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Koichi Hamada

Masaya Sakuragawa

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ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
27 Hillhouse Avenue
New Haven, Connecticut 06520

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CAPITAL FLIGHT, NORTH-SOUTH LENDING,
AND STAGES OF ECONOMIC DEVELOPMENT

Koichi Hamada
Yale University

Masaya Sakuragawa
Nagoya City University

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ABSTRACT

Why capital does not necessarily flow from capital-rich countries to capital-poor countries as conventional international trade theory would predict? This paper studies this question in a dynamic model that incorporates an asymmetric information structure similar to the structure in Gertler-Rogoff (JME 1990). As long as the degree of asymmetric information in the capital market is identical between the North and the South, then both regions will converge to a steady state with the same capital intensity, and the income gap between them will be eventually eliminated. However, if the asymmetric information exists only in the South, the income discrepancy between the North and the South will be magnified by international capital movements, and the timing of capital market liberalization in the South needs to be decided with extreme caution.

KEY WORDS: Asymmetric information, Capital flight, Underdevelopment trap

JEL Classification: D82, F21

1. Introduction

One of the crucial questions for the present world economy is, as has been argued by Lucas (1990), why capital does not necessarily flow from capital-rich countries to capital-poor countries in contrast to what conventional international trade theory would predict. International capital movements do not seem to play a sufficient role in narrowing the gap in factor prices between countries in the North and those in the South. Behind this puzzle is hidden the fact that real rates of interest in poorer countries are not necessarily higher than those in richer countries. Instead, as McKinnon (1991) shows, real rates of interest are positively correlated with real rates of growth.

The economics of asymmetric information may provide a clue to explain these phenomena. An insightful article by Gertler and Rogoff (1990) applies the economics of imperfect information to the issue of North-South lending in the presence of the threat of capital flight. Entrepreneurs have access to risky projects as well as safe lending abroad. The ex-post outcome of domestic risky projects is verifiable by a lender, but the entrepreneur's allocation of funds between a domestic risky project and lending abroad is not verifiable. Gertler-Rogoff's two-period analysis of the consequence of the moral hazard problem associated with this information structure derives many illuminating results. For example, capital does not necessarily flow from the capital-rich North to the capital-poor South, but may move in the reverse

direction. They also find that the empirical relationship between the relative indebtedness and per capita income is congruent to the prediction of their theoretical model. On reading their article, the reader would naturally be curious about the dynamic implications: Does the dynamic process of North-South lending not lead to further impoverishment of the South? May a rise in wealth in a rich country in fact work to suppress investment in a poor country? What is the appropriate timing for a developing country to liberalize its international capital flows?

The purpose of this paper is to study these questions in a dynamic overlapping generation model that incorporates the essential features of information structure analyzed in the Gertler-Rogoff model. In particular, we derive the following results: As long as the degree of incompleteness in the capital market due to asymmetric information is identical between the North and the South, then both regions will converge to a steady state with the same capital intensity, and the income gap between them will be eventually eliminated. However, if the asymmetric information exists only in the South, the income discrepancy between the North and the South will be magnified by international capital movements, and the timing of capital market liberalization in the South needs to be decided with extreme caution because the premature liberalization would lead the South into a trap of underdevelopment economic stagnation.

The model we will develop below is an overlapping generations model with information structure identical to Gertler and Rogoff (1990). People live for two periods, and consume only in the second period. A fraction of them are entrepreneurs who receive no endowment but have access to an investment project. The other fraction are lenders who supply labor in the first period and lend their labor income to other agents.

In order to analyze intergenerational transfers of wealth following Bernanke and Gertler (1989) or Sakuragawa (1991), we introduce two sectors of production: the consumption-goods sector and the investment-good sector. In the consumption-goods sector, production takes place instantaneously by employing labor and capital good with neoclassical production technology under certainty. In the investment-good sector transformation of the consumption goods into capital good takes one period of gestation and involves the same kind of uncertainty that was analyzed by Gertler and Rogoff. We also assume the existence of externalities due to social overhead capital following the procedure frequently used in endogenous growth literature (e.g. Romer, 1986). Labor income of lenders in the first period is earned by working with capital stock accumulated by the old generation. This serves as the key linkage between the economic activities of successive generations and that enables to study the dynamic process of economic development.

The following messages arise from our framework. First, the difference in the information structure in the credit market has important as well as long-enduring effects on the economic func-

tion of international capital movements. The difference in the initial conditions of income or wealth brings forth a serious consequence, particularly when the difference in the information structure exists between the North and the South. Many developing countries seem to lack the infrastructure of information networks, appropriate accounting and legal systems that facilitates effective monitoring. Thus we may presume that monitoring cost in these countries are much larger than those in developed countries. Such a difference in information structure can be a crucial factor that explains the reverse flow of capital from the South to the North. The difference in informational infrastructure seriously influences the long-run development path. "Convergence" could be the norm of a world economy that consists of national economies with different initial levels of wealth but with similar information structure in the credit market; "divergence of economic growth rates" (Lucas, 1988) would be a norm of the world economy consisting of national economies with different degree of imperfect information.

Integration of capital market between the North where verification costs are negligible and the South where information structure is imperfect will trigger capital movements from the South to the North. International capital mobility allows the North to choose a higher investment level, but induces the South to choose a lower level. "Capital flight" gives rise to a wider wage gap between the countries. In the North the wage rate becomes higher; in the South it becomes lower than it would be in autarky. This can be interpreted as an example of the result

noted by Hart (1975) that the introduction of an additional market may hurt economic welfare in the presence of incomplete information.

Moreover, the long-run state could depend on the initial wealth level of the South. The effects of the difference in the information structure will be aggravated if there is a large gap in wealth levels. If the gap in initial wealth between the North and South at the moment of integration is sufficiently large, the strong demand for loans from the North will absorb all of the savings of the South, and eventually leads to the economic collapse of the South. Thus, our analysis implies that capital market liberalization at an early stage of development under imperfect information may be harmful for low-income countries and that the timing of liberalization should be chosen with extreme caution. This more or less supports the discussion of the appropriate order of economic liberalization by McKinnon (1991).

Finally, in the world with imperfect capital market, we may talk about the endogenous financial repression in addition to the regulatory financial repression extensively studied by McKinnon (1973, 1991). Our analysis of the closed economy suggests that the real rate of interest is reduced under asymmetric information even without any government regulation on the ceilings of interest rates. Entrepreneurs are discouraged from pursuing productive projects because the burden of incomplete credit market falls upon them. This may be one of the reasons we observe the stylized fact that the real rate of interest is positively correlated to the real rate of economic growth.

In Section 2, we present the structure of our dynamic model. In Section 3 we derive the property of the optimal debt contract of a small open economy to which the (risk-free) rate of interest is exogenously given from abroad. In Section 4, we consider a closed economy in which the interest rate is endogenously determined. In the closed economy under asymmetric information, the interest rate is reduced, but the investment level chosen by entrepreneurs is the same as under symmetric information. Only lenders incur the agency cost.

In Section 5, we start the analysis of a dynamic two-country model of capital movements. If both the North and the South suffer the same degree of hazard due to asymmetric information, but the South has a poorer initial endowment of capital, and hence of income, at the initial moment, the income levels of the North and the South will converge to the same steady-state. In contrast to this convergence property, striking results emerge, as indicated above, in the model studied in Section 6, where only the South is assumed to have an incomplete capital market while no capital-flight problem exists in the North. Capital moves from the South to the North, the wage gap widens rather than narrows after the integration of capital markets, and the country that opens its capital market too prematurely may be trapped into underdevelopment.

2. A Dynamic Model with Capital Flight

Let us construct an overlapping generations version of the Gertler-Rogoff model of a small open economy. The information

structure and the notations are similar to those in theirs. In this section, we start with a single-country version by explaining several modifications we have added to their model. At each period $t = 0, 1, \dots, \infty$, a continuum of agents are born who live for two periods. Agents are either lenders or entrepreneurs, α ($0 < \alpha < 1$) being the fraction of entrepreneurs and $1-\alpha$ that of lenders.

In order to build a coherent intergenerational model, we now introduce two sectors in production: a consumption-good sector and an investment-good sector. The consumption-good sector instantaneously produces the consumption good by combining capital with labor under a standard neoclassical production technology. The investment-good sector transforms the consumption good into capital by a risky investment project which is endowed to each entrepreneur.

It is assumed that the consumption good cannot be directly used as an input for the production of the consumption good. The consumption good should be transformed into capital by the investment-good sector. Capital is an intermediate good for the production of the consumption good and cannot be consumed directly. Thus, the consumption-good sector is a final good sector and the investment-good sector an intermediate good sector. Production in the consumption-good sector takes place instantaneously. Production in the investment-good sector takes one period of gestation. Capital depreciates fully after one period and the consumption good perishes between the periods. (See Figure 1 for the timing of production.)

At any period the consumption good is produced in the consumption-good sector under the neoclassical production technology with constant returns to scale that employs two factors, capital and labor. Because the production technology is homogeneous in degree one, output of the consumption good can be described in terms of the action of a single, aggregate, price-taking firm. Denoting per capita (more rigorously, per efficiency unit labor) capital at period t as an input to the production of the consumption good by k_t , and per capita social overhead capital that enhances the general productivity level by \bar{k}_t , we write the production function as¹

$$(1) \quad y_t = k_t^\beta \bar{k}_t^{1-\beta}, \text{ where } 0 < \beta < 1.$$

The consumption-good producer rents capital and hires labor in spot markets. Due to the assumption of constant returns to scale with respect to private inputs, factor payments exhaust output when each factor is paid its private marginal product. Equating the social per capita capital and the private per capita capital such that $k_t = \bar{k}_t$, we obtain the wage rate as $(1-\beta)k_t$ and the rental price of capital as β .

Each entrepreneur is endowed with no consumption good in either period of her life. In the first period of her life, she has access to only one investment project which yields a random

¹Our specification of the production function follows Bencivenga and Smith (1991).

Although we do not derive endogenous growth, the specification has two advantages: (i) it simplifies calculation and facilitates the global analysis of the nonlinear dynamics, and more importantly (ii) it leads to the possibility of interesting multiple equilibria in the last section.

return in the second period. Each investment project yields θ units of the capital good with probability $\pi(x_{t+1})$ and zero otherwise between periods if x_{t+1} units of the consumption good are invested as an input.² $\pi(\cdot)$ is increasing, strictly concave, and continuously differentiable, such that $\pi(0) = 0$, $\pi(\infty) = 1$, and $\pi'(0) < \infty$. The returns to investment projects are independently and identically distributed across entrepreneurs. Alternatively, each entrepreneur has access to another investment opportunity with a rate of return r . In an open economy setting r is interpreted as the risk-free interest rate earned from investing in foreign countries. Entrepreneurs consume only in the second period of life and hence maximize their expected second period consumption $E_t(c_{t+1})$, where c_{t+1} is the consumption at period $t+1$ and E_t denotes the expectation operator conditional on information available at period t .

Each lender supplies a fixed amount of labor, $1/(1-\alpha)$ units of labor, to the consumption sector and receive $(1-\beta)k_t$ units of the consumption good as the wage rate in the first period of his life.³ Lenders also consume only in the second period of life and hence save the whole earning either by lending to entrepreneurs or by investing in the safe asset as in entrepreneurs. Lenders maximize the expected second period consumption of

²For example, installation of the computer system improves the accuracy of calculations required for production, which leads to an increase in the success probability of the project. Recently, Atkeson (1991) and Innes (1991) take a similar approach.

³This is only for notational simplicity. Qualitatively nothing is changed if you assume that each lender supplies one unit of labor.

$E_t(c_{t+1})$. Both types of agents are protected by the limited liability constraint such that $c_t \geq 0$.

The information structure is essentially the same as that assumed by Gertler-Rogoff. Each entrepreneur has access to two kinds of investment opportunities: the risky project and the safe asset. The ex-post outcome of the risky project is verifiable, but the profit from the safe asset is not verifiable to lenders. Lenders are able to observe the amount of loans of the borrower, but cannot observe the borrower's allocation of funds between the two investment opportunities. The allocation of funds is strictly within her private information, which creates a moral hazard problem.

Presumably, the underlying assumption in Gertler and Rogoff is that investment in financial assets is more difficult to verify than investment in production. If loans are invested in production, there will be physical counterparts that tell the outcome of projects, such as machines, plants, or raw materials. On the other hand, if they are invested in financial assets abroad, it will be relatively easy to hide the balance abroad as well as the profit from it.

3. The Optimal Contract

When an entrepreneur invests, she must raise loans from lenders. Thus, the amount of loans b_t must satisfy

$$(2) \quad b_t \geq x_{t+1} ,$$

where b_t is the amount of loans supplied to an entrepreneur born at t . The entrepreneur issues a state-contingent contract which

pays Z_t in the event when the project yields θ , and Q_t in the event when the project yields nothing. The contract must be structured to be specified only contingent on the verifiable project outcome. The individual rationality condition requires that the contract must compensate the lenders for the risk-free interest rate r per unit, such that

$$(3) \quad \pi(x_{t+1})Z_t + [1 - \pi(x_{t+1})]Q_t = rb_t .$$

(Individual Rationality IR)

The entrepreneur's expected second-period consumption is

$$(4) \quad \pi(x_{t+1})[\theta\beta - Z_t] - [1 - \pi(x_{t+1})]Q_t + r(b_t - x_{t+1}) .$$

where the first two terms represent the expected net return from the project and the third term the return from the safe asset.

Since the borrower's choice of the level x_{t+1} is not verifiable, contracts must be structured so as to induce borrowers to reveal the choice of x_{t+1} truthfully. The incentive compatibility implies

$$(5) \quad x_{t+1} = \operatorname{argmax} (4) .$$

(Incentive Compatibility IC)

Since that $[\theta\beta - (Z_t - Q_t)] > 0$, which we prove below, the second-order condition for internal maximum with respect to x_{t+1}

$$\pi''(x_{t+1})[\theta\beta - (Z_t - Q_t)] < 0 ,$$

is always satisfied. Thus the incentive compatibility condition

(5) is replaced by the first-order condition:

$$(6) \quad \pi'(x_{t+1})[\theta\beta - (Z_t - Q_t)] = r .$$

Since the payment in the bad state Q_t cannot exceed his second period verifiable income, zero,

$$(7) \quad Q_t \leq 0 .$$

The problem of the entrepreneur and the lenders

$$\max_{x_{t+1}, b_t, Z_t, Q_t} \quad (4)$$

subject to (2) (3) (6) and (7).

The optimal contract is represented by two equations:

$$(8) \quad Z_t = r \frac{x_{t+1}}{\pi(x_{t+1})} , \quad (\text{IR})$$

$$(9) \quad \pi'(x_{t+1}) [\theta\beta - Z_t] = r , \quad (\text{IC})$$

and

$$(10) \quad Q_t = 0 ,$$

where (8) implies the individual rationality condition for lenders and (9) a restatement of the incentive compatibility condition. (For the proof see Appendix of Gertler and Rogoff.)

Finally, we check the second-order condition. From (8), (9) and (10), it follows that

$$\theta\beta - (Z_t - Q_t) = \theta\beta \frac{\pi'(x_{t+1})}{\pi(x_{t+1}) + x_{t+1}\pi'(x_{t+1})} > 0$$

for any $x_{t+1} > 0$.

If information is perfect, the individual entrepreneur will choose the investment level x_{t+1} that satisfies the first best condition

$$(11) \quad \pi'(x_{t+1}) \theta\beta = r .$$

Equations (8) and (10) imply that the payment in the good state is to be strictly greater than that in the bad state, such that $Z_t > Q_t = 0$. The comparison between (9) and (11) tells that, the

investment level in the information-constrained economy is strictly below the first-best level.

Now consider briefly an equilibrium in a small open economy.

The relationship between the per capita aggregate capital k_t at period t and the borrower's investment level x_t determined in period $t-1$ is given by

$$(12) \quad k_t = \alpha \theta \pi(x_t) .$$

Three equations (8), (9) and (12) define, given the level of x_t , a temporary equilibrium in terms of k_t , Z_t and x_{t+1} in the small open economy under asymmetric information. The economy becomes a capital importer at time t if the demand for investment αx_t exceeds the supply of funds that is equal to $(1-\beta)k_t$.

The stationary equilibrium of this economy is characterized by the two equations without time subscripts:

$$(13) \quad Z = \frac{rx}{\pi(x)} ,$$

and

$$(14) \quad z = \theta\beta - \frac{r}{\pi'(x)} ,$$

both of which are the restatement of (8) and (9).

Let us introduce the following assumption as made by Gertler and Rogoff in order to assume the internal equilibrium.

Assumption 1

$$(15) \quad \theta\beta\pi'(0) - r > 0 .$$

Figure 2 draws both IR and IC relationship on (x, Z) space. Equation (13) is drawn as the upward-sloping IR curve through the

origin, and equation (14) as the downward-sloping IC curve. The IC curve intersects the horizontal axis at the first-best level x^{FB} that satisfies (11) at the stationary state. Given Assumption 1, Z is strictly positive, and the investment level in the information-constrained economy is strictly less than the first-best x^{FB} . As is seen from (12), per capita aggregate capital and thus the wage rate in the information constrained economy are lower than their first best level that are respectively x^{FB} and $(1-\beta)k^{FB}$.

4. A Closed Economy with Unverifiable Investment Outcomes

Let us consider a closed economy version of the model, in which the interest rate is endogenously determined. Let r_t denote the interest rate prevailing between period t and period $t+1$. Now each lender saves the first-period earning only by lending to others. Each entrepreneur has access to only one investment project, and in addition she can possibly lend the borrowed funds from lenders to other entrepreneurs or even to other lenders. Each lender can verify the profits of investment projects directly funded by him, but cannot verify the profits of investment projects indirectly funded.

The IC condition and the IR condition of this economy are respectively replaced by

$$(16) \quad \pi'(x_{t+1})(\theta\beta - Z_t) = r_t ,$$

and

$$(17) \quad Z_t = r_t \frac{x_{t+1}}{\pi(x_{t+1})} .$$

The market clearing condition in the loan market is given by

$$(18) \quad \alpha x_{t+1} = (1 - \beta)k_t ,$$

where the L.H.S is the aggregate quantity of loan demand and the R.H.S the aggregate quantity of loan supply. The risk-free interest rate is determined to satisfy (18). Strictly speaking, there is no security which guarantees r_t as a one-period (gross) interest with certainty. r_t is merely the "calculated" price which each lender must be compensated per unit of lending. If a financial intermediary is formed as an institution, it will provide r_z per unit of lending to lenders with certainty and r_z will be naturally interpreted as the "deposit rate of interest." However, intermediation is redundant in the sense that it does not improve the welfare of any agent in the national economy.

Incorporating (12) into (18) gives

$$(19) \quad x_{t+1} = (1-\beta)\theta\pi(x_t) ,$$

which describes the full dynamics of this economy.

Let us introduce another assumption that ensures the internal stationary equilibrium.

Assumption 2

$$(1-\beta)\theta\pi'(0) > 1 .$$

If we denote the variables of the closed economy by subscript "A", we obtain at the stationary state

$$(16S) \quad \pi(x_A)(\theta\beta - Z_A) = r_A$$

and

$$(17S) \quad Z_A = r_A \frac{x_A}{\pi(x_A)}$$

$$(19S) \quad x_A = (1-\beta) \theta \pi(x_A) ,$$

By Assumption (2), (19S) has a unique interior stationary equilibrium.

As Figure 3 illustrates, given Assumption 2, there exists an interior solution. Given $x_0 > 0$, there exist a unique dynamic path converging monotonically to x_A . The sequence of k_t is determined according to (10). Other two variables r_t and Z_t evolve, respectively in terms of x_t , as

$$(20) \quad Z_t = \theta \beta \frac{\pi'(x_{t+1})}{\pi(x_{t+1}) + x_{t+1} \pi'(x_{t+1})} ,$$

and

$$(21) \quad r_t = \theta \beta \frac{\pi'(x_{t+1}) \pi(x_{t+1})}{\pi(x_{t+1}) + x_{t+1} \pi'(x_{t+1})} ,$$

where both variables are positive and decreasing in terms of x_{t+1} .

On the other hand, the symmetric information case is described by three equations (10), (18), and

$$(22) \quad \theta \beta \pi'(x_{t+1}) = r_t ,$$

Comparing the two economies between in presence of and absence of asymmetric information, you will easily find that, the stationary state value of x (and hence k) as well as the dynamics of x_t (and hence k_t) are the same so long as the initial conditions are the same.

The difference in the information structure between the two economies is reflected in the interest rate. Denoting the values

on the comparison of information with superscript "*" and those under asymmetric information without it, we obtain the following result on the comparison of stationary states. We denote variables at the stationary state by those with subscript "A".

Proposition 1

The interest rate in the information-constrained case is lower than the interest rate under complete information, such that $r_A^* > r_A$.

Intuition behind Proposition 1 is as follows: If the same interest r_t would be quoted between the closed and the open economy, the investment level must be smaller in the information-constrained economy, as is easily seen by comparing (9) and (11) with $Z_t > 0$. However, the aggregate savings are predetermined when the investment level is chosen in the constructual arrangement. Hence, the investment levels must be the same between the two economies. It follows that market clearing mechanism in the loan market makes the rate of interest is lower in the information-constrained economy.

The agency cost is entirely incurred by lenders in such a manner that the interest rate earned by lending is below the case with complete information. In the case of a closed economy, there is no loss due to incomplete information, and there is only a difference in the distribution of profits between borrowers and lenders.

The contrast of this result to the result for the small open economy reveals the fact that openness or the possibility of capital flight is instrumental for the emergence of aggregate agency costs for a national economy in this setting. In a small open economy, there is a national loss due to under-investment arising from incomplete information. Wages at the stationary are lower than the first-best under the complete market. The agency costs are shared between lenders and entrepreneurs.

The issue of "financial repression" has been extensively discussed by McKinnon (1973, 1991) concerning many experiences in developing countries. Proposition 1 can be interpreted as an alternative explanation of the phenomena of financial repression from the standpoint of incomplete information rather than that of government regulations on interest rates.

We may illustrate the situation by a concrete functional form of $\pi(x)$.

Example 1

$$\pi(x) = \frac{x}{x+1} ,$$

This gives a convenient example because it satisfies all the conditions, $\pi(0) = 0$, and $\pi(x) \rightarrow 1$ as $x \rightarrow \infty$.

Assumption 2 implies

$$(1 - \beta)\theta > 1 ,$$

and under this assumption we obtain
at the stationary state.

$$\bar{x} = x = x^* = (1-\beta)\theta - 1 > 0 ,$$

$$r = \frac{\beta}{(1-\beta)[(1-\beta)\theta + 1]} < r^* = \frac{\beta}{(1-\beta)^2\theta} ,$$

$$Z = \frac{\beta}{(1-\beta)[(1-\beta)\theta + 1]} .$$

5. Two-Country Model with Identically Incomplete Information Structure

Let us embed the dynamic structure we have just introduced in the two-country general equilibrium model where incomplete information prevails in both countries. The two countries are identical except for the levels of initial wealth. In the beginning, the North is richer than the South, which is represented by the initial levels of capital. Let us denote variables of the North by those with superscript "*" and those of the South by those without the superscript. In the following, we discuss the effect of integration of capital market. In our framework, it makes no difference whether trade of goods is permitted, because the relative price of the capital goods to the consumption goods is a constant, β , throughout time.

Suppose that the capital markets of the two countries are integrated, so that the market clearing condition in the integrated capital market implies

$$(23) \quad (1-\beta)k_t + (1-\beta)k_t^* = \alpha x_{t+1} + \alpha x_{t+1}^* .$$

The IC condition and the IR condition of the South are

$$(24a) \quad \pi'(x_{t+1}) (\theta\beta - Z_t) = R_t ,$$

and

$$(24b) \quad Z_t = R_t \frac{x_{t+1}}{\pi(x_{t+1})} ,$$

respectively, where R_t is the world interest rate prevailing between period t and period $t+1$. Similarly, we have in the North

$$(25a) \quad \pi'(x_{t+1}^*) (\theta\beta - Z_t^*) = R_t ,$$

and

$$(25b) \quad Z_t^* = R_t \frac{x_{t+1}^*}{\pi(x_{t+1}^*)} .$$

where Z_t^* represents the payment for the good state of the project in the North.

Capital accumulates in both countries to satisfying

$$(26a) \quad k_t = \alpha \theta \pi(x_t) ,$$

and

$$(26b) \quad k_t^* = \alpha \theta \pi(x_t^*) .$$

The seven equations of (23), (24a), (24b), (25a), (25b), (26a) and (26b) define the world economy. Incorporating (24b) into (24a), collecting terms, we obtain

$$(27) \quad R_t = \theta\beta \frac{\pi'(x_{t+1}) \pi(x_{t+1})}{\pi(x_{t+1}) + \pi'(x_{t+1}) x_{t+1}} ,$$

Similarly, incorporating (25b) into (25a), rearranging terms, we obtain

$$(28) \quad R_t = \theta\beta \frac{\pi'(x_{t+1}^*) \pi(x_{t+1}^*)}{\pi(x_{t+1}^*) + \pi'(x_{t+1}^*) x_{t+1}^*} ,$$

If investment projects are undertaken in both countries,

(27) and (28) implies

$$(29) \quad x_t^* = x_t ,$$

since the R.H.S of (27) and (28) are decreasing function of the argument.

Substituting (26a) and (26b) into (23), we rewrite the capital market clearing condition by

$$(30) \quad (1-\beta)\theta[\pi(x_t) + \pi(x_t^*)] = x_{t+1} + x_{t+1}^* .$$

First, let us characterize a stationary equilibrium, in which all variables are held constant through time. Two stationary variables (x, x^*) satisfy the 45 degree line obtained from (29), and

$$(31) \quad (1-\beta)\theta\pi(x) + (1-\beta)\theta\pi(x^*) = x + x^* .$$

in the (x, x^*) space.⁴ As Figure 4I illustrates, there exists a unique interior stationary state $x = x_A$, and $x^* = x_A^*$, hence $x = x^*$. Concerning the stationary state, nothing is changed compared with the closed economy. At the stationary state capital never flows between countries.

Next let us examine dynamics. Incorporating (29) into (30), we derive a set of two dynamic equations:

$$(32a) \quad (1-\beta)\theta\pi(x_t^*) = x_{t+1}^* ,$$

and

$$(32b) \quad (1-\beta)\theta\pi(x_t) = x_{t+1} .$$

The whole dynamics is illustrated in Figure 4II. Suppose that the initial condition (x_0, x_0^*) is given in an interior. At period 1, the economy jumps to the intersection of the 45 degree line defined by equation (29) and the straight line which is diagonal to the 45 degree line of attainable pair of (x_1, x_1^*) that is defined by (x_0, x_0^*) and equation (30) in such a way that

$$(33) \quad x_1^* = -x_1 + (1-\beta)\theta[\pi(x_0) + \pi(x_0^*)] .$$

⁴See the Appendix [A] for the property of the curve of equation (31).

From point B, the economy converges to the stationary state A on the 45 degree line. Even in the transition, international capital movements never arise because the economy goes through the 45 degree line from period 1 on. The interior stationary state is dynamically stable. Any difference in the initial wealth levels never affects the long-run state.⁵

Proposition 2: The initial level of wealth will not affect the long-run equilibrium, if the degree of asymmetric information is identical between the North and South.

6. The Case Where Only the South Suffers from Incomplete Information

We depart from the two-country model above by assuming that only in the South capital market is incomplete. According to our setting, lenders can verify any kinds of profits of entrepreneurs in the North, while lenders can verify only the outcome of investment projects in the South. We could support this assumption by arguing that the legal system to enforce property rights, monitoring technology of banks, or accounting principle are less developed in many of the developing countries. In contrast, in developed countries institutional arrangements which enforce verification of earned profits are developed.

⁵A more gradual movement of transition would be derived in a model where each entrepreneur works and earns wage incomes in the first period of life.

Strictly speaking, we have three other stationary states in the present model, $(0, x_A)$, $(x_A, 0)$, and $(0, 0)$. For example, if and only if the initial condition satisfies $x_t = 0$ and $x_t^* > 0$, $(0, x_A)$ is eventually realized.

Start from the world under autarky. One can easily find that the resource allocations of the two countries are unchanged from those analyzed in Section 4. At a stationary state, the values of x_t^* and x_t (and hence k_t^* and k_t), accordingly the wage rates, are identical between the North and the South. As Proposition 1 shows, the interest rate is higher in the North than in the South. Suppose now that the capital markets are integrated.

The optimality condition of northern entrepreneurs who behave under complete information simply implies

$$(34) \quad \theta\beta\pi'(x_{t+1}^*) = R_t .$$

Except that (34) replaces (25a) and (25b), no other conditions are necessary from the equations of the two country models in Section 5.

First, let us focus attention on interior equilibria. Six equations (23), (24a), (24b), (26a), (26b) and (34) define interior equilibria. (24a), (24b) and (34) imply

$$(35) \quad \pi'(x_t^*) = \pi'(x_t) \frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)} .$$

By the implicit function theorem there exists a continuously differentiable function $\mu : [0, \infty) \rightarrow [M, \infty)$ such that

$$(36) \quad \pi'(\mu(x_t)) = \pi'(x_t) \frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)}$$

with three properties: (i) $\mu'(\cdot) > 0$ for any $x_t > 0$, (ii) $x_t^* > x_t$ for any $x_t > 0$, and (iii) $\mu(0) = M > 0$.⁶

Equation $x_t^* = \mu(x_t)$, which we call IA equation, decides the investment allocation between the two countries. Again, equation (30), which we call CM equation, describes the dynamics of investment that clears the world capital market. These equations fully describe the internal dynamic path of investment.

First, examine the stationary states. As in Figure 5I, $x^* = \mu(x)$ is drawn upward sloping curve UV and situated in the northwest region of the 45 degree line. Stationary states are obtained at the intersections of (31) that is drawn as curve $x_A x_A^*$

⁶First, we derive $\mu'(\cdot) > 0$ for any $x_t > 0$. Total differentiation of (35) yields

$$\frac{d\mu x_t}{dx_t} = \frac{[\pi'(x_t)]^2 [\pi'(x_t)x_t - \pi(x_t)] + [\pi(x_t)]^2 \pi''(x_t)}{\pi''(x_t^*) [\pi(x_t) + x_t \pi'(x_t)]^2} > 0 .$$

since $\pi''(\cdot) < 0$ and $\pi'(x_t)x_t - \pi(x_t) < 0$.

Second, we prove that $\mu(x_t) > x_t$ for any $x_t > 0$. Suppose not, to the contrary that, $x_t^* \leq x_t$. From (35) and the fact that

$$\frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)}$$

is strictly less than unity for any $x_t > 0$, this is valid only if

$$\pi'(x_t^*) < \pi'(x_t) ,$$

which is a contradiction.

Third, we prove that $\mu(0) > 0$. Using l'Hopital's rule,

$$\lim \frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)} = \lim \frac{\pi'(x_t)}{2\pi'(x_t) + x_t \pi''(x_t)} = 0.5 .$$

Hence, (31) implies, at $x = 0$,

$$\pi'(x^*) = 0.5\pi'(0) < \pi'(0) .$$

Therefore, x^* must be positive because $\pi'(\cdot)$ is decreasing. Q.E.D.

and $x^* = \mu(x)$. As Figure 5II illustrates, there may be a single stationary state like I, or internal stationary states (I and I'). If $\mu(0)$ is too large, there may be no interior stationary state.

Denoting variables in the integrated world by those with subscript "I" in contrast to subscript "A" that corresponds to the state of autarky, we derive three results concerning the stationary states. Those propositions hold regardless of whether stationary states may be unique or multiple.

Proposition 3

In any interior stationary states, the per-capita level of investment (accordingly of capital) becomes higher than the autarky level for the North, and lower than the autarky level for the South.

$$x_I < x_A = x_A^* < x_I^* \text{ for any interior stationary states.}$$

This proposition is obvious from Figure 5I or 5II.

Proposition 4

Capital flows from the South to the North.

Proof. It is sufficient to prove $(1-\beta)\theta\pi(x_I^*) < x_I^*$. Suppose to the contrary that

(i) $(1-\beta)\theta\pi(x_I^*) \geq x_I^*$.

Denote

$$(1-\beta)\theta \frac{\pi(x)}{x}$$

by $\eta(x)$. $\eta(x)$ is a decreasing function of x , and $\eta(x_I^*) \geq 1$.

From Proposition 3, $x_I < x_I^*$, so that we obtain

$$(ii) \quad \eta(x_I) > \eta(x_I^*) \geq 1.$$

(iii) implies that

$$(1-\beta)\theta[\pi(x_I^*) + \pi(x_I^*)] > x_I + x_I^* ,$$

which violates (31), which is a contradiction. Q.E.D.

Proposition 5

The rate of interest R at the stationary state of the integrated world is between the autarky interest rates. (The autarky interest rate of the North is higher than that of the South.)

$$r_A < R < r_A^* ,$$

Proof. The R.H.S of (21) is strictly decreasing in x_{t+1} .⁷

Since one has $x_I < x_A$ from Proposition 2, one obtains $r_A < R$ by comparing (21) and (27).

Similarly, comparing (22) and (34) and using the fact of $x_A^* < x_I^*$ that is implied by Proposition 3, one obtains $R < r^*$. Q.E.D.

Capital moves from the South to the North. Capital movements take place because southern lenders would substitute their funds

⁷We derive

$$\frac{dr}{dx} = \frac{\pi''\pi^2 - \pi'^2(\pi - \pi'x)}{(\pi'x + \pi)^2} < 0 ,$$

since $\pi - x\pi' > 0$.

from domestic to foreign lending for a higher rate of return. Integration of the two capital markets leads to an equalization of the interest rates across countries. Southern lenders face a higher interest rate, and northern lenders a lower rate. The changes in interest rates reduce the investment level of the South, and increase that of the North. Thus, "capital flight" triggers a unwelcome discrepancy of the investment levels between the two countries.

Propositions 3 through 5 are not due to the particular specification of information structure. So long as we assume two countries which differ in the extent of asymmetric information, we obtain similar results. Alternatively, Sakuragawa (1993) derives similar propositions by assuming two countries which differ in the monitoring costs under the setting of Townsend's (1979) costly-state-verification.

Finally, in order to illustrate the property of the dynamics of our model, by specifying $\pi(\cdot)$ as in Example 1 in Section 4, we present the following Proposition.

Proposition 6

Suppose that $\pi(x) = x/(1+x)$, then there is a unique interior stationary state if $(1-\beta)\theta > \sqrt{2}$, and there are two interior stationary states if $(1-\beta)\theta \leq \sqrt{2}$.

The proof proceeds in two steps. First, we show that $\mu(x)$ is strictly convex. Second, we show that, $\mu(0) < x_A$ if $(1-\beta)\theta$

$> \sqrt{2}$, and $\mu(0) \geq x_A$ if otherwise.⁸ (See the Appendix [B] for the formal proof.) Both cases are illustrated in Figure 6I and 6II.

Examination of corner solutions is necessary, because the dynamic process may eventually lead to a corner solution even if the initial condition is interior. From (35), $\mu(\cdot)$ is strictly increasing and $\mu(0) = \sqrt{2} - 1$. Allowing for the possibility of corner solutions, investment allocation (IA) condition (35) is replaced by three different investment allocation conditions:
 Case (a): If $x_t^* > \sqrt{2} - 1$ and $x_t > 0$ (namely, in the internal positive quadrant)

$$(37a) \quad \pi'(x_t^*) = \pi'(x_t) \frac{\pi(x_t)}{\pi(x_t) + \pi'(x_t)x_t} ,$$

as drawn by curve UV in figure 6I or 6II.

Case (b): If $x_t^* \leq \sqrt{2} - 1$ and $x_t = 0$ (on the segment OU)

$$(37b) \quad \pi'(x_t^*) \geq \pi'(x_t) \frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)} ,$$

Case (c): If $x_t^* > \sqrt{2} - 1$ or $x_t = 0$.⁹

⁸As long as $\mu'''(\cdot) > 0$, we can derive the same qualitative result as Proposition 6 for a more general form of the probability function.

⁹Strictly speaking, (37b) applies also to the case where $x_t^* \leq \sqrt{2} - 1$ and $x_t > 0$, (37c) where $x_t^* = 0$. However, as will be seen below, such cases would never arise if the initial condition is given in an interior.

$$(37c) \quad \pi'(x_t^*) \leq \pi'(x_t) \frac{\pi(x_t)}{\pi(x_t) + x_t \pi'(x_t)} .$$

In Case (a), the loan market clearing condition (23) is unchanged. Let the inverse function of (36) be denoted by the inverse function

$$(39) \quad x_t = \mu^{-1}(x_t^*) ,$$

where $\mu^{-1}(\cdot) > 0$ and $\mu^{-1}(\sqrt{2} - 1) = 0$.

The whole economy in Case (a) is composed of the two difference equations:

$$(40a) \quad (1-\beta)\theta[\pi(\mu^{-1}(x_t^*)) + \pi(x_t^*)] = \mu^{-1}(x_{t+1}^*) + x_{t+1}^* ,$$

and

$$(40b) \quad (1-\beta)\theta[\pi(x_t) + \pi(\mu(x_t))] = x_{t+1} + \mu(x_{t+1}) .$$

In Case (b), (30) reduces to (33a); in Case (c), (30) reduces to (33a).

The whole system is illustrated in Figures 7I and 7II in the space of (x, x^*) . We consider two distinct figures according to whether (i) $(1-\beta)\theta > \sqrt{2} - 1$ (Figure 7I) or (ii) $(1-\beta)\theta < \sqrt{2} - 1$ (Figure 7II). Suppose that the initial condition is given in an interior, such that $x_0 > 0$ and $x_0^* > \sqrt{2} - 1$. The formal analysis of the dynamic process is left to the Appendix [C].

Let us consider Figure 7I first. Inside $x_A x_A^*$, capital accumulation takes place in the northeast direction; outside $x_A x_A^*$ in the southwest direction. Given any interior initial condition like (x_0, x_0^*) , the world economy jumps to a point like S in the next period, and afterwards converges to the new stationary state I, the unique interior stationary state is globally stable. Accordingly, in the North the wage rate increases, while in the

South it decreases. Integration of the capital markets widens rather than narrows the wage discrepancy between the two countries at the new stationary state. The rental price of capital β will be the same regardless of economic environments.

Suppose that the capital markets are integrated when the world is at the stationary state under autarky (x_A, x_A^*) . In the next period, the economy jumps to a point like T, and afterwards converges to I that is given by the intersection of curve $x_A x_A^*$ and the curve $x^* = \mu(x)$. At I, x^* is higher, and x is smaller than at A, and capital continues to flow from the South to the North during the convergence process because the dynamic process continues along $x^* = \mu(x)$.

Next, consider Figure 7II. Here the initial wealth level of the South affects the long-run stationary equilibrium. There are two internal equilibria. We call the high equilibrium H whose coordinate is (x_H, x_H^*) , and the low equilibrium L, whose coordinate is denoted by (x_L, x_L^*) . Let us define a curve PQ through L, which describes the locus of the initial wealth pair x_0, x_0^* such that

$$(41a) \quad x_L + x_L^* = (1-\beta) \theta [\pi(x_0) + \pi(x_0^*)] .$$

PQ divides the (x, x^*) into two regions.¹⁰ If the initial condition of (x_0, x_0^*) satisfies,

$$(41b) \quad x_L + x_L^* < (1-\beta)\theta[\pi(x_0) + \pi(x_0^*)] .$$

(x_0, x_0^*) is located in the northwest of PQ. Conversely, if the initial condition satisfies,

$$(41c) \quad x_L + x_L^* > (1-\beta)\theta[\pi(x_0) + \pi(x_0^*)] ,$$

(x_0, x_0^*) is located northeast of PQ. If the initial pair of wealth is located in the northeast of PQ, the economy can afford to jump to a point on LV and eventually converges to H. If the initial pair of wealth is in the southwest of PQ at period 1 the economy has to jump to some point UL and eventually converges to $(0, x_A^*)$ because UL is outside curve $x_A x_A^*$ and, accordingly, capital decumulates along $x = \mu(x^*)$.

If the South is not too poor relative to the North at the initial moment, the world converges to the state H. On the other hand, if the South is sufficiently poor at the initial moment, liberalization of the domestic capital market to the world capi-

¹⁰PQ is downward sloping and convex to the origin because from (41a), implicitly we can derive

$$\frac{dx_0^*}{dx_0} = - \frac{\pi'(x_0)}{\pi'(x_0^*)} < 0 ,$$

and

$$\frac{d^2x_0^*}{dx_0^2} = - \frac{\pi''(x_0)}{\pi'(x_0^*)} > 0 .$$

tal market drives the South down to the worst state of the underdevelopment trap where $x = 0$ in the long-run.¹¹

In the integrated world, if the initial wealth gap between the North and the South is too large, the North's demand for capital is sufficiently large relative to the South's demand. This tends to increase the world interest rate to the level at which southern entrepreneurs find it unprofitable to invest in their projects, which in turn, makes it advantageous for southern lenders to invest all of their funds abroad. In such a case, "complete" capital flight would take place, which leads to a collapse of the South, that is, to a state with zero domestic production, zero saving, and hence zero national income. The effects of the difference in the information structure are aggravated if there is a substantial gap of wealth at the moment of integration.

In the closed economy, on the other hand, this threshold property never arises. Under autarky southern lenders are obliged to lend their funds to domestic entrepreneurs who would offer a smaller interest rate because the lenders have no any other investment opportunities. However, if the capital market is opened, southern lenders have a chance to lend to the North, which triggers the disastrous capital flight. This is an example of the result by Hart (1975) that the introduction of another market may harm the welfare if the market structure is incomplete.

¹¹A similar dynamic pattern in an information constrained economy is seen in Sakuragawa (1992), and in a model of nonlinear savings behavior as seen in Fukao and Hamada (1992).

Integration of the world capital market eventually deteriorates the welfare of the South.

We may recall historical examples. From the late 1970s to the early 1980s, a sizeable amount of domestic deposits in the Latin American countries flowed to banks in the United States, impeding economic development in the Latin American countries. Most oil money earned by the OPEC countries was invested in developed countries, and only a fraction of it was invested in developing countries. These phenomena are most naturally understood in the framework of incomplete information.

The analysis in this section bears a significant policy implication on the timing of capital market deregulation for developing countries. The government of the South may as well take account of the possibility that liberalization will make the domestic country worse off. Even in the case in which liberalization may eventually be desirable to the South for some reasons,¹² timing of integration is crucial. Liberalization in the capital market in an early stage of development could possibly drive the domestic country down to the underdevelopment trap. If the South is in developing process, too hasty liberalization of capital markets is not desirable to the North. The North can exploit the benefits of integration by capital imports after the economy of the South has passed a stage of "take off."

¹²Even the higher equilibrium H , wages in the South are lower than in autarky. However, since the rate of interest in the South is higher, there may be incentives for lenders to liberalize capital market.

7. Concluding Remarks

We have presented a simple dynamic model of North-South lending with the possibility of capital flight. In a dynamic version of the two-country model by Gertler and Rogoff, in which information structures of the North and the South are identical, the world economy converges to a harmonious steady state regardless of the initial wealth levels of the North and the South. This conclusion is dramatically altered when we depart from their assumption and introduce a difference in information structures between the North and the South. Under the integrated world capital market, capital moves from the South to the North, and the South may be trapped into an underdevelopment trap if its initial wealth is below a critical level. Thus the timing of capital liberalization becomes a crucial issue.

We have seen that the difference in information structure exerts a serious and long lasting impact on the development process. The creation of infrastructures including information networks, a legal system, an accounting system, and the development of monitoring skill are thus crucially important. The fact that hegemonies in the world economy always had well developed capital markets may have been more than mere coincidence.

Incidentally, a similar analysis with similar economic implications could be conducted using the macroeconomic model studied by Williamson (1987), and Greenwood and Williamson (1989) based on the costly state verification approach originated by Townsend (1979) and others. (For that type of analysis of international capital movements, see Sakuragawa 1993).

If there is a difference in information structures, initial conditions in wealth or in income do indeed matter. The liberalization of international capital flows that would benefit a developing country at later stages could push the country down to stagnation and underdevelopment trap if effected prematurely.

Recent studies emphasize the role of human capital and externalities to explain the lack of sufficient capital flows from the North to the South (see, for example, Lucas 1990; Barro, Mankiw and Sala-i-Martin 1992). In contrast, we have emphasized the role of asymmetric information and incentive compatibility. We do not consider the two approaches as necessarily contradictory because legal, accounting, and organizational structure are important ingredients of human capital as public goods.

Our analysis is based on many simplifying assumptions, and its policy implications should be taken with caveats. There are several ways to generalize the model so as to be more applicable to practical policy questions. For example, an extension of this model by introducing commodity trade will be able to shed light on the issue of the order of liberalization in various markets of developing economy (McKinnon 1991, Sakuragawa 1993). A more realistic and gradual time path could be studied in a model where an entrepreneur earns wage incomes in the first period of her life. Also, the effect of taxation on international resource allocation (e.g. Frenkel, Razin and Sadka, 1991) will be viewed from a new angle if one superimposes upon it the information structure of incomplete capital market that is considered in this paper. We hope that the simple analysis given in this paper will

be a preliminary step to clarify the important relationship between incomplete information and international credit market.

Appendix

[A] Properties of the capital accumulation CA curve, equation (31)

Total differentiation of (31) yields

$$(i) \quad \frac{dx^*}{dx} = - \frac{(1-\beta)\theta\pi'(x) - 1}{(1-\beta)\theta\pi'(x^*) - 1} ,$$

where we assume

$$(ii) \quad (1-\beta)\theta\pi'(x^*) \neq 1 .$$

First, consider the region of $x \in [0, x_A]$. At $x = 0$, there are two x^* 's which satisfy (31), 0 and x_A^* . Ignoring the trivial solution, $x^* = 0$, we derive, at $x = 0$,

$$(iii) \quad \frac{dx^*}{dx} > 0$$

since $(1-\beta)\theta\pi'(0) > 1$ and $(1-\beta)\theta\pi'(x_A^*) < 1$ are satisfied. At any $x \in (0, x_A)$, it follows that

$$(iv) \quad (1-\beta)\theta\pi(x) - x = x^* - (1-\beta)\theta\pi(x^*) > 0$$

from (31). Hence, $(1-\beta)\theta\pi(x^*) < 1$ is always satisfied over $x \in (0, x_A)$. Of course, Assumption (ii) is satisfied.

Obviously, there exists a unique x^+ , such that $0 < x^+ < x_A$, satisfying

$$(v) \quad (1-\beta)\theta\pi'(x^+) = 1 .$$

We can distinguish between three cases, (a) $dx^*/dx > 0$ for $0 < x < x^+$, (b) $dx^*/dx = 0$ at $x = x^+$, and (c) $dx^*/dx < 0$ for $x^+ < x < x_A$, since $\pi(\cdot)$ is increasing and strictly concave.

Differentiation of the L.H.S. of (i) with respect to x gives since $\pi(\cdot)$ is strictly concave. The curve (31) is strictly concave over $x \in (0, x_A)$.

$$(vi) \quad \frac{d^2x^*}{dx^2} = - \frac{(1-\beta)\theta\pi''(x)}{(1-\beta)\theta\pi'(x^*) - 1} < 0 ,$$

Second, consider the region of $x^* \in [0, x_A^*]$. From the symmetric property of x and x^* in (31), qualitatively the same analysis will follow.

Finally, we check, at $x = x_A$,

$$(vii) \quad \frac{dx^*}{dx} = -1 ,$$

since $x = x^*$.

[B] The proof of Proposition 6.

(35) becomes

$$(i) \quad \frac{1}{(1 + x_t^*)^2} = \frac{1}{(1 + x_t)(2 + x_t)} .$$

Total differentiation yields

$$\frac{dx_t^*}{dx_t} = \frac{2x_t + 3}{2x_t^* + 2} > 0$$

and

$$\frac{d^2x_t^*}{dx_t^2} = \frac{1}{x_t^* + 1} > 0 .$$

From (i), at $x_t = 0$, $x_t^* = \sqrt{2} = 1$. From (i), $\mu(0) = \sqrt{2} - 1$, and from Example 1 in Section 4, $x_A^* = (1-\beta)\theta - 1$. The intersection is unique if $\mu(0) < x_A^*$, that is, $\sqrt{2} < (1-\beta)\theta$, and the intersections are two if $\mu(0) > x_A^*$, that is, $\sqrt{2} > (1-\beta)\theta$. Q.E.D.

[C] The dynamics of x_t and x_t^*

First, examine the dynamics of x_t . From the above analysis, we can define $m(x_{t+1})$ by

$$m(x_{t+1}) = x_{t+1} + \mu(x_{t+1}) \text{ if (a) } x_{t+1}^* > \sqrt{2} - 1 \text{ and } x_{t+1} > 0 .$$

It is easy to see

$$m'(\cdot) = 1 + \mu'(\cdot) > 1 ,$$

$$m''(\cdot) = \mu''(\cdot) > 0 , \text{ and}$$

$$m(0) = \mu(0) = \sqrt{2} - 1 .$$

Next, define $n(x_t)$ by

$$n(x_t) = (1-\beta)\theta[\pi(x_t) + \pi(\mu(x_t))] \text{ if (a) } x_t^* > \sqrt{2} - 1 \text{ and } x_t > 0 .$$

Again it is easy to see

$$n'(\cdot) = (1-\beta)\theta[\pi' + \pi'\mu'] > 0 ,$$

$$n''(\cdot) = (1-\beta)\theta[\pi'' + \pi''\{\mu'\}^2 + \pi'\mu''] < 0 , \text{ and}$$

$$n(0) = (1-\beta)\theta(\sqrt{2} - 1)/\sqrt{2} .$$

We have two cases according to whether (I) $(1-\beta)\theta > \sqrt{2} - 1$

(Figure 8I) or (II) $(1-\beta)\theta < \sqrt{2} - 1$ (Figure 8II). In case (I),

if $x_0 (> 0)$ is below (above) the stationary state x_L , x_t in-

creases (decreases). In case (II), the system bifurcates at x_L .

If x_t is below x_L or above x_H , x_t decreases, while if x_t is below x_H and above x_L , x_t increases.

Next examine the dynamics of x_t^* . Define $m^*(x_{t+1})$ by

$$m^*(x_{t+1}^*) = x_{t+1}^* + \mu^{-1}(x_{t+1}^*) \text{ if (a) } x_{t+1}^* > \sqrt{2} - 1 \text{ and } x_{t+1}^* > 0 ,$$

$$= x_{t+1}^* \text{ if (b) } x_t^* \leq \sqrt{2} - 1 \text{ and } x_t = 0 ,$$

$$\text{or (c) } x_t^* > \sqrt{2} - 1 \text{ and } x_t = 0 .$$

It is easy to see

$$m^{*'}(\cdot) = 1 + \mu^{-1'}(\cdot) > 1 \text{ if (a), and } = 1 \text{ if (b) or (c),}$$

$$m^{*''}(\cdot) = \mu^{-1''}(\cdot) > 0 \text{ if (a),}$$

$$m^*(\sqrt{2} - 1) = \sqrt{2} - 1 .$$

Alternatively, define $n^*(x_t)$ by

$$n^*(x_t) = (1-\beta)\theta[\pi(x_t) + \pi(\mu^{-1}(x_t))]$$

$$\begin{aligned}
& \text{if (a) } x_t^* > \sqrt{2} - 1 \text{ and } x_t > 0 , \\
& = (1-\beta) \theta \pi(x_t) \text{ if (b) } x_t^* \leq \sqrt{2} - 1 \text{ and } x_t = 0 , \\
& \text{or (c) } x_t^* > \sqrt{2} - 1 \text{ and } x_t = 0 .
\end{aligned}$$

It is easy to see

$$\begin{aligned}
n^{*\prime}(\cdot) &= (1-\beta) \theta [\pi' + \pi' \mu^{-1}] > 0 \text{ if (a),} \\
&= (1-\beta) \theta \pi' > 0 \text{ if (b) or (c).} \\
n^{*\prime\prime}(\cdot) &= (1-\beta) \theta [\pi'' + \pi'' \{\mu^{-1}\}^2 + \pi' \mu''^{-1}] < 0 \text{ if (a),} \\
&= (1-\beta) \theta \pi'' < 0 \text{ if (b) or (c),} \\
n^*(\sqrt{2} - 1) &= (1-\beta) \theta (\sqrt{2} - 1) \sqrt{2} .
\end{aligned}$$

For the case of (a) and (b), we have two cases according to whether (I) $(1-\beta)\theta > \sqrt{2}$ (Figure 9I) or (II) $(1-\beta)\theta < \sqrt{2}$ (Figure 9II). In case (I), if x_t^* is below (above) x^* , x_t^* is increasing (decreasing). In case (II) if x_t^* is below x_A^* or above x_L^* and below x_H^* , x_t^* is increasing, while if x_t^* is above x_A^* and x_L^* or above x_H^* , x_t^* is decreasing.

On the other hand, for the case of (b) and (c), the whole dynamics is the same as in Figure 3.

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FIGURE 1

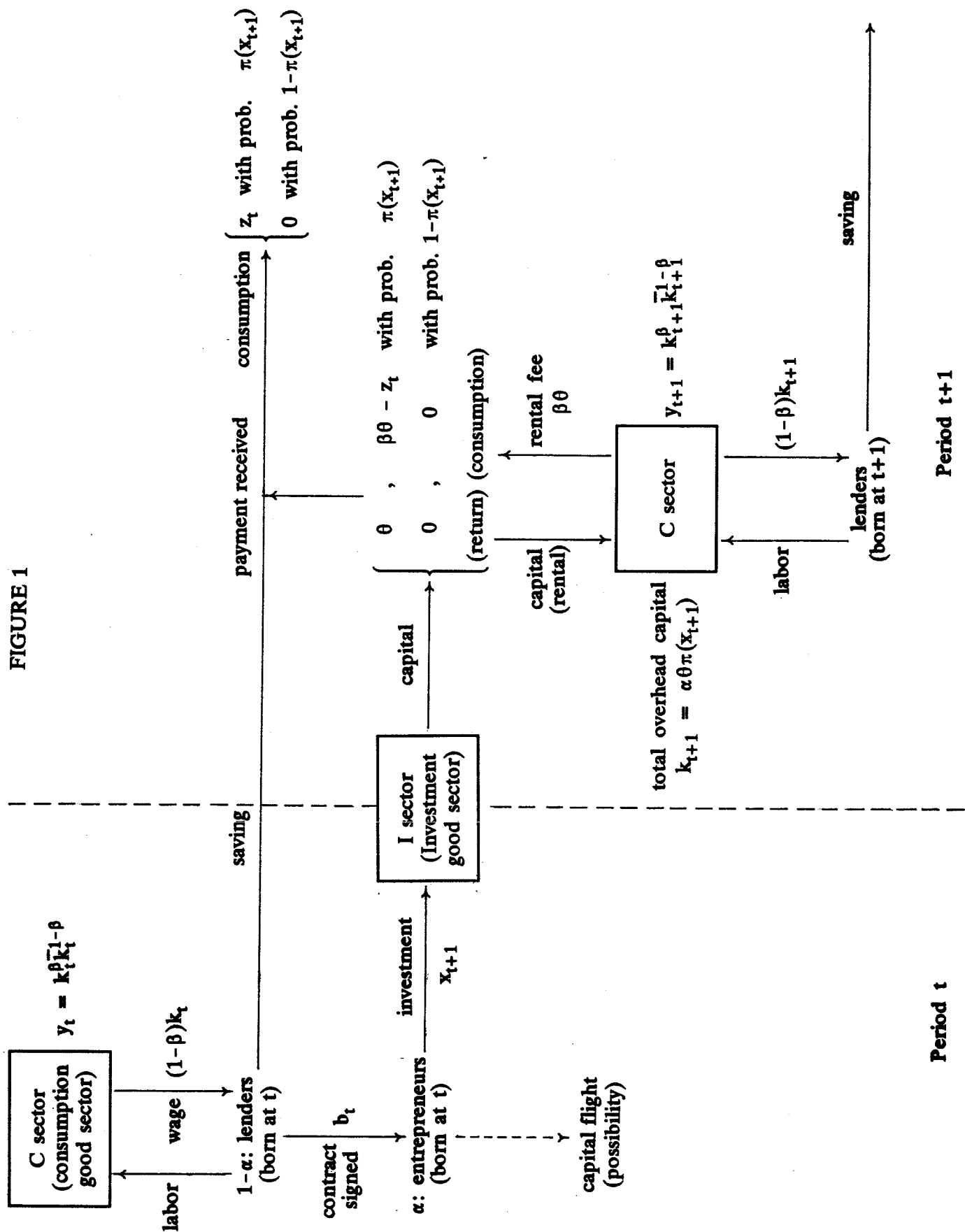


FIGURE 2

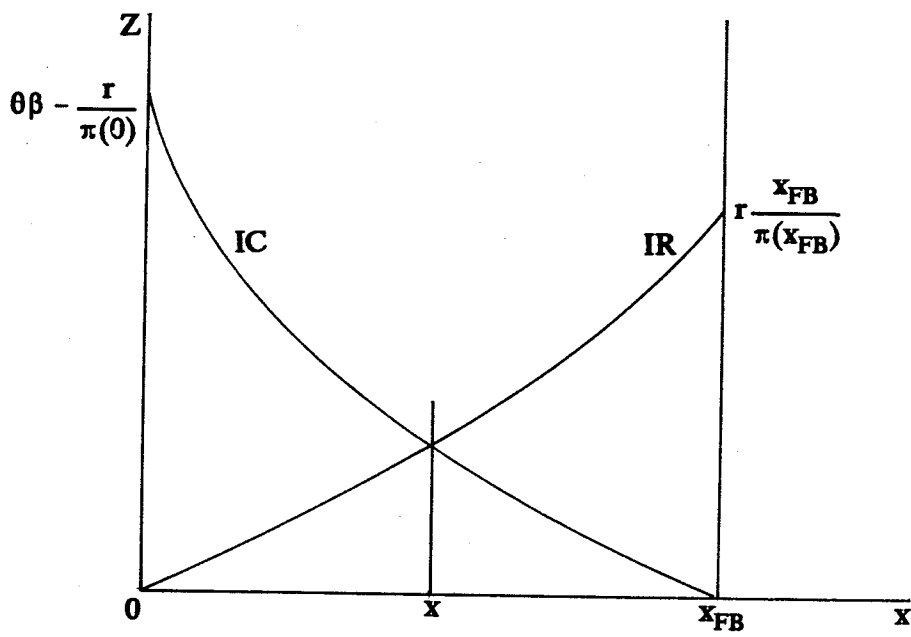


FIGURE 3

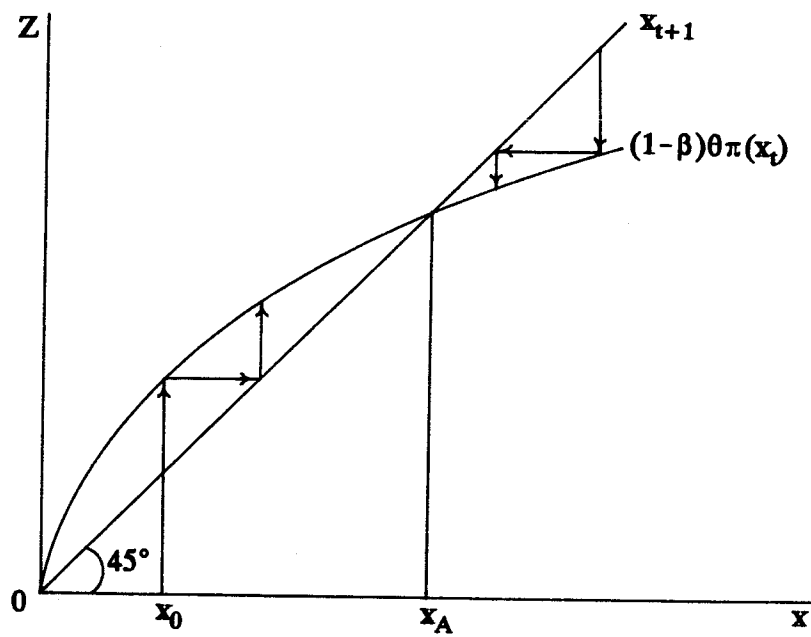


FIGURE 4I

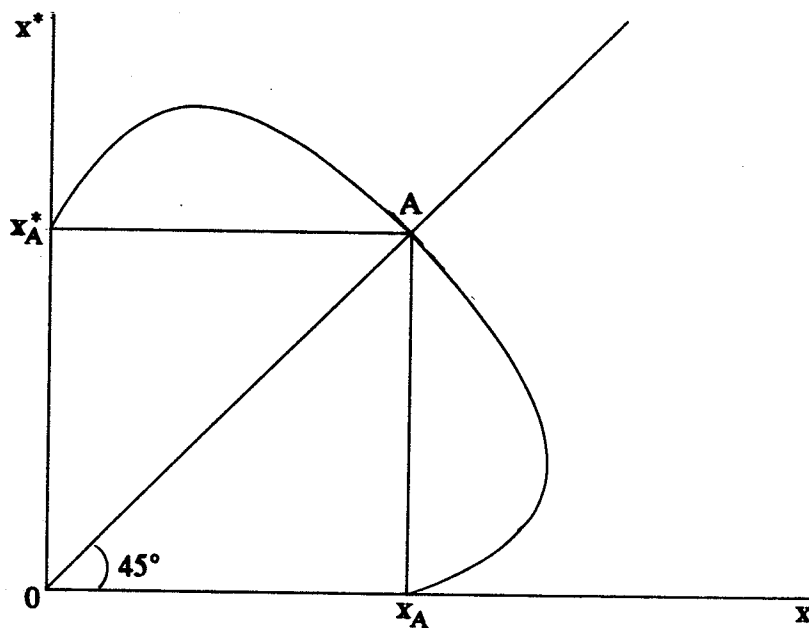


FIGURE 4II

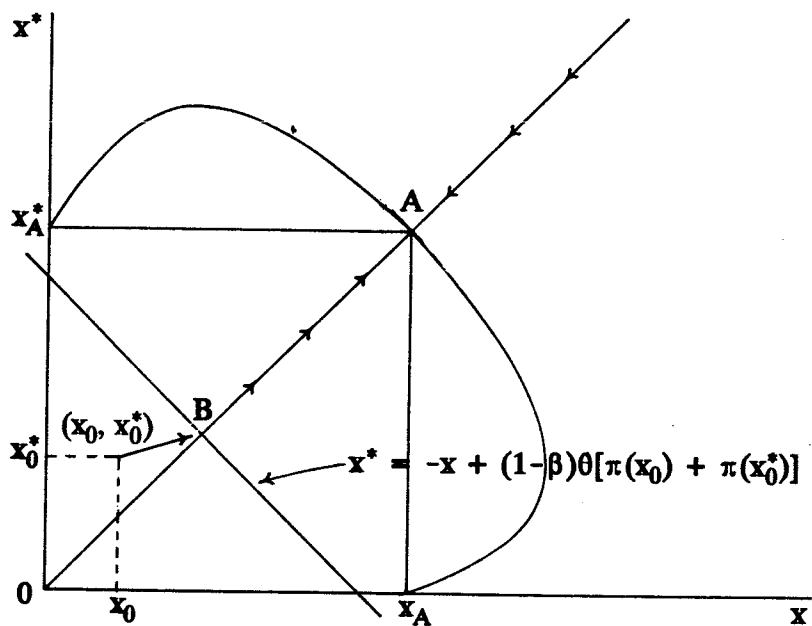


FIGURE 5I

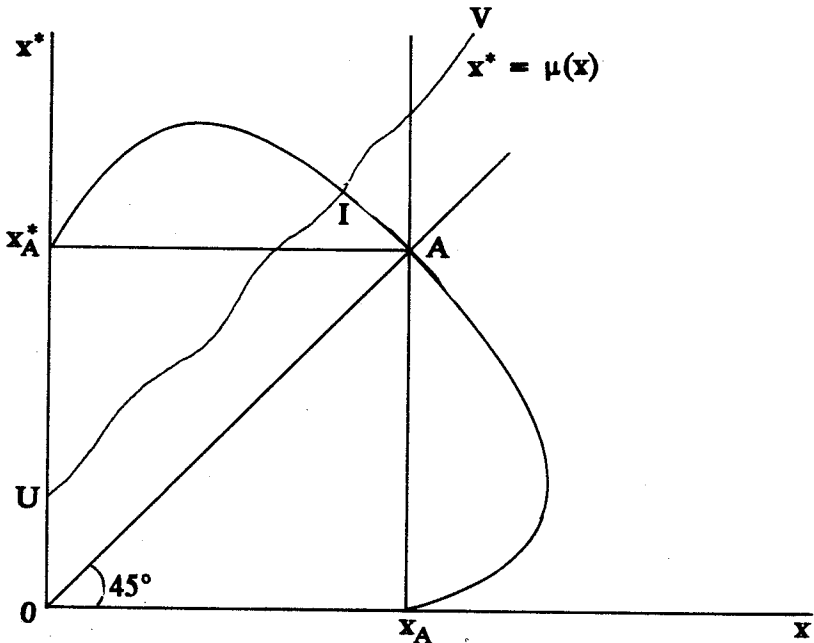


FIGURE 5II

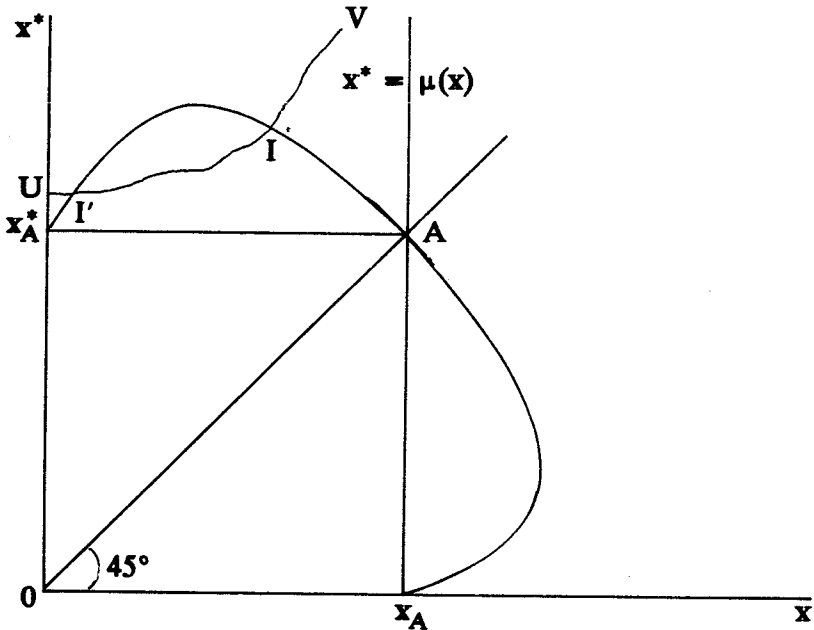


FIGURE 6I (i) $(1-\beta)\theta > \sqrt{2}$

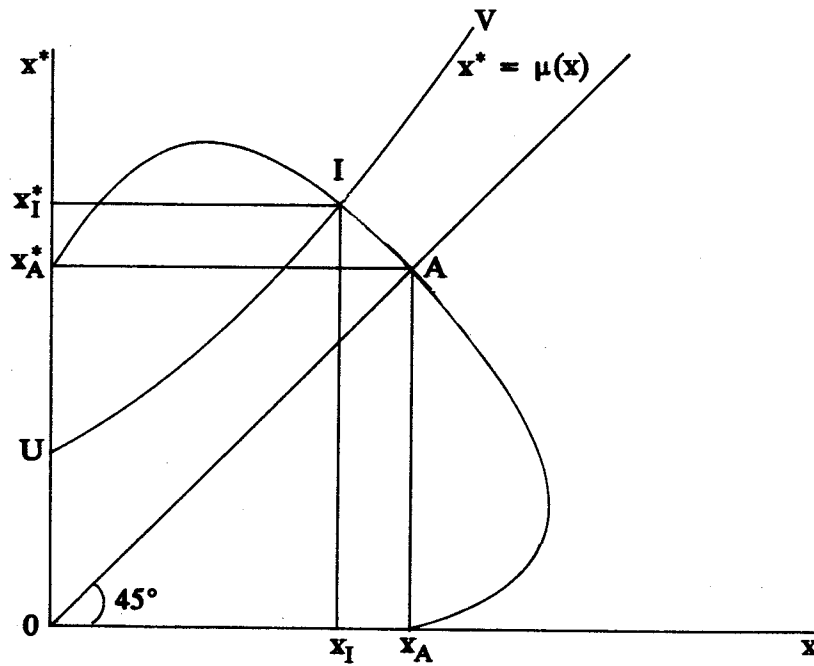


FIGURE 6II (ii) $(1-\beta)\theta < \sqrt{2}$

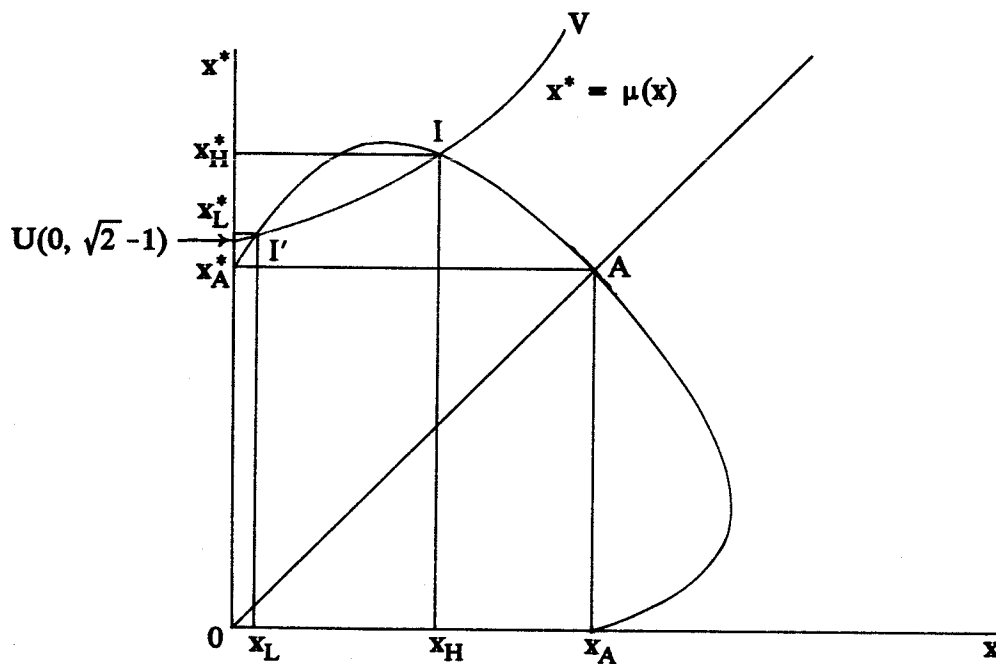


FIGURE 9I (i) $(1-\beta)\theta > \sqrt{2}$

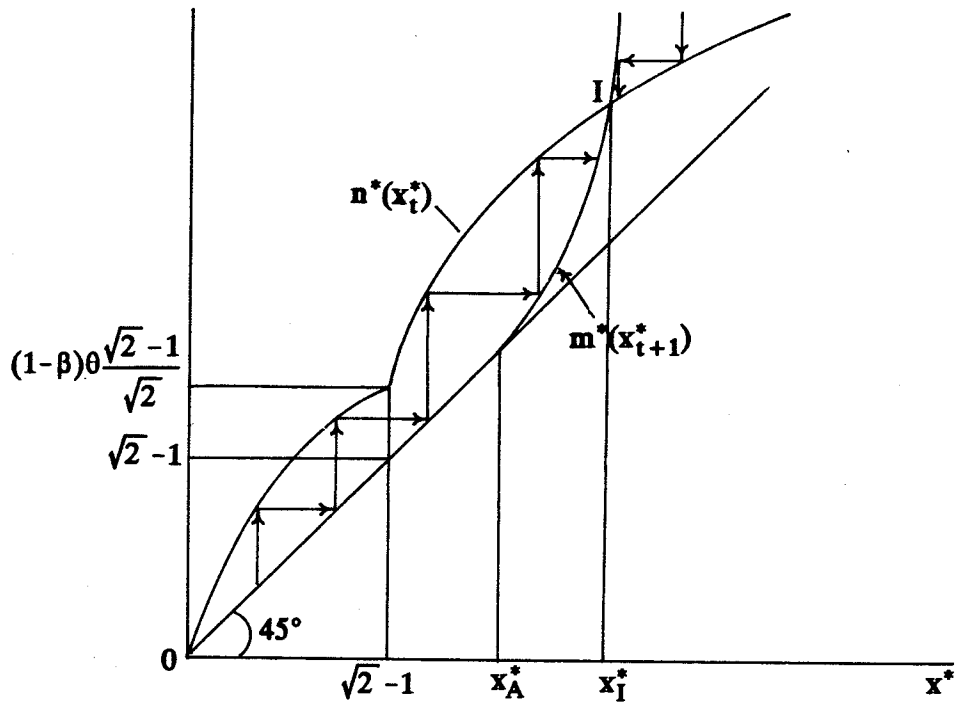


FIGURE 9II (ii) $(1-\beta)\theta < \sqrt{2}$

