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Water movement in layered soils — A simulation model

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Summary

A simulation model for infiltration of water in layered soils, written in CSMP (Continuous System Modeling Program), is described.

The influence of the occurrence of a compacted layer or a loosened topsoil on the infiltration behavior is checked. It is concluded that this behavior can be predicted if soil parameters are available.

In an appendix special attention is paid to the problem of choosing the proper size of the compartments in which the soil is divided and the necessary averaging procedure.

At last the magnitude of the time steps is discussed.

Introduction

Tillage is practiced to control weeds and to influence soil structure or the physical properties of the soil. These physical properties are of primary importance to plant growth, because they influence the mechanical resistance to root growth, the possible rate of intake of water, the chance of pool formation during rainfall, the availability of the soil water and the amount of oxygen in the soil.

In this paper the main interest is in the movement of water through the soil as influenced by plowing and the possible occurrence of hardpans under the plowed layer.

The variation in volumetric water content (Θ in $cm^3 . cm^{-3}$) of a soil, both in time and space, is usually described by the second order partial differential equation:

$$\frac{\delta \Theta}{\delta t} = \frac{\delta}{\delta x} K(\Theta) \frac{\delta P^*}{\delta x}) \qquad (1)$$

in which $K(\Theta)$ = hydraulic conductivity of the soil in cm . day^{-1}, and $\frac{\delta P^*}{\delta x}$ = gradient of the hydraulic potential in cm $H_2O . cm^{-1}$.

As the hydraulic conductivity, $K(\Theta)$, depends on the volumetric water content, it is impossible to obtain from Eq. 1 an analytical expression for the change of the water content with time and depth, and the problem must be solved by means of numerical methods.

Procedures to compute the infiltration into soils have been developed by Philip (1955) and Hanks and Bowers (1962). The former solution requires a uniform soil and a constant initial water content throughout the soil, the latter one requires constant boundary conditions throughout the computation.

In this paper a numerical method is presented, which yields dynamic temporal and spatial knowledge of the water status of any non-uniform one-dimensional soil system which can be divided into homogeneous layers of describable conductivity and matric suction as a function of the water content.

The method is presented in the language CSMP (Continuous System Modeling Program), which improves readability to a large extent by providing a large number of subroutines, especially for the handling of numerical integration.

The simulation model

The simulation program for the unsaturated flow of water in soils is given in Table 1.

To describe the movement of the soil water, a model of a soil column of unit area, divided into a number of compartments, not necessarily of the same size, is considered.

In Fig. 1 a schematic representation of a slab from the middle of the column is given. For convenience the given compartments are referred to as 1, 2 and 3.

At any moment the volume of water in compartment 2 (VOLW2 in cm^3) is defined as an integral with the formal statement:

VOLW2 = INTGRL(IVOLW2,NFLR2)

in which IVOLW2 = initial amount of water in compartment 2 in cm^3, and NFLR2 = net flow rate into compartment 2 in cm^3 . cm^{-2} . day^{-1}.

The net flow rate is calculated from the flow rates over boundary 1 and 2 (V1 and V2 in cm^3 . cm^{-2} . day^{-1}) with:

NFLR2 = V1 — V2

It is assumed that the water in each compartment is distributed homogeneously, so that the volumetric water content (WC in cm^3 . cm^{-3}) is calculated by dividing the amount of water with the thickness of the compartment (TCOM in cm):

WC1 = VOLW1/TCOM1

WC2 = VOLW2/TCOM2

To calculate the velocities of flow, it is assumed that the flow of water occurs from the middle of one compartment to the middle of the adjacent one and is governed by Darcy's law. This law states that the velocity is in the same direction as and propor-

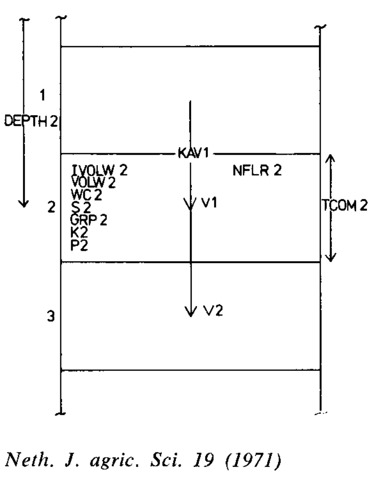

Fig. 1. Schematic representation of a part from the middle of the soil column.

tional to the driving force. Since the driving force is proportional to a potential gradient, the velocities (V1 and V2 in $cm^3 . cm^{-2} . day^{-1}$) are calculated with:

V1 = KAV1 × (P2—P1)/(0.5 × (TCOM1 + TCOM2))
V2 = KAV2 × (P3—P2)/(0.5 × (TCOM2 + TCOM3))

in which KAV1, KAV2 = average conductivity in $cm . day^{-1}$; and P1, P2, P3 = pressure equivalence of the hydraulic potential in cm H_2O.

The average conductivities are calculated from the conductivities of the two compartments involved. When the thickness of the compartments is in the order of centimeters, this method of averaging may be critical. It is shown in Appendix 1 that the arithmetic average is a good choice. Hence:

KAV1 = (K1 + K2)/2
KAV2 = (K2 + K3)/2

The conductivity of each compartment is obtained from an experimentally determined conductivity curve and the volumetric water content of the compartment with:

K1 = AFGEN(KTB1, WC1)
K2 = AFGEN(KTB2, WC2)

The AFGEN function interpolates linearly in the given tabulated functions, entered in the program in the following form:

FUNCTION KTB1 = (0.05,1.E–10), (0.10,1.E–5), (0.15,5.3E–3)

This statement presents the relation between the conductivity, the last figure of each pair and the volumetric water content, the first figure of each pair. By entering different relations for each compartment it is possible to introduce a layered soil. The actual interpolation is then most conveniently done by a TWOVAR function, which enables the simultaneous use of water content and depth as independent variables. This function is not given in the manual, but is described in detail by Luke (1968).

In the same way the matric suction of the compartments (S1, S2 in cm H_2O) is obtained from tabulated functions, which again may differ for different compartments, with:

S1 = AFGEN(SUTB1, WC1)
S2 = AFGEN(SUTB2, WC2)

If an hydraulic head is present, the water in the soil may be above atmospheric pressure but the relation between the volumetric water content of the soil and its matric suction is generally only given in the region below atmospheric pressure. The compressibility of water is so low, that for a completely saturated soil the potential increases practically with an infinite rate with increasing water content. Such an anomaly in the suction curve does not exist in practice, because always some air is included, which is compressed according to Boyle's law. Hence the suction curve may be extended in the region above atmospheric pressure with a finite slope.

To arrive at the hydraulic potential (P1, P2 in cm H_2O), the gravity potential must be added to the matric suction. This gravity potential (GRP1, GRP2 in cm H_2O) is calculated with respect to the depth of the bottom of the column as:

GRP1 = DEPTHT — DEPTH1
GRP2 = DEPTHT — DEPTH2

in which DEPTHT = total length of the column in cm; and DEPTH1, DEPTH2 = distance from the middle of the compartment to the soil surface in cm.

Thus:

P1 = S1 + GRP1
P2 = S2 + GRP2

It can be seen that all variables are calculated from the state of the system at any moment. Hence, the velocities depend only on that state and are independent of each other.

To obtain a reasonable total length of the column and a reasonable solution, twenty compartments are introduced, the first ten being 2 cm each, the next five 4 cm each and the last five 6 cm each. Hence, all calculations must be performed twenty times. This is most easily done with FORTRAN DO loops. To perform in that case all integrations simultaneously, the following formal statements are introduced:

```
VOLW1 = INTGRL(IVOLW1,NFLR1,20)
/REAL IVOLW(20), VOLW(20), NFLR(20)
/EQUIVALENCE (IVOLW(1), IVOLW1), (NFLR(1), NFLR1), (VOLW(1), VOLW1)
```

The flow rates over the first and the last boundary must be calculated separately, according to the imposed boundary conditions. These may be any time-dependent potential or flux rate.

The flow rate into the first compartment (FLR(1) in $cm^3 . cm^{-2} . day^{-1}$) equals the rainfall (RAIN in $cm . day^{-1}$), which is introduced as a function of time, with:

```
RAIN = AFGEN(RAINTB,TIME)
```

Pool formation can be accounted for by introducing another integral, which keeps track of the water on the soil (PLDPTH in cm), as follows:

```
PLDPTH = INTGRL(0.,RAIN–FLR(1))
FLR(1)  = FCNSW(PLDPTH, AMIN1(RAIN, FLOW), AMIN1(RAIN, FLOW),
          FLOW)
FLOW    = KAV(1) × (PS–P(1))/(0.5 × TCOM(1))
```

in which

FCNSW = CSMP function switch, which takes the value of the second argument, if the first argument $<$ 0, the third one, when the first = 0, and the fourth one when the first $>$ 0.

AMIN1 = functional statement, which takes as output the smallest of the two arguments.

PS = pressure equivalence of the hydraulic potential at the soil surface in cm H_2O.

PS is calculated as the sum of the matric suction at the soil surface (SSURF in cm H_2O), the gravity potential (DEPTHT) and the hydrostatic pressure of the water above soil (PLDPTH) with:

```
PS = SSURF + DEPTHT + PLDPTH
```

It is assumed that when there is a layer of water on top of the soil, or when it is raining, there is always a thin layer saturated at the surface. This means that in that case SSURF is always zero.

The number and the thickness of the compartments is choosen in such a way that the phenomenon that is studied does not affect the water content of the last compartment appreciably. Hence the flow rate over the last boundary is then always zero:

```
FLR(NL+1) = 0.
```

in which NL = number of compartments considered.

This constancy of the water content must of course be tested in the actual calculations.

An alternative boundary condition is obtained by assuming a stationary water table at the bottom of the column.

The amount of water infiltrating into the soil column (CUMINF in cm) is obtained by integrating the upper flow with:

```
CUMINF = INTGRL(0.,FLR(1))
```

Since the conductivities are in $cm . day^{-1}$, the controls on the TIMER card are also

```
*****************INITIAL PART OF THE PROGRAM*****************
INITIAL
NOSORT
FIXED NL, I, N, J
PARAMETER     NL = 20.
PARAMETER     GRAV = 1.
*
STORAGE       S(30)
STORAGE       PSTAR(30)
STORAGE       DEPTH(30)
STORAGE       TCOM(30)
STORAGE       COND(30)
STORAGE       FLR(30)
STORAGE       CND(30)
STORAGE       WC(30)
STORAGE       WCI(30)
*******************DEFINITION OF THE INVARIABLE GEOMETRY***************
*             AREA IS SET TO UNITY (CM**2)
TABLE         TCOM(1-20) = 10*2.,5*4.,5*6.
              DEPTH(1) = .5*TCOM(1)
       DO 190 I=2,NL
              DEPTH(I) = DEPTH(I-1)+.5*(TCOM(I-1)+TCOM(I))
  190         CONTINUE
              DEPTHT=0.
       DO 191 N= 1,NL
              DEPTHT = DEPTHT+TCOM(N)
  191         CONTINUE
TABLE         ADEPTH(1-6) = 0.,10.,10.1,12.,12.1,100.
STORAGE       NXYC(6) ,ADEPTH(6),NXYS(6)
FIXED         ITAG1,ITAG2
INCON         ITAG1 = 0
INCON         ITAG2 = 0
**********************DEFINITION OF THE HYDRAULIC PROPERTIES OF SOIL***********
FUNCTION      SUTB1 = .03,300000.,.06,20000.,.09,2800.,.12,900.,.15,   ...
       420.,.18,240.,.21,170.,.24,120.,.27,92.,.30,74.,.33,62.,.36,    ...
       54.,.39,43.,.42,31.,.45,6.,.459,0.,1.459,-6000.
FUNCTION      SUTB2 = .03,300000.,.06,20000.,.09,2800.,.12,900.,.15,   ...
       420.,.18,240.,.21,170.,.24,120.,.27,92.,.30,74.,.33,62.,.36,    ...
       54.,.39,43.,.42,31.,.45,6.,.459,0.,1.459,-6000.
FUNCTION      SUTB3 = .03,900000.,.06,150000.,.09,34000.,.12,900.,     ...
       .15,2500.,.18,950.,.21,440.,.24,210.,.27,120.,.30,77.,.33,48., ...
       .36,24.,.39,2.8,.394,0.,1.39,-7000.
FUNCTION      SUTB4 = .03,900000.,.06,150000.,.09,34000.,.12,900.,     ...
       .15,2500.,.18,950.,.21,440.,.24,210.,.27,120.,.30,77.,.33,48., ...
       .36,24.,.39,2.8,.394,0.,1.39,-7000.
FUNCTION      SUTB5 = .03,39935.,.06,33485.,.09,27035.,.12,20585.,     ...
       .15,14135.,.18,7685.,.21,3190.,.24,1675.,.27,665.,.30,331.,     ...
       .31,258.,.32,212.,.33,175.,.34,143.,.35,116.,.36,94.,.37,75.,   ...
       .38,59.,.39,45.,.40,36.,.41,28.,.42,21.,.43,15.,.44,10.,.45,5.,...
       .46,0.,1.46,-5000.
FUNCTION      SUTB6 = .03,39935.,.06,33485.,.09,27035.,.12,20585.,     ...
       .15,14135.,.18,7685.,.21,3190.,.24,1675.,.27,665.,.30,331.,     ...
       .31,258.,.32,212.,.33,175.,.34,143.,.35,116.,.36,94.,.37,75.,   ...
       .38,59.,.39,45.,.40,36.,.41,28.,.42,21.,.43,15.,.44,10.,.45,5.,...
       .46,0.,1.46,-5000.
FUNCTION      COTB1 = .03,1.1E-7,.06,2.E-6,.09,1.4E-5,.12,4.E-4,.15,   ...
       1.1E-3,.18,2.4E-3,.21,2.5E-3,.24,3.E-3,.27,1.9E-2,.30,7.1E-2,  ...
       .33,1.7E-1,.36,.31,.39,.69,.42,1.68,.45,10.6,.459,11.5
FUNCTION      COTB2 = .03,1.1E-7,.06,2.E-6,.09,1.4E-5,.12,4.E-4,.15,   ...
       1.1E-3,.18,2.4E-3,.21,2.5E-3,.24,3.E-3,.27,1.9E-2,.30,7.1E-2,  ...
       .33,1.7E-1,.36,.31,.39,.69,.42,1.68,.45,10.6,.459,11.5
FUNCTION      COTB3 = .03,1.1E-10,.21,1.1E-10,.24,2.3E-7,.27,2.7E-3,   ...
              .30,.03,.33,.15,.36,.59,.39,.97,.394,1.1
FUNCTION      COTB4 = .03,1.1E-10,.21,1.1E-10,.24,2.3E-7,.27,2.7E-3,   ...
              .30,.03,.33,.15,.36,.59,.39,.97,.394,1.1
FUNCTION      COTB5 = .03,1.5E-10,.17,1.5E-10,.18,6.35E-5,.19,8.72E-5,...
       .20,2.E-4,.21,4.85E-4,.22,8.12E-4,.23,1.E-3,.24,1.68E-3,.25,   ...
       2.37E-3,.26,6.2E-3,.27,1.5E-2,.28,1.88E-2,.29,3.24E-2,.30,     ...
       5.35E-2,.31,8.E-2,.32,.12,.33,.18,.34,.20,.35,.36,.36,.57,     ...
```

Table 1. CSMP program for unsaturated water flow in layered soils.

```
      .37,.73,.38,.86,.39,1.27,.40,1.96,.41,2.24,.42,2.88,.43,3.74,  ...
      .44,3.96,.45,4.2,.46,4.2
FUNCTION      COTB6 = .03,1.5E-10,.17,1.5E-10,.18,6.35E-5,.19,8.72E-5,...
      .20,2.E-4,.21,4.85E-4,.22,8.12E-4,.23,1.E-3,.24,1.68E-3,.25,   ...
      2.37E-3,.26,6.2E-3,.27,1.5E-2,.28,1.88E-2,.29,3.24E-2,.30,     ...
      5.35E-2,.31,8.E-2,.32,.12,.33,.18,.34,.20,.35,.36,.36,.57,     ...
      .37,.73,.38,.86,.39,1.27,.40,1.96,.41,2.24,.42,2.88,.43,3.74,  ...
      .44,3.96,.45,4.2,.46,4.2
TABLE         WCI(1-20) = .17017,.17050,.17084,.17117,.17150,.23897,  ...
              .30600,.30630,.30658,.30685,.30728,.30781,.30836,.30891,...
              .30945,.31020,.3113,.3124,.3135,.3146
      DO 148 I= 1,NL
              IVOLW(I) = WCI(I)*TCOM(I)
  148         CONTINUE
**********************DEFINITION OF THE BOUNDARY CONDITIONS******************
PARAMETER     WCSAT1 = .459
PARAMETER     WCSAT2 = .46
              WCSURF = WCSAT1
              CNDS = TWOVAR(COTB1,6,ADEPTH,NXYC,0.,WCSURF,ITAG1)
DYNAMIC
NOSORT
/     REAL  IVOLW(30),VOLW(30),NFLR(30)
/     EQUIVALENCE (IVOLW(1)'IVOLW1)'(NFLR(1)'NFLR1)'(VOLW(1)'VOLW1)
              VOLW1 = INTGRL(IVOLW1,NFLR1,20)
FUNCTION      RAINTB = 0.,0.,.01,48.,.10,48.,.1001,0.,1.,0.
              RAIN = AFGEN(RAINTB,TIME)
*************CALCULATION OF HYDRAULIC PROPERTIES OF THE LAYERS***********
      DO 3 N = 1,NL
              WC(N) = VOLW(N)/TCOM(N)
      COND(N) = TWOVAR(COTB1,6,ADEPTH,NXYC,DEPTH(N),WC(N),ITAG1)
      S(N) = TWOVAR(SUTB1,6,ADEPTH,NXYS,DEPTH(N),WC(N),ITAG2)
              PSTAR(N) = -1.*S(N) +(DEPTHT-DEPTH(N))*GRAV
    3         CONTINUE
      DO 13 N = 2,NL
              CND(N) =    (COND(N-1)+COND(N))/2.
   13         CONTINUE
      DO 10 N = 2,NL
              FLR(N) = CND(N)*((PSTAR(N-1)-PSTAR(N))/(.5*(TCOM(N)+    ...
              TCOM(N-1))))
   10         CONTINUE
      SSURF= TWOVAR(SUTB1,6,ADEPTH,NXYS,0.,WCSURF,ITAG2)
              CND(1)=(CNDS+COND(1))/2.
              FLOW = CND(1)*(PSTARS-PSTAR(1))/(0.5*TCOM(1))
              FLR(1) = FCNSW(PLDPTH,AMIN1(RAIN,FLOW),AMIN1(RAIN,FLOW),...
              FLOW)
              PLDPTH = INTGRL(0.,RAIN-FLR(1))
              FLR(NL+1) = 0.
      DO 5 N = 1,NL
              NFLR(N) = FLR(N)-FLR(N+1)
    5         CONTINUE
              CUMINF = INTGRL(0.,FLR(1))
***********************OUTPUT CONTROL****************
              A=IMPULS(0.,PRDEL)
              IF (A*KEEP.LT.0.5) GOTO 6
  104 FORMAT (13F10.4/12F10.4)
      WRITE(6,105)
  105 FORMAT(1H ,33HWATERCONTENT FOR DIFFERENT DEPTHS)
      WRITE (6,104)(WC(N),N=1,NL)
      WRITE(6,101)
  101 FORMAT(1H ,45H HYDRAULIC PRESSURE HEAD FOR DIFFERENT DEPTHS)
  106 FORMAT(10F13.4)
      WRITE(6,106) (PSTAR(N),N = 1,NL)
    6         CONTINUE
METHOD        MILNE
FINISH        WC1 = -.5,WC1 = 1.5,TELLER = 10000.
TIMER         FINTIM = 1.0,PRDEL = 0.01
END
STOP
```

Table 1 (continued).

in days. In this case the finish time is 1. day and the interval at which output is requested, defined by a print delay, is 0.01 day, i.e.:

TIMER FINTIM = 1., PRDEL = 0.01

The integration is performed with the predictor-corrector METHOD of MILNE, which chooses its own time step, according to an upper boundary at the difference between the predicted and the corrected value of the fastest changing integral. For the definition of the output, the CSMP PRINT and the FORTRAN WRITE capability are used.

Results

Simulation runs are made with three different soil types representative for the plowed and unplowed light humous sandy soils of the no-tillage experiment in Achterberg (Bakermans and de Wit, 1970). These are the following: an unplowed soil with a saturated conductivity of 4.2 cm . day^{-1}; a soil, consisting of a plowed deck with a saturated conductivity of 11.5 cm . day^{-1}, overlying an unplowed subsoil with a saturated conductivity of 4.2 cm . day^{-1}; the same plowed soil, but with a compacted plow zone between 10 and 12 cm having a saturated conductivity of 1.1 cm . day^{-1}.

While no data were available for the unsaturated conductivities of these soils, use was made of the experimental formula of Rijtema (1969), to calculate the values of the conductivity from the suction curves.

The suction curves (pF-curves) and conductivity curves are given in tabulated form in the program in Table 1. For the unplowed soil in SUTB5 and COTB5, for the compacted layer in SUTB3 and COTB3 and for the plowed soil in SUTB1 and COTB1.

The simulated rain is gradually increasing from zero at the onset of the simulation to 20 mm . h^{-1} at time 0.01 day, stays then the same on to 0.10 day and is further on absent.

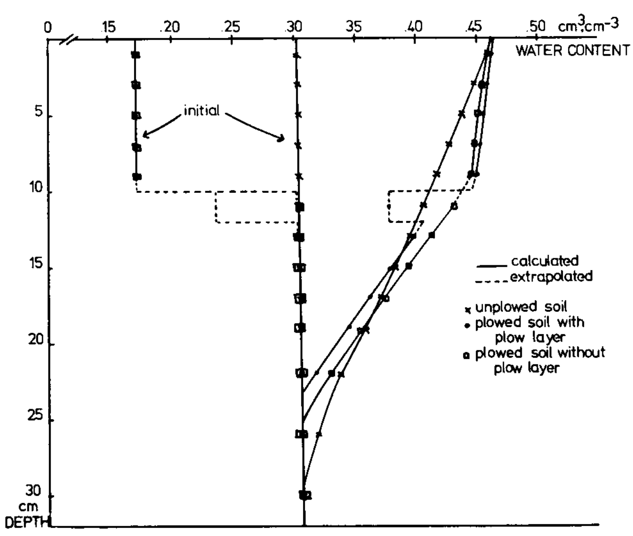

Fig. 2. Water content profiles for the three different soil types at time zero (initial) and at the end of the simulated rain, i.e. 0.10 days.

In all cases the profile started at an equilibrium situation, assuming a ground water table at 3.00 m, which means that the top layer of the soil is about at field capacity, but has not for all soil types the same water content. The initial water content profiles for the first 28 cm of the profile are shown in Fig. 2. As is explained earlier, the water contents are calculated in the middle of the compartments. In the homogeneous soil these points are connected by a solid line. On the boundary between different layers the exact shape is not known and the extrapolation is given as a dashed line. If one is interested in more detail, smaller compartments have to be introduced.

In Fig. 2 are also shown the water content profiles at 0.1 day, hence at the end of the simulated rainfall.

Least water has entered the unplowed soil with the lowest saturated conductivity. The water has however reached a greater depth, because the initial water content was highest.

Plowing of the soil leads to a faster intake of water, whereas the greater steepness of the conductivity curve causes a somewhat steeper wetting front. The occurrence of the hardpan prevents, to a certain extent, leakage to the subsoil. The difference in maximum intake rate of the water is demonstrated in Fig. 3, where the height of the water above soil is given as a function of time, for the three soil types. The unplowed soil is covered by a pool of 2.25 cm at the end of the rainfall. There remains water on that soil until 0.35 day, so that the upper part of the soil is still saturated at that time, which may result in oxygen deficiency.

After plowing the height of the water above soil does not exceed 1 cm. The compacted layer causes a somewhat slower intake, which leads to a more pronounced pool influence, although the effect is small.

It is obvious that although the total pore volume is little affected by plowing, there is a clear distinction in behaviour under heavy rain between the plowed and the unplowed soil. This is entirely due to the complete different pore size contribution, i.e. the percentage of large pores being greater in the plowed soil (Ouwerkerk and Boone, 1970).

The situation 7.2 hours after the end of the simulated rainfall is given in Fig. 4. As can be expected from Fig. 3, all three profiles started already drying at the top, while the lower part of the profile is still wetting. It should be noted that the effect of hysteresis is not taken into account, the same suction curve being used for both processes.

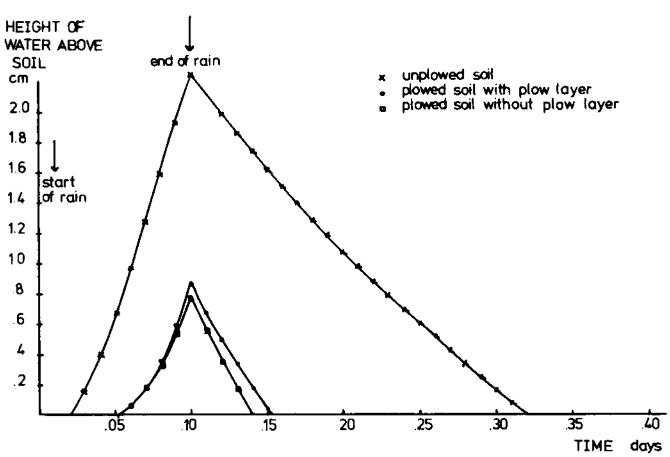

Fig. 3. Height of water above the soil, for the three soil types, as a function of time.

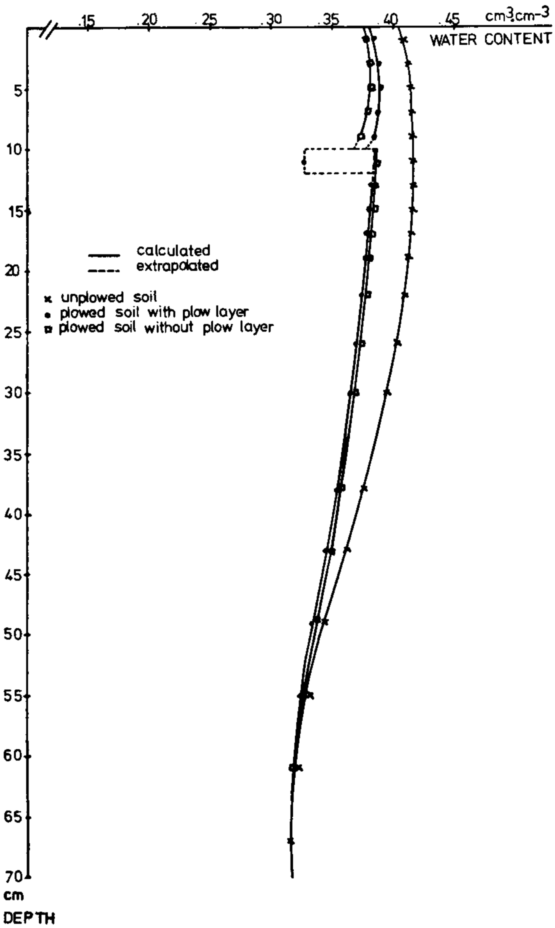

Fig. 4. Water content profiles for the three soil types 7.2 hours after the end of the simulated rain.

The highest water content is found in the unplowed soil, caused by the extended occurrence of the pool.

The differences between both plowed soils are not striking. However the hardpan leads to a slower drainage of the top soil, so that more water is left there.

Although from these figures significant differences in behavior are shown among the various treatments, it is difficult to draw definite conclusions because of the uncertainty in the magnitude of the soil parameters. It is however shown that, if sufficient accurate data are available, it is possible to predict the behavior of the soil under different moisture regimes and management practices.

Appendix 1: The average conductivities and the size of the compartments

The method of Milne was used to perform the integration along the time axis. This method chooses its own time step (Appendix 2), according to a rather strict error criterion, which is described in the CSMP Manual. Accordingly the steps are small enough to consider the integration in time as a continuous process.

However the integration in depth is done in a discontinuous manner because the column is divided in a relatively small number of compartments. This means that the calculated flow of water throughout the column may not only depend on soil parameters but also on the depth of the compartments and the method of averaging the conductivities between the compartments. The most simple way to evaluate whether artefacts are introduced, is by executing the program for smaller and smaller compartment sizes until the results do not change appreciably any more.

Simulated results with compartment sizes of 4, 2, 1 and 0.5 cm are given in Fig. 5, for the unplowed soil, to show that the present compartment size of 2 cm gives acceptable results.

The method of averaging the conductivity between two compartments and the size of the compartments may also be evaluated in another way. For this purpose the partial differential equation in which Darcy's law and the conservation equation are combined is considered:

$$\frac{\delta \Theta}{\delta t} = \frac{\delta}{\delta x} (K(\Theta) \frac{\delta P^*}{\delta x}) \tag{1}$$

in which: Θ = volumetric water content in $cm^3 . cm^{-3}$; t = time in seconds; x = distance in cm, positive in the direction of flow; $K(\Theta)$ = hydraulic conductivity in $cm . sec^{-1}$; P^* = the pressure equivalence of the hydraulic potential in cm H_2O.

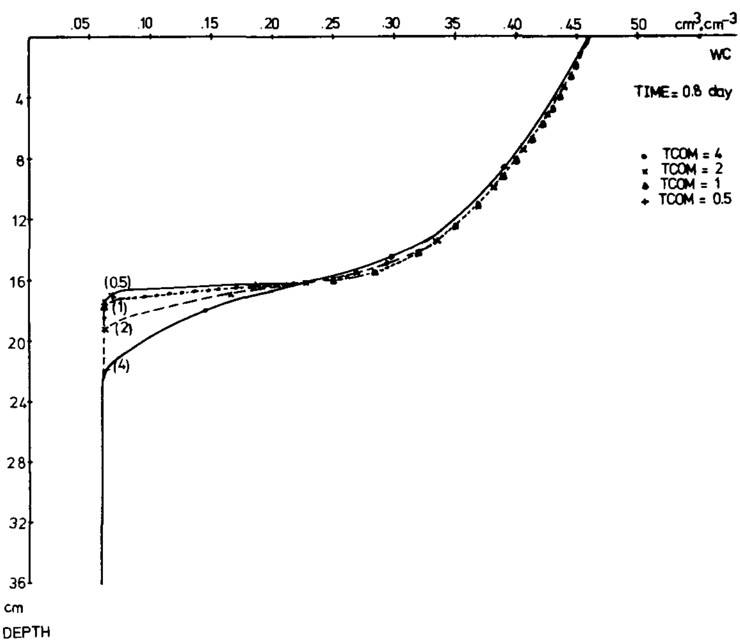

Fig. 5. The influence of the size of the compartments on the infiltration profile.

For vertical flow the hydraulic potential in this equation is equal to the sum of the matric potential and the gravity potential.

Replacing the gradient in matric potential by a gradient in moisture content the equation reads:

$$\frac{\delta\Theta}{\delta t} = \frac{\delta}{\delta x}\left(D(\Theta)\frac{\delta\Theta}{\delta x} + K(\Theta)\frac{\delta h}{\delta x}\right) \tag{2}$$

in which: $D(\Theta)$ = diffusivity in $cm^2 . sec^{-1}$ = $K(\Theta)/C(\Theta)$; $C(\Theta)$ = the differential moisture capacity $d\Theta/dP_v$ in $(cm\ H_2O)^{-1}$; P_v = matric potential in cm H_2O; h = position in the gravity field.

In the absence of gravity influence, as for horizontal flow, the equation reduces to:

$$\frac{\delta\Theta}{\delta t} = \frac{\delta}{\delta x}\left(D(\Theta)\frac{\delta\Theta}{\delta x}\right) \tag{3}$$

For a uniform non-swelling soil with a uniform initial water content and wetted at one side, the boundary conditions are:

$\Theta = \Theta_i$ for $x = 0$ and $t \geqslant 0$ (f = final)

$\Theta = \Theta_f$ for $x > 0$ and $t = 0$ (i = initial)

By applying the Boltzmann transformation, $1 = x/\sqrt{t}$, to Eq. 3 and the boundary conditions, the partial differential equation in x and t, is transformed into an ordinary second-order differential in 1:

$$\frac{d}{d1}\left(D(\Theta)\frac{d\Theta}{d1}\right) + \frac{1}{2}\frac{d\Theta}{d1} = 0. \tag{4}$$

with boundary conditions: $\Theta = \Theta_i$ for $1 = \infty$; $\Theta = \Theta_f$ for $1 = 0$.

Eq. 4 can be rewritten as:

$$\frac{d^2\Theta}{d1^2} = -1./D(\Theta)\left(\frac{1}{2}\frac{d\Theta}{d1} + \frac{dD(\Theta)}{d1}\frac{d\Theta}{d1}\right) \tag{5}$$

This equation may be solved with a CSMP program with 1 as the independent semi-continuous variable. This results in a relation between 1 and Θ, and this relation may be compared with the similar relation obtained from the 'compartmentalized' soil, as discussed in the paper. In this way it can be judged whether the method of averaging between the compartments and whether the size of the compartments are reasonable.

The relation between 1 and Θ is obtained by integrating Eq. 5 twice, so that the dynamic part of the CSMP program reads as follows:

WC2D = —1./D × (L/2 × WC1D + D1D × WC1D) (6)

Eq. 6 for the second-order differential of the water content is identical to Eq. 5, WC2D standing for $d^2\vartheta/d1^2$, WC1D for $d\vartheta/d1$ and D1D for dD/d1. The variable L is introduced as the independent variable with the statement:

```
RENAME TIME = L
```

The differential quotient of D is calculated with the CSMP function:

```
D1D = DERIV(D1DI,D)
```

D1DI being equal to $(dD/d1)_{1=0}$.

The first-order differential quotient is then given by:

$$WC1D = INTGRL(WC1DI,WC2D) \quad (7)$$

in which WC1DI stands for $(d\Theta/d1)_{1=0}$.

The value of WC itself is obtained by:

$$WC = INTGRL(WCI,WC1D) \quad (8)$$

in which WC stands for Θ and WCI for Θ_f.

Philip (1955) used the sorptivity to characterize a soil with respect to its infiltration behaviour. The sorptivity (S in cm . sec$^{-\frac{1}{2}}$) is defined as:

$$S = \int_{1=0}^{\infty} (\Theta - \Theta_i)d1 \quad (9)$$

The sorptivity is calculated in the program with:

$$S = INTGRL(0.,WC) \quad (10)$$

During the computation the value of Θ_i is not known, so that the value of S obtained from Eq. 10 must be reduced afterwards by the rectangle $\Theta_i \times 1$. Of course equal values for S do not imply similar shapes of the $\Theta - 1$ relations.

The initial values D1DI and WC1DI are needed to start the calculation. These two values are connected by means of the chain rule:

$$(dD(\Theta)/d1)_{1=0} = (d\Theta/d1)_{1=0} \times (dD(\Theta)/d\Theta)_{1=0}.$$

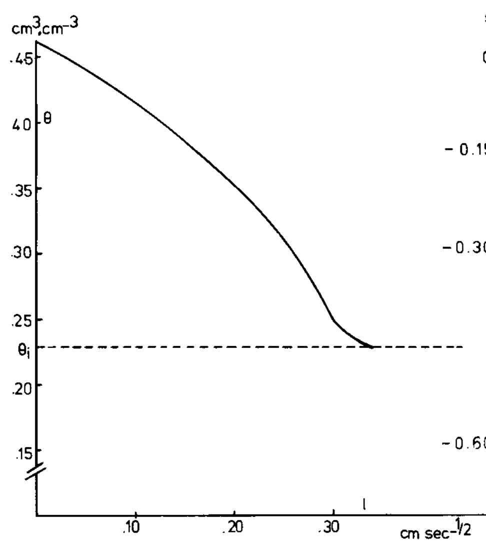

Fig. 6. The relation between $x\sqrt{t}$ (l) and water content (ϑ) with an initial water content (ϑ_i) of 0.228.

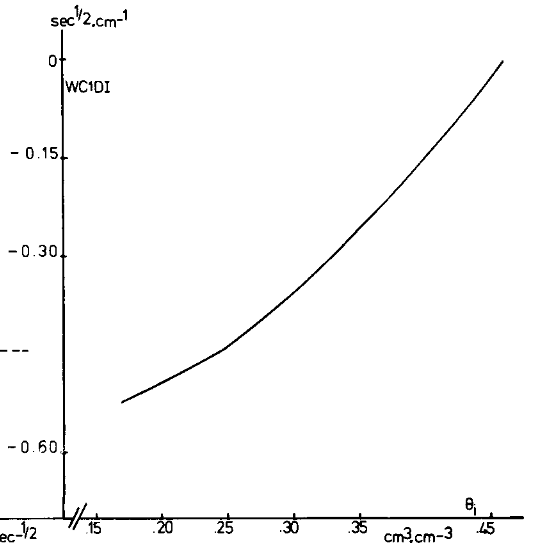

Fig. 7. The relation between the initial slope of the ϑ–l curve (WC1DI) and the initial water content (ϑ_i).

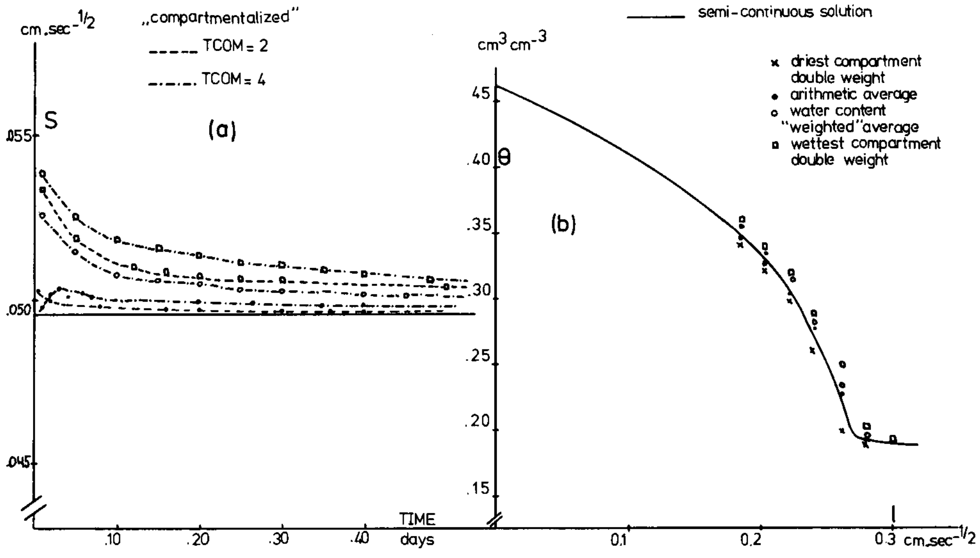

Fig. 8. The sorptivity (S) as a function of time (a) and the relation between $x\sqrt{t}$ (l) and water content (Θ) (b) for different averaging procedures compared with the semi-continuous solution.

The value of $(dD(\Theta)/d\Theta)_{l=0}$ is known, because the relation between D and Θ is known. Choosing a value for $(d\Theta/dl)_{l=0}$ provides a value for $(dD(\Theta)/dl)_{l=0}$.

Introducing in the CSMP program an arbitrarily chosen value of WC1DI, a relation between Θ and l is obtained, as shown in Fig. 6, for WC1DI = — 0.425. The constant

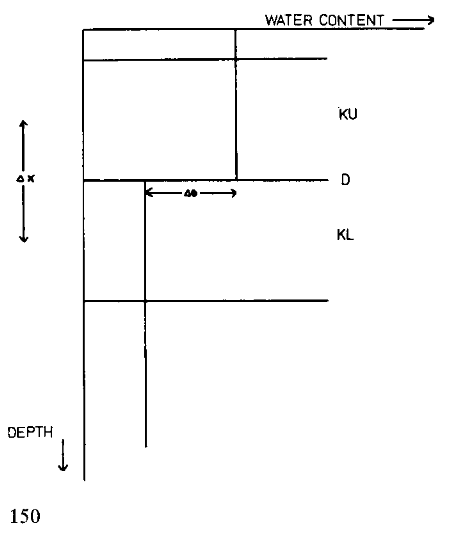

Fig. 9. Schematic representation of two adjacent compartments during infiltration.

value Θ_i, which is approached with increasing 1 equals the initial water content of the soil.

By executing the program several times, with a range of initial slopes, the relation between WC1DI and Θ_i is found. This relation is graphically presented in Fig. 7, for a final water content $\Theta_f = 0.46$ $cm^3 . cm^{-3}$ and the unplowed soil.

To obtain the relation between Θ and 1 for any given initial water content (Θ_i) of the soil column, the proper value of WC1DI is read from Fig. 7 and entered in the CSMP program. The value of the sorptivity (S) is also obtained in this manner. In Fig. 8a this sorptivity is compared with the sorptivity obtained from the 'compartimentalized' program, using three averaging procedures for the conductivities and compartment sizes of 2 and 4 cm. At early stages the values of the 'compartimentalized' program differ considerably from the proper value, especially with the coarse grid, because of the small number of compartments involved. In all cases however, the sorptivities approach the proper value even with the largest compartments. It appears that the arithmetic average gives the best results within the shortest time.

In Fig. 8b the Θ–1 curves for the three methods of averaging are compared with the one obtained from the semi-continuous integration. Although there are small differences at the dry end of the column, the arithmetic average gives here also reasonable results.

Appendix 2: The magnitude of the time step

When the method of Milne or the method of Runge-Kutta is used for integration, the CSMP program chooses its own time step, according to a rather strict error criterion. In the examples given here, the time step varies between 4.5×10^{-6} and 6.25×10^{-4} day.

These small time steps are due to the high values of the diffusivity, especially at high water contents. The size of the time step itself may be evaluated as follows.

Consider a situation as presented in Fig. 9, where a soil at the top touches a somewhat drier soil at the bottom. In the absence of gravity influence, the net flow rate into the bottom compartment equals:

$$\text{NFL} = \text{D} \times \frac{\triangle\Theta}{\triangle \text{x}}$$, since the flow out of that compartment is zero.

The system will start to oscillate when in one time step the change in water content is greater than $0.5 \times \triangle\Theta$. This implies that:

$$\text{D} \times \frac{\triangle\Theta}{\triangle \text{x}} \times \triangle \text{t} = \triangle\Theta \times \triangle \text{x} \times 0.5 \qquad \text{or}$$

$$\triangle \text{t} = \frac{(\triangle \text{x})^2}{\text{D}} \times 0.5$$

Hence, in the absence of gravity, the time step is proportional to the thickness of the compartments squared and depends furthermore on the diffusivity, which depends again on the water content. For a given soil it is obvious that the same accuracy is obtained after a short time (t) at a shallow depth (x), as after a longer time (nt) at a greater depth (n^2x). This reasoning holds only if the system is stability controlled as is the case with these distributive systems.

If gravity is involved, the net flow rate into the lower compartment equals the sum of the net diffusion flow and the net gravitational flow:

$$NFL = NDF + NGF$$

The net diffusion flow is still the same:

$$NDF = D \times \frac{\triangle \Theta}{\triangle x}$$

The net gravitational flow equals:

$$NGF = (KL–KU) \times 1/1$$

in which KL = hydraulic conductivity of the lower compartment, and KU = hydraulic conductivity of the upper compartment.
As KU may be approximated by:

$$KU = KL - \frac{dK}{d\Theta} \times \triangle \Theta,$$

in which $\frac{dK}{d\Theta}$ is the slope of the conductivity curve, the total net flow rate equals:

$$NFL = D \times \frac{\triangle \Theta}{\triangle x} + \frac{dK}{d\Theta} \times \triangle \Theta$$

This system will start to oscillate when:

$$(D \times \frac{\triangle \Theta}{\triangle x} + \frac{dK}{d\Theta} \times \triangle \Theta) \times \triangle t = \triangle \Theta \times \triangle x \times 0.5$$

or

$$\triangle t = \frac{0.5\,(\triangle x)^2}{D + dK/d\Theta \times \triangle x}$$

From the foregoing equation it is clear that in the presence of gravity influence the time step is proportional to $\triangle$x–squared in situations where

$$D >> dK/d\Theta \times \triangle x$$

and is proportional to $\triangle$x in situations where:

$$dK/d\Theta \times \triangle x >> D$$

The given formula applied to the saturated unplowed soil from this paper gives that the time step is proportional to $\triangle$x in situations where:

$$\triangle x >> D \times d\Theta/dK, \text{ i.e.}$$

$$\triangle x >> 4200 \times \frac{0.46}{4.24} = 453.6 \text{ cm}$$

TCOM (cm)	DELT (days)
0.5	1.95×10^{-4}
1	7.85×10^{-4}
2	3.25×10^{-3}
4	1.25×10^{-2}

Table 2. The magnitude of the time step for simulation runs with different compartment sizes. Calculated as the average between time = 0. and 0.5 day.

This means that for all relevant situations in the scope of this paper the time step is proportional to $\triangle$x squared, as is shown in Table 2 for the unplowed soil.

Only in soils where the diffusivity at saturation is very low, the time step becomes proportional to $\triangle$x.

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