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ECONOMIC GROWTH CENTER

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PUBLIC FINANCE IN MODELS OF ECONOMIC GROWTH

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Abstract

The recent literature on endogenous economic growth allows for effects of fiscal policy on long-term growth. If the social rate of return on investment exceeds the private return, then tax policies that encourage investment can raise the growth rate and levels of utility. An excess of the social return over the private return can reflect learning-by-doing with spillover effects, the financing of government consumption purchases with an income tax, and monopoly pricing of new types of capital goods. Tax incentives for investment are not called for if the private rate of return on investment equals the social return. This situation applies in growth models if the accumulation of a broad concept of capital does not entail diminishing returns, or if technological progress appears as an expanding variety of consumer products.

In growth models that incorporate public services, the optimal tax policy hinges on the characteristics of the services. If the public services are publicly-provided private goods, which are rival and excludable, or publicly-provided public goods, which are non-rival and non-excludable, then lump-sum taxation is superior to income taxation. Many types of public goods are subject to congestion, and are therefore rival but to some extent non-excludable. In these cases, income taxation works approximately as a user fee and can therefore be superior to lump-sum taxation. In particular, the incentives for investment and growth are too high if taxes are lump sum. We argue that the congestion model applies to a wide array of public expenditures, including transportation facilities, public utilities, courts, and possibly national defense and police.

The recent literature on endogenous economic growth has provided some insights into why countries grow at different rates over long periods of time. In some of these models, the government's choices of tax rates and expenditure levels influence the long-term growth rates. The present paper discusses these types of fiscal effects within a variety of models that can generate long-term growth endogenously.

The models that we consider assume a closed economy and share a common perspective with respect to household choices on consuming and saving. We begin with the standard model of the representative, infinite-lived household, which seeks to maximize overall utility, as given by

$$(1) \quad U = \int_0^{\infty} u(c)e^{-\rho t} dt$$

where c is consumption per person and $\rho > 0$ is the constant rate of time preference.

Population is constant and the momentary utility function is given by

$$(2) \quad u(c) = c^{1-\theta}/(1-\theta), \quad \theta > 0$$

so that marginal utility has the constant elasticity $-\theta$. Households hold the quantity $a(t)$ of real assets (denominated in units of consumables) in the form of claims on physical or human capital or internal loans. The real rate of return on assets, in units of future consumables per unit of current consumables per unit of time, is $r(t)$. Thus, the household's budget constraint determines the change over time in assets to be

$$(3) \quad \dot{a} = ra - c$$

(The term, $r \cdot a$, includes returns on human capital—that is, labor income—as well as returns on physical capital.)

As is well known, the first-order condition for the maximization of utility in equation (1) subject to the budget constraint in equation (3) requires the growth rate of consumption per person, denoted γ_c , to be

$$(4) \quad \gamma_c = \dot{c}/c = (1/\theta)(r-\rho)$$

Equivalently, the real rate of return must satisfy

$$(5) \quad r = \rho + \theta \cdot \gamma_c$$

That is, r equals the required premium in future consumption over current consumption. This premium exceeds the rate of time preference, ρ , by a term that equals the product of γ_c and θ , the reciprocal of the intertemporal elasticity of substitution. The higher θ the more the household must be compensated for deferring consumption. The upward-sloping line in Figure 1 plots the relation between the interest rate and the growth rate implied by the preference relation in equation (5). For our purposes, the positive slope of this line is the main content of the standard model of household saving over an infinite horizon.

Production with Constant Returns to Broad Capital

The simplest model that generates growth endogenously is one where production, y , is linear in a broad concept of capital, k ; that is, the "Ak model":

$$(6) \quad y = Ak$$

Because the economy is closed, $k=\dot{a}$ holds in equilibrium.

Producers that can borrow and lend at the rate $r(t)$ seek to maximize the present value of net revenues,

$$(7) \quad \text{Net revenues} = \int_0^{\infty} \{(Ak - \eta i) \cdot \exp[-\int_0^t r(s) ds]\} dt$$

where η is the constant cost of capital in units of consumables and i is investment, measured as a quantity of capital goods purchased. (It is convenient for the subsequent analysis not to normalize η to unity.) With zero depreciation, the change in the capital stock is given by $\dot{k} = i$. The first-order optimization condition entails

$$(8) \quad r = A/\eta$$

because A/η is the constant rate of return on investment (with the division by η converting units of capital into units of consumables). It is the private rate of return on investment that enters into equation (8), but the social return equals the private return in this model.

The horizontal line in Figure 1 shows the relation between the interest rate and the growth rate implied by the production condition in equation (8). The line is horizontal because the growth rate does not affect the rate of return on investment in this model.

Combining equations (5) and (8) yields the solution for the constant growth rate of per capita consumption

$$(9) \quad \gamma_c = [(A/\eta) - \rho]/\theta$$

This result can be read off the intersection of the two lines in Figure 1. The growth rate is positive if $(A/\eta) > \rho$. (We assume the condition, $\rho > (A/\eta)(1-\theta)$, which ensures that utility is bounded.). The growth rates of k and y can be shown, using the budget constraint and the appropriate transversality conditions, to equal the growth rate of c , which is given in equation (9). There is no transitional dynamics and the economy is always in a position of constant, steady-state growth.¹ The outcomes are Pareto optimal in this model because the social return on investment equals the private return.

A number of endogenous growth models in the literature amount to a theory of A/η , which is the private rate of return on investment. In some of these models, considered in the following sections, the social return differs from the private return, so that the decentralized equilibrium is not Pareto optimal. Also, some models allow for a transition to the steady-state growth path, whereas others have no transition.

Learning-by-Doing Models with Spillovers

The Arrow (1962)–Romer (1986) learning-by-doing model assumes that production per worker, y , depends on own capital per worker, k , and also on the average (or in some versions the aggregate) of the capital stocks of other producers, \bar{k} . The effects from \bar{k} can represent the uncompensated spillovers of knowledge or ideas from

¹Rebelo (1990) worked out a two-sector version of the Ak model with two types of productive capital, k_1 and k_2 , one of which could be human capital. The properties of the steady-state growth rate are similar to those for the one-sector model. In the two-sector model, however, there is a transition from an initial ratio of k_1 to k_2 to the steady-state ratio. The nature of the transition in two-capital-goods models of this type is discussed in Mulligan and Sala-i-Martin (1991).

one producer to another. In a simple case, the production function takes the Cobb–Douglas form,

$$(10) \quad y = A \cdot k^{1-\alpha} (\bar{k})^\alpha, \quad 0 < \alpha < 1$$

that is, production is subject to diminishing returns in k for fixed \bar{k} , but constant returns with respect to k and \bar{k} together. The private rate of return on investment in this model is $(1-\alpha)(A/\eta)(\bar{k}/k)^\alpha$, which equals $(1-\alpha)(A/\eta)$ in the equilibrium where $k = \bar{k}$.² As in the Ak model, the rate of return is independent of the growth rate, as shown by the horizontal line in Figure 1. The steady–state growth rate follows from equation (9), substituting $(1-\alpha)(A/\eta)$ for A/η . As with the Ak model, the growth rate always equals this value; that is, there is no transition to the steady state. The growth rate falls short of the Pareto optimal rate, which is found by using the social rate of return on investment, A/η , instead of the private return.

One way to attain the social optimum is to subsidize production (which corresponds here to the income on broad capital) at the rate $\alpha/(1-\alpha)$. Thereby the private rate of return on investment becomes A/η , which equals the social return. The subsidy would have to be financed with a lump–sum tax, which could be a consumption tax because the model lacks a labor–leisure choice. Alternatively, the government could

²If \bar{k} is the aggregate stock of capital (corresponding, say, to the total of knowledge), then \bar{k}/k equals the number of producing units in the economy. In that case, a higher number of producing units implies a scale benefit that raises the economy's per capita growth rate. Increases over time in the number of units, which could be implied by population growth, then lead to rising per capita growth rates. These implications do not follow if \bar{k} represents the average of capital per worker in the economy. For example, in Lucas (1988), \bar{k} corresponds to the average person's human capital. One interpretation here is that the knowledge spillover relates to the ability of the person whom one happens to encounter; if meetings are random, then the average ability would matter.

subsidize purchases of capital goods (an investment–tax credit), so that buyers of capital pay only $(1-\alpha)\eta$ for each unit. Again, the private return on investment would be A/η .

Models with Public Services and Taxes

Barro (1990) constructs a growth model that includes public services as a productive input for private producers. We consider three versions of this type of model: publicly–provided private goods, which are rival and excludable; publicly–provided public goods, which are non–rival and non–excludable; and publicly–provided goods that are subject to congestion. The third category of public goods, which are rival but to some extent non–excludable, includes highways, water and sewer systems, courts, and so on. We argue later that the congestion model may also be appropriate to security services, such as national defense and police. Activities like education and health can be represented by some combination of the first two types of models.

In the first model, based on publicly–provided private goods, each producer has property rights to a specified quantity of public services. The services are rival but excludable; therefore, an individual producer cannot trespass on or congest the services provided to others. If G is the aggregate quantity of government purchases, then $g = G/n$ is the quantity allocated to each producer, where n is the number of producers (or firms). In the case of a Cobb–Douglas technology, the production function is

$$(11) \quad y = Ak^{1-\alpha}g^\alpha$$

Hence, production is subject to diminishing returns with respect to the private input, k , for given g , but is subject to constant returns with respect to k and g together. In this

setting, an individual producer regards his individual allotment of public services, g , as fixed when choosing the quantity of private input, k .

The government runs a balanced budget and, in one version of the model, levies a proportional tax at rate $\tau = g/y$ on the quantity of output, y . Because each unit of g requires the government to use one unit of resources (measured in units of consumables), the natural efficiency condition for determining the size of the public sector is $\partial y / \partial g = 1$. In the case of a Cobb–Douglas technology, the government that seeks to maximize the utility of the representative household turns out to satisfy this condition even in the second–best case in which expenditures are financed by the distorting tax on output. It can be readily verified from equation (11) that $\partial y / \partial g = 1$ implies $g/y = \alpha$.³

The marginal product of capital can be determined from equation (11) as

$$(12) \quad \partial y / \partial k = (1-\alpha)A^{1/(1-\alpha)}(g/y)^{\alpha/(1-\alpha)}$$

where $\partial y / \partial k$ is computed for a given value of g . The condition $g/y = \alpha$ can be substituted on the right side of equation (12) if the size of the public sector is optimal. The private rate of return on investment is found by multiplying $\partial y / \partial k$ by $(1-\tau)/\eta$, where τ is the marginal tax rate on output (and hence, on the income from capital) and η is again the cost of a unit of capital in terms of consumables. (If $g/y = \alpha$ and the government levies a proportional tax on y , then $\tau = \alpha$.) Once again, the rate of return on investment is independent of the growth rate, as shown by the horizontal line in Figure 1.

The growth rate of the economy follows by substituting the expression, $(1-\tau)(\partial y / \partial k)/\eta$, for A/η in equation (9). Note that $\partial y / \partial k$ depends on g/y in equation

³If $g/y = \alpha$ and $\tau = g/y$, then the representative firm ends up with zero profit in this model. That is, the benefit from the public services just balances the tax bill. The number of firms is then indeterminate.

(12); that is, this model brings in a dependence of the growth rate on the quantity of the government's productive services. If g/y and τ are constant, then the model again has no transition period; that is, the growth rate always equals the steady-state growth rate.

If the size of the government is optimal, so that $g/y = \alpha$, then the private and social returns on investment would coincide if the marginal tax rate, τ , were zero. If $\tau > 0$, then the private return falls short of the social return as in the learning-by-doing models. Hence, the growth rate in a decentralized economy is too low from a social perspective. A Pareto optimal outcome can be achieved by shifting to a lump-sum tax (which can be a consumption tax in this model) or by subsidizing the purchase of capital goods. If the private price of a unit of capital were $\eta \cdot (1-\tau)$ —that is, subsidized in the proportion τ —then the private return on investment would coincide with the social return and, therefore, the decentralized growth rate would equal the socially optimal rate. Of course, the subsidy to purchases of capital would have to be financed by a lump-sum tax; if such a tax were feasible then it could have been used in the first place to finance the government's purchases of goods and services.

The second version of the model treats public services as Samuelson (1954)—style, non-rival, non-excludable public goods. In this case, the aggregate quantity of government purchases, G , replaces the per capita quantity, g , in each producer's production function:

$$(13) \quad y = Ak^{1-\alpha}G^\alpha$$

Equation (13) implies that the aggregate of public services, G , can be spread in a non-rival manner over all of the n producers. Because of this non-rivalry, the marginal

product of public services is the effect of a change in G on aggregate output, $Y = y \cdot n$.⁴ Productive efficiency now requires this revised marginal product of public services to equal one. (In the Cobb–Douglas case, this condition still leads to the result, $G/Y = \alpha$.)

The comparison between private and social returns to investment is similar to that in the model in which the public services were publicly–provided private goods. Suppose, for example, that the size of the government satisfies productive efficiency, but that government spending is financed with a proportional tax on output, y . Then the privately determined growth rate is again below the socially optimal rate, and a Pareto optimal situation can be attained by shifting to a lump–sum tax.

The third version of the model (suggested by Ken Judd) allows for congestion of the public services. In this case, the public good is rival but not excludable. Suppose that the government services available to an individual producer involve the ratio of total purchases, G , to the aggregate of private input, K . For example, G could represent total highway mileage (or the size of a fishing pond) and K the total of highway traffic (or the number of fishermen). For a given G , the quantity of public services available to a producer declines if other producers raise their levels of usage, as represented by their levels of input.

Formally, the production function for each producer in the Cobb–Douglas case would be

$$(14) \quad y = Ak \cdot (G/K)^\alpha$$

⁴With free entry and no fixed costs, the number of firms, n , would be infinite in this model. If fixed costs were introduced, the number of firms could be determined from a zero–profit condition. However, this model—or one where n is set exogenously—has the same type of counter–intuitive scale effect that appeared in some of the learning-by-doing models: an increase in the number of firms, n , raises the per capita growth rate.

where $0 < \alpha < 1$ and k again represents the inputs provided by an individual producer. Equation (14) says that individual production, y , satisfies constant returns to private inputs, k , as long as the government maintains a given state of congestion of the public facilities; that is, as long as the government maintains the ratio of G to K . Equivalently, aggregate production, $Y = ny$, exhibits constant returns with respect to $K = nk$ and G , but diminishing returns to K for given G because of the increase in congestion of the public services.

The crucial new element in equation (14) is that an individual's decision to expand own capital, k , and hence output, y , congests the facilities available for other producers. With no user fee—that is, under lump-sum taxation—this distortion leads to the usual excessive use of the "public" good. In particular, the private rate of return to investment now exceeds the social return, and hence, the decentralized growth rate is too high.

The production function in equation (14) implies that congestion depends on the expenditure ratio, G/Y . In this case, the user fee that internalizes the congestion distortion is a proportional tax on output or income at rate $\tau = G/Y$. That is, a tax at this rate equalizes the private and social rates of return on investment, and therefore leads to a Pareto optimal growth rate. (The tax also yields just enough revenue to satisfy the government's budget constraint in each period.)

One lesson for public finance is that taxes on output or income—or equivalent user fees—are well matched to services that entail congestion. The congestion model applies readily to highways and other transportation facilities, water and sewer systems, courts, etc. The applicability of the model to government activities is, however, much broader if the setting applies also to national and domestic security.

National defense is often regarded as the prototypical non-rival (and non-excludable) public good. That is, in equation (13), the non-rival input G might

represent the national security services conveyed by defense expenditures.⁵ Thompson (1974) disputed this interpretation by arguing that defense expenditures were subject to a form of congestion. In particular, the state of national security depends on defense expenditures, G , in relation to the level of the external threat. This threat (that is, the incentive to threaten) depends, among other things, on the size of the prize available to external aggressors. The prize is, in turn, proportional to the domestic capital stock, K . Therefore, the variable G/K in equation (14) could represent the effective level of national security. Similarly, if G pertains to the domestic security services provided by police, prisons, and so on, then G/K could describe the effective level of domestic security. The conclusion is that the congestion model of government services applies to a substantial portion of the government's productive expenditures. This result is important because the congestion model favors income taxation—as an approximation to an appropriate user fee—over lump-sum taxation.⁶

Barro (1990) also considered government consumption purchases, G^C , which entered into household utility functions. These activities did not affect production opportunities and therefore did not affect the social rate of return on investment. On the other hand, if government expenditure is financed by a proportional income tax, then an increase in G^C/Y raises the marginal tax rate, τ , and thereby lowers the private rate of return on investment. It follows that an increase in G^C/Y lowers the economy's steady-state growth rate, γ (in an absolute sense and also relative to the socially optimal rate). This result follows even if the rise in G^C/Y is warranted from the

⁵Instead of entering directly into the production function, the security of property rights could appear as a determinant of investment. The implications for growth and investment end up being similar.

⁶In some cases, a direct user fee—such as a tax on gasoline or a charge for sewer usage—would be superior to an income tax. However, in many situations, such as national defense and police services, direct fees would be infeasible, and income or property taxes would be superior to lump-sum taxes.

standpoint of maximizing the utility of the representative household. That is, an increase in G^C/Y can be consistent with an increase in utility that accompanies a decrease in the growth rate.

Barro (1991) provided evidence on the relation between government spending and economic growth in a sample of 98 countries over the period 1960 to 1985. Government consumption, G^C , was measured by government consumption purchases as reported in the standard national accounts, less the amounts spent on national defense and education. The assumption was that G^C proxied for public services that enter into household utility functions. The data indicated a significantly negative relation between G^C/Y and the growth rate of per capita real GDP.

Public investment, G^i , was taken as a proxy for the government's activities that enter into production functions. The empirical results indicated little relation between G^i/Y and economic growth, especially if the ratio of private investment to GDP was held constant. One interpretation of this finding is that public investment is not very important for economic growth. An alternative explanation, however, is that governments are optimizing and are therefore going to the point where the marginal effect of public investment on the growth rate is close to zero. In this case, we would find little relation across countries between the growth rate and the share of GDP that went to public investment.

Models with Varieties of Capital Goods and Imperfect Competition

Technological progress is a central element in many models of long-term economic growth. Romer (1987, 1990) modeled technological change by applying the analyses of varieties of products from Dixit and Stiglitz (1977) and Ethier (1982) to endogenous growth models. Related models in an international context have been developed by Grossman and Helpman (1991, Chapter 3). In these models, technological progress

corresponds to an expansion of the number of types of capital goods.⁷ An increase in the number of types—that is, inventions—requires purposive activity in the form of research and development. Firms get compensated for their R&D activity through the retention of monopoly power over the use of their inventions. Therefore, the models involve elements of imperfect competition.

The household consumption side of these models is the same as that used before. To illustrate the production side, assume that producers of "basic goods," y , use the quantity x_j of a variety of capital inputs, $j = 1, \dots, N$, where N is the number of varieties available at time t . The production function is

$$(15) \quad y = A \cdot L^\alpha \cdot \sum_{j=1}^N (x_j)^{1-\alpha}, \quad 0 < \alpha < 1$$

where L is labor input. This functional form exhibits additive separability across the x_j 's. Thus, new types of capital goods are different from, but neither better nor worse than, the old ones. The subsequent analysis treats $N(t)$ as continuous in time, rather than discretely varying. The continuous case can be modeled formally by replacing the sum in equation (15) by an integral over quantities of capital goods over a range of types. For given N , equation (15) implies constant returns in the various x_j 's and L together, but diminishing returns in the x_j 's with L held fixed. In the subsequent analysis, L is regarded as constant for the representative producer (that is, aggregate labor supply is given and the real wage rate adjusts to ensure full employment).

⁷It is possible to work out similar models in which technological progress involves improvements in the quality of capital goods rather than increases in the number of varieties. See Aghion and Howitt (1990) and Grossman and Helpman (1991, Chapter 4).

The producers of basic goods are perfect competitors and face the rental price R_j for the capital good of type j . The marginal product of capital is

$$(16) \quad \partial y / \partial x_j = A \cdot (1-\alpha) \cdot L^\alpha \cdot (x_j)^{-\alpha}$$

The first-order condition for the maximization of profit equates this marginal product to R_j ; hence, the demand for x_j is

$$(17) \quad x_j = L \cdot \left[\frac{A(1-\alpha)}{R_j} \right]^{1/\alpha}$$

Producers of consumables are perfect competitors and can use one unit of basic goods, y , to generate one unit of consumables, c . Accordingly, the price of c in units of y is still unity.

A producer of a new type of capital good, say type j , must first incur a fixed cost for research and development to generate a design. Each design is assumed to cost the constant amount β in units of the basic good, y . Hence, the assumption is that the technology for doing research is the same as the technology for producing basic goods and consumables. Once the cost β is incurred, the producer of capital good j is assumed to maintain a perpetual monopoly over the production of this good. This production involves the constant marginal cost η as in previous models. For simplicity, η is assumed to be the same for capital goods of all types. Producers of good j rent the quantity x_j of their infinitely-durable capital at the price R_j to the producers of basic goods.

The present value of profits for a capital-goods producer who begins production of good j at time 0 is

$$(18) \quad \Pi = -\beta - \eta \cdot x_j(0) + \int_0^{\infty} \{ [R_j x_j - \eta \dot{x}_j] \cdot \exp[-\int_0^t r(s) ds] \} dt$$

where x_j is given from the demand function in equation (17). The term $-\beta$ is the cost of R&D and the term $-\eta \cdot x_j(0)$ is the cost of producing the discrete amount $x_j(0)$ when the good is first introduced. The subsequent path $x_j(t)$ is assumed to be differentiable, so that the term $-\eta \dot{x}_j(t)$ is the cost of production at each $t > 0$. The term $R_j x_j$ is the flow of rental income.

We limit attention to the steady state where the interest rate, $r(s)$, equals the constant r . In that case, the first-order conditions for maximizing the expression in equation (18) subject to equation (17) imply a markup formula for the monopoly rental:

$$(19) \quad R_j = r\eta/(1-\alpha)$$

That is, the ratio of the present value of the rental price, R_j/r , to the marginal cost, η , equals the constant markup ratio, $1/(1-\alpha)$. Because η and α are independent of j , the rental price R_j is the same for all j . The value $R = R_j$ is constant in the steady state (when r is constant). Correspondingly, the quantity x_j is independent of t or j :

$$(20) \quad x = x_j = L \cdot \left[\frac{A(1-\alpha)^2}{r\eta} \right]^{1/\alpha}$$

Because x_j is constant (in the steady state), the term \dot{x}_j is zero in equation (18). That is, the creator of a new design at time 0 produces the entire quantity of capital $x_j(0)$ at that time; production at later dates equals zero. Nonzero production of existing types of capital goods would occur in the steady state if the model were extended to allow for depreciation of capital stocks or growth of the labor force.

There is free entry into the business of creating a new design and using it to produce capital goods. Therefore, the present value of profits shown in equation (18) must end up being zero. Using the constancy of $x_j = x$, $R_j = R$, and $r(s) = r$, the zero-profit condition is

$$(21) \quad \beta = x \cdot [(R/r) - \eta] = x\eta\alpha/(1-\alpha)$$

where the formula for R/r from equation (19) was used. Combining equations (20) and (21) to eliminate x leads to a condition for the interest rate,

$$(22) \quad r = A(1-\alpha)(1-\alpha)^{1-\alpha}(\eta)^{-(1-\alpha)}\alpha^\alpha(\beta/L)^{-\alpha} \\ = (\partial y / \partial x_j) \cdot (1-\alpha) / \eta = (\partial y / \partial x_j) / (R/r)$$

where the calculations use the formula for $\partial y / \partial x_j$ from equation (16) and for R/r from equation (19). The right side of equation (22) is the private rate of return on investment, which equals the marginal product of capital divided by the monopoly price of capital, R/r . As in previous models, this rate of return is independent of the growth rate, as shown by the horizontal line in Figure 1. Recall from equation (19) that R/r is the multiple $1/(1-\alpha)$ of the marginal cost, η . This excess of the monopoly price over marginal cost lowers the private rate of return on investment (relative to what it would have been in a competitive situation in which price equaled marginal cost) and correspondingly lowers the interest rate, r .

The household optimization problem still implies from equation (5) that the steady-state interest rate satisfies the condition, $r = \rho + \theta\gamma$, as indicated by the upward-sloping line in Figure 1. Therefore, the steady-state values of r and γ are still determined by the intersection of the two lines shown in the figure. Equivalently,

equation (9) holds with A/η replaced by the expression for the private rate of return on investment from the right side of equation (22). Hence, the steady-state growth rate is now

$$(23) \quad \gamma = (1/\theta)[A(1-\alpha)(1-\alpha)^{1-\alpha}(\eta)^{-(1-\alpha)}\alpha^\alpha(\beta/L)^{-\alpha} - \rho]$$

One interesting implication is that a decrease in the cost of doing research, β , raises the steady-state growth rate.⁸

The model also determines the steady-state value of $x = k/N$, which equals $(\beta/\eta) \cdot (1-\alpha)/\alpha$ from equation (21). The economy can begin at an arbitrary value of x ; that is, an arbitrary ratio of total capital to the number of types of capital. Then the model implies a transition to the steady-state ratio, along the lines of the analysis in Mulligan and Sala-i-Martin (1991).⁹

It turns out, given the Cobb-Douglas specification for production, that the choice of x —which equals the quantity of capital relative to the number of designs—coincides with the value that would be chosen by a social planner. (See Romer [1987] for a discussion.) The decentralized outcome involves, however, the private rate of return on investment, which depends on the monopoly price of capital, $R/r = \eta/(1-\alpha)$, whereas

⁸The model contains a pronounced scale effect in that an increase in the labor force, L , leads to an increase in the steady-state growth rate. Moreover, as in some of the learning-by-doing models (n. 2) and the model with public goods (n. 4), growth in the labor force would lead to continuing increases in the growth rate of per capita output. The reason is that the cost of doing research, β , represents a lump-sum expense that can be spread across a market of arbitrary size. Hence, by expanding the size of the market, an increase in L represents a decrease in the effective cost of research, β/L , which leads to a higher growth rate. These effects might not arise if the model contained other costs that rose as an innovation was spread over a larger scale of operations. As a related matter, Becker and Murphy (1989) argue that the benefits of increased specialization are offset by costs involved with the increasing span of control.

⁹The model is like the two-sector production framework (see n. 1), except that the state variables are now the stock of capital, k , and the number of designs, N , which corresponds to accumulated research or knowledge capital.

the related social planner's problem involves the social rate of return, which depends on the true marginal cost of capital, η . The excess of the monopoly price over the competitive one implies that the private rate of return on investment falls short of the social return, and hence, that the steady-state growth rate in the decentralized economy is below the socially optimal rate. In this sense, the results are similar to the findings in the learning-by-doing model with spillovers and in some of the models with an income tax. In all three types of models—learning by doing with spillovers, taxation of income from capital (in models where government services are not subject to congestion), and varieties of capital goods in an environment of imperfect competition—the key element is the shortfall of the private rate of return on investment from the social rate of return. It follows that a Pareto optimum can be attained in each model if the government raises the private rate of return on investment to the social rate of return without introducing other distortions. As in the two earlier models, this outcome can be achieved in the capital-varieties model either by subsidizing the purchase of capital goods (at the rate α) or by subsidizing the income on capital (at the rate $\alpha/[1-\alpha]$).

A natural policy that does not work in the model with varieties of capital goods is to subsidize research. Note from equation (22) that the private rate of return on investment depends on the cost of research through the term $(\beta)^{-\alpha}$. It would be possible to raise the private rate of return to the social rate with the appropriate subsidy on research; namely, the private cost β^* would have to be the fraction $(1-\alpha)^{1/\alpha}$ of the social cost β . The problem with this policy is that it distorts the choice of quantity of capital, k , versus the number of varieties, N . That is, the value $x = k/N$ is no longer Pareto optimal if research is subsidized. (Recall that the decentralized model generates the appropriate value of x in the case where the production function is Cobb Douglas.) With the subsidy to research, private producers would have too much incentive to create new types of capital goods.

Romer's (1990) extended model motivates a subsidy to research by adding a learning-by-doing, spillover effect that is specific to the research sector. In the language of the present model, the extension is that the cost of an additional design (or invention), β , declines with the total number of existing designs, N .¹⁰ Although a creator of a new product is assumed to maintain a monopoly position with respect to the production of the good, this creator is assumed to have no property rights over the added knowledge that helps everyone's future research. The model in which uncompensated learning-by-doing effects were transmitted via the accumulation of capital implied that a subsidy to capital accumulation could be helpful. Analogously, if the learning-by-doing effects are tied to research activity, then a subsidy to research could be beneficial.

The Association between the Interest Rate and the Growth Rate

Equation (5), which came from utility maximization, implies a relationship in the steady state among the interest rate, r , the preference parameters ρ and θ , and the growth rate, γ . In particular, given ρ and θ , an increase in r goes along with an increase in γ , as shown by the upward-sloping line in Figure 1.

The interest rate, r , also equals the steady-state value of the private rate of return on investment. The production models considered thus far determine the return on investment as a function of technological parameters, spillover effects in production, quantities of public services, taxes that affect the net income from capital or the cost of

¹⁰Continuing growth in N would lead to continuing declines in β , which would lead in the present model to continuing increases in the growth rate. Romer (1990) assumes, however, that the research sector is intensive in human capital, relative to the sector that produces basic goods. Specifically, he assumes that the required amount of human capital in the research sector is proportional (for given N) to the number of new designs created. In this case, the growth of real wages offsets the effect of increasing N and leads to a constant cost of designs, β , in the steady state. Then the steady-state growth rate is again constant.

purchasing capital, and the costs of doing research. In all of these models, one variable that does not enter into the formulas for the steady-state rate of return on investment is the steady-state growth rate, γ . Therefore, the relation between the rate of return on investment and the growth rate is a horizontal line, as shown in Figure 1.

Consider a change in any of the parameters that influence the steady-state private rate of return on investment. The interest rate, r , ends up changing by the same amount as this rate of return. Therefore, for given ρ and θ , the household formula, $r = \rho + \theta\gamma$ from equation (5), implies that γ moves in the same direction. Thus, shifts on the production side of the model generate positively correlated movements in r and γ .

Consider, on the other hand, changes in the preference parameters, ρ and θ . The steady-state r is already determined by the production side, which fixes the steady-state private rate of return on investment. Therefore, r does not change and γ moves in the direction opposite to shifts in ρ and θ . In other words, shifts on the utility side of the model generate uncorrelated movements in r and γ .

Putting the results together, the implication is that the models predict positively correlated movements in r and γ . This conclusion is troublesome because this correlation is difficult to detect empirically, whether one interprets γ as the per capita growth rate of consumption or as the per capita growth rate of other variables, such as output and capital. In particular, the data suggest, as a first approximation, that the per capita growth rate of consumption is uncorrelated with r .

The results about the correlation of r and γ reflect an asymmetry in the model whereby the consumers' required premium on future consumption (equation [5]) is increasing in γ but the private rate of return on investment is independent of γ . The latter property no longer holds if investment entails adjustment costs; in that case, the private rate of return on investment tends to be diminishing in γ . This extension is useful because it can eliminate the apparently counterfactual prediction of a positive

correlation between r and γ . The correlation ends up depending on whether the preponderance of shifts comes from the utility side or the production side.

Adjustment Costs for Investment

The effects of adjustment costs in a growth model can be illustrated by introducing these costs into the Ak model of production. Suppose, following Abel and Blanchard [1983], that the internal adjustment cost, $\eta i \cdot \phi(i/k)$, subtracts from the producer's flow of net revenue, $Ak - \eta i$, which appears in equation (7). The assumptions are $\phi(0) = 0$, $\phi' > 0$, and $\phi'' \geq 0$. Because ηi is the direct outlay on investment, the formulation amounts to specifying the total cost of these purchases as $\eta i \cdot [1 + \phi(i/k)]$. That is, $\phi(i/k)$ is the proportionate premium paid on each unit of investment goods. The functional form implies, as seems reasonable, that a doubling of i and k doubles the total cost of investment goods. As before, the change in the capital stock is given by $\dot{k} = i$.

Producers again optimize by setting the private rate of return on investment equal to r . In the steady state, this condition is now

$$(24) \quad r = (1/q) \cdot (A + \eta \gamma^2 \phi')$$

where γ is the steady-state growth rate. The variable q is the shadow price of capital in place in units of consumables. The term $A + \eta \gamma^2 \phi'$ is the marginal revenue product of capital, taking out of the effect of k on the adjustment cost, $\eta i \cdot \phi(i/k)$ (and noting that $\gamma = i/k$). Because of the adjustment cost, q exceeds η when $\gamma = i/k$ is positive; in particular, q is given in the steady state by

$$(25) \quad q = \eta(1 + \phi + \gamma \phi')$$

It can be shown that the private rate of return on investment, given on the right side of equation (24), is diminishing in γ .¹¹ The downward-sloping curve in Figure 2 shows this relationship.¹² The upward-sloping solid line in the figure, which appeared also in Figure 1, is the preference relation from equation (5).

From the side of preferences, an increase in ρ or θ shifts the preference schedule upward and leads therefore to an increase in r and a decrease in γ . That is, preference shifts generate an inverse correlation between r and γ . From the side of production, an increase in the private rate of return on investment, induced say by an increase in A , shifts the production schedule upward. Consequently, r and γ increase. In other words, shifts on the production side lead to a positive correlation between r and γ . The overall association between r and γ depends on the relative importance of shocks to preferences and production. In particular, if the preference parameters are relatively stable, then the model still predicts a positive correlation between r and γ .

Varieties of Consumer Goods

In a previous section, we allowed for technological progress in the form of new varieties of capital goods. The analog on the consumer side is technological progress in the shape of new types of consumer goods. This setting is, in fact, the one considered originally by Dixit and Stiglitz (1977) in a static context.¹³

¹¹The transversality condition requires $r > \gamma$, where r is given by equation (24). This condition implies that the right side of equation (24) is diminishing in γ .

¹²We get a similar result within the model with varieties of capital goods if the cost of inventing a new product, β , rises with the rate of introduction of new products. This effect occurs within Romer's (1990) model because the research sector is intensive in human capital. An increase in the growth rate, which corresponds to a higher rate of introduction of new products, raises the real wage rate for human capital and thereby increases the cost of doing research.

¹³See Grossman and Helpman (1991, Chapter 3) and Xie (1990) for dynamic models that allow for an increasing variety of consumer goods.

The household's momentary utility function is now modified from equation (2) to

$$(26) \quad u = \left\{ \sum_{i=1}^M (c_i)^{(1-\theta)/\phi/(1-\theta)} \right\}^\phi$$

where $M(t)$ is the number of varieties of consumer products available at time t . If $\phi=1$, then the utility function is additively separable in the c_i . If $\phi < 1$, which we assume, a new product is substitutable for old ones in the sense that an increase in the quantity of the new good lowers the marginal utility of each of the existing products. We assume $0 < \theta < 1$ and $\theta + \phi > 1$ to get well-behaved demand functions for individual goods. (These conditions ensure concavity of the utility function.) Total utility, U , is still given from equation (1) with u substituted for $u(c)$ and ρ still treated as the constant rate of time preference.

We can interpret the conditions on the preference parameters by considering a case in which the quantities c_i are the same for all i at a given point in time. The expression for momentary utility from equation (26) can then be written as

$$(27) \quad u = \frac{1}{(1-\theta)^\phi} \cdot M^{(\theta+\phi-1)} c^{(1-\theta)}$$

where $c = Mc_i$ is the household's total consumption. Thus, the condition $\theta + \phi > 1$ implies that variety is desirable (u rises with M for given c). The condition $\phi < 1$ means that u is homogeneous of degree less than one with respect to M and c (that is, with respect to an increase in c with c_i held constant). This property corresponds to the negative effect of new goods on the marginal utility of the old goods. If this cross effect were absent—that is, if $\phi=1$ —then u would be linear homogeneous in M and c .

To see the implications for intertemporal substitution, note that equation (27) implies that the negative of the growth rate of marginal utility is

$$(28) \quad -(1/u')(du'/dt) = \theta(\dot{c}/c) - (\theta + \phi - 1)(\dot{M}/M)$$

The reduction in marginal utility over time deters households from postponing consumption; effectively, the rate of time preference is ρ plus the expression in equation (28). If the number of types of goods does not change, so that $\dot{M}/M = 0$, then the effective rate of time preference is $\rho + \theta(\dot{c}/c)$, as in equation (5). If M rises over time, then the effective rate of time preference includes the term, $-(\theta + \phi - 1)(\dot{M}/M)$. This term reduces the effective rate of time preference because future periods have more varieties of goods and are therefore more desirable for consumer expenditure (because $\theta + \phi > 1$). If $(\dot{c}/c) = (\dot{M}/M) = \gamma$ (that is, if c_i , the consumption of a given variety is constant), then the effective rate of time preference becomes $\rho + (1 - \phi)\gamma$, which is independent of γ if $\phi = 1$. Because u is linear homogeneous in this case in c and M (equation [27]), households do not require a premium above ρ to postpone consumption. On the other hand, if $\phi < 1$, the required premium, $(1 - \phi)\gamma$, is increasing in the growth rate (but by less than the amount $\theta\gamma$ that would apply if M did not grow).

Consider now the household's problem of maximizing overall utility. Suppose that P_i is the consumer price (measured in units of the basic good, y) for type i , $i = 1, \dots, M$. For a good with constant P_i over time, the first-order conditions for maximizing utility imply (as a generalization of equation [4])

$$(29) \quad \dot{c}_i/c_i = (1/\theta) \cdot [r - \rho - (1 - \phi)(\dot{M}/M)].$$

The relative quantities consumed of goods i and j are determined from the household's first-order optimization conditions as functions of the relative prices:

$$(30) \quad c_i/c_j = (P_j/P_i)^{\phi/(\theta+\phi-1)}$$

If the expenditure share of good i is not too large (so that income effects from changes in P_i on the demand for c_i can be neglected), equation (30) and the household's budget constraint imply that the elasticity of demand for c_i with respect to P_i is approximately equal to $-\phi/(\theta+\phi-1)$. The condition $\theta+\phi>1$ implies that the elasticity is negative and the condition $0<\theta<1$ implies that the elasticity is greater than one in magnitude.

The production of consumer goods of M types is modeled analogously to the production of capital goods of N types in the capital-varieties model discussed before. A producer of consumer goods of type i pays the lump-sum cost b (measured in units of basic goods, y) for research to generate a design. Once this cost is paid, the quantity c_i is produced by the monopoly producer at constant marginal cost ν (in units of y). Because the demand for c_i has constant elasticity, the monopoly price turns out to be a constant markup on marginal cost:

$$(31) \quad P_i = \frac{\nu\phi}{(1-\theta)}$$

The condition $\theta+\phi>1$ implies $P_i>\nu$.

The monopoly producer of good i receives the flow of net revenue, $c_i(P_i-\nu)$. With free entry to the creation and production of new products, the present value of this flow must equal the lump-sum cost of a design, b . In the steady state, where r is constant, this zero-profit condition dictates the level of c_i , which turns out to be

$$(32) \quad c_i = \frac{rb(1-\theta)}{\nu(\theta+\phi-1)}$$

Because all goods are treated symmetrically in the utility function and in the costs of production, equations (31) and (32) determine P_i and c_i independently of the index i .

In the steady state, the quantity consumed, c_i , of a good of given type is determined from equation (32) and is constant over time (for any time interval over which the good is available). But then equation (29) implies

$$(33) \quad r = \rho + (1-\phi)(\dot{M}/M)$$

Therefore, if $0 < \phi < 1$, the relation between r and the growth rate of varieties, \dot{M}/M , is positively sloped.

Per capita consumption, c , equals Mc_i , where c_i is the consumption of any of the available varieties, $i = 1, \dots, M$. In the steady state, c_i is constant but M , and hence, c grow at the rate γ . That is, the growth of per capita consumption corresponds entirely to growth in varieties and not at all to growth in the per capita consumption of a particular type of good. As discussed before, the effective rate of time preference in this situation is $\rho + (1-\phi)\gamma$, which appears on the right side of equation (33). This condition implies a positive relation between r and γ in the steady state. Thus, the preference relation is still positively sloped, as shown by the dashed line in Figure 2. (The slope is less than that of the old preference relation, shown by the solid line, because $(1-\phi) < \theta$.)

Basic goods, y , are produced competitively in accordance with the Ak model with adjustment costs, as discussed in the preceding section. Hence, equations (24) and (25) imply a negative relation between r and γ in the steady state, as shown by the downward-sloping curve in Figure 2.

The correlation between the steady-state values of r and γ depends again on the relative importance of shifts to preferences and production. Equation (33) shows that the relevant shifts to preferences now involve the parameters ρ and ϕ . (The parameter θ does not matter because the quantity consumed of a particular variety, c_i , is constant in the steady state.) We can interpret the parameter ϕ in terms of the technological opportunities for discovering new types of consumer products. A value of ϕ close to one signals the potential for generating new goods that are nearly independent of the old ones, whereas a value of ϕ below one means that the introduction of new goods makes the old ones less useful. Thus, we can think of an increase in ϕ as an improvement in the technology of developing consumer goods.¹⁴ This type of shift lowers the value of r that corresponds to each value of γ (for $\gamma > 0$) in the preference schedule shown in Figure 2. Therefore, r declines and γ increases. On the other hand, a technological improvement on the production side tends, as before, to raise r and γ .¹⁵ Thus, the overall pattern of association between r and γ depends on whether technological shifts tend to apply mainly to consumer products or to producer goods.

In the model with varieties of capital goods, the choice of quantity of capital relative to the number of varieties, $x = k/N$, turned out not to be distorted (with Cobb–Douglas production functions). The outcomes were not Pareto optimal overall

¹⁴In the present framework we have to think of a shift in ϕ that applies to the existing types of goods as well as to new ones. This setting may be a satisfactory approximation to a modified framework that fixes the interrelations between the existing goods while allowing for changes in the way that new goods substitute for old ones.

¹⁵We could have introduced a parameter ψ , analogous to ϕ , into the production function (equation [15]) in the model with varieties of capital goods. Then a higher ψ signifies a more favorable technological climate in the sense that there exist potential new types of capital goods that are less substitutable for the old types. If $\psi = 1$, then output is homogeneous of degree one with respect to total capital, $k = Nx$, and the number of types, N . There are diminishing (or increasing) returns in k and N if $\psi < 1$ (or $\psi > 1$). The previous treatment assumed $\psi = 1$. If $\psi < 1$, then the possibilities for discovering new types of capital goods are sufficiently limited so that diminishing returns apply and the growth rate is zero in the steady state. If $\psi > 1$, then increasing returns prevail and the growth rate rises continually.

because the monopoly price for capital goods lowered the private rate of return on investment below the social return. In effect, the excess of the monopoly price of capital over the competitive price acted like a tax on the income from capital. The growth rate was depressed, exactly as it would have been if the government had levied an explicit tax on the income from capital.

In the setting with varieties of consumer goods, the monopoly price of each good exceeds the competitive price. It turns out again that the determination of the quantity of consumption relative to the number of varieties, $c_1 = c/M$, is not distorted. (This result depends on the specification of utility in equation (27); see Dixit and Stiglitz (1977) and Judd (1985) on the distortions that can arise more generally.) The excessive price of consumables works just like a consumption tax. But in the present model, which lacks a labor-leisure choice, a consumption tax is not distorting.¹⁶ For this reason, the results in the model with a variety of consumer goods (and with no distortions on the production side) are Pareto optimal. In particular, the growth rate, γ , and the value of $c_1 = c/M$ coincide with the choices that would be made by a social planner who sought to maximize the utility of the representative consumer.

Concluding observations

We studied the role of tax policy in various models of endogenous economic growth. If the social rate of return on investment exceeds the private return, then tax policies that encourage investment can raise the growth rate and thereby increase the utility of the representative household. An excess of the social return over the private return can reflect learning-by-doing with spillover effects, the financing of government

¹⁶There would be distortions if some goods were produced under conditions of monopoly, whereas others were produced competitively. This setting would arise, for example, if the monopoly on new varieties of goods were not perpetual. See Judd (1985) for a discussion of the case in which monopoly rights, possibly enforced by patents, persist only for a finite interval.

consumption purchases with an income tax, and monopoly pricing of new types of capital goods. On the other hand, tax incentives to investment are not called for if the private rate of return on investment equals the social rate of return. This situation applies in growth models if the accumulation of a broad concept of capital does not entail diminishing returns and in some cases when technological progress appears as an expanding variety of consumer products.

In growth models that incorporate public services, the optimal tax policy hinges on the characteristics of the services. If the public services are publicly-provided private goods, which are rival and excludable, or publicly-provided public goods, which are non-rival and non-excludable, then lump-sum taxation is superior to income taxation. Many types of public services, such as transportation facilities, public utilities, courts, and possibly national defense and police services, are subject to congestion. That is, the goods are rival, but non-excludable to varying degrees. In these cases, income taxation works approximately as a user fee and can therefore be superior to lump-sum taxation. In particular, the incentives for investment and growth are too high if taxation is lump sum.

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FIGURE 1: Schedules for Preferences and Production

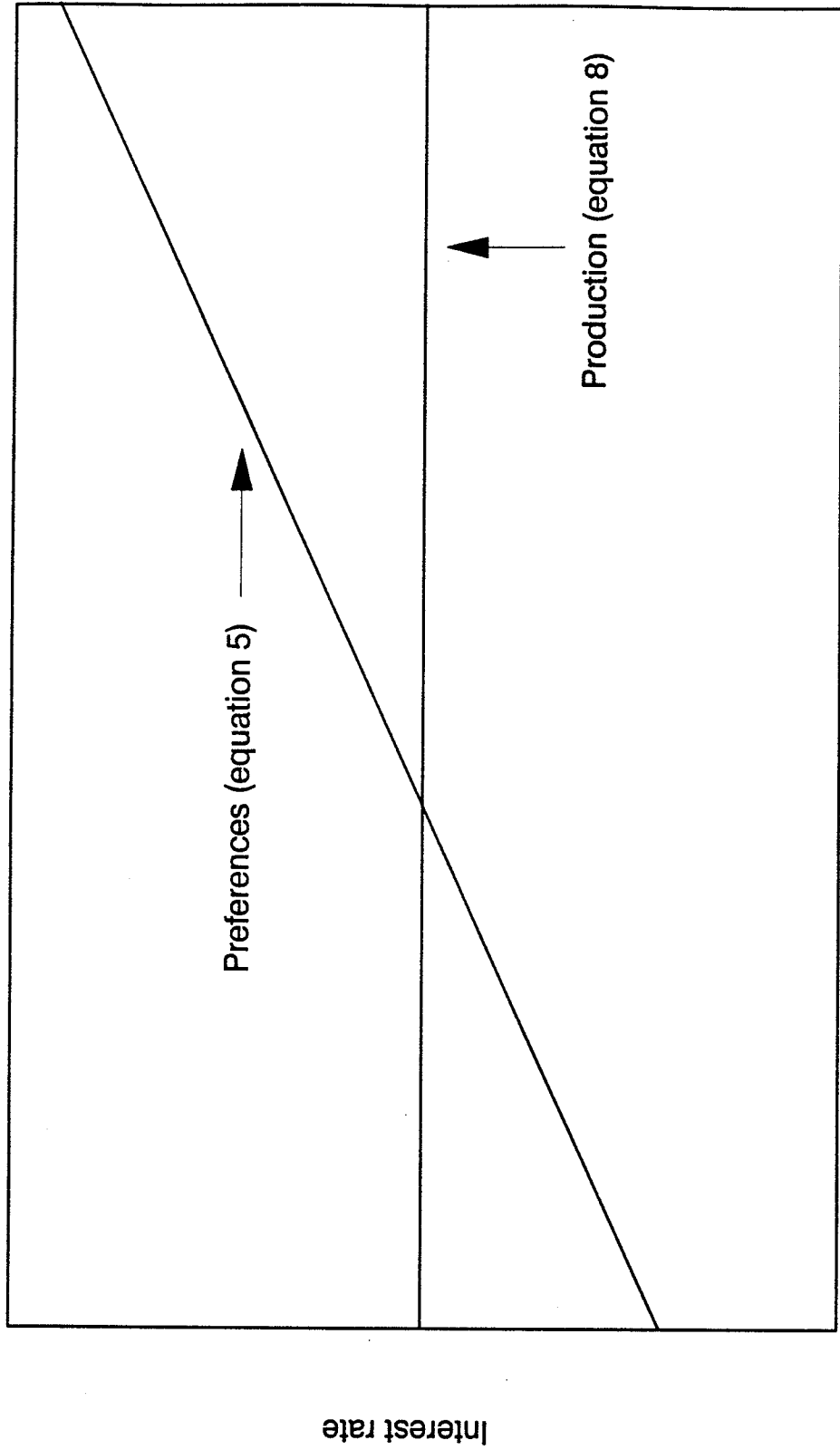


FIGURE 2: Schedules for Preferences and Production
with Adjustment Cost for Investment

