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Discrete and continuous treatment of local buckling in stainless steel elements

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Abstract: Cross-section classification is an important concept in the design of metallic structures, as it addresses the susceptibility of a cross-section to local buckling and defines its appropriate design resistance. For structural stainless steel, test data on cross-section capacity have previously been relatively scarce. Existing design guidance has been developed based on the limited experimental results and conservative assumptions, generally leading to unduly strict slenderness limits. In recent years, available test data for stainless steel cross-sections have increased significantly, enabling these slenderness limits to be re-assessed. In this paper all available stainless steel test data have been collected and additional moment-rotation curves have been presented. The study covers both cold-formed and welded plated elements as well as CHS. Following analysis of the test results, new slenderness limits for all loading conditions have been proposed and statistically validated. In addition to re-assessment of the current slenderness limits, a new approach to the treatment of local buckling in structural elements – the Continuous Strength Method – has been outlined. The Continuous Strength Method (CSM) is based on a continuous relationship between cross-section slenderness and deformation capacity and is applied in conjunction with accurate material modelling. The method enables more rational and

precise prediction of local buckling than can be achieved with the traditional cross-section classification approach, thus allowing better utilization of material and more economic design.

Keywords: Cross-section classification, Deformation capacity, Eurocodes, Local buckling, Moment-rotation, Non-linear material, Rotation capacity, Slenderness limits, Stainless steel.

1. INTRODUCTION

The concept of cross-section classification as a means of codified treatment for local buckling of cross-sections that are partly or fully in compression was originally developed for materials that closely follow an idealised bilinear stress-strain response such as carbon steel. The existence of a sharply defined yield point, beyond which a sudden drop in stiffness occurs and hence instability is triggered, defines distinct behavioural groups, based on whether the attainment of this yield stress in any part of the cross-section is limited by the occurrence of local buckling. For the fundamental case of pure compression, cross-section failure may occur either by material yielding and inelastic local buckling in the case of stocky cross-sections (class 1-3) or by local buckling at an average stress below the yield stress for slender cross-sections (class 4). For cross-sections in bending, failure may occur by local buckling prior to reaching the yield stress in the case of slender sections (class 4), by inelastic local buckling above the elastic moment capacity but below the plastic moment capacity following extreme fibre yielding for intermediate sections (class 3), or by inelastic local buckling above the plastic moment capacity following extensive yielding for stocky sections (classes 1-2). Distinction is made between class 1 and 2 cross-sections depending on whether they can sustain their plastic moment with increasing deformation and allow sufficient moment redistribution to take place in the structure for a collapse mechanism to form, in which case plastic analysis may be applied (class 1), or local buckling limits their deformation capacity (class 2) and elastic analysis need be applied.

Given the relatively recent emergence of stainless steel as a structural material, efforts have been made to maintain consistency with carbon steel design guidance. However, unlike carbon steel, stainless steel exhibits a rounded non-linear stress-strain relationship with no strictly defined yield point (Fig. 1) and hence no sharp behavioural transition occurs at any specific stress, thereby complicating any design process traditionally based on a characteristic stress level [1]. This complexity is overcome by defining the yield point as the stress level corresponding to 0.2% permanent strain $\sigma_{0.2}$, and assuming bilinear stress-strain behaviour for stainless steel as for carbon steel. The substantial differences in the structural response between the two materials are neglected in favour of simplicity, generally resulting in conservative slenderness limits for stainless steel cross-sections.

As an alternative to this approach, the Continuous Strength Method (CSM) [2] is outlined in Section 6 of the present paper – this method represents a departure from the traditional cross-section classification methodology, but is more rationally based and offers more accurate prediction of local buckling behaviour, particularly for materials exhibiting a high degree of strain hardening, such as stainless steel.

2. CURRENT SLENDERNESS LIMITS FOR STAINLESS STEEL CROSS-SECTIONS

The classification process employed in the current codified treatment of local buckling for stainless steel cross-sections mirrors that applied to carbon steel. Squash load F_y, elastic moment capacity M_{el} and plastic moment capacity M_{pl} of stainless steel cross-sections are defined with respect to the conventional yield (0.2% proof) stress $\sigma_{0.2}$, and relevant classes are based on susceptibility to local buckling and cross-sectional deformation capacity as for carbon steel. Cross-sectional response is assumed to relate to the behaviour of its most slender plate element, thereby neglecting the interaction of constituent plate elements, which are individually classified according to their width-to-thickness ratios, as compared to codified slenderness limits. These limits depend on each element's boundary conditions (internal or outstand), manufacturing process (welded or cold-formed) and stress gradient.

Determination of slenderness limits is ideally based on relevant experimental results; stub column and/or bending tests for the derivation of the class 3 limit and bending tests only for the class 2 and class 1 limits. However, unlike carbon steel, only a limited number of test results existed at the time of the development of most stainless steel design codes. These included 46 stub column and 11 bending tests on cross-sections comprising internal and outstand (either welded or cold-formed) plate elements, reported by Johnson and Winter [3], Wang and Winter [4] and Yamada et al. [5], which were primarily intended to investigate the effective width of class 4 plate elements in compression. These tests are appropriate only for the derivation of the class 3 limit for plate elements in compression. The remaining slenderness limits for compressed elements were derived based on engineering judgment and assumed analogies with carbon steel, whereas the relevant classification limits for plate elements in bending and combined compression and bending were based on adjustment to the respective compression limits by appropriate buckling factors derived from elastic solutions of perfect plates subjected to stress gradients. Hence the existing class 1 and class 2 limits are largely unverified, while the class 3 limit has been experimentally justified but only by a relatively limited number of test results.

3. ADDITIONAL EXPERIMENTAL RESULTS

Since the development of Eurocode 3: Part 1.4 [6], considerable further research has been conducted on structural stainless steel. Many additional experimental results on cross-sectional resistance, that were not available during the development of the code, now exist. These include both stub column and bending tests, which can be used to assess the applicability of current slenderness limits.

3.1 Stub column tests

A total of 183 stub column tests results have been gathered from published sources and utilized to establish the appropriate class 3 limit for stainless steel internal elements, outstand elements, angles and CHS in compression. The section types considered, number of tests conducted, material grade and relevant references, are shown in Table 1.

3.2 Three and four point bending tests

A total of 62 bending tests have been collected in order to derive class 3 and class 2 limits for stainless steel cross-sections (see Table 2). Moment-rotation or moment-curvature relationships were not reported for all tests, but, where available, these have been used for the determination of class 1 limits.

3.3 Previously unreported moment-rotation curves

Gardner and Nethercot [23] conducted nine 3-point bending tests on SHS and RHS. Displacements were measured at five points (at the point of loading and at a distance of 50 mm from either side of the end supports), but only moment versus vertical midspan deflection curves were reported. In the present paper the measured displacements at the ends of the beams have been utilized to derive the corresponding rotation at the loaded cross-section and the relevant moment-rotation curves are shown in Fig. 2.

It should be noted that the main objective of the experimental program was attainment of the maximum moment, shortly after which many of the tests were stopped. Hence, the maximum recorded rotations from the tests, which do not necessarily reflect the full rotation capacities of the specimens, have been obtained. However, due to the limited availability of experimental results on rotation capacity, the aforementioned moment-rotation curves, although conservative, have been utilized for the assessment of appropriate class 1 limits. The results are shown in Fig. 13 where a distinction is made between those tests that reached their full rotation capacity and those where the test was stopped prior to reaching this point.

4. ASSESSMENT OF EXISTING SLENDERNESS LIMITS

In this section, existing slenderness limits are compared with all published stainless steel experimental results to assess their applicability. Determination of the slenderness parameters follows the provisions of EN 1993-1-4 [6]. Experimental results are shown in Figs 3-11 and 13-15. The corresponding class limits for carbon steel and stainless steel, as specified by EN 1993-1-1 [25] and EN 1993-1-4 [6] respectively, are also depicted for comparison. For all cross-sections the relevant response characteristic ($F_u/A\sigma_{0.2}$, M_u/M_{el}, M_u/M_{pl}, or rotation capacity R) is plotted against the slenderness of the most slender constituent element in the cross-section, where F_u and M_u are the ultimate loads and moments as determined from the experiments and M_{el} and M_{pl} are the conventional elastic and plastic moment capacities. In determining element slenderness, due account of the stress distribution and element support conditions has been made through the buckling factor k_σ, as defined in EN 1993-1-5 [26]. The following symbols are employed: c =

compressed flat width; d = outer diameter; t = element thickness; $\varepsilon = [(235/f_y)(E/210000)]^{0.5}$. Note that this definition of ε for stainless steel, as given in EN 1993-1-4 [6] differs from that given in EN 1993-1-1 [25], to allow for variation in Young's modulus. However, given that the difference in Young's modulus between stainless steel and carbon steel is small (approximately 5%), in comparison to the scatter of experimental data, the authors believe that in the interest of simplicity and harmonisation the familiar carbon steel definition of $\varepsilon = (235/f_y)^{0.5}$ could be utilized for both materials.

4.1 Class 3 slenderness limit

Cross-sections capable of reaching their yield stress ($\sigma_{0.2}$ in the case of stainless steel) prior to the onset of local buckling are classified as class 3 or better. Both stub column and bending tests are utilized to assess appropriate class 3 limits, as shown in Figs 3-8.

In Figs 5 and 8, carbon steel data have been included for comparison purposes as will be discussed in Section 5. The carbon steel data were obtained from stub column tests on CHS conducted by O'Shea and Bridge [27] and Elchalakani et al. [28] and on cross-sections comprising flat internal elements conducted by Akiyama et al. [29] and Uy [30].

4.2 Class 2 slenderness limit

Cross-sections capable of reaching their full plastic moment capacity are classified as class 2 or better. A series of bending tests have been utilized to assess the current class 2 limits as illustrated in Figs 9-11. For CHS (Fig. 9) and cross-sections comprising internal parts in compression (Fig. 11), a reasonable number of test results exist, but for outstand elements, test data are scarcer. Fig. 10 shows results from six I-section beam tests. No bending tests have been reported on cross-sections comprising cold-formed outstand elements to date.

4.3 Class 1 slenderness limit

Cross-sections capable of reaching and maintaining their plastic moment capacity with sufficient deformation capacity to be used in plastic design, are classified as class 1. The term deformation refers either to the rotation at the theoretical plastic hinge location (i.e. most heavily stressed cross-section in the case of specimens under a moment gradient e.g. 3-point bending tests), or to the constant curvature developed in the uniform moment region of specimens under pure bending (typically achieved in the central region of symmetrical 4-point bending tests). In either case the deformation capacity is quantified through Eq. (1), where k_u (θ_u) is the curvature k (rotation θ) at the point at which the falling branch of the moment-curvature (moment-rotation) curve falls below M_{pl}, and k_{pl} (θ_{pl}) is the elastic curvature (rotation) corresponding to M_{pl} as illustrated in Fig. 12.

$$R = \frac{k_u}{k_{pl}} - 1 \text{ (based on M-k relationship);} \qquad R = \frac{\theta_u}{\theta_{pl}} - 1 \text{(based on M-}\theta\text{ relationship)} \quad (1)$$

In the absence of a codified deformation capacity requirement for class 1 stainless steel cross-sections (no rules for plastic global analysis are given in Eurocode 3: Part 1.4 [6]), the equivalent carbon steel requirement of R = 3 [31, 32] has been adopted. In some cases, experiments were stopped upon attainment of the maximum moment capacity and the full deformation capacity was not reached. Since there is no accurate way of extrapolating the reported experimental curves, the deformation capacity in those cases was conservatively defined by using the maximum reported curvature k_{max} (for 4-point bending tests) or rotation θ_{max} (for 3-point bending tests) in place of k_u (θ_u) in Eq. (1). The tests results are shown in Figs 13-15; the specimens that reached their full deformation capacity are depicted with shaded symbols, while those that did not are depicted with blank symbols.

5. PROPOSAL FOR NEW SLENDERNESS LIMITS

Analysis of the presented test data reveals that current slenderness limits for stainless steel are overly conservative and that harmonisation with the equivalent carbon steel limits may be justified. For class 1 and class 2 limits, Figs 9-15 indicate that the equivalent carbon steel limits may be safely adopted for stainless steel, although the number of tests reported for outstand elements in compression is rather limited and further test results are required.

The current class 3 limits for stainless steel angles and outstand elements is unduly strict and all reported test results support the adoption of the equivalent carbon steel limits. Furthermore, the distinction made in Eurocode 3: Part 1-4 between cold-formed and welded outstand parts is not clearly justified by the relevant test results. Although the cold-formed outstand elements generally exhibit superior capacity to their welded counterparts in the stocky range, attributed to the enhanced corner properties brought about by the forming process and the less severe residual stresses, the disparity in response becomes insignificant at higher slendernesses. Hence, it is recommended that the current carbon steel class 3 limit for outstand elements (including both welded and cold-formed elements) be adopted for both cold-formed and welded stainless steel outstand elements. Regarding the class 3 limit for CHS in bending, Fig. 6 suggests that the current stainless steel slenderness limit of 280 is reasonable. However no test data exist in the vicinity of this limit and any extrapolation should be conducted with caution. The current corresponding slenderness limit for carbon steel CHS of 90 is clearly inappropriate for stainless steel, and its suitability for carbon steel has also been questioned [33]. Finally, the comparison between carbon steel and stainless steel test data for CHS and internal elements in compression, shown in Figs 5 and 8, demonstrate that stainless steel CHS and flat internal elements in compression perform similarly to their carbon steel counterparts.

The class 3 slenderness limit for CHS in compression currently lies at 90 for both carbon steel [25] and stainless steel [6], and this value is supported by the presented test data. For flat internal elements in compression, the current class 3 slenderness limit for stainless steel is conservative, but harmonisation with the carbon steel limit is not supported by the test data, and an intermediate limit is therefore proposed.

The above recommendations are summarised in Table 3, where specific slenderness limits are given for each behavioural class and for all loading conditions. The recommended class 2 and class 3 slenderness limits for internal and outstand (both cold-formed and welded) elements in compression, CHS in compression and bending and the class 3 limit for angles in compression have been validated by means of statistical analysis according to Annex D of EN 1990 [34] against all available test data summarised in Tables 1 and 2 and were deemed safe for design in conjunction with a partial safety factor of γ_{M1}=1.1, as specified in EN 1993-1-4 [6]. For consistency, it is proposed that the effective width formulae specified in EN 1993-1-4 [6] be modified to Eqs (2) and (3), which have also been statistically validated according to [34]:

$$\rho = \frac{1}{\bar{\lambda}_p} - \frac{0.188}{\bar{\lambda}_p^{\,2}} \le 1 \qquad \text{for outstand elements (cold-formed or welded)} \qquad (2)$$

$$\rho = \frac{0.772}{\bar{\lambda}_p} - \frac{0.079}{\bar{\lambda}_p^{\,2}} \le 1 \qquad \text{for internal elements (cold-formed or welded)} \qquad (3)$$

where ρ is the reduction factor for local buckling and $\bar{\lambda}_p$ is the element slenderness, as defined in [6].

For elements under stress gradients, the slenderness limits recommended in Table 3 have been derived on the basis of modification of the proposed limits for compression by appropriate buckling factors k_σ [26], though due to lack of experimental data, no statistical validation has been performed. Moreover, no statistical analysis has been carried out for the proposed class 1 limits due to the highly scattered nature of the rotation capacities of the sections. Poor correlation between rotation capacity and codified slenderness parameters was also reported for carbon steel specimens [31, 32], where the scatter was attributed to the effect of other parameters, such as moment gradient, material properties and the interaction of constituent plate elements. Hence, as for carbon steel [31, 32], a degree of engineering judgement has been required for the assessment of the class 1 limits given in Table 3. It is recommended that the proposed slenderness limits (Table 3) and effective width formulae (Eqs (2) and (3)) be incorporated into future revisions of EN 1993-1-4.

6. THE CONTINUOUS STRENGTH METHOD

Adoption of the revised slenderness limits proposed in the previous section results in more efficient structural design for stainless steel elements and greater consistency with carbon steel. However, the achievable level of accuracy and design efficiency in cross-section capacity predictions is limited by the simplifying assumptions involved in the classification process. Use of the $\sigma_{0.2}$ proof stress as the maximum attainable stress by the cross-section and the resulting failure to account for the actual strain-hardening behaviour of the material leads to unnecessary conservatisms and inefficient design, particularly for relatively stocky cross-sections that may fail at stresses far beyond $\sigma_{0.2}$, as evidenced in Figs 3-7 and Fig. 11. For a material with a high initial cost like stainless steel, such conservatism is detrimental for its more widespread usage in construction. For improved efficiency and to reflect the true behaviour of the material, departure from the traditional classification process seems justified.

The Continuous Strength Method (CSM) outlined below is a novel approach to the treatment of local buckling in metallic cross-sections, which does not utilize the effective width concept, does not assume the traditional bilinear material behaviour and allows for better exploitation of the material. It is based on the deformation capacity of the cross-section in question, as predicted by an experimentally derived design curve relating the strain at which local buckling occurs, denoted ε_{LB}, to the cross-section slenderness. This deformation capacity is utilized in conjunction with an accurate stress-strain law to obtain the maximum attainable stress σ_{LB} corresponding to the local buckling strain ε_{LB}. Additional features of the method include explicit allowance for the beneficial influence of strain-hardening incurred during the forming process on the strength of the corner regions of cold-formed cross-sections and generalizations of the method to cover member instabilities and interaction of various loading conditions [35]. The method has also been successfully applied to aluminium alloy, high strength steel [36] and carbon steel design [2]. It should be noted that the Continuous Strength Method deals primarily with the fundamental loading cases associated with normal stresses (i.e. pure compression [37], bending and interaction of compression and bending). The shear buckling resistance of stainless steel cross-sections has been examined elsewhere [38].

6.1 Basic concepts and design curves

The basic concept underpinning the CSM is that the occurrence of local buckling is the only physical limit to the exploitation of material's strain-hardening capacity. Indeed, the continuous nature of stainless steel's stress-strain law and the absence of a sharply defined yield point, beyond which a dramatic loss of stiffness occurs and instability is triggered, implies a continuous variation in the maximum attainable stress by a cross-section, which is not limited by $\sigma_{0.2}$. Hence the maximum attainable stress at failure is not a material-specific stress (i.e. the $\sigma_{0.2}$ proof stress), but is rather a continuous function of material properties, geometry of the cross-section and imposed stress gradient, all of which are incorporated into a cross-section slenderness parameter.

For plated cross-sections, the relevant cross-section slenderness parameter is related to the plate slenderness $\bar{\lambda}_p$ of the most slender constituent element (which is also the case for the traditional classification approach) as given by Eq. (4), whilst the corresponding cross-section slenderness for CHS $\bar{\lambda}_c$ is given by Eq. (5) [39]:

$$\bar{\lambda}_p = \sqrt{\frac{\sigma_{0.2}}{\sigma_{cr}}} = \frac{\sqrt{12(1-\nu^2)}\sqrt{235}}{\pi\sqrt{210000}\sqrt{k_\sigma}}\frac{b}{t\varepsilon} \tag{4}$$

$$\bar{\lambda}_c = \frac{\sigma_{0.2}}{\sigma_{cr}} = \frac{\sqrt{3(1-\nu^2)}235}{2\cdot 210000}\frac{(d-t)}{t\varepsilon^2} \tag{5}$$

where σ_{cr} is the elastic critical buckling stress of the plate element or CHS, b is the flat element width measured between centrelines of adjacent faces, d is the CHS outer diameter, t is the plate or CHS thickness, ν is Poisson's ratio, E is Young's modulus, $\varepsilon = [(235/f_y)(E/210000)]^{0.5}$ and k_σ is a buckling factor allowing for differing boundary and loading conditions (i.e. stress gradients). For CHS no buckling factor is applied and Eq. (5), which is derived for CHS in compression, is conservatively adopted for bending.

Based on numerous stub column tests and validated FE results, empirical equations have been derived relating the normalised cross-section deformation capacity to the cross-section slenderness. The deformation capacity of a cross-section is defined by Eq. (6) as the compressive strain corresponding to ultimate load ε_{LB}, obtained by dividing the axial shortening δ_u at ultimate load (as shown in Fig. 16) by the stub column's initial length L_0. This deformation capacity is normalised to the elastic strain corresponding to the 0.2% proof stress $\sigma_{0.2}$, henceforth denoted ε_0, as defined by Eq. (7). Adjustment to the definition of deformation capacity (Eq. (6)) is made for slender sections to account for post-buckling effects [37].

$$\varepsilon_{LB} = \frac{\delta_u}{L_0} \tag{6}$$

$$\varepsilon_0 = \frac{\sigma_{0.2}}{E} \tag{7}$$

The resulting normalised deformation capacity is approximated as a function of cross-section slenderness ($\bar{\lambda}_p$ or $\bar{\lambda}_c$) by Eq. (8) for plated sections and Eq. (9) for CHS, as reported in [35] and [40] respectively:

$$\frac{\varepsilon_{LB}}{\varepsilon_0} = \frac{1.43}{\bar{\lambda}_p^{\,2.71-0.69\bar{\lambda}_p}} \tag{8}$$

$$\frac{\varepsilon_{LB}}{\varepsilon_0} = \frac{0.18}{\bar{\lambda}_c^{\,1.24+1.70\bar{\lambda}_c}} \tag{9}$$

A comparison between the normalised deformation capacity $\varepsilon_{LB}/\varepsilon_0$ predicted from Eq. (7) and the actual deformation capacity obtained from test results is shown in Fig. 17 for plated cross-sections and Fig. 18 for CHS. The proposed design curves may be seen to provide a good fit to the test data and can hence be used to predict the maximum strain ε_{LB} attainable by the cross-section prior to the occurrence of local buckling with acceptable accuracy. Having obtained the local buckling strain, the next step in the design procedure is the determination of the corresponding stress (or stress distribution in the case of a bending resistance calculation) via an accurate material model as described in the following section.

6.2 Material behaviour

Stainless steel exhibits a rounded stress-strain relationship with no sharply defined yield point, as illustrated in Fig. 1. Traditionally its stress-strain relationship has been described by the Ramberg-Osgood model [41], as modified by Hill [42]. However the Ramberg-Osgood model has been found to overestimate the stress at high strains, thereby compromising the accuracy of any strain-based design method. An improved material model, based on a two stage Ramberg-Osgood equation [43, 44], as modified by Gardner and Nethercot [9], is utilized in the CSM. This adopted compound Ramberg-Osgood model is defined by Eqs (10) and (11):

$$\varepsilon = \frac{\sigma}{E_0} + 0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^n \qquad \text{for } \sigma \le \sigma_{0.2} \tag{10}$$

$$\varepsilon = \frac{(\sigma - \sigma_{0.2})}{E_{0.2}} + \left(\varepsilon_{t1.0} - \varepsilon_{t0.2} - \frac{\sigma_{1.0} - \sigma_{0.2}}{E_{0.2}}\right)\left(\frac{\sigma - \sigma_{0.2}}{\sigma_{1.0} - \sigma_{0.2}}\right)^{n'_{0.2,1.0}} + \varepsilon_{t0.2} \qquad \text{for } \sigma > \sigma_{0.2} \tag{11}$$

where E_0 and $E_{0.2}$ are the Young's modulus and the tangent modulus at 0.2% offset strain, respectively, $\sigma_{0.2}$ and $\sigma_{1.0}$ are the proof stresses at 0.2% and 1% offset strains, respectively, $\varepsilon_{t0.2}$

and $\varepsilon_{t1.0}$ are the total strains at $\sigma_{0.2}$ and $\sigma_{1.0}$, respectively and n and $n'_{0.2,1.0}$ are strain hardening exponents.

Eqs (9) and (10) express strains as a function of stress, whereas the CSM requires the evaluation of stress values corresponding to strain (i.e. deformation capacity) values. Since Eqs (10) and (11) cannot be exactly inverted to express stress as a function of strain, Gardner and Ashraf [36] provides stress values as a function of strain values for the most commonly used structural stainless steels in a tabulated format. Alternatively, a recently developed approximate inversion of the aforementioned material model proposed by Abdella [45] can be adopted.

The corner regions of cold-formed stainless steel sections exhibit higher material strengths than the flat portions of the sections due to cold-work during forming. Whilst these strength enhancements can be significant [46], they are relatively localised and thus, for simplicity, have not been considered in the design method presented herein.

6.3 Cross-section resistance

Having determined the local buckling strain ε_{LB} and the corresponding local buckling stress σ_{LB}, cross-section compression resistance is calculated through Eq. (12):

$$N_{c,Rd} = \sigma_{LB} A_g \tag{12}$$

For cross-section resistance in bending, the same principles apply as for compression; the relevant cross-section slenderness is evaluated (Eq. (3) or (4)) and is then substituted in the respective design expression (Eq. (7) or (8)) to obtain the cross-section deformation capacity.

Having specified the outer fibre strain limit and assuming a linear through-depth strain distribution, the corresponding stress distribution is obtained from the material model presented in Section 6.2 and hence the moment capacity can be calculated by means of Eq. (13):

$$M_{c,Rd} = \int_A \sigma y \, dA \tag{13}$$

where y is the distance from the neutral axis of the section. It should be noted that the test data considered include only doubly symmetric cross-sections in bending. However, the same methodology is, in principle, equally applicable to non-symmetric cross-sections.

The integration (Eq.(13)) can be avoided by introducing the generalised shape factor a_g originally developed by Mazzolani [47] for the determination of the moment resistance of aluminium alloy beams in bending. The generalised shape factor is a means of incorporating both geometric and material properties (i.e. the stress-strain curve) into a unique parameter and is calculated using Eq. (14):

$$a_g = A_1 + A_2\varepsilon_0 + A_3 a_p + A_4\varepsilon_0 a_p \tag{14}$$

where ε_0 is defined by Eq. (6), a_p is the familiar geometric shape factor and A_1, A_1, A_1 and A_4 are numerical coefficients depending on the local buckling strain ε_{LB}, which can be obtained from the tables given in [35], [36] and [37] for doubly symmetric cross-sections in bending and the most commonly used stainless steel grades. The moment resistance can thus be calculated from Eq. (15), where W_{el} is the elastic modulus of the cross-section:

$$M_{c,Rd} = \sigma_{0.2} a_g W_{el} \tag{15}$$

6.4 Validation of the method and comparison to codified predictions

The resistance predicted by the CSM has been compared to the results of all available stub column test data (Table 1) and bending test data (Table 2). The comparison between the CSM predictions and equivalent predictions based on the provisions of Eurocode 3: Part 1.4 (both normalised by the relevant experimental resistance) reveals the superiority of the CSM over the Eurocode provisions, regarding both accuracy and consistency, as shown in Table 4.

Statistical analysis of the method has been carried out according to Annex D of EN 1990 [34] and it has been concluded that the CSM may be safely adopted for the design of CHS and plated sections with a slenderness value $\bar{\lambda}_p$ less than 1.8 in both compression and bending. It should be noted that, despite CHS bending capacity being over-predicted by the CSM, the approach is found to be statistically reliable [34] owing to material over-strength and a low coefficient of variation. The resistance of very slender sections is under-predicted by the CSM; for such cases ($\bar{\lambda}_p$>1.2), application of the conventional effective width approach may result in more efficient design. A comparison of the predicted compressive capacities r_t (normalised to $A\sigma_{0.2}$) according to the CSM and EN 1993-1-4 with the experimental results r_e is given in Fig. 19. The CSM design model may be seen to be more consistent than the Eurocode approach over the full range of slenderness, and the shortcomings of EN 1993-1-4 whereby compressive resistance is limited to $A\sigma_{0.2}$ are clearly illustrated in Fig. 19 by the vertical distribution of data points in the region r_t = 1.0.

7. CONCLUSIONS

A comprehensive assessment of the current treatment of local buckling in stainless steel elements according to EN 1993-1-4 has been carried out. All relevant experimental results have been gathered, analysis of which has highlighted conservatism within the current design process. Based on the experimental results, new statistically validated slenderness limits for each behavioural class and for all loading conditions, have been proposed. The new slenderness limits allow more efficient exploitation of the material and greater harmonisation with the corresponding slenderness limits for carbon steel. Furthermore, the existing effective width formulae have been updated to maintain consistency with the proposed slenderness limits. It is recommended that the proposed slenderness limits and effective width formulae be adopted in future revisions of EN 1993-1-4.

Re-assessment of the slenderness limits for stainless steel has also highlighted the differences in structural response between carbon steel and stainless steel, and in particular, the shortcomings associated with limiting the maximum compressive stress to the material 0.2% proof stress. Hence, as an alternative to the cross-section classification approach, the Continuous Strength Method has been outlined. The proposed method, which has been statistically validated according to EN 1990 for both compression and bending of CHS and plated sections, offers more accurate and consistent predictions of resistance than the current Eurocode provisions, thereby leading to more efficient design, particularly for stocky cross-sections. It is envisaged that the proposed method may be adopted as an alternative to cross-section classification for the treatment of local buckling in future revisions of EN 1993-1-4.

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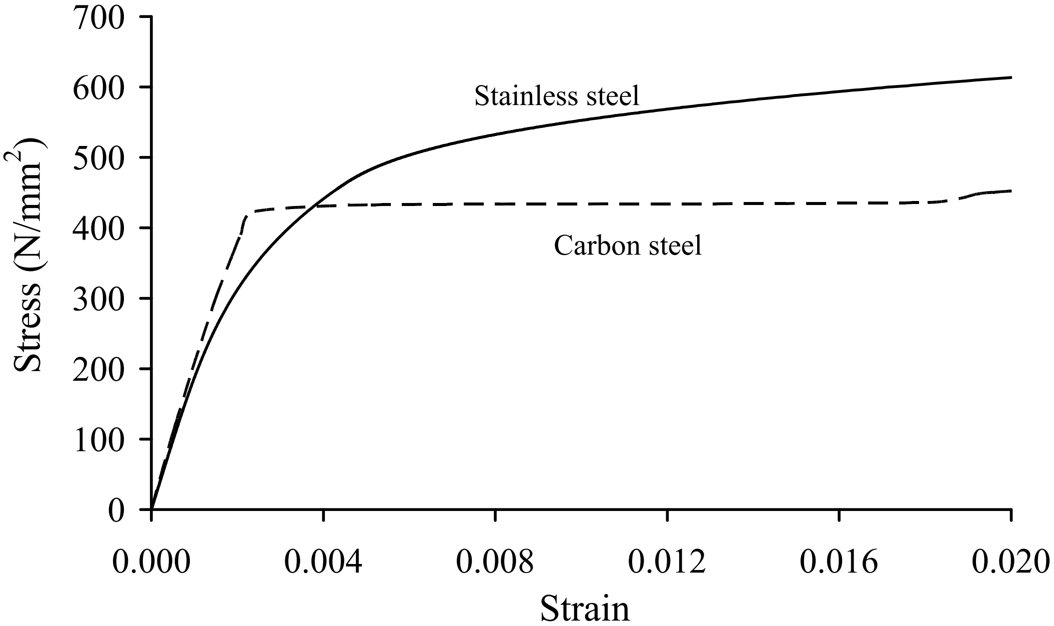

Fig. 1: Indicative stainless steel and carbon steel stress-strain behaviour.

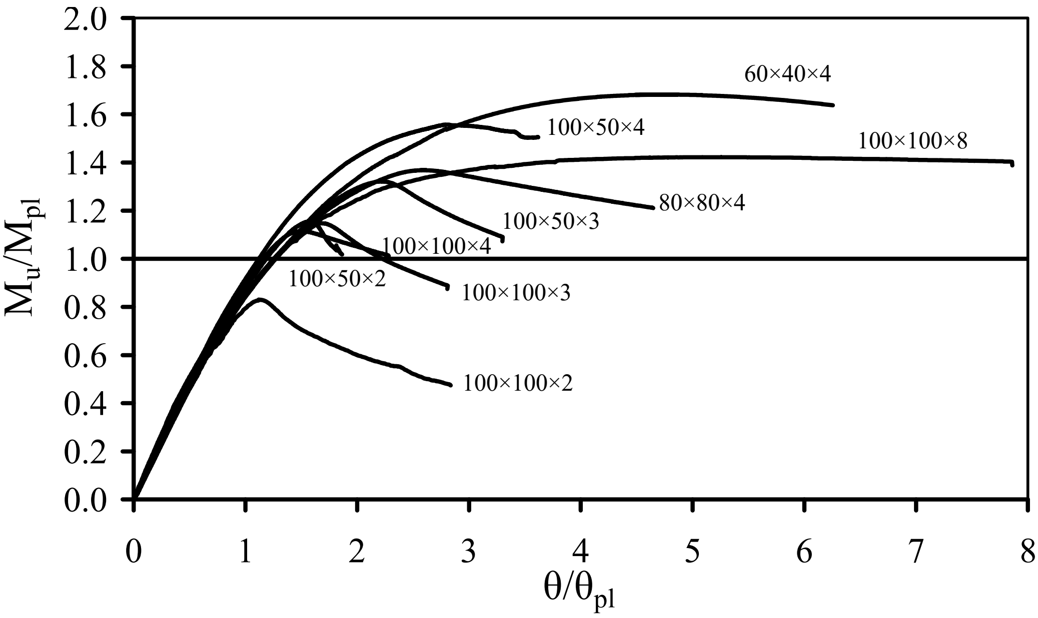

Fig. 2: Moment-rotation curves of SHS and RHS under 3-point bending.

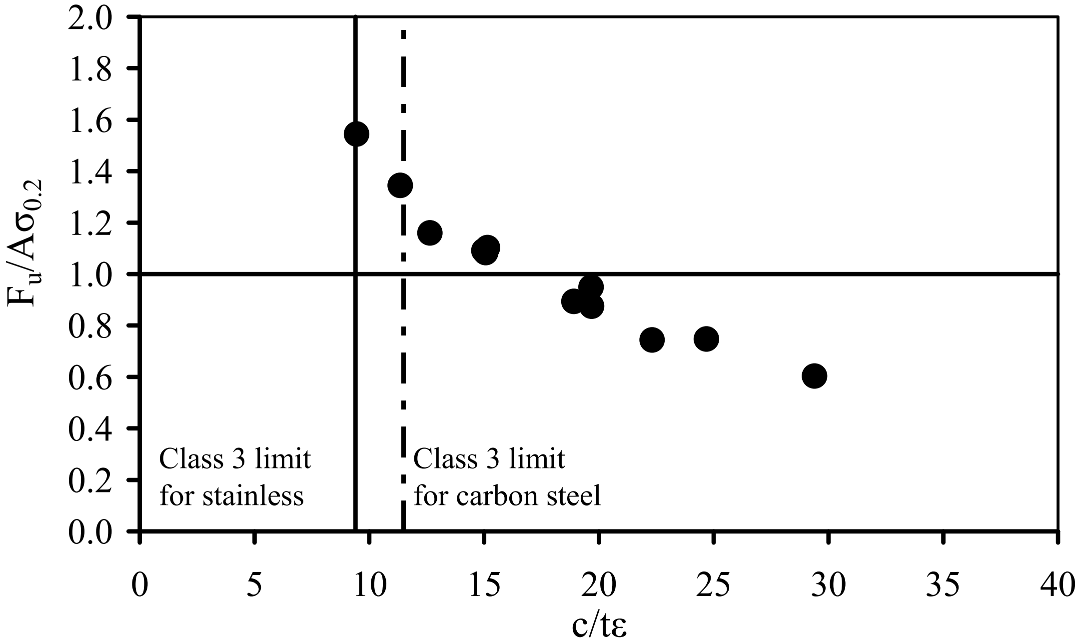

Fig. 3: Class 3 limit for angles in compression.

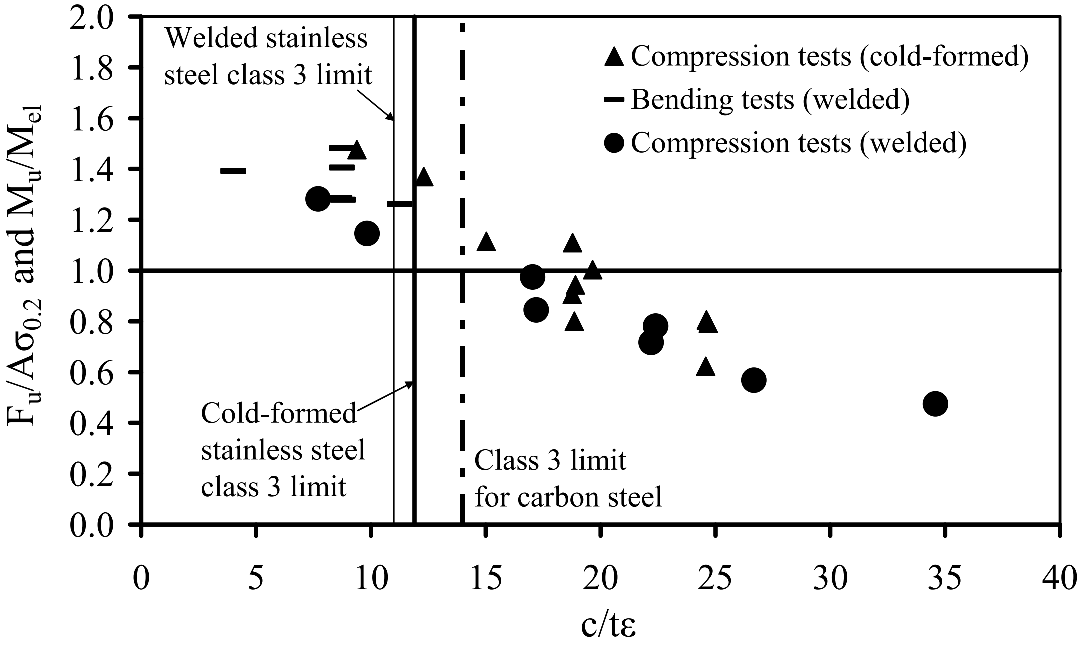

Fig. 4: Class 3 limit for outstand elements in compression.

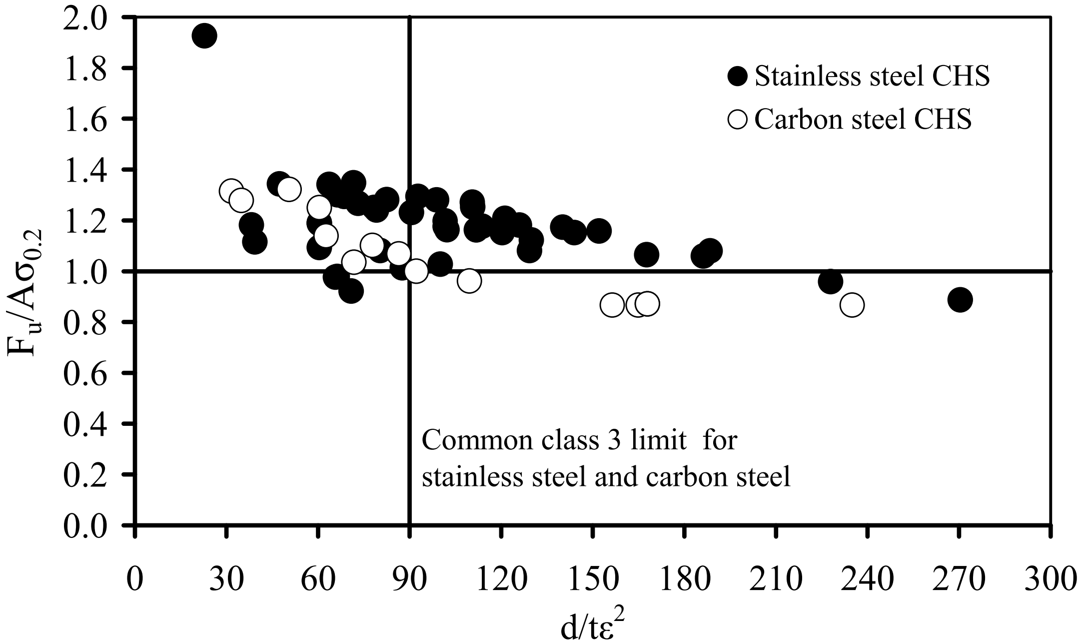

Fig. 5: Class 3 limit for CHS in compression.

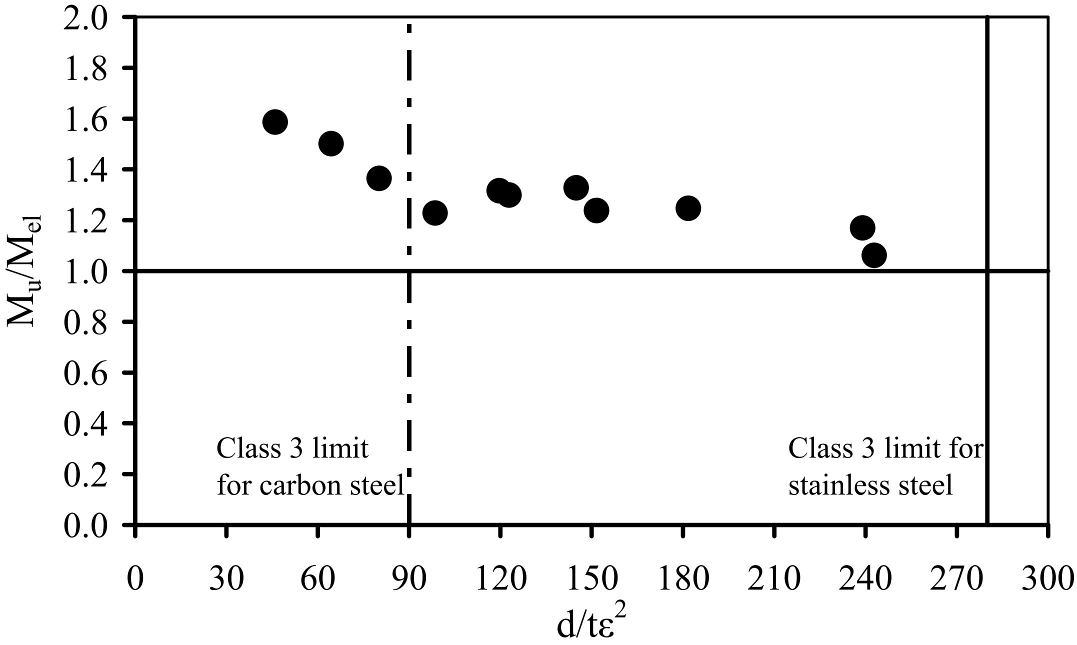

Fig. 6: Class 3 limit for CHS in bending.

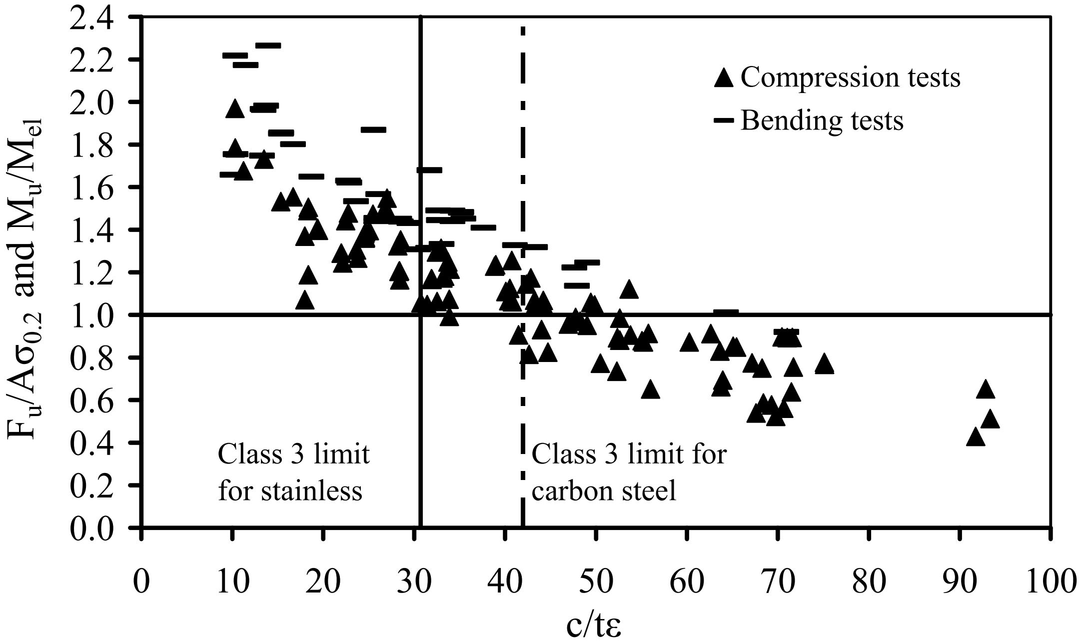

Fig. 7: Class 3 limit for internal elements in compression.

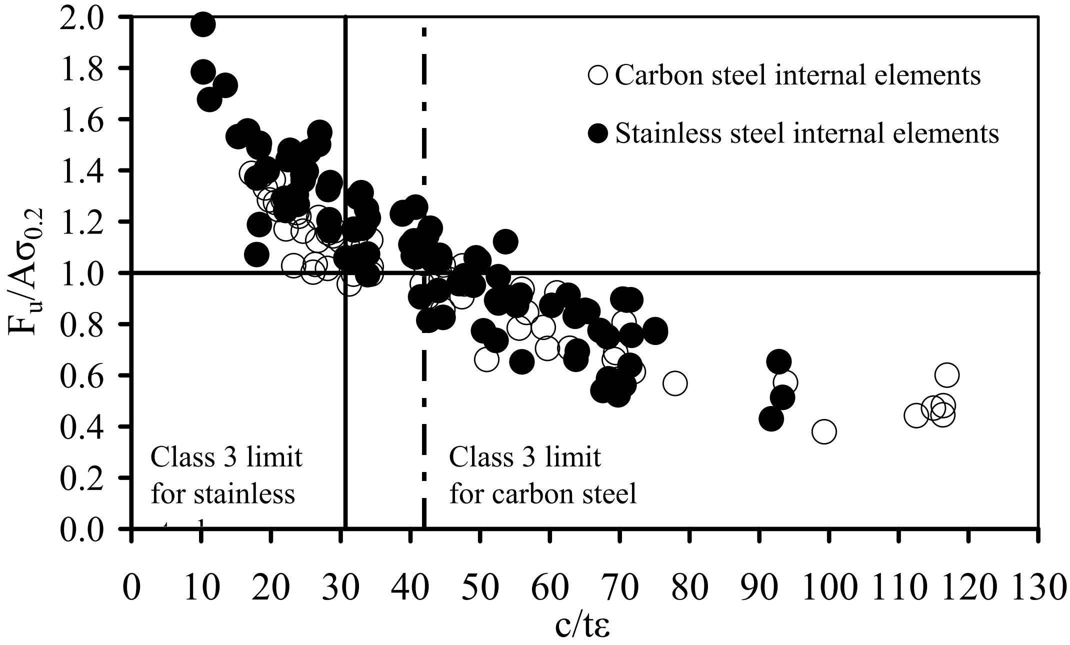

Fig. 8: Class 3 limit for carbon steel and stainless steel internal elements in compression (stub column tests only).

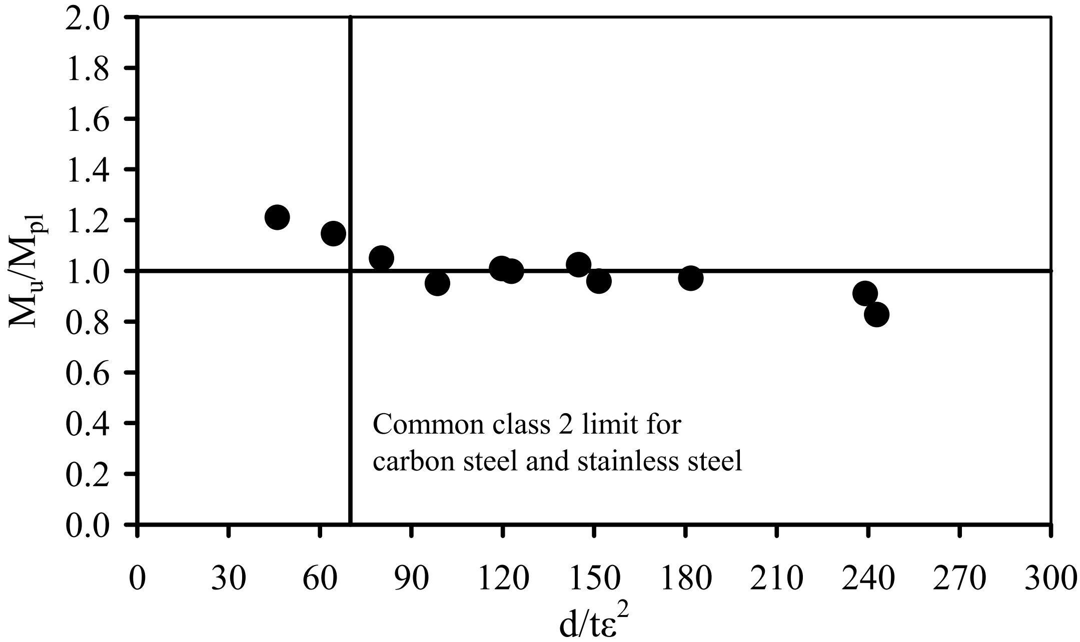

Fig. 9: Class 2 limit for CHS in bending.

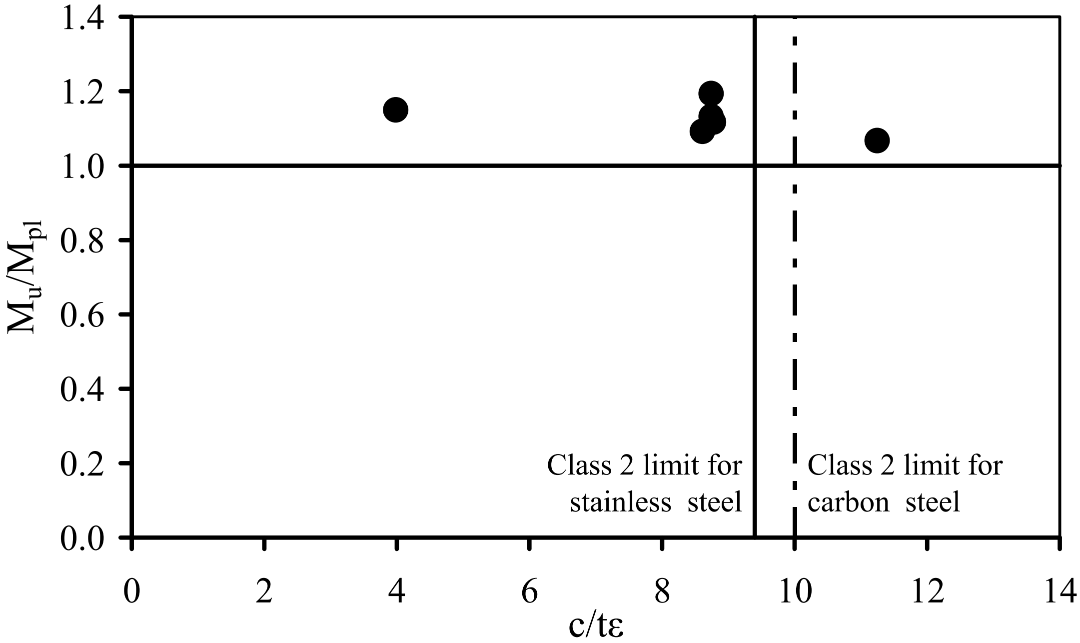

Fig. 10: Class 2 limit for welded outstand elements in compression (I-section tests).

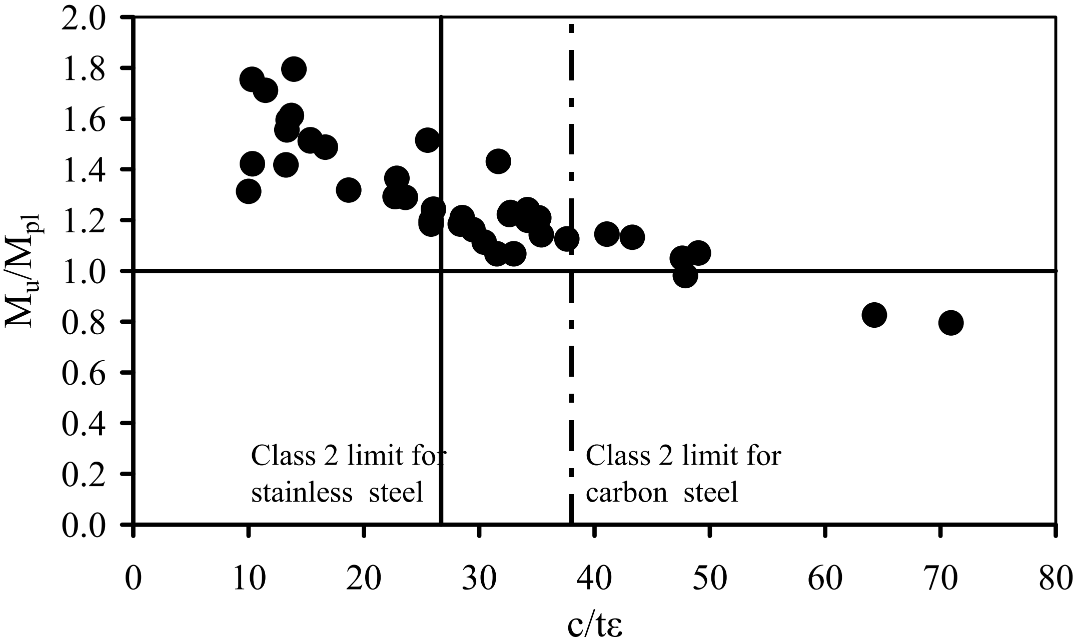

Fig. 11: Class 2 limit for internal elements in compression (SHS and RHS bending tests).

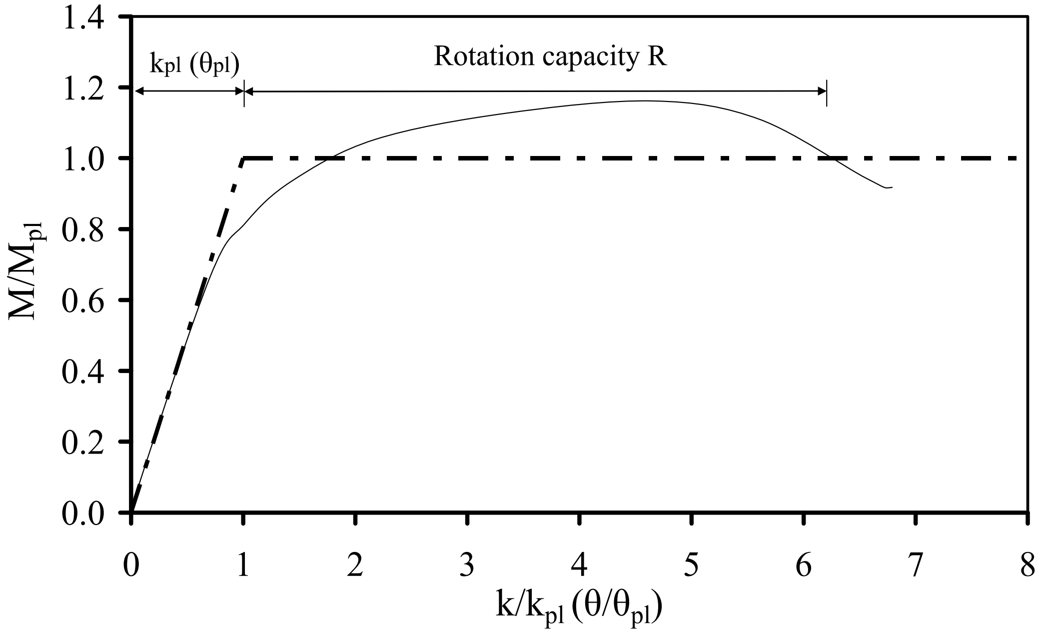

Fig. 12: Definition of deformation capacity.

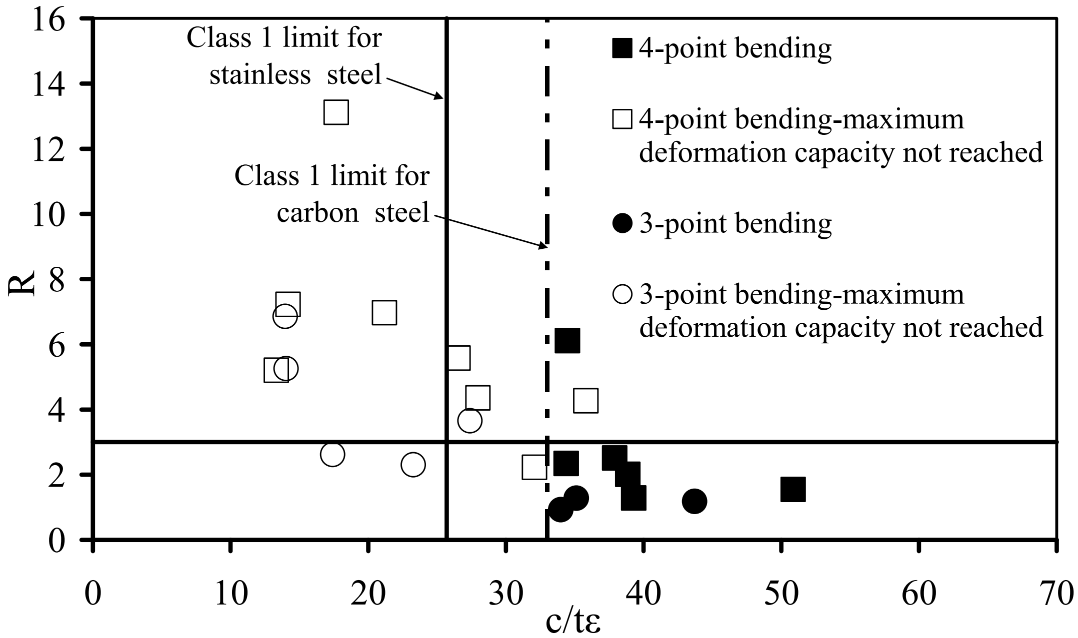

Fig. 13: Class 1 limit for internal elements in compression.

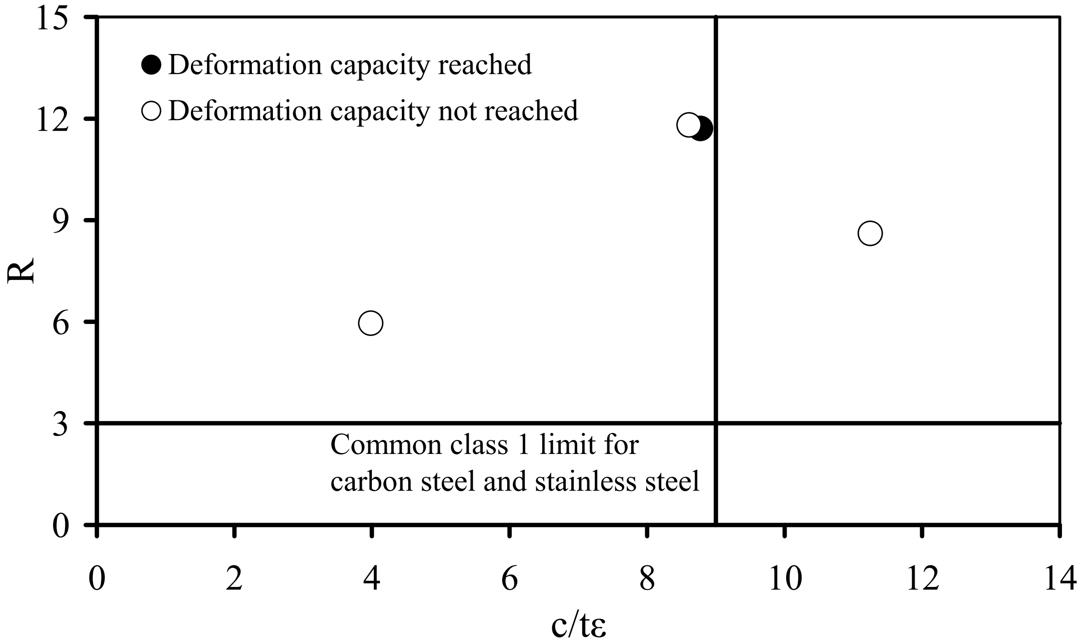

Fig. 14: Class 1 limit for welded outstand elements in compression.

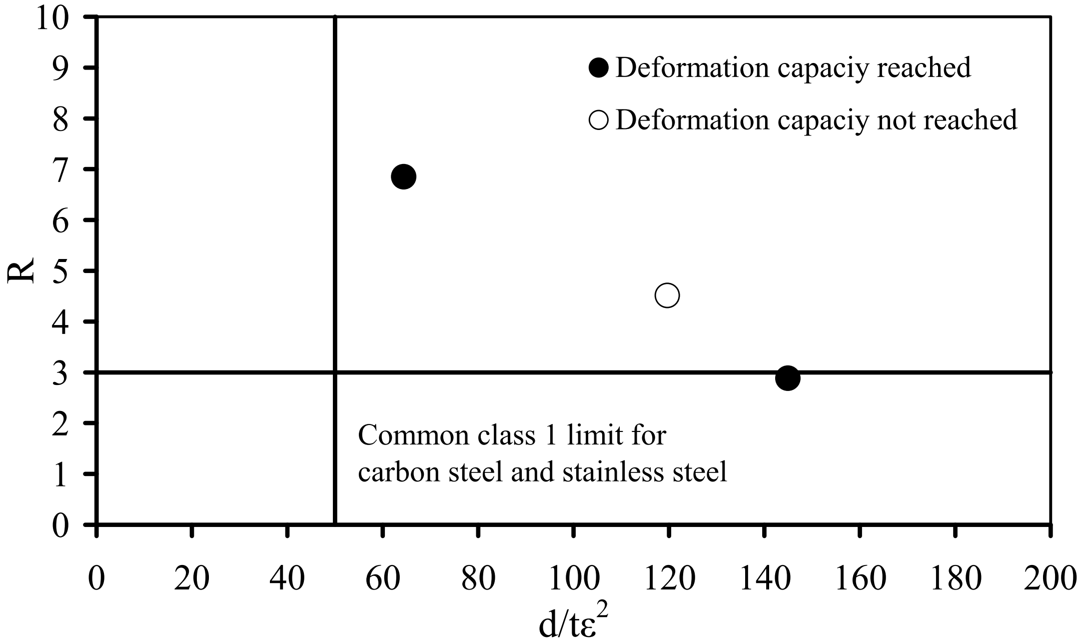

Fig. 15: Class 1 limit for CHS in bending.

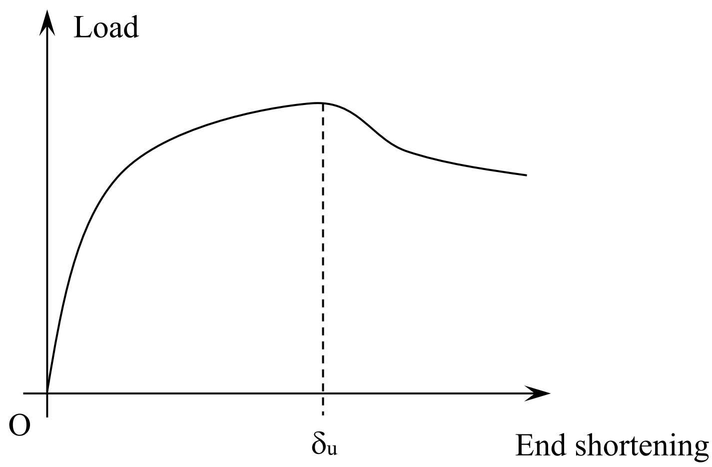

Fig. 16: Typical stub column load-end shortening curve.

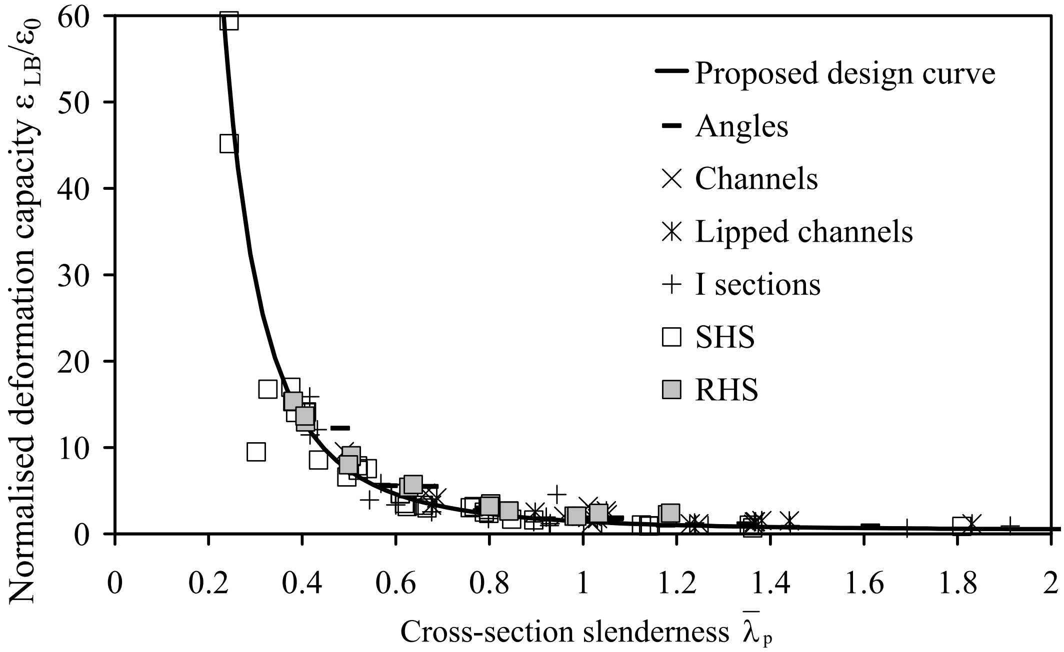

Fig. 17: Normalised deformation capacity against cross-section slenderness for plated cross-sections

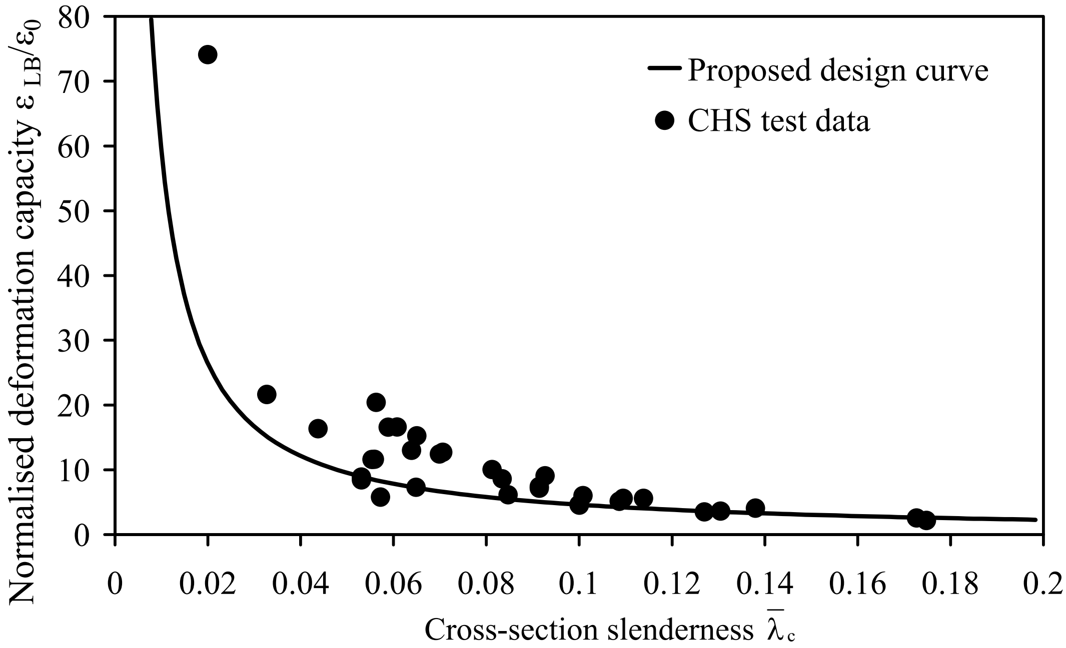

Fig. 18: Normalised deformation capacity against cross-section slenderness for CHS

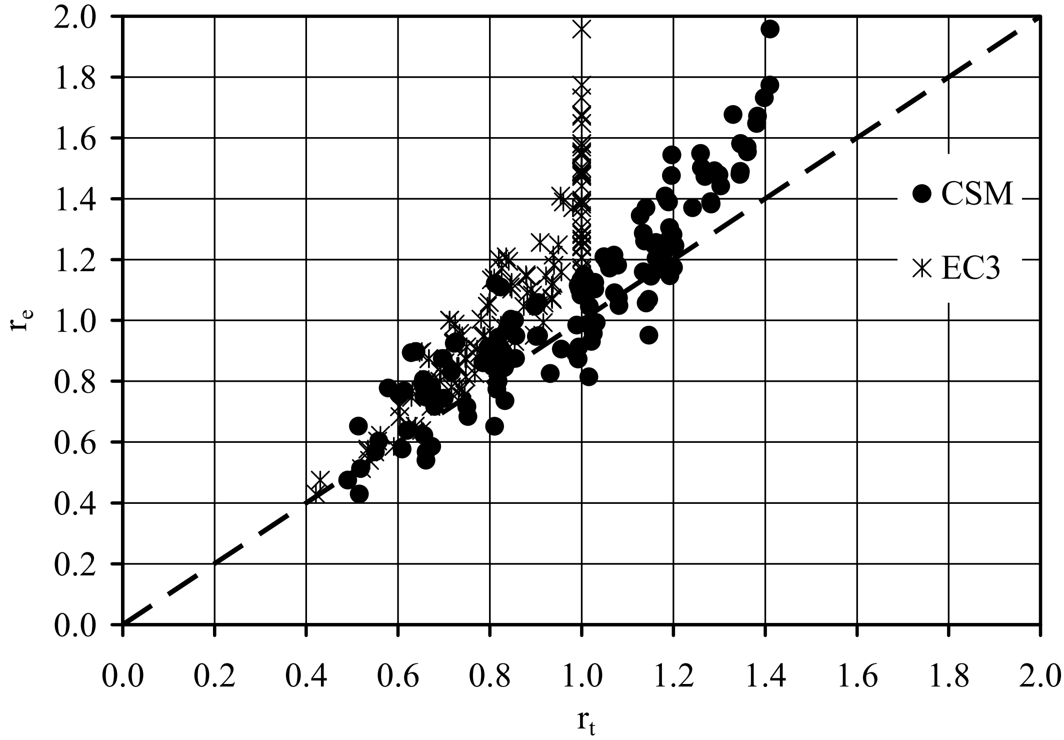

Fig. 19: Experimental and theoretical normalised compression resistances for EN 1993-1-4 and CSM.

Table 1: Stub column tests

Reference	Section type	No. of tests	Material grade
Kuwamura [7]	Angle	12	1.4301 1.4318
	Channel	11	
	Lipped channel	12	
	I (welded)	16	
	SHS	12	
	CHS	10	
ECSC [8]	I (welded)	4	1.4301 1.4462
Gardner and Nethercot [9]	SHS	17	1.4301
	RHS	16	
	CHS	4	
Talja and Salmi [10]	SHS	1	1.4301
	RHS	2	
Rasmussen and Hancock [11]	SHS	2	1.4306
	CHS	3	
Liu and Young [12]	SHS	4	1.4301
Young and Liu [13]	RHS	8	1.4301
Young and Lui [14]	SHS	6	Duplex
	RHS	3	
Young and Ellobody [15]	SHS	2	1.4462
	RHS	3	
Gardner et al. [16]	SHS	4	1.4318
	RHS	4	
Young and Hartono [17]	CHS	4	1.4301
Burgan et al. [18]	CHS	3	1.4435 1.4541
Bardi and Kyriakides [19]	CHS	18	1.4410
Lam and Gardner [20]	CHS	2	1.4401

Table 2: Three and four point bending tests

Reference	Section type	No. of tests	Material grade
Rasmussen and Hancock [21]	SHS CHS	1 1	1.4306
Talja and Salmi [10]	SHS RHS	3 6	1.4301
ECSC [8]	I (welded) CHS	4 11	1.4301 1.4462
Real and Mirambell [22]	SHS RHS I (welded)	2 2 2	1.4301 1.4306
Gardner and Nethercot [23]	SHS RHS	5 4	1.4301
Zhou and Young [24]	SHS RHS	8 7	1.4301 Duplex
Gardner et al. [16]	SHS RHS	2 4	1.4318

Table 3: Carbon steel (CS), stainless steel (SS) and proposed slenderness limits for compression elements

Element		Class 1			Class 2			Class 3		
		CS limit	SS limit	Proposed limit	CS limit	SS limit	Proposed limit	CS limit	SS limit	Proposed limit
Internal element in compression		33ε	25.7ε	33ε	38ε	26.7ε	35ε	42ε	30.7ε	37ε
Internal element in bending		72ε	56ε	72ε	83ε	58.2ε	76ε	124ε	74.8ε	90ε
Internal element subject to bending and compression	$a > 0.5$ or $\psi > -1$	$\frac{396\varepsilon}{13a-1}$	$\frac{308\varepsilon}{13a-1}$	$\frac{396\varepsilon}{13a-1}$	$\frac{456\varepsilon}{13a-1}$	$\frac{320\varepsilon}{13a-1}$	$\frac{420\varepsilon}{13a-1}$	$\frac{42\varepsilon}{0.67+0.33\psi}$	$15.3\varepsilon\sqrt{k_\sigma}$	$18.5\varepsilon\sqrt{k_\sigma}$
	$a \le 0.5$ or $\psi \le -1$	36ε/a	28ε/a	36ε/a	41.5ε/a	29.1ε/a	38ε/a	$62\varepsilon(1-\psi)\sqrt{-\psi}$	$15.3\varepsilon\sqrt{k_\sigma}$	$18.5\varepsilon\sqrt{k_\sigma}$
Cold-formed outstand element in compression		9ε	10ε	9ε	10ε	10.4ε	10ε	14ε	11.9ε	14ε
Cold-formed outstand subject to bending and compression (tip in compression)		9ε/a	10ε/a	9ε/a	10ε/a	10.4ε/a	10ε/a	$21\varepsilon\sqrt{k_\sigma}$	$18.1\varepsilon\sqrt{k_\sigma}$	$21\varepsilon\sqrt{k_\sigma}$
Cold-formed outstand subject to bending and compression (tip in tension)		$\frac{9\varepsilon}{a\sqrt{a}}$	$\frac{10\varepsilon}{a\sqrt{a}}$	$\frac{9\varepsilon}{a\sqrt{a}}$	$\frac{10\varepsilon}{a\sqrt{a}}$	$\frac{10.4\varepsilon}{a\sqrt{a}}$	$\frac{10\varepsilon}{a\sqrt{a}}$	$21\varepsilon\sqrt{k_\sigma}$	$18.1\varepsilon\sqrt{k_\sigma}$	$21\varepsilon\sqrt{k_\sigma}$
Welded outstand element in compression		9ε	9ε	9ε	10ε	9.4ε	10ε	14ε	11ε	14ε
Welded outstand subject to bending and compression (tip in compression)		9ε/a	9ε/a	9ε/a	10ε/a	9.4ε/a	10ε/a	$21\varepsilon\sqrt{k_\sigma}$	$16.7\varepsilon\sqrt{k_\sigma}$	$21\varepsilon\sqrt{k_\sigma}$
Welded outstand subject to bending and compression (tip in tension)		$\frac{9\varepsilon}{a\sqrt{a}}$	$\frac{9\varepsilon}{a\sqrt{a}}$	$\frac{9\varepsilon}{a\sqrt{a}}$	$\frac{10\varepsilon}{a\sqrt{a}}$	$\frac{9.4\varepsilon}{a\sqrt{a}}$	$\frac{10\varepsilon}{a\sqrt{a}}$	$21\varepsilon\sqrt{k_\sigma}$	$16.7\varepsilon\sqrt{k_\sigma}$	$21\varepsilon\sqrt{k_\sigma}$
Angles in compression		-	-	-	-	-	-	11.5ε	9.1ε	11.5ε
CHS in compression		$50\varepsilon^2$	$50\varepsilon^2$	$50\varepsilon^2$	$70\varepsilon^2$	$70\varepsilon^2$	$70\varepsilon^2$	$90\varepsilon^2$	$90\varepsilon^2$	$90\varepsilon^2$
CHS in bending		$50\varepsilon^2$	$50\varepsilon^2$	$50\varepsilon^2$	$70\varepsilon^2$	$70\varepsilon^2$	$70\varepsilon^2$	$90\varepsilon^2$	$280\varepsilon^2$	$280\varepsilon^2$

(a=depth of plastic compression zone over total element width, ψ=elastic compressive over elastic tensile stress ratio, k_σ defined in EN 1993-1-5)

Table 4: Comparison of test results with EN 1993-1-4 and CSM

Section type	Loading type	EN 1993-1-4 (2006)		CSM		CSM/EN predictions
		Mean EN/Test	COV	Mean CSM/Test	COV	
Plated sections	Compression	0.81	0.14	0.93	0.11	1.15
Plated sections	Bending	0.69	0.13	0.94	0.11	1.35
CHS	Compression	0.85	0.15	1.00	0.09	1.18
CHS	Bending	0.81	0.07	1.06	0.05	1.30