Biomathematics
Innovation
Group
Modeling Biological Phenomena

Quantifying the Shape of a Plant (*Brassica rapa*) During Growth Under Different Light Intensities Using Fractal Geometry

Matthew J. Cotroneo, SUNY Geneseo

GENESEO

Introduction

The process by which plants allocate resources to different structures is guided by complex underlying mechanisms. Understanding how plants adapt to stressors by varying allocation can provide important information for developing sustainable agricultural processes. The goal of this study was to model and parameterize how the structure of plants changes in response to varying light intensities, from low insufficient light levels up to light levels certainly sufficient for optimal growth.

The structure of plants was captured by their fractal dimension. Fractal dimension, a measure of how objects fill space, can provide a wholistic understanding of plant shape and how that shape changes as the plants react to their environments. Std. Strain *Brassica rapa* are a model organism for assays into plant morphology as their quick life cycle (less than 1 month under ideal circumstances) allows for rapid experimentation. Furthermore, extensive use in scientific studies means that these plants are well understood and can provide robust, repeatable data.

Methods

In order to obtain real fractal dimension growth curves for a plant, Std. Strain *Brassica rapa* were grown under three different lighting conditions over 23 days with eight replicates each. Light conditions varied based on intensity and were as follows: low (40 µEinsteins), medium (75 µEinsteins) and high (100 µEinsteins). Plants in each treatment were scanned using a NextEngine 3D Laser Scanner UHD with a resolution of 16K points/in^2 at 9, 12, 16, 19 and 23 days after planting. By the end of the 23 days, plants had reached the end of their growth cycle. An algorithm was developed to determine the fractal dimension of each scan using a 3d box-counting method. The box-counting method involves determining the minimum number of cubes, N_ϵ, with side length ϵ required to completely cover the object. This process is repeated for increasingly small ϵ. Then, a log-log plot of N_ϵ vs $1/\epsilon$ is constructed for each of these coverings and the fractal dimension, D, is defined as the slope of the data.

Due to the imperfect nature of the LIDAR scanner, although the overall structure of the plant is well captured, small discontinuities are sometimes present in the portion of scans representing stems and petioles (Fig. 1). As a result the log-log plot of N_ϵ vs $1/\epsilon$ is not a single linear line, but rather possesses two or three distinct slopes depending on the range of $1/\epsilon$ considered. In order to best capture the structure of these plants without noise from the discontinuities, D, was determined using intermediate values of ϵ, ignoring cube counts for cubes with sides longer than 10% of the plant's height as well as cubes with sides smaller than the thickness of a petiole.

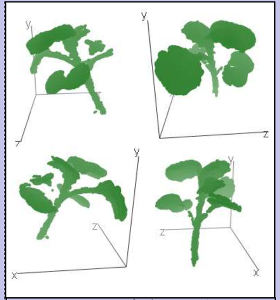


Fig. 1. Four views of a (rotating by 90° between each view) 3D reconstruction of a young plant scanned . Notice that while most of the structure is well captured by the scanner, there are a few spots in thin petioles where data was not properly captured. This plant has fractal dimension $D = 1.250481$.

Experimental Results

Fig 2a. Plot of fractal dimensions over time for plants grown under the low light condition. A nonlinear regression of the form $FD = a(1 - e^{c*day})$ with $a = 1.66002 \pm 0.7974$, ($p = 1.55 * 10^{-13}$) and $c = 0.13769 \pm 0.02542$, ($p = 4.63 * 10^{-5}$) was fit to the mean fractal dimension of all the plants at each time step.

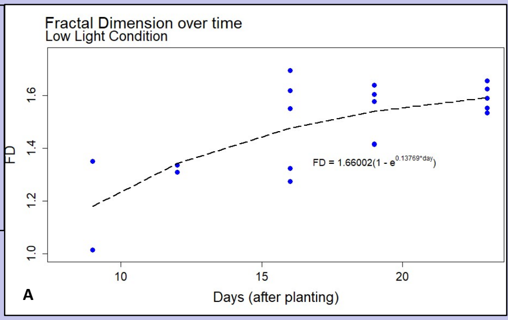


Fig 2b. Plot of fractal dimensions over time for plants grown under the mid light condition. A nonlinear regression of the form $FD = a(1 - e^{c*day})$ with $a = 1.64012 \pm 0.7487$, ($p = 8.40 * 10^{-13}$) and $c = 0.16009 \pm 0.02542$, ($p = 5.395 * 10^{-5}$) was fit to the mean fractal dimension of all the plants at each time step.

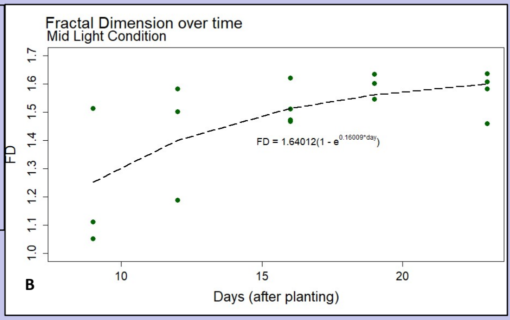
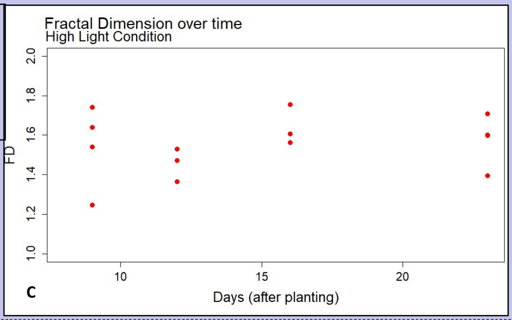


Fig 2c. Plot of fractal dimensions over time for plants grown under the high light condition. The fractal dimension of plants over time here was not well fit by any regression.



Model

A diffusion-limited aggregation model was developed to simulate plant growth. At the top of the simulated space, light intensity was set to 74 or 42 (out of 100) to simulate the mid and low light conditions, respectively. Light intensity in each cell slightly decreases with increasing distance from the top level. Stem pieces are added vertically in a single column at a constant rate; leaf pieces, are added at each time step with probability NL, undergo a random walk from the edge of the simulated space until they attach to the stem. Leaf pieces already attached can spawn new a leaf piece horizontally adjacent to themselves with probability GL. GL is scaled by the light intensity at the existing leaf's location. Furthermore, new leaf pieces can only be added a maximum of MD cells from the stem. Each plant is allowed to grow for 14 time steps in all simulations. After time step 9, no random walk leaves are added, new leaf pieces can only be produced from existing leaf pieces. The fractal dimension for plant is calculated at each time step using the same method as for the real plants.

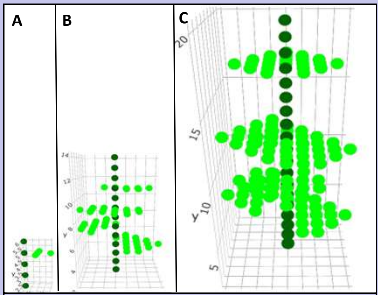


Fig 3. A visual representation of a plant grown with conditions $NL = 0.85, GL = 0.95, MD = 5$ and light intensity 74. **A:** The plant at time step 2 with $D = 1.0803$. **B:** The plant at time step 7 with $D = 1.4687$. **C:** The plant at time step 14 with $D = 1.5429$.

Model Results

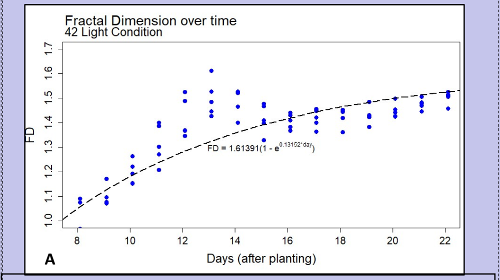


Fig 4a. Plot of fractal dimensions over time for simulated plants grown under the 42 light condition, grown with parameters $NL = .95$, $GL = .75, MD = 6$. A nonlinear regression of the same form as that for the low light real plants was fit to the data with $a = 1.63914 \pm 0.03110$, ($p < 2 * 10^{-16}$) and $c = 0.13769 \pm 0.02542$, ($p < 2 * 10^{-1}$). Within errors specified, these values agree with those for the real low light plants' fit parameters.

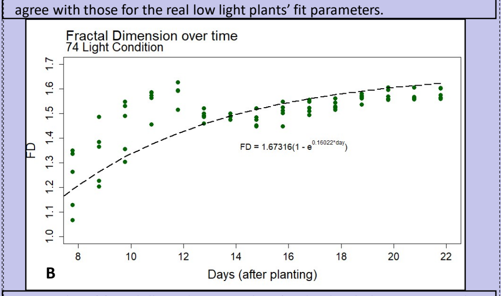


Fig 4b. Plot of fractal dimensions over time for simulated plants grown under the 74 light condition, grown with parameters $NL = .75$, $GL = .85, MD = 5$. A nonlinear regression of the same form as that for the mid light real plants was fit to the data with $a = 1.67316 \pm 0.03171$, ($p < 2 * 10^{-1}$) and $c = 0.16022 \pm 0.00966$, ($p < 2 * 10^{-16}$). Within errors specified, these values agree with those for the real mid light plants' fit parameters.

Conclusion

Results show that the model can generate plants with a similar fractal dimension to real life plants. Furthermore, the model can reliably grow simulated plants based on light intensity available and by varying the parameters $NL, GL,$ and MD (Fig. 4a, Fig. 4b). No attempt was made to generate plants under the high light condition as no fit was possible for those data. Altogether, fractal dimension proves to be an effective measure of plant complexity and morphology and this model shows promise for simulating fractal dimension over time under varying parameters.

Work for future endeavors will include parameterizing the response of *Brassica rapa* plants to other conditions such as below ground nutrients available. Assays into the sensitivity of the model to changes in $NL, GL,$ and MD may also reveal further information on what influences how plants adapt to available light intensity.

Acknowledgements

Special thanks to Dr. Gregg Hartvigsen and Dr. Christopher Leary for input and support throughout the research process. Further thanks to Dr. George Briggs for the use of his lighting rig.

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