SINGLE INVERTED PENDULUM SYSTEM CONTROLLER: SYSTEM STUDY AND COMPARISON

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This thesis covers the experimental study of the design of a controller for the single inverted pendulum system. This study will focus mostly on two methods to control a single inverted pendulum system, and the comparison between them. In this study, a real single inverted pendulum system will also be built. The study will be undertaken using the following steps.

Firstly, according to Newton's laws of motion, the mathematical model of the inverted pendulum system is established. That will make it easier to obtain the system's transfer function and state space equation, so that the system's stability and controllability can be analyzed with the help of MATLAB.

Secondly, the two methods, which are the proportional–integral–derivative PID double closed loop and the pole placement technique will be used to build the controller separately. The control algorithm simulation uses MATLAB/Simulink to obtain the step

response curve of the system and to evaluate the system's performance.

Thirdly, the single inverted pendulum system will be built in the lab, and an attempt will be made to make the system balance itself by tuning the PID parameters.

Finally, by comparing these two methods, advantages and disadvantages will be demonstrated, and suggestions for controlling optimally the single inverted pendulum system will be given.

Accepted by: ____________________, Chair

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CHAPTER I INTRODUCTION

General Area of Concern

The inverted pendulum system is a classic problem in the area of control systems. It is often used to demonstrate key concepts in linear control such as the stabilization of unstable systems. Since the system is inherently nonlinear, it has also been useful in illustrating some of the ideas in nonlinear control. In this system, an inverted pendulum is attached to a cart equipped with a motor that drives it along a horizontal track. What needs to be done is to make sure the pendulum does not fall and can stay up straight by moving the cart left or right. The study of the inverted pendulum has significance for a variety of research problems in control, such as the biped robot walking problem, flight attitude adjustment of a rocket, aircraft landing, the stability of offshore oil platforms, etc.

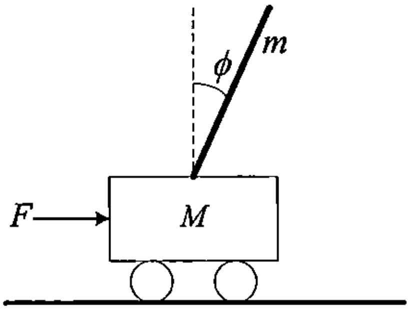


Figure 1.1. Single inverted pendulum system.

Objectives

This study aims at analyzing the mathematical model of a single inverted pendulum system and comparing the different control methods. Basically, there are three main steps in this study, which are studying the mathematical model, designing the controllers, and testing the control results.

Significance of the Study

The inverted pendulum is a classic problem in dynamics and control theory and is widely used as a benchmark for testing control algorithms. To design a stabilizing controller for a single inverted pendulum is a typical problem in control system design based on the state space approach. A stabilized pendulum is useful to show the layperson the power of the state-space theory as a control technique (Furuta, Kajiwara & Kosuge, 1980). The inverted pendulum system is a typical unstable, higher-order, multivariable, strongly coupled non-linear system. It has two main purposes: firstly, as an inherently unstable nonlinear system, the inverted pendulum control system is the ideal platform to carry out a variety of tasks in the teaching of control theory, and in doing research. Many typical problems in control, such as nonlinear behavior, robustness, follow-up, etc. are included in the study of the inverted pendulum system. Secondly, due to the simplicity of the inverted pendulum as a controllable device, new control methods can be tested on the inverted pendulum to see if they are able to deal with a system that is nonlinear and unstable.

Definition of Terms

1. Inverted pendulum

An inverted pendulum is a pendulum which has its mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally. Unlike a normal pendulum which is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright.

2. Controllability and observability

Controllability and observability represent two major concepts of modern control system theory. These concepts were introduced by R. Kalman in 1960. As to controllability, in order to be able to do whatever we want with the given dynamic system under control input, the system must be controllable. As to observability, in order to see what is going on inside the system under observation, the system must be observable. The observability and controllability of a system are mathematical duals (Kalman, 1959).

3. Root locus

The root locus is a way of presenting graphical information about a system's behavior when the controller is working. The root locus is a widely used tool for design of closed loop systems, and it has the virtue of being a good design tool for continuous time systems (where you work in the s-plane, or the complex plane) and for sampled (computer controlled) systems (where you work in the z-plane). Root locus analysis is a graphical method for examining how the roots of a system response function change with variation of a certain system parameter, commonly the gain of a feedback system.

4. Pulse-Width Modulation (PWM)

Pulse-width modulation (PWM), or pulse-duration modulation (PDM), is a commonly used technique for controlling power to inertial electrical devices, made practical by modern electronic power switches. The average value of voltage (and current) fed to the load is controlled by turning the switch between supply and load on and off at a fast pace. The longer the switch is on compared to the off periods, the higher the power supplied to the load is. The term "duty cycle" describes the proportion of "on" time to the regular interval or "period" of time; a low duty cycle corresponds to low power, because the power is off for most of the time. Duty cycle is expressed in percent and 100% is fully on.

CHAPTER II REVIEW OF LITERATURE

Historical Background

In 1976, Shozo Mori, Hiroyoshi Nishihara and Katsuhisa Furuta successfully controlled a single pendulum system, considering the overall characteristics including its nonlinear property, which represented the real system more closely (Mori, Nishihara & Furuta, 1976).

In 1980, Katsuhisa Furuta, Hiroyuki Kajiwara and Kazuhiro Kosuge designed a controller to stabilize a double inverted pendulum on an inclined rail. The controller was designed by means of computer aided design, or CAD (Furuta, Kajiwara & Kosuge, 1980).

In 1984, K. Furuta, T. Ochiai, and N. Ono realized the control of a triple inverted pendulum, consisting of three arms. This pendulum was a good analogy to a human standing on a single leg without a foot, and the results of the paper contributed to the study of a biped locomotive machine (Furuta, Ochiai & Ono, 1984).

In 1992, a robust swing-up control using a subspace projected from the whole state space was proposed by K. Furuta, M. Yamakita, and S. Kobayashi. Based on the projected state space or pseudo-state, the control input is determined depending on the partitioning of the state as a bang-bang type control. The control algorithm is applied for a new type of pendulum (TI Tech pendulum), and the effectiveness and robustness of the proposed control are examined by experiments (Furuta, Yamakita & Kobayashi, 1992).

In 1997, Gordillo compared LQR, which is Linear-quadratic regulator, and the control method based on a genetic algorithm, and drew the conclusion that the traditional control method was much better than the genetic algorithm.

Now, most research on the inverted pendulum is conducted in Asia, such as at Beijing Normal University, the University of Science and Technology of China, Beijing University of Aeronautics and Astronautics (Beihang University) in China, Tokyo Institute of Technology, Tokyo Denki University, Tokyo University, in Japan, and Pusan National University, and Chungnam National University in Korea. Besides the above research centers, St. Petersburg University in Russia, Russian Academy of Sciences, Poznan University in Poland, University of Florence in Italy, and so on also have correlative research in this field.

Software Review

In this study, Simulink 6.3 (MATLAB 7.1) and LabVIEW 2011 served as the simulation tool and the actual controller respectively. Both of them provide a great convenience for analyzing and building control systems. MATLAB, developed by MathWorks, is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numerical computation. Using MATLAB, technical computing problems can be solved faster than with traditional programming languages, such as C, C++, and FORTRAN. MATLAB also stands out for its extraordinary expandability. More functions are able to be added to

MATLAB by the implementation of toolboxes, one of which is Simulink. Simulink, which is a tool for modeling, simulating and analyzing multidomain dynamic systems, is also from MathWorks. It is widely used in control theory and digital signal processing for multidomain simulation and Model-Based Design. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive MATLAB or be scripted from it. One of the most important reasons why Simulink was chosen for the theoretical analysis and simulation required by this thesis is that its program is very easy to interpret from the block diagram, which is a basic and common way to represent a system's principal functions and their relationships in the engineering world of control. The version of MATLAB and Simulink used in this thesis are version MATLAB 7.1 and Simulink 6.3, released in 2005.

LabVIEW, short for Laboratory Virtual Instrumentation Engineering Workbench, is a system design platform and development environment for a visual programming language from National Instruments. It uses the graphical language for programming, which is also known as the "G" language. G is a high-level, dataflow graphical programming language designed to develop applications that are interactive, execute in parallel and multicore. LabVIEW combines the convenient graphical development environment with the powerful but flexible G language to make the program on LabVIEW quite intuitive and readable. Besides, LabVIEW's performances in data acquisition, instrument control, and industrial automation are extremely impressive. Thus, both advantages of LabVIEW make it a good choice to serve as the core controller of the inverted pendulum system. The latest version of LabVIEW, which is version LabVIEW 2011, released in August 2011, is used in this thesis,

Control Methods Review

In this study, there are two control methods that will be used: Proportional-Integral-Derivative control and pole placement. Even though PID control is a classical control theory, while the pole placement is a modern one, both of them show good performance in system control.

A proportional–integral–derivative control, or PID control, is the most commonly used closed loop feedback method in control systems. Even complex industrial control systems may comprise a control network whose main control building block is a PID control module (Johnson & Moradi, 2005). It was the first, as well as the only, controller to be mass produced for the high-volume market that existed in the process industries.

A PID controller calculates an "error" value as the difference between a measured process variable and a desired set-point. It attempts to minimize the error by adjusting the process control inputs. The block diagram of closed loop systems with PID controller is shown in Figure 2.1.

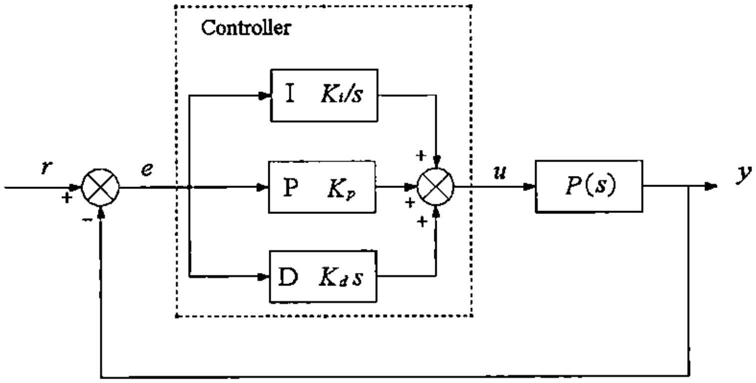


Figure 2.1. Block Diagram of Closed Loop Systems with PID Controller

In the figure above, the control signal u for the system is formed entirely from the error e (Åström & Murray, 2008). It is pretty clear that the PID algorithm involves three separate parameters: the proportional, the integral and the derivative values, denoted P, I, and D. That is why the PID control is also called "Three-Term Control". The input/output relation for an ideal PID controller with error feedback is:

$$u = K_p e + K_i \int_0^t e(\tau)d\tau + K_d \frac{de}{dt} = K_p \left(e + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{de}{dt} \right). \quad (1\text{-}1)$$

Each of the terms or parameters in the PID control system has its own function. P, or proportional control, has an immediate response to the input error of the system. This means that once the sensor senses the disturbance, the difference between the set and sensed value, or error will be sent into the proportional controller. Then the proportional controller will produce a control signal, which abates the error instantly, and the strength depends on the coefficient K_p. It is simple, but the static error of the system response cannot be eliminated. Usually, the static error can be reduced by increasing the K_p. However, the stability and dynamic performance of the system will decrease if K_p is too large. I, or integral control, can help to remove the static error of the response. As long as the input error e is not equal to zero, it can keep modifying the control signal u cumulatively until the error e reaches zero. Integral control overcomes the shortcomings of proportional control by eliminating static error without the use of excessively large controller gain. The integral time constant T_i reflects the strength of the integral action. The larger T_i is, the more integral action will be added. What's more, a larger T_i can reduce the overshoot, although the time used to eliminate the static error will be longer. Therefore, the cost of implementing integral control is weakening the rapidity of the system. D, or derivative control, uses the rate of change of an error signal as an input and introduces an element of prediction into control action to nip the error in the bud. So derivative control reduces the overshoot and overcomes the oscillation of a system (Lu, Lin & Zhou, 2009). To sum up, each of the three terms of PID control brings some benefits and shortcomings. In order to achieve an ideal and satisfying target, the three parameters, P, I and D, have to act coordinately and properly to work as a whole.

Pole placement, or full state feedback, is a method implemented in feedback control system theory to place the closed-loop poles of a plant in pre-determined locations in the s-plane, or complex plane. Placing poles is desirable because the location of the poles corresponds directly to the eigenvalues of the system, which control the characteristics of the response of the system.

Usually, the system must be considered controllable in order to implement this method. Represent the closed-loop input-output transfer function in the manner shown below:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}.$$

A is called the "state matrix" and is decided by the system itself. B is the "input matrix". C is the "output matrix" and D is the "feed through or feed forward matrix" (Liu & Tang, 2008). The poles of the system are the roots of the characteristic equation given by $|s\mathbf{I} - \mathbf{A}| = 0$. In order to assign the poles, what needs to be done is to find out the feedback matrix **K** and substitute the **A** with **A+BK** so that the poles can be successfully relocated on the left half side of the complex plane. By doing this, the system can become stable. The roots of the full state feedback system are given by the characteristic equation $\det\left[s\mathbf{I} - (\mathbf{A} + \mathbf{BK})\right]$ (Liu & Tang, 2008). One compares the terms of this equation with those of the desired characteristic equation to obtain the K matrix elements.

Hardware

1. Incremental Encoder

An encoder is an electromechanical device that converts linear or rotary displacement into digital or pulse signals. An incremental encoder generates a pulse for each incremental step in its rotation. Unlike the absolute encoder, the incremental encoder does not output absolute position. The most common type of incremental encoder uses two output channels (A and B) to sense position and usually one channel is shifted by 90 electrical degrees from the other. The direction "CW" or"CCW" can be determined by evaluating the signal levels relative to the alternate channel at the time of the rising edge.

For example, in Figure 2.2, when the encoder is rotated clockwise, each rising edge of channel A occurs when channel B is low. This gives a negative count.

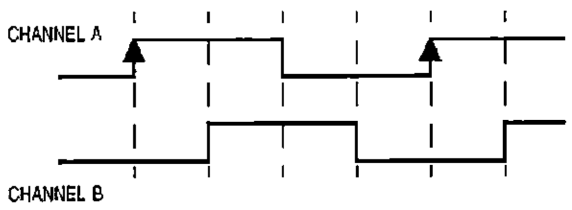


Figure 2.2. When the Encoder Is Rotating Clockwise.

When the encoder is rotated anti-clockwise, each rising edge of channel A occurs when channel B is high. This gives a positive count.

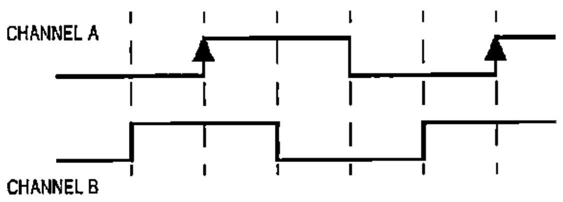


Figure 2.3. When the Encoder Is Rotating Counter-clockwise.

2. H-Bridge Circuit

An H bridge is an electronic circuit that enables a voltage to be applied across a load in either direction. These circuits are often used in robotics and other applications to allow DC motors to run forwards and backwards (Li & Wang, 2005).

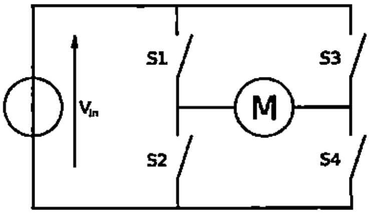


Figure 2.4 Structure of an H-bridge.

The H-Bridge circuit is usually used to reverse the polarity of the motor, as well as to brake the motor. Table 2.1 is the truth table of the H-Bridge.

Table 2.1. Truth Table of the Operation of H-Bridge circuit.

S1	S2	S3	S4	Status
1	0	0	1	Forward
0	1	1	0	Reversed
0	0	0	0	Free Runs
0	1	0	1	Brake
1	0	1	0	Brake

The H-bridge is available as an integrated circuit. It is built using a multi-technology process which combines bipolar and CMOS control circuitry with DMOS power devices on the same monolithic structure. It is ideal for driving both DC and stepper motors.

3. National Instruments PCI-6229

The National Instruments PCI-6229 is a low-cost multifunction M-Series data acquisition (DAQ) board optimized for cost-sensitive applications. It has four 16-bit analog outputs, 48 digital I/O, 32-bit counters, digital triggering, correlated DIO (32 clocked lines, 1 MHz), NIST-traceable calibration certificate and more than 70 signal conditioning options. The PCI-6229 pinout is shown in Figure 2.5 and the timer/counter pins are shown in Table 2.2.

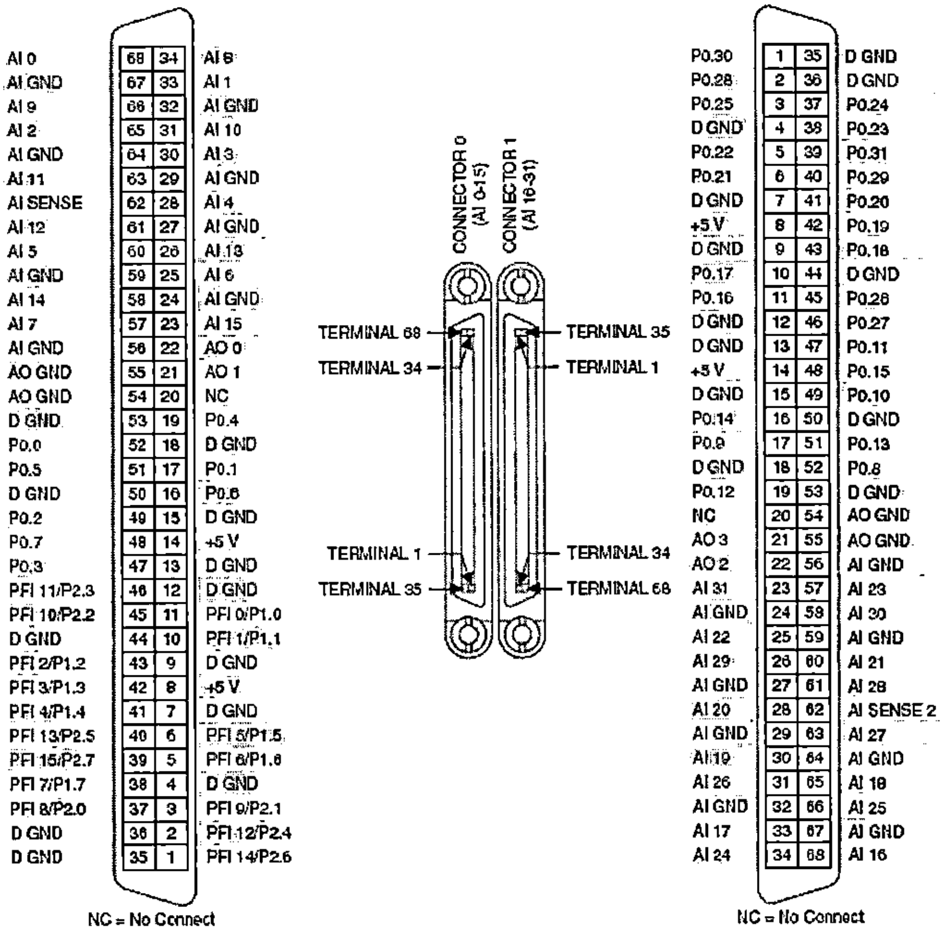


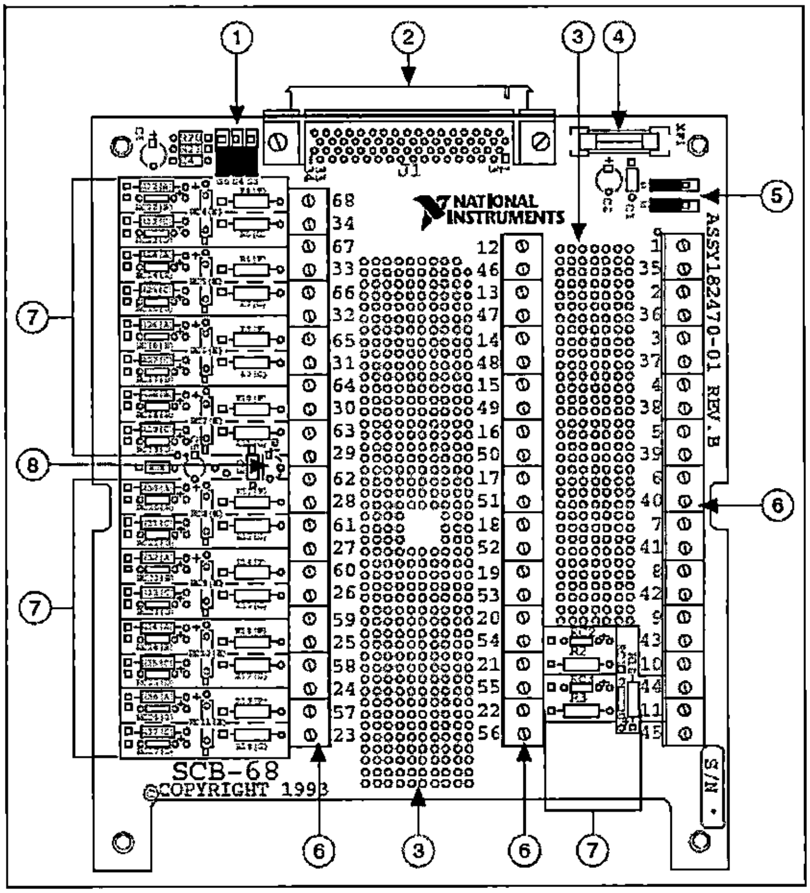
Figure 2.5. PCI-6229 Pinout.

Table 2.2. PCI-6229 Timer/Counter Pins

Counter/Timer Signal	Default Connector 0 Pin Number (Name)
CTR 0 SRC	37 (PFI 8)
CTR 0 GATE	3 (PFI 9)
CTR 0 AUX	45 (PFI 10)
CTR 0 OUT	2 (PFI 12)
CTR 0 A	37 (PFI 8)
CTR 0 Z	3 (PFI 9)
CTR 0 B	45 (PFI 10)
CTR 1 SRC	42 (PFI 3)
CTR 1 GATE	41 (PFI 4)
CTR 1 AUX	46 (PFI 11)
CTR 1 OUT	40 (PFI 13)
CTR 1 A	42 (PFI 3)
CTR 1 Z	41 (PFI 4)
CTR 1 B	46 (PFI 11)
FREQ OUT	1 (PFI 14)

4. National Instrument SCB-68

The NI SCB-68 is a shielded I/O connector block for interfacing I/O signals to plug-in data acquisition (DAQ) devices with 68-pin connectors. Combined with the shielded cables, the SCB-68 provides rugged, very low-noise signal termination. It is compatible with single- and dual-connector NI X Series and M Series devices with 68-pin connectors. The connector block is also compatible with most NI E, B, S, and R Series DAQ devices. The printed circuit board diagram is shown in Figure 2.6.



1	Switches S3, S4, and S5	5	Switches S1 and S2
2	68-Pin I/O Connector	6	Screw Terminals
3	Breadboard Area	7	Signal Conditioning Areas
4	Fuse (800 mA, 250 V, 5 - 20 mm)	8	Temperature Sensor

Figure 2.6. SCB-68 Printed Circuit Board Diagram

CHAPTER III METHODOLOGY

Mathematical Modeling

Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. Therefore, in my study, I will build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. I will build the mathematical model of the single inverted pendulum by studying and analyzing its movement mechanism and get the mathematical model according to the basic mechanical motion. In this research, Newton's laws of motion will be used to build the mathematical model of the single inverted pendulum system so that it can be convenient to analyze the system's controllability and observability.

As has been mentioned in the previous chapter, the inverted pendulum system is a typical unstable, higher-order, multivariable, strongly coupled non-linear system. So, in order to simplify the analysis and build its mathematical model, the following assumptions are made:

a. All components in the system are considered to be rigid.

b. The air resistance and the friction in each joint are ignored.

c. The entire mass of the pendulum is concentrated at its center of mass.

The physical parameters and values of the system prototype are tabulated as follows:

Table 3.1 Values and Parameters of the Inverted Pendulum System.

m (kg)	Pendulum's mass	0.18
M (kg)	Cart's mass	0.94
l (m)	Pendulum's half length	0.30
J $(kg{\cdot}m^2)$	Pendulum's moment of inertia	0.0054
F (N)	Force on the cart	
x (m)	Cart's position	
Φ (rad)	Pendulum's angle	

Figure 3.1 is the model of the inverted pendulum.

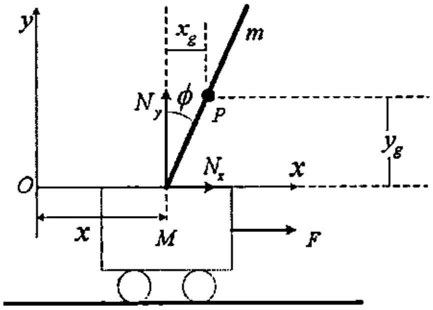


Figure 3.1 The Mechanical Analysis of the Inverted Pendulum System

In the rectangular coordinate system above, x_g and y_g are the distances from P to N_y and N_x, which are the vertical and horizontal force the cart gives to the pendulum respectively. Therefore, the kinetic equation of pendulum's rotating around its center of gravity is:

$$J\ddot{\phi} = N_y x_g - N_x y_g = N_y l \sin\phi - N_x l \cos\phi \tag{2-1}$$

where J is the pendulum's moment of inertia.

$$J = \frac{1}{12} m(2l)^2 = \frac{1}{3} ml^2 \tag{2-2}$$

For the pendulum's center of mass:

$$m \frac{d^2(x + l\sin\phi)}{dt^2} = N_x \tag{2-3}$$

$$m \frac{d^2(l\cos\phi)}{dt^2} = N_y - mg \tag{2-4}$$

For the cart (the horizontal direction):

$$M \frac{d^2 x}{dt^2} = F - N_x \tag{2-5}$$

According to (2-3) and (2-5),

$$(M + m)\ddot{x} + ml\ddot{\phi}\cos\phi - ml\dot{\phi}^2 \sin\phi = F \tag{2-6}$$

According to (2-1), (2-3) and (2-4),

$$(J + ml^2)\ddot{\phi} + ml\ddot{x}\cos\phi - mgl\sin\phi = 0 \tag{2-7}$$

Thus, we can get the precise kinetic equations of the inverted pendulum system are as followed;

$$\ddot{\phi} = \frac{(M + m)mgl\sin\phi - mlF - m^2 l^2 \dot{\phi}^2 \sin\phi}{J(M + m) + m^2 l^2 (1 - \cos\phi) + Mml^2} \tag{2-8}$$

$$\ddot{x} = \frac{(J + ml^2)F + (J + ml^2)ml\dot{\phi}^2 \sin\phi - m^2 l^2 g \sin\phi\cos\phi}{J(M + m) + m^2 l^2 (1 - \cos\phi) + Mml^2} \tag{2-9}$$

Since ϕ is usually quite small while the inverted pendulum is working, we could assume that here $\sin\phi \approx \phi$ and $\cos\phi \approx 1$. So the simplified precise kinetic equations are:

$$\ddot{\phi} = \frac{mgl(M+m)}{(M+m)J + Mml^2}\phi - \frac{ml}{(M+m)J + Mml^2}F \quad (2\text{-}10)$$

$$\ddot{x} = -\frac{m^2gl^2}{(M+m)J + Mml^2}\phi + \frac{J + ml^2}{(M+m)J + Mml^2}F \quad (2\text{-}11)$$

After substituting the variables with values from Table 2-1, we can get:

$$\ddot{\phi} = 27.86\phi - 2.538F \quad (2\text{-}10)$$

$$\ddot{x} = -1.343\phi + 1.015F \quad (2\text{-}11)$$

Taking the Laplace transforms of the above equations, we have:

$$s^2\Phi(s) = 27.86\Phi(s) - 2.538F(s) \quad (2\text{-}12)$$

$$s^2X(s) = -1.343\Phi(s) + 1.015F(s) \quad (2\text{-}13)$$

So the transfer functions are:

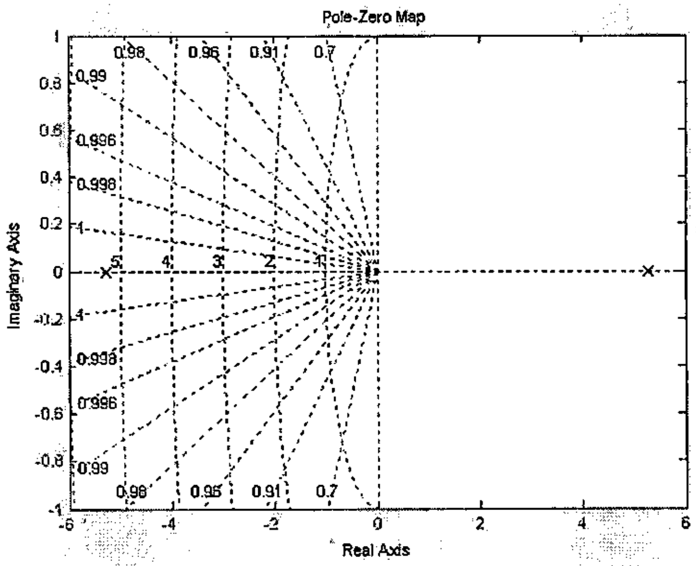
$$\frac{\Phi(s)}{U(s)} = \frac{-2.538}{s^2 - 27.86} \quad (2\text{-}14)$$

$$\frac{X(s)}{\Phi(s)} = \frac{9.800 - 0.400s^2}{s^2} \quad (2\text{-}15)$$

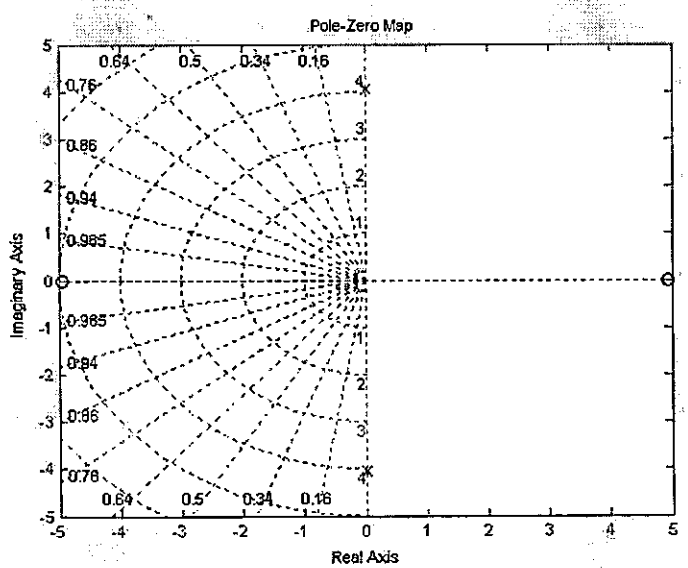
System Stability

The first step in designing compensation for any plant is to observe the closed loop unity feedback response to check for stability. Many systems are unstable in open loop but stable in closed loop configuration. The other way round is also possible that the system is stable in the open loop but unstable in closed loop, although this case is rare.

Figure 3.2 is the pole zero map of the open loop system (○ is a zero and × is a pole), which shows that one of the poles of the transfer function lies on the right half side of the s-plane. Thus the system is absolutely unstable. For the closed loop situation, the root locus of the system in Figure 3.3 indicates that the closed loop system is not stable either because whatever is the value of the loop gain, one branch of the locus remains on the right half side of the s-plane. This reveals that this system can never be controllable by unity feedback (Lu, Lin & Zhou, 2009).

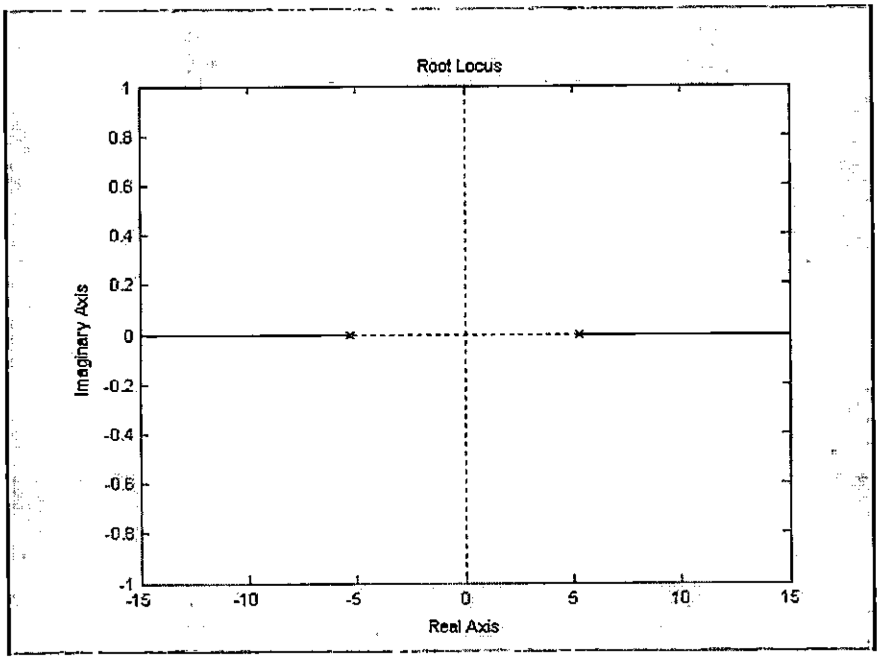


$$\frac{\Phi(s)}{U(s)} = \frac{-2.538}{s^2 - 27.86}$$

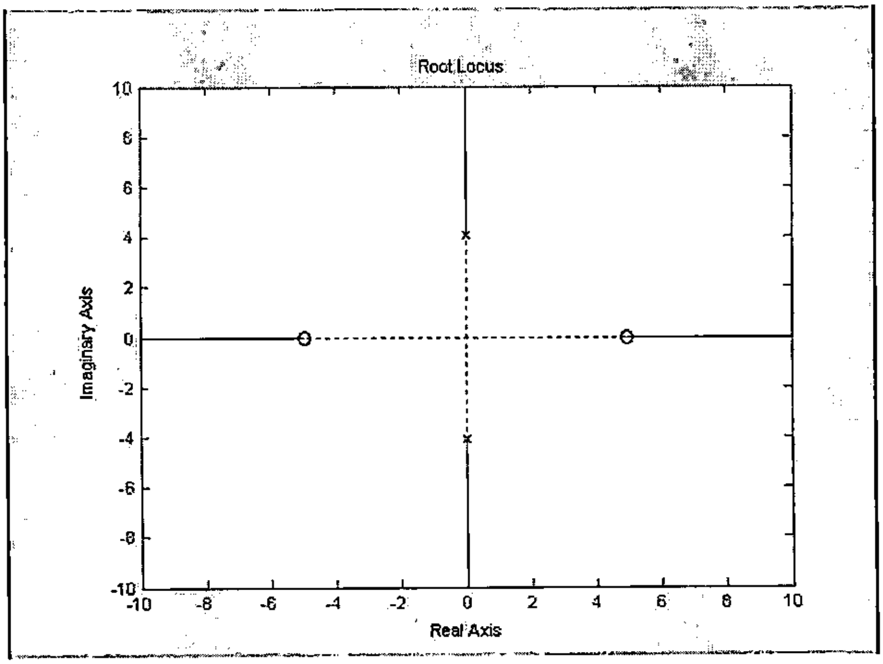


$$\frac{X(s)}{\Phi(s)} = \frac{9.800 - 0.400s^2}{s^2}$$

Figure 3.2 The Pole Zero Maps of the Open Loop System.



$$\frac{\Phi(s)}{U(s)} = \frac{-2.538}{s^2 - 27.86}$$

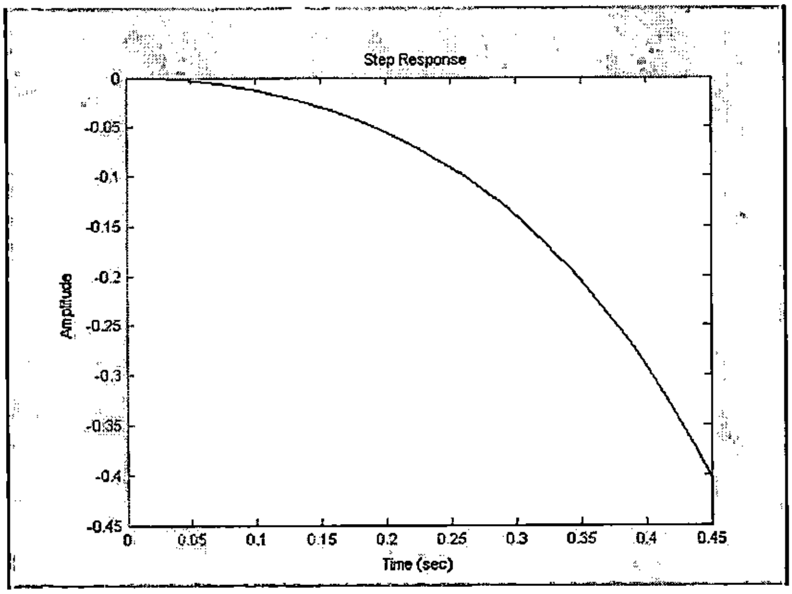


$$\frac{X(s)}{\Phi(s)} = \frac{9.800 - 0.400s^2}{s^2}$$

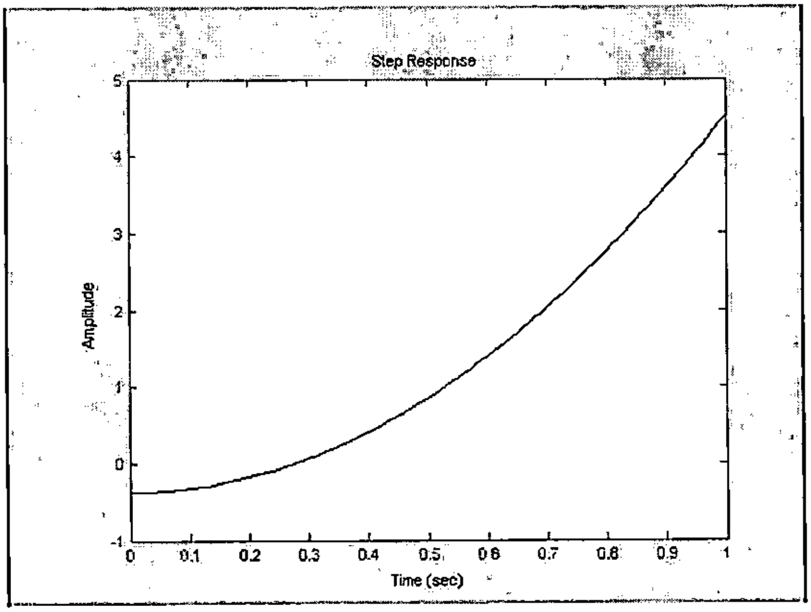
Figure 3.3 The Root Locus of the Closed Loop Systems

In order to prove my judgment above, the step response of the systems are shown in

Figure 3.4.



$$\frac{\Phi(s)}{U(s)} = \frac{-2.538}{s^2 - 27.86}$$



$$\frac{X(s)}{\Phi(s)} = \frac{9.800 - 0.400s^2}{s^2}$$

Figure 3.4 The Step Response of the Open Loop Systems.

So, to sum up, the uncompensated systems are unstable, whether they are open loop or closed loop.

Control Strategy

1. PID control.

For the single inverted pendulum system, double closed loop control system will be implemented. There are two variables that need to be controlled: the angle of the pendulum, and the position of the cart. The block diagram is shown below in which $G_1(s)$, the angle, is the transfer function of the inner loop while $G_2(s)$, the position, is that of the outer loop.

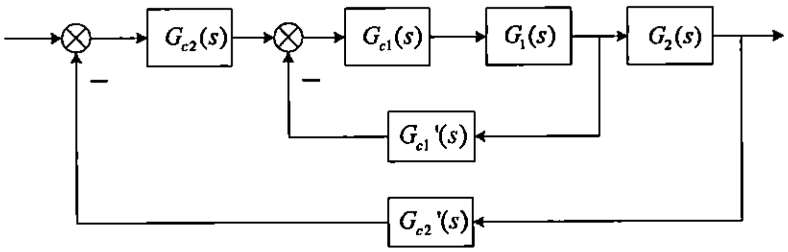


Figure 3.5. Block Diagram of Double Closed Loop Control

For the angle control, the expected control targets are below (standard parameters in engineering):

1. Static error $\dot{e}_{ss} \le \frac{1}{30}$;
2. The system uses the optimum damping coefficient 0.707;
3. Overshoot $M_p \le 5\%$;
4. Settling time $t_s \le 2s$;

Therefore, $K = -\frac{1}{\dot{e}_{ss}} = -30$ and $\zeta = 0.707$.

For the purpose of moving the root locus to the left side, which would make the system more stable, a zero can be added. So the controller could be:

$$G_{c1}(s) = K$$

$$G_{c1}'(s) = K_1 s + K_2$$

The closed loop transfer function is:

$$W_1(s) = \frac{G_{c1}G_1}{1+G_{c1}'G_{c1}G_1} = \frac{\frac{-2.538K}{s^2-27.86}}{1+\frac{-2.538K(K_1 s+K_2)}{s^2-27.86}} = \frac{-2.538K}{s^2-2.538KK_1 s-27.86-2.538KK_2}$$

$$\because K = -30$$

$$\therefore W_1(s) = \frac{76.14}{s^2+76.14K_1 s-27.86+76.14K_2}$$

The standard second order transfer function model is

$$W_1(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\therefore \omega_n^2 = 76.14,\ 2\zeta\omega_n = 76.14K_1,\ \omega_n^2 = -27.86+76.14K_2$$

$$\because \zeta = 0.707$$

$$\therefore \omega_n = 8.726 rad/s, K_1 = 0.1621, K_2 = 1.366$$

$$\therefore G_{c1}(s) = -30,\ G_{c1}'(s) = 0.1621s+1.366$$

$$\therefore W_1(s) = \frac{76.14}{s^2+12.34s+76.14}$$

One checks if the compensated system has met the expectations:

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.707\pi}{\sqrt{1-0.707^2}}} = 4.33\% \le 5\%$$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.707\times 8.726} = 0.648s \le 2s$$

So the controller is acceptable.

For the position control, the control object is now

$$\begin{aligned} W_1(s)G_2(s) &= \frac{76.14}{s^2+12.34s+76.14} \times \frac{9.800-0.400s^2}{s^2} \\ &= \frac{9.8\left(-0.04082s^2+1\right)}{s^2\left(0.01313s^2+0.1621s+1\right)} \end{aligned}$$

Since the order of the system is high, the higher order terms are going be removed by choosing a suitable crossover frequency ω_c. Rewriting the $W_1(s)G_2(s)$:

$$W_1(s)G_2(s) = \frac{9.8\left[-(0.2020s)^2+1\right]}{s^2\left[(0.1146s)^2+0.1621s+1\right]}$$

$$\therefore \omega_1 = \frac{1}{T_1} = \frac{1}{0.2020} = 4.950 rad/s,\ \omega_2 = \frac{1}{T_2} = \frac{1}{0.1158} = 8.636 rad/s$$

Therefore, if let $\omega_c < \omega_1 < \omega_2$, then:

$$-(0.2020s)^2+1 \approx 1;\ (0.1158s)^2+0.1653s+0.1020 \approx 0.1653s+0.1020$$

$$\therefore W_1(s)G_2(s) \approx \frac{9.8}{s^2(0.1621s+1)}$$

Check if the truth that $\omega_c < \min\{\omega_1,\omega_2\}$ for $W_1(s)G_2(s)$.

$$W_1(j\omega)G_2(j\omega) \approx \frac{9.8}{-\omega^2(0.1621j\omega+1)} = \frac{9.8}{-0.1621j\omega^3-\omega^2}$$

$$\because \left|W_1(j\omega_c)G_2(j\omega_c)\right| = 1$$

$$\therefore \left|\frac{9.8}{-0.1621j\omega^3-\omega^2}\right| = 1$$

$$\therefore \omega_c \approx 2.97 rad/s < \min\{\omega_1,\omega_2\} = \min\{4.950, 8.726\} = 4.950$$

Therefore, the simplification is reasonable.

By using a PD controller, the root locus can be relocated to the left side of s-plane and this will also make the system become a standard type II system.

$$G(s)=\frac{K(\tau s+1)}{s^2(Ts+1)}$$

So, $G_{c2}(s)=K_p(\tau s+1)$ and $G_{c2}'(s)=1$.

$$\therefore W_1(s)G_{c2}(s)G_2(s)=\frac{9.8K_p(\tau s+1)}{s^2\left(0.1621s+1\right)}$$

In practical use, the ratio of τ and T is defined as h, i.e. $h=\frac{\tau}{T}$. The parameter h here reflects the system's comprehensive performance and 5 is usually a better value for h. So the value of τ is:

$$\tau=hT=5\times 0.1621=0.8105$$

$$9.8K_p=\omega_1\omega_c=\frac{\omega_c}{\tau}=\frac{1}{0.8105}=1.234$$
$$\therefore K_p=\frac{1.234}{9.8}=0.1260$$
$$\therefore G_{c2}(s)=0.1260(0.8105s+1)=0.1020s+0.1260$$

Therefore, the open loop transfer function of the whole double loop feedback system is:

$$G(s)=G_{c2}(s)W_1(s)G_2(s)=\frac{76.14(-0.4s^2+9.8)(0.1020s+0.1260)}{s^2(s^2+12.34s+76.14)}$$
$$=\frac{-3.107s^3-3.837s^2+76.11s+94.02}{s^4+12.34s^3+76.14s^2}$$

The root locus of $G(s)$ is shown in Figure 3.6. Apparently all the poles are in the left half plane, which means that the system is stable when the locus is on the left half plane.

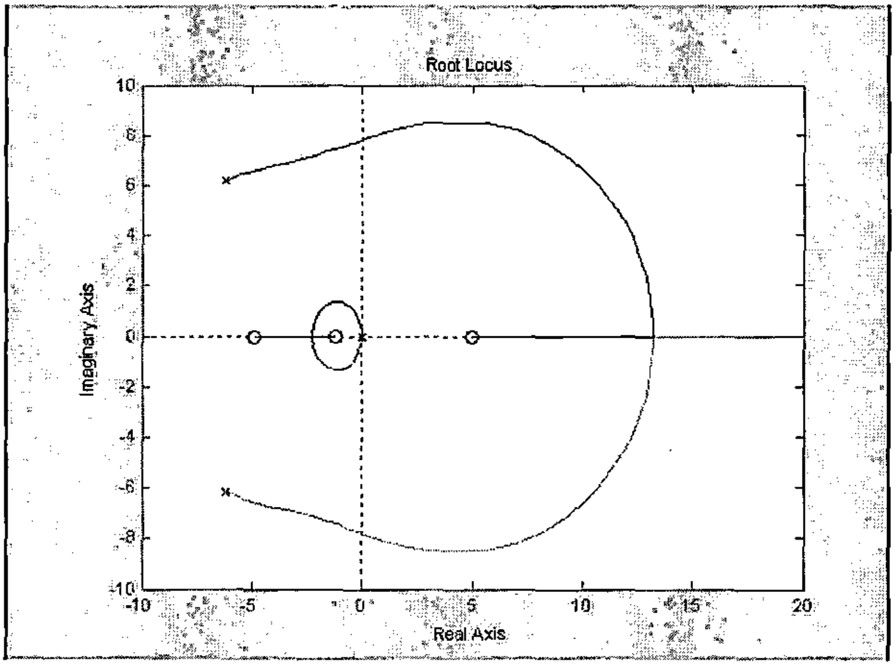


Figure 3.6. The Root Locus of the Whole Closed Loop System

In order to see the step response of the closed loop system, the simulation is done on the Simulink in MATLAB. The full diagram of the inverted pendulum system is created based on the block diagram shown in Figure 3.5 and all the transfer functions that were just derived.

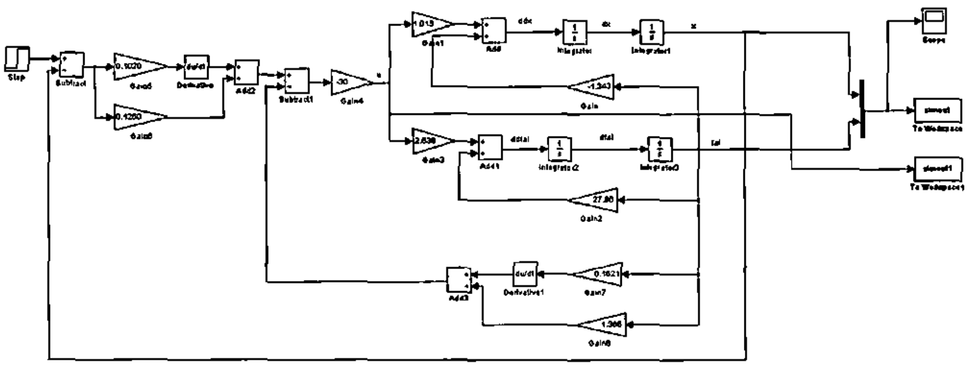


Figure 3.7. Simulation Diagram of the Single Inverted Pendulum System (For larger figure please refer to the Appendix).

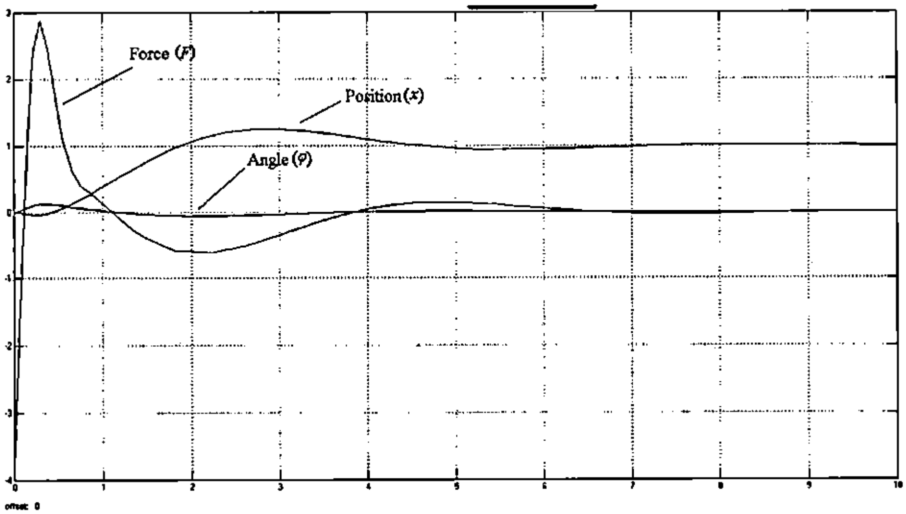


Figure 3.8. Step Response of the System (For larger figure please refer to the Appendix).

So the system will get back to the stable state in about 6 seconds.

2. Pole placement

The expectations of the system are still the same as the ones in PID control, which are:

a. Take the optimum damping coefficient 0.707

b. Overshoot $M_p \le 5\%$

c. Settling time $t_s \le 2s$

Actually, if the damping coefficient $\zeta = 0.707$, the M_p will definitely be less than 5%. $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.707\pi}{\sqrt{1-0.707^2}}} = 4.33\% \le 5\%$. If the optimum damping coefficient is taken, usually the corresponding overshoot will be satisfying. As for the t_s,

$t_s = -\dfrac{\ln(0.05\sqrt{1-\zeta^2})}{\zeta\omega_n}$ are required to be less than 2 seconds. So,

$$t_s = -\frac{\ln(0.05\sqrt{1-\zeta^2})}{\zeta\omega_n} \le 2$$

i.e.

$$\omega_n \ge -\frac{\ln(0.05\sqrt{1-\zeta^2})}{2\zeta} = -\frac{\ln(0.05\times\sqrt{1-0.707^2})}{2\times0.707} = 2.364 rad/s$$

Take $\omega_n = 2.4 rad/s$, so the closed-loop dominant poles are:

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{1-\zeta^2}\,\omega_n j = -1.7 \pm 1.7j$$

As for the non-dominant poles, whichever are on the left side of the complex plane, and that are farther from the *imaginary* axis than $s_{1,2}$ can be taken. Here $s_{3,4} = -10$.

When finishing the mathematical modeling, we will get the system state space expression below:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{cases}$$

According to the block diagram of the state feedback control system, shown in Figure 3.2, the state feedback matrix, *K*, can be calculated easily.

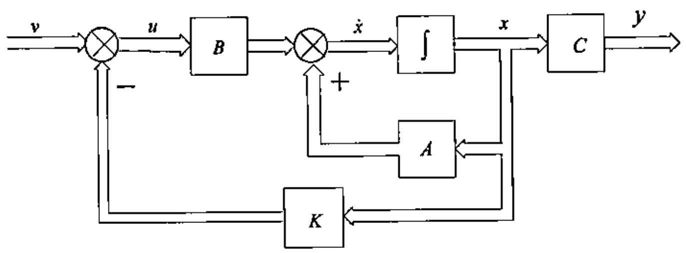


Figure 3.9. Block diagram of state feedback control system.

As is explained in the previous part, two variables, angle and position, are selected to control the whole system. Recall the differential equation (2-10) and (2-11) and write the state space expression of the system.

$$\ddot{\phi} = 27.86\phi - 2.538F \qquad (2\text{-}10)$$

$$\ddot{x} = -1.343\phi + 1.015F \tag{2-11}$$

Assume $x_1 = x,\ \dot{x}_1 = x_2;\ x_3 = \phi,\ \dot{x}_3 = x_4;\ u = F$

Then

$$\dot{x}_2 = \ddot{x} = -1.343\phi + 1.015F = -1.343x_3 + 1.015u$$
$$\dot{x}_4 = \ddot{\phi} = 27.86\phi - 2.538F = 27.86x_3 - 2.538u$$

$$\therefore \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -1.343x_3 + 1.015u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = 27.86x_3 - 2.538u \end{cases}$$

i.e.

$$\begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1.343 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 27.86 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.015 \\ 0 \\ -2.538 \end{bmatrix} u \\ \mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \end{cases}$$

$$\therefore \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1.343 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 27.86 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 1.015 \\ 0 \\ -2.538 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Before calculating the feedback matrix **K**, the controllability and observability of the system need to be checked (Liu & Tang, 2008). The necessary and sufficient condition of a system's being controllable and observable is that both the controllability and observability matrix are full rank.

Since

$$\mathbf{M} = \left[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \cdots, \mathbf{A}^{n-1}\mathbf{B}\right]$$

$$\mathbf{N} = \left[\mathbf{C}, \mathbf{CA}, \mathbf{CA}^2, \cdots, \mathbf{CA}^{n-1}\right]^T$$

in this inverted pendulum system:

$$\mathbf{M} = \left[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \mathbf{A}^3\mathbf{B}\right] = \begin{bmatrix} 0 & 1.015 & 0 & 3.408 \\ 1.015 & 0 & 3.408 & 0 \\ 0 & -2.538 & 0 & -70.1 \\ -2.538 & 0 & -70.71 & 0 \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1.343 & 0 \\ 0 & 0 & 27.86 & 0 \\ 0 & 0 & 0 & -1.343 \\ 0 & 0 & 0 & 27.86 \end{bmatrix}$$

$$\therefore \text{Rank}(\mathbf{M}) = \text{Rank}(\mathbf{N}) = 4$$

Therefore, the system is both controllable and observable.

Assume the feedback matrix $\mathbf{K} = \left[K_0 \quad K_1 \quad K_2 \quad K_3\right]$, then the closed-loop characteristic polynomial is $f(\lambda) = \det[\lambda\mathbf{I} - (\mathbf{A} + \mathbf{BK})]$.

$$\begin{aligned} \therefore f(\lambda) = {} & \lambda^4 + \left(2.538K_3 - 1.015K_1\right)\lambda^3 + \left(2.538K_2 - 27.86 - 1.015K_0\right)\lambda^2 \\ & + 24.87K_1\lambda + 24.87K_0 \end{aligned}$$

According to the expectations of the system, the revised system will have four poles: $s_{1,2}=-1.7\pm1.7j,\ s_{3,4}=-10$. So the expected closed-loop characteristic polynomial is:

$$\begin{aligned}\therefore f^{*}(\lambda) &= [\lambda-(-1.7+1.7j)][\lambda-(-1.7-1.7j)][\lambda-(-10)]^{2}\\ &= \lambda^{4}+23.4\lambda^{3}+173.78\lambda^{2}+455.6\lambda+578\end{aligned}$$

Compare $f^{*}(\lambda)$ and $f(\lambda)$:

$$\begin{cases}2.538K_3-1.015K_1=23.4\\ 2.538K_2-27.86-1.015K_0=173.78\\ 24.87K_1=455.6\\ 24.87K_0=578\end{cases}$$

$$\therefore\begin{cases}K_0=23.24\\ K_1=18.32\\ K_2=88.74\\ K_3=16.55\end{cases}$$

$$\therefore \mathbf{K}=[K_0\quad K_1\quad K_2\quad K_3]=[23.24\quad 18.32\quad 88.74\quad 16.55]$$

Therefore, the closed-loop state space expression is:

$$\begin{cases}\dot{\mathbf{x}}=(\mathbf{A}+\mathbf{BK})\mathbf{x}+\mathbf{B}v=\begin{bmatrix}0 & 1 & 0 & 0\\ 23.59 & 18.59 & 88.73 & 16.80\\ 0 & 0 & 0 & 1\\ -58.98 & -46.50 & -197.36 & -42.00\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ 1.015\\ 0\\ -2.538\end{bmatrix}v\\ \mathbf{y}=\mathbf{Cx}+\mathbf{D}v=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ 0\end{bmatrix}v\end{cases}$$

The step response is also simulated by the Simulink in MATLAB. The diagram and step response chart are shown in Figure 3.10 and 3.11 respectively (For larger figures please refer to the Appendix).

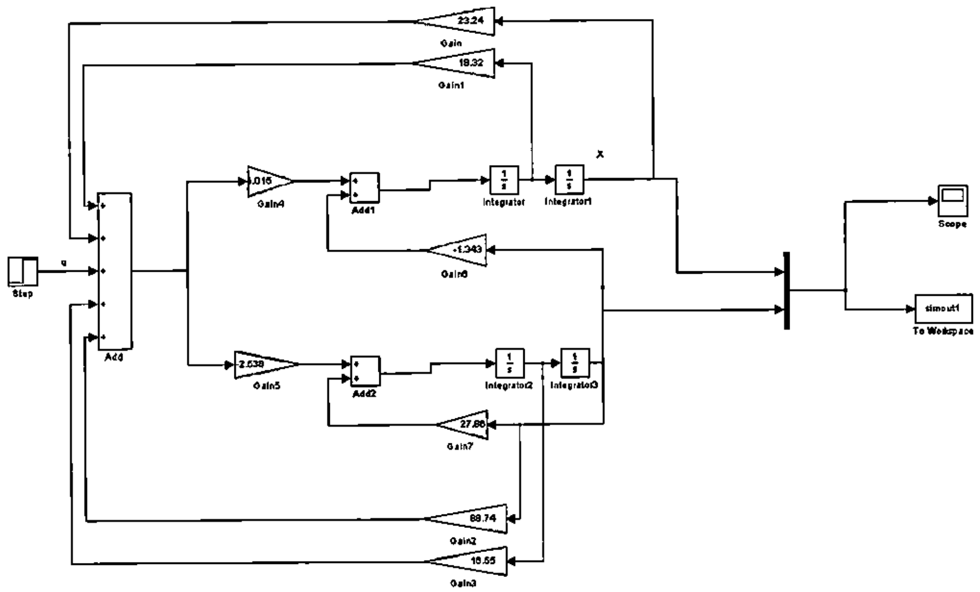


Figure 3.10. Simulation Diagram of the Single Inverted Pendulum System.

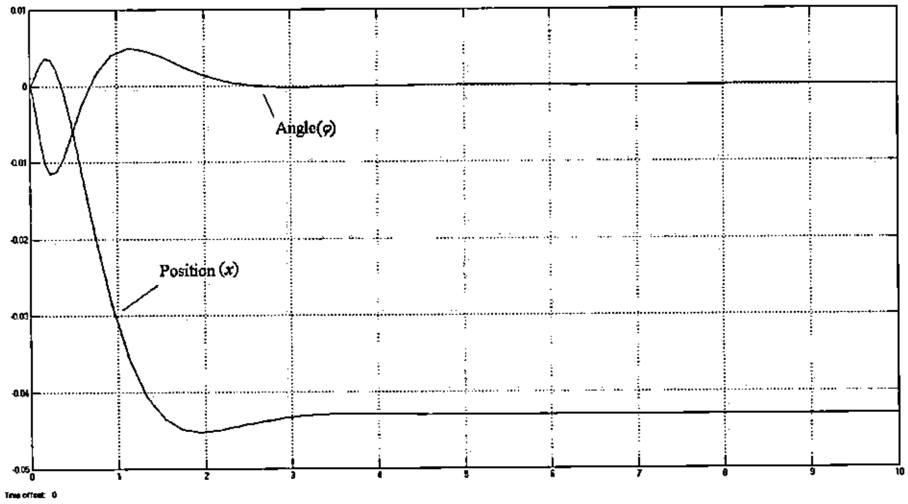


Figure 3.11. Step Response of the System.

So the system will get back to the stable state in about 3.5 seconds.

Instruments and Software Used for Actual System

1. HP Core 2 PC
2. US Digital Incremental Encoder S2-1000-236-NE-D-D
3. NI-PCI-6229
4. NI-SCB-68
5. H-Bridge LMD 1820
6. LabVIEW 2011

Wiring Diagram for Hardware

The whole system is going to be controlled by the LabVIEW program in the PC. Due to the limitations of the lab, only single closed-loop feedback could be implemented in the single inverted pendulum system that was built in the lab. This system senses the angle of pendulum and adjusts the speed of the motor that drives the cart accordingly. The schematic diagram is shown in Figure 3.12.

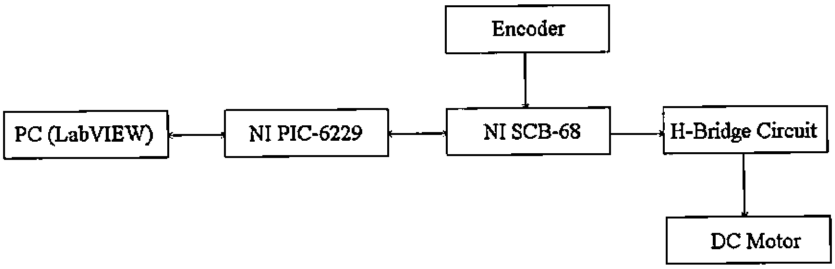


Figure 3.12. Schematic Diagram of the System.

A PC with the LabVIEW program works as the core controller. It receives the signal from the encoder and sends out the PWM, or Pulse Width Modulated signal to control the speed of the motor after a series of calculations. The National Instruments PCI-6229 is a multifunction M Series data acquisition (DAQ) board. This board has two counters, Counter 0 and Counter 1. Counter 0 is used to read the pulse signal from the encoder and convert the two-phase pulse trains to an angle value, while Counter 1 is used to generate the PWM signal. The LabVIEW program is shown in Figure 3.13.

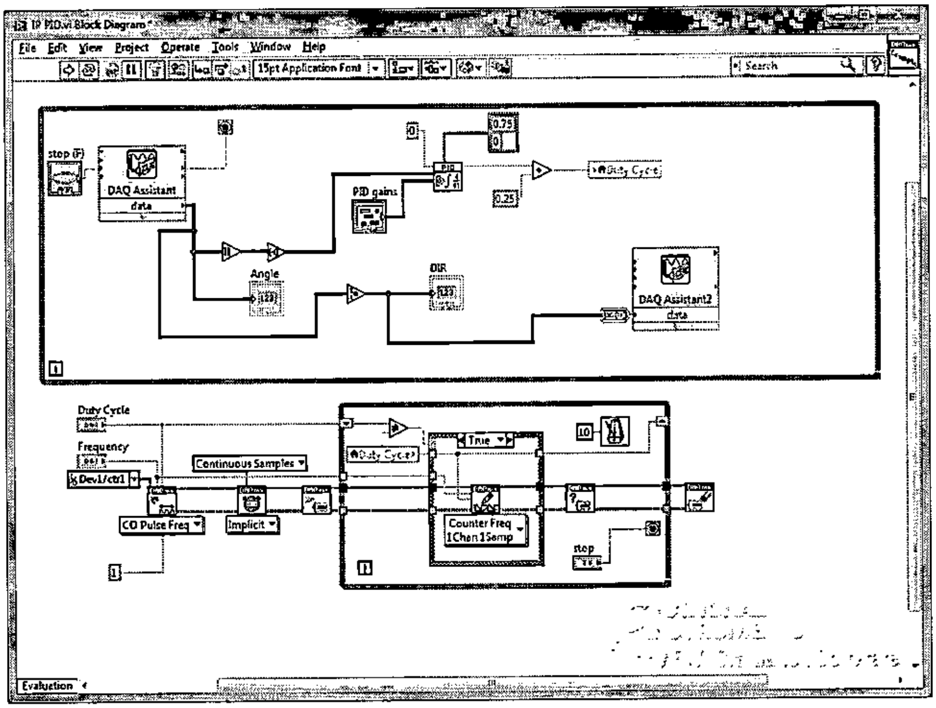


Figure 3.13. LabVIEW Program for the System.

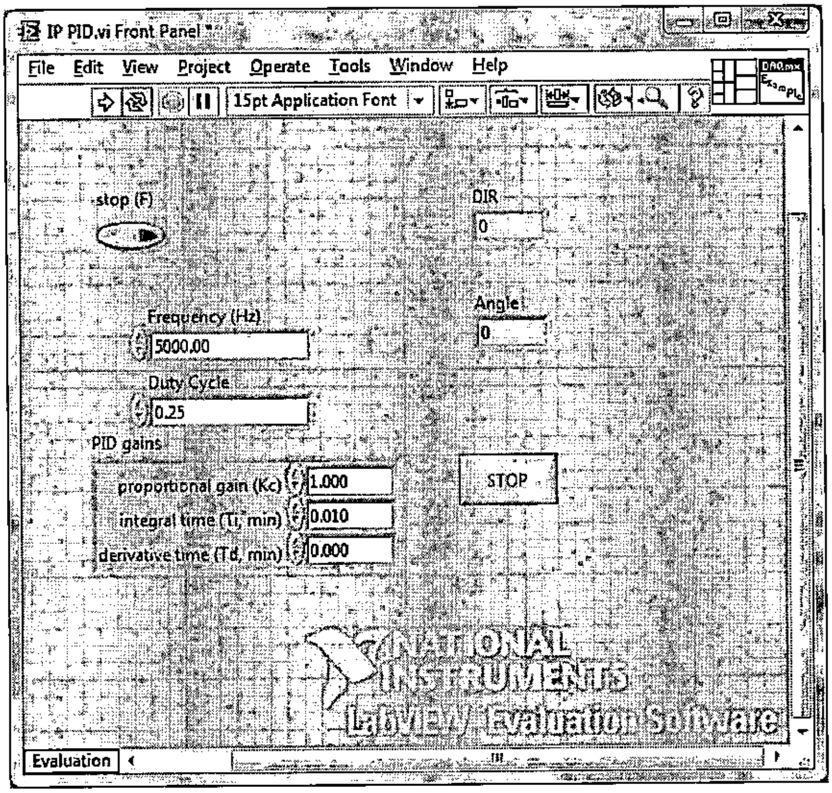


Figure 3.14. Front Panel of the Program

The program basically has two parts. The part on the top is for reading the encoder and the other part is for generating the PWM signal. These two programs are connected with each other by the angle variable. Once the angle has been changed, the width of the pulse train which is generated will change accordingly by the PID control algorithm. Besides, the program also generates a direction signal, which is used for the H-Bridge drive circuit to decide whether the motor should rotate clockwise or counter-clockwise. The H-Bridge motor drive circuit and the program will be explained in detail later.

The PCI-6229 from National Instrument, as was mentioned, is a data acquisition (DAQ) board. Three components are used for the whole system: Counter 0, Counter 1, and Digital Output 0.0. The pins that are used for this system are PFI 8 and PFI 10, which are the Channel A and Channel B for the encoder on Counter 0, PFI 13, which is the output port for PWM signal on Counter 1, and P0.0, which is the digital output used to send digital signal to control the direction of the DC motor connected to the following H-Bridge circuit. Together with the DAQ board is the National Instrument SCB-68. SCB-68 is a shielded I/O connector block for interfacing I/O signals to plug-in data acquisition (DAQ) devices with 68-pin connectors. The corresponding terminals for PFI 8, PFI 10, PFI 13 and P0.0 are Pin 37, Pin 45, Pin 40 and Pin 52 respectively.

The US Digital S2-1000-236-NE-D-D encoder is an incremental encoder with 1000 cycles per revolution (CPR). Output channels A and B from the encoder (pins 3 and 5 respectively) are connected to the Pin 37 and Pin 45 on the NI SCB-68. Besides, the encoder also needs an external five-volt power supply.

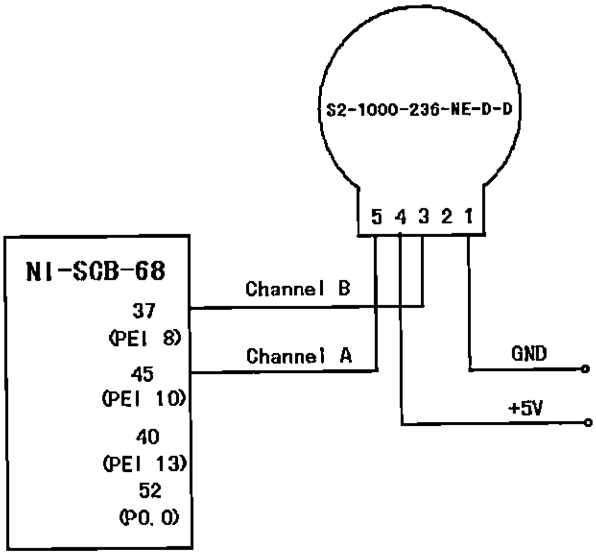


Figure 3.15. Wiring Diagram for Encoder.

The LMD18200T H-Bridge from National Semiconductor is a DC motor drive circuit to control the speed by PWM signal. This H-Bridge circuit accepts up to 55V supply voltage and is able to deliver up to 3A of output current. The device contains the 4-switch setup required for our motor control, along with protection diodes and circuitry to ensure there is never a short from the supply to ground. There are five pins connected to this drive. Pin 6 and pin 7 are for the external power supply which provides a voltage of 15V. Pin 5 is for the input of the PWM signal and is connoted to Pin 40, which is the output of Counter 1, on NI SCB-68. Pin 3 is connected to Pin 52, a digital output on NI SCB-68 to get the motor's direction information. Last but not least, the output pins, Pin 2 and Pin 10, are connected to the two terminals on the DC motor.

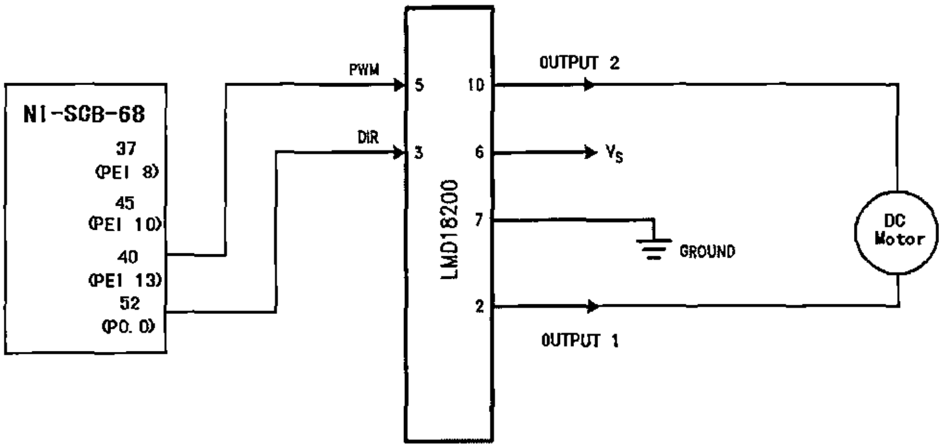


Figure 3.16. Wiring Diagram for LMD 1820

Therefore, to sum up, the entire wiring diagram for the whole system is shown in Figure 3.17 (For larger figure please refer to the Appendix).

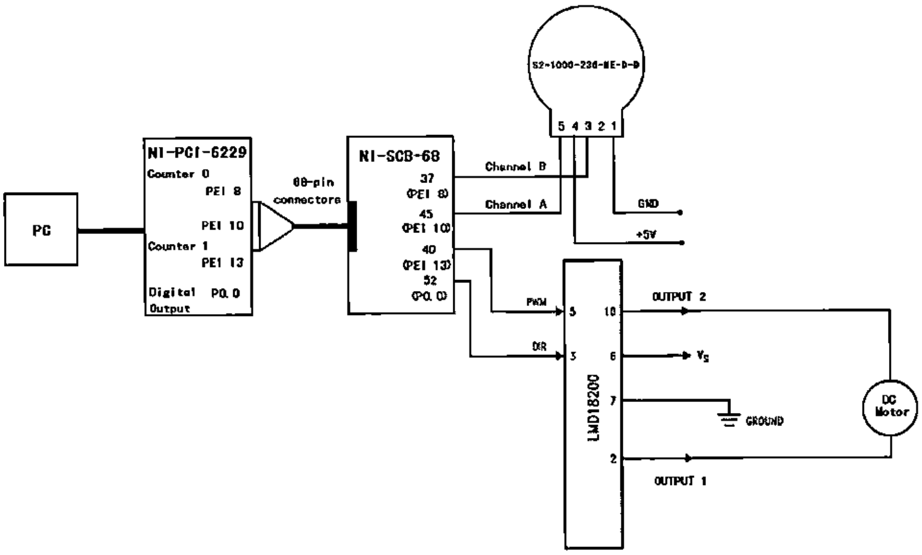


Figure 3.17. Wiring Diagram for the Single Stage Inverted Pendulum System.

Programming

As is shown and mentioned in Figure 3.13 (For larger figure please refer to the Appendix), the whole program is basically divided into two parts, one for reading signals from encoder and the other for generating the adjustable pulse train, or PWM signals. For the first part, two modules from LabVIEW 2011 are used, which are DAQ Assistant and PID toolkit.

The DAQ Assistant is an easy-to-use graphical interface for configuring measurement tasks and channels and for customizing timing, triggering, and scales without programming. Using the DAQ Assistant can help to configure a measurement task for all of the DAQ applications and then generate code to configure and use the task in the application program. Right click on the blank area of Block Diagram window and select Express, Input, and then DAQ Assist, referring to Figure 3.18.

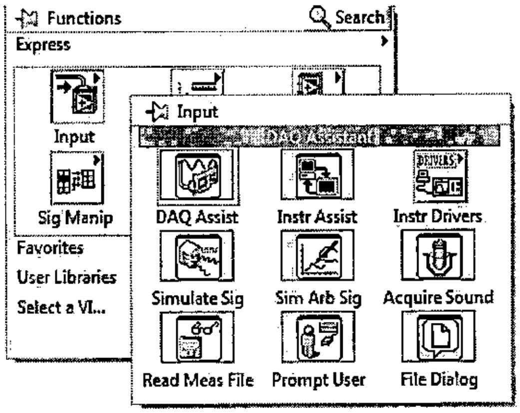


Figure 3.18. Inserting the DAQ Assistant.

Then the DAQ Assistant Creating Wizard in Figure 3.19 shows up. Since the program is going to read the angular signals from encoder by Counter 0 on NI-PCI-6229, go to Acquire Signals, Counter Input, Position, and then Angular. After hitting "Next", the supported physical channels will be shown. Here, Counter 0 is selected.

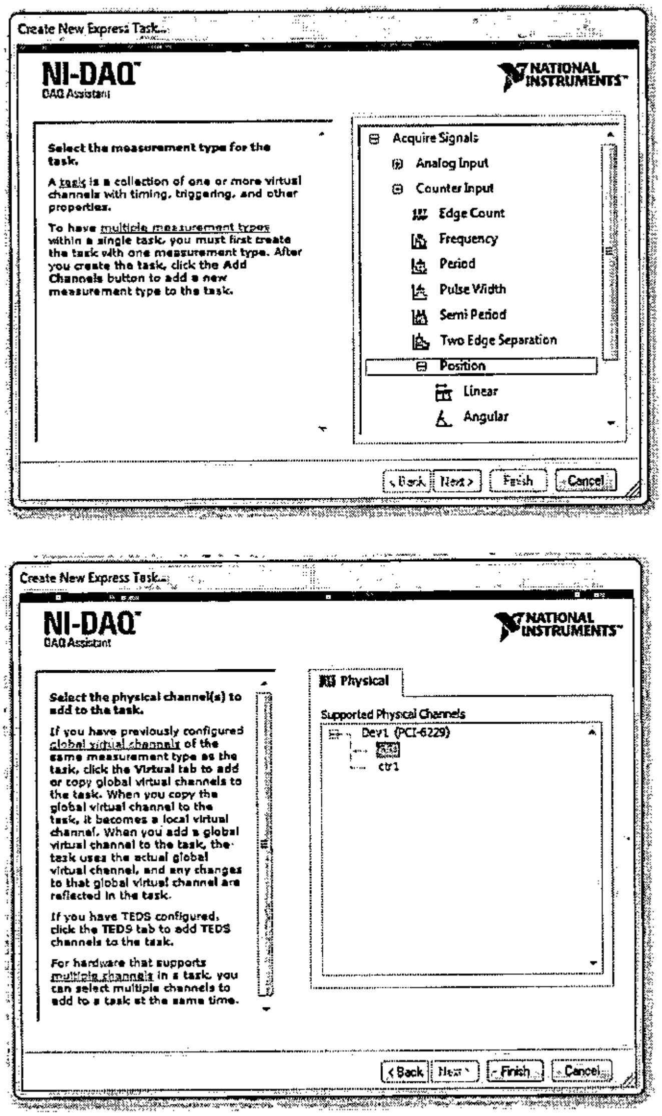


Figure 3.19. DAQ Assistant Creating Wizard for Angular Position Input.

After hitting Finish, the DAQ Assistant has been created successfully, and what is done next is to double click on the DAQ Assistant icon and configure the parameters for the encoder. See Figure 3.20. After putting in the information and parameters of the encoder, we can test the encoder to see if it works as the way we wish by hitting the Run button at the top, and the measured value will be shown. Hit OK when everything works fine.

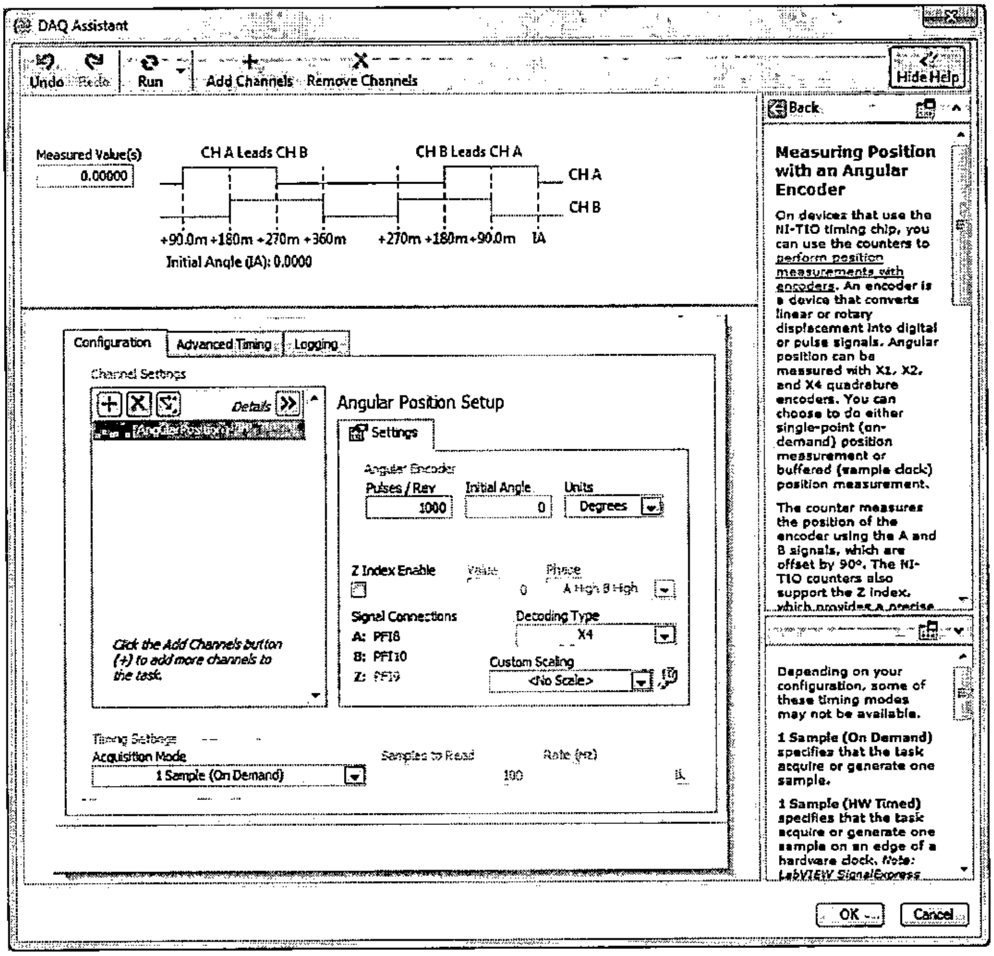


Figure 3.20. Configuration of the DAQ Assistant.

What is next is to build the VI to generate the direction information by using another DAQ Assistant. This time in the DAQ Assistant creating wizard, we go to Generate Signals, Digital Output, and then Line output. See Figure 3.21. After hitting Next, Port 0/Line 0 is selected in the list of supported physical channels. Hit Finish when it is done.

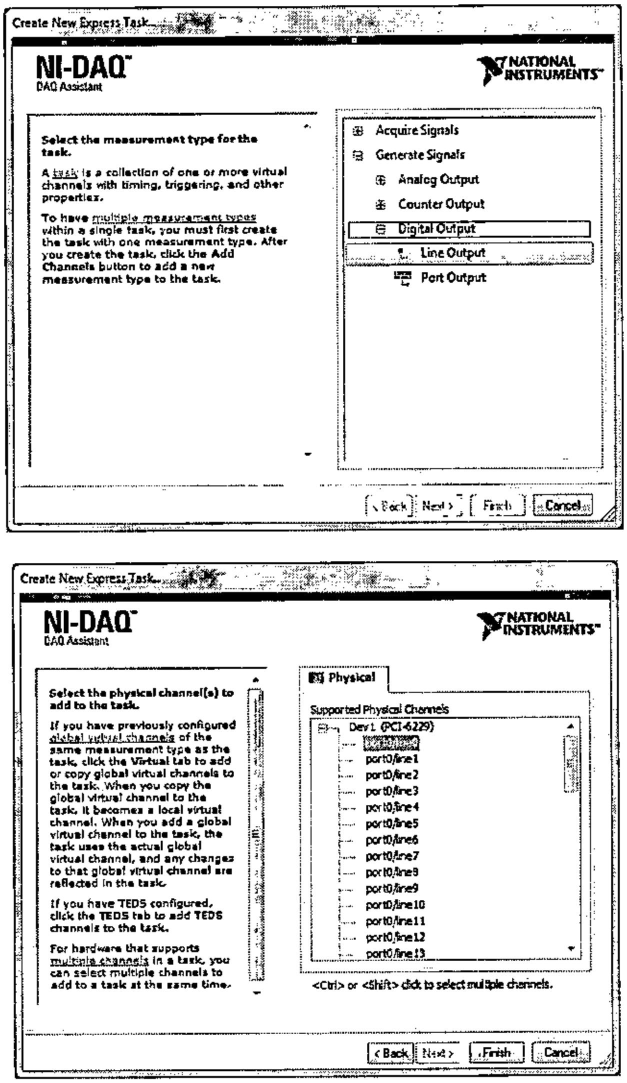


Figure 3.21. DAQ Assistant Creating Wizard for Digital Output.

PID toolkit from National Instruments hardware can be used to develop LabVIEW control applications. Use I/O hardware, like a DAQ device, FieldPoint I/O modules, or a GPIB board, to connect the PC to the system that needs to control. Right click on the blank area of Block Diagram window and select Control Design & Simulation, PID, and then PID.vi. Place the VI in blank area and connect the parameters it needs to realize the PID algorithm, like PID gians, set point, process variable, output range and output.

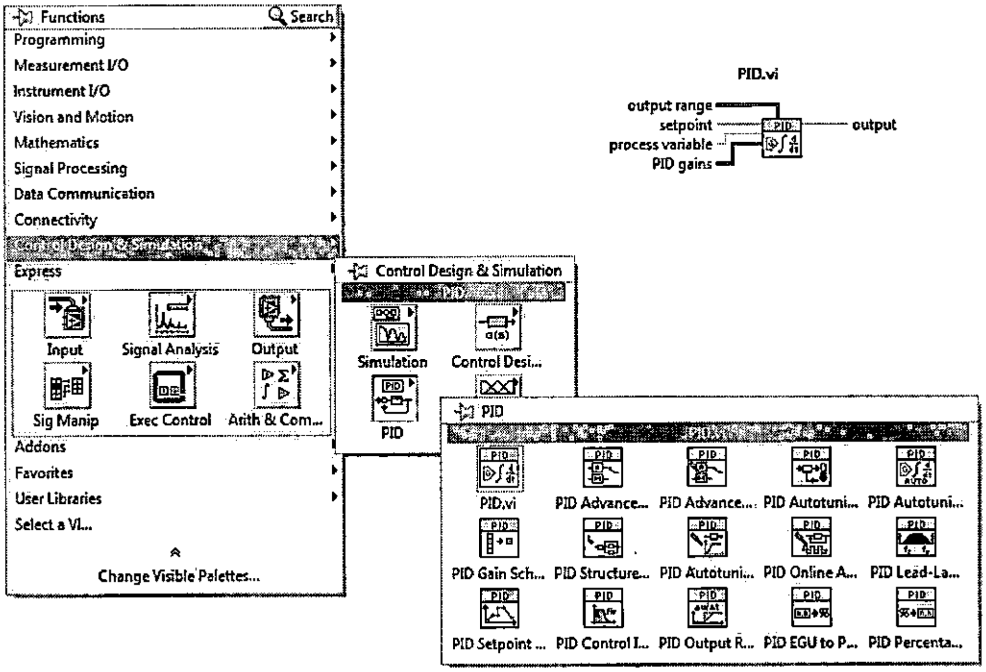


Figure 3.22. PID Toolkit from National Instruments

Recalling Figure 3.13, the second part of the program aims at generating adjustable pulse trains, or PWM signals. As a matter of fact, the DAQ Assistant used in the first part of the program is be able to generate pulse trains. It is convenient and does not involve too much programming work. However, taking advantage of the DAQ Assistant makes it nearly impossible to change the duty cycle for the PWM while the program is running. The duty cycle of the PWM signal that DAQ Assistant generates has to be preset before running the program. Consequently, NI-DAQmx programming is used to take the DAQ Assistant's place in this case.

NI-DAQmx is the next generation driver for the data acquisition hardware from National Instruments. It contains a LabVIEW Application Programming Interface (API) which allows customers to create applications for their own device. It is easy to use and has many new features such as improved ease of use, faster development time, multithreaded measurements and increased accuracy of measurements. Also, the data acquisition application in LabVIEW and NI-DAQmx is quite straightforward. NI-DAQmax can be found when you right click on the blank area of Block Diagram window and select Mesurement I/O and then DAQmx. See Figure 3.23.

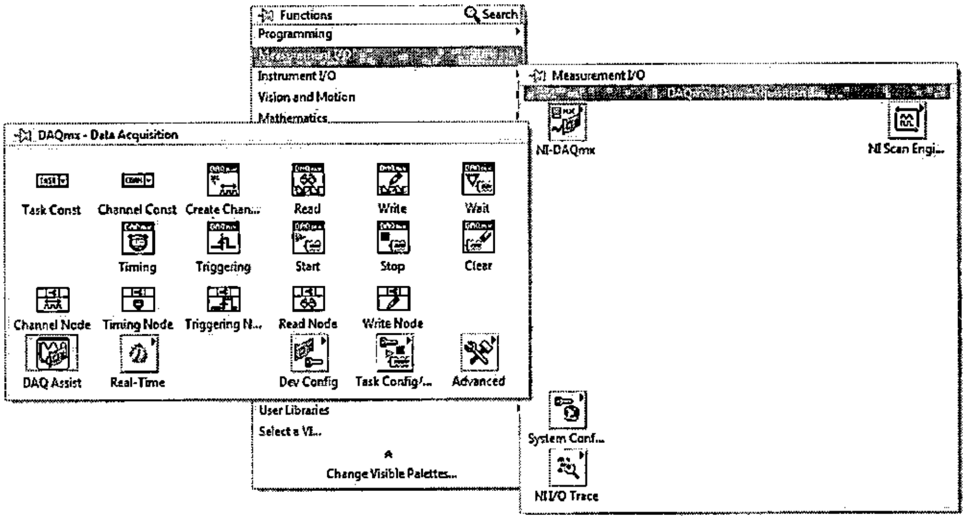


Figure 3.23. NI-DAQmx.

Before actually starting to generate PWM signals, the physical channel has to be created first. So choose "DAQmx Create Virtual Channel.vi" and initialize the parameters of the PWM signals such as frequency and duty cycle. In this case, the duty cycle generated by the PID controller is used. Then DAQmx Timing VI is used to configure the duration of the pulse generation. The Implicit instance should be used when no sample timing is needed, such as in counter tasks like pulse train generation. Additionally, choose Continuous as the sample mode. Finally, call the DAQmx Start VI. This VI begins the pulse train generation and loop continuously until the user presses the

Stop button. The duty cycle control on the front panel is checked every iteration of the loop. If it has been changed, the new duty cycle is set using the DAQmx Write VI. If the duty cycle has not been changed, the false case of the case structure executes, and nothing is updated.

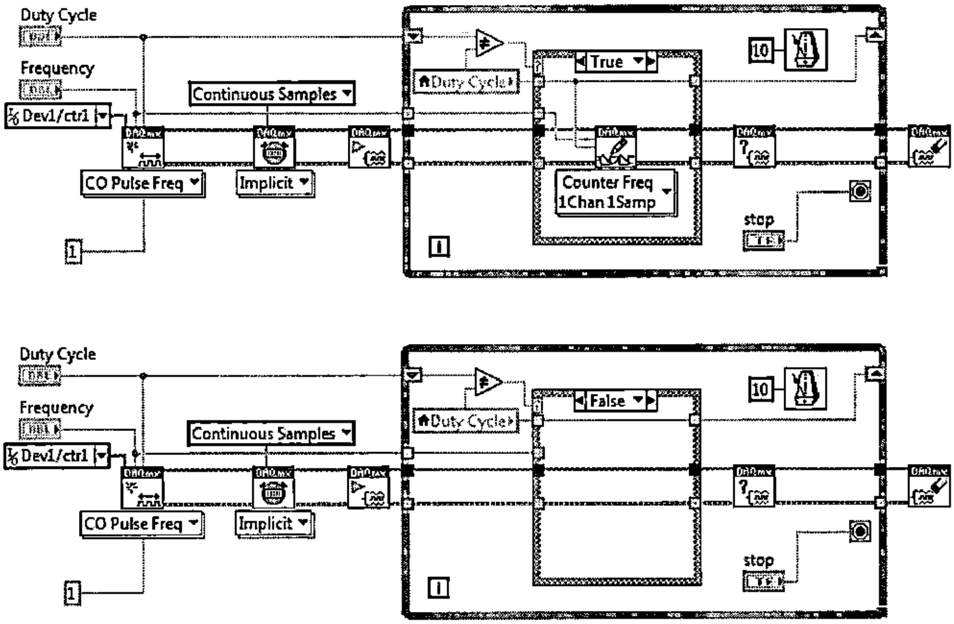


Figure 3.24. The "True" and "False" Case of the DAQmx Program.

CHAPTER IV FINDINGS

Introduction

After setting up both the mechanical and electrical system, it is the time to test the each component of the system. The input signal from the encoder will be tested first, followed by the output PWM signals. Different duty cycles will be assigned to test if the width of the pulse train can be modified accordingly. Then, the motor will be driven by the mutative PWM signal. The motor speed should change with duty cycle changes. After the motor can be well controlled by the PWM signal, the change of angle will be directly used to control the speed of motor. This means that the disturbance of angle is able to be converted to the change of the duty cycle of PWM signal, and then the new duty cycle will be reflected on the width of the PWM signal so that the voltage across the motor will be changed and therefore the motor will run at a new speed. Finally, whether the newly built inverted pendulum system can balance itself will be tested.

Testing

1. The encoder.

Run the LabVIEW program and turn the shaft of the encoder manually. The real-time curve in Figure 4.1 shows the angle can be sensed precisely and the response time is acceptable.

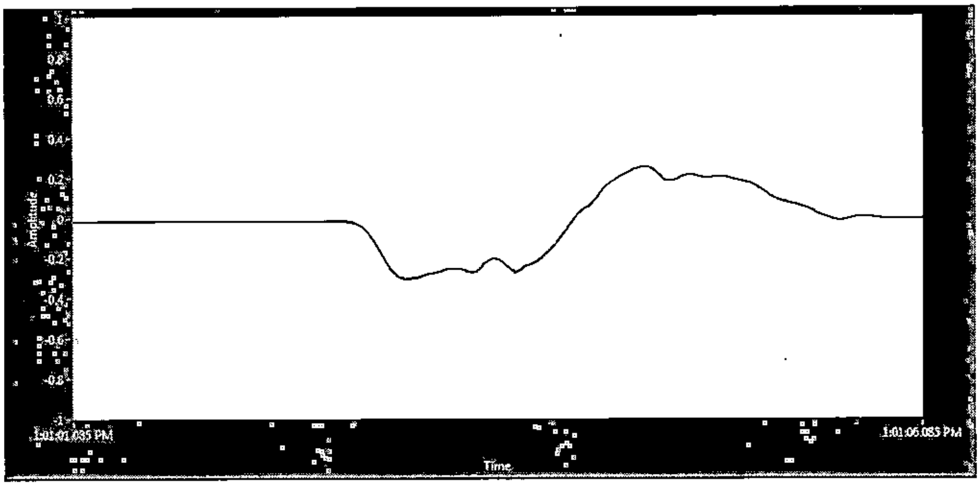


Figure 4.1. Test of the Encoder.

2. The PWM signal.

Run the LabVIEW program and change the value of duty cycle on the front panel of the program. The PWM signal shown on the oscilloscope indicates that the PWM signal is well adjusted by the assigned duty cycle.

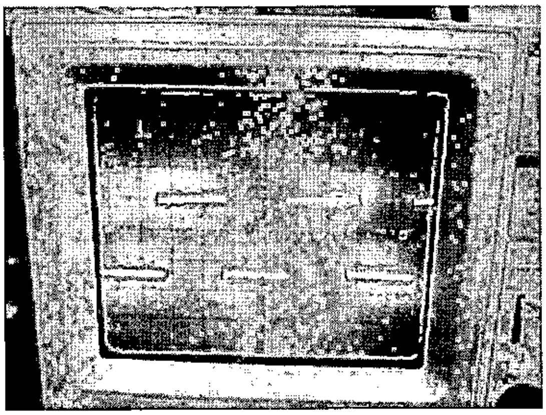


Figure 4.2. PWM Signal with Duty Cycle of 0.61

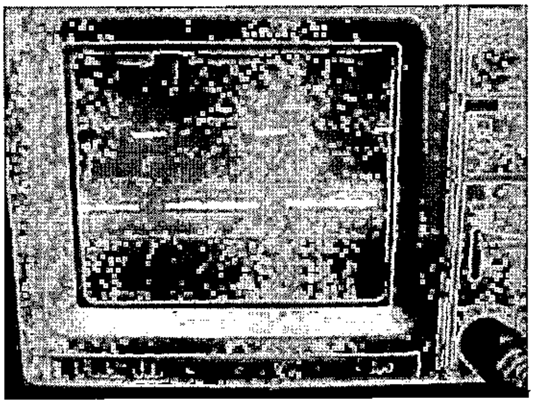


Figure 4.3. PWM Signal with Duty Cycle of 0.23.

3. The speed control of the motor.

Run the LabVIEW program and change the value of duty cycle on the front panel of the program. The speed of the motor increases and decreases with the duty cycle goes higher and lower. When the direction is set to be 1, the motor will rotate clockwise, and 0 for the counter-clockwise.

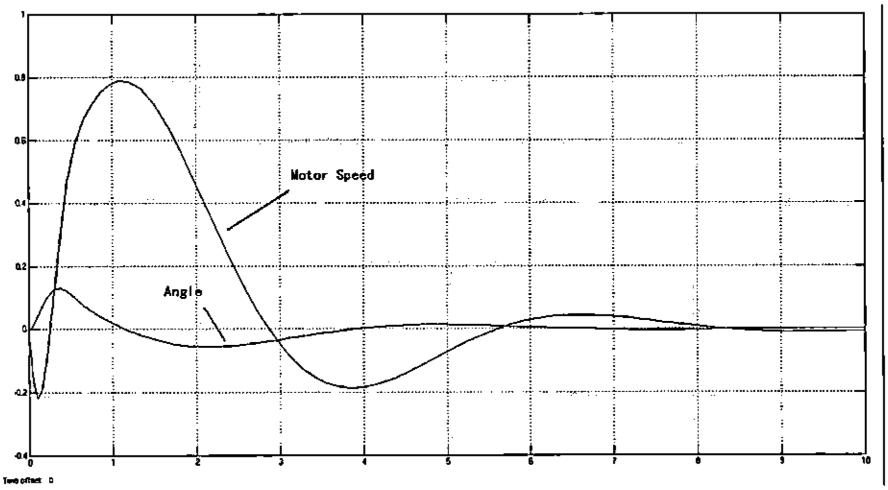


Figure 4.4. Test the Motor Controlling by Setting the Duty Cycle.

4. Control the speed of the motor by changing the angle.

Run the LabVIEW program and manually rotate the shaft of the encoder, the speed of the motor can be well modified. When the angle becomes negative, the direction of the motor gets changed immediately. Also, the more offset on the angle, the higher speed the motor will get. In Figure 4.5, the angle varies like a sine function to maintain a continuous change both on angular position and direction.

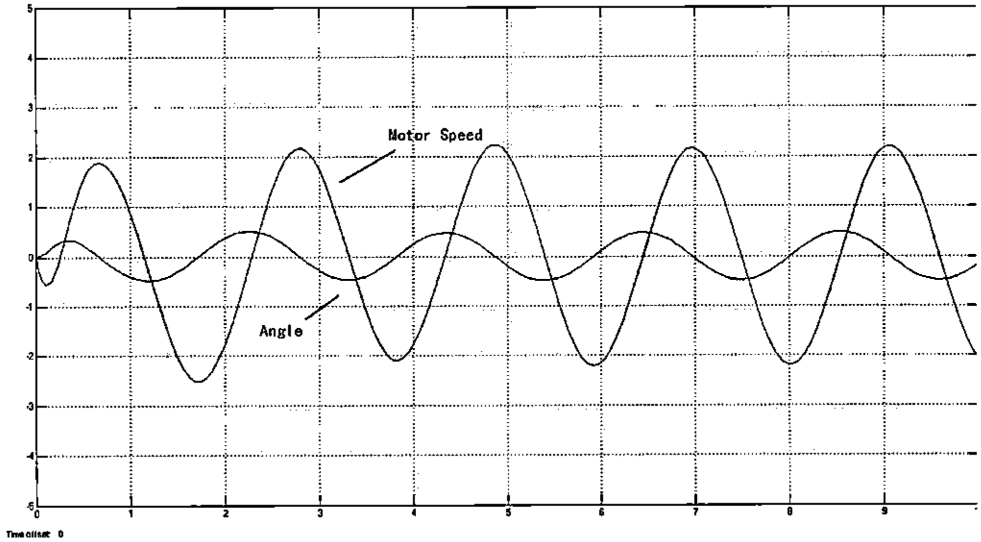


Figure 4.5. Motor Speed Varies from the Angular Position.

5. Let the inverted pendulum balances itself.

In order to let the inverted pendulum get balanced, proper PID parameters should be assigned. The values of PID parameters can be obtained by PID tuning. However, the tests show that the real system fails to fulfill the initial task which is to balance a single inverted pendulum system; this failure is due to poor mechanical design of the real system. Generally speaking, the track on which the cart slides should be longer. And the friction between the cart and the track should be as low as possible so that once the pendulum falls, the cart could slide in the opposite direction immediately. Also, due to the difficulty of installing another encoder to sense the position of the cart, the double closed loop feedback control was not able to be used. Surely the single closed loop feedback control will work and can balance the system theoretically, but the lack of the outer loop will increase the difficulty of control. Besides, the PID tuning techniques are based more on experience and repeated testing. So, all these reasons led to the failure of the real system.

Advantages & Disadvantages

Due to the time limits and conditions in the lab, the two control methods cannot be tested on the real inverted pendulum system. However, from the simulation, it is apparent that both methods have advantages and disadvantages.

For the PID control, the principle and algorithm of the control method are quite straightforward. Each part of the controller has different functions and as long as they can coordinate perfectly, the system will become stable. What's more, it is also quite easy for people to tune the PID parameters while the system is working and the performance of the response is fast and clear. This is a quite classic and traditional control method and all the analysis of the system is in the time domain. However, the calculation of each parameter of the controller is very complicated. That requires the designer to be very familiar with not only classic control theory but also physics and mathematics. Once some of the parameters like ω_c, ζ or K are not taken properly, almost all the work should be redone. Usually it will consume a lot of time to finally find suitable and proper parameters for the system. As a matter of fact, the work I have done in the previous chapter also includes the pole placement of root locus in time domain. When choosing the proper controllers for the system, I got help from the root locus to decide whether to use PI control or PD control.

For the pole placement, the calculation is not a problem any more. The calculations involved in the PID control method sometimes require a lot of skill, but the full state feedback makes the calculation much easier to understand. The full state feedback requires that the system be written in a different way, which to most people is a little bit awkward and difficult to understand. However, once one fully understands the theory and how the full state feedback can revise an unstable system, it will not be a problem to design the controller at all. In a word, this method tries to find a feedback matrix **K** to modify the state matrix **A** so that the new system can be stable. Nevertheless, not all the sate variables are easily measured. Some of them even have no way to measured. That makes this control method not practicable directly. But, with the introduction of a "state observer", this shortcoming can be overcome. So, the non-measurable nature of the state variables is one of the biggest disadvantages it has.

CHAPTER V CONCLUSION

Theoretical Study

A single inverted pendulum system is an unstable, higher-order, multivariable and strongly coupled non-linear system. It is a model that has been abstracted from the areas of robotics, aerospace, and nonlinear control among others. This study shows two ways to control the system, PID control and pole placement.

Apparently, from the previous study and designs, to control the system in a classic way like PID control is feasible. But there has to be a series of simplifications to reduce the order of the system. Without the simplifications, the theoretical calculation for the controller will be even more difficult and complicated. Also, trying to find out the proper way to make the simplifications needs several trials and cannot be done at one time. Controller should be designed and tested and then modified by observing and analyzing the response curves. Unlike the traditional way of finding out the PID parameters of a PID controller, root locus is used as an aid to design the controller. By relocating the poles of the system, whether the system is stable or not can be directly demonstrated, instead of studying the mathematical equations.

Pole placement, or full state feedback, is based on the modern control theory. Since a system can only be stable when all of its poles are in the left half side of the complex plane, what has to be done is to relocate the poles and just move them to left half side of the complex plane by adding a feedback matrix **K**. It does not involve too much mathematical calculation and obviously does not need simplification when dealing with the higher order systems as compared with the PID control. In some more complicated systems, some of their outputs might even be immeasurable. It would be a little difficult to make use of the full state feedback. However, by designing the state observer, this control method can be implemented anyway.

Practical Implementation

Even though the theoretical study has taken many practical environments and conditions into consideration, there will always be a difference between the mathematical model and the real system. In real world, some of the parameters obtained from an ideal condition need to be modified while testing the system. For the real system in this study, it does not accomplish the goal of balancing itself because of the poor mechanical design of the real system. For the other parts in the system, however, they work pretty well. The system successfully generates the controllable PWM signals that follow the change of angle. By taking advantage of the PWM technology, the change of angle can directly control not only the speed but also the direction of the DC motor.

Therefore, once some improvements could be done on the mechanical design of the whole system, for example lightening the cart and reducing the friction, it would definitely be good for the system to balance itself.

Summary

The study of a single inverted pendulum system is basically successful, even though the pendulum cannot balance itself due to time limitation and the poor mechanical design of the system. Two control methods are implemented and used to design the controller of a single inverted pendulum system. The simulation results indicate that the design has reached the expected targets, and the two different control methods are compared, which are the two main goals of the whole study.

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APPENDIX

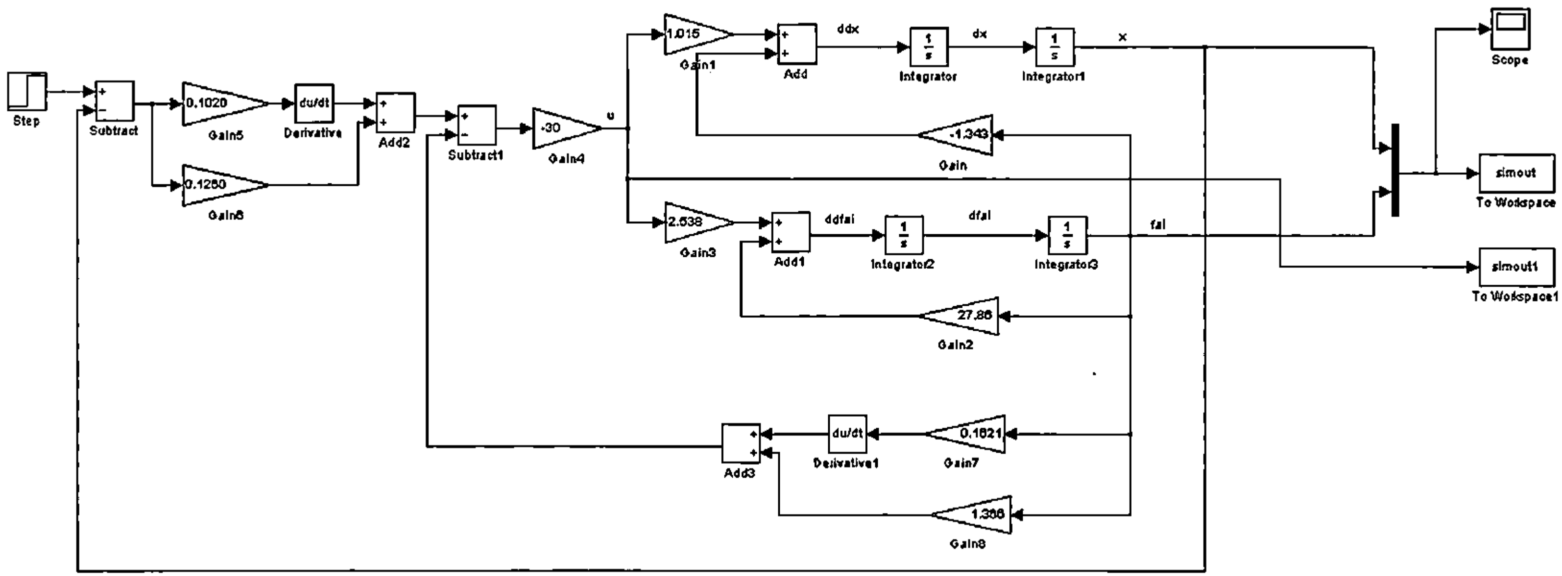

Figure 3.7. Simulation Diagram of the Single Inverted Pendulum System.

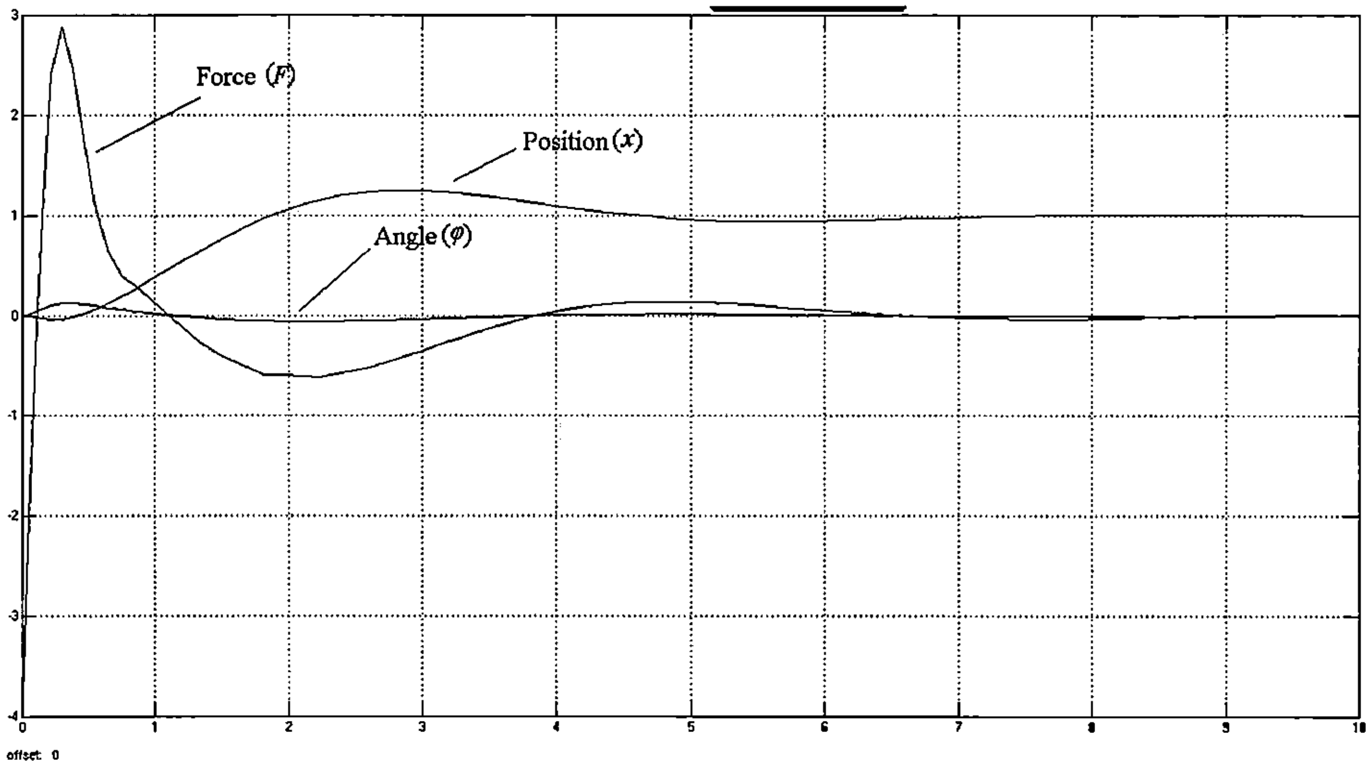

Figure 3.8. Step Response of the System.

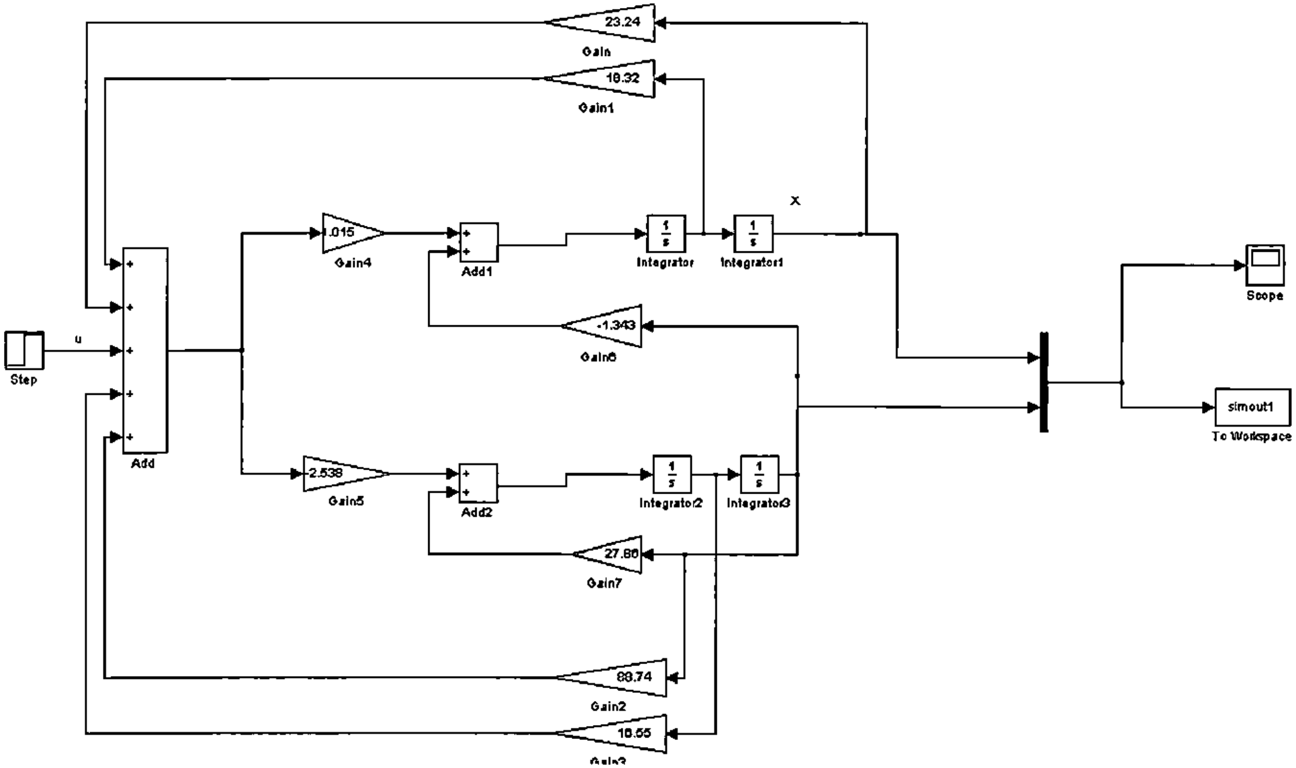

Figure 3.10. Simulation Diagram of the Single Inverted Pendulum System.

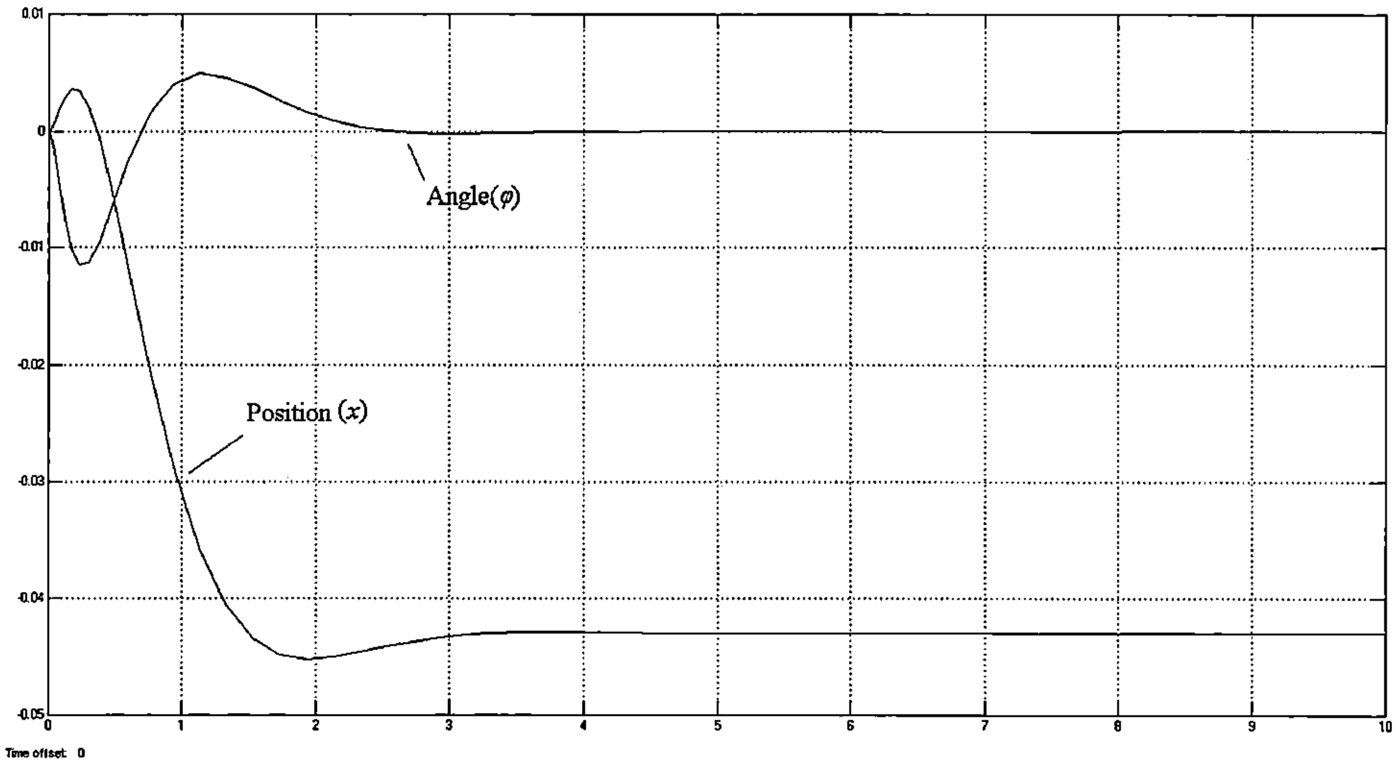

Figure 3.11. Step Response of the System.

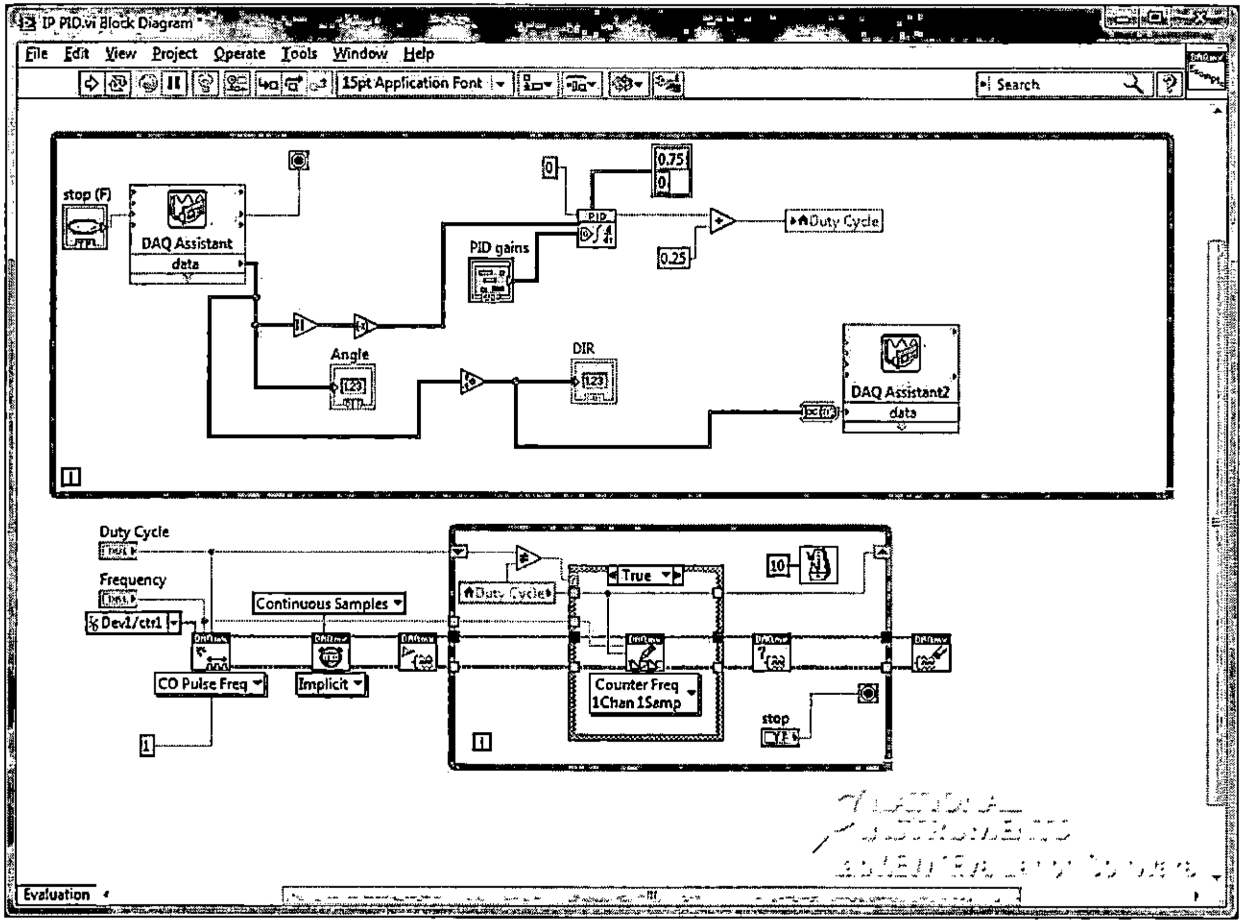

Figure 3.13. LabVIEW Program for the System.

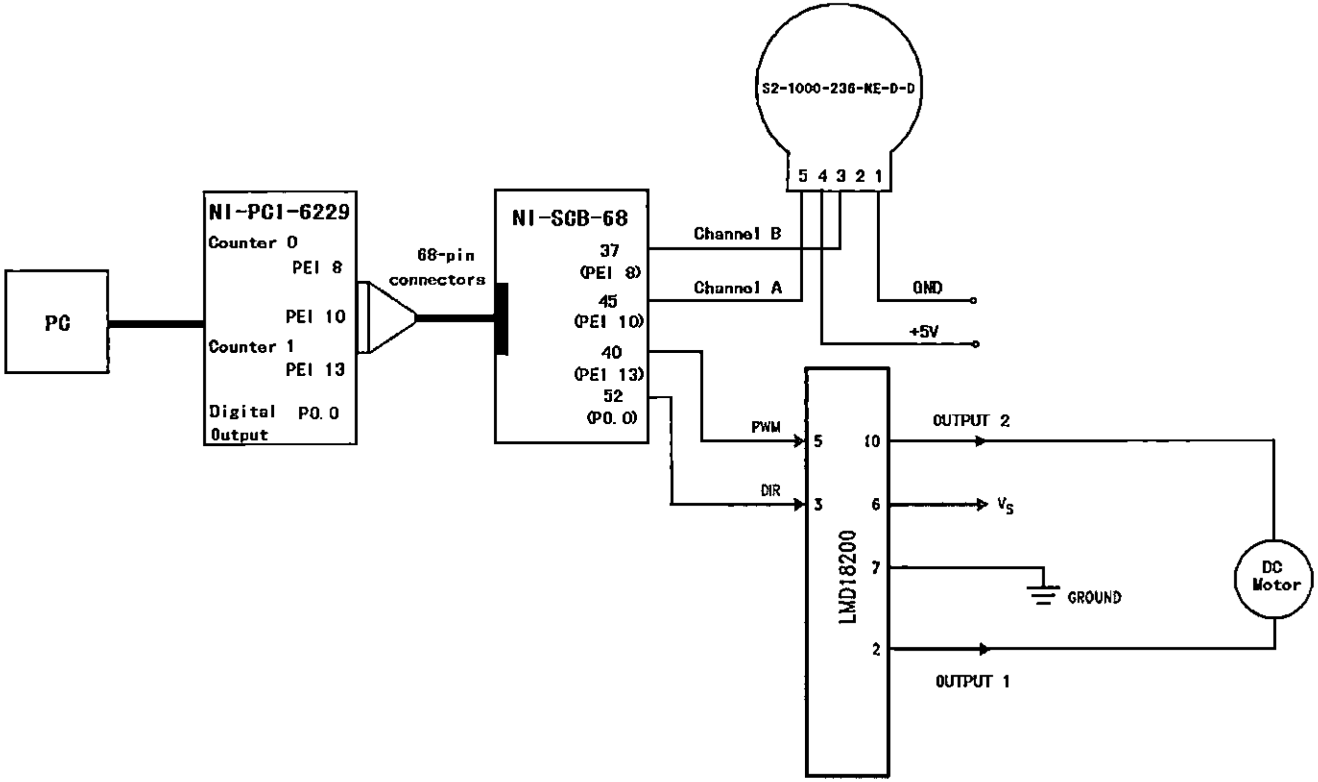

Figure 3.17. Wiring Diagram for the Single Stage Inverted Pendulum System.