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First *Kepler* results on compact pulsators – VIII. Mode identifications via period spacings in g-mode pulsating subdwarf B stars

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ABSTRACT

We investigate the possibility of nearly equally spaced periods in 13 hot subdwarf B (sdB) stars observed with the *Kepler* spacecraft and one observed with *CoRoT*. Asymptotic limits for gravity (g-)mode pulsations provide relationships between equal-period spacings of modes with differing degrees ℓ and relationships between periods of the same radial order n but differing degrees ℓ. Period transforms, Kolmogorov–Smirnov tests and linear least-squares fits have been used to detect and determine the significance of equal-period spacings. We have also used Monte Carlo simulations to estimate the likelihood that the detected spacings could be produced randomly.

Period transforms for nine of the *Kepler* stars indicate $\ell = 1$ period spacings, with five also showing peaks for $\ell = 2$ modes. 12 stars indicate $\ell = 1$ modes using the Kolmogorov–Smirnov test while another shows solely $\ell = 2$ modes. Monte Carlo results indicate that equal-period spacings are significant in 10 stars above 99 per cent confidence, and 13 of the 14 are above 94 per cent confidence. For 12 stars, the various methods find consistent period spacings to within the errors, two others show some inconsistencies, likely caused by binarity, and the last has significant detections but the mode assignment disagrees between the methods.

We use asymptotic period spacing relationships to associate observed periods of variability with pulsation modes for $\ell = 1$ and 2. From the *Kepler* first-year survey sample of 13 multiperiodic g-mode pulsators, five stars have several consecutive overtones making period spacings easy to detect, six others have fewer consecutive overtones but period spacings are readily detected, and two stars show marginal indications of equal-period spacings. We also

examine a g-mode sdB pulsator observed by *CoRoT* with a rich pulsation spectrum, and our tests detect regular period spacings.

We use Monte Carlo simulations to estimate the significance of the detections in individual stars. From the simulations, it is determined that regular period spacings in 10 of the 14 stars are very unlikely to be random, another two are moderately unlikely to be random and two are mostly unconstrained.

We find a common $\ell = 1$ period spacing spanning a range from 231 to 272 s allowing us to correlate pulsation modes with 222 periodicities and that the $\ell = 2$ period spacings are related to the $\ell = 1$ spacings by the asymptotic relationship $1/\sqrt{3}$. We briefly discuss the impact of equal-period spacings which indicate low-degree modes with a lack of significant mode trappings.

Key words: stars: oscillations – subdwarfs.

1 INTRODUCTION

Asteroseismology is the process in which stellar pulsations are used to discern the physical condition of stars. The process includes matching stellar models to observations, associating periodicities with pulsation modes and examining where the models succeed and fail. Slight mismatches between observations and models can provide insights into new physics or add constraints to previously assumed conditions. Examples include using deviations from sinusoidal variations to constrain convective depths (Montgomery 2005) and using deviations from equally spaced overtones to discern interior composition gradients (e.g. Kawaler & Bradley 1994; Degroot et al. 2010). In many cases, the best results are achieved for stars with highly constrained observations. Observational constraints can include the usual spectroscopic measurements (log g, $T_{\rm eff}$ and some compositional constraints), number and characterization of periodicities (frequency or period, amplitude, phase and pulse shape) as well as frequency multiplets and equal-period spacings which associate specific periodicities with pulsation modes.

In brief, non-radial pulsations (periodicities) are characterized by three quantized numbers (modes) n, ℓ and m. These represent the number of radial nodes (n), surface nodes (ℓ) and azimuthal surface nodes (m). In the asymptotic limit for $n \gg \ell$, gravity (g) modes should be equally spaced in period for consecutive values of n according to the expression

$$\Pi_{\ell,n} = \frac{\Pi_o}{\sqrt{\ell(\ell+1)}}n + \epsilon, \tag{1}$$

where Π_o and ϵ are constants, in seconds (see Unno et al. 1979; Tassoul 1980; Smeyers & Tassoul 1987; Aerts, Christensen-Dalsgaard & Kurtz 2010, among others). The period spacings between two consecutive overtones are

$$\Delta\Pi_\ell = \frac{\Pi_o}{\sqrt{\ell(\ell+1)}}, \tag{2}$$

where $\Delta\Pi_\ell = \Pi_{\ell,n+1} - \Pi_{\ell,n}$. Because of geometric cancellation (Dziembowski 1977; Reed, Brondel & Kawaler 2005), $\ell = 1$ and 2 modes are the most likely non-radial modes to be observed and the specific relations between them are

$$\Pi_{n,\ell=2} = \frac{\Pi_{n,\ell=1}}{\sqrt{3}} + C, \tag{3}$$

where C is a constant that is expected to be small and is zero if $\epsilon_2 = \epsilon_1$, and

$$\Delta\Pi_{\ell=2} = \frac{\Delta\Pi_{\ell=1}}{\sqrt{3}}. \tag{4}$$

The asymptotic approximation applies to periods within completely homogeneous stars. However real stars, particularly compact stars for which gravitational settling is important, and hot stars, in which radiative levitation is important, develop compositional discontinuities where the mean molecular weight changes. The *transition zones* of compositional changes can work as a reflective wall which confines pulsations to specific stellar regions. This 'trapping' of pulsation modes changes the spacing between consecutive overtones compared to the average spacing $\Delta\Pi$. Trapped modes can be used to deduce structural changes associated with chemical transitions (e.g. Kawaler & Bradley 1994; Costa et al. 2008).

The period spacing relations are independent of m and are applied under the assumption of $m = 0$ periodicities. In the case of extremely slowly rotating stars, this may be a valid assumption. However for stars which complete several revolutions within a set of observations, pulsations will create frequency multiplets. To first order, these multiplets will have $2\ell + 1$ components spaced at

$$\nu_{n,\ell,m} = \nu_{n,\ell,0} + m\Omega\left(1 - C_{n,\ell}\right),$$

where Ω is the rotation frequency and $C_{n,\ell}$ is the Ledoux constant (Ledoux 1951).

In this paper, we apply equations (1)–(4) to g-mode pulsations observed in hot subdwarf (sdB) variables. sdB variables were first discovered in 1996 and now consist of two well-established classes. These are the short-period pressure (p-)mode pulsators which are designated V361 Hya stars (Kilkenny et al. 1997) and longer period gravity (g-)mode pulsators designated V1093 Her stars (Green et al. 2003). There are also hybrid pulsators, sometimes called DW Lyn stars after that prototype (Schuh et al. 2006), which show both types of variations. About 50 V361 Hya pulsators have been detected (Østensen et al. 2010a) with about half receiving various amounts of follow-up data (see e.g. Reed et al. 2007). However, observational constraints on pulsation modes are extremely rare for the V361 Hya class, occurring only twice using multiplets (Reed et al. 2004; Baran et al. 2009). Time-resolved spectroscopy, sometimes coupled with multicolour photometry, has had some limited success (Telting & Østensen 2004, 2006; Reed et al. 2009; Baran et al. 2010). See sections 2.2–2.4 of Østensen (2010) for a recent review of these methods. The lack of observational constraints has led model-matching efforts to proceed by using the forward method, which consists of matching observed periods to those of models, with the closest fit, within spectroscopic constraints, being deemed the correct one (for a review, see Charpinet et al. 2009). Progress on g-mode pulsators has been slow because of the difficulties in observing many pulsation cycles for periodicities of 1–3 h in extremely blue stars from the ground. With the acquisition

Table 1. Properties of the stars in this paper. Columns 1 and 2 supply the KIC number and a more common name, Column 3 lists the observing period in quarter and month, Columns 4 and 5 provide the *Kepler* magnitudes (Kp) and estimated contamination factors (F_{cont}), Columns 6 and 7 supply spectroscopic parameters, Column 8 lists if the star is in a known binary (RE = reflection effect binary, EB = eclipsing binary, EV = ellipsoidal variable binary and the inferred companion is in parentheses) and Column 9 lists references from this series of papers. RHØ indicates Østensen et al. (2010b) and CH10 indicates Charpinet et al. (2010).

KIC	Name	Q	Kp	F_{cont}	T_{eff}	$\log g$	Binary	Ref
2697388	J19091+3756	2.3	15.39	0.149	23.9(3)	5.32(3)	–	I, III
2991403	J19272+3808	1	17.14	0.601	27.3(2)	5.43(3)	RE(dM)	I, V
3527751	J19036+3836	2.3	14.86	0.081	27.9(2)	5.37(9)	–	I, III
5807616	KPD 1943+4058	2.3	15.02	0.332	27.1(2)	5.51(2)	–	I, III, IV
7664467	J18561+4319	2.3	16.45	0.879	26.8(5)	5.17(8)	–	I, III
7668647	FBS 1903+432	3.1	15.40	0.226	27.7(3)	5.45(4)	–	VI, VII
8302197	J19310+4413	3.3	16.43	0.256	26.4(3)	5.32(4)	–	VI, VII
9472174	2M1938+4603	0	12.26	0.022	29.6(1)	5.42(1)	EB(dM)	I, RHØ
10001893	J19095+4659	3.2	15.85	0.710	26.7(3)	5.30(4)	–	VI, VII
10553698	J19531+4743	4.1	15.13	0.385	27.6(4)	5.33(5)	–	VI, VII
10670103	J19346+4758	2.3	16.53	0.450	20.9(3)	5.11(4)	EV(WD)	I, III
11179657	J19023+4850	2.3	17.06	0.129	26.0(8)	5.14(13)	RE(dM)	I, V
11558725	J19265+4930	3.3	14.95	0.028	27.4(2)	5.37(3)	–	VI, VII
–	KPD 0629−0016	–	14.91[a]	–	27.8(3)	5.53(4)	–	CH10

[a] *CoRoT* observation.

of long time series photometric data from satellites, such as *Kepler* and *CoRoT*, detailed asteroseismology of the V1093 Her pulsators is now possible.

The *Kepler* spacecraft has a primary mission to find Earth-size planets within the habitability zone around Sun-like stars (Borucki et al. 2010). To do this, the spacecraft continuously examines roughly 150 000 stars in search of transits. As a byproduct of that search, high-quality photometric observations are obtained which have proven extremely useful for the study of variable stars (Koch et al. 2010; Prsa et al. 2011). The *Kepler* spacecraft has two effective integration times: a short-cadence (SC) integration near 1 min and a long-cadence integration near 30 min. The first year of the *Kepler* mission was dedicated to a survey phase where many target buffers were assigned to SC observations, which switched targets on a monthly basis (Jenkins et al. 2010). Papers I through VII (Kawaler et al. 2010a,b; Østensen et al. 2010c, 2011; Reed et al. 2010; van Grootel et al. 2010; Baran et al. 2011) of this series along with Østensen et al. (2010b) describe the search and resulting detections of periodicities in compact stars from *Kepler* survey phase observations. Papers I and VI provide a spectroscopic analysis and pulsation overview of all compact stars observed with *Kepler*; Papers II, III, V, VII and Østensen et al. (2010b) describe the specific pulsation periods of the pulsators and Paper IV associates a model with one pulsator using the forward method. These papers serve as a complete introduction to pulsating sdB stars using *Kepler* SC observations. Additionally, *CoRoT*[1] has observed one V1093 Her pulsator, KPD 0629−0016 (hereafter KPD 0629; Charpinet et al. 2010).

2 DETECTION AND SIGNIFICANCE OF REGULAR PERIOD SPACINGS

In Paper III (Reed et al. 2010), we identified 26 of 27 periodicities for KIC 10670103 as $\ell = 1$ or 2 using the relations of equations (2)–(4). In this section, we search all V1093 Her stars with space-based (13 *Kepler* and 1 *CoRoT*) observations and apply significance tests.

The basic information for the 14 stars of this study is provided in Table 1. This includes Kepler Input Catalog (KIC) numbers, stellar designations from other sources and spectroscopic properties from Papers I and VI (except for KPD 0629, which are from Charpinet et al. 2010).

From our work with KIC 10670103, we were expecting $\ell = 1$ period spacings near to 250 s and there are several other stars (particularly KIC 8302197 and KIC 10001893) which trivially show equal-period spacings (or multiples thereof) very near to this value. Since V1093 Her stars have small ranges for T_{eff} and $\log g$, we anticipated that all our targets should have $\ell = 1$ period spacings near to 250 s. We took the dual approach of Winget et al. (1991), to search for regular period spacings using period transforms (PT) and Kolmogorov–Smirnov (KS) tests. The PT is an unbiased test where power spectra are converted to period spectra and then a Fourier transform (FT) is taken of that. Peaks in the PT indicate common period spacings. We used the g-mode region from 0 to 1000 µHz from our power spectra. The PT method is sensitive to the number of periods from which to find correlations. Winget et al. (1991) did this for the pulsating DOV star PG 1159−035 (also known as GW Vir), for which 125 periods were detected. Conversely, our richest g-mode pulsator only has 46 periodicities and our poorest a meagre seven. As such, our expectations were low and we were happily surprised by the success of this method. For 10 stars (shown in the left-hand panels of Fig. 1) the $\ell = 1$ peak corresponding to a regular period spacing near 250 s is readily picked out and in five of these, we can deduce the $\ell = 2$ peak as well using equation (4). We then fitted the PT with a non-linear least-squares technique to determine the period spacing values and errors for each one. For KIC 3527751, an alias occurs for $\Delta\Pi_1 + \Delta\Pi_2$ and KIC 11558725's peaks are split because of small period spacings (possibly related to rotational multiplets). KIC 8302197 and KIC 9472174 do not have any peaks that stand out. For KIC 8302197, this most likely occurs because of the few periods (nine) and for KIC 9472174 this is likely related to the short data series (9.7 d) and the complexity within the FT caused by binarity.

The KS test is a non-parametric test that compares a sample distribution [$F_n(x)$] with a reference distribution (equation 1 in our case; Chakravarti, Laha & Roy 1967). The KS test has proven useful with
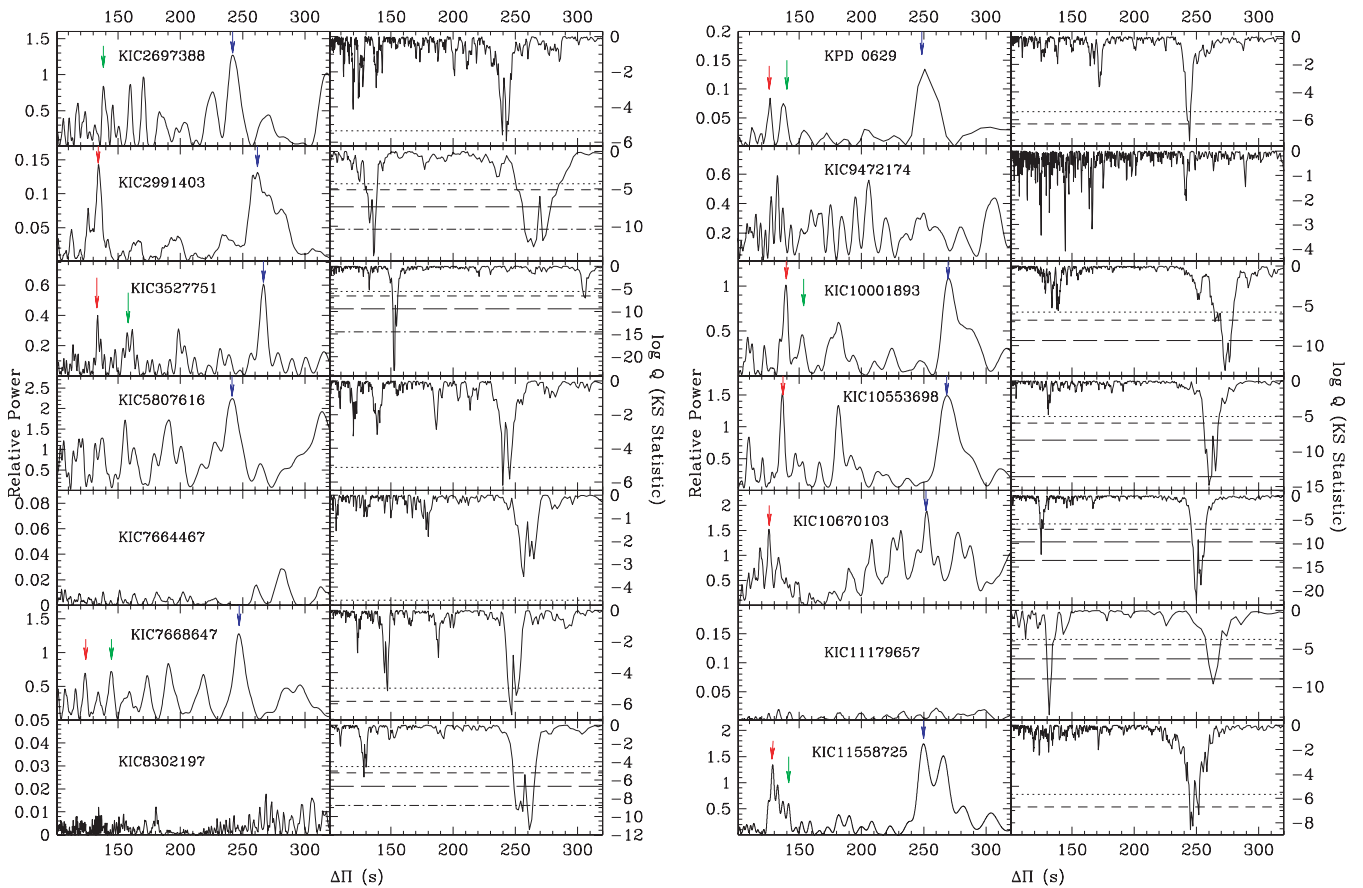
Figure 1. Left-hand panels: indications of regular period spacings using PTs. Blue arrows indicate the $\ell = 1$ spacings, green arrows the $\ell = 2$ spacings and red arrows aliases. Right-hand panels: the KS test applied to the detected periodicities. Confidence levels of 90 per cent (dotted line), 95 per cent (short dashed line), 99 per cent (long dashed line) and 99.9 per cent (dot–dashed line) are shown. These confidence levels are calculated for the range of period spacings between 100 and 400 s, assuming uniformly distributed random periods.

white dwarf pulsators (Kawaler 1988; Winget et al. 1991). The KS test uses previously detected pulsation periods as input and so has a selection effect caused by our detections. Any such effect should be small as the data are nearly gap-free, and so period detections should be accurate. However, some stars show small, marginally unresolved periodicities and these could skew the results as they are sometimes included and other times excluded in the period lists. We applied the KS test for equal-period spacings between 50 and 800 s. The results for the range of 100–300 s are shown in the right-hand panels of Fig. 1. Unlike the PT test, the KS test has a local minimum for all stars for period spacings near 250 s, except for KIC 3527751, where it detects spacings of 152 s. The PT test for KIC 3527751 shows both the $\ell = 1$ and 2 peaks, with the $\ell = 1$ having a higher amplitude, whereas the KS test only detects an extremely strong $\ell = 2$ spacing. The $\ell = 2$ spacing is within the errors of both the PT test and a linear least-squares fit and matches the value determined using equation (4) based on our $\ell = 1$ determinations. However, only for the KS test does it dominate. The $\Delta\Pi = 241$ s detection for KIC 9472174 is not significant. KIC 9472174 is a fairly rich pulsator with a large range of periods and in a known short-period binary. There are many small spacings between 100 and 150 s, and it could be that many of these are parts of rotationally split multiplets. With a binary period of 3.0 h, any rotational multiplets would have splittings of ≈ 92 μHz, which means that they would overlap asymptotic spacings and disrupt our period spacing detection techniques. While the PT test was not useful for KIC 8302197 as it has

too few periods, the KS test readily found a regular period spacing. All of the KS results show some quantity of multiple peaks caused by deviations in the period spacings. In PG 1159, these are attributed to mode trapping. Stars which show weak secondary peaks indicative of $\ell = 2$ spacings include KICs 2991403, 5807616, 7664467, 7668647, 10553698, 11558725 and KPD 0629. Surprisingly, the $\ell = 2$ spacings for KIC 10670103 produce an insignificant peak, even though we previously detected eight $\ell = 2$ modes including five consecutive overtones.

Using the period spacings found in the PT and KS tests (or integer multiples thereof), we identified periods as $\ell = 1$ or 2, or unknown.[2] We then did a least-squares straight line fit to each $\ell = 1$ or 2 series, arbitrarily assigning n values such that n was not negative[3] and satisfying equation (3) between the $\ell = 1$ and 2 series. The period spacings found using all three methods were in agreement and in Table 2 we use those from the linear least-squares fits, for which the errors are the most straightforward.

2.1 Monte Carlo tests

From the PT and KS tests, we already have strong evidence that nearly all of these stars have regular period spacings. However,

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[2] Periods and mode identifications appear in Tables 4–17 in the Supporting Information.

[3] Except for KIC 5807616, where n is chosen to match Paper IV.

Table 2. Period spacings determined from linear least-squares fits. Column 1 provides the KIC number (KPD designation for the *CoRoT* star), Columns 2 and 3 are the $\ell = 1$ and 2 period spacings (errors in parentheses), and Columns 4–8 provide the total number of periods, the number assigned as $\ell = 1$ and 2, and the number of consecutive $\ell = 1$ and 2 overtones. Parenthetic numbers in Column 6 indicate the number of modes which are ambiguous between $\ell = 1$ and 2 identifications. They are not counted as $\ell = 2$ in Column 6. The last column provides the percentage of Monte Carlo simulations that produced a match to the observations.

Star	$\Delta\Pi_1$	$\Delta\Pi_2$	N	N_1	N_2	NC_1	NC_2	MC per cent
2697388	240.07 (0.27)	138.54 (0.16)	36	16	13 (2)	4	3	0.04
2991403	268.52 (0.74)	153.84 (1.19)	16	7	4	5	0	0.009[a]
3527751	266.10 (0.38)	153.57 (0.12)	38	15	14 (5)	2	2	0.018
5807616	242.12 (0.62)	139.13 (0.38)	22	11	6 (3)	3	3	23.0
7664467	260.02 (0.77)	–	7	6	1	0	–	0.16
7668647	248.15 (0.44)	144.71 (0.57)	18	12	5 (2)	2	0	0.0014
8302197	257.70 (0.56)	–	9	9	–	2	–	0.0007
9472174	255.63 (0.30)	147.70 (0.69)	20	8	8 (1)	2	2	5.4[b]
10001893	268.53 (0.61)	154.74 (0.34)	26	18	9	12	3	0.0[c]
10553698	271.15 (0.54)	156.68 (0.31)	30	12	9 (6)	6	3	0.22
10670103	251.13 (0.31)	145.59 (0.26)	27	19	8	5	5	0.04
11179657	231.02 (0.02)	133.64 (0.40)	12	3	7 (1)	0	3	0.0002[d]
11558725	246.77 (0.58)	142.57 (0.14)	46	18[e]	13 (1)	8	2	2.2[f]
KPD0629	247.17 (0.48)	142.74 (0.30)	17	12	3 (2)	3	2	1.1

[a]Using just the $\ell = 1$ sequence with five consecutive overtones; [b]leaving three deviant $\ell = 2$ matches as unassigned; [c]10 million simulations produced no results which included 13 consecutive overtones; [d]assuming that f1 is $\ell = 1$; [e]periods f1 and f2 are counted as $\ell = 2$; [f]leaving the deviant periods f17 and f44 unassigned and assuming that f2 is $\ell = 2$.

according to stellar models (Charpinet et al. 2002, hereafter CH02), even period spacings are not anticipated. Since most of the pulsators have rich pulsation spectra, it is reasonable to question if the detections are just chance alignments. As a third test, we produced Monte Carlo simulations that randomly select periods to match with asymptotic sequences to within the errors. A number of observed periodicities, N, set to match what is observed, were randomly selected to fit within an observed range $P_{\min} \leq P \leq P_{\max}$. For KIC 8302197 and KIC 7664467, we compared the N randomly selected periods with a single sequence of the form $P_n = (P_{\min} - j\delta) + n \times \Delta\Pi \pm \sigma$. The quantity δ represents a small shift of the zero-point which was repeated j times until $P_0 \leq P_{\min} - \Delta\Pi$. The j value that produced the greatest number of matches was used and the n values were tested to see how many consecutive overtones were detected. σ is the difference allowed between the random and sequence periods and is chosen to be slightly bigger than what is observed. Choosing this value for σ makes our Monte Carlo simulations extremely conservative and the real probability of matching is much smaller since this σ is typically valid for only one or two periods, with most errors being much smaller. n is then stepped until $P_n > P_{\max}$. To generate an $\ell = 2$ appropriate sequence, we simply divided the $\ell = 1$ P_n by $\sqrt{3}$, as required by equation (3) and then extended $P_{\max}$ so that even the longest random periods could have $\ell = 2$ matches. Our code also ensured that only one randomly selected period matched each possible sequence period (eliminating double counting). A million sets of random periods were generated in each Monte Carlo simulation and the resulting matches were converted to percentages in column 9 of Table 2.

The Monte Carlo simulations indicate that 10 stars have less than a 1 per cent chance that their regular period spacings are the product of random chance. KPD 0629 has a 1.1 per cent chance and KIC 11558725 has a 2.2 per cent chance of occurring randomly from our Monte Carlo simulations. KIC 5807616 and KIC 9472174 are unconstrained from this test. Table 3 (and online supporting material) provide a complete list of periods and mode identifications for all 14 stars.

Table 3. Periods and period spacings for KIC 2697388. Identifications (Column 1) and periods are those from Paper III (Reed et al. 2010). Columns 3–5 provide the mode degree ℓ and the overtone fit to $P_\ell = P_{\ell o} + n \cdot \Delta P_\ell$ where n is arbitrarily chosen such that there are no negative values, except for KIC 5807616, where it is chosen to match Paper IV. Column 6 provides the difference between the observed and asymptotic relation periods, Column 7 lists the fractional period differences and Column 8 is the observed spacing $P(n_{\ell,i}) - P(n_{\ell,j})/(i - j)$. It is ambiguous whether $\ell = 1$ or 2 modes should be associated with f25, f23 and f11. f25 was not used for the $\ell = 1$ fit as it is most likely $\ell = 2$. This is a sample table of the online-only material. Tables for all 14 stars appear as Supporting Information with the online version of the article.

ID	Period (s)	ℓ	n_1	n_2	δP (s)	$\delta P/\Delta P$	Spacing (s)
f30	2757.118	2	–	15	19.393	0.140	140.7
f29	3008.732	2	–	17	−6.072	0.044	125.8
f28	3517.219	1	10	–	−8.790	−0.037	254.2
f27	3700.224	2	–	22	−7.277	−0.053	138.3
f26	3757.261	1	11	–	−8.815	−0.037	240.0
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–

3 ENSEMBLE AND MODEL COMPARISON

Fig. 2 shows our detected $\ell = 1$ period spacings with gravity and effective temperature. The temperatures span nearly 10 000 K while $\log g$ only covers 0.6 dex. Naturally, what is sought is a relationship between period spacings and physical properties, such as equation (5) of Kawaler & Bradley (1994) for white dwarfs. Such a relationship would allow the determination of properties based on period spacings alone. While white dwarfs and sdB stars are both compact stars, there is no a priori reason to expect that any correlations should exist for sdB stars. Fig. 31 of CH02 indicates that as envelope thickness decreases, the distance between trapped modes increases as do period spacings. However, the effect of trapped modes is increased
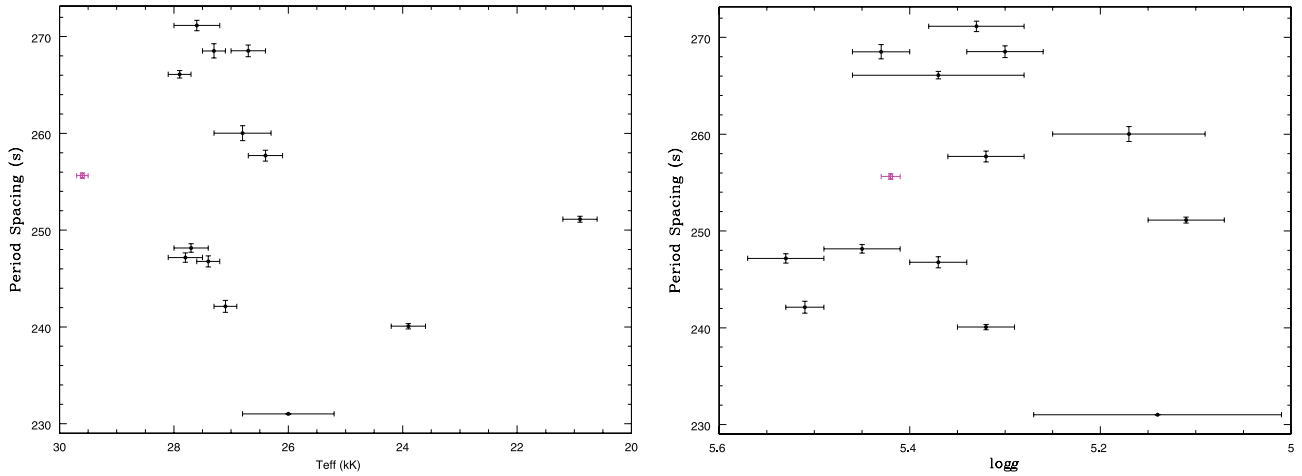
Figure 2. Period spacings compared with T_{eff} and log g. The magenta point indicates KIC 9472174, which is the only star for which PT, KS and MC tests were all inconclusive.

with decreasing envelope thickness, and so while there are more overtones between trapped modes, the impact of a trapped mode would be to eliminate any sequence of the form of equation (1) longer than three or four consecutive periods. Fig. 16 of CH02 indicates that the longest period spacings should occur where T_{eff} and log g are both small or both large, though they only test for g modes with $n \leq 9$, which may be too small for asymptotic relations. However, in Fig. 2 there do not appear to be any trends, either with gravity or with temperature. Since 10 stars have temperatures near 27 500 K yet period spacings that range from 242 to 271 s while the temperature extremes of the group have period spacings near the middle of this range, it would have to be deduced that temperature does not impact period spacings in sdB stars. No trends are obvious with log g either, though in this case the span is much smaller compared with the associated errors. Table 3 of CH02 indicates that period spacing should increase with decreasing envelope mass. It would be useful to compare the CH02 models with Paper IV, but unfortunately the CH02 paper calculates for $\ell = 3$ modes and Paper IV does not, making a direct comparison difficult. Appropriate stellar models will have to be produced to determine what the parameter(s) is (are) that affects the period spacings, but this paper is concerned

with interpreting observations and so we will not address modelling issues.

While CH02 examined period spacings for gravity (and pressure) modes, the model they used was significantly hotter than these stars. The model of Paper IV is obviously appropriate as it was made to match KIC 5807616 and so we compared it with our findings. Fig. 3 shows the model spacings for many of the $\ell = 1$ and 2 modes (black circles). The $\ell = 1$ period spacings range from $\approx$50 to 400 s with mode trapping dominating the spacings. In the sequence of 21 period spacings, the change between consecutive spacings is only twice smaller than 20 s while the rest are greater than 50 s. For comparison, the period spacings we selected for KIC 5807616 (which changed by less than 25 s for all $\ell = 1$ modes) are shown as blue triangles. Naturally, one could pick out just the peaks or troughs of the model and get more consistent period spacings that way, but you would only rarely get a sequence of three consecutive overtones. To test this assumption, we performed a blind test on 51 model $\ell = 1$ and 2 periods from Paper IV, including model sequences of 21 consecutive $\ell = 1$ and 30 $\ell = 2$ modes. Putting them in period order only (removing the model mode assignments) and using the observed period spacings as a guide, we assigned
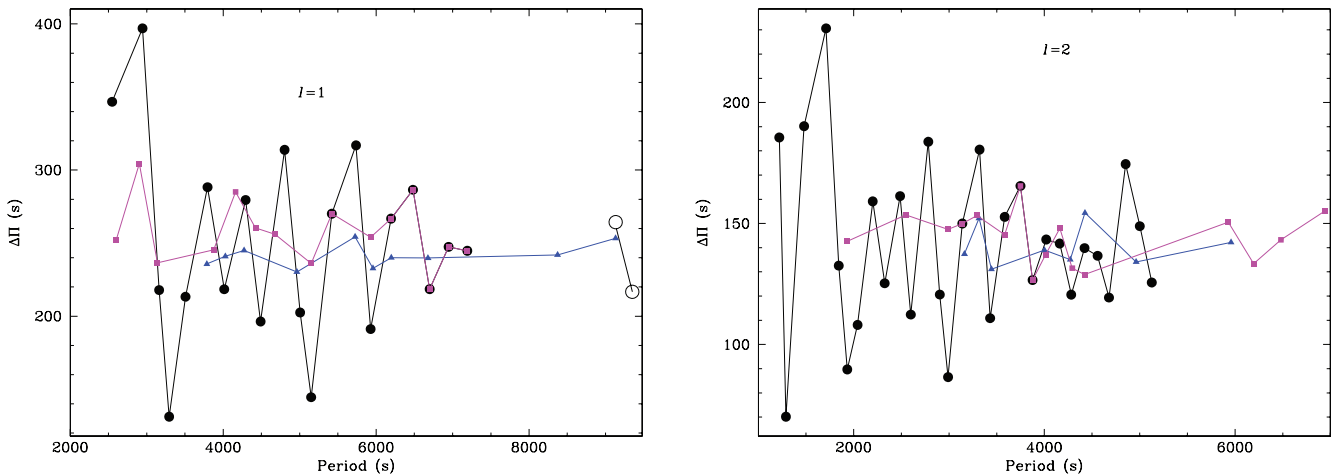

Figure 3. A comparison between model, observed and linearly fitted model data for KIC 5807616. Black circles are model periods from Paper IV (open circles were not used in the fit), blue triangles are those found from *Kepler* data and magenta squares are those found from a blind fit using the observed period spacing with model periods. If a magenta square is plotted over a black circle, then our blind fit matched the model's mode assignments.
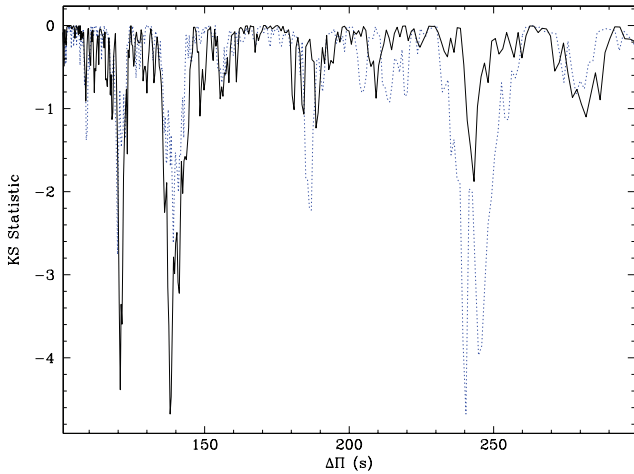
Figure 4. The KS test applied to 51 model periods from Paper IV. The dotted (blue) line is the KS test for KIC 5807616 from Fig. 1.

periods as $\ell = 1$ or 2 or left them unassigned. Allowing periods to deviate by up to 32 s from equal spacings (28 per cent more than the observed deviations), we assigned 15 $\ell = 1$ and 16 $\ell = 2$ modes (double counting eight periods, which were ambiguous between the modes). Of the 15 $\ell = 1$ assignments, eight were model $\ell = 1$ modes and of the 16 possible $\ell = 2$ mode assignments, eight were model $\ell = 2$ modes (two others were close). Our mode assignments from the model periods are shown as (magenta) squares in Fig. 3. When squares are plotted over circles, our blind test mode assignments match those of the model. As expected, this test indicates that mode identifications using equal-period spacings does not work well if there is any significant mode trapping since equation (1) biases us to selecting periods with small (or no) mode trapping. We also applied the KS test to the 51 model periods, and the results are shown in Fig. 4. The KS test preferentially detects $\ell = 2$ period spacings with a mild $\ell = 1$ period spacing. For comparison, the KS test for KIC 5807616's observed periods is shown as a dotted (blue) line and shows that the actual data have much stronger $\ell = 1$ period spacings. However, the period spacings detected in the models are about correct, indicating that perhaps with more subtle mode trapping, the model would better approximate the observations. In Table 6 of the Supporting Information, we show our mode assignments as well as those of Paper IV. We chose the radial order n to match the model at f11 = 4027 s. When mode assignments via regular period spacings and those from the model agreed, so did the radial order. Four of our 11 $\ell = 1$ mode assignments matched those of the model and seven of our nine $\ell = 2$ mode assignments matched. Again, this likely indicates that the star does not trap modes as significantly as the model predicts. Since this paper is concerned with observed mode identifications and period spacings, we leave a detailed model analysis to those best suited to do them.

4 SUMMARY

We tested 13 *Kepler*-observed and one *CoRoT*-observed g-mode pulsating sdB stars for consistent period spacings which can be used to observationally identify pulsation modes. We used two different spacing detection tests, a PT and KS test and a Monte Carlo (MC) significance test. The PT test identified 10 stars as having consistent $\ell = 1$ period spacings, and five of these also showed indications of $\ell = 2$ period spacings. Our KS test results clearly detected $\ell =$

1 constant period spacings in *all* our programme stars, except KIC 9472174, which has a period spectrum complicated by binarity (though the KS result does have an appropriate local minimum), and KIC 3527751, for which it finds a very strong $\ell = 2$ period spacing. A further five stars show local minima appropriate for $\ell = 2$ period spacings from their KS tests. Monte Carlo tests indicate that for 10 stars, our mode assignments (provided in the Supporting Information) are very likely correct. For three additional stars, a random cause for the spacings is below 6 per cent and KIC 5807616 has a 23 per cent chance that the equal-period spacings are being created randomly, as a worst-case scenario.

For all sample stars, except KIC 9472174, three of the four methods (PT, KS, MC and linear least-squares) find evidence for regular period spacings. 12 of the 14 stars have $\ell = 1$ and 2 period spacings which satisfy equations (4) and 11 stars have periods (45 periods in total) that satisfy equation (3), with C equal to zero. Combined, these provide a strong indicator that we are correctly identifying periodicities as $\ell = 1$ and 2 modes, rather than higher degrees which have different relations. For these 14 stars, we assigned a total of 222 of possible 317 periodicities as $\ell = 1$ or 2 modes. Such a large quantity of observationally constrained modes should prove exceedingly useful for stellar modelling.

Our results clearly show the value of long-duration space-based observations. While there have been some remarkable ground-based efforts to observe g-mode sdB stars, they have not resulted in sufficient detections to evaluate period spacings. Additionally, the *Kepler* results are solely from the survey phase of the mission. Longer duration observations should detect more pulsation periods, including higher degree ($\ell \geq 3$) modes, which we have not searched for at all.

5 CONCLUSION

In order for equation (1) to be useful, mode trapping must be small (or none). Since equation (1) produced a large fraction of significant mode assignments for nearly all of the stars we examined, mode trapping must be substantially reduced from what current models indicate. Fig. 16 of CH02 shows period spacings against both T_{eff} and $\log g$ for g-mode pulsations. Unfortunately, it only has $n \leq 9$, where evenly spaced periods are not expected. However, for higher n values, such a plot should show a flat surface. According to CH02, $\Delta \Pi$ shows a plateau of maximum values running from the lowest T_{eff} and $\log g$ to the highest T_{eff} and $\log g$. However, these results are significantly affected by mode trapping and so may not be a clear indicator of trends in period spacings. Minimal mode trapping could be an indicator that sdB stars are not as chemically stratified as models usually presume. Hu et al. (2009) examined the effects of diffusion on period spacing and their fig. 4 shows damped mode trapping, though for $\ell = 3$ modes. Further diffusion may work to remove a sharp mode-trapping boundary. Another possibility could be thermohaline convection, caused by an inverse μ gradient, as described by Théado et al. (2009) which could increase mixing and reduce chemical stratification.

Our assignments as $\ell = 1$ or 2 modes also add constraints to driving models. While Paper IV produced models with driven modes in the correct period range, previous modelling work (including Fontaine et al. 2006; Jeffery & Saio 2007; Hu et al. 2009, among others) had difficulties. These models preferentially found $\ell \geq 4$ to be driven (also at temperatures cooler than observed). This is contradicted by our results, which clearly follow equations (3) and (4), indicating $\ell = 1$ and 2 modes.

Prior to space-based data such as *Kepler* and *CoRoT*, it seemed unlikely that sdB asteroseismology using g modes to probe the core would bear fruit. The discovery of equal-period spacings will now have changed that as we can readily correlate modes with periodicities. The forward method of mode assignment is no longer necessary for these stars, which now provide a new modelling challenge. This challenge will be to model stars such as KIC 10670103, KIC 10001893 and KIC 10553698, which have lengthy sequences of successive overtones and equal-period spacings which show minimal indications of mode trapping, and provide tens of periods, each with secure mode assignments.

We anticipate that once longer duration *Kepler* data are available, many more pulsation periods will be detected. There are already typically too many periods to be accounted for solely using $\ell = 1$ and 2 modes and this problem will be compounded. It is anticipated that the extra periodicities will be accounted for using higher degree modes. Such an event will require more sophisticated techniques and tests for assigning modes to periodicities. However, the relatively simple tests of this paper have been sufficient to confirm that regular period spacings in g-mode sdB pulsators exist and provide useful constraints which stellar models can now aspire to fit.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article.

Tables 4–17. Periods and period spacings for all 14 stars.

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