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CONSUMPTION-SMOOTHING MODEL

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Abstract

This paper examines the long-term relationship that arises when external loans are used to smooth the consumption path of a risk-averse sovereign subject to endowment shocks. We assume seniority is legally enforced in lender countries, but that no third party can enforce sovereign loan contracts. We model the loan market as a repeated game in which contracts are always subject to renegotiation; the only credible punishments are renegotiation-proof changes in the path of future transfers. Simple debt contracts with initial free entry by lenders can support transfers that achieve permanent consumption smoothing that is efficient, subject to the perfection constraints.

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1. INTRODUCTION

Respect for sovereign immunity has long been recognized (e.g. Keynes (1924)) as a crucial constraint on lenders to sovereign states. The consequences of lenders' inability to seize collateral are seen in the history of sovereign lending. Overall payments on sovereign loans during the past century or so have not come close to discharging the original contractual obligations, and there have been many defaults as identified by historians.[1]

But though debt service has fallen far short of contractual obligations, lack of collateral has not meant that lenders did not recover their principal, on average. In fact, lending to sovereign nations has been, overall, quite profitable, with average returns comparing favorably with those on contemporaneous domestic government obligations in lender nations.[2] Even loans in default were frequently profitable ex post.[3]

When payment deviations or defaults occurred, it has been widely noted that there was generally no abrupt termination of the borrower-lender relationship as seen in domestic bankruptcy. Instead, "Settlement was achieved on a case-by-case basis through bilateral negotiation" (Eichengreen and Lindert (1989) p. 8). The relationship typically continued after renegotiation, with a modified sequence of transfers under the guise of partial repayments, new loans, debt repurchase, and so on. Indeed all parties might view a default as "excusable", an equilibrium phenomenon in appropriate states of the underlying international financial relationship.[4]

The equilibrium relationship that underlies the formal contract for a loan to a sovereign is the subject of our paper. Assuming legal enforcement of lender seniority but no collateral nor any exogenous punishments for non-performance, we consider how loans and repayments evolve when neither party to a loan can be forced to honor past payment commitments.

The first issue that arises is that of motivation. Why should sovereign borrowers with no collateral repay their lenders? One rationale, recently advanced by Bulow and Rogoff (1989a), is to avoid some kind of contemporaneous sanctions such as interference with intratemporal trade. Cases exist where the existence of this type of _quid pro quo_ is easy to infer.[5] However, elsewhere in the historical record (Eichengreen and Portes 1989b), and in the recent experience of Brazil, Ecuador and Peru (Sachs 1989 p. 26), there is evidence of marked reluctance on the part of lenders and/or their governments to interfere with a debtor's trade, when they could have done so as punishment for non-performance on foreign loans.

Another motive for repayment, identified in Eaton and Gersovitz (1981), is the desire of a risk-averse borrower for continuation of a consumption-smoothing relationship with a lender. We adopt this motive for our model, but we consider punishments different from the trigger strategy of Eaton and Gersovitz, in which permanent cutoff of access to the credit market is the penalty of borrower misbehavior.

We assume ex ante commitment by either borrowers or lenders is infeasible.[6] All debt contracts are in general subject to renegotiation, which we define as any deviation by a party from the terms of the contract or of the associated punishments for deviations.

For a sovereign borrowing equilibrium to be credible when renegotiation is feasible, it must be enforced by sufficiently strong punishments that are also credible. To be credible, the punishments that support the equilibrium given renegotiation must themselves be consistent with the self-interested choices of the parties jointly, given

2

that one of them has deviated from the equilibrium path (or from a previous punishment regime). Punishments are credible if they are what would be chosen as a result of renegotiation; by construction renegotiation will not induce deviation from the punishment. Such punishments are "renegotiation-proof" by several definitions in the recent literature (e.g. Farrell and Maskin (1990), Pearce 1987). The trigger strategy punishment in the model of Eaton and Gersovitz (1981), reversion to permanent autarky, is vulnerable to renegotiation since it can be Pareto dominated by other alternatives when there are positive net transfers in equilibrium; it is not "renegotiation-proof."

In this paper we characterize the equilibrium paths of the transfers made between the parties in the course of the consumption smoothing relationship underlying a succession of simple debt contracts, including the punishment paths that would be followed in equilibrium after a deviation. The equilibrium transfers are the result of renegotiation.

In our model, the economy has an infinite horizon, and the risk-averse borrower's endowment ("income") is exogenous and stochastic each period; borrowers do not invest domestically. Lenders are risk-neutral and competitive. All agents have the same information set.[7] In this problem, constrained optimal smoothing can be implemented with simple debt contracts and a strict seniority privilege for initial lenders. The ability to renegotiate transforms a formal simple debt contract into an equilibrium sequence of transfers quite different from what one would infer from the terms of the formal contract. Should one party deviate from the equilibrium, renegotiation implies a new equilibrium path of transfers, a punishment, that gives the deviant no gain from the deviation.

The operation of smoothing proceeds as intertemporal exchange between the parties, which can be modeled with only one good. Thus our dynamic model, which has intertemporal trade consisting of unilateral transfers of a single good under risk

3

aversion, is complementary to the model of Bulow and Rogoff (1989a) in which there is bilateral contemporaneous exchange of different goods (repayments for freedom from trade sanctions or from seizure of their traded commodity) under risk neutrality in a static bargaining equilibrium using the extensive form game of offers and counter-offers proposed by Rubinstein (1982). In departing from a strategic Nash bargaining model to determine the distribution of surplus in the subgame perfect equilibrium with renegotiation, we have a precedent in the (quite different) model of Hart and Moore (1988).[8]

Before we proceed with the more technical exposition of the model, we offer an informal overview of the borrowing-lending relationship with renegotiation in Section 2. An outline of the model follows in Section 3, and renegotiation-proof equilibrium consumption-smoothing transfer paths are derived in Section 4. The dynamic evolution of the equilibrium transfers for given initial conditions, and the dynamic equilibrium response to deviations by either of the parties, are discussed in Section 5. Implementation of the renegotiation-proof relationship using simple debt contracts is described in Section 6, and conclusions follow in Section 7.

2. OVERVIEW

It is easy to see how net transfers between borrower and lender that smooth the consumption of the risk-averse agent could be Pareto-superior to the complete absence of transfers in all periods, denoted "permanent autarky" here. But any transfers that occur must be supported in an equilibrium in which there is no means of current enforcement of commitment exogenous to the smoothing activity itself, such as military power or interference with trade.

The sole motivation for any voluntary unilateral transfer in this model is the future surplus to be had from continuation on the equilibrium path. Transfers are made by the borrower or lender only if she or he, by making the transfer, ensures sufficient

anticipated future transfers in return. To put this another way, the punishment for non-cooperation of any party is the loss of the positive net transfers he or she would receive in equilibrium in different states in the future. Because no party can be forced to make a transfer to another at any time, the largest penalty for a deviation from an agreed lending and repayment plan is limited to imposition of permanent autarky.

An equilibrium choice of strategies for all agents must be subgame perfect, because unilateral deviations are always possible. Therefore, we first restrict our attention to subgame perfect equilibria (Selten (1965, 1975)). After every possible history of actions by the lender and by the borrower, the strategy profile for the remaining repeated game for each is a best response to the strategy profile for the other. Thus, for punishments to be credible, they must be consistent with the self-interest of the punishing party subsequent to a deviation from the equilibrium path. That is, execution of the punishment must maximize the present value of surplus expected by the punishing party, given the history up to that point, and the strategies adopted by all other agents.

However, subgame perfect equilibria frequently involve strategy choices after some histories that, although best responses to the other agent's strategy, yield outcomes Pareto-inferior to other pairs of strategies. For example, Eaton and Gersovitz (1981) rely on trigger strategies to support non-trivial financial market equilibria without renegotiation of repayment obligations in a consumption-smoothing model. Reversion to credit market autarky is Pareto-dominated by a return to an initial path which provides some smoothing of borrower consumption across income states. If support of an equilibrium path requires a punishment that is Pareto-dominated by an alternative equilibrium path, a commitment technology is implicitly assumed to make the parties jointly adhere to the dominated path. Absent such commitment, the punishment is subject to renegotiation. Renegotiability of a

punishment implies that a subgame perfect equilibrium path that requires the punishment for attainment is not viable.

Only penalties that would survive renegotiation, given a deviation has occurred, provide either party with bargaining power in negotiations over transfers. The provider of a transfer has the power of a refusal to give now, and the recipient has the ability to impose a credible punishment later. A credible punishment, in this paper, is one which is subgame perfect and not subject to successful renegotiation.

Suppose, for example, that at some date the borrower is expected to pay a net transfer of ten to the lender. (The nominal debt contract could specify a larger amount; ten is what the lender anticipates in equilibrium.) Should she propose in pre-play communication to renegotiate her transfer to five, the lender can respond that he will punish her if she does. But were she to insist on making the reduced out-of-equilibrium transfer, then the lender would be happy to take it. However, he could retaliate for its inadequacy by in turn providing smaller transfers than he would have along the equilibrium path, at future dates.

The threat of this penalty is effective for blocking the proposed renegotiation if it ensures that her utility is lower despite her higher current consumption. The threat is credible because at any time in the future at which the punishment dictates that the lender reduce his net resource transfer, ("supply of new money"), the lender would at that time find it in his interest to do so. No alternate pair of strategies that the borrower might propose at that time could make the lender better off. Similarly, the borrower can threaten a credible retaliation for insufficient (that is, off-the-equilibrium path) net transfer from the lender. If a loan that is smaller than the equilibrium one at some date were made it would be taken, but future repayments would be reduced enough so that the lender is worse off offering such a loan.

In this repeated game of borrowing and lending for consumption-smoothing, renegotiation-proof equilibrium rules of the game or "strategy profiles" support paths

of transfers that deter credible punishments. In accordance with recent literature on repeated games and renegotiation, our equilibria could be called bargaining equilibria. We show that there are many renegotiation-proof bargaining equilibria for the consumption-smoothing model. By the definition of Farrell and Maskin (1989), all of our equilibria are also weakly renegotiation-proof, and strongly renegotiation-proof equilibria exist.

Among the bargaining equilibria in our model, we focus on the subset that is consistent with a competitive (free entry) lending market; that is, that the subsect gives the lender zero profit in the initial period of the relationship. We characterize the equilibrium path on the Pareto frontier of this subset, and the associated credible punishment paths.

Equilibrium consumption and international transfers in the repeated game evolve along a stochastic path that converges to a stochastic steady state. Depending upon the common rate of discount, different types of equilibrium paths may be observed. There may be no transfers in equilibrium, borrower consumption may be fully smoothed from the initial date forward, or consumption may be at least partially smoothed initially, and then either partially or fully smoothed in the steady state. If consumption is partially smoothed, then it is serially correlated, even though the borrower's endowment is independently distributed across periods.

This efficient equilibrium consumption-smoothing outcome can be achieved by renegotiation of debt contracts incorporating legally enforceable strict seniority privileges. The senior lender has the right to attach any payments by the borrower either for debt service or to purchase financial assets in lender countries.[9] These debt contracts can be at least as crude or simple as those that have been used for many years in international lending. They can be simple public documents consisting of a loan between named parties specifying single-valued repayment obligations customarily denoted as interest and/or principal repayment. When repayments

7

become due, all parties have the same information and are free to renegotiate any terms of repayment, if they jointly wish to do so. The seniority privilege is an extremely simple way for the punishments of the borrower by the lender to be enforced; other potential providers of consumption smoothing only need to observe if a lender has an outstanding debt claim to decide whether or not to deal with the borrower. The seniority privilege effectively gives a lender the power to cut off all lending to the borrower despite the competitive lending environment, and this power establishes the lender's bargaining position in renegotiations with the borrower.

The equilibrium debt contract under renegotiation locks the borrower and her initial creditor into a permanent relationship. A debt contract with formal repayment obligations that cannot be fulfilled with positive probability in finite time along the subsequent equilibrium path serves to establish this relationship in the presence of seniority privileges and potential entry. Renegotiation of the formal terms of simple debt contracts achieves a sequence of transfers dependent on the history of states of nature. In the absence of commitment opportunities for either lenders or the borrower, simple debt contracts with renegotiation suffice to sustain a path of net transfers that is efficient in the set of attainable (that is, subgame perfect) equilibria, even though the formal debt contracts are not state-contingent.[10]

3. MODEL AND NOTATION

The model economy has an infinite horizon in discrete time with periods numbered $t = 1, 2, \ldots$ There is a single risk-averse agent (borrower) and J risk-neutral agents (lenders), with $J \geq 1$. Each agent receives an exogenous endowment every period of a single non-storable good. The endowment of the risk-averse agent is stochastic, and there is a finite number of states of nature, denoted by $s = 1, \ldots N$. The endowment of each risk-neutral agent is the same for every period for every lender. The endowment received by the borrower in any given period and the

8

preferences of every agent, as well as all past and current actions of every agent, are common knowledge.

There are no external commitment opportunities available to any party. No agent can commit to make a transfer of resources to any other agent. Therefore any party can always simply choose not to make a transfer to another.

Each agent is infinitely-lived and maximizes a discounted stream of felicity of current consumption using a constant rate of time preference. For simplicity, we assume that the discount factor, β, is common across all agents, and that the endowment of the risk-averse agent is identically and independently distributed.

Assumption 1:   The utility function for the risk-averse agent is given by:

$$(1) \qquad U_0 = E \sum_{t=1}^{\infty} \beta^t u(c_t),$$

where felicity, u(c), is continuously differentiable, increasing, and strictly concave, and $0 < \beta < 1$. The expectation is taken with respect to the distribution of consumption plans, $(c_1, c_2, \ldots)$, conditional on current information. Period t endowments are observed before period t consumption occurs. Subscript 0 denotes the borrower.

Assumption 2:   The utility function for each risk-neutral agent is given by:

$$(2) \qquad U_j = E \sum_{t=1}^{\infty} \beta^t c_t, \text{ for } j = 1, \ldots J,$$

where the expectation again is taken conditional on the current information set.

The states of nature are ordered with increasing borrower endowment, so that $y^1 < y^2 < \ldots < y^N$, and we assume that each risk-neutral agent receives an endowment every period equal to y^N. We define the history of nature as $w_t = (y_1, \ldots, y_t)$, where y_t is the realization of the state of nature at time t.

By assumption all transfers of part of one's endowment to another agent are voluntary. There are no third parties to force such transfers nor can either agent force the other to make a transfer. Hence all agents can simple choose to consume their endowment streams. We define the surplus for an agent attained with a consumption plan as the difference between the utility achieved under that plan and the utility achieved under permanent autarky. The payoffs to agents from an equilibrium borrowing and repayment path are these surpluses. At time t, the borrower receives surplus,

$$(3) \qquad V_0(t) = \left[u\left(c_t^0\right) - u(y_t) \right] + E \sum_{i=1}^{\infty} \beta^i \left(u\left(c_{t+i}^0\right) - u(y_{t+i}) \right),$$

from the consumption plan, $\left(c_t^0, c_{t+1}^0, \ldots\right)$, and each risk-neutral agent receives surplus,

$$(4) \qquad V_j(t) = \tau_t^j + E \sum_{i=1}^{\infty} \beta^i \tau_{t+i}^j,$$

where $\tau_{t'}^j = \left(c_{t'}^j - y^N\right)$ is the net transfer received by agent j in period t'.

Assumption 3: After any history, each agent can always assure himself or herself non-negative surplus.

This model can be represented by an infinitely repeated game, in which the streams of transfers made by each agent to every other agent are strategies. We next introduce the notation used in the rest of the paper, adopted from Rubinstein (1980) and Abreu (1988).

The stage game

At each date, t, there is a simultaneous move stage game, $\left\{ (A_i)_{i=0}^J, (\pi_i)_{i=0}^J \right\}$, where A_i is the pure strategy set and π_i the stage-game payoff function for agent i. A

strategy in the stage game is an action, $a_i \in A_i$. a_i is the vector of transfers $a_{ij} \geq 0$ made by agent i to each other agent j where A_i is a simplex in R_+^J that depends upon the endowment of agent i. That is,

$$A_i(y) = \left\{ x \in R_+^J \,|\, \sum_{k \neq i} x_k < y, \text{ for } y = y^N \text{ for } i = 1, \ldots, J, \text{ and } y = y_t \text{ for } i = 0 \right\}$$

An action profile, a, is the vector $(a_0, \ldots, a_J)$. The stage-game payoff for each agent is a function, $\pi_i : A(y_t) \to R_+$, where $A(y_t) = A_0(y_t) \times A_1(y^N) \times \ldots \times A_J(y^N)$. (A varies only with y_t, because y^N is constant over time.) For the risk-averse agent, $\pi_0(a) = u\left(y_t - \sum_{j=1}^J a_{0j} + \sum_{j=1}^J a_{j0}\right) - u(y_t)$ and for risk-neutral agents,

$$\pi_j(a) = \left[\sum_{i \neq j} a_{ij} - \sum_{i \neq j} a_{ji} \right], j = 1, \ldots, J.$$

There is a single Nash equilibrium for this stage-game, in which each agent makes zero gross transfers to any other agent.

The Repeated Game

For the infinitely repeated game, we define a path (or punishment) to be the sequence of action profiles taken over all possible histories of nature. A path is a sequence, $\{a(w_t)\}_{t=1}^\infty$, where $a(w_t)$ specifies the transfers made by each agent at time t for each feasible history of nature, w_t. The set of all paths is denoted by Ω. A history of play is just the sequence of all past actions for all agents, $h_t \equiv (a(y_1), a(w_2), \ldots a(w_t))$. The set of all feasible histories of play up to and including date t is denoted H_t.

A pure strategy profile, σ, is a function from the set of histories to the space of action profiles defined by $\sigma(w_t) = (\sigma_0(w_t), \ldots, \sigma_J(w_t))$, where $\sigma(w_1) \in A(y_1)$ and for all $t \geq 1$, and $i = 0, \ldots, J$, $\sigma_i(w_t) : A^{t-1} \to A_i$, and

11

$A^{t-1} \equiv A(y_1) \, x \ldots x \, A(y_{t-1}) \in R_+^{J(J+1) \cdot (t-1)}$. The set of all strategy profiles is denoted Σ. The strategy profile, σ, specifies an initial path to be followed beginning in period 1 and paths, called punishments, to be initiated after some agent deviates from a previously initiated path or punishment. We denote an initial path by $s(\sigma)$ and punishments by $q(\sigma)$.

The payoff to agent i along the path $s(\sigma)$ at time t, is given by:

$$\pi_i(s(\sigma), w_t) + \upsilon_i(s(\sigma), w_t),$$

where $\pi_i(s(\sigma), w_t) \equiv \pi_i(\hat{a}(w_t))$ for $s(\sigma) = \{\hat{a}(w_t)\}_{t=1}^{\infty}$ and the continuation value, $\upsilon_i(s(\sigma), w_t)$ equals $E\left(\sum_{t'=t+1}^{\infty} \beta^{t'-t} \pi_i(s(\sigma), w_{t'}) \right)$, and the expectation is taken with respect to the distribution of $\{w_{t'}\}_{t'=t+1}^{\infty}$ conditional on w_t.

We will say that an initial path $s(\sigma)$ is supported by a set of punishments $\{q^i(\sigma)\}_{i=0}^{J}$ if

$$\pi_i(s(\sigma), w_t) + \upsilon_i(s(\sigma), w_t) \geq \overline{\pi}_i \, s(\sigma), w_t) + \upsilon_i(q^i(\sigma), w_t),$$

for all $i = 0, \ldots J$ and all feasible histories of nature, w_t for $t = 0, 1, \ldots$. $\overline{\pi}_i(s(\sigma), w_t)$ is the maximal payoff in the stage game attainable by agent i for any feasible choice of $a_i(w_t)$ given that other agents all play the action $a_j(w_t)$ prescribed by $s(\sigma)$. That is,

$$\overline{\pi}_i(s(\sigma), w_t) = \max_{\tilde{a}_i} \left\{ \pi_i \left(a_0(w_t), \ldots, a_{i-1}(w_t), \tilde{a}_i, a_{i+1}(w_t), \ldots, a_j(w_t) \right) \mid \tilde{a}_i \in A_i \right\},$$

where $\upsilon_i(q^i(\sigma), w_t)$ equals $E\left(\sum_{t'=t+1}^{\infty} \beta^{t'-t} \pi_i(q^i(\sigma), w_{t'}) \right)$.

We denote the set of all strategy profiles that are subgame perfect equilibria by Σ^P. We say that $\sigma \in \Sigma^P$ generates the initial path $s(\sigma)$. The set of all paths generated by members of Σ^P is Ω^P.

12

4. RENEGOTIATION OF REPAYMENTS AND NEW LOANS

Equilibrium lending and repayment in the presence of potential renegotiation are first analyzed for the case of an exclusive long-term relationship between the borrower and a single lender. Each loan or repayment is voluntary in the absence of third party enforcement of contracts. Since neither party can commit his or her future actions, each chooses a strategy that maximizes his or her payoff in the repeated game, taking the choice of strategy for the other as given after any history of previous actions. Therefore, we first restrict strategy profiles to be subgame perfect.

In the single Nash equilibrium for the stage-game, an agent maximizes his or her one-period payoff by making no positive transfer to the other agent. As is well known, in the infinitely repeated game, outcomes other than infinite repetition of Nash equilibrium play in the stage-game can be achieved by subgame perfect equilibria if the discount rate is low enough. At least partial smoothing of the consumption path of the risk-averse borrower is possible because a positive net transfer by one agent may be rewarded by future cooperation in the consumption-smoothing relationship by the other agent.

A player is deterred from failing to make a transfer that would be made on an equilibrium path by threats that the other player will not behave cooperatively in the future. Threatened punishments that support cooperation in equilibrium consist of withholding from the deviant positive transfers that would otherwise have been made. These threats are credible in the sense that they specify unilateral best responses for each agent to the other agent's strategy in each subgame reached through a deviation by one player.

Although neither player has an incentive to deviate singly from punishment strategies in a subgame perfect equilibrium, the vector of payoffs achieved by a punishment path may be Pareto-dominated by the payoffs provided by some other subgame perfect equilibrium path. This is the case for a trigger strategy punishment:

permanent noncooperative play (autarky) serves as a subgame perfect equilibrium punishment but is Pareto-dominated by some of the paths its threat supports for low enough discount rates. A threat of such a punishment might be considered incredible if players can communicate and agree to choose an alternative pair of strategies following a one-period deviation by one of them.

In general, the payoffs sustained by a subgame perfect equilibrium after some histories may be Pareto-dominated by the payoffs provided by another equilibrium. In applications of repeated games it is often assumed that pre-play communication leads the players to select an efficient path from the set of all paths generated by subgame perfect equilibrium profiles. As pointed out by several authors[11], the possibility of negotiation, that is, communication, raises the possibility of renegotiation of the punishments that support the equilibrium path. The importance of potential renegotiations between the players is that a subgame perfect punishment can deter deviation from the equilibrium transfer path only if it cannot be abandoned by mutual agreement. If a particular punishment is necessary to support an initial path and this punishment is subject to successful renegotiation, then the initial path, as well as the punishment path, cannot be considered viable. Because the agents are unable to commit themselves not to renegotiate punishment strategies in our model, only initial paths of transfers supported by punishment threats that cannot be renegotiated are considered to be equilibrium paths. We choose to call such punishments credible.

Our approach to modelling lending to a sovereign under potential renegotiation is to derive the equilibrium strategy profiles that survive the possibility of renegotiation between the agents. An equilibrium path of loans and repayments is a path of transfers that cannot be negotiated further. Because out-of-equilibrium behavior is also subject to renegotiation, equilibrium outcomes are subgame perfect equilibria incorporating credible punishments. If one imagines a sequence of explicit debt contracts each with negotiations over the amounts of actual repayments and new

14

loans, then the equilibrium outcome of these negotiations is the path generated by some subgame perfect equilibrium strategy profile incorporating credible punishment threats. The transfers actually made by each agent along the equilibrium path are those necessary to deter credible threats of noncooperation.

In our model, subgame perfect equilibrium strategy profiles incorporating punishments that cannot be renegotiated are called "renegotiation-proof." Alternative definitions of renegotiation-proof equilibria appear in the literature. The equilibria that we derive for our model satisfy the different criteria of renegotiation-proofness proposed by Farrell and Maskin (1989) and by Pearce (1987). The equilibrium profile is strongly renegotiation-proof (Farrell and Maskin) and is a consistent bargaining equilibrium (Abreu, Pearce and Stacchetti (1989)). In the next Section, we allow free entry to lenders in the model and show that an efficient subgame perfect equilibrium path emerges as a renegotiation-proof outcome.

We first characterize the set of all subgame perfect equilibrium paths and payoffs for the two players in our consumption-smoothing model. Then we show the existence of credible threats that support any equilibrium that is efficient in the set of all subgame perfect equilibrium paths and characterize these punishments. As a first step, the characterization of subgame perfect equilibria by Abreu (1988) is useful for our description of equilibrium lending and repayment in the presence potential renegotiation.

Following Abreu (1988), we present the following definitions and result.

DEFINITION: Let s, q^0, q^1 be paths in Ω^P. The simple strategy profile, denoted as $\sigma[s, q^0, q^1]$, specifies that

(i)   s is followed until an agent deviates singly from s,

15

(ii)   for each $i = 0, 1$, q^i is followed if agent i deviates singly from s or from q^j, $j = 0, 1$, whichever is the previously initiated path. No change in path occurs if the two agents deviate simultaneously.

DEFINITION: For every strategy profile, σ, define

(i) $\tilde{\upsilon}(\sigma, w_t, h_t) = \upsilon(r(\sigma), w_t)$, for $r \in \{s, q^0, q^1\}$, such that σ generates r in the history h_t,

(ii) $C(\sigma, w_t) = \{\tilde{\upsilon}(\sigma, w_t, h_t) \mid h_t \in H_t\}$ is the set of continuation values for σ, in the history of nature w_t, and

(iii) $\underline{\upsilon}_i(w_t) = \min\{\upsilon_i \mid (\upsilon_0, \upsilon_1) \in C(\sigma, w_t), \sigma \in \Sigma^P\}$ and

$\underline{\upsilon}^i(w_t) = \{\upsilon \mid \upsilon \in C(\sigma, w_t) \text{ and } \upsilon_i = \underline{\upsilon}_i(w_t), \sigma \in \Sigma^P\}$, for each $i = 0, 1$.

Infinite repetition of the stage-game Nash equilibrium (permanent autarky) is a subgame perfect equilibrium for this model. Because a transfer cannot be confiscated from either agent, permanent autarky provides the minimum payoff for an agent in any subgame perfect equilibrium strategy profile after he or she deviates from an initial path. Therefore, $\underline{\upsilon}_i(w_t)$ is zero for every w_t and each $i = 0, 1$.

The following characterization of subgame perfect equilibria combines propositions proved by Abreu (1988):

PROPOSITION (Abreu):

(i)   The simple strategy profile, $\sigma[s, q^0, q^1]$, is a subgame perfect equilibrium if and only if

$\bar{\pi}_i(s, w_t) + \upsilon_i(q^i, w_t) \leq \pi_i(s_i w_t) + \upsilon_i(s, w_t)$,

and

$\bar{\pi}_i(q^j, w_t) + \upsilon_i(q^i, w_t) \leq \pi_i(q^j, w_t) + \upsilon_i(q^j, w_t)$,

for each i, j = 0, 1 and every feasible w_t,

16

(ii)   The paths, s, q^0 and q^1, are in Ω^P if and only if they are each supportable by continuation values $v^0(w_t)$ and $v^1(w_t)$, where $v^0(w_t) \in \underline{v}^0(w_t)$ and $v^1(w_t) \in \underline{v}^1(w_t)$.

That is,

$$\overline{\pi}_i(r, w_t) + v^i(w_t) \leq \pi_i(r, w_t) + v_i(r, w_t),$$

for $i = 0, 1$ and $r \in \{s, q^0, q^1\}$,

(iii)   A path s is an element of Ω^P if and only if $\sigma[s, q^0, q^1]$ is a simple strategy profile such that $v(q^i, w_t) \in \underline{v}^i(w_t)$, for $i = 0, 1$ and every feasible w_t.

PROOF: Assumptions 1-4 of Abreu are satisfied by our game. (i) is Proposition 1 of Abreu, (ii) is implied by Propositions 2-4, and (iii) is a restatement of Proposition 5 of Abreu.

The proposition implies that a subset of all subgame perfect equilibrium profiles suffices to generate all paths in Ω^P. Any subgame perfect equilibrium path can be generated by a simple strategy profile incorporating punishments that give agents who deviate their worst possible subgame perfect outcomes. These punishments include reversion to permanent autarky along with many paths that Pareto-dominate permanent autarky. Therefore, most of the harshest punishments possible in perfect equilibria do not survive renegotiation proposals between players.

We next characterize the set of all payoffs achievable in the repeated game using subgame perfect equilibria. These are just the discounted streams of expected surpluses for the two players from all paths that are supportable by reversion to Nash equilibrium play (permanent autarky) for each given initial state of nature, y_i.

DEFINITION: Let $W(y) \equiv \{\pi(s(\sigma), y) + v(s(\sigma), y) \mid \sigma \in \Sigma^P\}$ for each $y \in \{y^1, \dots y^N\}$. $W(y)$ is the set of payoffs (ordered pairs) for all subgame perfect equilibria for the two-player game starting in state of nature y.

We prove the following Proposition:

PROPOSITION 1: For each $y \in \{y^1, .., y^N\}$, $W(y)$ is a convex and compact subset of R_+^2. The set Ω^P is convex and compact.

PROOF: We first show that Ω^P is convex. Let σ_1 and σ_2 be two elements of Σ^P. Because π_1 is linear and π_0 is convex in $(a_0 - a_1)$, the net transfer made in the stage-game, we have for $0 < \lambda < 1$,

$$\pi_0\big(\lambda s(\sigma_1) + (1-\lambda)s(\sigma_2), w_t\big) + v_0\big(\lambda s(\sigma_1) + (1-\lambda)s(\sigma_2), w_t\big)$$

$$\geq \lambda\big(\pi_0\big(s(\sigma_1), w_t\big) + v_0\big(s(\sigma_1), w_t\big)\big) + (1-\lambda)\big(\pi_0\big(s(\sigma_2), w_t\big) + v_0\big(s(\sigma_2), w_t\big)\big)$$

and

$$\pi_1\big(\lambda s(\sigma_1) + (1-\lambda)s(\sigma_2), w_t\big) + v_1\big(\lambda s(\sigma_1) + (1-\lambda)s(\sigma_2), w_t\big)$$

$$= \lambda\big(\pi_1\big(s(\sigma_1), w_t\big) + v_1\big(s(\sigma_1), w_t\big)\big) + (1-\lambda)\big(\pi_1\big(s(\sigma_2), w_t\big) + v_1\big(s(\sigma_2), w_t\big)\big),$$

for all w_t, each t. The path, $s = \lambda s(\sigma_1) + (1-\lambda)s(\sigma_2)$, is supportable by linear combinations of the same punishments that support $s(\sigma_1)$ and $s(\sigma_2)$ using the weights λ and $(1-\lambda)$, respectively. It follows that $W(y)$ is convex.

$W(y)$ is bounded because $u(c)$ is continuous, $0 < \beta < 1$, and the action set in each stage-game is bounded. To show that $W(y)$ is closed, take any payoff pair, $\hat{V}$, in $W(y)$ and a sequence $\{V^k\}_{k=1}^{\infty} \subseteq W(y)$ that converges to $\hat{V}$. We endow Ω^P with the product topology and select a sequence $\{s(\sigma_k)\}_{k=1}^{\infty} \subseteq \Omega^P$ such that $V(s(\sigma_k), y) = V^k$, where $V_i(s(\sigma_k), y) \equiv \pi_i(s(\sigma_k), y) + v_i(s(\sigma_k), y)$. Because A is a compact subset of R_+^2 for each state, y, and $\pi: \Omega \to R^2$ is continuous, $V: \Omega \to R^2$ is continuous. By Tychonoff's Theorem, the set Ω is compact; therefore, $\{s(\sigma_k)\}_{k=1}^{\infty}$ has a convergent subsequence. Without loss of generality, we can assume $\{s(\sigma_k)\}_{k=1}^{\infty}$

18

converges to a limit path $\hat{s}$. Because $V(s, y)$ is continuous, $V(\hat{s}, y) = \hat{V}$. It remains to show that $\hat{s} \in \Omega^P$. Suppose not. This implies that $\bar{\pi}_i(\hat{s}, y) + 0 > \pi_i(\hat{s}, y) + \upsilon_i(\hat{s}, y)$, for at least one i = 0,1 (by Abreu's results). However,

$$\pi_i\big(s(\sigma_k), y\big) + \upsilon_i\big(s(\sigma_k), y\big) \geq \bar{\pi}_i\big(s(\sigma_k), y\big) + 0, \text{ for every } k = 1, 2, \ldots ,$$

so that continuity of $V(s, y)$ leads to a contradiction and $\hat{s}$ is a perfect equilibrium path. Therefore, $W(y)$ is compact for all $0 < \beta < 1$ and each $y \in \{y^1, \ldots, y^N\}$ and Ω^P is compact in the product topology.

DEFINITION: Let the frontier of $W(y)$ be the set

$$\Big\{ V \in W(y) | V_0 = \max \big(\pi_0(s(\sigma), y) + \upsilon_0(s(\sigma), y)\big), \text{ such that}$$

$$V_1 \leq \pi_1(s(\sigma), y) + \upsilon_1(s(\sigma), y) \text{ and } \sigma \in \Sigma^P \Big\}.$$

The frontier of $W(y)$ implicitly defines a function $V_0(V_1; y)$, which gives the maximum payoff in state y for the borrower in any subgame perfect equilibrium profile given that the lender receives the payoff V_1 and $V_0(V_1; y)$ is in $W(y)$.

PROPOSITION 2: The frontier of $W(y)$ is downward-sloping and strictly concave.
PROOF: For $V \in W(y_t)$ such that $V_1 > 0$, a reduction in V_1 can be achieved in state y_t by choosing σ such that $s(\sigma)$ is unchanged for all dates $t' > t$ and $a_1(w_t)$ is increased. This $s(\sigma)$ is an element of Ω^P, and $V_0 = \pi_0(s(\sigma), w_t) + \upsilon_0(s(\sigma), w_t)$ is increased since u(c) is increasing. Strict concavity follows from strict concavity of u(c) and convexity of Ω^P.

Any pair of payoffs for the two agents sustainable by some subgame perfect equilibrium is attainable using an initial path in which only one of the players at a time makes a positive transfer to the other. This transfer is just the net transfer made in

19

any equivalent initial path along which simultaneous transfers are made by the agents. A unilateral transfer path is defined as a path such that at each date one of the agents makes no transfer to the other. It is never rational for the recipient of a positive transfer at some date to deviate from a unilateral transfer path. When an agent is a recipient, his or her surplus in the stage-game is maximized by making no transfer and accepting what is offered. Because the agent making a transfer currently consumes less than his or her endowment, the repeated game payoff of the transferor can be zero after some histories in a subgame perfect equilibrium. But he or she can receive at most zero surplus in the stage-game by deviating from a unilateral transfer path. Since the continuation value for either agent is always nonnegative, the current transferee realizes positive surplus from the relationship.

$W(y_t)$ is the set of all payoffs that are sustainable by subgame perfect equilibria at date t after any feasible history of actions, $h_t \in H_t$. Therefore, the surplus available to the two agents to divide at any date depends only on the current resources available because future endowments are independently distributed. In a subgame perfect equilibrium, the transfers made at any date generally depend upon the history of actions up to that date. Our notation suggests that the action profile $a(w_t)$ may also depend upon past states of nature rather than just on the current state of nature. In the next Section, we show that the transfers planned for each state of nature in the next period depend upon the current consumption of the borrower (hence, on the history of nature) when consumption-smoothing is incomplete along the equilibrium path.

We now show that any path in Ω^P is supported by punishments that sustain payoffs on the Pareto-frontier of $W(y)$. If an agent deviates at date t, then the punishment initiated at date t+1 is an efficient subgame perfect equilibrium path for each $y_{t+1} \in \{y^1, \ldots, y^N\}$. These threats are credible in the sense that both players

cannot do better by abandoning the proposed punishment for an alternative subgame perfect equilibrium path.

PROPOSITION 3:

(a) There exist paths $\overline{q}^0$ and $\overline{q}^1 \in \Omega^P$ such that $\pi(\overline{q}^0, w_t) + \upsilon(\overline{q}^0, w_t)$ and $\pi(\overline{q}^1, w_t) + \upsilon(\overline{q}^1, w_t)$ are Pareto efficient in the set $W(y_t)$ and $\pi_i(\overline{q}^i, w_t) + \upsilon(\overline{q}^i, w_t) = 0$, for $i = 0,1$, for each $y_t \in \{y^1, \ldots, y^N\}$ where $w_t = (w_{t-1}, y_t)$. It follows that $\upsilon_i(\overline{q}^i, w_t) = 0$, since $\upsilon_i(\overline{q}^i, w_t) = E\left[\pi_i(\overline{q}^i, w_t) + \upsilon_i(\overline{q}^i, w_t)\right]$

(b) For every $s \in \Omega^P$, the simple strategy profile $\sigma[s, \overline{q}^0, \overline{q}^1]$ is a subgame perfect equilibrium.

PROOF:

(a) By Proposition 1 and Proposition 2, $W(y_t)$ contains the closed intervals:

$$\left\{(V_0, V_1) \in R_+^2 \mid V_0 = 0 \text{ and } 0 \leq V_1 \leq \overline{V}_1(y_t)\right\},$$

and

$$\left\{(V_0, V_1) \in R_+^2 \mid 0 \leq V_0 \leq \overline{V}_0(y_t) \text{ and } V_1 = 0\right\}.$$

By Proposition 2,

$$\overline{V}_i(y_t) \equiv \max\left\{V_i \mid (V_0, V_1) = \pi(s, w_t) + \upsilon(s, w_t), s \in \Omega^P\right\}, \text{ for } i = 0, 1,$$

where the maximum exists by Proposition 1.

Because Ω^P is compact, there exist paths $\overline{q}^0, \overline{q}^1 \in \Omega^P$ yielding payoffs at date t

$$\pi(\overline{q}^0, w_t) + \upsilon(\overline{q}^0, w_t) = (0, \overline{V}_1(y_t)) \text{ and }$$

$$\pi(\overline{q}^1, w_t) + \upsilon(\overline{q}^1, w_t) = (\overline{V}_0(y_t), 0), \text{ respectively.}$$

Note that $\upsilon_i(\overline{q}^i, w_{t-1}) = 0$ and that $\overline{q}^0$ and $\overline{q}^1$ need not be unique.

21

(b)   For every $s \in \Omega^P$, $\pi_i(s, w_t) + \upsilon_i(s, w_t) \geq 0$ for every feasible w_t and for $i = 0, 1$. Because $\pi_i(\overline{q}^i, w_t) + \upsilon_i(\overline{q}^i, w_t) = 0$, the profile $\sigma(s, \overline{q}^0, \overline{q}^1)$ is a subgame perfect equilibrium.

Any pair of paths, $\overline{q}^0$ and $\overline{q}^1$, given by Proposition 3 provides the lowest continuation value to a deviant possible in any subgame perfect equilibrium, so that $\overline{q}^0$ and $\overline{q}^1$ support any path sustained by the threat of permanent autarky. The paths $\overline{q}^0$ and $\overline{q}^1$ are solutions to a dynamic programming problem that is deferred to the next Section. The problem is to maximize the continuation value of the agent who does not deviate over the set of all subgame perfect equilibrium paths subject to the constraint that the deviant agent receives a continuation value of zero.

If agent i deviates from an initial path $s \in \Omega^P$ at date t, the path $\overline{q}^i$ prescribes a sequence of transfers between the agents for all dates $t' > t$. Because the set of payoffs, $W(y_{t+1})$, depends only on y_{t+1}, the path $\overline{q}^i$ followed from date t+1 onwards is independent of the history of nature before t+1, w_t. Therefore, we use the notation $\overline{q}^i(t+1)$, to identify the punishment initiated at date t+1 in response to a single player deviation at date t. The paths, $\overline{q}^0(t+1)$ and $\overline{q}^1(t+1)$ are not necessarily unique, but the paths of net transfers made in either punishment are shown to be unique in Section 7 below.

Because the simple strategy profile, $\sigma[s, \overline{q}^0, \overline{q}^1]$, is a subgame perfect equilibrium, the following inequality must hold for $i = 0, 1$:

$$\pi_i(\overline{q}^i(t), w_t) + \upsilon_i(\overline{q}^i(t), w_t) \geq \overline{\pi}_i(\overline{q}^i(t), w_t) + \upsilon_i(\overline{q}^i(t+1), w_t).$$

Since

$$\pi_i(\overline{q}^i(t), w_t) + \upsilon_i(\overline{q}^i(t), w_t) = 0$$

and

$$v_i\left(\overline{q}^i(t+1), w_t\right) = 0,$$

the maximum stage-game payoff that agent i can realize by deviating from the punishment $\overline{q}^i(t)$, $\pi_i\left(\overline{q}^i(t), w_t\right)$, must be equal to zero. This implies that an agent who deviates from any initial path receives no transfer in any state of nature during the first period after playing noncooperatively.

The punishments $\overline{q}^0(t+1)$ and $\overline{q}^1(t+1)$ have the property that the deviant plays cooperatively beginning in period t+1 (after he or she deviates in period t) by making a non-negative transfer to the other player. The other player pays nothing, and receives all of the surplus from initiating an efficient subgame perfect equilibrium at date t+1 in each possible state of nature, y_{t+1}. If the deviant deviates again (fails to cooperate in period t+1) then $\overline{q}^i(t+2)$ is initiated. That is, the same $\overline{q}^i$ restarts in period t+2. The player who was cooperating in period t continues to make no transfers to the currently deviant player. To comply with the punishment, the deviant must make the initial equilibrium transfer that starts him or her on an efficient equilibrium path giving all of the surplus possible in a subgame perfect equilibrium to the initially cooperative player. Subsequently, the player who did not deviate resumes positive transfers consistent with the new punishment path given the evolution of states.

Without loss of generality, we can assume that all paths are unilateral transfer paths since any payoff vector in $W(y_t)$ is attainable this way and all paths are supportable by $\overline{q}^0$ and $\overline{q}^1$. We label the punishments using unilateral transfers that yield the deviant zero surplus and are on the frontier of $W(y_t)$, $\tilde{q}^0(t)$ and $\tilde{q}^1(t)$.

Farrell and Maskin (1989) define a subgame perfect equilibrium , σ, to be weakly renegotiation-proof if no member of the set of paths generated by σ in feasible histories of actions strictly Pareto-dominates another member of the set. A weakly

renegotiation-proof equilibrium is renegotiation-proof in the sense that if the players agree on a strategy profile σ at the outset and the history of play at date t means that a path q should be followed, they do not have a joint incentive to switch to another path generated by σ. In particular, they both cannot gain by abandoning a punishment for the initial path.[12]

A weakly renegotiation-proof equilibrium survives the possibility of such "internal" renegotiations (in the terminology of Pearce), but there may exist a path generated by another subgame perfect equilibrium strategy profile that is preferred by both agents in some history. Farrell and Maskin define a strongly renegotiation-proof equilibrium to be a weakly renegotiation-proof equilibrium such that none of the paths it generates in any feasible history of play is strictly Pareto-dominated by another weakly renegotiation-proof equilibrium.[13]

The properties of the sets of subgame perfect equilibrium paths and payoffs proved in Propositions 1 and 2 can be used to find punishments in Ω^P which support any path in Ω^P in a simple strategy profile that is weakly renegotiation-proof. Rather than prove this result, we allow the two agents to negotiate over the strategy profile that is adopted and show that a strongly renegotiation-proof equilibrium exists for all discount rates. In our model, the agents cannot commit themselves not to renegotiate the strategy profile they are following, mutually agreeing on another that incorporates credible threats of punishment for deviation. Also, by emphasizing initial paths that are efficient in the set of paths supported by credible threats, we anticipate assuming free entry by lenders in initial contracts in Section 7 below.

We call an equilibrium renegotiation-proof if it is renegotiation-proof by the Farrell and Maskin definition, replacing the weak Pareto rule with the strong rule by which one allocation dominates another only if it makes each party at least as well off and one party strictly better off. In our model the weak rule is unnecessary for existence of equilibrium.

24

PROPOSITION 4: There exists a subgame perfect equilibrium σ such that for every feasible w_t and $h_t \in H_t$, the payoff vector $\pi(r(\sigma), w_t) + \upsilon(r(\sigma), w_t)$ is Pareto efficient in $W(y_t)$, where $r(\sigma)$ is the path generated by σ in history h_t. The strategy profile $\sigma[s, \tilde{q}^0, \tilde{q}^1]$, where s is an efficient unilateral transfer path in Ω^P, is strongly renegotiation-proof.

PROOF: Because Ω^P is compact in the product topology, there exists an $r \in \Omega^P$ such that the payoff vector, $\pi(r, w_t) + \upsilon(r, w_t)$, is Pareto efficient in the set $W(y_t)$, for every $y_t \in \{y^1, \ldots, y^N\}$, for all $t \geq 1$. In particular, there is an initial path $s \in \Omega^P$ that yields any Pareto efficient payoff vector in $W(y_1)$. Since $\upsilon(s, w_{t-1}) = \beta E(\pi(s, w_t) + \upsilon(s, w_t))$ and Bellman's Principle holds, the payoff vector, $\pi(s, w_t) + \upsilon(s, w_t)$ is Pareto efficient in $W(y_t)$ for every $y_t \in \{y^1, \ldots, y^N\}$, for each feasible $w_{t-1}, t > 1$. (The expectation is taken with respect to the distribution of y_t.) Likewise, the payoff vectors sustained by $\tilde{q}^0(t')$ and $\tilde{q}^1(t')$ are Pareto efficient in $W(y_t)$ for all $t \geq t' \geq 1$.

Because the strategy profile, $\sigma[s, \tilde{q}^0, \tilde{q}^1]$, is in Σ^P for all $s \in \Omega^P$, it is a subgame perfect equilibrium for every $s \in \Omega^P$ that yields a payoff vector on the Pareto frontier of $W(y_1)$. After every feasible history of play and of nature, σ sustains a payoff vector on the Pareto frontier of $W(y_1)$. Therefore, $\sigma[s, \tilde{q}^0, \tilde{q}^1]$ is strongly renegotiation-proof for any efficient $s \in \Omega^P$.

Proposition 4 demonstrates the existence of an equilibrium that survives the possibility of renegotiation by mutual agreement. Furthermore, it shows that an equilibrium path of transfers that is efficient among those that are supported by trigger strategy punishment threats is also sustainable in a renegotiation-proof equilibrium. The equilibrium profile generates a path in every history of nature and of play that is Pareto-undominated by an other subgame perfect equilibrium path.

In our strongly renegotiation-proof equilibria, if agent i deviates by making a smaller transfer than required in the equilibrium path at some date (for any feasible history), then the other agent j refuses to provide a positive transfer to agent i until after agent i cooperates in his or her punishment. Agent j can only lower his or her utility by agreeing to abandon the punishment $\tilde{q}^i$ on the next date, and agent i cannot increase his or her payoff by again unilaterally deviating, this time from the path $\tilde{q}^i$. For any deviation by agent i from an efficient perfect equilibrium path, agent j can do no better than to carry out his or her part of the punishment $\tilde{q}^i$. This means that any attempt by agent i to renegotiate $\tilde{q}^i$ fails. A refusal to accept an attempted renegotiation of the punishment is credible because the other agent maximizes his or her payoff by reinitiating $\tilde{q}^i$ in the period after agent i make s smaller payment than prescribed by the punishment.

Pearce (1987) suggests an alternative definition of renegotiation-proofness which allows strategy profiles with Pareto-ranked continuation equilibria. In symmetric games, his definition requires that the path generated by the strategy profile after every history not be strictly Pareto dominated by the worst possible outcome under another strategy profile. In general, a strategy profile σ is called renegotiation-proof by his definition if there exists no other strategy profile σ^1 in Σ^P which generates paths in all histories at least as good in a Pareto sense as some path generated by σ and such that some worst equilibrium outcome for σ^1 strictly Pareto-dominates an equilibrium outcome for σ. Pearce emphasizes renegotiation between strategy profiles ("external renegotiation") to the exclusion of renegotiation within an equilibrium ("internal renegotiation"). Abreu, Pearce and Stacchetti (1989) define a strategy profile to be a consistent bargaining equilibrium if it is renegotiation-proof in the sense of Pearce (1987).

For our model, the strategy profile $\sigma\left[s, \tilde{q}^0, \tilde{q}^1\right]$ for any efficient subgame perfect equilibrium path, s, is a consistent bargaining equilibrium.[14] Although this concept of

renegotiation differs significantly from the approach taken by Farrell and Maskin, the choice of definition of renegotiation-proofness is not instrumental for our consumption-smoothing game. With the zero-sum stage-game, each definition yields the same set of efficient renegotiation-proof equilibria. The simple strategy profile $\sigma[s, \tilde{q}^0, \tilde{q}^1]$ is renegotiation proof (either definition) and sufficient[15] to support the efficient paths in Ω^P.

Because the initial paths generated by our renegotiation-proof equilibria are efficient among the set of subgame perfect equilibrium paths, we state two results for limits in the discount factor. The first follows from the Folk Theorem for repeated games with discounting proved by Fudenberg and Maskin (1986): There exists a value for the discount factor, $0 < \overline{\beta} < 1$, such that for all $\beta \geq \overline{\beta}$, every renegotiation-proof equilibrium path fully smooths the borrower's consumption. The second is that there exists another value for the discount factor, $0 < \hat{\beta} < 1$, such that for all $\beta \leq \hat{\beta}$, the only subgame perfect equilibrium is permanent autarky. This result is straightforward.

5. THE DYNAMICS OF INTERNATIONAL TRANSFERS

In this Section, we discuss the dynamics of lending and repayment in a strongly renegotiation-proof equilibrium path for the long-term debtor-creditor relationship. In anticipation of introducing free entry with many potential lenders, we characterize the initial path in which all of the surplus in the relationship in the first period goes to the borrower. This is the path chosen by the borrower when an exclusive relationship is formed at the outset in the presence of free entry. The punishment paths are found by solving a similar problem, giving all the surplus at the initiation of the punishment to the non-deviating party.

The equilibrium strategy profile, $\sigma[s, \tilde{q}^0, \tilde{q}^1]$, such that all the initial surplus in the exclusive relationship goes to the borrower and s is efficient in Ω^P is denoted σ^*. The problem of finding the path $s(\sigma^*)$ is to derive the solution to the dynamic program:

$$\max \left\{ \pi_0\left(s, w_t\right) + \upsilon_0\left(s, w_t\right) \right\}$$

Subject to $\pi_i\left(s, w_t\right) + \upsilon_i\left(s, w_t\right) \geq \overline{\pi}_i\left(s, w_t\right) + \upsilon_i\left(q^i(t+1), w_t\right) = 0$,

and subject to $\pi_1\left(s, y_1\right) + \upsilon_1\left(s, y_1\right) = 0$

for each i and for all w_t and t,

with respect to unilateral transfer paths, $s \in \Omega^P$.

This problem is identical to determining the path for a subgame perfect equilibrium whose outcome is Pareto-undominated by that for any other subgame perfect equilibrium for this model. Our first result is:

PROPOSITION 5: Along the initial path $s\left(\sigma^*\right)$, the borrower's consumption plan and the net transfers made between the agents are unique for each w_t.

PROOF: Because u(c) is strictly concave, so is $\pi_0\left(s, y_1\right) + \upsilon_0\left(s, y_1\right)$, and Ω^P is convex. Therefore, $\pi_0\left(s, w_t\right)$ is unique for every w_t for every solution path s, so that the path of net transfers supported by $s\left(\sigma^*\right)$ is unique.

Because the frontier of the set of subgame perfect equilibrium outcomes is concave and differentiable, we can state the dynamic program for finding $s\left(\sigma^*\right)$ as a straightforward concave programming problem. In their paper on implicit wage contracts, Thomas and Worrall (1988) analyze the dynamics of wages for an efficient contract in a consumption-smoothing problem similar to ours. They assume that exclusive relationships must be formed at the initial date and that any departure from the implicit contract leads to permanent reversion to the Nash equilibrium in the single period game. We use their results to describe the dynamics of borrower consumption and net transfers along the path $s\left(\sigma^*\right)$ for each w_t.

Because $s(\sigma^*)$ gives a unique action profile and unique ordered pair of continuation values for each history w_t, we can state the problem of determining $s(\sigma^*)$ in the following form. Let $V_1(w_t, y^\ell)$ denote the surplus the lender receives along the initial paths when state y^ℓ occurs at time $(t+1)$ after the history of nature w_t. Let $V_0(V_1; y^\ell)$ be the efficient frontier of all subgame perfect equilibrium payoffs in state y^ℓ. $V_0(V_1; y^\ell)$ denotes the surplus the borrower receives if V_1 is the payoff of the lender in state y^ℓ. The path $s(\sigma^*)$ is found by solving for each state $k = 1, \ldots N,$:

$$\text{maximize } \left(u(c_t) - u(y^k)\right) + \beta \, EV_0\left(V_1(w_t, y^\ell); y^\ell\right)$$

$$\text{with respect to } c_t, V_1(w_t, y^1), \ldots V_1(w_t, y^N)$$

$$\text{subject to } V_0\left(V_1(w_t, y^\ell); y^\ell\right) \geq 0, \text{ for all } y^\ell,$$

$$V_1(w_t, y^\ell) \geq 0, \text{ for all } y^\ell, \text{ and}$$

$$\left(y^k - c_t\right) + \beta \, EV_1(w_t, y^\ell) = V_1(w_{t-1}, y^k) \geq 0.$$

The expectation is taken over the distribution of y^ℓ, for $\ell = 1, \ldots, N$. The surplus of the borrower in the relationship is maximized with respect to her current consumption and the payoffs for the next period promised the lender in equilibrium.

Because this is a concave programming problem, the necessary conditions for an optimum are also sufficient. To find $s(\sigma^*)$, we set $V_1(y_1)$ equal to zero, as implied by initial free entry of lenders. The first-order conditions for maximization of the implied Lagrangian and the envelope condition yield:

$$u'(c(w_t)) = (1 + \varphi_\ell)u'(c(w_t, y^\ell)) - \psi_\ell,$$

$$\varphi_\ell \geq 0 \text{ and } \varphi_\ell \cdot V_0\left(V_1(w_t, y^\ell); y^\ell\right) = 0, \text{ and}$$

$$\psi_\ell \geq 0 \text{ and } \psi_\ell \cdot V_1(w_t, y^\ell) = 0,$$

for each $\ell = 1, \ldots, N$, where $c(w_t, y^\ell)$ is consumption at time t+1 in state ℓ, $\beta p_\ell \varphi_\ell$ is the Lagrange multiplier for the constraint $V_0(V_1(w_t, y^\ell); y^\ell) \geq 0$ (where p_ℓ is the probability that state ℓ occurs), and $\beta p_\ell \psi_\ell$ is the Lagrange multiplier for the constraint $V_1(w_t, y^\ell) \geq 0$. The punishment path $\tilde{q}^1(t)$ is found in the same way, reinterpreting t as the first period after the most recent lender deviation. The punishment path for a borrower deviation in period t - 1, $\tilde{q}^0(t)$, is found similarly, but maximizing lender profits with the borrower's surplus constrained to be non-negative.

The dynamics of $s(\sigma^*)$ are summarized in the following result adapted from several propositions in Thomas and Worrall (1988). For completeness, we offer a proof in the appendix that is much simpler in parts that the proofs in Thomas and Worrall. Our version extends readily to a model in which the borrower's endowment follows a Markov chain displaying first-order stochastic dominance.

PROPOSITION 6: For any history of nature, $w_t = (w_{t-1}, y_t)$, borrower consumption in the path $s(\sigma^*; w_t)$ is restricted to a closed internal, $[\underline{c}^k, \bar{c}^k]$, where $y_t = y^k$, and is given by

$$c_t = c(w_t) = \begin{cases} \bar{c}^k, \text{ if } c(w_{t-1}) > \bar{c}^k \\ c(w_{t-1}), \text{ if } \underline{c}^k \leq c(w_{t-1}) \leq \bar{c}^k \\ \underline{c}^k, \text{ if } c(w_{t-1}) \leq \underline{c}^k. \end{cases}$$

Furthermore, $\bar{c}^k > \bar{c}^\ell$ and $\underline{c}^k > \underline{c}^\ell$ for $y^k > y^\ell$ and $y^k \in [\underline{c}^k, \bar{c}^k]$ for each y^k, and $y^N = \bar{c}^N$, $y^1 = \underline{c}^1$.

PROOF: See appendix.

In an exclusive relationship that yields a Pareto-undominated payoff within the set of subgame perfect equilibrium outcomes for each state of nature, the consumption of the borrower and net transfer made by one party at each date each follow Markov chains. For large enough $\beta, (\beta \geq \overline{\beta})$, full consumption smoothing results, so that the transfer made each period in $s(\sigma^*)$ is identically and independently distributed. In this case, all of the intervals $[\underline{c}^k, \overline{c}^k]$ overlap for $k = 1, \ldots N$. If, on the other hand, β is less than $\hat{\beta}$, then for each k the interval is a single point, y^k and there is no smoothing. The upper end of any given interval, $\overline{c}^k$, is the consumption that the borrower realizes in state y^k in an efficient path if all the surplus in the relationship from that date forward goes to the borrower, and similarly, for the lender when borrower's consumption is $\underline{c}^k$.

For β between $\overline{\beta}$ and $\hat{\beta}$, the borrower's consumption follows a non-trivial Markov chain even though her endowment is identically and independently distributed across dates. This is intuitive because possible consumption levels next period are being planned in the current period, and the efficient path smooths these as much as possible subject to the absence of commitment and the limits on punishments. (No agent can be forced to provide any given transfer.) If a transfer in some state for the next period that provides the risk-averse borrower with identical consumption to that in the current period leaves both agents with non-negative surplus, then any divergence in the consumption levels, holding constant consumption in all other states next period, would reduce utility for at least one of the agents. History matters because planned consumption is smoothed between today and tomorrow to the greatest extent possible so that next period's transfers depend upon today's consumption and tomorrow's realized state of nature. The Markov dependence follows from forward-looking behavior, in contrast to the role of the history of actions.

Figure 1 portrays the intervals for possible values of the borrower's consumption in an efficient subgame perfect equilibrium path for an example such that

31

$\hat{\beta} < \beta < \bar{\beta}$ and y can take on four possible values. The borrower's endowment is recorded on the horizontal axis and her consumption on the vertical. The vertical bars denote the intervals, $\left[\underline{c}^k, \bar{c}^k \right]$.

The reader is invited to track a history for $s(\sigma^*)$ in Figure 1. Let the first period (period 1) endowment of the borrower be y^3. Her consumption at date 1 is given by $\bar{c}^3$. Now, if her period 2 endowment is y^3 or y^4, then her consumption is the same as in period 1. If her endowment instead falls to y^2 or y^1, then her consumption falls to either $\bar{c}^2$ or $\bar{c}^1$. Now suppose that her endowment takes the following history for six periods, $w_6 = (y^3, y^4, y^2, y^3, y^1, y^4)$. The borrower' consumption follows the plan, $(\bar{c}^3 \; \bar{c}^3, \bar{c}^2 \; \bar{c}^2, \bar{c}^1 \underline{c}^4)$. This is the path ABCDEF indicated by the dashed line in Figure 1. After y^1 has occurred for the first time, the transfers follow the stochastic steady state. If, for example, the endowment history $(y^3, y^4, y^2, y^3, y^1, y^4)$ immediately recurs starting in period 7, income-consumption combinations from periods 7 through 12 are the points GFHGEF, consumption being constant at $\underline{c}^4$ in the periods 7 through 10, then falling to $\bar{c}^1$ in period 11, returning to $\underline{c}^4$ again in period 12. There are only two consumption levels in the steady state in this example, $\bar{c}^1$ and $\underline{c}^4$. Because $\underline{c}^4$ is less than the higher consumption levels $\bar{c}^2, \bar{c}^3$ or $\bar{c}^4$ that might be observed in the transition, the latter never recur in the steady state. Consumption below $\bar{c}^1$ never occurs on the equilibrium path $s(\sigma^*)$, either in the transition or in the steady state.

In such cases of partial smoothing in the steady state, the net transfer to the borrower is larger at a given y_t if the endowment is falling $(y_t < y_{t-1})$ than if it is increasing, even though y is i.i.d. Her consumption is Markovian, as noted above, not i.i.d. as asserted by Grossman and von Huyck (1988). A testable implication of this model is that net transfers are positively related to their first lag, and negatively related to the first difference of the level of the endowment.

If $\beta \geq \bar{\beta}$, the steady state is fully smoothed at $\bar{c}^1$, as depicted in Figure 2. But the steady state is not reached until the first period in which y^1 occurs. If the initial

sequence of endowment realizations starting with period 1 is, for example, $\left(y^4, y^3, y^4, y^2, y^1\right)$, then consumption follows the path JKLMN in Figure 2, consumption permanently remaining at $\bar{c}^1$ after period 5.

The equilibrium path is enforced by the punishment paths in the simple strategy profile $\sigma^*\left[s, \tilde{q}^0, \tilde{q}^1\right]$. A punishment $\tilde{q}^i(t+1)$ starts in period $t+1$ whenever the borrower or lender singly deviates from the path (whether the equilibrium path $s(\sigma^*)$ or a punishment path) in force in period t. In a simple strategy profile a punishment is independent of the size of deviation taken, so a deviant would rationally choose to make a zero transfer.[16]

The punishment of one agent gives the other all of the maximum surplus available from the relationship, from $t+1$ onwards. The only social loss caused by a one-period deviation by the borrower is the reduction in feasible smoothing between periods t and $t+1$. In period $t+1$, cooperation by the borrower in her punishment is Pareto-undominated in the set of all subgame perfect equilibrium outcomes. If the borrower does not cooperate in the punishment, (i.e. if she deviates from $\tilde{q}^0(t+1)$), the punishment merely re-starts as $\tilde{q}^0(t+2)$. She can get at most zero surplus after she deviates from any strategy she may choose. Her continuation value from cooperation in the punishment $\tilde{q}^0(t+1)$ is the same as under permanent autarky.

To see how the punishments work, consider, for example, a deviation by the borrower from the equilibrium path $s(\sigma^*)$ in period 2 in the example illustrated in Figure 1, when the borrower's endowments are, as before, $\left(y^4, y^2, y^3, y^1, y^4\right)$ for periods 2 through 6. Instead of paying $y^4 - \bar{c}^3$ in period 2 and receiving positive surplus measured by $\bar{c}_3 - \underline{c}^4$, as at point B in Figure 1, the borrower deviates, paying nothing and consuming y^4 at point Q. But she loses surplus measured by $\bar{c}_3 - \underline{c}^4$ from the deviation.

When the borrower cooperates in her punishment $\tilde{q}^0(3)$, her consumption path after period 2 follows a new efficient path RSTF illustrated by the dashed lines in

33

Figure 3, and subsequently enters a stochastic steady state path of consumption identical to that observed on the initial equilibrium path $s(\sigma^*)$. But until period 5, consumption is lower than it would have been under $s(\sigma^*)$. The net social loss from the deviation is the loss, as of period 2, of the feasible smoothing between consumption in period 2 and the vector of possible consumption levels in period 3. A comparison of Figures 1 and 3 shows that, given the realization y^2 occurred in period 3, consumption was increased when it was higher (in period 2) and decreased when it was lower (in period 3) by the deviation in period 2. What happens if the borrower does not cooperate in $\tilde{q}^0(3)$ in period 3? In period 4, the endowment draw is y^3. If the borrower cooperates in the new punishment $\tilde{q}^0(4)$, she pays $y^3 - \underline{c}^3$ and rejoins at point S the path $\tilde{q}^0(3)$ discussed immediately above. If the borrower continues to deviate, then the punishment restarts as $\tilde{q}^0(5)$ in period 5. Instead of making the (below steady state) transfer in period 5, prescribed by $\bar{q}(3)$ that would raise the borrower's consumption to point T, the lender continues his moratorium on transfers, so the borrower consumes only $\underline{c}^1$. If the borrower then cooperates in period 6 by making a transfer of $y^4 - \underline{c}^4$, the punishment path subsequently follows the same path as $s(\sigma^*)$.

In the steady state at maximum income y^4 the borrower is left with zero surplus under $s(\sigma^*)$ or any punishment of her, $\tilde{q}^0(\cdot)$, in contrast to the dynamic situation in period 2, discussed above, where compliance leaves her positive surplus. But she still gains nothing from deviation from the equilibrium transfer $(y^4 - \underline{c}^4)$.

The punishment path does not necessarily eventually converge to the path $s(\sigma^*)$, as in the above example. The plan followed under $\tilde{q}^0(t+1)$ if the borrower deviates in the steady state in period t for the case shown in Figure 2, with endowment realizations $(y_t, y_{t+1}, y_{t+2}, y_{t+3}) = (y^4, y^3, y^4, y^1)$, follows the dashed path shown in Figure 4. By deviating, the borrower consumes y^4 in period t instead of $\bar{c}^1$, but then suffers a fall in consumption to $\underline{c}^3$, with consumption constant at $\underline{c}^4$

thereafter, permanently below the steady state level $\bar{c}^1$ on the equilibrium path $s(\sigma^*)$. Since in y^4 compliance with consumption smooth at $\bar{c}^1$ yields positive surplus measured by $\bar{c}^1 - \underline{c}^4$, the borrower in this case is strictly worse off in period t by deviating and obtaining zero surplus.

For deviation in period t by the lender, the punishment $\tilde{q}^1(t+1)$ means that all surplus goes to the borrower in t+1, just as in the initial period of the relationship in the same state as w_{t+1}, under free entry. Thus the punishment is just the re-initiation of the equilibrium path. In Figure 2 with $y_t = y^2$, consider lender deviation in the steady state from his equilibrium transfer $\bar{c}^1 - y^2$ stipulated by $s(\sigma^*)$. If the next four endowment realizations are (y^3, y^4, y^2, y^1) then the punishment path $\tilde{q}^1(t+1)$ passes through points KLMN, following the same sequence of transfers as in the initial path $s(\sigma^*)$ illustrated in Figure 2 for periods 2 through 5, discussed above, thereafter keeping borrower consumption constant at $\bar{c}^1$. The lender loses, (ex ante), $\bar{c}^2 - \bar{c}^1$ from the deviation in this example. Had the lender's deviation from $s(\sigma^*)$ occurred in state y^1, the entire burden of the social loss would have fallen on the borrower, but the lender would still have gained nothing from the deviation.

If a punishment of the borrower $\tilde{q}^0(\cdot)$ were in force in the steady state in this case (see Figure 4) with current state $y_t = y^1$, lender deviation from $\tilde{q}^0(\cdot)$ leads to a new lender punishment path $\tilde{q}^1(t+1)$ which, when followed causes a loss in lender surplus as of period of t of $\bar{c}^1 - \underline{c}^4$. Borrower consumption rises in t + 1 to at least $\bar{c}^1$, the steady state level under $s(\sigma^*)$, and converges to that level from above in the steady state.

The above punishments are extremely simple. There is a moratorium on transfers to the deviant until the deviant complies with the punishment path starting or re-starting in the current period. This path transfers all the surplus anticipated in the relationship to the aggrieved party. Along the punishment path, the borrower's consumption is smoothed relative to last period's consumption to the maximum extent

feasible, in general delaying convergence to the stochastic steady state to the advantage of the aggrieved party. This is shown in Figure 2 for a case of deviation by the lender, and in Figure 3 for deviation by the borrower. After the moratorium is lifted, the deviant in most cases continues to receive reduced net transfers for some periods.

The meaning of renegotiation-proofness of the simple strategy profile $\sigma^*[s, \tilde{q}^0, \tilde{q}^1]$, and the contrast with trigger strategies, are illuminated by Figure 5. Proposition 6 implies that the maximum payoff that the lender can get in any perfect equilibrium is increasing in the current state y_t, and the maximum payoff that the borrower can attain over all perfect equilibria is decreasing in y_t. Figure 5 shows the frontiers of surpluses from the repeated game for the two extreme endowment states y^1 and y^N and for a β such that a cooperative game exists $\left(\beta \geq \hat{\beta}\right)$. The intersection of the frontier, $V_0\left(V_1, y^1\right)$ with the horizontal axis at point C shows the surplus from the repeated game on the equilibrium path $s(\sigma^*)$ in period t if $y_t = y^1$. In that state the lender is making the unilateral transfer to the borrower, who receives all the surplus.

If, on the other hand, the borrower deviated the previous period then the punishment $\tilde{q}^0(t)$ yields surpluses represented by point A. The borrower gets no surplus. The lender, who withholds the transfer he would have made had the borrower not deviated, gets zero payoff in the stage game but all the surplus, OA units of expected present value of profit, from the future transfers along the punishment path.

Similarly point B shows the surpluses that accrue in the steady state on the equilibrium path $s(\sigma^*)$ in period t if $y_t = y^N$. The borrower, who makes the transfer in this case, gets no surplus; she transfers it all to the lender. If in any state w_t the lender alone has deviated at the most recent date k at which he would in equilibrium have made a transfer, then the surpluses are the ordered pair at point D. The lender gains nothing, and the borrower gets no payoff in the stage game but positive surplus

from future transfers along the punishment path $\tilde{q}^1(k+1)$. The horizontal distance DC measures the social loss in terms of the loss in borrower utility from deviation by the lender when the current state is y^1. (Remember y is i.i.d. and OD is the surplus V_0 when the stage-game payoffs are zero and a punishment of the lender is in effect next period.) The vertical distance BA shows the loss in expected profit to the lender from borrower deviation when the current state is y^N. The social cost of non-cooperative behaviour by either party is positive, in general, because a feasible opportunity for smoothing the risk-averse borrower's consumption between the current period and the next is foregone.

Permanent reversion to autarky as in trigger strategy punishments would leave the surpluses at point O. The distances AO and OD represent the gains from "renegotiations" of trigger strategy punishments by the borrower and lender respectively for deviation in current state y_t. Since A and D both Pareto dominate O, the trigger strategies are not renegotiation-proof under alternative current conceptions, including those of Farrell and Maskin (1989) and of Pearce (1987).

6. IMPLEMENTATION VIA SIMPLE DEBT CONTRACTS WITH SENIORITY:

THE ROLE OF THE DEBT BURDEN

The renegotiation-proof consumption smoothing transfers described above can be implemented via simple debt contracts, between the borrower and a competitive lender in a lending market with free entry, subject to the seniority privilege. The seniority privilege means that if any contractually specified loan repayment obligation has not been fulfilled, the lender has an enforceable right to any international financial transfers (repayments, investments, or loans) made by the debtor. Seniority is common knowledge.

The loan contract specifies the identities of the parties, and the sequence of formal repayment obligations (principal plus interest). The net transfers actually

made ("loan repayments" less "new loans") are renegotiated between borrower and senior lender, so that net transfers in equilibrium are determined by $s(\sigma^*)$ regardless of the details of the formal contract. Renegotiation here has an especially simple form. No explicit sequence of offers and counteroffers, familiar in simultaneous bargaining games, is necessary. The essential act of renegotiation is merely the unilateral transfer (loan or repayment) made by one party and received by the other. If the transfer deviates from the equilibrium transfer, a punishment phase is then initiated by the recipient. But it is in the interests of the recipient to accept any positive transfer that the deviant offers; renegotiation does not entail any refusal of an offered transfer. The equilibrium renegotiation is enforced by the credible threats embodied in the punishments $\tilde{q}^0$ and $\tilde{q}^1$.

The punishment $\tilde{q}^1$, which motivates the lender to make a positive net transfer in equilibrium when the borrower's endowment realization is low, is simply imposition of a debtor repayment moratorium whenever new loans are less than the equilibrium level. The moratorium lasts until a new positive-valued loan is made, consistent with the efficient path, that yields all surplus to the debtor in the current state $y^i, i = 1, 2, ..., N$. This loan is the same as the initial loan in the same state y^i under free entry into a new lending relationship.

Similarly, the punishment $\tilde{q}^0$, initiated when the endowment realization is high and net repayments fall short of the equilibrium level, is a moratorium on new loans. This moratorium persists regardless of shocks to the borrower's endowment, until such time as a new positive repayment is made to the lender with a magnitude consistent with the efficient path that yields all surplus to the lender in the state prevailing at the time of the repayment.

If the lender's punishment of a deviant borrower under $\tilde{q}^0$ is to support the efficient smoothing relationship with a competitive lending market, the borrower must be unable to obtain any funds from any other competitive lenders during the

punishment. This means that the formal full repayment obligation contractually specified for any period (the payment that must be made for the lender's seniority right to lapse) must be at least as large as the maximum present value of repayments that could be required up to that period under the punishment $\tilde{q}^0$. Otherwise, a new competitive lender could in some state buy out the old lender's seniority by paying off the contractually specified obligation, loan a little more, and still make non-negative profits. Anticipation of this would render the punishment incredible.

Free entry into initial loan contracts ensures that the equilibrium strategy profile $\sigma^*[s, \tilde{q}^0, \tilde{q}^1]$ is unique for each w_t. Assuming in addition strict seniority privileges, the equilibrium loan contract effectively gives permanent seniority to an exclusive lender (or, equivalently, a group of lenders). Consider the situation depicted in Figure 1. If a loan contract is initiated in period 1 in the second highest state, with endowment y^3, the value of the competitive initial loan is the expected present value of the repayments, $y^4 - \bar{c}^3$, which occur in those immediately subsequent periods, in any, in which y^4 occurs before the next realization of a lower value of y. On the equilibrium path, the borrower's consumption in those periods equals her consumption in the first year of the loan; consumption is completely smooth over those periods.

Suppose instead that the loan is formally paid off if there is a finite number n of successive occurrences of y^4 and equal repayments of the loan after the first period. Then the initial loan, and initial consumption would have to be a little lower than if n were infinite. Furthermore if y^4 should recur in period n+2, consumption would rise to y^4. (No new smoothing loans would be taken until y falls below its maximum value.) If n is infinity, transfers with the same zero expected present value completely smooth the consumption that occurs until y first falls below y^3, and at a higher level. So an infinite n is strictly preferred by the risk-averse borrower. Thus the option of eventually paying off the loan is, in equilibrium, sold off by the borrower at the initial period of borrowing in exchange for a larger loan .

More generally, the borrower chooses a contract such that her consumption is smoothed as much as it can be across dates and histories of nature subject to the constraints imposed by subgame perfection. In particular, she chooses a contract such that her first period consumption cannot be further increased without lowering consumption in some future history in which it is already equal to or lower than her first period consumption along the equilibrium path. In some future histories, her consumption along an equilibrium path can be higher than her first period consumption, but in these histories the individual rationality constraint will be binding (that is, her surplus over permanent autarky consumption will equal zero). If there is a positive probability that her debt will go to zero in finite time, then the borrower is anticipating that in some future histories her consumption will be higher than it is today and she will be getting positive surplus. She can reduce her consumption in these histories further and raise her (lower) present consumption by eliminating the possibility of paying off the debt.

In a loan contract, permanent seniority is achieved by ensuring that the repayment obligation (principal plus interest) exceeds, for any possible sequence of states, the value of the maximum net repayments possible along the equilibrium path. This means the interest contractually specified for the loan must exceed the maximum equilibrium next-period repayment. In the transition to the stochastic steady state, the maximum net repayments possible rise monotonically, in general, along the equilibrium path so that the debt burden must rise to maintain a permanent relationship. However, if any state of nature recurs an increase in the debt burden is unnecessary and once the stationary state is reached further increases are also unnecessary. The minimum interest rate payment (principal plus interest) that needs to be charged, applied to the entire accumulated balance, declines in equilibrium each period during the transition. This appears consistent with the tendency for spreads in

interest rates for a new borrower to decline over time, as found empirically by Ozler (1988a).

7. CONCLUSION

We have modelled international lending to a sovereign as a repeated game played between a risk-neutral lender and a risk-averse borrower. In this model, one agent makes a unilateral transfer of a single good to the other at any date in anticipation of future reciprocal cooperation in the consumption-smoothing relationship. In contrast with the model of Bulow and Rogoff (1989) in which trade sanctions are exchanged for contemporaneous repayments each period, renegotiation of a formal simple debt contract is not appropriately modelled as a strategic Nash bargaining game. Actions by the two players are sequential, not simultaneous, so that repayments by the borrower and new loans by the lender are made to deter threatened punishment consisting of future noncooperative play by the other agent. Strategies, including punishments, are not only subject to unilateral deviation by either agent, but must also survive the possibility of joint agreements to abandon them in favor of alternative strategies in any history.

We have shown that any efficient subgame perfect equilibrium path for the two-person repeated game can be supported by an equilibrium strategy profile that is renegotiation-proof under alternative (and very different) current definitions in the game theory literature. In our model, the lender and the borrower are treated in an essentially symmetric fashion: neither agent can commit his or her future actions. The borrower makes transfers to the lender to assure future consumption-smoothing inflows adequate to leave her at least as well off in every history as she would be in permanent autarky. Likewise, the lender makes a positive transfer only if, looking forward from that date on, he obtains non-negative expected profit by doing so; the lender cannot commit to make an insurance contract. This contrasts with models in

41

which one-sided commitment by the lender is possible. Asymmetries arise in our model in the difference in attitudes towards risk (that generate gains from trade) and by assuming free entry in the initial contracts (which gives all of the initial surplus to the borrower).

With free entry by lenders in the presence of a seniority privilege, the initial equilibrium contract creates an exclusive long-term relationship between the borrower and a lender. Standard simple debt contracts can be used to establish a permanent relationship, with renegotiation of the formal crude terms of the contract assuring that an efficient path of transfers is followed, subject to the perfection constraints. In this repeated setting, with a sufficiently low discount rate, the debtor-creditor relationship is not the simple one that a standard debt contract specifies. However, state-contingent contracts are unnecessary to achieve at least partial smoothing in the presence of potential renegotiation of a long-term relationship.

The formal contract serves only to create the permanent relationship; a simple debt contract used in equilibrium will be one such that the debt burden at all dates, including the first, is large enough that it can never be paid off in any possible history of nature following the equilibrium path. We believe that this sheds some light on the nature of debt relationships in general. A crude contract (incorporating no state-contingent clauses) is sufficient to sustain an efficient intertemporal allocation for parties with different attitudes towards risk under uncertainty. This is true even though there is no asymmetry of information (induced, for example, by costly verifiability of the borrower's endowment) as in one-period credit market models in the literature.

Our assumption of a strict seniority privilege might seem to imply that an external authority, such as a creditor country government, is necessary for our results. In an extension of this study, we show that the efficient equilibrium path can be supported by a renegotiation-proof equilibrium strategy profile under free entry by

lenders in the absence of any external enforcement. Seniority is assumed in this paper so that we may concentrate on renegotiation and the dynamics of the consumption-smoothing relationship.

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APPENDIX

PROOF OF PROPOSITION 6:

Note that the function $V_0(V_1; y_1)$, which gives the Pareto frontier of the set of all SPE payoffs (for feasible V_1), is differentiable on the interior of its domain (the interval $[0, \overline{V}_1(y_1)]$ such that $V_0(V_1; y_1) \geq 0$) because u(c) is concave and differentiable and the set of paths Ω^P is convex and compact. We can form the Lagrangian for the concave programming problem for finding $s(\sigma^*)$ with multipliers, $\beta p_\ell \varphi_\ell$ and $\beta p_\ell \psi_\ell$ for the constraints $V_0 \geq 0$ and $V_1 \geq 0$, respectively, for each state $y_{t+1} = y^\ell$, and multiplier λ_k for the constraint

$$\left(y^k - c_t\right) + \beta \mathrm{EV}_1\left(w_t, y^\ell\right) = V_1\left(w_t\right).$$

The necessary and sufficient conditions for an optimum are

$$\lambda_k = u'\left(c\left(w_t\right)\right),$$

$$-\lambda_k = \psi_\ell + \left(1 + \varphi_\ell\right) \frac{dV_0\left(V_1; y^\ell\right)}{dV_1},$$

$$-\lambda_k = \frac{dV_0\left(V_1; y^\ell\right)}{dV_1}, \text{ and}$$

$$\lambda_k \geq 0, \ \varphi_\ell \geq 0, \ \psi_\ell \geq 0, \ \varphi_\ell \cdot V_0\left(V_1; y^\ell\right) = 0, \text{ and } \psi_\ell \cdot V_1 = 0.$$

We define $\underline{c}_k$ and $\overline{c}^k$ by

$$u'\left(\underline{c}_k\right) = -\frac{dV_0\left(\overline{V}_1^k; y^k\right)}{dV_1}, \text{ where } V_0\left(\overline{V}_1^k; y^k\right) = 0,$$

$$\text{and } u'\left(\overline{c}_k\right) = -\frac{dV_0\left(0; y^k\right)}{dV_1}.$$

44

Recall that $V_0(V_1; y)$ is downward-sloping and strictly concave.

The first-order conditions imply that

(i)   if $V_0(w_t, y^\ell) = 0$, then

$$u'\big(c(w_{t-1}, y^k)\big) = (1 + \varphi_\ell)u'\big(c(w_t, y^\ell)\big)$$
$$\geq u'\big(c(w_t, y^\ell)\big),$$

so that $c(w_{t-1}, y^k) \leq c(w_t, y^\ell) = \underline{c}^\ell$, where $w_t \equiv (w_{t-1}, y^k)$.

(ii)   if $V_1(w_t, y^\ell) = 0$, then

$$u'\big(c(w_{t-1}, y^k)\big) = -\psi_\ell + u'\big(c(w_t, y^\ell)\big)$$

$$\leq u'\big(c(w_t, y^\ell)\big),$$

so that $c(w_{t-1}, y^k) \geq c(w_t, y^\ell) = \bar{c}^\ell$,

and (iii) if both V_0 and V_1 exceed zero in state ℓ after history w_t, then

$$c(w_{t-1}, y^k) = c(w_t, y^\ell), \text{ where } \underline{c}^\ell \leq c(w_t, y^\ell) \leq \bar{c}^\ell.$$

To show that $\bar{c}^N > \bar{c}^{N-1} > \ldots > \bar{c}^1$ and $\underline{c}^N > \underline{c}^{N-1} > \ldots > \underline{c}^1$, we use a contradiction. Assume $\max_j \{\bar{c}^j\} \neq \bar{c}^N$. We have by definition, $(y^N - \bar{c}^N) + \beta E(V_1 | c_t = \bar{c}^N) = 0$, where the expression $E(V_1 | c_t = \bar{c}^N)$ is well-defined by the necessary conditions above. By these conditions, we also know that $E(V_0 | c_t = \bar{c}^N) > 0$ if $\bar{c}^k > \bar{c}^N$ for some k. Therefore, $y^N - \bar{c}^N < 0$, and $(y^k - \bar{c}^k) + \beta E(V_1 | c_t = \bar{c}^k) = 0$.

Let $\bar{c}^k = \max_j \{\bar{c}^j\}$, so that the first-order conditions imply that $E(V_1 | c_t = \bar{c}^k) = 0$. Therefore $y^k = \bar{c}^k$. But $\bar{c}^k > \bar{c}^N > y^N$ implies that $y^k > y^N$, a contradiction.

Next, take $\bar{c}^{N-1}$ and assume that $\exists k \neq N$ such that $\bar{c}^k > \bar{c}^{N-1}$. We have

$$(y^k - \bar{c}^k) + \beta E(V_1 | c_t = \bar{c}^k) < (y^{N-1} - \bar{c}^{N-1}) + \beta E(V_1 | c_t = \bar{c}^{N-1}), \qquad \text{b e c a u s e}$$

45

$\beta E\left(V_1 | c_t = \bar{c}^{N-1}\right) > 0$ by the first-order conditions and $y^k < y^{N-1}$. This contradicts the assumption that both sides of the inequality are zero.

By repetition of the argument, we have that $\bar{c}^j \leq \bar{c}^k$ whenever $j < k$. To show that $\bar{c}^j$ is strictly less than $\bar{c}^k$ when $y^j < y^k$, suppose that $\bar{c}^j = \bar{c}^k$. The first order conditions imply that

$$E\left(V_1 | c_t = \bar{c}^j\right) = E\left(V_1 | c_t = \bar{c}^k\right).$$

But, both

$$\left(y^k - \bar{c}^k\right) + \beta E\left(V_1 | c_t = \bar{c}^k\right) = 0 \text{ and } \left(y^j - \bar{c}^j\right) + \beta E\left(V_1 | c_t = \bar{c}^j\right) = 0$$

cannot hold since $y^j < y^k$. Therefore, we have the order $\bar{c}^1 < \bar{c}^2 < \ldots \bar{c}^N$, and by a symmetric argument using V_0, that $\underline{c}^1 < \underline{c}^2 < \ldots < \underline{c}^N$.

To show that $y^N = \bar{c}^N$, note that $\bar{c}^N = \max_j \left\{\bar{c}^j\right\}$ implies that $E\left(V_1 | c_t = \bar{c}^N\right) = 0$, so that $y^N = \bar{c}^N$, and, similarly, $y^1 = \underline{c}^1$.

Because $V_i\left(w_t, y^k\right) \geq 0$ for every state k and each $i = 0, 1$, the equations, $\left(y^k - \bar{c}^k\right) + \beta E\left(V_1 | c_t = \bar{c}^k\right) = 0$ and $u\left(\underline{c}^k\right) - u\left(y^k\right) + \beta E\left(V_0 | c_t = \underline{c}^k\right) = 0$, imply that $\underline{c}^k \leq y^k \leq \bar{c}^k$ for every $k = 1, \ldots, N$.

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FOOTNOTES

[1]Lindert and Morton (1989) examined 1552 external bonds of ten borrowing governments (approximately the top ten borrowers over the past 30 years) including those outstanding in 1850 or floated between then and 1970, following all through to settlement or the end of 1983. Defaults were not only common but widespread in their sample; most of the countries had some defaults in each of the periods 1820-1929 and the 1930s (Figure 2.2 p. 61). A detailed summary of experience by country is presented in their Table 2.8 pp. 92-98.

[2]Eichengreen and Portes (1989b) examined 125 London overseas issues and a sample of 250 United States foreign issues floated in the 1920s. (Nearly half of the latter, by value, lapsed into default (p. 233)). In their samples British bonds had an overall internal rate of return of 5 percent, higher than domestic investments, (Eichengreen and Portes 1989a p. 77) while United States loans to national governments had an internal rate of return of 4.6 percent, compared to the 4.1 percent yield on United States treasury bonds over the 1920s (pp. 35, 38). These yields were, however, substantially below those offered ex ante, which were generally between 7 and 8 percent (p. 27). Overall the bonds in the Lindert and Morton (1989) sample proved profitable; the average 2 percent ex ante premium over domestic government bonds became a 0.42 percent premium ex post (p. 77). Further, they find (p. 56) that "there is no clear evidence of a systematic difference in realized returns" between the bonds of their ten borrower governments and United States domestic-corporate bonds.

[3]Eichengreen and Portes (1989b p. 234) report that, in their 1920s samples, "The typical default reduced the internal rate of return by 4.3 percent for dollar loans, but by 1.4 to 2.3 percent on sterling loans." They note, for example, that all sterling loans to Brazil in that period went into default, but they yielded positive internal rates of return between 1.1 and 2.3 percent.

[4]For an early expression of this view, see Wallich (1943) The term "excusable default" is from Grossman and Van Huyck (1988). The insight that "defaults" might not always violate the underlying equilibrium relationship helps explain the findings of Lindert and Morton (1989) and

53

Eichengreen (1989) that defaulters have not generally suffered subsequent discrimination in credit terms, and also the finding of Ozler (1988) for loans 1968-81 that the average penalty for past defaults, though statistically significant, was only a small fraction of the spread.

[5]Diaz-Alejandro (1983) illuminates the differential treatment of Britain and the United States by Brazil and Argentina in the 1930s. Each preferentially repaid the country with which she had a large net trade surplus, and therefore greater concern with market access. For other cases see Bulow and Rogoff (1989a).

[6]If the lender can make credible commitments, he can offer an insurance contract which, in some states, forces him to pay more than the expected present value of future net payments to him. This case, in which no long-run relationship need arise, is examined under symmetric information by Worrall (1990) and for asymmetric information by Atkeson (1988).

[7]Other studies assume asymmetric information about a borrower's aggregate debts, (Kletzer 1984), the borrower's attributes (e.g. Cole, Dow and English (1989), Eaton (1989), and Kletzer (1989)) or the borrower's actions (e.g. Atkeson (1988), who also assumes full pre-commitment to state-contingent payments on the part of lenders with two-period lives). For a critical evaluation of the importance of adverse selection and moral hazard in international lending see Eaton, Gersovitz and Stiglitz (1986) and Kletzer (1987). Kletzer (1989) also discusses seniority privileges in a consumption-smoothing model at length and examines properties of equilibria in a version of this model with an exogenous distribution of bargaining power assumed.

[8]Hart and Moore (1988) study a model in which the players renegotiate an incomplete contract for the exchange for a single unit of a good under uncertainty. Each player can insist on exogenous enforcement of the existing contract, and renegotiation occurs when there are gains from trade that will not be realized under the contract. In a subgame perfect equilibrium, the surplus attained through renegotiation is not divided between the players as in a strategic Nash bargaining game; in one version, it all goes to the player who is willing to trade under the terms of the existing contract. In our model, there is no external enforcement and payments are made to ensure future cooperation by the

other player; the value of such cooperative play depends upon the equilibrium strategies being followed. The division of the surplus in the relationship varies with the history of play and of nature.

[9]This legal provision ensures that the consumption-smoothing arrangement does not "unravel" in the fashion demonstrated by Bulow and Rogoff (1989b). This strong assumption is sufficient but not necessary. In an extension of this paper (Kletzer and Wright (1990)) we shall show that the equilibrium derived here can be supported by renegotiation-proof punishments if there is no legal protection of this type for any agent.

[10]The literature on credit markets has emphasized informational imperfections for motivating the use of standard (non-state-contingent) debt contracts in one-period models with risk-aversion and uncertainty (see, for example, Diamond (1984) or Gale and Hellwig (1985)). An assumption such as costly verification of the borrower's endowment is not needed to assure that simple debt contracts are efficient in our model of a long-term debtor-creditor relationship.

[11]See for example, Farrell (1983), Farrell and Maskin (1989), Pearce (1987), Abreu and Pearce (1989), and also Asheim (1988), van Damme (1989), Bergin and MacLeod (1989), and Benoit and Krishna (1988) among others.

[12]Bernheim and Ray (1989) also suggest a definition of renegotiation-proofness, called internal consistency, which coincides with weak renegotiation-proofness.

[13]Evans and Maskin (1989) prove that for generic two-person finite stage-games, an efficient weakly renegotiation-proof equilibrium exists for the infinitely repeated game for low discount rates.
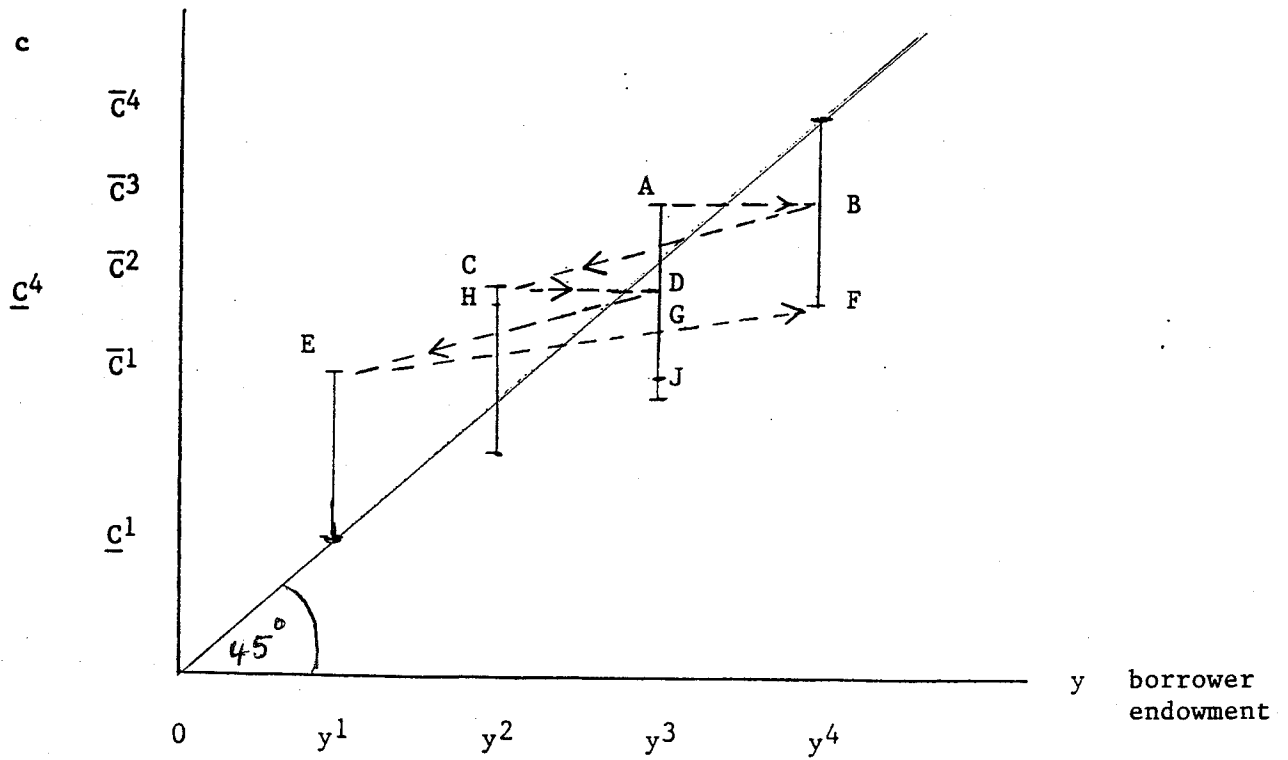
[14]It can be shown that any path in Ω^P can be generated by a strategy profile in Σ^P which is a consistent bargaining equilibrium. In general, these profiles will use punishments that are Pareto-unranked with $\tilde{q}^0$ and $\tilde{q}^1$.

[15]Other punishments than $\tilde{q}^0$ and $\tilde{q}^1$ yielding payoffs on the frontier of $W(y)$ are possible if the discount factor is close enough to one. These are the cases in which full smoothing of the borrower's consumption is possible in a subgame perfect equilibrium.

[16]It follows that an agent deviates only in states in which he or she would make a positive transfer under the simple strategy profile, given the history h.
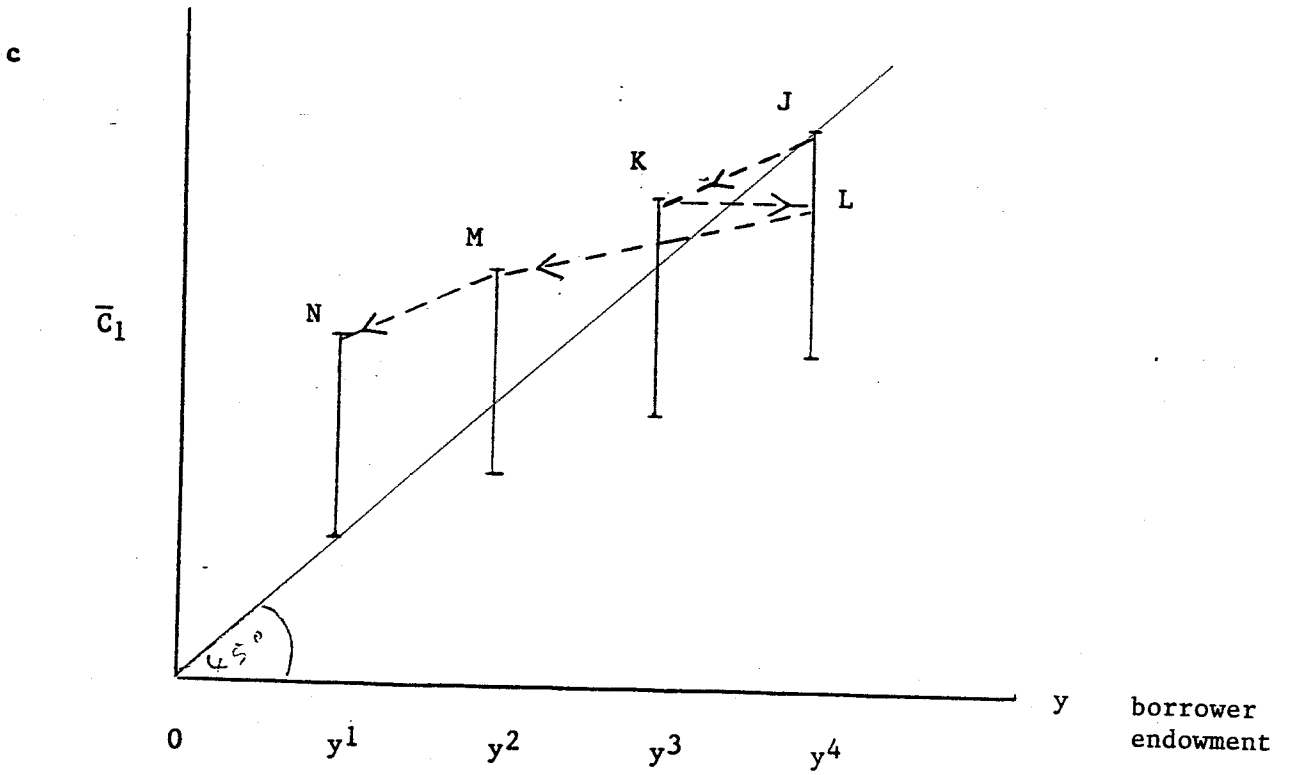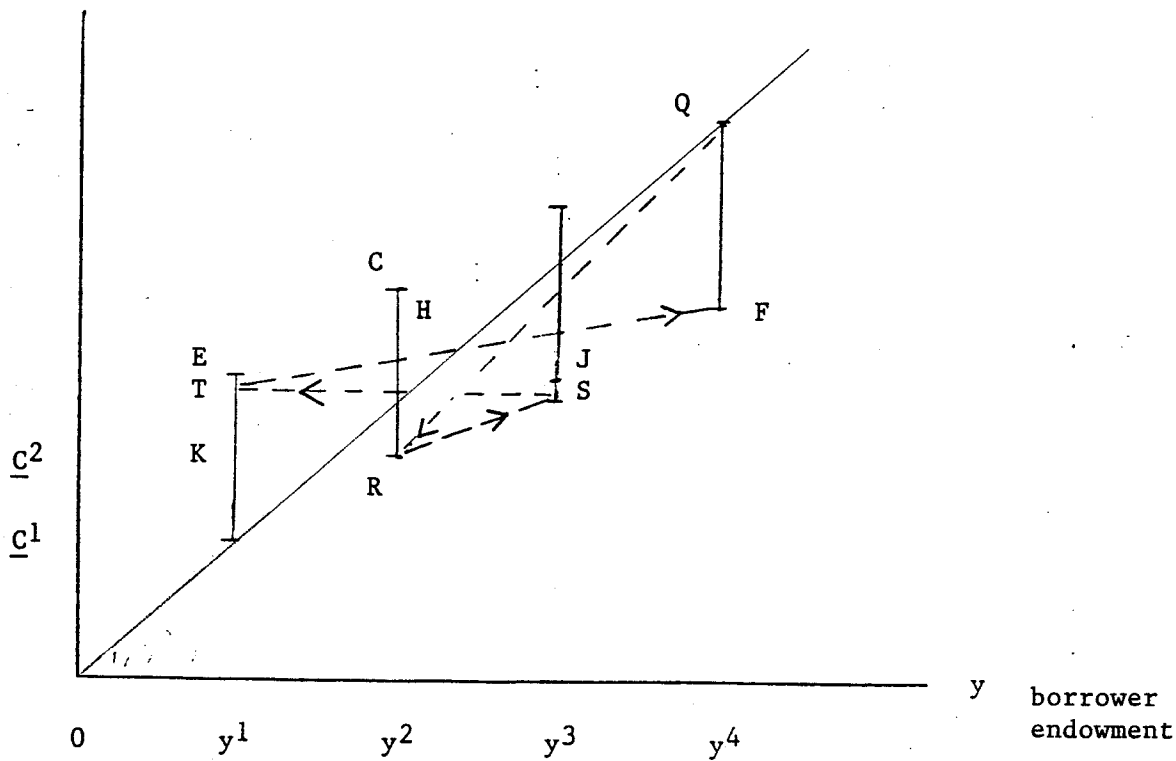
FIGURE 1

borrower
consumption

borrower
endowment

borrower
consumption

FIGURE 2

c

$\overline{C}_1$

45°

0   y^1 y^2 y^3 y^4 y   borrower
                                              endowment

borrower
consumption

FIGURE 3

59

FIGURE 4

FIGURE 5

SURPLUSES IN EXTREME STATES AND THE GAINS FROM RENEGOTIATIONS OF TRIGGER STRATEGY PUNISHMENTS

Lender Surplus V_1

B.

A

O

D

C

V_0

$V_0(V_1; y^N)$

$V_0(V_1; y^1)$

Borrower Surplus

61