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Abstract

We argue a source of time-varying term premium (TVTP) in Japanese government bond market, and show that it is interest rate smoothing that causes empirical failures of expectation theory of term structure of interest rates. We estimate a regime switching ARCH model where an interest rate smoothing regime can be identified. Based on a model of time-inconsistency by Missale and Blanchard (1994), we further focus on a role of debt maturity in TVTP, which is an alternative to an ARCH process.

Our robust empirical evidences support the expectation theory in Japanese government bond market. Moreover, in comparison with the ARCH process, debt maturity turns out to be a reliable proxy for the TVTP. This shows a possibility of debt management policy in Japan: fiscal authority takes advantage of the debt maturity for price stability which is a target of monetary policy. It sharply contrasts with an evidence for ineffectiveness of the U.S. debt management policy by Wallace and Warner (1996).

Key Words: Time-Varying Term Premium, Interest Rate Smoothing, Regime Switching ARCH Model, Debt Maturity, Reputation Equilibrium.

JEL Classification Codes: E42, E43, E52.

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Introduction

Term structure of interest rates has been studied intensively, partly because this is a relatively accessible area to study. In particular, expectation theory is attractive in simplicity. However, there remains an empirical puzzle. A yield spread should not be theoretically a significant variable in an equation of which the dependent variable is an excess holding yield, but regression results show that, indeed, it is quite significant, which is a stylized fact in the G7 countries (Hardouvelis (1994)). A clue to the problem was provided for us by a seminal work of Mankiw and Miron (1986), who have noted that central bank's nominal interest rate smoothing policy increases a difficulty in predicting future rates.

Recently, from the viewpoint of Mankiw and Miron a number of researchers are trying to explain causes of the puzzle (Rudebusch (1995), Fuhrer (1996), Balduzzi, Bertola and Foresi (1997), Balduzzi, Bertola, Foresi and Klapper (1998), Roberds, Runkle and Whiteman (1996), Roley and Sellon (1996)). Their interests are in the U.S. Federal Reserve, probably stimulated by an influential paper of Goodfriend (1991). They are also at the first stage, by which Campbell and Shiller (1991) means that each "thoroughly explores the validity of the simple expectations theory before undertaking a detailed study of the source of predictable time variation in excess returns." Indeed, these works thoroughly examine daily Federal Funds rate and the target rate. They seem to result in predicting actual interest rate data successfully.

However, in order for us to draw a conclusion that the smoothing policy impedes the expectation theory of term structure, time-varying term premium (afterwards TVTP) is indispensable. The fact is ignored by the previous researches, more or less. Therefore, proceeding into the second stage as mentioned by Campbell and Shiller (1991), we need to specify what is a source of the TVTP.

The TVTP has been dealt with by a time series analysis of autoregressive conditional heteroskedasticity (ARCH) model, as represented by Bollerslev, Chou and Kroner (1992). Among some ARCH applications, Engle, Lilien and Robins (1987) had the most influences on sequential researches. They estimated an ARCH-M model, where a conditional variance is a determinant of current term premium, concluding that term premium is far from time invariant. According to their estimation, once a conditional variance is included as a regressor, an effect of the yield spread on the term premium certainly becomes weaker. Yet, the spread is still significant at a significance levels 10%. It follows that in order to recover the expectation theory, a role of an ARCH process is so limited.

Therefore, we propose an alternative to the ARCH of understanding the TVTP. It is through debt maturity, a hypothesis that is based on a model of time-inconsistency by Missale and Blanchard (1994). Debt maturity is an instrument of debt management policy by fiscal authority. The government has originally an inflation bias stemming from an incentive to decrease the real debt burden. In order to compensate for the real loss of holding a government bond, the public requires an inflation premium for the return. What the government can do to prevent the nominal interest rate from increasing is to shorten the debt maturity. That is because debt maturity would be a commitment of the government against the inflation bias. As a result, depending on length of the debt maturity, the required inflation premium would be reflected in the TVTP.

Thus, fiscal authority can take advantage of the debt management for the purpose of price stability, which is a target of monetary policy. Indeed, against the misalliance of both fiscal and monetary policies, there are some institutional devices which have contributed to fiscal discipline. For example, the U.S. Federal Reserve and the Treasury agreed on "the Accord" in 1951, and in the U.K., numerous indexed bonds have been increasingly issued since the Thatcher's era. Contrary to such fiscal discipline, there may be quite enough room for the debt management policy in case of Japan, where the Ministry of Finance has exercised its authority over both fiscal and monetary aspects of the state. Consequently, Japanese government bond market is likely to be influenced by the debt management policy. In other words, the debt maturity may be a source of the TVTP in the market.

Based on another look at debt management above, we argue which source of the TVTP is significant in Japanese case, ARCH process or debt maturity. Taking into account the TVTP, moreover, we aim to show that a reason for the empirical failure of expectation theory is interest rate smoothing.

Our methodology for these goals is somewhat roundabout because of a problem in Japanese data: the Bank of Japan (thereafter BOJ) has never officially announced a target rate of policy instrument, unlike the Federal Reserve (Rudebusch (1995)). First, in order to identify when the BOJ had an intention to smooth the nominal interest rate, we must rely on an index for interest rate smoothing, which we call Yoshikawa index (Yoshikawa (1993)). Apart from the observations, next, we estimate a regime switching ARCH model. The regressors are both a proxy for debt management policy and a yield spread. We test a significance of the yield spread in the term structure equation, considering a regime identification.

Finally, in order to confirm robustness of our estimation results, we compare the Yoshikawa index with an estimated posterior probability of a smoothing regime.

As a result, a regime switching ARCH model is found to provide a good fit for Japanese government bond data. Our robust empirical evidences support the expectation theory in term structure of interest rates in the market. Moreover, an ARCH process turns out not to be necessarily an essential factor in analyzing the TVTP, while the debt maturity must be a reliable proxy for the TVTP. The result coincides with a time-inconsistency model of Missale and Blanchard (1994).

The plan of this paper is as follows: we begin in Section 1 a description of interest rate smoothing policy, with an emphasis on Japanese experience. In Section 2, we provide a statistical basis for our theory. We look at Japanese debt management policy in Section 3. Section 4 explains our empirical method, a regime switching ARCH model, and its application to Japanese data. Finally, we discuss our conclusion in Section 5.

1 Interest Rate Smoothing

Let us look at short-term interest rate in Japan. The policy instrument of the Bank of Japan (thereafter BOJ) is officially nominal rate of call money, which corresponds to the Federal Funds rate in the U.S. In Figure 1, a dotted line illustrates the actual call rate since June 1958 and a thick solid line shows our presumed target of the call rate, as later described in detail. We hereafter use monthly average rate of collateralized overnight call money. We see an increased number of periods in the constant call rate until the 1980's, probably due to some institutional regulations concerning to coordination and pricing [2]. However, the call rate's steady movement around some fixed values also marks more months for the latter periods, when there remains few regulations. It means that the smoothing phenomenon is not necessarily stemming from the pricing regulations in the call money market.

Is the interest rate smoothing phenomenon peculiar to Japan? Bernanke and Mishkin (1992) made some international comparisons of representative macroeconomic variables, concluding that Japan has been the most successful among the other major countries in stabilizing nominal interest rates. Figure

[2] The Federation of Bankers' Association of Japan (FBAJ) had arranged the call rate level from July 1957 to September 1967. Moreover, a quotation procedure for pricing, the so-called "Tatene," had been adopted from May 1961 to April 1979. However, since the 1970's call money market has been considered as the most advanced in deregulation among Japanese financial markets (Ito(1992)).

1 may reflect the stabilizing policy by the BOJ. In addition, Clarida, Gali and Gertler (1997) estimate policy reaction function in G3 (the U.S., Germany and Japan) countries since 1979, such as the form $r_t = (1-\rho)r_t^* + \rho r_{t-1} + v_t$ where actual and targeted nominal short-term interest rates are respectively denoted by r_t and r_t^*. Their estimates of coefficient ρ represent a degree of interest rate smoothing. The values are 0.92 in case of the Fed, 0.91 of the Bundesbank and 0.93 of the BOJ. Therefore, the BOJ has certainly committed the smoothing policy at least to the same extent as the other central banks.

1.1 Japanese Index for Interest Rate Smoothing

Before moving to Japanese index for interest rate smoothing, we have to mention Japanese monetary policy in general. Okina (1993) explains how Japanese daily market operation works from the point of view of a central banker. In regard to its institutions, such as a lagged reserve maintenance system of monthly unit or market operations through inter-bank lending markets, the system is the same as the U.S. Fed [3]. Not surprisingly in terms of secrecy of central bank, there have been no publications of target figures of policy instruments in Japan. We have only "forecasts" of money supply growth (Cargill, Hutchison and Ito (1997)). Consequently, in order to follow policy intention of the BOJ in each economic phase, we must rely on another resource of policy decisions.

Fortunately, we can trace how and when the BOJ has made nominal interest rate smoothed, according to Yoshikawa (1993). Yoshikawa introduces nominal wage rigidity of Taylor type into a model with rational expectation like Goodfriend (1987) and Barro (1989). Monetary policy stances are there described as two ways: 1) interest rate pegging or 2) active and 'dynamic' operation. When interest rate pegging policy is taken in his model, both real output and price level become independent of money demand shock. Moreover, the shock is one-to-one reflected in money supply. On the contrary, real output is then affected by independent change in money supply.

What Yoshikawa does is first to group his whole sample into a pair of 'easy' and 'tight' money periods, based on when the discount rate was first either lowered or raised. Within each pair of period, next, he plots monthly co-movement of call rate and other macroeconomic variables, such as money supply (M2+CD), index of industrial production (IIP) and consumer price index (CPI). Finally, comparing the

[3] However, to more extent than the FRB, Japanese monetary policy board has been nullified ever since its founding. In response to such a criticism, amendment of the Bank of Japan Law has taken effect in April 1997.

observations with the above theoretical implications, he identifies the concerned periods. As a result, he found that, among his sample of 389 months from June 1958 at the beginning of the Iwato boom to October 1990, there were 195 months during which the nominal interest rate was observably pegged. These periods are illustrated at Table 1. Figure 1 also shows both actual and presumed target rates

Table 1: Yoshikawa Index

Beginnings	Ends	Length of Months	Presumed Target Rates (%)[1]
Dec. 1958	Nov. '59 [2]	12	8.4
Dec. 1959	Jul. '60 [2]	8	8.4
Aug. 1960	Jan. '61 [2]	6	8.4
Feb. 1961	Jun. '61	5	8.03
May 1963	Nov. '63	7	7.3
Jul. 1964	Dec. '64	6	10.95
Nov. 1965	May '67	19	5.84
Jan. 1969	Jun. '69	6	7.3
Oct. 1969	Sep. '70	12	8.25 and 8.5
Aug. 1972	Dec. '72	5	4.5
Nov. 1974	Mar. '75	5	13
Feb. 1976	Feb. '77	13	7
Apr. 1978	Mar. '79	12	first half of 4's
May 1981	Dec. '85	56	7 and first half of 6's
May 1987	Mar. '89	23	3's
[3] Sep. 1990	Jun. '91	10	8
Aug. 1992	Jan. '93 [2]	6	4
Feb. 1993	Aug. '93	7	3
Oct. 1993	Mar. '95	18	2
Oct. 1995	Aug. '96	11	0.4

Notes: 1) The presumed target rates are not considered by the original paper Yoshikawa (1993). 2) Yoshikawa (1993) separates successive periods in the same presumed target rates into two, the latter of which corresponds to a period when the official discount rate change occurred in the beginning month. 3) The periods below the line are identified according to level of the call rate itself, not the rules of Yoshikawa (1993), the sample of which is till October 1990. We automatically choose them, that is, if the call rate fluctuates within less than $\pm 0.5\%$ around our presumed target values during more than 3 months, then we select the period as that of interest rate smoothing.

according to "Yoshikawa index (afterwards YI)" as we have named it.

1.2 Diagnostic Experiments of Yoshikawa Index

In order to confirm a validity of the YI for the interest rate smoothing, let us submit the index to several diagnostic experiments. There may be some doubt whether the index merely shows seasonality in the interest rate. In regard to this doubt, Table 2 indicates monthly frequency of the index YI. We cannot

Table 2: Monthly Frequency of Yoshikawa index

	The Number of Periods											
Sample	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Jun. ’58 to Oct. ’90 [1)]	16	17	16	14	17	16	15	16	16	16	18	18
Jun. ’58 to Aug. ’96 [2)]	21	22	21	18	21	20	18	20	19	21	23	23
	The Number of Beginnings											
Jun. ’58 to Oct. ’90 [1)]	1	1		1	3		1	2		1	2	2
Jun. ’58 to Aug. ’96 [2)]	1	2		1	3		1	3	1	3	2	2
	The Number of Ends											
Jun. ’58 to Oct. ’90 [1)]		1	3		1	2	1		1		2	3
Jun. ’58 to Aug. ’96 [2)]	1	1	4		1	3	1	2	1		2	3

Notes: 1) The original sample in Yoshikawa (1993). 2) Our extended sample.

see any seasonal pattern in smoothing policy. We can conclude that the index does not have any seasonal cycle property.

Indeed, let us inspect what the BOJ observed in, for example, a pair of easy and tight money periods from May 1987 to June 1991. In Figure 2, a horizontal axis is measured by the call rate and a vertical line is by an annual money supply (M2+CD) growth [4]. During May 1987 to May 1989, the call rate stayed around 3% to 4%, while the M2+CD was volatile. Reading "Quarterly Economic Outlook" that is a BOJ's official publication, we can see the BOJ was paying careful attention to price levels, which were likely to reflect a series of incidents: 1) an unexpected appreciation in yen even after the Louvre Agreement in Feb. 1987, 2) the Black Monday in Oct. 1987, 3) a steep increase in land prices from 1988 and 4) an introduction of the consumption tax in April 1989. For the latter periods of Sep. 1990 to Jun. 1991, the call rate remained near 8% in spite of the M2+CD's oscillation. Similarly, the Quarterly Economic Outlook then says that the BOJ inspected price levels, in order to calm fears of out-of-control inflation. Thus, in both periods the BOJ was concerned about price levels, instead of seasonality in the interest rate.

[4] Yoshikawa (1993) says,

> May 1987 through March 1989 (23 months): The interest rate was basically pegged at 3%. Output growth recovered from −1% (May 1987) to 11% (March 1988) and stayed high afterward. Throughout this period inflation was very stable. Changes in the money supply therefore mainly reflected output shocks.
>
> April 1989 through October 1990 (19 months): The interest rate was actively raised. Output growth declined from 7% (May 1989) to zero (March 1990) but recovered to 8% (October 1990) again. Inflation was stable at 3% during this period.

Further, we examine another diagnostic experiment of the index. It is an empirical evidence of Takeda (1996), which exploits an implication: velocity of money is more volatile in an interest rate smoothing regime than otherwise. The implication is deduced from a model of Walsh (1984) and is also induced from a careful observation of Goodhart (1989). Walsh (1984) showed that coefficients in money demand function can depend on both variance of bond price and covariance between price level and bond price. In his model, a money supply rule that allows greater fluctuation in bond price, as had been seen in Chairman Volker's era of the U.S, would result in all the more increase in bond price volatility through less interest-elastic money demand. On the other hand, Goodhart (1989) observed that velocity of money is more stable during the periods of the endogenous money supply than otherwise. Based on the implication, Takeda (1996) empirically showed that variance of the money velocity during the YI is significantly smaller than that in dynamic operation periods. In addition, the statistical significance is not true of any other index else than the YI, which is identified according to level of the call rate or difference between the call and the official discount rates. The evidence shows the YI is a valid index for the interest rate smoothing, even if we make use of another framework else than Yoshikawa (1993). We will adopt the index in estimating term structure.

2 Time-Varying Term Premium

In this section we discuss a cause of empirical failure in expectation theory. Central to the problem is the TVTP. In order to focus on a role of the TVTP, we wish to give a good account of the failure with as simple a theory as possible. Accordingly, we take an example of one and two-period discount bonds. We can easily extend our theory to a case of N-period coupon bonds, like government bond.

Each yield of two bonds are denoted by r and $r^{(2)}$. Then, there is a relationship as follows:

$$r_t^{(2)} = \theta + \lambda r_t + (1-\lambda) E_t r_{t+1}$$

$$r_{t+1} = E_t r_{t+1} + \epsilon_{t+1},$$

where a term premium θ is for the moment assumed to be constant. Expectation theory means that $\lambda = \frac{1}{2}$. Arranging the above equation, we can get another equation where a dependent variable is an excess holding yield, $r_t - (2r_t^{(2)} - r_{t+1})$.

$$r_t - (2r_t^{(2)} - r_{t+1}) = -\frac{\theta}{1-\lambda} + \frac{2\lambda - 1}{1-\lambda}(r_t^{(2)} - r_t) + \epsilon_{t+1}. \tag{1}$$

Similarly, expectation theory says a coefficient of the yield spread is 0. The left side in Equation (1) means an excess return in a case of holding the two-period bond relative to rolling over the one-period bond. As a consequence, expectation theory implies that the present yield spread cannot forecast an excess holding return.

Now suppose the term premium is time-varying $\theta = \theta_t$. Then, least square estimator of a coefficient $\hat{\beta}$ of the yield spread, $r_t^{(2)} - r_t$ is,

$$\begin{aligned}\hat{\beta} \equiv \frac{2\lambda - 1}{1-\lambda} &= \frac{Cov(r_t - (2r_t^{(2)} - r_{t+1}),\ r_t^{(2)} - r_t)}{Var(r_t^{(2)} - r_t)} \\ &= \frac{-8\sigma^2(\theta_t) - 4\rho\sigma(\theta_t)\sigma(E_t\Delta r_{t+1})}{4\sigma^2(\theta_t) + \sigma^2(E_t\Delta r_{t+1}) + 4\rho\sigma(\theta_t)\sigma(E_t\Delta r_{t+1})}.\end{aligned}$$

The notations $\sigma^2(\cdot)$ and $\sigma(\cdot)$ respectively mean variance and standard deviation. $E_t\Delta r_{t+1}$ means $E_t(r_{t+1}) - r_t$. ρ is a correlation coefficient between θ_t and $E_t\Delta r_{t+1}$.

Suppose $\sigma^2(E_t\Delta r_{t+1}) \to \infty$, then

$$plim\hat{\beta} = 0.$$

On the other hand, if an extreme smoothing policy makes the interest rate a martingale process, that is $\sigma^2(E_t\Delta r_{t+1}) = 0$, the coefficient $\hat{\beta}$ would turn out to be -2. The difference in the coefficient is what Mankiw and Miron (1986) empirically showed: volatility of the nominal short-term interest rate was greater before 1914 when the Federal Reserve System was founded in the U.S., and estimate of the spread's coefficient is significantly equal to the theoretical value.

Moreover, a coefficient of the term premium $-\frac{1}{1-\lambda}$ does not depend on the degree of interest rate smoothing, unlike that of the yield spread.

$$\begin{aligned}-\frac{1}{1-\lambda} &= \frac{Cov(r_t - (2r_t^{(2)} - r_{t+1}),\ \theta_t)}{Var(\theta_t)} \\ &= -2\end{aligned}$$

If we can find some proxies for the TVTP, then an effect of the variables on the excess holding yield would be invariant to interest rate smoothing policy.

Indeed, the above statistical theory shows that a cause of the empirical failure in expectation theory lies in the interest rate smoothing policy. However, what is more important is that time variation of term premium is a necessary condition for the empirical failure of the expectation theory [5]. Contrary to the necessity, most of previous literature scrutinizing the Fed's market operation assumes a time invariant term premium, for example Roberds, Runkle and Whiteman (1996). Even otherwise, Balduzzi, Bertola and Foresi (1997) merely take into account a biweekly periodicity due to reserve maintenance periods. Fuhrer (1996) also misunderstands regime switching in monetary policy stance as the time variation of term premium, though each of them is a separate concept. Without doubt, it is not until specifying sources of the TVTP θ_t that we can explain why the yield spread has significant effect on the excess holding yield.

3 Another Look at Debt Management Policy

We will give a new view on the TVTP: debt maturity indicates the time variation. Before explaining the hypothesis, we discuss government bond market in Japan. Figure 3 indicates each amount of domestic government bonds classified by authorizing laws. Construction and special government bonds are authorized for the purposes of financing fiscal expenditures and deficits, respectively, while refinancing bond is for redeeming previously issued bonds. Figure 3 shows that the refinancing bond has had a strong tendency to rise rapidly since the 1990's. The Japanese Ministry of Finance (MOF) turns out to have been faced with a difficulty in refinancing the bonds.

Meanwhile, Figure 4 shows a ratio of long-term coupon bonds since April 1981. The figure is defined as a ratio of coupon bonds with maturity of 6, 10, 15, and 20 years, relative to all the market holding stock of Japanese government bonds. Figure 4 indicates that while, until the first half of 1990's, the ratio has been cyclical through the peak to the trough, it has been decreasing markedly since 1993 by more than 5%. The shortening of debt maturity structure is in progress.

Both figures clearly capture a fact that in these years the Japanese MOF has intentionally replaced the redeemed long-term bonds with the shorter term refinancing bonds. Why has the MOF adopted such a debt management policy? One explanation is given by Missale and Blanchard (1994). The

[5] McCallum (1994) showed in a rational expectation model, that even if the expectation theory itself is true, the reduced form of term structure can deviate from what the theory predicts, when both the term premium is subject to AR (1) process, and short-term interest rate operation is taken.

model is entirely different from a traditional literature of public debt management policy (Tobin (1963), Friedman (1992), Roley (1982), Modigliani and Sutch (1966, 1967), Agell and Persson (1992)), because it is concerned with a time-inconsistency problem pioneered by Barro and Gordon (1983).

3.1 Debt Maturity as a Commitment for Price Stability

We discuss Missale and Blanchard (1994) model briefly. According to the model, government is supposed to minimize its loss function associated with inflation, while the public responds to the government's policy by forming expectations concerning inflation. The loss function at period t consists of three parts: 1) costs of inflation, 2) output effect from unexpected inflation and 3) distortion of taxation. The government is also faced with an accumulation equation of debt, where real debt value depends on debt maturity through unexpected inflation, given a spending and taxation decision. At the beginning of period t, government inherits previous debt and decides the maturity of debt. The public forms their rational expectation concerning inflation, based on their best knowledge including length of the maturity the government has chosen. Finally, the government chooses a rate of inflation in the period.

In the repeated game situation, although the best outcome is clearly a zero-inflation equilibrium achieved by a precommitment of the government, the outcome is time-inconsistent, not a Nash equilibrium. In order to attain the zero-inflation equilibrium for infinite horizons, the public's reputation for the government would be sufficient.

In order for the government to attain the first best outcome, the government would solve for its problem backward in time, conditional on the public's punishment: if the government cheats and brings about unexpected inflation this period, in next period and on the public will take a time-consistent expectation of inflation. Comparing a loss from the punishment with a once gain from the inflation, the government must decide the debt maturity. Because the longer the debt maturity the higher is the incentive to inflate, the maximum of debt maturity needs to be bounded so that the reputation equilibrium would be better off than a cheat equilibrium.

The maximum maturity is also a decreasing function of debt burden, because an increase in the debt level means a larger reward from unexpected inflation and as a result, the government does not need to increase the debt maturity. According to an empirical analysis in Missale and Blanchard (1994), an adverse relation between average maturity and a ratio of debt to GDP is significant in some countries where debt overburden has been a problem, such as Ireland, Italy and Belgium.

Thus, the model shows that a decrease in debt maturity can be a signal for government's commitment against inflating so as to depreciate the real debt burden. This implies that the government's commitment for price stability would lower the public's inflation premium, which is inevitably required for nominal government debt because of the inflation bias. The inflation premium is thought of as a part of the TVTP. Consequently, the time variation of term premium is partly caused by the debt management policy, an instrument of which is debt maturity. If Missale and Blanchard (1994) model fits into government's behavior, then the debt maturity will be a proxy for the TVTP.

3.2 Japanese Evidence

We apply a few implications of Missale and Blanchard (1994) model to Japan, where as described above, the MOF has been faced with the refinancing problem.

Figure 5 shows plots of the long-term debt ratio and a ratio of market holding government bonds to nominal GDP. An adverse movement is revealed with a negative correlation of -0.57 in our whole sample and -0.97 after 1993 when the long-term debt ratio has began to rapidly drop. The negative correlation serves as an evidence in favor of Missale and Blanchard's theory. Similarly, Figure 6 shows plots of the long-term debt ratio and GDP deflator measured as an annual growth rate. The plots indicate a nearly positive relation, where correlation coefficients are 0.42 in our whole sample and 0.84 since 1993.

From these statistical facts, it is quite reasonable that especially since 1993, a role of government debt maturity has been played in price stability in Japan, as Missale and Blanchard (1994) says. As a result, the debt maturity can be a proxy for the TVTP in Japan. We will utilize the long-term debt ratio as an independent variable in our regression of term structure.

4 Regime Switching Model

Now we are ready to estimate a term structure equation. What we should consider is, as described above, two causes: 1) interest rate smoothing regime, and 2) debt maturity as a proxy for the TVTP. The statistical inference in Section 2 showed that a coefficient of the yield spread (estimate $\hat{\beta}$) in the regression would be different, depending on which regime monetary policy lies in. In an extreme interest rate smoothing regime, the coefficient would be equal to -2, while otherwise it would be significantly 0.

The feature leads us to adopt a regime switching model where an interest rate smoothing regime can be identified.

In addition, we have to consider an effect of the TVTP. Previous literature has taken into account an ARCH process in error term (Engle, Lilien and Robins (1987)). In order to compare a relative empirical performance between an ARCH process and debt maturity, we introduce an ARCH process into a regime switching model.

Therefore, our method is a regime switching ARCH model where the long-term debt ratio is a regressor. It is a simplified model of Markov regime switching ARCH model developed by Hamilton and Susmel (1994) and applied by Cai (1994). The model can produce a posterior probability of each regime, which will be an evidence to check the propriety of our index for smoothing YI.

4.1 Regime Switching Model of Term Structure

Our model is quite simple. Monetary policy consists of two regimes 0 or 1, where interest rate smoothing policy is explicitly operated or otherwise, respectively. We call each regime " smoothing " and " leaving-alone " afterwards. At the beginning of each period, the public expects where the present regime will be, conditional on their best knowledge at that point. For simplicity, suppose that the public forms expectation concerning regime every period in the same way: a probability of smoothing regime 0 is a parameter π independent of which regime is. We also assume that a disturbance in each regime follows an i.i.d. normal distribution, $N(0, 1)$. Moreover, in order to exploit an effect of ARCH modeling within our regime switching model, we allow the error term to have different ARCH processes in each regime.

After all, the following is our model:

$$y_t = x_t' \beta_{S_t} + u_t$$

$$u_t = \sqrt{h_t} v_t$$

$$v_t \overset{i.i.d.}{\sim} N(0,\ 1)$$

$$h_t = \gamma_0 + \gamma_1 S_t + \sum_{i=1}^{m} (\alpha_{0i} + \alpha_{1i} S_t) u_{t-i}^2$$

A coefficient β_{S_t} is different in regime, denoted by $S_t = 0, 1$. The error term u_t has an ARCH (m) structure. The variance h_t only depends on the present regime S_t. For example, a coefficient of u_{t-i} for

$i = 1, \cdots m$ is α_{0i} if monetary policy lies in the smoothing regime this period, while $\alpha_{0i} + \alpha_{1i}$ in case of the leaving-alone regime.

4.2 Estimation

We estimate the regime switching ARCH model, where a dependent variable y_t is an excess holding period yield (afterwards EHPY). The data is calculated with a linearization method of Shiller (1990) [6]. Figure 7 indicates the data. The statistics shows a low serial correlation of the EHPY [7]. Taking into consideration the low correlation, we won't include lagged values of the EHPY among the regressors.

Meantime, our independent variables x_t are a yield spread and the long-term debt ratio. The yield spread (YS) is a difference between a long-term and a short-term yields. For each yield, we use a circulation yield of government bond with longest maturity, and a yield of three-month public and corporate bonds with repurchase agreement (the so-called Gensaki), respectively. As described in Section 3, the long-term debt ratio is defined as a ratio of long-term government bond to all the market holding stock of Japanese government bonds. It is an instrument of debt management policy, so that we abbreviate the name to DM. Only for the DM variable, an Augmented Dickey Fuller test shows that we cannot reject the non-stationarity [8]. Consequently, we differentiate the variable in the first order to have a new series ΔDM substituted for the raw data DM.

What we first have to do is to choose our benchmark model. The candidates are two: a switching-ARCH (1) or a switching-ARCH (2) [9]. In order to fix our AR order, we rely on Akaike Information Criterion (AIC) and Schwarz Bayes Information Criterion (SBIC) [10]. These statistics (AIC, SBIC) are $(-263.19, -280.84)$ in case of ARCH (1), and $(-267.73, -288.56)$ in ARCH (2). As a result, we select a

[6] See Appendix A. 2.

[7] Also see Appendix A. 2.

[8] See Appendix A. 3.

[9] Switching-ARCH (3) model cannot converge even if we try some sets of appropriate starting values, so that we exclude it from our benchmark model candidates.

[10] We list the log likelihood function at Appendix B.

benchmark switching-ARCH (1) model:

$$Ehy_t = \begin{cases} \beta_{00} & +\beta_{01}\ Ys_t & +\beta_{02}\ \Delta DM_t + u_t & if\ regime = \ 0 \\ \beta_{10} & +\beta_{11}\ Ys_t & +\beta_{12}\ \Delta DM_t + u_t & if\ regime = \ 1 \end{cases}$$

$$\begin{aligned} u_t &= \sqrt{h_t} v_t \\ v_t &\overset{i.i.d.}{\sim} N(0,\ 1) \end{aligned}$$

$$h_t = \quad \gamma_0 \quad +\gamma_1\ S_t \quad +(\alpha_{01} + \alpha_{11} S_t)\ u_{t-1}^2.$$

We estimate the benchmark via a via maximum likelihood method. Then each monetary regime is identified as either smoothing or leaving-alone, based on a restriction: in a regime 1, interest rate volatility is more than in another regime 0, that is $\gamma_1 > 0$. Owing to the identifying restriction, we can name a regime 0 a smoothing regime.

In the process of estimating each model, we try to adopt some appropriate starting values so that the estimation could converge after a finite number of iteration. In principle, we use estimates of an ordinary ARCH model without any switching. The exception is a coefficient β_{11} of the yield spread in the leaving-alone regime 1. We set the value for 0, which would be somewhat in favor of the expectation theory, as explained in Section 2. As for some parameters concerning switching, we apply a theory of Walsh (1984) to the values; since in the leaving-alone regime volatility of interest rate would be more than in the smoothing one, the parameters γ_1 and α_{11} should be positive. Consequently, we give them a small positive number 0.01. Finally, for the probability π of the smoothing regime, we substitute a sample frequency of months in the smoothing regime, judging from the YI, that is $\frac{131}{184}$.

In the following results, parenthesis includes standard error of each variable, and $*$ means a variable is significant at a 5% significance level.

$$Ehy_t = \begin{cases} \begin{array}{cccl} 0.0022 & +7.65\ Ys_t & +6.60\ \Delta DM_t + u_t & if\ regime = \ 0 \\ (0.0067) & (1.83)^* & (1.47)^* & \\ -0.0048 & +0.95\ Ys_t & -1.35\ \Delta DM_t + u_t & if\ regime = \ 1 \\ (0.011) & (3.66) & (2.46) & \end{array} \end{cases}$$

$$h_t = \begin{array}{ccc} 0.00026 & +0.0052\ S_t & +(0.18 - 0.093 S_t)\ u_{t-1}^2 \\ (0.00017) & (0.0012)^* & (0.14) \quad (0.28) \end{array}$$

$$\pi = \begin{array}{l} 0.385121. \\ (0.098)^* \end{array}$$

The estimation tells us some interesting points. In the term structure equation, we can find a sharp contrast between each regime. In the smoothing regime, both the yield spread and the long-term debt ratio are significant, while in the leaving-alone regime neither accounts for the excess return. In particular, the coefficients of both variables in the smoothing regime are significantly larger than 2, a value that our theory in Section 2 predicts. On the other hand, the conditional variance equation h_t does not have a significant variable, except for a constant term which depends on the regime. The positive effect means that the leaving-alone regime creates more volatility of interest rates than the smoothing one does, a result that agrees with a theory of Walsh (1984) and a stylized fact by Goodhart (1989). The ex ante probability π of the smoothing regime 0 is also significantly estimated to be around 39%. However, the value is unexpectedly lower than an average percentage 71%, a figure that we can count from the YI.

4.3 Result

Next, we further select a model through likelihood ratio test, within some nested models of our benchmark. Each of them corresponds to a case among any combinations, depending on whether it is with or without an ARCH process (that is, $\alpha_{01} = 0, \alpha_{11} = 0$), whether there is any switching or not in the ARCH process ($\alpha_{11} = 0$), whether there is any switching or not in constant terms of the ARCH process ($\gamma_1 = 0$), and whether there are any switching or not in a term structure ($\beta_{00} = \beta_{10}, \beta_{01} = \beta_{11}, \beta_{02} = \beta_{12}$). The combination amounts to twelve cases including the benchmark model. In addition, in order to reconfirm the validity of the long-term debt ratio as a proxy for term premium, whether there are any debt maturity effects or not ($\beta_{02} = 0, \beta_{12} = 0$) is examined separately. Accordingly, besides the benchmark, we prepare twelve nested models in total.

The likelihood ratio test shows that in comparison with the benchmark, we can reject nine nested models at a significance level 10%. There remain three cases which cannot be rejected: case 1) no switching in the ARCH process; case 2) neither switching in the ARCH process nor in the term structure; and case 3) without the ARCH process. Not surprisingly, the rejected models include a case where the long-term debt ratio is excluded. Table 3 indicates the results of likelihood ratio test and estimates of each parameter. For reference, as well as three cases, it contains the rejected case 4) without the long-term debt ratio.

Now let us summarize our results:

1. A constant term in the ARCH process significantly changes according to monetary regime. The figure in the leaving-alone regime 1 is higher than in the smoothing regime 0. It reflects higher

Table 3: Comparison of Alternative Models

Parameter	Benchmark	Case 1	Case 2	Case 3	Case 4
γ_0	0.00026	0.00027	0.000021	0.000078	0.00019
	(0.00017)	(0.00017)	(0.000023)	(0.000046)*	(0.00014)
γ_1	0.0052	0.0050	0.0047	0.0051	0.0043
	(0.0012)*	(0.00096)*	(0.00069)*	(0.00064)*	(0.00072)*
α_{01}	0.18	0.15	0.19		0.0027
	(0.14)	(0.087)*	(0.097)*		(0.019)
α_{11}	-0.093				0.19
	(0.28)				(0.17)
β_{10}	-0.0048	-0.0052		-0.0033	-0.0076
	(0.011)	(0.011)		(0.0096)	(0.0092)
β_{11}	0.95	1.29		3.21	2.62
	(3.66)	(3.56)		(2.46)	(2.41)
β_{12}	-1.35	-1.26		0.56	
	(2.46)	(2.36)		(1.68)	
β_{00}	0.0022	0.0024	-0.0064	-0.00087	0.0072
	(0.0067)	(0.0068)	(0.0023)*	(0.0031)	(0.0055)
β_{01}	7.65	7.55	5.66	7.32	7.28
	(1.83)*	(1.85)*	(0.84)*	(0.91)*	(1.50)*
β_{02}	6.60	6.62	4.33	7.57	
	(1.47)*	(1.46)*	(0.73)*	(0.81)*	
π	0.39	0.38	0.23	0.20	0.22
	(0.098)*	(0.096)*	(0.065)*	(0.051)*	(0.066)*
Log Likelihood	252.19	252.14	249.07	251.26	249.10
Likelihood Ratio		0.11	6.24	1.87	6.18
p-value		(0.74)	(0.18)	(0.39)	(0.045)

Notes :

1) $*$ shows that a variable is significant at a significance level 5%. 2) Each case corresponds to the following restrictions: case 1) no switching in the ARCH process; case 2) neither switching in the ARCH process nor in the term structure; case 3) without the ARCH process; and case 4) without the long-term debt ratio.

volatility of interest rate in the former regime than in the latter. However, the ARCH process itself does not show any switches in regime.

2. The public's prior probability π of the smoothing regime 0 is estimated at 20% to 39%. The probability is extremely low, compared with the YI.

3. As for the expectation theory, we are quite successful in recovering the empirical evidence. The coefficients β_{11} of the yield spread in the leaving-alone regime 1 are not significant in all the cases. On the contrary, all the coefficients β_{01} in the smoothing regime 0 are significant. This robust evidence strongly shows that a lot of the empirical failures in the expectation theory plausibly result from ignoring a monetary regime switching.

4. As explained in Section 2, even if we take into account the TVTP, the effect of the debt maturity would not be different in regime. The theoretical value is 2 in both regimes. Concerning this point, our estimates show a complicated result. In the leaving-alone regime 1, we cannot reject the null hypothesis that the coefficient β_{12} is equal to 2, while in the smoothing regime 0 we can clearly reject the null $\beta_{02} = 2$. It may suggest that there remains a problem.

5. The case 3 without the ARCH process is not rejected against the benchmark model. Previous literature has treated the TVTP as an ARCH process, typically seen in Engle, Lilien and Robins (1987). However, this result says that the ARCH process is not crucial for modeling a time variation in term premium. Interestingly, the conclusion is agreed with by Cai (1994) who worked with the U.S. data.

6. On the contrary, the debt maturity turns out to be indispensable for the term structure. The case 4 where the long-term debt ratio is omitted, is statistically rejected against our benchmark model. It clearly contrasts with an analysis of the U.S. by Wallace and Warner (1996). They estimate a simple GARCH model, and cannot detect any significant relations between the U.S. Federal debt maturity and the excess holding period yield. Wallace and Warner's results may be interpreted as an evidence of ineffective debt management policy in the U.S., from a viewpoint of time-inconsistency problem of Missale and Blanchard (1994). The difference between the U.S. and Japan seems to be related to an independence of central bank. Further research will be required.

4.4 Posterior Probability of Regimes

The estimation above produces a by-product, a posterior probability of each regime. The probability is given, as described above, based on an identifying restriction: volatility of interest rate is more in the leaving-alone regime than in the smoothing one. Even if the estimation result supports our hypothesis: the interest rate smoothing policy is responsible for persistent empirical failures in the expectation theory, then the validity would be weakened by our incorrect identification of regimes. Therefore, we have to examine whether the probability coincides with the YI.

The posterior probability can be easily calculated in our simplified model [11]. Figures 8-11 indicate posterior probabilities of the smoothing regime 0 in each case. In each figure, a shaded area on an upper horizontal axis shows that the months belong to the smoothing regime of the YI, while a shaded area on a lower axis mean the leaving-alone regime. Obviously, the posterior probability is so volatile that it does not look steady around a fixed level for a while [12]. However, both Figure 10 and 11 show some striking differences in the probability between peaks and troughs. Making use of this feature, a tendency to stay near a zero probability is adopted as our criterion of the leaving-alone regime. Similarly, a tendency to stay up at a higher probability is our criterion of the smoothing regime.

In terms of the criteria, we observe figures of the posterior probability. In Appendix C, we chronologize what Figures 8-11 indicate, following dates of the YI. Summing up the chronology, we may consider the posterior probability to approximately grasp the YI. To be concrete, the whole cases are clustered into two classes: the benchmark and the case 1 often give us the probability which fits into the smoothing regime only, while the cases 2 and 3 mostly coincide with the leaving-alone regime only. It seems to reflect a difference in the estimated probability of the smoothing regime between the former (39% and 38%) and latter cases (23% and 20%), as seen in Table 3.

[11] See Appendix B.

[12] Our regime identification is somewhat dissatisfactory. It seems to be due to both the YI and our switching regime ARCH model. For the former, another approach will be replaced. Clarida, Gali and Gertler (1997), for example, estimate stochastic processes of both actual interest rate and the targeted rate vis a vis GMM. On the other hand, the latter problem will be solved by a Markov regime switching model. Instead, we assumed that an ex ante probability of a regime is independent of which regime is. The simplification would make state partition all the coarser. Both approaches will hold value.

5 Conclusion

Our robust empirical evidences support the expectation theory in term structure of interest rates in Japanese government bond market. It sharply contrasts with so many previous researches concerning the G7 countries. What is new in our analysis is both a switching in regimes of monetary policy and a role of debt maturity in time-varying term premium (TVTP). Our switching-ARCH model gives a posterior probability of a smoothing regime, which is reasonable in terms of an anecdotal Yoshikawa Index (YI) (Yoshikawa (1993), Walsh (1984), Goodhart (1989)). Moreover, in comparison with an ARCH process (Engle, Lilien and Robins (1987), Cai (1994)), the debt maturity turns out to be a reliable proxy for the TVTP (Wallace and Warner (1996)).

In our interpretation, the debt maturity of government plays a role in achieving a reputation equilibrium with price stability, instead of the so-called inflation bias (Missale and Blanchard (1994)). The optimal equilibrium contributes to lowering investors' inflation premium required for the government bond rate. Because the premium is reflected in the TVTP, the debt maturity would be a proxy for the TVTP after all. The inference is supported by our evidences concerning a long-term debt ratio in Japan.

This shows a possibility of a debt management policy in Japan: for price stability which is a target of monetary policy, fiscal authority takes advantage of the debt maturity. However, it seems to be impossible in the U.S. and the U.K. bond markets, because in 1951 the Federal Reserve and the Treasury agreed on "the Accord" in the U.S., and numerous indexed bonds have been increasingly issued since the Thatcher's era in the U.K. As an evidence for the ineffectiveness of the U.S. debt management policy, Wallace and Warner (1996) cannot detect any significant relations between the Federal debt maturity and the excess holding period yield.

Such a debt management based on a model of time-inconsistency is an ordinary phenomenon in each country? Some governments conduct it forcefully, like the Japanese Ministry of Finance; others cannot do institutionally like the U.S. Treasury or practically like the U.K. one. How about another country? The question is an important issue, which is closely related to an independence of central bank. It remains a further research.

A Data Appendix

A.1 Data Description

Table 4: Data Description

Government Bond	Amount of both long and medium-term coupon bonds, and discount bond, excluding that of both underwriting by the Trust Fund Bureau and subscription by the BOJ. Measured in a hundred million yen. Sample from May '81 to Feb. '96, total 178 months. Source: *Monthly Report of Public and Corporate Bonds* (the Bond Underwriting Association), cited by *Annual Statistical Report of Securities* (the Tokyo Stock Exchange).
Long-Term Government Bond	Amount of 6, 10, 15 and 20-year maturity bonds. Measured in a hundred million yen. Sample from May '81 to Feb. '96, total 178 months. Source: *Monthly Report of Public and Corporate Bonds* (the Bond Underwriting Association), cited by *Annual Statistical Report of Securities* (the Tokyo Stock Exchange).
Long-Term Yield	A circulation yield of government bond with longest maturity at the end of month (the Tokyo Stock Exchange) . Sample from Mar. '77 to Feb. '96, total 228 months. Source: *Monthly Report of Public and Corporate Bonds* (the Bond Underwriting Association).
Short-Term Yield	A yield of 3-month public and corporate bonds with repurchase agreement at the end of month. Sample from Mar. '77 to Feb. '96, total 228 months. Source: *Monthly Statistical Economic Report* (the Bank of Japan).

A.2 Estimate of Excess Holding Period Yield

We calculate our dependent variable, an excess holding period yield, according to a linearization method of Shiller (1990) [13].

A holding period return on a bond $h_i(t, t', T)$ is a return at present t over some holding period $t' - t$ less than the bond's maturity $T - t$. The subscript i means that the bond is a par bond $i = p$ or a discount bond $i = d$. An excess holding period return $\Phi_{h,i}(t, t', T)$ is defined as a difference between an expected holding period yield $E_t h_i(t, t', T)$ and a spot rate $r_i(t, t')$.

[13] Campbell, Lo and MacKinlay (1997) is also familiar with some basic concepts of a fixed-income security.

Meanwhile, the holding period yield is represented in terms of duration $D_i(\cdot)$:

$$h_i(t,t',T) = \frac{D_i(T-t)r_i(t,T) - [D_i(T-t) - D_i(t'-t)]r_i(t',\ T)}{D_i(t'-t)}.$$

We also approximate a long-term bond like government bond, as a par bond with an infinite maturity. Further, assuming that an investor's holding period is infinitesimal $t' = t+1$, we get $r_p(t,t+1) \approx r_d(t,t+1)$. By definition, a duration $D_p(\infty)$ is a reciprocal of a spot rate $r_p(t,\infty)$.

From these definitions and approximations, we can have a formula of the excess holding period yield as follows:

$$\Phi_{h,p}(t,t+1,\infty) = E_t h_p(t,t+1,\infty) - r_d(t,t+1)$$

$$\begin{aligned} h_p(t,t+1,\infty) &= \frac{D_p(\infty)r_p(t,\infty) - [D_p(\infty) - D_p(1)]r_p(t+1,\infty)}{D_p(1)} \\ &= 1 + r_p(t+1,\infty) - \frac{r_p(t+1,\infty)}{r_p(t,\infty)}. \end{aligned}$$

For the above formula, we substitute actual data of Japan. We use a circulation yield of Japanese government bond with longest maturity for the long-term yield $r_p(t,\infty)$ [14]. As for the short-term yield $r_d(t,\ t+1)$, supposing that one period corresponds to three months, a rate of three-month bond with repurchase agreement (the so-called Gensaki) is adopted.

Figure 7 indicates our data of the excess holding period yield. Table 5 also shows some univariate statistics of the excess holding period yield. The details of our data are described in Appendix A. 1 Data

Table 5: Statistics of Excess Holding Period Yield

Mean	Standard Deviation	Skewness	Kurtosis
0.0051	0.062	-0.15	1.30

Serial Correlations

Order 1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
0.067	0.073	-0.10	-0.22	-0.082	-0.062	-0.11	0.058	0.087	0.10

Description.

[14] We have another data for the long-term yield: an over-the-counter tone figure. Even if we substitute this for the circulation yield, our results turn out not to be essentially altered.

A.3 Unit Root Test

We test stationarity of all the variables, which is needed for precluding spurious causation in our estimation. We run an Augmented Dickey Fuller test, where a significance of the coefficient ρ matters in the following equation:

$$\Delta y_t = \mu + \beta t + \rho y_{t-1} + \alpha_1 \Delta y_{t-1} + \cdots + \alpha_p \Delta y_{t-p} + u_t.$$

The results are shown in Table 6. Regardless of the number p of lag order, we cannot reject the non-stationarity of a variable DM. Consequently, in our empirical analysis we use a difference of first order ΔDM.

Table 6: Augmented Dickey-Fuller Test

Variable	$p = 0$	$p = 1$	$p = 2$	$p = 3$	$p = 4$
EHY	-13.7^{**}	-9.6^{**}	-9.0^{**}	-9.6^{**}	-8.4^{**}
YS	-2.8	-2.6	-3.3^{*}	-3.4^{**}	-3.2^{*}
DM	-2.2	-2.1	-2.3	-2.2	-1.9

Note: $**$ and $*$ show a rejection of the null hypothesis $H_0 : \rho = 0$ at significance levels of 5% and 10%, respectively.

B Log Likelihood Function and Posterior Probability

As in the text, we assume that our model is two-regime switching ARCH (1), where a disturbance follows an i.i.d. normal distribution. Let the regime at date t be indexed by an unobserved random variable $S_t = 0, 1$. We also suppose that a probability π of a regime switching is independent of which regime is. A vector θ parenthesizes all the concerned parameters.

Then, a density of y_t conditional on $S_t = j$ and $S_{t-1} = i$ is

$$f(y_t | S_t = j \ S_{t-1} = i \ ; \theta) = \frac{1}{\sqrt{2\pi}\sqrt{h_t}} \exp^{-\frac{(y_t - x_t' \beta_{S_t})^2}{2h_t}} .$$

A joint density-distribution function of y_t, S_t and S_{t-1} is also

$$P(y_t, \ S_t = j, \ S_{t-1} = i \ ; \theta) = f(y_t | S_t = j, \ S_{t-1} = i \ ; \theta) P(S_t = j | S_{t-1} = i ; \theta) P(S_{t-1} = i \ ; \theta),$$

where under our assumption,

$$P(S_t = j | S_{t-1} = i ; \theta) = P(S_t = j \ ; \theta) = \pi, \ for \ j = 0, \ 1.$$

Summing the joint density functions bears a density function of y_t,

$$f(y_t \ ; \theta) = \sum_{j=0}^{1} \sum_{i=0}^{1} P(y_t, \ S_t = j, \ S_{t-1} = i \ ; \theta).$$

Substituting those for the density function of y_t, we obtain a log likelihood function,

$$L(\theta) = \sum_{t=1}^{T} \log f(y_t \ ; \theta).$$

Similarly, a posterior probability of a regime is easily attained as follows:

$$\begin{aligned}
P(S_t = j \ | y_t \ ; \theta) &= \frac{P(y_t, \ S_t = j \ ; \theta)}{f(y_t \ ; \theta)} \\
&= \frac{\sum_{j=0}^{1} P(y_t, \ S_t = j \ S_{t-1} = i \ ; \theta)}{f(y_t \ ; \theta)} \\
&= \frac{\sum_{i=0}^{1} f(y_t | S_t = j, S_{t-1} = i ; \theta) P(S_t = j | S_{t-1} = i ; \theta) P(S_{t-1} = i ; \theta)}{\sum_{j=0}^{1} \sum_{i=0}^{1} f(y_t | S_t = j, S_{t-1} = i ; \theta) P(S_t = j | S_{t-1} = i ; \theta) P(S_{t-1} = i ; \theta)}.
\end{aligned}$$

C Comparison of Posterior Probability and Yoshikawa Index

1. 1981-Dec. '85:

   During this period in the smoothing regime, both Figures 8 and 9 show persistently high probability.

2. '86-Apr. '87:

   Figures 8 and 9 also clearly grasp a switch from the smoothing to the leaving-alone regimes. Moreover, Figures 10 and 11 show nearly zero probability in the leaving-alone regime. Especially in Figure 11, the probability levels out at zero during most of the period.

3. May '87-Mar. '89:

   Although the YI indicates the smoothing regime then, neither figure fits absolutely into the YI. However, all the figures show a sharp increase in the probability at the end of the period.

4. Apr. '89-Aug. '90:

   Figure 10 grasps a switch from the smoothing to the leaving-alone regimes. The figure also shows nearly zero probability in the leaving-alone regime. Moreover, in Figure 11 the probability levels out at zero for five successive months.

5. Sep. '90-Jun. '91:

   In the short period, we can see a heap in all the figures. It tells us that all the figures fit well into the smoothing regime of the YI.

6. Jul. '91-Jul. '92:

   In the same way as in the previous leaving-alone periods, Figure 10 shows nearly zero probability in the leaving-alone regime.

7. Aug. '92-Jan. '93, Feb. '93-Aug. '93 and Oct. '93-Mar. '95:

   In spite of the smoothing regime, neither figure fits into the YI.

8. Apr. '95-Sep. '95:

   In all the figures, there is a rapid drop of the probability. Moreover, Figures 10 and 11 show persistently low probability in the leaving-alone regime. Especially in Figure 11 the probability levels out at zero for three successive months.

9. Oct. '95-Aug. '96:

   In spite of the smoothing regime, neither figure fits into the YI.

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