

Yale University

EliScholar – A Digital Platform for Scholarly Publishing at Yale

Discussion Papers

Economic Growth Center

2-1-1990

Debt, Deficits and Inflation: An Application to the Public Finances of India

Willem H Buiter

Urijit R. Patel

Follow this and additional works at: <https://elischolar.library.yale.edu/egcenter-discussion-paper-series>

Recommended Citation

Buiter, Willem H and Patel, Urijit R., "Debt, Deficits and Inflation: An Application to the Public Finances of India" (1990). *Discussion Papers*. 601.

<https://elischolar.library.yale.edu/egcenter-discussion-paper-series/601>

This Discussion Paper is brought to you for free and open access by the Economic Growth Center at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact elischolar@yale.edu.

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 593

DEBT, DEFICITS AND INFLATION:
AN APPLICATION TO THE PUBLIC FINANCES OF INDIA

Willem H. Buiter
Yale University

Urjit R. Patel
Yale University

February 1990

Notes: This paper was prepared for the ISPE/NIPFP Conference, January 11-13, 1990, in New Delhi.

Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comments.

Mr. Patel is a Fourth Year Graduate Student in the Department of Economics.

DEBT, DEFICITS AND INFLATION: AN APPLICATION TO THE PUBLIC FINANCES OF INDIA

Willem H. Buiter and Urjit R. Patel

ABSTRACT

The paper studies the solvency of the Indian public sector and the eventual monetization and inflation implied by stabilization of the debt-GNP ratio without any changes in the primary deficit.

The nonstationarity of the discounted public debt suggests that indefinite continuation of the pattern of behavior reflected in the historical discounted debt process is inconsistent with the maintenance of solvency. This message is reinforced by the recent behavior of the debt-GNP ratio and the ratio of primary surplus plus seigniorage to GNP.

Our estimates of the base money demand function suggest that even maximal use of seigniorage will not be sufficient to restore solvency.

DEBT, DEFICITS AND INFLATION: AN APPLICATION TO THE PUBLIC FINANCES OF INDIA

OUTLINE

1. Introduction
2. Some Facts Concerning Debt, Deficits and Seigniorage
3. Is there a Threat to the Solvency of the Public Sector?
4. Policy Options to Avoid Insolvency
5. Conclusion

Notes

References

DEBT, DEFICITS AND INFLATION: AN APPLICATION TO THE PUBLIC FINANCES OF INDIA

Willem H. Buiter and Urjit R. Patel

1. INTRODUCTION

The state of Indian public finances appears perilous. In recent years the public sector financial deficit has been rising in real terms and as a proportion of GNP. The same is true for the primary or noninterest deficit. The real values of the public debt and the public debt-GNP ratio are rising sharply. Even the discounted public debt, that is the present value of the public debt (discounted to some common base year), is rising steadily. The question naturally arises as to whether this pattern of debt and deficits is sustainable.

There are four reasons why a rising public debt burden may be of concern. The first of these is financial crowding out. If there is no debt neutrality, the substitution of borrowing for current taxes (even lump-sum) on labor income will tend to raise private consumption. In an economy with full utilization of resources this will lead either to the displacement of private investment and other interest-sensitive forms of private spending or to an increase in the deficit on the current account of the balance of payments. This aspect of public debt and deficits will not be addressed in what follows.

The second reason relates to tax smoothing. Even if there is "first-order" debt neutrality, the option of running budget deficits or surpluses may be valuable if there are no lump-sum, nondistortionary tax-transfer schemes available. Temporary deficits and surpluses permit changes in the time profile of the distortionary (dead weight) losses (or collection costs) associated with nonlump-sum taxes and transfers. Under

rather restrictive separability and homogeneity assumptions, a tax smoothing prescription emerges (see e.g. Barro [1979]): the ratio of distortionary tax receipts to the tax base is expected to be constant over time. If there is no first-order debt neutrality, financial crowding out concerns may prevent otherwise desirable tax smoothing. For reasons of space this issue too is not considered further in what follows.

The third concern, which is addressed in this paper, relates to the eventual monetization of persistent deficits and thus to their potential inflationary consequences. The last concern is the possibility of insolvency or bankruptcy of the national Exchequer. This too will be dealt with in what follows.

The question of how to evaluate the sustainability of a government's fiscal financial strategy has been explored intensively in recent years (see e.g. Buiters [1983a, b; 1985; 1989a, b], Anand and van Wijnbergen [1989], Hamilton and Flavin [1986], Grilli [1989] and Wilcox [1989]).

After presenting a brief review of some of the key facts concerning recent budgetary developments in Section 2, we turn in Section 3 to a systematic analysis of whether the current and recent behavior of key budgetary and related time series is sustainable. Since our conclusion is that a continuation of current patterns would eventually lead to insolvency of the Exchequer, we turn in Section 4 to the consideration of alternative policy options to avoid insolvency. These fall into three categories: first policies aimed at ensuring larger primary surpluses or smaller primary deficits; second policies to reduce the interest cost of borrowing; and third increased recourse to seigniorage or the inflation tax. Our main conclusions are summarized in Section 5.

2. SOME FACTS CONCERNING DEBT, DEFICITS AND SEIGNIORAGE

Our data cover the period 1970/71 to 1986/87. The most striking fact is the dramatic increase in the total (internal and external) public debt-GNP ratio since 1980/81, from 32.8 percent in that year to 52.4 percent in 1986/87--an average annual increase in the ratio of 3.3 percentage points. There is no reason to believe that there has been a significant downturn in the ratio since then, or even that the rate of increase has been reduced markedly. Figure 1 and Table 1 show this striking pattern quite clearly.

Our total public debt figure (NTD) covers both internal (or domestic) and external (or foreign) debt, and subtracts official foreign exchange reserves. The domestic subtotal includes domestic private sector holdings of central government debt (CDD), state government debt (SDD) and public enterprise debt (PDD). Intrapublic sector assets and liabilities are netted out. It is important that the liabilities of public enterprises (and of the "holding companies" created in the mid seventies) be included in the public debt total, since ultimately these liabilities are de facto or de jure the responsibility of the state or central Treasuries. The banking sector, other than the Reserve Bank, was excluded from our definition of the public sector despite its being publicly owned. The decomposition of the domestic public debt is given in Table 2. Note that it is the domestic debt that accounts for most of the increase in total government indebtedness.

The foreign debt (TFD) figures in Table 3 include public and publicly guaranteed long-term debt (as defined and given in the World Bank's World Debt Tables), use of IMF credit, and an estimate of public and publicly guaranteed short-term debt. Foreign exchange reserves *R* are subtracted from TFD to get net foreign debt (NTFD).

FIGURE 1: DEBT-GNP RATIO, 1970/1-1986/7

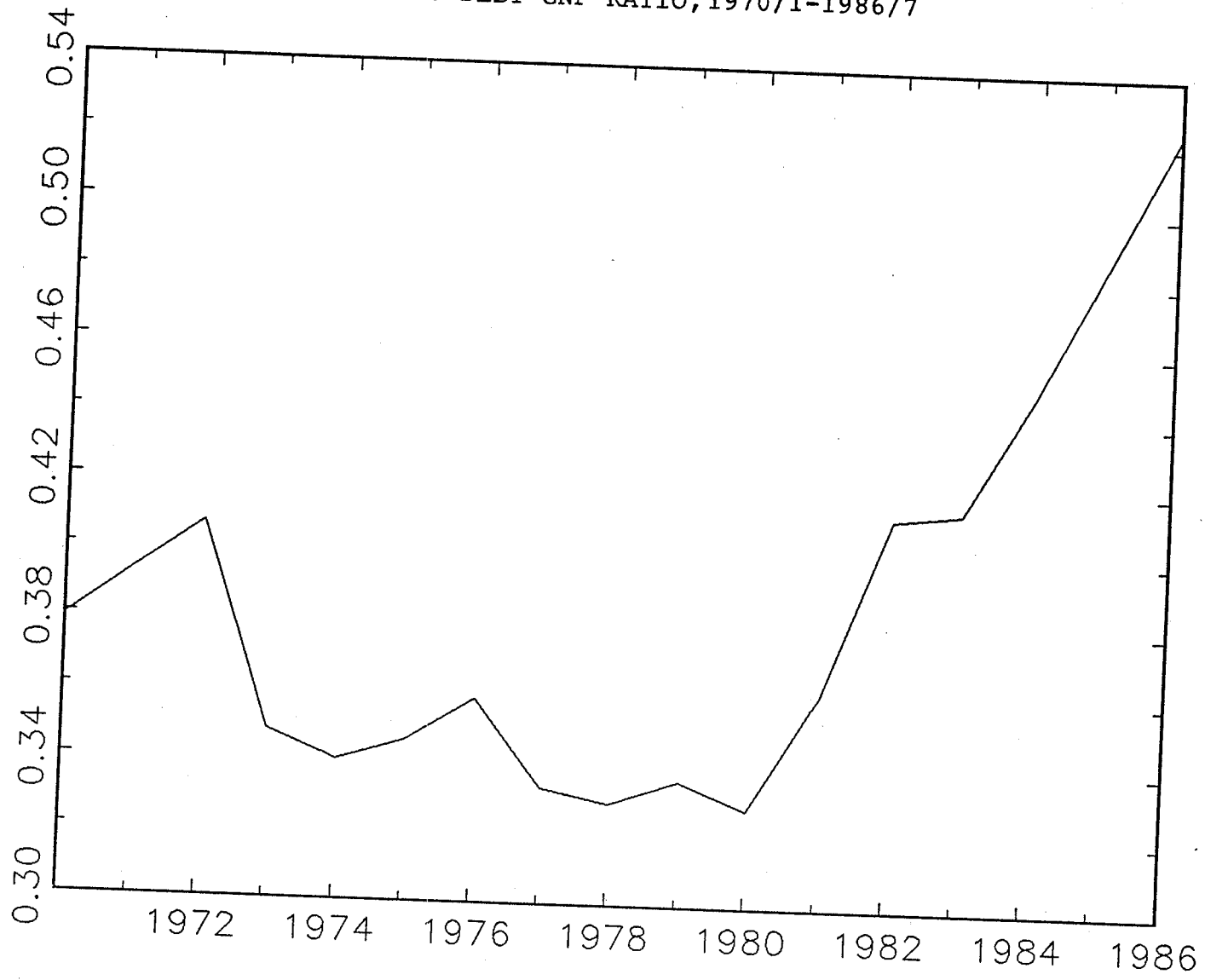


TABLE 1: % GNP

	NTD	NTDD	NTFD
1970/71	37.9	23.9	14.0
1971/72	39.3	24.7	14.6
1972/73	40.7	25.7	15.0
1973/74	34.8	21.4	13.4
1974/75	34.0	20.8	13.2
1975/76	34.6	21.1	13.5
1976/77	35.8	23.4	12.4
1977/78	33.3	23.5	9.8
1978/79	32.9	24.7	8.2
1979/80	33.6	25.7	7.9
1980/81	32.8	24.7	8.1
1981/82	36.1	25.5	10.6
1982/83	41.2	28.9	12.3
1983/84	41.4	29.3	12.1
1984/85	44.8	31.3	13.7
1985/86	48.6	33.6	15.0
1986/87	52.4	35.9	16.5

SOURCES:

1. Report of the Committee to Review the Working of the Monetary System, April 1985, RBI, Bombay.
2. India, Bureau of Public Enterprises, Public Enterprises Survey: Annual Report on the Working of Industrial and Commercial Undertakings of the Central Government, Volumes for 1970/1 to 1986/7.
3. Report on Currency and Finance, RBI, Volumes for 1977/8-1987/8.
4. Economic Survey, Government of India, 1988/9.
5. World Debt Tables: External Debt of Developing Countries, 1988/9, Volumes II and III, Country Tables, Washington, DC.

DEFINITIONS OF VARIABLES:

NTD \equiv NTDD + NTFD (see notes to Tables 2 and 3).

TABLE 2: DOMESTIC PRIVATE HOLDINGS OF CENTRAL GOVERNMENT, STATE AND PUBLIC ENTERPRISE LIABILITIES, 1970/1-1986/7 (% OF GNP)

	CDD	SDD	PEDD	NTDD
1970/71	18.5	5.1	0.3	23.9
1971/72	19.1	5.3	0.3	24.7
1972/73	19.9	5.4	0.4	25.7
1973/74	16.4	4.8	0.2	21.4
1974/75	15.5	4.7	0.6	20.8
1975/76	15.1	5.2	0.8	21.1
1976/77	16.8	5.4	1.2	23.4
1977/78	17.2	5.2	1.1	23.5
1978/79	17.9	5.4	1.4	24.7
1979/80	18.4	5.5	1.8	25.7
1980/81	17.7	5.2	1.8	24.7
1981/82	18.6	5.1	1.8	25.5
1982/83	21.3	5.4	2.2	28.9
1983/84	21.4	5.5	2.4	29.3
1984/85	22.6	5.9	2.6	31.1
1985/86	24.7	6.1	2.8	33.6
1986/87	26.5	6.3	3.1	35.9

SOURCES:

1. Report of the Committee to Review the Working of the Monetary System, April 1985, RBI, Bombay.
2. India, Bureau of Public Enterprises, Public Enterprises Survey Annual Report on the Working of Industrial and Commercial Undertakings of the Central Government, Volumes for 1970/1-1986/7.
3. Report on Currency and Finance, RBI, Volumes for 1977/8-1987/8.
4. Economic Survey, Government of India, 1988/9.

DEFINITIONS OF VARIABLES:

$NTDD \equiv CDD + SDD + PEDD$.

CDD: Internal debt of Central Government except special securities issued to the Reserve Bank of India, Treasury bills issued to the Reserve Bank of India and to State Governments; plus Small Savings Scheme; plus Five-Year Time Deposits; plus Provident Funds etc; minus loans and debentures to Public Enterprises.

SDD: Internal debt of State Governments less Ways and Means Advances from the Reserve Bank of India; plus Provident Funds; less loans to Public Enterprises.

PEDD: Rupee denominated debt of Public Enterprises not held by Central Government or States.

**TABLE 3: FOREIGN LIABILITIES AND ASSETS OF THE
PUBLIC SECTOR, 1970/1-1986/7 (% OF GNP)**

	TFD	R	NTFD
1970/71	15.4	1.4	14.0
1971/72	16.2	1.6	14.6
1972/73	16.5	1.5	15.0
1973/74	14.8	1.4	13.4
1974/75	14.4	1.2	13.2
1975/76	15.9	2.4	13.5
1976/77	16.4	4.0	12.4
1977/78	15.2	5.4	9.8
1978/79	14.2	6.0	8.2
1979/80	13.5	5.6	7.9
1980/81	12.4	4.3	8.1
1981/82	13.2	2.6	10.6
1982/83	15.2	2.9	12.3
1983/84	15.2	3.1	12.1
1984/85	17.1	3.4	13.7
1985/86	18.2	3.2	15.0
1986/87	19.5	3.0	16.5

SOURCES:

1. World Debt Tables: External Debt of Developing Countries, 1988/9, Volumes II and III, Country Tables, Washington, DC.
2. Economic Survey, Government of India, 1988/9.

DEFINITIONS OF VARIABLES:

NTFD \equiv TFD - R.

TFD: Public and Publicly Guaranteed Long-Term debt plus use of IMF Credit plus imputed Short-Term Public Debt*.

R: Official foreign exchange reserves plus SDRs.

* We assumed that the Public Sector's share of total short-term external debt was the same as its share of total long-term debt.

Note: External debt data in World Debt Tables is on a calendar year basis. The figures in Tables 1 and 3 "apportion" the calendar year figures to financial years (April 1 to March 31) e.g. the figure for financial year 1986/7 is three-quarters of the calendar year 1986 figure plus one-quarter of the calendar year 1987 figure.

Table 4 shows that the public sector deficit as a proportion of GNP has been rising since 1973/4 with the exception of two dips in 1977/8 and 1981/2. The primary (noninterest) deficit as a proportion of GNP shows a similar pattern, more than doubling between 1973/4 and 1986/7 to 9.8 percent. Interest payments reached 4.1 percent of GNP in 1986/7. Use of seigniorage has been nonnegligible, averaging 2.4 percent of GNP over the last five years of our sample. Table 5 gives the time series for GNP growth, inflation, exchange rate depreciation and interest rates. Figures for the discounted debt, discounted primary deficit and discounted seigniorage--which are key ingredients in our solvency tests--are given in Table 6 and Figure 2.

3. IS THERE A THREAT TO THE SOLVENCY OF THE PUBLIC SECTOR?

A key question when evaluating the fiscal options open to the Indian government is whether a continuation of past and present policies is consistent with government solvency. To answer this question we start from the consolidated public sector budget identity given in equation (3.1). It consolidates general government (central, state and local) with the public enterprise sector and the central bank.

$$(3.1) \quad \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - B_{t-1}}{P_t} + \frac{V_t(\bar{B}_t^* - \bar{B}_{t-1}^*)}{P_t} - \frac{V_t(F_t^* - F_{t-1}^*)}{P_t}$$

$$\equiv C_t + A_t - T_t + i_{t-1} \frac{B_{t-1}}{P_t} + i_{t-1}^* \frac{V_t}{P_t} (\bar{B}_{t-1}^* - F_{t-1}^*) - \rho_{t-1} K_{t-1}$$

**TABLE 4: THE PUBLIC SECTOR DEFICIT, ITS COMPONENTS
AND SEIGNIORAGE, 1970/1-1986/7 (% OF GNP)**

	DEFICIT	PRIMARY DEFICIT	INTEREST PAYMENTS	SEIGNIORAGE
1970/71	5.8	3.9	1.9	1.1
1971/72	7.3	5.3	2.0	1.1
1972/73	6.9	5.0	1.9	1.0
1973/74	5.6	3.9	1.7	2.2
1974/75	5.9	4.2	1.7	0.7
1975/76	6.7	4.7	2.0	0.3
1976/77	7.1	4.8	2.3	1.4
1977/78	6.9	4.9	2.0	1.9
1978/79	7.9	5.6	2.3	2.2
1979/80	8.3	5.7	2.6	2.8
1980/81	10.0	7.6	2.4	1.7
1981/82	9.6	6.9	2.7	1.7
1982/83	10.7	7.8	2.9	2.2
1983/84	10.7	7.7	3.0	3.1
1984/85	12.4	9.1	3.3	1.3
1985/86	11.6	8.2	3.4	2.7
1986/87	13.9	9.8	4.1	2.7

SOURCES:

1. Economic Survey, Government of India, Volumes 1972/3-1988/9.
2. Report on Currency and Finance, RBI, Volumes 1980/1-1987/8.
3. Statistical Appendix: Supplement to the RBI Occasional Papers, Volume 3(1), June 1982, Monetary Policy in India: Issues and Evidence.

TABLE 5: SELECTED ECONOMIC INDICATORS

	(1)	(2)	(3)	(4)
	Growth GNP %	%Δ in GDP Deflator	%Δ in CPI	Deprec. Ex. Rate %
1970/71	6.15	2.99	4.19	0.0
1971/72	2.14	5.43	5.36	0.32
1972/73	-0.54	11.00	11.44	2.74
1973/74	3.74	18.89	27.00	1.72
1974/75	0.20	17.97	16.77	1.44
1975/76	9.67	-2.87	-10.77	8.49
1976/77	1.61	6.59	8.91	3.29
1977/78	8.37	3.41	2.90	-4.20
1978/79	6.71	2.06	3.59	-4.17
1979/80	-4.51	15.56	12.13	-1.58
1980/81	6.48	18.71	12.58	-2.27
1981/82	6.48	10.01	8.82	13.13
1982/83	3.47	7.23	9.91	7.88
1983/84	7.61	8.36	11.15	7.05
1984/85	3.20	7.14	5.01	15.27
1985/86	6.11	7.53	8.85	2.90
1986/87	6.07	5.60	7.61	4.42

TABLE 5, continued

	(5) Yield on L.T. Gov't Securities (Rupee)	(6) Advance Rate % (Rupee)	(7) Official Creditors Rate % (US\$)	(8) All Creditors Rate % (US\$)
1970/71	5.15	7.75	2.18	2.40
1971/72	5.64	8.50	1.93	2.03
1972/73	5.37	8.50	1.48	1.83
1973/74	5.37	8.75	1.80	1.98
1974/75	6.16	11.25	2.15	2.28
1975/76	6.28	14.00	2.68	2.85
1976/77	6.25	14.00	3.48	3.70
1977/78	6.25	13.50	2.33	2.80
1978/79	6.43	13.00	2.05	2.45
1979/80	6.59	14.75	2.65	4.28
1980/81	6.97	16.50	2.60	4.78
1981/82	7.23	16.50	4.35	5.35
1982/83	7.73	16.50	5.70	7.15
1983/84	8.24	16.50	5.13	6.70
1984/85	9.22	16.50	5.83	6.48
1985/86	9.94	16.50	5.03	5.73
1986/87	10.19	16.50	5.18	5.55

TABLE 5, continued

SOURCES:

1. Columns (1) and (2): IMF, International Financial Statistics, Yearbook, 1989, Washington, DC.
2. Column (3): Economic Survey, Government of India, Volumes 1969/70-1987/8.
3. Column (4): IMF, International Financial Statistics (IFS), Monthly, 1970-1988, Washington, DC.
4. Column (5): Report on Currency and Finance, RBI, Volumes for 1970/1-1987/8.
5. Column (6): Reserve Bank of India Bulletin, Monthly, Volumes for 1970/1-1987/8.
6. Columns (7) and (8): World Debt Tables: External Debt of Developing Countries, 1988/9, Volumes II and III, Country Tables, Washington, DC.

TABLE 6: DISCOUNTED* DEBT, PRIMARY DEFICIT AND SEIGNIORAGE, 1970/1-1986/7
 (current Rupees discounted to 1970/1)

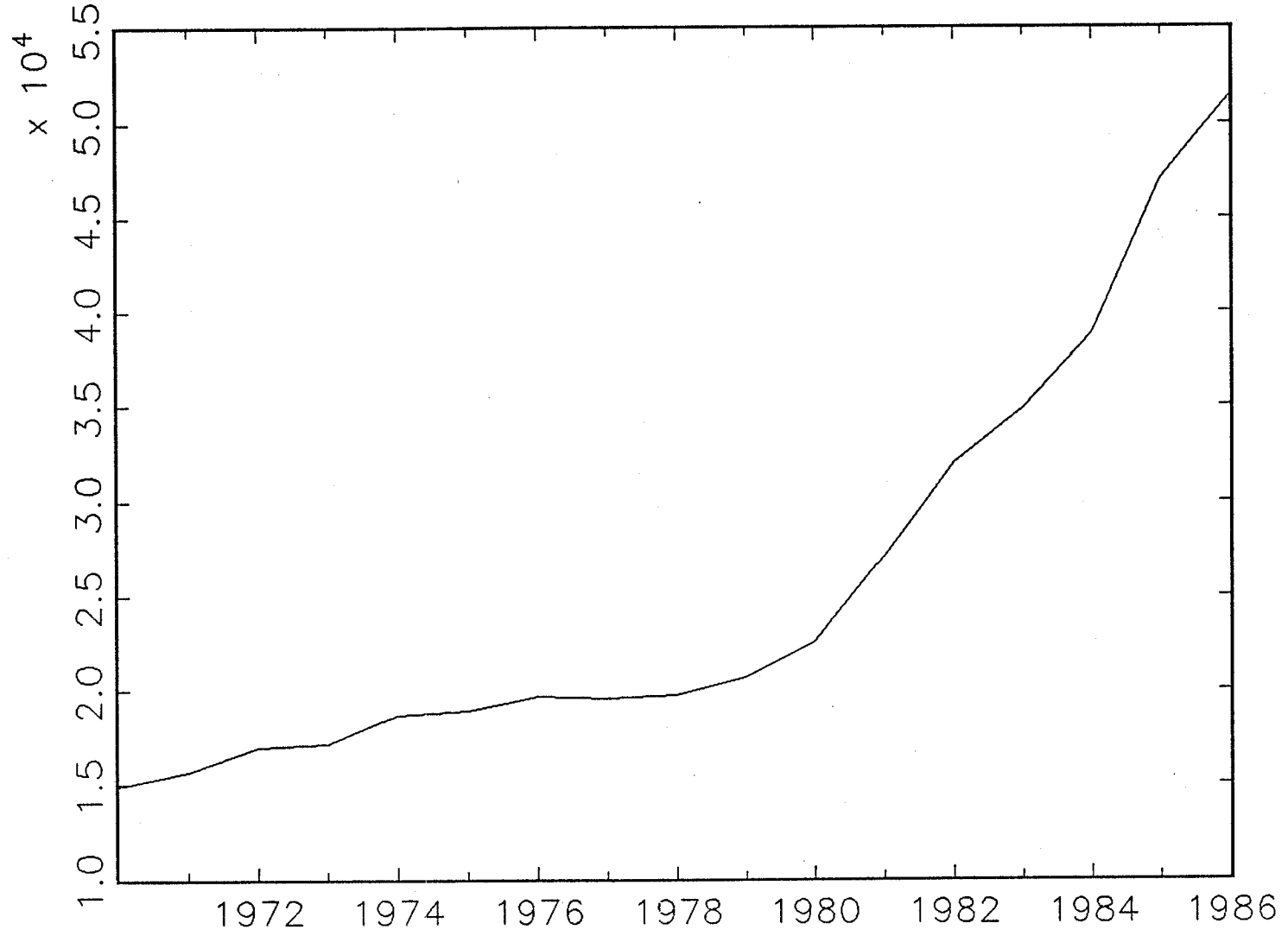
	(CR.RS)		
	NTD	PRIMARY DEFICIT	SEIGNIORAGE
1970/71	14887	1542	426
1971/72	15660	2118	451
1972/73	16966	2083	433
1973/74	17162	1924	1063
1974/75	18668	2283	3788
1975/76	18909	2580	185
1976/77	19644	2652	793
1977/78	19542	2893	1114
1978/79	19698	3314	1338
1979/80	20672	3532	1701
1980/81	22529	5231	1160
1981/82	27065	5167	1257
1982/83	31955	6022	1695
1983/84	34834	6500	2594
1984/85	38820	7923	1115
1985/86	47045	7980	2655
1986/87	51427	9618	2636

* Discounted using the Long-Term Government Bond Yield.

SOURCES:

Same as for Tables 1-5.

FIGURE 2: DISCOUNTED DEBT, 1970/1-1986/7, CR.RS



M is the nominal stock of base money, B the stock of domestic currency denominated public debt, \tilde{B}^* the stock of foreign currency denominated public debt, F^* the stock of foreign exchange reserves, K the public sector capital stock valued at current reproduction cost, C government consumption, A public sector gross domestic capital formation, T net current revenue, i the domestic nominal interest rate, i^* the foreign nominal interest rate, V the foreign exchange rate, P the domestic price level and ρ the cash rate of return on public sector capital.

For simplicity our analysis is cast in terms of one-period public debt, and the interest rate on international reserves is assumed to be the same as that paid on foreign debt, but these simplifications are unimportant. Let $B^* \equiv \tilde{B}^* - F^*$ be net official foreign debt.

It is sometimes helpful (but without behavioral significance) to rewrite this identity in terms of the behavior over time of stocks and flows per unit of GDP. This yields

$$(3.2) \quad b_t + b_t^* \equiv b_{t-1} \frac{(1 + i_{t-1})}{(1 + \pi_{t-1})(1 + n_{t-1})} + b_{t-1}^* \frac{(1 + i_{t-1}^*)(1 + \epsilon_{t-1})}{(1 + \pi_{t-1})(1 + n_{t-1})} \\ + c_t + a_t \tau_t - \frac{\rho_{t-1}}{1 + n_{t-1}} k_{t-1} - \sigma_t .$$

Lower-case stocks and flows are the corresponding upper-case quantities

expressed as a proportion of GDP. $\pi_{t-1} \equiv \frac{P_t - P_{t-1}}{P_t}$; $n_{t-1} \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}$;

$\epsilon_{t-1} \equiv \frac{V_t - V_{t-1}}{V_{t-1}}$; $\sigma_t \equiv \frac{M_t - M_{t-1}}{P_t Y_t}$. Y denotes real output.

We shall refer to $c_t + a_t - \tau_t - \rho_{t-1}k_{t-1} \equiv \delta_t$ as the **primary public sector deficit** per unit of GDP. It is the conventionally measured consolidated public sector deficit net of any interest payments or interest income. Total public debt as a fraction of GDP will be denoted $d \equiv b + b^*$. σ_t , the increase in the nominal stock of base money as a fraction of GDP will be referred to as **seigniorage**. Equation (3.2) can be rewritten as (3.3). γ is the proportional depreciation rate of the real exchange rate, r the domestic real interest rate, r^* the foreign real interest rate and $\tilde{r} \equiv r - n$.

$$(3.3) \quad d_t \equiv (1 + \tilde{r}_{t-1})d_{t-1} + \delta_t \\ + \frac{b_{t-1}^*}{1 + n_{t-1}} \left[(1 + r_{t-1}^*)(1 + \gamma_{t-1}) - (1 + r_{t-1}) \right] - \sigma_t$$

Finally, defining the **augmented primary deficit**

$$\bar{\delta}_t \equiv \delta_t + \frac{b_{t-1}^*}{1 + n_{t-1}} \left[(1 + r_{t-1}^*)(1 + \gamma_{t-1}) - (1 + r_{t-1}) \right] \quad \text{we get}$$

$$(3.4) \quad d_t \equiv (1 + \tilde{r}_{t-1})d_{t-1} + \bar{\delta}_t - \sigma_t .$$

Solving (3.4) recursively forward in time and, letting E_t denote the expectation operator conditional on information at time t we get:

$$(3.5) \quad d_t \equiv \sum_{i=0}^{\infty} E_t \prod_{j=0}^i \frac{1}{1 + \tilde{r}_{t+j}} \left[\bar{\delta}_{t+1+i} + \sigma_{t+1+i} \right] + \lim_{i \rightarrow \infty} E_t \prod_{j=0}^i \frac{1}{1 + \tilde{r}_{t+j}} d_{t+1+i} .$$

Let $q_i \equiv \prod_{j=0}^i (1 + \tilde{r}_j)^{-1}$, $q_{-1} = 1$ be the discount factor from period zero to period $t+i$. Equation (3.5) can be rewritten as

$$(3.5') \quad d_t = \sum_{i=0}^{\infty} E_t \frac{q_{t+i}}{q_{t-1}} \left[-\delta_{t+1+i} + \sigma_{t+1+i} \right] + \lim_{i \rightarrow \infty} E_t \frac{q_{t+i}}{q_{t-1}} d_{t+1+i} .$$

The terminal condition we impose on (3.5') to obtain the government solvency constraint or present value budget constraint is

$$(3.6) \quad \lim_{i \rightarrow \infty} \frac{1}{q_{t-1}} E_t q_{t+i} d_{t+1+i} \leq 0 .$$

In what follows we shall in fact assume that (3.6) holds with strict equality i.e. "supersolvency" is not considered. $\frac{1}{q_{t-1}} q_{t-1+i} d_{t+i}$ is the present discounted value at time t of government debt in period $t+i$. Equation (3.6) states that the expectation, at t , of the present value of future public debt goes to zero in the limit. It makes no difference of course whether we express (3.6) in terms of the debt-GDP ratio and real interest rates net of real growth rates; in terms of real debt and real interest rates; in terms of debt measured in home currency and nominal interest rates in terms of home currency; or in terms of debt measured in terms of foreign currency and nominal interest rates in terms of foreign currency.

When (3.6) holds (with equality), equation (3.5') becomes the familiar government solvency constraint or public sector present value budget constraint:

$$(3.7) \quad d_t = \sum_{i=0}^{\infty} \frac{E_t q_{t+i}}{q_{t-1}} \left[-\bar{\delta}_{t+1+i} + \sigma_{t+1+i} \right]$$

or

$$(3.7') \quad D_t = \sum_{i=0}^{\infty} E_t \left[-\bar{\Delta}_{t+1+i} + S_{t+1+i} \right]$$

where $D_t \equiv q_{t-1} d_t$, the present value at time zero of public debt at time t , $\bar{\Delta}_{t+1+i} \equiv q_{t+i} \bar{\delta}_{t+1+i}$, the present value at time zero of the primary deficit at time $t+1+i$ and $S_{t+1+i} \equiv q_{t+i} \sigma_{t+1+i}$, the present value at time zero of seigniorage at time $t+1+i$.

The condition in (3.6) (holding with equality) can be rewritten as

$$(3.6') \quad \lim_{i \rightarrow \infty} E_t D_{t+i} = 0.$$

The original budget identity in (3.4) can be rewritten as

$$(3.4') \quad D_t \equiv D_{t-1} + \bar{\Delta}_t - S_t.$$

THE MEANING OF THE SOLVENCY CONSTRAINT

In a finite horizon economy with a finite terminal date T , the solvency constraint is the requirement that public debt in the last period be nonpositive i.e.

$$(3.8) \quad b_T \leq 0 .$$

In an infinite horizon economy that is not dynamically inefficient, the natural analogue of (3.8) is (3.6) or (3.6'). This ensures that in the infinite horizon economy--as in the finite horizon economy--the existing debt ultimately is serviced (note not paid off) by current and future primary surpluses or by current and future seigniorage.

If the economy is dynamically inefficient, which will be the case if the interest rate is below the growth rate forever ($r_t < n_t$ for all t), there is no convincing case for requiring (3.6) or (3.6') to hold. Ponzi games can be viable indefinitely in a dynamically inefficient system: the government can, each period, pay the interest on its existing debt by further borrowing. We assume in what follows that while the interest rate can be below the growth rate for extended finite periods of time, the Indian economy is not dynamically inefficient, and that there are no social free lunches to be earned by increasing the public debt (see also Abel et al. [1989]).

Note that the terminal condition (3.6) or (3.6') can be rewritten as

$$(3.9) \quad \lim_{i \rightarrow \infty} d_t E_t \prod_{j=0}^i \left[\frac{1}{1 + r_{t+j}} \right] \left[\frac{d_{t+j+1}}{d_{t+j}} \right] \leq 0 .$$

With a positive initial stock of debt ($d_t > 0$) equation (3.9) is satisfied only if ultimately the debt is expected to grow at a rate less than the interest rate $\frac{1}{1 + r_{t+j}} \frac{d_{t+j+1}}{d_{t+j}} < 1$.¹ Since

$\frac{d_{t+j+1}}{d_{t+j}} \frac{1}{1+r_{t+j}} \equiv 1 + \frac{\bar{\delta}_{t+j+1} - \sigma_{t+j+1}}{(1+r_{t+j})d_{t+j}}$, it is clear that solvency requires,

eventually, positive values for $\sigma_t - \bar{\delta}_t$, the sum of seigniorage and the augmented primary surplus. This is only a necessary condition, of course.

The flows of seigniorage and primary surpluses should satisfy (3.7) or (3.7').

To prevent the solvency requirement (3.6) or (3.7) from being satisfied trivially we must purge market values and discount rates of the influences of actual default and of the market's assessment of the risk of future default. The discount rates used to obtain the discounted public debt series used in the solvency tests should therefore be net of any risk premiums reflecting the market's perception of the possibility of default. Similarly, the value of the debt used in the solvency tests should be gross of any discount due to default risk. When the market value of the debt is endogenous and potentially variable even without any default risk (as will e.g. be the case with long-term fixed interest debt), it may not be empirically simple to assess the influence of market perceptions of default risk on the pricing of the debt. When the debt has a fixed nominal market value in the absence of default risk there is of course no empirical problem. We assume in what follows that in our sample the market does not give any weight to the possibility of default on the Indian public debt.

Note that solvency is a very weak criterion with which to evaluate the sustainability of fiscal and financial policy. A government can be solvent even though its real debt and even its debt relative to GDP or GNP grows without bound. If the long-run growth rate of the debt-GDP ratio while positive is less than the long-run value of \tilde{r} i.e. less than the excess of

the interest rate over the growth rate in the long run, then eventually unbounded debt-GDP ratios can still be consistent with solvency.

The reason why this remarkable fiscal high wire act may be possible is that our approach to solvency thus far has ignored the growing excess burden associated with ever higher distortionary taxes and the rising real resource cost of extracting an ever rising tax burden from the private sector.

Steady growth in the debt-GDP ratio and even an eventually unbounded debt-GDP ratio may be consistent with continued debt service and solvency because the government is (implicitly) assumed to be able to tax away in lump-sum fashion (i.e. without distortions or collection and enforcement costs) any amount up to the sum of GDP plus the interest it pays on the public debt.² Since the "tax base" is not GDP alone but GDP plus debt interest, there is nothing logically inconsistent about debt service outstripping GDP, even by indefinitely increasing amounts.

While this point is logically correct, its practical relevance is likely to be slight. If dead weight losses, excess burdens or collection costs are an increasing and strictly convex function of the real tax take or of the tax-GDP ratio, then only finite debt-GDP ratios are feasible.

While our weak solvency criterion only implies that discounted debt D_t cannot have a positive stochastic or deterministic trend, a stricter and very plausible practical solvency criterion in addition states that the undiscounted debt-GDP ratio d_t cannot have a positive stochastic or deterministic trend.

Under neither solvency criterion is it necessary for the public sector ever to run conventionally measured public sector financial surpluses even if the initial debt is positive.

Consider the three budget surplus measures given below.

The conventional financial surplus (CFS):

$$(3.10a) \quad CFS \equiv T + \rho K - [C + A] - \left[i \frac{B}{P} + i^* \frac{V}{P} B^* \right] .$$

The operational financial surplus (OFS):

$$(3.10b) \quad OFS \equiv T + \rho K - [C + A] - \left[\left[\frac{(1+i)}{(1+\pi)(1+n)} - 1 \right] \frac{B}{P} \right. \\ \left. + \left[\frac{(1+i^*)(1+\epsilon)}{(1+\pi)(1+n)} - 1 \right] \frac{V}{P} B^* \right] .$$

The primary financial surplus (PFS):

$$(3.10c) \quad PFS \equiv T + \rho K - [C + A] .$$

Neither under the weak solvency criterion nor under the practical solvency criterion is it necessary for a government with a positive stock of public debt and $r > n$ ever to run conventional financial surpluses. It may choose to do so. It may be desirable or even optimal to do so, permanently or temporarily, but it is not required for solvency.

Ignoring seigniorage our practical solvency criterion (but not the weak solvency criterion) implies that at some point positive operational financial surpluses are required. If seigniorage is allowed for, the practical solvency criterion need not imply the eventual necessity of operational (or asset-revaluation and real-growth-corrected) financial surpluses.

With positive debt and $r - n$, the weak solvency criterion and *a fortiori* the practical solvency criterion require, if seigniorage is ignored, that

primary surpluses be achieved at some point. Again, recourse to seigniorage could reverse this conclusion.

TESTING FOR SOLVENCY

The only testable implication of the weak solvency requirement is equation (3.6) or (3.6'): the unconditional expectation of the discounted public debt should be zero (or nonpositive).

Since we do not have a structural model of the economic and political processes governing the evolution of the Indian public debt, we are restricted to a rather mechanical "data description" which aims to answer the following questions:

1. Is there a stable (reduced form) data generating process (DGP) that describes the behavior of the discounted public debt?
2. Is this DGP (covariance) stationary?
3. If it is covariance stationary, is its unconditional mean equal to zero?

If the DGP is not covariance stationary, then its unconditional expectation will be (almost surely) nonzero. Establishing nonstationarity would therefore imply that the policies pursued during the sample period would, if they were adhered to into the indefinite future, either imply supersolvency or ultimate insolvency of the government. We can dispose easily of the supersolvency possibility. The initial debt is certainly positive, and the case of characteristic roots less than -1 can be ruled out. Note that a finding of a nonstationary DGP does not mean that there will be government insolvency; only that in the absence of policy or other changes that render the DGP stationary, bankruptcy of the Treasury will result. In Section 4 we consider some of the policy options that would eliminate the specter of insolvency. Ex post the solvency constraint will be satisfied either through

changes in the paths of the augmented primary surplus and of seigniorage or by default i.e. write downs in the value of the outstanding debt. Our analysis also is silent on the important issue as to when and how the threat of insolvency (given unchanged policies) will either compel changes in the process governing primary surpluses and seigniorage or result in de facto or de jure repudiation of (part of) the debt.

If the DGP is covariance stationary, its unconditional mean will be zero if the univariate representation of the stochastic process governing it is strictly indeterministic. If the process has a deterministic component, its unconditional mean may of course be nonzero even if the process is stationary.

What are the testable implications of the solvency constraint for the sum of the augmented primary surplus $-\bar{\delta}_t$ and seigniorage σ_t ? Let $x_t \equiv -\bar{\delta}_t + \sigma_t$, and $X_t \equiv q_{t-1}x_t$. Since if the solvency constraint is satisfied we have

$$(3.11) \quad D_t = \sum_{i=0}^{\infty} E_t X_{t+1+i},$$

it follows that stationarity of X_t , the present discounted value of seigniorage plus augmented primary surplus, is necessary but not sufficient for solvency. The infinite sum of stationary stochastic processes may be nonstationary. If that were the case for $\{X_t\}$, then D_t would be nonstationary and solvency would fail. Even if the expectation of the infinite sum of future X_{t+i} is stationary, it need not equal D_t and again solvency would fail. An example of such a process for X_t is given in Wilcox [1989] whose approach is followed closely in this section of our paper.

Let $X_t = aX_{t-1} + \eta_t$ with $|a| < 1$, and η_t white noise. This implies

$$(3.12) \quad E_t \sum_{i=0}^{\infty} X_{t+1+i} = \frac{\alpha}{1-\alpha} X_t .$$

Can it be true that $D_t = \frac{\alpha}{1-\alpha} X_t$? If this were the case, we would have

$$D_t - D_{t-1} = -X_t + \frac{\alpha}{1-\alpha} \eta_t .$$

From (3.4') however we know that

$$D_t - D_{t-1} \equiv -X_t .$$

Therefore the first-order stationary autoregressive process for the discounted sum of seigniorage and the augmented primary surplus is inconsistent with solvency.

We could estimate the stochastic process governing X_t , calculate

$$E_t \sum_{i=0}^{\infty} X_{t+1+i} \text{ and test whether this equaled } D_t . \text{ An equivalent but slightly}$$

more direct method is to estimate the stochastic process governing D_t and to test whether $\lim_{i \rightarrow \infty} E_t D_{t+i} = 0$. Hamilton and Flavin [1986] pioneered these tests of the discounted debt. Wilcox [1989] and Grilli [1989] extended their approach. Our analysis involves a statistical generalization of Wilcox's approach using techniques developed by Phillips and Perron [1988].

Wilcox assumes that D_t can be represented by the following potentially multivariate *ARIMA* process:

$$(3.13) \quad [1 - \rho(L)] \left[(1 - L)^d Z_t - \alpha_0 \right] = [1 - \theta(L)] e_t .$$

$\rho(L)$ is a ρ th order polynomial, $\theta(L)$ is a q th order polynomial, Z_t is a random vector whose first element is D_t , a_0 is a vector of constants, and e_t is a vector white noise process. $(I - L)^d Z_t$ is a covariance-stationary series i.e. the series Z is integrated of order d . We assume that $(I - \rho(L))$ and $(I - \theta(L))$ have all their roots outside the unit circle. Since $\rho(L)$ and $\theta(L)$ are thus assumed to satisfy the conditions for stationarity and invertibility, (3.13) has the autoregressive representation

$$(3.14) \quad \eta(L) \left[(I - L)^d Z_t - a_0 \right] = e_t$$

where

$$(3.15) \quad \eta(L) = \sum_{i=0}^{\infty} \eta^i L^i = \left[(I - \theta(L)) \right]^{-1} \left[(I - \rho(L)) \right].$$

Note that the order of the autoregressive process in (3.14) and (3.15) is potentially infinite. This representation is operational only if it can be approximated by a finite-order autoregressive process. a_0 is the unconditional expectation of $(I - L)^d Z_t$.

The solvency tests of Wilcox's statistical model in (3.15) consist of two parts, of which only the first may be necessary. First test whether Z_t is stationary. By assumption $(I - L)^d Z_t$ is stationary. So Z_t is stationary if the order of integration d is less than $\frac{1}{2}$. In a univariate representation of (3.14) with D_t as the only random variable, a finding that the process governing D_t has a unit root (is integrated of order 1) would imply that the D_t process is inconsistent with government solvency. A finding that $d < \frac{1}{2}$ i.e. that Z_t is stationary can still be consistent with insolvency provided

the first element of a_0 is nonzero. If we permit supersolvency ($\lim_{i \rightarrow \infty} E_t D_{t+i} < 0$) as well as solvency ($\lim_{i \rightarrow \infty} E_t D_{t+i} = 0$) then nonstationary processes involving negative values only of D_t would be permissible. A stationary process for D_t with the first element of a_0 negative would also be consistent with (super)solvency. Neither of these two supersolvency cases is a practical possibility for the Indian economy, and they are indeed rejected by the data. If D_t is stationary, the second test therefore is whether the first element of a_0 is zero.

IS THE INDIAN DISCOUNTED PUBLIC DEBT STATIONARY?

Our empirical tests of the stationarity of the Indian public debt are complicated by two problems. First our annual time series is short indeed: 17 annual observations from fiscal year 1970/71 to fiscal year 1986/87. Limited degrees of freedom means low power for all our tests. Second, there are problems about the choice of interest rate to use for discounting the debt. If expected holding period rates of return (interest coupon payments plus capital gains per Rupee invested) were the same for all debt instruments regardless of maturity, currency denomination or other characteristics, there would be no problems as the holding period rate of return on any debt instrument could be used.

This assumption of "uncovered interest parity" for all debt instruments fails heroically in tests involving industrial country financial claims. We should certainly not expect it to be valid for the case of India where domestic financial markets are characterized by frequent and prolonged nonprice rationing.

To make our findings as robust as possible under the circumstances, we perform four versions of the stationarity tests--two involving domestic Rupee interest rates and two involving dollar interest rates.

Our first domestic rate is a government borrowing rate (the Government Bond Yield) while the second is a quasigovernment lending rate--the Advance Rate--which relates to the State Bank of India's prime lending rate which regulates all interest rates for the various categories and classes of advances granted by the Bank. The tests involving the Advance Rate tilt the odds towards rejecting nonstationarity as it exceeds sometimes by a wide margin the rate actually paid by the government on its outstanding liabilities (see Table 5).

The two dollar interest rates are the "All Creditors" rate from the World Bank World Debt Tables and the "Official Creditors" rate from the same source.

With all four interest rates our conclusion is the same. Discounted public debt is nonstationary: the process characterizing discounted public debt over our sample has a unit root in each case. The pattern of behavior that produces the public debt process is therefore not sustainable.

In his empirical work Wilcox [1989] implements the univariate³ special case of equation (3.14) given in (3.16).

$$(3.16) \quad D_t = a_0 + a_1 t + \sum_{i=1}^{\infty} \beta_i D_{t-1} + e_t$$

The error term e_t is assumed to be i.i.d. Eventual insolvency will occur if (at least) one of the following conditions holds:

1. The roots of $1 - \beta(L)$ do not all lie outside the unit circle.

2. $a_1 \neq 0$, that is there is a deterministic trend (one expects a positive coefficient i.e. no supersolvency).
3. $a_0 \neq 0$, that is even though the D_t process is stationary, its unconditional expectation is nonzero (again one expects $a_1 > 0$ i.e. no supersolvency).

Following Perron and Phillips [1987] we generalize Wilcox's approach to handle a wider class of stochastic processes for the error term (see also Phillips [1988], and Phillips and Perron [1988]). Our D_t equation is:

$$(3.17) \quad D_t = a_0 + a_1 t + \beta D_{t-1} + u_t .$$

The error term u_t can belong to a very wide class of stochastic processes. $\{u_t\}_0^\infty$ is a weakly stationary sequence of random variables satisfying the following conditions (see Phillips [1988]).

$$E(u_0) = 0 ;$$

$$E|u_0|^{\beta+\epsilon} < \infty \text{ for some } \beta > 2 ;$$

$\{u_t\}_0^\infty$ is strong mixing with mixing numbers a_m which satisfy

$$\sum_{m=1}^{\infty} a_m^{1-2/\beta} < \infty .$$

These conditions allow for many weakly dependent time series and include a broad class of data generating mechanisms such as finite order *ARMA* models under very general conditions.

The original Dickey–Fuller test for the presence of a unit root was developed for autoregressive representations of known order and normal i.i.d. errors (Dickey and Fuller [1979]). The augmented Dickey–Fuller test assumed an autoregressive representation of known order plus a time trend and normal i.i.d. errors (Dickey and Fuller [1981]). Said and Dickey [1984] showed that the Dickey–Fuller t -test for a unit root can be applied to *ARIMA* ($p, 1, q$) models provided the lag length in the autoregression increases with the sample size T at a rate less than $T^{\frac{1}{3}}$. The Phillips–Perron tests are nonparametric and can be applied to a wider class of processes than the three other tests.⁴

Our null hypothesis for equation (3.17) is that $\beta = 1$, and $a_1 = 0$ within a maintained hypothesis that permits a nonzero drift a_0 . If we fail to reject the null, the discounted public debt is nonstationary which would imply insolvency if the process persisted into the indefinite future. If the null is rejected but we find a significantly positive value of a_1 , that is a positive deterministic trend, in the discounted debt series, eventual insolvency still looms. If the null is rejected and we cannot reject $a_1 = 0$ and $\beta < 1$, then finding a positive drift ($a_0 > 0$) again would imply eventual insolvency.

The following three test statistics given in Table 7 are derived in Phillips and Perron [1988] for the null that $\beta = 1$ and $a_1 = 0$. $Z(\beta)$ makes use of the standardized and centered least squares estimates of β . $Z(t_\beta)$ makes use of the t statistic on β , t_β (for $\beta = 1$), and $Z(F_g)$ is the regression " F -test" of Dickey and Fuller [1981] for the more general class of

TABLE 7: UNIT ROOT TEST FOR DISCOUNTED DEBT

Discounted Series	$Z(\beta)$	$Z(t_\beta)$	$Z(\phi_g)$
D_1	1.62	1.20	12.26
D_2	-3.37	-1.10	1.08
D_1^*	-3.74	-1.39	2.46
D_2^*	-2.17	-1.00	4.31
Critical (95%)	-17.9	-3.60	7.24
Values ** (99%)	-22.5	-4.38	10.61

** These critical values are for sample size 25. Our sample size is 17. Using the critical values appropriate to the larger sample biases the test towards rejection of the null.

D_1 is the debt measured in Rupees discounted at the Long-Term Government Bond yield.

D_2 is the debt measured in Rupees discounted at the average advance rate.

D_1^* is the debt measured in U.S. dollars discounted at the foreign all creditors dollar interest rate.

D_2^* is the debt measured in U.S. dollars discounted at the foreign official creditors dollar interest rate.

error processes in (3.17). These three statistics have the same limiting distributions for a very wide class of error processes as the statistics developed by Dickey and Fuller for the case of i.i.d. errors. The critical values of the three statistics are therefore the same and can be found in Fuller [1976], and Dickey and Fuller [1981]. We also provide the point estimates and standard errors for the three parameters a_0 , a_1 and β in Table 8.

With the exception of the $Z(\hat{\phi}_g)$ test on D_1 (debt in Rupees discounted at the Government's Long Bond Yield), all the evidence points to nonstationarity of the discounted debt series. The rejection of the (single) unit root hypothesis in the case of the $Z(\hat{\phi}_g)$ statistic for the D_1 series occurs because the discounted debt appears to be more nonstationary than can be captured by a single unit root.

When we perform the Phillips-Perron tests for the first difference of D_1 , we obtain the results given in Table 9. The null of a unit root in the first difference of D_1 cannot be rejected. The hypothesis of nonstationarity for D_1 therefore cannot be challenged.

ARE THE INDIAN DEBT-GNP RATIO AND THE PRIMARY DEFICIT-GNP RATIO STATIONARY?

To complement the findings of nonstationarity of the discounted public debt, Table 10 below presents unit root tests for the debt-GNP ratio NTD/GNP and for the discounted value of the sum of the augmented primary surplus and seigniorage X . The point estimates of a_0 , a_1 and β are given in Table 11.

The debt-GNP ratio is nonstationary. For the discounted sum of seigniorage and augmented primary surplus, we also fail to reject the null that $\beta = 1$ and $a_1 = 0$.

TABLE 8: POINT ESTIMATES OF a_0 , a_1 AND β FOR THE DISCOUNTED DEBT*

	a_0	a_1	β
D_1	-133.9 (73.4)	200.3 (184.6)	1.10 (0.083)
D_2	1462.8 (3651.5)	52.6 (43.0)	0.92 (0.073)
D_1^*	9691.9 (7124.2)	667.4 (414.7)	0.78 (0.158)
D_2^*	6952.0 (8435.1)	566.8 (376.3)	0.87 (0.131)

*Standard errors are given in brackets below coefficient estimates.

TABLE 9: UNIT ROOT TESTS FOR ΔD_1

	$Z(\beta)$	$Z(t_\beta)$	$Z(\frac{1}{g})$
	-11.69	-3.28	4.03
Critical (95%)	-17.9	-3.60	7.24
Values* (99%)	-22.5	-4.38	10.61

*These critical values are for sample size 25, (see Table 7).

TABLE 10: UNIT ROOT TESTS FOR NTD/GNP AND X

	$Z(\beta)$	$Z(t_\beta)$	$Z(\frac{1}{g})$
NTD/GNP	0.04	0.02	5.08
X [*]	-12.89	-3.16	4.39
Critical (95%)	-17.9	-3.60	7.24
Values ^{**} (99%)	-22.5	-4.38	10.61

* The discount rate used is the yield on long-term government securities.

** These critical values are for sample size 25 (see Table 7).

TABLE 11: POINT ESTIMATES OF a_0 , a_1 AND β FOR NTD/GNP AND X*

	a_0	a_1	β
NTD/GNP	0.014 (0.039)	0.0034 (0.0015)	0.9883 (0.552)
X	-2681.56 (9246.76)	-298.68 (101.84)	0.18 (0.26)

* Standard errors are given in brackets below coefficient estimates.

4. POLICY OPTIONS TO AVOID INSOLVENCY

From equation (3.5) reproduced below, the options for restoring solvency are clear:

$$(4.1) \quad d_t \equiv \sum_{i=0}^{\infty} E_t \prod_{j=0}^i \frac{1}{1 + r_{t+j}} \left[-\bar{\delta}_{t+1+i} + \sigma_{t+1+i} \right] + \Omega_t$$

where

$$(4.2) \quad \Omega_t \equiv \lim_{i \rightarrow \infty} E_t \prod_{j=0}^i \left[\frac{1}{1 + r_{t+j}} \right] d_{t+1+i} .$$

The present value of the "solvency gap" (as a proportion of GDP) is given by Ω_t . For solvency we require $\Omega_t \leq 0$ or, ruling out the case of supersolvency, $\Omega_t = 0$. With or without solvency, Ω_t follows a martingale

$$(4.3) \quad E_t \Omega_{t+1} = \Omega_t .$$

To get some sense of the magnitude of the solvency gap, it is helpful to calculate the permanent flow equivalent to the stock Ω_t . Let \tilde{r}_t^{∞} be the long interest rate net of the real growth rate i.e.

$$(4.4) \quad \tilde{r}_t^{\infty} = \left[\sum_{i=0}^{\infty} E_t \prod_{j=0}^i \left[\frac{1}{1 + r_{t+j}} \right] \right]^{-1} .$$

\tilde{r}_t^{∞} is the yield on a consol or perpetuity which pays a constant coupon of one unit of output each period when the single-period discount rates are \tilde{r}_{t+j} ,

and the expectations theory of the term structure of interest rates holds.

$$\tilde{r}_t^{\infty} \Omega_t \equiv \tilde{r}_t^{\infty} \left[d_t - \sum_{i=0}^{\infty} E_t \prod_{j=0}^i \left[\frac{1}{1 + r_{t+j}} \right] \left[-\bar{\delta}_{t+1+i} + \sigma_{t+1+i} \right] \right] \text{ is the smallest}$$

constant fraction of GDP that--if it were to be devoted this period and every period in the future to eliminating the solvency gap--would succeed in closing the gap. It is the "permanent correction" i.e. sustained change in the augmented primary surplus or seigniorage (as a proportion of GDP) that must be made to the government's fiscal, financial and monetary plans to avoid insolvency.

Noting that

$$(4.5) \quad \bar{\delta}_t = \delta_t + \frac{b_{t-1}^*}{1 + n_{t-1}} \left[(1 + r_{t-1}^*)(1 + \gamma_{t-1}) - (1 + r_{t-1}) \right]$$

and

$$(4.6) \quad \delta_t = c_t - \tau_t + a_t - \frac{\rho_{t-1}}{1 + n_{t-1}} k_{t-1},$$

we see that there are four broad types of policy options for reducing the augmented primary deficit: 1) reductions in government consumption spending; 2) increases in net current revenues; 3) reducing public sector capital formation and/or increasing government cash revenues from the stock of public sector capital; and 4) shifting the composition of the public debt between internal and external debt. It is important to note that the different spending and revenue items that make up δ cannot be expected to be

behaviorally independent of each other either in the short run or in the long run.

The first two options for reducing the primary deficit are conceptually simple and politically difficult. Proposals for widening the current revenue base through an expenditure tax, a broadly based value-added tax or a broadly based income tax have been made for decades with little to show for it. Wealth taxes such as a land tax or a tax on urban real estate are also administratively feasible but unpopular among the politically influential classes. Replacing nonauctioned import quotas with auctioned quotas or tariffs is another relatively straightforward policy measure for raising current revenues.

The net cash return on the government's capital stock ρk need bear no relation whatsoever to the social returns to public sector capital. Cash returns are relevant for our purposes because no matter how desirable the project it will have to be financed, and the financial implications of the project will influence the overall financial position of the government including its solvency.

For many forms of public sector capital, the cash returns are persistently negative. Public enterprises with secular operating losses would be one example. Social overhead capital or infrastructure that does not yield revenues directly (say through tolls and other user charges) but absorbs current resources for maintenance is another. To the extent that infrastructure boosts real GNP and thus widens the tax base and tax revenues for given tax schedules, its contribution to the Exchequer would be through τ rather than through ρk . In principle such indirect revenue contributions can (and should) be allowed for (see Buiter [1989c]), but the necessary

information (including estimates of the effect of increments to public sector capital on the tax base) is not in practice available.⁵

Noting that $K_t - K_{t-1} = A_t - wK_{t-1}$ where $0 < w < 1$ is the depreciation rate of public sector capital, we can rewrite the budget identity as follows:

$$(4.7) \quad d_t - k_t = (1 + \bar{r}_{t-1})(d_{t-1} - k_{t-1}) + c_t - \tau_t - \sigma_t \\ + \left[\frac{(1 + r_{t-1}^*)(1 + \gamma_{t-1}) - (1 + r_{t-1})}{1 + n_{t-1}} \right] b_{t-1}^* \\ + \left[\frac{(\tau_{t-1} - (\rho_{t-1} - w))}{1 + n_{t-1}} \right] k_{t-1} .$$

The solvency constraint can in turn be rewritten as:

$$(4.8) \quad d_t - k_t = \sum_{i=0}^{\infty} E_t \frac{q_{t+i}}{q_{t-1}} \left[[\tau_{t+1+i} - c_{t+1+i}] + \sigma_{t+1+i} \right] \\ + \sum_{i=0}^{\infty} E_t \frac{q_{t+i}}{q_{t-1}} \left[1 + r_{t+i} - (1 + r_{t+i}^*)(1 + \gamma_{t+i}) \right] \frac{b_{t+i}^*}{1 + n_{t+i}} \\ + \sum_{i=0}^{\infty} E_t \frac{q_{t+i}}{q_{t-1}} (\rho_{t+i} - w - r_{t+i}) \frac{k_{t+i}}{1 + n_{t+i}} .$$

The debt total emphasized in (4.7) and (4.8) is public debt net of the value of the public sector capital stock at current reproduction costs. The first term on the r.h.s. of equation (4.8) is the present value of future

seigniorage and the primary current account or consumption account surplus of the public sector.

If all public sector assets and liabilities earned the same expected cash rate of return as that used in the present value calculations (i.e. r_{t+i}), then the last two terms on the r.h.s. of (4.8) vanish. If $\rho_{t+i} - w$, the net of depreciation cash rate of return on public sector capital, exceeds r_{t+i} , the marginal cost of domestic borrowing, then smaller primary current surpluses and/or less seigniorage are required to achieve solvency. A value of r_{t+i} in excess of $\rho_{t+i} - w$ worsens the solvency of the government.

The second term on the r.h.s. of equation (4.8) allows for differences between internal and external borrowing rates. Clearly if the expected real rate of return on domestic debt exceeds that on foreign debt ($1 + r_{t+j} > (1 + r_{t+j}^*)(1 + \gamma_{t+j})$) or equivalently if $1 + i_{t+j} > (1 + i_{t+j}^*)(1 + \epsilon_{t+j})$, then switching from domestic debt to foreign debt (either through an open-market operation today or by switching the future internal-external financing mix towards external borrowing) will strengthen the solvency of the government. Of course such an operation may not be feasible say because the foreign interest rate does not represent the marginal cost of increased external borrowing. This will e.g. be the case if there is external credit rationing.

The final option open to the government to restore its solvency without repudiation is to increase the present value of future seigniorage.

Seigniorage σ_t can be written in a number of equivalent ways:

$$\begin{aligned}
 (4.9) \quad \sigma_t &\equiv \frac{\Delta M_t}{P_t Y_t} \\
 &\equiv \frac{\Delta M_t}{M_t} m_t = \mu_{t-1} m_t \\
 &\equiv \Delta m_t + \left[(1 + \pi_{t-1})(1 + n_{t-1}) - 1 \right] \frac{m_{t-1}}{(1 + n_{t-1})(1 + \pi_{t-1})} .
 \end{aligned}$$

Here m denotes base money per unit of GDP.

The first of the three expressions on the r.h.s. of (4.9) emphasizes the real resources appropriated by the government in period t by running the printing presses. The second breaks down total seigniorage revenue as the product of a "seigniorage tax rate" $\frac{\Delta M}{M} \equiv \mu$ and a "seigniorage tax base" $m \equiv \frac{M}{PY}$ the reciprocal of the income velocity of circulation of base money. The third expresses the value of the resources extracted by the government as the sum of the increase in the stock of money per unit of output Δm_t and the change in the money output ratio that would have occurred with a constant nominal money stock because of inflation and real output growth (approximately $(\pi_{t-1} + n_{t-1})m_{t-1}$). Some authors reserve the term "inflation tax" for $\pi_{t-1} m_{t-1}$, but we shall use the terms seigniorage and inflation tax interchangeably.

In steady state we assume that

$$(4.10) \quad 1 + \mu = (1 + n)(1 + \pi) .$$

We wish to investigate the relationship between σ , the amount of seigniorage the government wishes to extract, and the rate of inflation. This

requires a model of the demand for base money. Base money is the sum of currency CU and commercial bank reserves RES i.e. $M = CU + RES$.

For the purposes of this paper it is sufficient to work with an aggregate demand for base money function which is not "built up" from its constituent components CU and RES . Using annual data from 1960/61 to 1986/87, we estimated a base money demand equation in velocity form. V is the ratio of GNP to the monetary base (the income velocity of circulation of base money). In our notation $V_t \equiv \frac{M_{t-1}}{P_t Y_t} = \frac{m_{t-1}}{(1+n_{t-1})(1+\pi_{t-1})}$.⁶ The result is given in equation (4.11).

$$(4.11) \quad \Delta V_t = 48.282 - 0.151 V_{t-1} + 2.384 \pi_{t-1} \\ (3.151) \quad (-2.082) \quad (2.422) \\ - 6.850 \ln Y_{t-1} + 0.188 t \\ (-3.075) \quad (2.545)$$

$R^2 = 0.65$; $SE = 0.32$; $F(4, 20) = 9.15$ (0.0002); $DW = 2.21$. Conventionally calculated t statistics are given in brackets below the coefficient estimates. SE is the equation standard error. The probability value for the F test whose null is that the population R^2 is zero is 0.0002 . The LM test for autocorrelated residuals (from lags 1 to 3) gives $\text{Chi}^2(3) = 1.07$. The F -form suggested by Harvey [1981] gives $F(3, 17) = 0.25$. There is therefore no evidence of residual autocorrelation. White's [1980] test for heteroskedasticity gives $F(8, 11) = 0.83$. The Chi^2 test for normality of the residuals yields $\text{Chi}^2(2) = 0.94$.⁷

The estimated equation suggests that a higher rate of inflation raises velocity (reduces the demand for base money). A possible interpretation of the last two terms on the r.h.s. of equation (4.11) is that positive deviations of output from trend are associated with reductions in velocity.

Inferences about the long-run rate of inflation implied by the need for seigniorage revenue are most easily made when the money demand or velocity equation has a steady state. Equation (4.11) almost qualifies. In steady state output grows at the constant proportional rate n . Over the sample period the mean growth rate of real output is 0.034 . The last two terms on the r.h.s. of equation (4.11) can in steady state be written as:

$$-6.850 \ln Y_0 - 6.850nt + 0.188t .$$

While the two opposing trends don't quite cancel each other out when $n = 0.034$, a reasonable first approximation would permit us to evaluate long-run velocity by ignoring the offsetting time trends. This yields the long-run velocity equation:

$$(4.12a) \quad V = 6.463 + 15.782\pi .$$

Alternatively we can evaluate both $\ln Y$ and t at their sample means. This yields

$$(4.12b) \quad V = 6.312 + 15.782\pi .$$

In what follows we work with the numerical estimates given in equation (4.12b).

Steady-state seigniorage as a proportion of GNP is, from equation (4.9), given by

$$(4.13) \quad \sigma = [(1 + \pi)(1 + n) - 1] V^{-1} .$$

Unlike the long-run money demand functions that have $\frac{m}{(1+n)(1+\pi)}$ or $\ln\left[\frac{m}{(1+n)(1+\pi)}\right]$ linear in π , the long-run base money demand function we have estimated does not have a long-run seigniorage Laffer curve. Across steady states seigniorage increases with the rate of inflation as long as $9.47n < 6.312$. The reason is (speaking somewhat loosely) that the elasticity of long-run money demand $\frac{m}{(1+n)(1+\pi)}$ with respect to the inflation rate is less than unity in absolute value for all inflation rates.⁸ A greater need for long-run seigniorage therefore unambiguously implies a higher long-run rate of inflation: $\pi = \frac{n - 6.312\sigma}{15.782\sigma - (1+n)}$.

To get a sense of the magnitude of the correction in $x = -\delta + \sigma$, the sum of the augmented primary surplus and seigniorage, we calculate what constant value of x would stabilize the debt-GNP ratio at some given value d for given constant values of the real interest rate net of the growth rate $r - n$. Since $x = (r - n)d$, the figures in Table 12 follow immediately.

The actual value of x_t in 1985/6 was -5.5 percent of GNP (see Table 4). It reached -7.1 percent of GNP in 1986/7. Even in the optimistic case in which d is stabilized at its 1986/87 value of just over 50 percent of GNP and the interest rate exceeds the growth rate by only one percentage point, x_t would have to be raised by between 6.0 and 7.6 percentage points of GNP permanently.

If the debt-GNP ratio is not stabilized until it reaches some higher level or if the long-run growth rate of GNP falls short of the long-run real interest rate by more than one percentage point, the required permanent fiscal or seigniorage correction can be considerably higher.

$\hat{\sigma}$, the maximal steady-state seigniorage that can be extracted given our estimate of the money demand function, is given by $\hat{\sigma} = \frac{1+n}{\beta} = \frac{1+n}{15.782}$. With a four percent long-run real growth rate, $\hat{\sigma} \approx 0.066$ that is 6.6 percent of GNP. This amount of seigniorage will only be extracted in the limit as the rate of inflation goes to infinity. Without a reduction in the augmented primary deficit, stabilizing the debt-GNP ratio at 50 percent through increased use of seigniorage alone will not be possible even when the excess of the interest rate over the growth rate is only one percentage point. In that case seigniorage would have to be raised by between 6.0 and 7.6 percent of GNP to 8.70 or 10.30 percent of GNP. The nonexistence of equilibrium in this case suggests the possibility of a hyperinflation.

The steady-state inflation rate implied by a continuation of the current share of seigniorage in GNP (2.7 percent, and therefore a reduction in the primary deficit of between 6.0 and 7.6 percent of GNP) would, with a four percent long-run real growth rate of GNP, be 21.2 percent per year. Lowering the long-run share of seigniorage in GNP to one percent would, with a four percent long-run real growth rate, reduce the long-run inflation rate to 2.6 percent per annum. Raising the state-state share of seigniorage in GNP to four percent would, with a four percent real growth rate, imply a long-run rate of inflation of 52.0 percent per year. These figures should of course be taken with a fairly substantial pinch of salt. Our demand function for base money leaves much to be desired. We offer it as an illustration of a methodology that we believe to be useful. Further empirical refinement is

**TABLE 12: DEBT-GNP RATIO STABILIZING VALUES OF SEIGNIORAGE
PLUS AUGMENTED PRIMARY SURPLUS**

$r - n$	d	50%	75%	100%
1%		0.5%	0.75%	1.0%
2		1.0	1.5	2.0
3		1.5	2.25	3.0

necessary before confident statements can be made about the trade-offs faced by India's policy makers.

The seigniorage concept analyzed here can be referred to as one component of the anticipated inflation tax. Anticipated inflation may also have effects on the primary deficit. A tax system that incorporates nominal progression will, with any positive rate of inflation, yield increased real tax revenues if the tax brackets are not fully index-linked. This "fiscal drag" has been argued to be dominated at high rates of inflation by the negative effect on real tax collections of delayed payment of taxes and inadequate interest penalties for late payment. This Tanzi effect (Tanzi [1978]) is further complicated through the existence of nominal specific taxes and tariffs, nominal cash limits on spending etc.

If the government has long-dated fixed interest debt outstanding which is denominated in domestic currency, then an unanticipated increase in the rate of inflation will act as an unanticipated capital levy on such assets. This can be viewed as a de facto repudiation of part of the public debt. Even with domestic currency denominated short maturity debt or long-dated but variable interest debt, an unexpected capital levy can be imposed if an unexpected increase in the general price level can be engineered. In a very open economy this can be achieved through a devaluation of the nominal exchange rate, but in the case of India the very limited openness of the economy prevents this from being a practical policy option.

Like formal repudiation, de facto repudiation through a deliberately engineered unexpected reduction in the market value of the public debt is a serious policy option that should be considered along with policy measures for reducing the primary deficit and along with the increased use of seigniorage. Long-dated public debt, even when index-linked (which is not the case in

India), and to a lesser extent short-dated debt is already "contingent debt", even in the absence of repudiation risk: its real value can change, both in anticipated and unanticipated ways, because of a whole range of shocks that have not been engineered deliberately or accidentally by the policy makers.

One of the likely consequences of (partial) repudiation, (as of a capital levy on bond holders or a decline in the market value of long-dated debt due to an unexpected increase in the rate of inflation) is to add a risk premium to the interest rate the government must pay on new debt issues. In extreme cases the government might, for a while, not be able to issue new debt at all. These costs have to be balanced against the cost of reducing the primary deficit and of making increased use of seigniorage. There is no general case, in equity or efficiency, for exempting the owners of public debt from sharing the burden of adjustment to unforeseen contingencies in preference to owners of human capital, physical capital or land and in preference to the beneficiaries of public spending.

5. CONCLUSION

The statistical solvency tests and the estimated demand for base money function suggest two conclusions. First continuation of recent patterns of behavior will eventually threaten the solvency of the government. Second the option of using the inflation tax to close part of the budgetary gap appears to be a limited option indeed. Small increases in the share of seigniorage in GNP will have a high cost in terms of additional long-run inflation and even maximal use of the inflation tax will not be sufficient to close the solvency gap.

The first of these conclusions does not stand or fall with our formal tests for solvency. Visual inspection of Figures 1 and 2, and Tables 1 and 4 make it abundantly clear that the fiscal situation is perilous. It is primarily the rapid increase in the internal public debt that signals the crisis to come.

Unless measures to reduce the primary deficit are taken, a fiscal crisis is bound to come. Where and when it will strike cannot be predicted with certainty. Often a fiscal crisis first manifests itself in the foreign exchanges. Actual or imminent international reserve exhaustion is a common trigger for emergency measures including recourse to IMF standby financing and the conditionality this implies. Such foreign exchange crises can happen even if, as in the case of India, the external debt burden of the country is quite modest.

The fiscal retrenchment that appears to be required is large and will be painful. The political and economic challenge is to implement the required combination of spending cuts, and of tax and other revenue increases in a way that does least damage to the economy's growth prospects, and that protects the weakest and poorest citizens.

NOTES

¹This condition can of course be stated in several equivalent ways such as

$$\frac{B_{t+j+1} + S_{t+j+1} B_{t+j+1}^*}{B_{t+j} + S_{t+j}^* B_{t+j}} < 1 + i_{t+j}$$

$$\frac{(B_{t+j+1} + S_{t+j+1} B_{t+j+1}^*) P_{t+j}}{(B_{t+j} + S_{t+j}^* B_{t+j}^*) P_{t+j+1}} < 1 + r_{t+j} .$$

²In an open economy this statement implies the assumption of a "source based" system of taxation which taxes all income streams originating within the national economy. With a residence-based view of taxation we instead tax all income accruing to domestic residents i.e. GNP and the simple story we tell here would not work.

³Strictly speaking, treating time as a purely deterministic process, our model is a bivariate version of equation (3.14) with a zero variance innovation in the second equation.

⁴The tests were performed using a program written by Mico Loretan in GAUSS.

⁵If money losing public enterprises can be privatized at a price exceeding the present discounted value of the cash flow of these enterprises (on the assumption that they remain in the public sector), such privatization can strengthen the government's balance sheet.

⁶The estimation and tests were done using PC Give.

⁷Note that M_{t-1} is the money stock at the end of period $t-1$.

⁸This would be exactly correct if $n = 0$.

REFERENCES

Andrew B. Abel, Gregory Mankiw, Lawrence H. Summers and Richard J. Zeckhauser [1989], "Assessing Dynamic Efficiency: Theory and Evidence," *Review of Economic Studies* 56 January 1-19.

Ritu Anand and Sweder van Wijnbergen [1989], "Inflation and the Financing of Government Expenditure: An Introductory Analysis with an Application to Turkey," *The World Bank Economic Review* 3 17-38.

Robert J. Barro [1979], "On the Determination of the Public Debt," *Journal of Political Economy* 87(5), Part 1, October, 940-971.

Willem H. Buiter [1983a], "Measurement of the Public Sector Deficit and Its Implications for Policy Evaluation and Design," *International Monetary Fund Staff Papers* 30 June, 306-349.

_____ [1983b], "The Theory of Optimum Deficits and Debt," in Federal Reserve Bank of Boston Conference Series No. 27, The Economics of Large Government Deficits, October, 4-69.

_____ [1985], "A Guide to Public Sector Debt and Deficits," *Economic Policy* 1 November, 14-61.

_____ [1989a], "The Arithmetic of Solvency," in W.H. Buiter, Principles of Budgetary and Financial Policy, 149-159, London: Harvester Wheatsheaf.

- ____ [1989b], "Some Thoughts on the Role of Fiscal Policy in Stabilization and Structural Adjustment in Developing Countries," in in W.H. Buiter, Principles of Budgetary and Financial Policy, London: Harvester Wheatsheaf.
- ____ [1989c], "Can Public Spending Cuts be Inflationary?" in W.H. Buiter, Principles of Budgetary and Financial Policy, London: Harvester Wheatsheaf.
- D.A. Dickey and W.A. Fuller [1979], "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," Journal of the American Statistical Association 74 427-31.
- ____ [1981], "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," Econometrica 49 1057-72.
- W.A. Fuller [1976], Introduction to Statistical Time Series, New York: Wiley.
- Vittorio Grilli [1989], "Seigniorage in Europe," in M. de Cecco and A. Giovannini (eds.), A European Central Bank?, Cambridge University Press, 53-79.
- James D. Hamilton and Marjorie A. Flavin [1986], "On the Limitations of Government Borrowing: A Framework for Empirical Testing," American Economic Review 76 September, 808-819.

Andrew C. Harvey [1981], The Econometric Analysis of Time Series, Oxford:

Philip Allan.

Pierre Perron and Peter C.B. Phillips [1987], "Does GNP have a Unit Root? A Reevaluation," *Economics Letters* 23 139-145.

Peter C.B. Phillips [1988], "Regression Theory for Near-Integrated Time Series," *Econometrica* 56(5), September, 1021-1043.

_____ and Pierre Perron [1988], "Testing for a Unit Root in Time Series Regression," *Biometrika* 75(2), 335-46

S.E. Said and D.A. Dickey [1984], "Testing for Unit Roots in Autoregressive-Moving Average of Unknown Order," *Biometrika* 71 599-607.

Vito Tanzi [1978], "Inflation, Real Tax Revenue and the Case of Inflationary Finance: Theory with an Application to Argentina," *International Monetary Fund Staff Papers* 25 September.

H. White [1980], "A Heteroskedasticity-Consistent Covariance Estimator and A Direct Test for Heteroskedasticity," *Econometrica* 48(4), 817-838.

David W. Wilcox [1989], "The Sustainability of Government Deficits: Implications of the Present Value Borrowing Constraint," *Journal of Money, Credit and Banking* 21 August, 291-306.