



Shear behaviour of reinforced concrete beams under impact loads by the Lumped Damage framework

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ABSTRACT. Impact loadings such as vehicle collisions or falling rocks on a structure could lead it to collapse and, eventually, to fatal victims. Among possible alternatives to analyse this issue, Lumped Damage Mechanics is an interesting option due to its formulation and easiness for practical application. Therefore, this paper presents a simplified Lumped Damage model to evaluate reinforced concrete structures under impact loads with shear failure mode. In the proposed approach, the damage variable is considered as an output parameter. Such damage variable takes values between zero and one, which quantifies the concrete cracking due to impact loading. The proposed formulation was applied in experiments of reinforced concrete beams under impact loads that presents shear collapse. The obtained results showed good accuracy between the proposed model and the actual structural behaviour. Moreover, a possible flowchart for practical applications is also presented. Since the model parameters are easily associated to inelastic phenomena, the proposed formulation might become accessible to engineers in practice.

KEYWORDS. Impact load; Reinforced concrete beams; Shear failure; Lumped Damage Mechanics.



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INTRODUCTION

Accidents involving impact loads often occur on reinforced concrete structures. Some usual accidents are vehicles that collide on structures, explosions, rock-fall, falling of heavy objects or parts of another structure in the construction phase, and impact of ships in docks. In such cases, engineers often need a consistent and immediate result of the structural response.

During the impact, the structural response can be obtained by Fracture Mechanics [1] or Continuous Damage Mechanics [2]. However, such theories are applied in a finite element analysis that present an excessive computational cost and depict results that difficult their use for several engineering applications, making their practical application unfeasible in many cases. On the other hand, Lumped Damage Mechanics (LDM) combines and applies fundamental concepts of Continuous Damage Mechanics and Fracture Mechanics, such as the strain equivalence hypothesis and the Griffith criterion, with the concept of plastic hinges. LDM is an interesting alternative for several civil engineering applications. This theory was initially proposed by Flórez-López [3] to analyse reinforced concrete structures under seismic loads, and the first book about LDM was published in 2015 [4]. LDM has already been used to satisfactorily analyse several structures with actual engineering applications, such as simple and reinforced concrete structures, bridges and masonry arches [5-14]. Recently, Uchoa et al. [15] proposed a LDM based methodology to estimate the loss of flexural stiffness in simple concrete beams subjected to the four-point bending experimental test.

The collapse of reinforced concrete (RC) beams subjected to impact loads can be caused by bending moment [16, 17] or shear force [18-22]. Li et al. [23] and Pham et al. [24] showed that the contact stiffness has limited effect in impulse and midspan displacement response. Previous studies have proposed numerical models to evaluate behaviour response of the RC beams under low impact velocity [25-29]. Fan et al. [30] proposed a finite element analysis in order to predict flexural and shear responses of RC beams and columns subjected to low impact velocity. However, due to differences in RC beams behaviour under low and high impact velocities, there are not enough studies in this theme, especially for high velocities. According to Zhao et al. [22], shear failure in RC beams has not yet been adequately understood, unlike bending failure. Kishi et al. [18] showed that RC beams that under static load fail by bending, may fail by shear when subjected to high impact loads. In order to evaluate the ability of RC beams to resist shear under impact loads, simplified analytical model have been proposed [18-21]. Nonlinear analyses using finite elements can also be applied. For instance, Zhao et al. [22] used an elastoplastic damage model with three-dimensional finite elements. This type of application results in a high computational cost and complex interpretation of the numerical results.

In the light of the foregoing, this paper presents an alternative approach based on LDM that estimates the response of RC beams subjected to shear failure due to impact loads. The proposed formulation is obtained by the thermodynamic of irreversible processes. Since the proposed formulation is quite simple, structural safety might be easily analysed with practical engineering criteria. Finally, the accuracy of the proposed LDM model is verified by comparing the obtained results with experimental responses [22].

LUMPED DAMAGE MECHANICS

General overview

Consider a finite beam element between nodes i and j , as the one depicted in Fig. 1a. The deformed shape of the beam element can be described by the flexure rotations ϕ_i and ϕ_j at the nodes i and j , respectively. Both flexure rotations are set in the matrix of generalised deformations, defined as:

$$\{\Phi\} = \{\phi_i \quad \phi_j\}^T \tag{1}$$

where the superscript T means 'transpose of'.

In order to take into account the inelastic phenomena, the finite element is now understood as an assemblage of an elastic-plastic-damage Timoshenko beam with two inelastic hinges at its edges, as shown in Fig. 1b. Note that the bending inelastic effects are lumped at the hinges and the shear ones are distributed along the beam element. For RC beams, the damage variables d_i and d_j account for the bending concrete cracking, the plastic flexure rotations ϕ_i^p and ϕ_j^p quantify the yielding of the longitudinal reinforcement, the damage variable d_s measures the shear concrete cracking and the plastic distortion γ^p accounts for the yielding of the transversal reinforcement (Fig. 1c).

The matrix of generalised stresses $\{\mathbf{M}\}$ is defined as a set of bending moments, M_i and M_j , which are conjugated with generalised deformations ϕ_i and ϕ_j (Fig. 1c).

$$\{\mathbf{M}\} = \{M_i \quad M_j\}^T \tag{2}$$

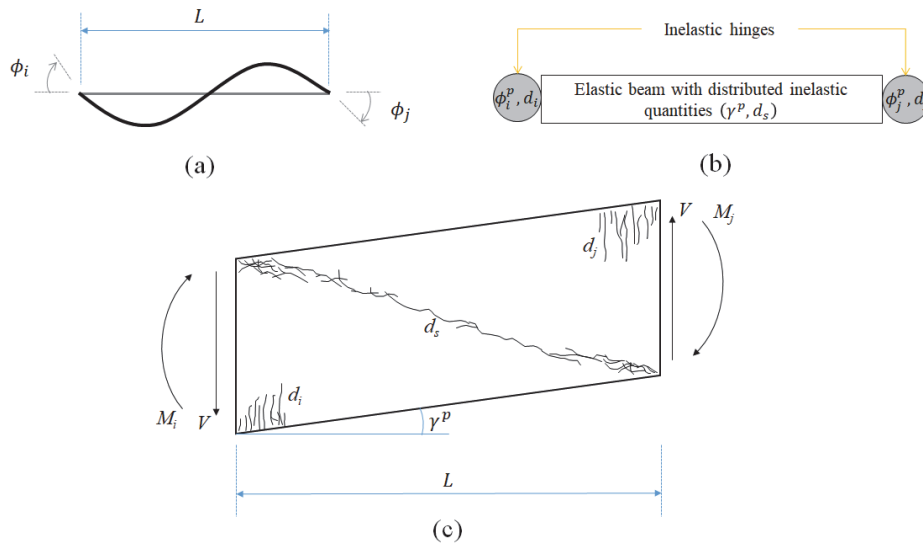


Figure 1: Lumped Damage beam: (a) deformed shape, (b) finite element and (c) inelastic variables.

According to the deformation equivalence hypothesis [4], the matrix of generalised deformations can be expressed as:

$$\{\Phi\} = \{\Phi^e\} + \{\Phi^d\} + \{\gamma^d\} + \{\Phi^p\} + \{\gamma^p\} \tag{3}$$

where $\{\Phi^e\}$ is the matrix of elastic deformations, given by [4]:

$$\{\Phi^e\} = \begin{bmatrix} \frac{L}{3EI} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix} \{\mathbf{M}\} + \begin{bmatrix} \frac{1}{LGA} & \frac{1}{LGA} \\ \frac{1}{LGA} & \frac{1}{LGA} \end{bmatrix} \{\mathbf{M}\} \tag{4}$$

being E the Young's modulus, G the shear modulus, I the inertia moment and A the cross section area; $\{\Phi^d\}$ is the matrix that represents the deformation of the beam due to bending cracking in concrete by means of damage variables (d_i e d_j) in each hinge, given by [4]:

$$\{\Phi^d\} = \begin{bmatrix} \frac{Ld_i}{3EI(1-d_i)} & 0 \\ 0 & \frac{Ld_j}{3EI(1-d_j)} \end{bmatrix} \{\mathbf{M}\} \tag{5}$$

$\{\gamma^d\}$ is the matrix that represents the beam distortion caused by diagonal shear cracks through the damage variable d_s , expressed as [4]:

$$\{\gamma^d\} = \begin{bmatrix} \frac{d_s}{LGA(1-d_s)} & \frac{d_s}{LGA(1-d_s)} \\ \frac{d_s}{LGA(1-d_s)} & \frac{d_s}{LGA(1-d_s)} \end{bmatrix} \{\mathbf{M}\} \tag{6}$$



$\{\Phi^p\}$ is the matrix that quantifies the deformation caused by the yielding of the longitudinal reinforcement, i.e.:

$$\{\Phi^p\} = \{\phi_i^p \quad \phi_j^p\}^T \quad (7)$$

and $\{\gamma^p\}$ is the matrix that represents the plastic distortion of the beam caused by the yielding of the transversal reinforcement (Fig. 1c), given by:

$$\{\gamma^p\} = \{\gamma^p \quad \gamma^p\}^T \quad (8)$$

Hence, the elastic relation is obtained by substituting (4-8) in (3):

$$\begin{aligned} \{\Phi - \Phi^p - \gamma^p\} &= [\mathbf{F}_f(d_i, d_j) + \mathbf{F}_s(d_s)]\{\mathbf{M}\} = \\ &= \begin{bmatrix} \frac{L}{3EI(1-d_i)} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI(1-d_j)} \end{bmatrix} \{\mathbf{M}\} + \begin{bmatrix} \frac{1}{LGA(1-d_s)} & \frac{1}{LGA(1-d_s)} \\ \frac{1}{LGA(1-d_s)} & \frac{1}{LGA(1-d_s)} \end{bmatrix} \{\mathbf{M}\} \end{aligned} \quad (9)$$

where $[\mathbf{F}_f(d_i, d_j)]$ and $[\mathbf{F}_s(d_s)]$ are the flexibility matrices due flexural and shear cracking, respectively.

Thermodynamic approach

Considering a beam element with a certain level of flexure and shear damage as well as plastic rotations and distortion, the total thermodynamic potential is given by the Helmholtz's free specific energy [34, 35]:

$$\chi = \frac{1}{2} \{\Phi - \Phi^p - \gamma^p\}^T [\mathbf{E}(\mathbf{D})] \{\Phi - \Phi^p - \gamma^p\} \quad (10)$$

being χ the total thermodynamic potential and $[\mathbf{E}(\mathbf{D})]$ the stiffness matrix of the damaged element, given by:

$$[\mathbf{E}(\mathbf{D})] = [\mathbf{F}_f(d_i, d_j) + \mathbf{F}_s(d_s)]^{-1} \quad (11)$$

where $(\mathbf{D}) = (d_i, d_j, d_s)$ is the set of the damage variables.

For a proper definition, the model must be thermodynamically admissible. This condition is achieved through Clausius-Duhem inequality, which is also called as non-negative dissipation. Thus, considering that the process is isothermal, such inequality is expressed as:

$$\{\mathbf{M}\}^T \{\dot{\Phi}\} - \dot{\chi} \geq 0 \quad (12)$$

where the first term is the variation of the internal energy involved in the process and the second one is the variation of the thermodynamic potential.

Assuming that the total thermodynamic potential can be linearized around the current values of the state variables, then:

$$\dot{\chi} = \left\{ \frac{\partial \chi}{\partial \{\Phi\}} \right\}^T \{\dot{\Phi}\} + \left\{ \frac{\partial \chi}{\partial \{\Phi^p\}} \right\}^T \{\dot{\Phi}^p\} + \left\{ \frac{\partial \chi}{\partial \{\gamma^p\}} \right\}^T \{\dot{\gamma}^p\} + \left\{ \frac{\partial \chi}{\partial \{\mathbf{D}\}} \right\}^T \{\dot{\mathbf{D}}\} \quad (13)$$

Therefore, the Clausius-Duhem inequality is rewritten as:



$$\left(\{\mathbf{M}\}^T - \left\{ \frac{\partial \chi}{\partial \{\Phi\}} \right\}^T \right) \{\dot{\Phi}\} - \left\{ \frac{\partial \chi}{\partial \{\Phi^p\}} \right\}^T \{\dot{\Phi}^p\} - \left\{ \frac{\partial \chi}{\partial \{\gamma^p\}} \right\}^T \{\dot{\gamma}^p\} - \left\{ \frac{\partial \chi}{\partial \{\mathbf{D}\}} \right\}^T \{\dot{\mathbf{D}}\} \geq 0 \quad (14)$$

Eqn. (14) must be obeyed in any thermodynamic process, including one that is fully reversible, where the inelastic effects are null. This can be observed since the derivation of the total thermodynamic potential in relation to the total generalised deformations results in the generalised stress matrix, according to the following expression:

$$\left\{ \frac{\partial \chi}{\partial \{\Phi\}} \right\} = [\mathbf{E}(\mathbf{D})] \{\Phi - \Phi^p - \gamma^p\} = \{\mathbf{M}\} \quad (15)$$

The other differential relations of thermodynamic potential in relation to state variables are given by the following expressions:

$$\left\{ \frac{\partial \chi}{\partial \{\Phi^p\}} \right\} = -[\mathbf{E}(\mathbf{D})] \{\Phi - \Phi^p - \gamma^p\} = -\{\mathbf{M}\} \quad (16)$$

$$\left\{ \frac{\partial \chi}{\partial \{\gamma^p\}} \right\} = -[\mathbf{E}(\mathbf{D})] \{\Phi - \Phi^p - \gamma^p\} = -\{\mathbf{M}\} \quad (17)$$

$$\left\{ \frac{\partial \chi}{\partial \{\mathbf{D}\}} \right\} = \begin{Bmatrix} -\frac{LM_i^2}{6EI(1-d_i)^2} \\ -\frac{LM_j^2}{6EI(1-d_j)^2} \\ \frac{(M_i + M_j)^2}{2LGA(1-d_s)^2} \end{Bmatrix} = \begin{Bmatrix} -Y_i \\ -Y_j \\ -Y_s \end{Bmatrix} = \{\mathbf{Y}\} \quad (18)$$

It is observed that $\{\mathbf{M}\}$, $-\{\mathbf{M}\}$, $-\{\mathbf{M}\}$ and $\{\mathbf{Y}\}$ are the thermodynamics variables associated with $\{\Phi\}$, $\{\Phi^p\}$, $\{\gamma^p\}$ and $\{\mathbf{D}\}$, respectively, being Y_i and Y_j the damage driving moments of the hinges i and j , and Y_s the damage driving moment of the shear cracking along the beam [4]. Therefore, by substituting the expressions (15-18) in the inequality (14) and assuming the predominance of shear effects, then:

$$\{\mathbf{M}\}^T \{\dot{\gamma}^p\} + Y_s \dot{d}_s \geq 0 \quad (19)$$

The inequality (19) must be obeyed throughout the structural analysis. Therefore, the positivity of each term must be ensured separately, since both inelastic phenomena (plastic deformations and damage) can occur non-simultaneously (considering any application, not only for RC beams). In the first term of (19) it is noted that the internal stress present in the matrix $\{\mathbf{M}\}$ always have the same signs of the generalised plastic distortion rate, therefore, the positivity of the first term is verified. Since the physical and geometric properties of the structural element under analysis, as well as the damage variable, are always positive, then the positivity of the second term is achieved.

PROPOSED MODELLING OF IMPACT PROBLEMS

In order to analyse the nonlinear response of simply supported beams under impact loads, Fujikake et al. [17] performed experiments in which a known mass body (hammer) is released from four different heights. With these experiments, an analytical model based on the energy balance of a two-degree-of-freedom mass-spring-damping system was

presented with the purpose of quantify the behaviour of RC beams under impact loads. Thus, the energy dissipated (E_p) during the impact is given by [17]:

$$E_p = \int P(w)dw = \frac{m_2 v^2}{2} - \frac{m_1 m_2}{2(m_1 + m_2)} v^2 + (m_1 + m_2) g w_{\max} \quad (20)$$

being m_1 and m_2 the beam and hammer masses respectively, g the acceleration of gravity, w_{\max} the maximum displacement (mid-span), and v the impact velocity, given by:

$$v = \sqrt{2gb} \quad (21)$$

where b is the drop height.

It is worthy noted that the analytical Eqn. (20), presented by Fujikake et al. [17], has been successfully applied on shear impact problems in the technical literature [18-21]. Therewith, by equalling both energy dissipation equations, i.e. (19) and (20), for a simply supported beam as the one depicted in Fig. 2 and assuming, for the sake of simplicity, that shear damage and plastic distortion after impact are characterised by their final values (d_s and γ^p), then:

$$\frac{m_2 v^2}{2} - \frac{m_1 m_2}{2(m_1 + m_2)} v^2 + (m_1 + m_2) g w_{\max} = M_j \gamma^p + \frac{M_j^2}{2LGA(1-d_s)^2} d_s \quad (22)$$

being the bending moment M_j calculated with the mean impact force P_m (Fig. 3):

$$P_m = \frac{I_p}{T_d} \quad \therefore I_p = \int_0^{T_d} P(t) dt \quad (23)$$

where I_p is the impulse and T_d is the impact duration.

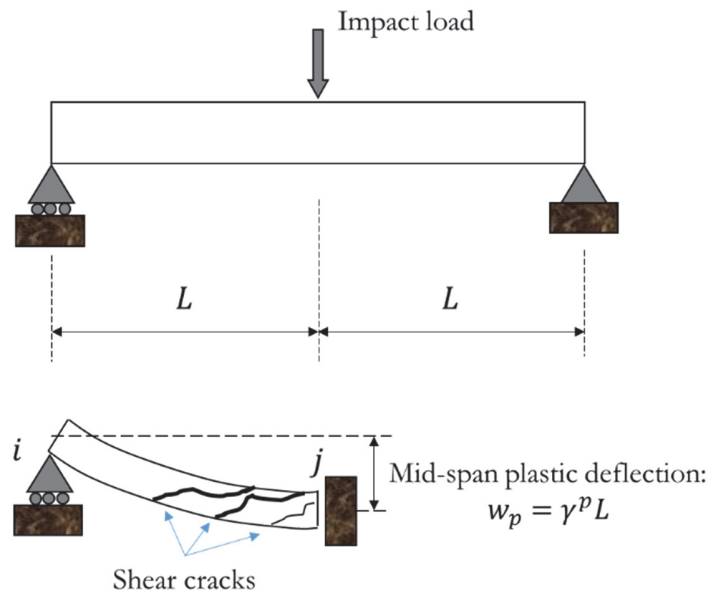


Figure 2: Simply supported beam under impact load and its mathematical model in the aftermath.

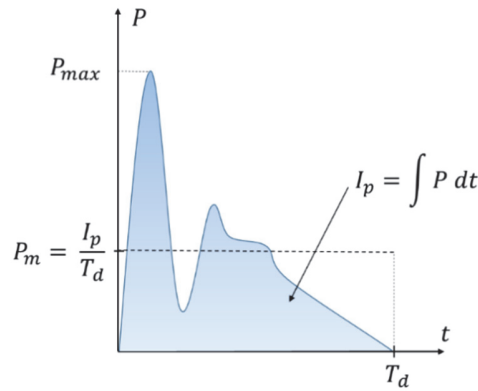


Figure 3: Force-time graph for impact loads.

RESULTS AND DISCUSSION

Experiments performed by Zhao et al. [22] and Bhatti et al. [37] were used in order to analyse the proposed shear impact model.

Zhao et al. [22]

The beams present 3m span ($2L$), longitudinal reinforcement composed of four bars with 20mm in the tension side and two bars with 16mm in the compression side, with yield stress of 495.5MPa and ultimate tension of 620.2MPa. The stirrups consisted of 6mm bars spaced by 30cm, with yield stress 344.7MPa and ultimate tension of 550.4MPa [22]. Other properties of the beams are presented in Tab. 1, as well as some dynamic response values.

Beam – impact weight (kg) – drop height (m)	Cross section (cm)	Compressive strength of concrete (MPa)	Mean impact force (kN)
Beam C-1700-4.60		32.14	205.22
Beam C-1300-5.56	20×50	30.25	296.88
Beam C-868-7.14		26.26	286.71

Table 1: Properties of the analysed beams.

Therefore, for each beam, the damage variable (d_s) and the plastic distortion are obtained from Eqn. (9) and (22). This calculation process was performed to evaluate the applicability of the proposed formulation. The damage and the plastic distortion calculated on each beam is shown in Tab. 2. Fig. 3 shows a scheme with the crack pattern after impact. The obtained damage values are high, meaning that a severe cracking level in the three beams, which is consistent with the experimentally observed cracking patterns (Fig. 4). Furthermore, by knowing the value of the plastic distortion (γ^p), it is possible to determine the plastic displacement of the beam (Fig. 2):

$$w_p = \gamma^p L \quad (24)$$

In order to illustrate the proposed approach, the solution of Beam C-1700-4.60 [22] is described as follows. Firstly, Eqn. (9) must be rewritten to agree with the mathematical model of the beam (Fig. 2). Therewith, it is assumed that the inelastic effects due to bending moment are negligible. Then, the constitutive relation for a simply supported beam is given as:

$$\{\Phi - \gamma^p\} = [\mathbf{F}_f + \mathbf{F}_s(d_s)]\{\mathbf{M}\} \quad (25)$$

which leads to:

$$\phi_j = \gamma^p + \frac{L}{3EI} M_j + \frac{1}{LGA(1-d_s)} M_j \quad (26)$$

Beam – impact weight (kg) – fall height (m)	Damage (d_s)	Plastic distortion (γ^p)	Estimated plastic displacement (cm)	Experimental plastic displacement (cm) [22]
Beam C-1700-4.60	0.9681	0.0750729	11.3	Not Available
Beam C-1300-5.56	0.9603	0.0402814	6.0	≅ 5.2
Beam C-868-7.14	0.9601	0.0404670	6.1	≅ 5.0

Table 2: Damage and plastic distortion results for the analysed beams.

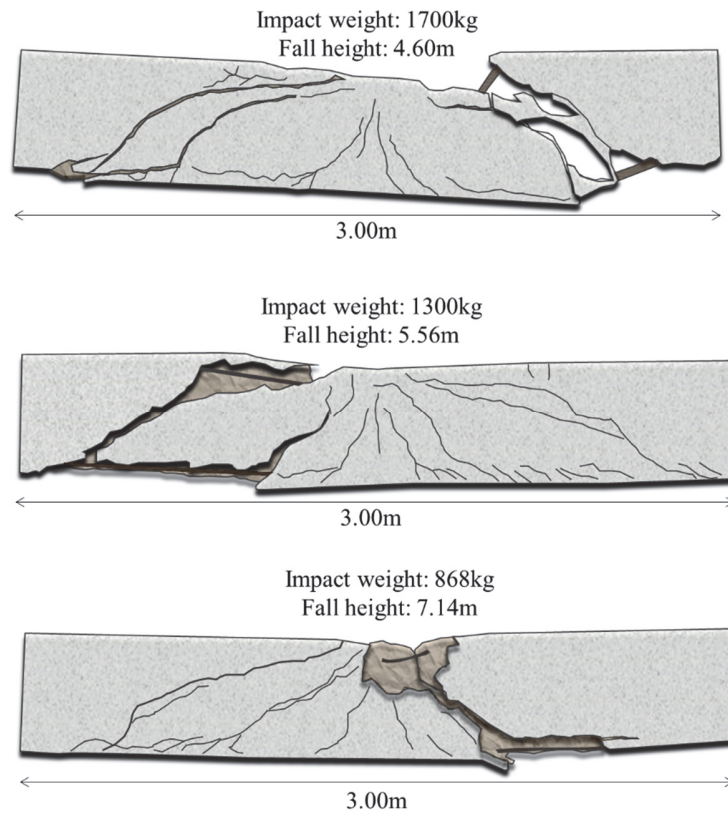


Figure 4: Beam cracking patterns after impact. (Source: adapted from Zhao et al. [22]).

Since Zhao et al. [22] presented the maximum displacement (w_{\max}) of the beam, Eqn. (26) can be rewritten as:

$$\phi_j = \frac{w_{\max}}{L} = \gamma^p + \frac{L}{3EI} M_j + \frac{1}{LGA(1-d_s)} M_j \quad (27)$$

Note that Eqn. (27) is a simple rearrangement of Eqn. (9) according to the analysed problem. The Young's modulus can be estimated by any design code regulation. In this paper, the Brazilian code [36] was used i.e.



$$E = 5600\sqrt{f_{ck}} \tag{28}$$

being f_{ck} the compressive strength of concrete, given in MPa.

For Beam C-1700-4.60 [22], Eqns. (27) and (22) give, respectively, the following relations (units in N and m):

$$0.07866666667 = \gamma^p + 0.001163539185 + \frac{0.00007756927894}{(1 - d_s)} \tag{29}$$

$$17227.43033 = 153914.9250\gamma^p + \frac{5.969534875d_s}{(1 - d_s)^2} \tag{30}$$

Then, by solving this nonlinear system (29-30) the shear damage (d_s) and plastic distortion (γ^p) are, respectively, 0.9681 and 0.0750729. Finally, by using Eqn. (24) the plastic displacement (w_p) is estimated in 11.3 cm.

Bhatti et al. [37]

The beams presented span of 2.0 m and cross section with 20 cm of basis and 40 cm of height. The concrete compressive strength is 41.2 MPa and its Young’s Modulus is 25.7 GPa. All beams casted by Bhatti et al. [37] present two rebars of 35 mm as longitudinal reinforcement at both sides and two types of transversal reinforcements: 6 mm @ 15.0 cm (type A) and 6 mm @ 7.5 cm (type B). Both longitudinal and transversal reinforcements present yield stress and Young’s modulus of 373 MPa and 206 GPa, respectively. The impact velocity and the mean impact force on each beam is given in Tab. 3.

Beam – impact velocity (m/s)	Mean impact force (kN)
A-4.6	397.35
A-6.5	463.05
A-8.4	635.66
B-4.6	419.47
B-7.4	509.01
B-9.3	614.62

Table 3: Properties of the analysed beams.

Therefore, damage and plastic distortion were obtained by applying the same procedure previously illustrated (Tab. 4). Fig. 5 shows cracking pattern presented in the beams after impact.

Beam – impact velocity (m/s)	Damage (d_s)	Plastic distortion (γ^p)	Estimated plastic displacement (cm)	Experimental plastic displacement (cm) [37]
A-4.6	0.9446	0.0066656	0.667	≅ 0.362
A-6.5	0.9505	0.0107417	1.074	≅ 0.590
A-8.4	0.9516	0.0184035	1.840	≅ 0.944
B-4.6	0.9327	0.0070843	0.708	≅ 0.203
B-7.4	0.9469	0.0143020	1.430	≅ 0.663
B-9.3	0.9579	0.0201287	2.013	≅ 1.235

Table 4: Damage and plastic distortion results for the analysed beams.

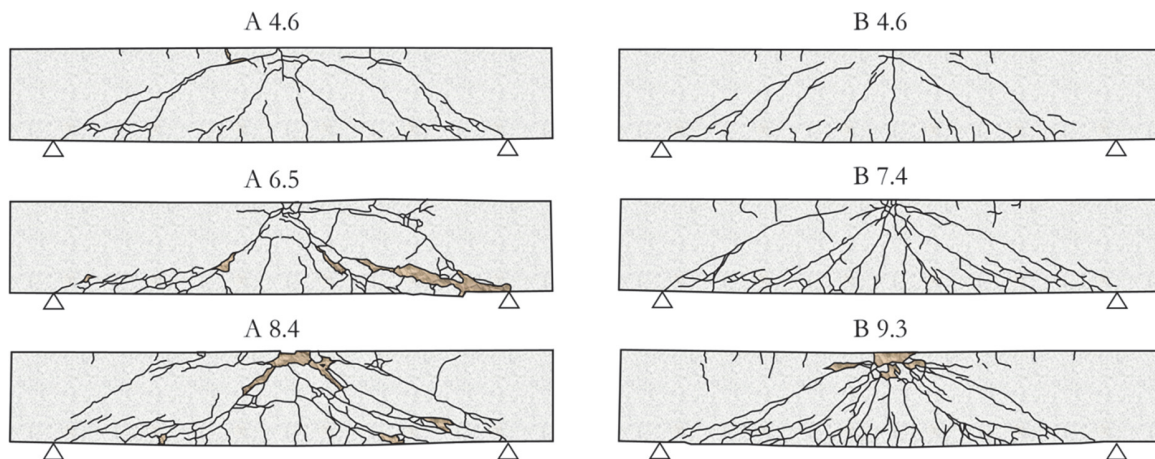


Figure 5: Beam cracking patterns after impact. (Source: adapted from Bhatti et al. [37]).

Note that the calculated plastic displacements are quite close to the experimental ones. The absolute difference between the estimated and experimental responses varies from 0.305 cm to 0.846 cm, approximately. Note that in all cases the estimated response is greater than the experimental one. Therefore, the proposed procedure can estimate the actual structural behaviour analytically. Since the proposed formulation can be applied to real problems quite easily, a sequence of actions is proposed based on the flowchart shown in Fig. 6. Thus, during accidents involving impact load on reinforced concrete structures, the field engineer can obtain the necessary parameters for the model damage calculation, which are: data of the structure characteristics, obtained in the design; data of the impact nature that can be measured with an analysis of where the accident occurred and the application of basic concepts of Physics. Thus, from the level of permanent deformation of the structure, it is possible to calculate the plastic distortion. Then the damage (d_s) and maximum displacement (v_{max}) values are calculated based on Eqn. (22). Finally, as observed in the experiments of Zhao et. al. [22], the damage variable indicates how severe the impact load was and, then, could be used to create practical engineering criteria.

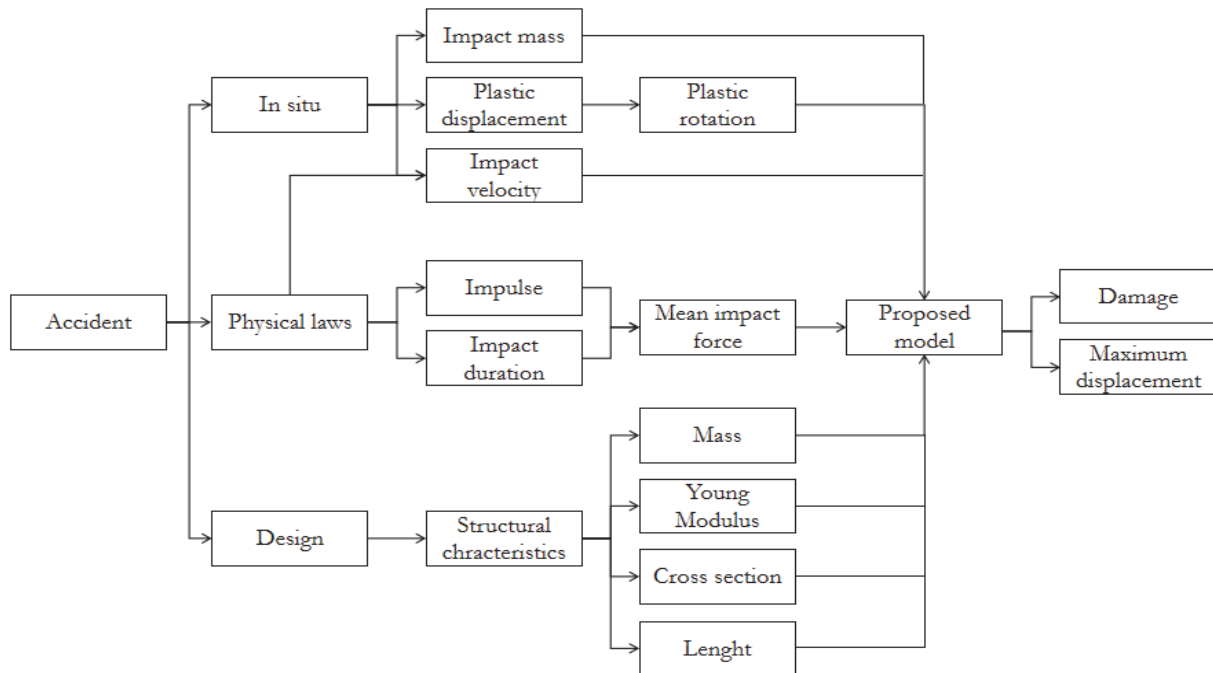


Figure 6: Demonstrative flowchart of the structural accident assessment procedure.



CONCLUSIONS

The proposed shear impact model was able to obtain damage, plastic distortion and plastic displacement values for each beam tested by Zhao et al. [22] and Bhatti et al. [37]. This damage variable is associated with the cracking level of the beam due to the shear effect. Analogously, the calculated plastic distortion is associated with the stirrup yielding. Note that the damage and plastic displacement values calculated with the proposed formulation are in agreement with the experimental observations of Zhao et al. [22].

A possibility for practical applications is represented by a flowchart (Fig. 6) that describes the procedure for obtaining the necessary data and then indicates their application in the proposed model, obtaining the damage values and maximum displacement of the structure. By means of the damage value, it is possible to objectively evaluate cracking level of the structural element and then determine its type of rehabilitation. Therefore, further experiments are needed to corroborate and validate the accuracy of the proposed formulation.

Notwithstanding, it is expected that with a larger experimental campaign it will be possible to adjust equations that describe the concrete crack resistance and reinforcement yielding, similar to that performed by Teles et al. [29]. Additionally, with more results it is possible to relate damage ranges with repair levels, similar to the ones proposed by Flórez-López et al. [4] for static loadings.

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