research Institute. This work is an example of the kind of unique contribution that people with our backgrounds can bring to the area of optical communications. We have modified an algorithm that is used to suppress noise jamming in an adaptive antenna array to cancel WDM crosstalk in photodetector currents that come from a grating demultiplexer and photodetector array. The result is that the capacity of a WDM network can be more than doubled if just a few photodetectors are employed in each receiver. We presented our initial results at the OFC last February, and had a paper appear in *Electronics Letters* in August. These articles assume that the channels are close enough to cause linear crosstalk, but not close enough to cause nonlinear crosstalk (i.e. not spaced on the order of the electronic bandwidth of the receiver). We are currently examining the nonlinear crosstalk case, the effects of noise from an EDFA preamplifier, and the case where different channels use different bit-rates. We will consider the use of a neural network to cancel nonlinear crosstalk. Our immediate goal is to submit a more complete paper to the *Journal of Lightwave Technology* by January.

The final two projects to be described are both military applications of

estimation and detection theory. The first is detection of weak optical and infrared (IR) targets, such as a missile, in a heavy, nonhomogeneous and nonstationary optical clutter background. This work is being sponsored by the Naval Weapons Center. The student doing this work has identified a new model for the IR sensor noise, and is currently addressing the problem of online calibration to the nonuniformity of IR sensors in an array. Other goals on this project are to statistically characterize the clutter in real data supplied by the Naval Weapons Center, and to compare nonparametric detection techniques to detection techniques based on the wavelet transform in the presence of jitter. Since the wavelet transform can offer both temporal and spatial information, we think it may be useful to detect jitter or motion between image frames.

The other detection and estimation project, which has just begun, is the application of the time-frequency and wavelet transforms to linearly-constrained wideband adaptive arrays. A linearly-constrained adaptive array is one which has fixed temporal responses in the directions of friendly transmitters but is otherwise free to reduce antenna gains in the directions of unfriendly transmitters. The research is sponsored by DoD labs for the development of the next generation of Electronic Counter-Counter Measures (ECCM) for space-based sensors. The major contribution of this work will be improvement of the transient response of the array processor in stationary and non-stationary environments while simultaneously satisfying a requirement for limited computational resources.

The last two projects described both involve the wavelet transform. I have recently begun to investigate potential applications of the wavelet transform in optical communications. This is motivated by the success of the wavelet transform in the processing of non-stationary signals and by the fact that the wavelet transform has recently been shown to give very good approximations to fiber-dispersed versions of arbitrary transmitted pulse shapes, both with and without chirping (L.R. Watkins and Y.R. Zhou, "Modelling of optical waveforms using wavelets," Submitted to Optics Letters). One potential research problem, for example, is pulse equalization schemes that are based on the wavelet expansion of a dispersed pulse.

4 Conclusions

I am excited about our group's research. The work on the RIA topic is progressing well and has yielded interesting results. We anticipate no problems in completing the proposed research. We are also getting some meaningful results in the other research projects and we have identified some fertile ground for future research, namely noise in optical networks and applications of the wavelet transform to optical communications. In the coming year we will place particular emphasis on contract development, mainly to provide for continued support of graduate students. There is no question that the RIA has been and continues to be critical to the launching of my research program.

Polarization mode dispersion effects on phase and polarization diversity receivers

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ABSTRACT

The objective of this paper is to examine the combined effects of weak phase noise and polarization mode dispersion (PMD) on a coherent receiver employing phase and polarization diversity reception. The receiver is assumed to be subjected to the following: transmitter and receiver polarization misalignment relative to the principal states of the optical fiber, phase noise, polarization mode dispersion, and shot noise. The receiver outputs are investigated for ASK demodulation using square-law and envelope detection. The results show that for the assumed receiver configuration, square law detection provides an output which is independent of PMD, phase noise, and polarization misalignment. Envelope detection results in a receiver output which is dependent on all of these parameters. Furthermore, when phase noise and PMD are simultaneously present, the resulting probability of bit error is no greater than the probability of bit error under worst-case operating conditions when polarization mode dispersion and phase noise are absent.

1. INTRODUCTION

Interest in the effects of polarization mode dispersion (PMD) on optical fiber communication systems is steadily increasing as the prospect of PMD-limited communications becomes more plausible. A significant amount of work has been devoted to characterizing PMD^{1, 2}, and its band limiting effect on direct detection systems³. However, very little work has addressed the effects of PMD on coherent receivers. The simultaneous presence of both phase noise and PMD results in a depolarization of the lightwave presented to the receiver. Aside from an inherent pulse spreading, depolarization is of no real consequence to direct detection systems since these systems detect incident power. However, the effects of depolarization on coherent communication systems may be more serious and have not yet been thoroughly investigated. Our effort investigates the effects of PMD in concert with phase noise on coherent processing components which are polarization sensitive. Specifically, we study these effects on a combined polarization and phase diversity receiver, a structure considered to be a viable solution to both the phase noise and polarization matching problems faced by coherent communication systems⁴. Figure 1 depicts the polarization diversity receiver structure assumed for the analysis. The receiver consists of two orthogonally polarized processing channels, each with a phase diversity receiver, where the phase diversity receivers employ either square-law or envelope demodulation.

PMD is defined as the differential group delay between field components that are aligned with the output principal polarization states of a length of optical fiber. In the presence of phase noise, PMD induces a time varying differential phase between field components at the output of the fiber. This leads to a reduction in coherence between field components and causes the polarization of the output signal to fluctuate along an arc on the Poincaré sphere. The rate of fluctuation is slow relative to typical bit rates in high speed systems. Our approach to the investigation of depolarization was to divide the depolarization arc into a discrete set of purely polarized states, and to evaluate the probability of bit error for each of these discrete states. We assume that appropriate averaging over these discrete states will approximate the bit error rate for a truly fluctuating differential phase.

The paper is organized as follows. In the next section we derive expressions for the receiver output for both configurations. We restrict the analysis to the case of continuous wave (CW) transmission (i.e., the transmission of a "1" bit in ASK), and neglect the effects of intersymbol interference since we are interested in effects other than pulse spreading. Section III discusses the analysis of the receiver in the presence of shot noise. For square law detection, we find that receiver performance is identical to the performance of the receiver in the absence of PMD. For envelope detection, receiver performance is found to depend on the power distribution to each branch of the receiver. Using numerical techniques, we show the dependence of the probability of bit error on a selection of polarization states that correspond to a depolarized signal. The convergence time of our numerical technique limited the study to low signal-to-noise ratio (SNR) values.

2. THEORY

The transmitted polarization state, the principal states⁵ of the fiber, and the polarization axes of the diversity channels are assumed to be linearly polarized in this investigation. Let us express the complex envelope of the transmitted signal in terms of orthogonal field components, ϵ_x and ϵ_y .

$$\epsilon_x = \sqrt{P_s} \cos(\gamma) e^{j(\phi(t) + \theta)} \quad \epsilon_y = \sqrt{P_s} \sin(\gamma) e^{j(\phi(t) + \theta)} \quad (1)$$

where P_s is the intensity of the transmitted signal, γ is the angle between the polarization of the input signal and the fast axis of the principal states, θ is an arbitrary constant phase term, $\phi(t)$ is the phase noise of the laser, and x and y are aligned with the principal axes of the optical fiber. The x and y components of the signal will travel through a fiber of length L with group velocities v_x and v_y respectively. If the axes of the polarization diversity receiver are at an angle ρ with respect to the principal states of the optical fiber, the received signal at the input to each diversity receiver will be:

$$\epsilon_1 = A_1 e^{j(\phi(t - \frac{L}{v_x}) + \theta)} + B_1 e^{j(\phi(t - \frac{L}{v_y}) + \theta)} \quad (2)$$

$$\epsilon_2 = A_2 e^{j(\phi(t - \frac{L}{v_x}) + \theta)} + B_2 e^{j(\phi(t - \frac{L}{v_y}) + \theta)} \quad (3)$$

where

$$A_1 = \sqrt{P_s} \cos\gamma \cos\rho \quad B_1 = \sqrt{P_s} \sin\gamma \sin\rho \quad (4)$$

$$A_2 = -\sqrt{P_s} \cos\gamma \sin\rho \quad B_2 = \sqrt{P_s} \sin\gamma \cos\rho \quad (5)$$

The LO is assumed to be aligned 45 degrees relative to the axis of the receiver; hence, the LO power is evenly split between the channels of the receiver. The LO signal into each receiver is given by

$$L_1 = L_2 = \sqrt{\frac{P_{LO}}{2}} e^{j\phi_{LO}(t)} \quad (6)$$

Consider one channel of the polarization diversity receiver (PDR), say channel 1 (PDR1), shown in Figure 1. The PDR consists of a hybrid, followed by photodetectors on each branch, ideal low pass filters, demodulators and a summing junction. The inputs to PDR1 are L_1 and ϵ_1 . Upon photodetection, signal a_2 becomes

$$\begin{aligned} a_2 &= R |a_1|^2 + N_s \\ &= R \left[\frac{1}{\sqrt{2}} \left(\epsilon_1 + \sqrt{\frac{P_{LO}}{2}} e^{j\phi_{LO}} \right) \right] \left[\frac{1}{\sqrt{2}} \left(\epsilon_1^* + \sqrt{\frac{P_{LO}}{2}} e^{-j\phi_{LO}} \right) \right] + N_s \\ &= \frac{R}{2} \left[D_1^2 + \frac{P_{LO}}{2} + 2D_1 \sqrt{\frac{P_{LO}}{2}} \cos(d_1) \right] + N_s \end{aligned} \quad (7)$$

where $D_1 = |\epsilon_1|$, $d_1 = \arg(\epsilon_1) - \phi_{LO}$, and where N_s represents shot noise. The shot noise is assumed Gaussian. Since $P_{LO} \gg P_s$ in a coherent system, the power spectral density is flat with height $S_n = qRP_{LO}/4$. The ideal low pass filters

which follow the photodetectors are assumed to suppress the intersymbol interference and pass undistorted all of the signal power within the pass band of the filter. Upon filtering, the noise, n_a , at the output of the filter will have power

$$E[n_a^2] = S_n B = \frac{QR P_{LO} B}{4} \quad (8)$$

since the laser linewidth is assumed to be small in comparison to the bit rate, the signal portion of the current, i_s , will have the form

$$i_s = \frac{R}{2} \left[\frac{P_{LO}}{2} + D_1^2 + 2D_1 \sqrt{\frac{P_{LO}}{2}} \cos(d_1) \right] \quad (9)$$

In (9), the first term is a dc current and can be rejected by coupling capacitors. Since $P_{LO} \gg P_s$, the second term is negligible in comparison to the third term⁶. Therefore, the current of interest is

$$a_3 = RD_1 \sqrt{\frac{P_{LO}}{2}} \cos(d_1) + n_a \quad (10)$$

In a similar way, signal b_3 is given by

$$b_3 = RD_1 \sqrt{\frac{P_{LO}}{2}} \sin(d_1) + n_b \quad (11)$$

The response of the remainder of the circuit depends on the demodulation scheme which is employed. Both envelope detection and square law detection schemes will be examined. For the moment, we shall neglect shot noise to isolate those effects introduced exclusively by PMD. In a later section, the effects of PMD on both detection schemes will be analyzed with shot noise included.

2.1. Square law detectors

Neglecting shot noise, when square law detectors are used, a_4 and b_4 have the form

$$a_4 = |a_3|^2 = \frac{R^2 D_1^2 P_{LO}}{2} \cos^2(d_1) \quad b_4 = |b_3|^2 = \frac{R^2 D_1^2 P_{LO}}{2} \sin^2(d_1) \quad (12)$$

The output of a PDR using square law detectors is $a_4 + b_4$ which becomes:

$$C_1 = a_4 + b_4 = \frac{R^2 D_1^2 P_{LO}}{2} \quad (13)$$

Applying the following identity to D_1^2 ,

$$|X e^{jx} + Y e^{jy}|^2 = X^2 + Y^2 + 2XY \cos(x - y) \quad (14)$$

we find that

$$D_1^2 = A_1^2 + B_1^2 + 2A_1B_1 \cos\left(\phi\left(t - \frac{1}{v_x}\right) - \phi\left(t - \frac{1}{v_y}\right)\right) \quad (15)$$

$$= P_s [\cos^2\gamma \cos^2\rho + \sin^2\gamma \sin^2\rho + 2\cos\gamma\cos\rho\sin\gamma\sin\rho\cos(\Psi_x - \Psi_y)]$$

where

$$\Psi_x = \phi\left(t - \frac{L}{v_x}\right) + \theta + \phi_{LO}(t) \quad \Psi_y = \phi\left(t - \frac{L}{v_y}\right) + \theta + \phi_{LO}(t) \quad (16)$$

So,

$$C_1 = \frac{R^2 P_{LO}}{2} [A_1^2 + B_1^2 + 2A_1B_1 \cos(\Psi_x - \Psi_y)] \quad (17)$$

For PDR2, by symmetry, it is evident that

$$C_2 = \frac{R^2 P_{LO}}{2} [A_2^2 + B_2^2 + 2A_2B_2 \cos(\Psi_x - \Psi_y)] \quad (18)$$

The output of the polarization and phase diversity receiver is given by $C_3 = C_1 + C_2$, or

$$C_3 = \frac{R^2 P_{LO}}{2} [A_1^2 + B_1^2 + A_2^2 + B_2^2 + 2A_1B_1 \cos(\Psi_x - \Psi_y) + 2A_2B_2 \cos(\Psi_x - \Psi_y)] \quad (19)$$

The output expression may be simplified further. Recalling equations (4) and (5), we find that

$$A_1^2 + A_2^2 + B_1^2 + B_2^2 = P_s \quad A_1B_1 + A_2B_2 = 0 \quad (20)$$

Therefore

$$C_3 = \frac{R^2 P_{LO} P_s}{2} \quad (21)$$

which is independent of phase noise, polarization mode dispersion, and the polarization orientation of the system components.

2.2 Envelope detectors

In the case when envelope detectors are employed, we have for PDR1

$$a_4 = |a_3| = \frac{RD_1\sqrt{P_{LO}}}{\sqrt{2}} |\cos(d_1)| \quad b_4 = |b_3| = \frac{RD_1\sqrt{P_{LO}}}{\sqrt{2}} |\sin(d_1)| \quad (22)$$

and

$$C_1 = \frac{RD_1\sqrt{P_{LO}}}{\sqrt{2}} [|\cos(d_1)| + |\sin(d_1)|] = RD_1\sqrt{P_{LO}} \cos(\tau_1) \quad (23)$$

where

$$\tau_1 = \tan^{-1} |\tan(d_1)| - \frac{\pi}{4} \quad (24)$$

By symmetry we find that

$$C_2 = RD_2 \sqrt{P_{LO}} \cos(\tau_2) \quad (25)$$

where

$$\tau_2 = \tan^{-1} |\tan(d_2)| - \frac{\pi}{4} \quad (26)$$

Here, $D_2 = |\epsilon_2|$, and $d_2 = \arg(\epsilon_2) - \phi_{LO}$. The output of the receiver is given by

$$C_3 = C_1 + C_2 = R\sqrt{P_{LO}} [D_1 \cos(\tau_1) + D_2 \cos(\tau_2)] \quad (27)$$

The variables in (27) which depend on PMD are D_1 , D_2 , τ_1 , and τ_2 . When PMD is not present in the system, then

$$D_1 = A_1 + B_1 = \sqrt{P_s} |\cos(\gamma - \rho)| \quad (28)$$

$$D_2 = A_2 + B_2 = \sqrt{P_s} |\sin(\gamma - \rho)| \quad (29)$$

$$d_1 = d_2 \quad (30)$$

and the output of the receiver reduces to

$$C_3 = R\sqrt{P_{LO}P_s} \cos(\tau_1) [|\cos(\gamma - \rho)| + |\sin(\gamma - \rho)|] \quad (31)$$

When PMD is present, an explicit dependence of the output on Ψ_x and Ψ_y can be shown to be:

$$C_3 = R\sqrt{\frac{P_{LO}P_s}{2}} [|\cos\gamma \cos\rho \cos\Psi_x + \sin\gamma \sin\rho \cos\Psi_y| + |\cos\gamma \cos\rho \sin\Psi_x + \sin\gamma \sin\rho \sin\Psi_y| \\ + |-\cos\gamma \sin\rho \cos\Psi_x + \sin\gamma \cos\rho \cos\Psi_y| + |-\cos\gamma \sin\rho \sin\Psi_x + \sin\gamma \cos\rho \sin\Psi_y|]$$

In (32), ψ_x and ψ_y are correlated since Ψ_x is a delayed version of Ψ_y . Let

$$\Psi_D = \Psi_y - \Psi_x = \phi\left(t - \frac{L}{v_y}\right) - \phi\left(t - \frac{L}{v_x}\right) \quad (33)$$

Since $\phi(t)$ is modeled as a Brownian motion, then Ψ_D is a zero mean Gaussian noise process whose standard deviation is parameterized as σ in this study⁷. Smaller standard deviations are interpreted in one of three ways: either as an increase in the coherence time of the transmitter (i.e., a reduction in phase noise), a reduction in PMD, or as a reduction in fiber length.

We compute the probability of bit error conditioned on various values of Ψ_D . Our range of values was determined as follows. In the presence of random polarization mode coupling, PMD increases as the square root of the length of the fiber⁸. Reported results suggest that typical values for PMD range between 0.1 and 2.0 ps/ $\sqrt{\text{km}}$ ^{3,9}. We assume a nominal

value of $1 \text{ ps}/\sqrt{\text{km}}$. Then, for a very long fiber of length 10,000 km, the total group delay difference between polarization modes will be given by

$$|t_1 - t_2| \approx 100 \text{ psec} \quad (34)$$

The standard deviation, σ , of the phase difference Ψ_n between polarization modes for the parameters above may be determined to be near 5 degrees for a laser bandwidth of 10 MHz from the relationship⁷

$$\sigma^2 = 2\pi B_L |t_1 - t_2| \quad (35)$$

We allow Ψ_n to range between -3σ and 3σ , or rather, between -15 and 15 degrees.

3. ANALYSIS WITH SHOT NOISE

3.1 Square law detection

In this section, the effects of shot noise are considered. We proceed by first examining the case of square law detectors. Following reference 4, we first normalize the signal and noise to unity noise variance after photodetection. This can be performed without loss of generality in the bit error analysis since the noise from each channel is identically distributed. Note that the sum of the mean squares of the branches is always given by

$$\sum_{i=1}^4 m_i^2 = R^2 P_{10} P_s \quad (36)$$

This sum is independent of PMD, although the contribution from each branch will generally depend on PMD.

By conditioning results on Ψ_x , Ψ_y , γ , and ρ , and since the branch signals are normalized to unity variance, the output from each branch of the receiver is distributed according to a noncentral chi square distribution. Therefore, the probability density function (PDF) of the square root of the variable at the input to the threshold detector is a noncentral chi random variable with four degrees of freedom.

The general form of the noncentral chi distribution is given as follows⁴. Let us consider an N dimensional Gaussian vector, each component of which is an independent Gaussian stochastic variable g_i with mean m_i and unit variance ($i = 1, 2, \dots, N$). The probability density function of the variable

$$\chi = \sqrt{\sum_{i=1}^N g_i^2} \quad (37)$$

is given by:

$$P(\chi) = \frac{\chi^{\frac{N}{2}} I_{\frac{N-1}{2}}(A\chi) e^{-\frac{\chi^2 + A^2}{2}}}{A^{\frac{N-1}{2}}} \quad (38)$$

where I represents the modified Bessel function of the first kind and A is the noncentral parameter defined as

$$A^2 = \sum_{i=1}^N m_i^2 \quad (39)$$

Since the output PDF is a function of the sum of the mean squares of each component, and since that mean-square is always given by (39), we find that the probability of bit error reported in⁴ still holds for square law detectors in the presence of PMD.

3.2 Envelope detection

In general, when envelope detection is used, the output of each branch, conditioned on γ , ρ , Ψ_x , and Ψ_n , will be Ricean. However, when the transmitted signal is zero in any branch, the random variable for that branch degenerates to a Rayleigh-distributed random variable. The k th moment of Ricean random variable r (or Rayleigh, when $s = 0$) is given as⁹:

$$E(r^k) = (2\sigma^2)^{\frac{k}{2}} e^{-\frac{s^2}{2\sigma^2}} \frac{\Gamma\left(\frac{n+k}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} F\left(\frac{n+k}{2}, \frac{n}{2}, \frac{s^2}{2\sigma^2}\right) \quad k \geq 0 \quad (40)$$

where $F(x, y, z)$ is the confluent hypergeometric function, $\Gamma(x)$ is the gamma function, and $n=2$ for narrowband noise. The bit error rate may be determined for a receiver using envelope detection by using the expression for moments in (40) with results reported by Beaulieu^{10,11}. In his work, Beaulieu outlines an approach for computing the cumulative distribution function (CDF) and PDF of a sum of independent, arbitrarily distributed functions using a convergent series. The algorithm requires the evaluation of the characteristic function of each distribution at countably many uniformly spaced points. Characteristic functions are not known in closed form for either the Ricean or the Rayleigh distribution. However, they can be estimated from a Taylor series expansion in terms of their respective moments:

$$\Phi(\omega) = \sum_{n=0}^{\infty} \frac{m_n}{n!} (j\omega)^n \quad (41)$$

where m_n is the n^{th} moment of the distribution. Therefore, using (40) with (41) as described in^{10,11}, we have a means for obtaining probability of bit error for the receiver under consideration.

The probability of bit error conditioned on Ψ_x , Ψ_n , γ , and ρ is given by

$$P_E(\Psi_x, \Psi_n, \gamma, \rho) = p(0) P_{E/0} + p(1) P_{E/1}(\Psi_x, \Psi_n, \gamma, \rho) \quad (42)$$

where $p(0)$ and $p(1)$ are the probabilities of transmitting a 0 or 1, respectively, and where $P_{E/0}$ and $P_{E/1}$ are the conditional probabilities of bit error given that a 0 or 1 was transmitted, respectively. For a fixed threshold and SNR, $P_{E/0}$ is constant with respect to Ψ_x , Ψ_n , γ and ρ , whereas $P_{E/1}$ has a dependency on these same parameters. Since PMD is modeled through variations in Ψ_n , PMD can only affect the bit error rate through the term $P_{E/1}$. Physically, this is evident since PMD will not affect system performance when power is not transmitted. Therefore, in this paper, we investigated only the term $P_{E/1}(\Psi_x, \Psi_n, \gamma, \rho)$, and neglected all other terms in (42).

Our approach in the analysis was to compute the probabilities of bit error, $P_{E/1}$ conditioned on Ψ_x , Ψ_n , γ , and ρ . By evaluating these conditional probabilities of error over the expected range of values for each of the four parameters conditioned, we were able to demonstrate that PMD yields conditional probabilities of bit error which are always at least as small as worst case conditional probabilities of bit error when PMD and phase noise are absent (i.e., when $\Psi_n=0$). The next section discusses the results obtained in the investigation of $P_{E/1}$.

3.3 Numerical results

A computer was used to evaluate the mathematical expressions for the output of a receiver with envelope demodulation. A relatively low SNR of 17 dB was used due to the computer model's slow convergence times for high SNR. In order to quantify conditioned values of $P_{E/1}$, a decision threshold was required. Through experimentation, we discovered that the optimal threshold for an ML receiver depends on the conditioning parameters Ψ_x , Ψ_y , γ , and ρ . In fact, the CDF of the receiver output varies significantly as a function of these parameters. Interestingly, the "worst case" CDF (that is, the CDF yielding the highest probability of error) occurs when all power is concentrated in one branch of the receiver. As an example, this occurs when $\Psi_x = \Psi_y = \gamma = \rho = 0$. We assumed that a designer would establish a threshold based on this worst case to minimize the maximum probability of bit error as γ and ρ vary in time.

By employing the techniques described in^{10,11}, and for the signal-to-noise ratio assumed in this investigation, we were able to compute the CDF for the worst case given that a 1 was sent, as well as the complementary CDF given that a 0 was sent. By determining the intersections of these two curves, we arrived at the optimal threshold for a maximum likelihood receiver. The CDF and complementary CDF are shown in Figure 2, and illustrate that the optimum threshold for the SNR in this study was about 10.97. We point out that the optimal threshold determined in this fashion was much higher (in percent of the SNR) than anticipated in light of the thresholds recommended by Siudzak⁴. This difference is almost certainly due to the low SNR values assumed in this paper.

After establishing an appropriate threshold for the receiver, we proceeded to calculate the probability of bit error conditioned on various values of Ψ_x , Ψ_y , γ , and ρ . We determined that the conditioned probability of bit error is completely determined by the power split of the received signal between the four branches of the receiver. Therefore, if the received signal power is divided into branch powers of P1 and P2 for PDR1, and P3 and P4 for PDR2, then receiver performance will be identical for any combinations of these powers among the four branches. As an example, we present Figures 3 and 4. In Figure 3, curve A represents the power difference between the PDR's, $(P1+P2) - (P3+P4)$. Curve B represents the difference in power between the two branches in PDR1, P1-P2, and curve C represents the power difference between the two branches in PDR2, P3-P4. These results were obtained by fixing $\Psi_x = \Psi_y = 0$, varying γ from 0 to π , and varying ρ according to the relationship $\rho = \pi - \gamma$. For the example, we find that the signal power is distributed only between branch 1 of PDR1 and branch 3 of PDR2, and that the relative power division is periodic with γ . Figure 4 provides the corresponding conditional probability of bit error as γ varies. Note that the probability of bit error is completely determined by the split in power between these two branches. Interestingly, the conditional probability of bit error is largest when all power is distributed to one PDR, and is smallest as the power distribution between the two branches equalizes.

We also observed a less intuitive result; an equal division of received power among the four branches is not the optimal distribution in power. In other words, the smallest probability of bit error is not obtained when power is equally split between all four branches, but rather is obtained for a non-uniform distribution in power. Experimentation also led to the formerly mentioned conclusion that the highest probability of bit error is obtained when all received signal power is delivered exclusively to any one of the four branches of the receiver. We make use of this fact in our final evaluation of the effects of PMD.

The investigation continued with a look at the maximum polarization change induced by PMD through changes in the relative group velocity between the principal state components of the received lightwave. We noted that the maximum shift in polarization, as measured by the arc distance on a Poincaré sphere, is totally determined by the polarization orientation of the transmitter relative to the principal states of the fiber, γ . In fact, depolarization is maximized when the transmitted signal power is equally divided between the principal states of the optical fiber. For the linearly polarized transmitter assumed in this analysis, depolarization is maximized when $\gamma = \pi/4$ or $3\pi/4$; Depolarization is not evident when γ takes on values of either 0, $\pi/2$, or π . These latter conditions physically correspond to the case when the transmitted power is injected completely into one of the principal states of the optical fiber. Our results are valid to first order, since principal states produce no depolarization to first order, but may occur as second order effects [5]. Figure 5 illustrates results which show the maximum differential polarization over the range in Ψ_x . Results are shown in the $\gamma\rho$ plane.

Since we were interested in estimating the maximum effect of PMD on receiver performance, we naturally investigated the changes in $P_{E/1}$ that were induced at conditions corresponding to maximum depolarization. Therefore, we fixed $\gamma = \pi/4$ and examined the effects of varying Ψ_x for all possible receiver polarization orientations (i.e., $0 \leq \rho \leq \pi$).

The results from this investigation are illustrated in Figures 6 and 7 for the cases $\Psi_x = 0$ and $\Psi_x = \pi/4$, respectively. We found that in the cases corresponding to maximum depolarization which were considered in this study, the presence of PMD improved receiver performance. This result is directly attributable to the change in the distribution of received power between the branches induced by the presence of PMD.

One important result which was observed was that receiver performance in the presence of PMD was always better than the worst case (e.g. $\Psi_x = \Psi_y = \gamma = \rho = 0$) performance of the receiver in the absence of PMD and phase noise. When considering the effects of PMD on the power distribution, this result is obvious. PMD has the effect of depolarizing the output of the lightwave, causing power to be distributed between at least two channels, independent of γ and ρ . Since worst case performance occurs only when all power is distributed to one branch, a receiver in the presence of PMD will always perform at least as well as the worst case receiver without PMD.

4. CONCLUSIONS

The extent to which a combined polarization and phase diversity receiver for ASK demodulation is sensitive to PMD depends upon the detection law used. For square law detection, the receiver output is theoretically insensitive to PMD in the presence of transmitter phase noise. These receivers are also insensitive to any polarization misalignment of the transmitter and receiver relative to the principal states of the fiber. If envelope detection is employed, the output of the receiver varies as a function of the polarization misalignment, the phase noise and the PMD. We found that receiver performance was determined completely by the division of the received power among the branches of the receiver; worst case performance occurs when all power is distributed only to one branch of the receiver, and best performance occurs for some non-equal division in received power. The combined effect of phase noise and PMD distributes the received signal power over at least 2 branches of the receiver, so that the performance of the receiver in the presence of PMD improved in comparison to the worst case performance when PMD and phase noise are absent.

5. ACKNOWLEDGMENTS

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6. REFERENCES

1. Foschini, G.J., and Poole, C.D., "Statistical Theory of Polarization Dispersion in Single Mode Fibers," *Journal of Lightwave Technology*, Vol. 9, No. 11, November 1991, pp. 1439-1456.
2. Rashleigh, S., "Origins and Control of Polarization Effects in Single-Mode Fibers," *Journal of Lightwave Technology*, Vol. LT-1, No. 2, June 1983, pp. 312-331.
3. Wagner, R.E., and Elrefaie, A.F., "Polarization-Dispersion Limitations in Lightwave Systems," OFC '88, TU16.
4. Siudzak, J., and Van Etten, W., "BER Evaluation for Phase and Polarization Diversity Optical Homodyne Receivers Using Noncoherent ASK and DPSK Demodulation," *JLT*, Vol. 7, No. 4, April 1989, pp. 584-599.
5. Poole, C.D., and Wagner, R.E., "Phenomenological Approach to Polarization Dispersion in Long Single-Mode Fibers," *Electronics Letters*, Vol. 22, pp. 1029-1030, 1986.
6. Kazovsky, L.G. "ASK Multipoint Optical Homodyne Receivers," *Journal of Lightwave Technology*, Vol. LT-5, No. 6, June 1987, pp. 770-791.
7. Barry, J., and Lee, E., "Performance of Coherent Optical Receivers," *Proceedings of the IEEE*, Vol. 78, No. 8, August 1990.

8. Gisin, N., Von der Weid, J.P., and Pelloux, J.P., "Polarization Mode Dispersion of Short and Long Single-Mode Fibers," *Journal of Lightwave Technology*, Vol. 9, No. 7, July 1991, pp. 821-827.
9. Proakis, J.G., *Digital Communications*, McGraw-Hill Book Company, 1983, p.32.
10. Beaulieu, N., "An Infinite Series for the Computation of the Complementary Probability Distribution Function of a Sum of Independent Random Variables and Its Application to the Sum of Rayleigh Random Variables," *IEEE Transactions on Communications*, Vol. 38, No. 9, September 1990, pp. 1463-1474.
11. Beaulieu, N., "Analysis of Equal Gain Diversity on Nakagami Fading Channels," *IEEE Transactions on Communication*, Vol. 39, No. 2, Feb 1991.
12. Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, McGraw Hill Book Company, 1984.

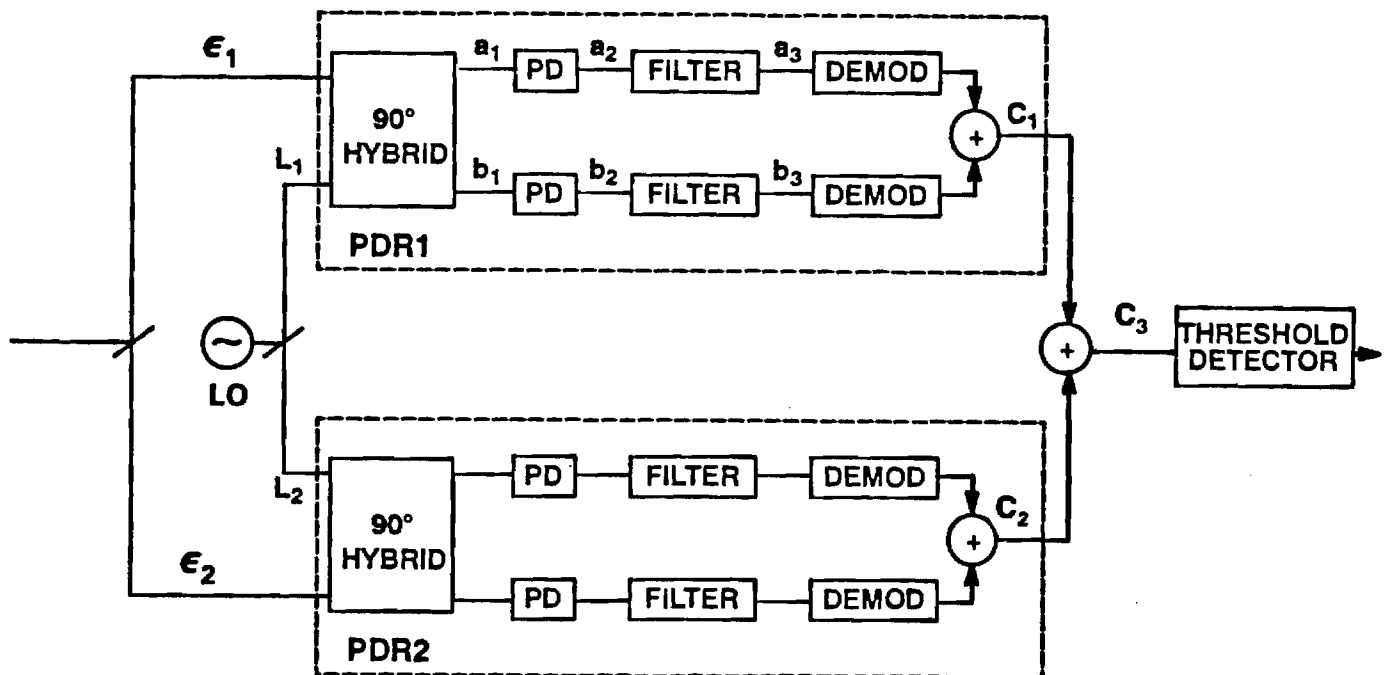


Figure 1. Phase and polarization diversity receiver

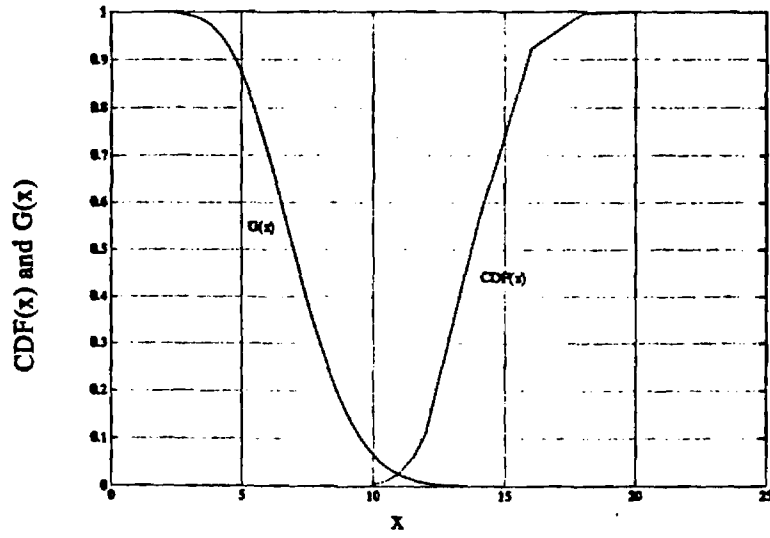


Figure 2. CDF(x) and complementary CDF, G(x), for SNR = 17 dB

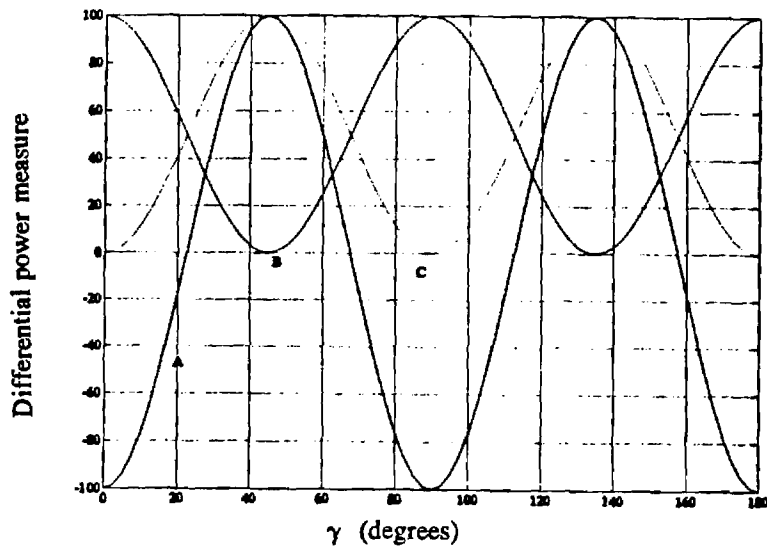


Figure 3. Differential branch power

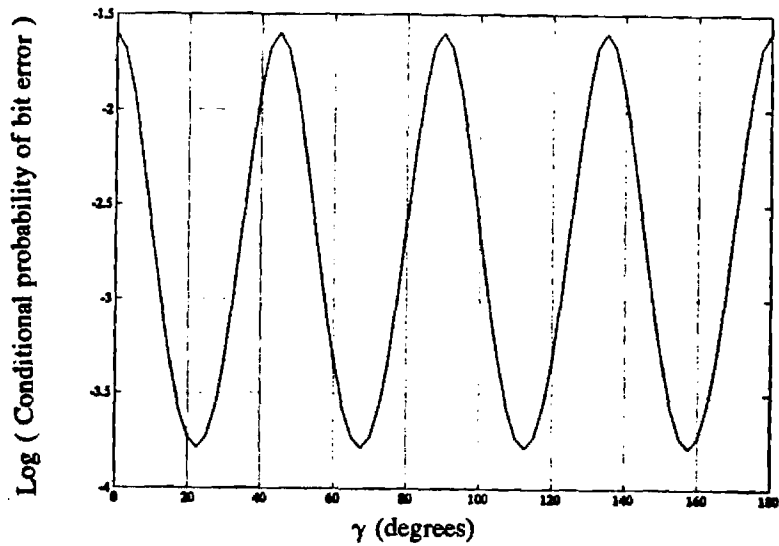


Figure 4. Conditional probability of bit error, $\Psi_x = \Psi_n = 0$, $\rho = \pi - \gamma$

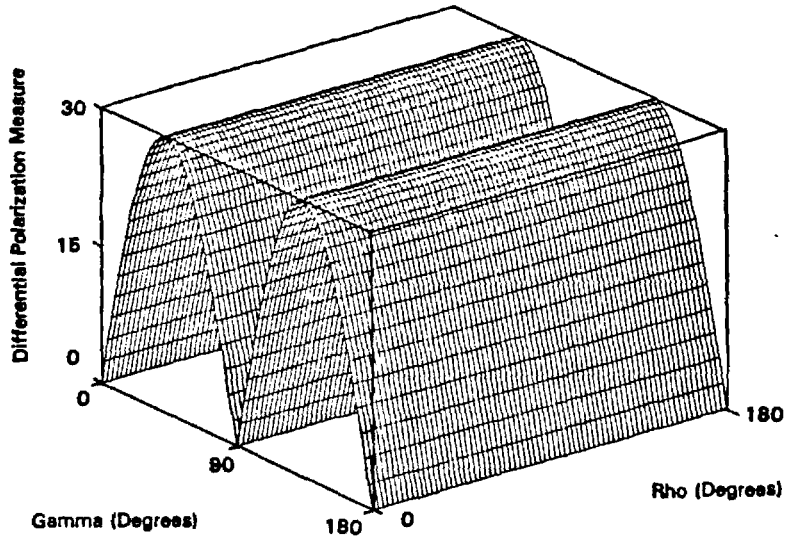


Figure 5. Differential polarization measure

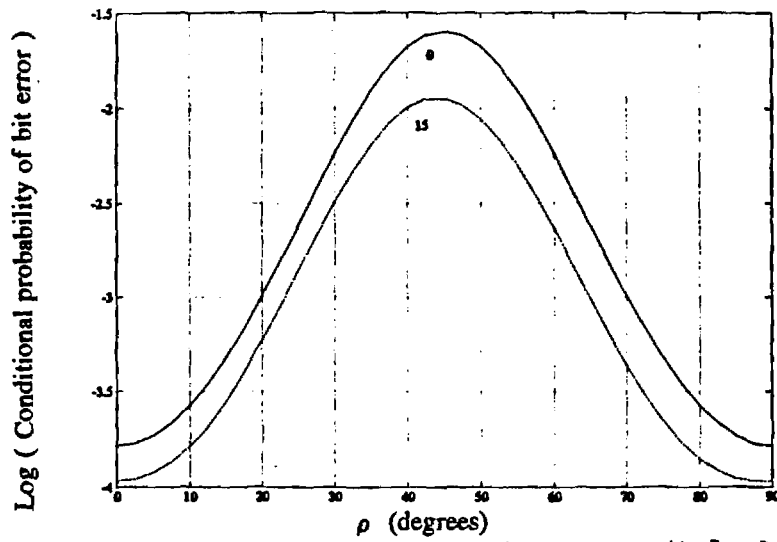


Figure 6. Conditional probability of bit error, $\gamma = \pi/4$, $\Psi_x = 0$

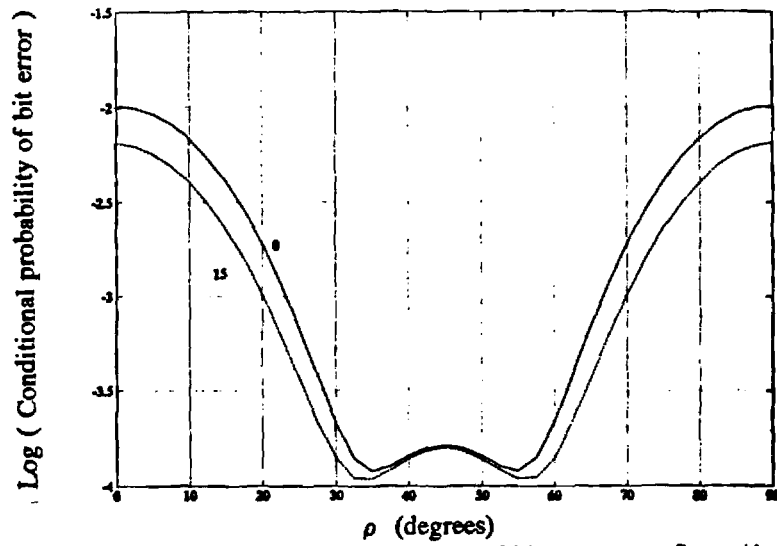


Figure 7. Conditional probability of bit error, $\gamma = \Psi_x = \pi/4$

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3. Award Dates (MM/YY)	From: 08/91 To: 01/94
4. Institution and Address	GA Tech Res Corp - GIT Administration Building Atlanta GA 30332
5. Award Number	9110363
6. Project Title	Effects of Polarization Mode Dispersion on Coherent Optical Receivers

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PART II: SUMMARY OF COMPLETED PROJECT FOR RESEARCH INITIATION AWARD # 9110363

This funding supported two projects: effect of polarization mode dispersion (PMD) on polarization and phase diversity receivers and effect of EDFA preamplifier noise on an optical serial-to-parallel (STP) converter that uses second harmonic generation. In the polarization project, we determined that only receivers that use envelope detection, not square law detection, are affected by the incident polarization, and hence by PMD. We applied a numerical technique, Beaulieu's method, to calculate the distributions (which are quite non-Gaussian) of the signals at the threshold detector. The optimal threshold as a function of incident polarization was found, and the bit error rates for a fixed threshold with varying polarization were computed. A journal article was written and reviewed twice for the Journal of Lightwave Technology before we discovered a flaw in a basic assumption in late 1994. We are considering redoing the analysis. In the STP converter, the conversion followed by photodetection constitutes a fourth order nonlinearity. The strengths of the different beat noises were determined, and the BER was computed using Beaulieu's method. We have a great deal of confidence in these results, and a paper should be ready for submission within a month.

PART III: TECHNICAL INFORMATION
FOR RESEARCH INITIATION AWARD # 9110363

1. Pratt, T.G. and Ingram, M.A., "Polarization mode dispersion effects in phase and polarization diversity receivers," *Proceedings of SPIE OE/Fibers '92, Conference on Multigigabit Fiber Communications*, Vol. 1787, pp. 390-401, Boston, MA, September 1992.
2. Wang, S.C and Ingram, M.A., "Performance analysis of an optical serial-to-parallel converter with EDFA preamplification," presently being internally reviewed, to be submitted to *Journal of Lightwave Technology*.

PART IV -- FINAL PROJECT REPORT -- SUMMARY DATA ON PROJECT PERSONNEL

(To be submitted to cognizant Program Officer upon completion of project)

The data requested below are important for the development of a statistical profile on the personnel supported by Federal grants. The information on this part is solicited in response to Public Law 99-383 and 42 USC 1885C. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. You should submit a single copy of this part with each final project report. However, submission of the requested information is not mandatory and is not a precondition of future award(s). Check the "Decline to Provide Information" box below if you do not wish to provide the information.

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Asian					1					
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Hispanic										
Pacific Islander										
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