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TOWARDS THE IMPLEMENTATION OF THE DESIRABLE RULES OF MONETARY  
COORDINATION AND INTERVENTIONS

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and

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Towards the Implementation of Desirable Rules of Monetary  
Coordination and Interventions

Abstract

Adopting the decomposition of the world system into subsystems of the sum of and the difference of variables, the paper considers the characteristics of optimal rules of monetary coordination and exchange-rate intervention in a symmetric, two-country Dornbusch model. Global monetarism concerns the sum or average system: Discussions of misalignment of exchange rate concerns the difference variables. It is shown that the results by Poole (QJE 1970) have a strong analogy in this framework. No or little intervention is desirable if dominant country-specific shocks are in IS curves; interventions to keep exchange rates are desirable if dominant country-specific shocks are in LM curves.

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Towards the Implementation of Desirable Rules of Monetary  
Coordination and Interventions  
Shin-ichi Fukuda  
Koichi Hamada

I. Introduction

Nowadays people often claim the existence of misalignments of exchange rates among major currencies, and they grope for macroeconomic coordination and intervention rules. At the Tokyo Summit in May 1986, the leaders of seven countries agreed to request their finance ministers to review their economic objectives and forecasts collectively using indicators such as gross national growth rates, inflation rates, interest rates, and so forth.

It is one thing to name important indicators; quite another to implement an effective policy rule based on these indicators. In the present world, economy where there are various kinds of disturbances -- supply or demand, monetary or real, domestic or foreign, and temporary or permanent -- policy authorities are obliged to rely on some form of monetary rules and intervention rules if they wish to turn the declaration of the summit into an operational scheme.

The purpose of this paper is to characterize the nature of simple, appropriate rules for monetary management in the world economy under uncertainty. We will confine our attention to choices among simple feedback rules. Even though sophisticated discussions of complex dynamic rules, contingent both upon various state variables and upon the reaction of the other player, are intellectually fascinating, we

consider that the examination of simpler rules is at least as important as that of sophisticated ones. Simpler rules have a strong advantage in that they can be more easily explained to policy makers and more easily understood by them so that the possibility of adopting one of the simpler rules is much higher.

We must distinguish two types of policy discussions in international macroeconomics on two issues: The first issue is the need for global monetary coordination, addressed in the proposal of McKinnon (1984) that the monetary authorities of the major countries should agree to provide stable aggregate monetary growth to the world economy in order to stabilize the world price level. The second issue is possible remedies for exchange misalignment, emphasized by Williamson (1983). The discussion of misalignment is concerned with the relative prices of currencies, and, accordingly, with differences among national economies.

We will primarily rely on a stochastic two-country version of the model with sluggish price movements developed by Dornbusch (1976) and also of the neoclassical flex-prices model where output varies due to the discrepancy between the actual and the expected price level (e.g. Weber 1981). We shall adopt the method of Aoki (1981) for decomposing variables into their sums and differences in a two country model following Miller (1982) and Buiter (1986).<sup>1</sup> In order to keep our analysis transparent and focus on the qualitative nature of desirable rules, we will deal with a two country situation with a symmetric economic structure, assuming identical parameters of characterizing economic behavior between the two countries. Only stochastic

disturbances and accordingly policy reactions to these disturbances can differ.

Under these assumptions, we will show that the subsystems of the Dornbusch model with sluggish prices as well as those of a neoclassical model have structures similar to the closed Keynesian model under disturbances that was analyzed by Poole (1970). In the system of average variables of our two-country model, Poole's original results naturally hold. The world average or global money supply target is more desirable when synchronized disturbances in the IS curve dominate; the target for the average interest rate target is more desirable when synchronized disturbances in money demand such as a worldwide financial innovation dominate. More interesting is the fact that analogy holds in the difference system as well. No or little intervention is more desirable when country specific disturbances are mainly in the IS curve, including disturbances due to changing competitiveness in trade; extensive intervention in such a way as to slow down the movement of exchange rates or to reduce the difference in interest rates is more desirable when the difference, or country-specific, disturbances in the LM curve dominate.

There have been many contributions on optimal intervention rules under uncertainty, notably Boyer (1978), Turnovsky (1983), Weber (1981), and many articles in Bhandari (1985). By adopting a symmetric structure between countries, our paper focuses on the interaction of overall disturbances and country-specific disturbances. Thus the results by Weber (1981) are generalized and set in a more symmetric framework. We

also show the effectiveness of feedback rules on interest rate differentials. Such rules have been relatively neglected compared with those that feedback from exchange rates. The link between this kind of model and Poole's analysis was already suggested by Buitier and Eaton (1985), but our analysis shows that the link is much stronger than one might imagine. Some regularities found in the simulation analysis by McKibbin and Sachs (1986) can be better understood by our complementary, analytical approach.

In Section II, we will develop the basic framework of the two-country Dornbusch model, in Section III we will characterize the optimization problem in the decomposed subsystems of the sum and the difference. In Section IV, we will compare the results of the Dornbusch model with a neoclassical model where prices clear the goods market instantaneously while the supply of output depends on unexpected price changes. Section V summarizes the results and the Appendix sketches the possible extension of our approach to n-country case.

## II. The Basic Framework

In order to illustrate the advantage of decomposing the system into averages and differences, consider a simple two-country version of the Dornbusch model. Every parameter is symmetric between the two countries, but disturbances and policy variables may differ. Here  $m$ ,  $p$ ,  $y$ , and  $s$  refer respectively to the logarithm of nominal money supply, price level, real income, and nominal spot exchange rate (the price of the foreign currency in terms of the home currency).  $i$  is the nominal rate of interest. All the variables with asterisks indicate the

variables for the foreign country. Subscripts refer to time periods. There are three types of random disturbances to the system: an LM shock  $\epsilon_t$ , an IS shock  $\eta_t$  and a price shock  $\mu_t$ . Throughout the paper we shall assume that  $\epsilon_t$ ,  $\eta_t$  and  $\mu_t$  (similarly  $\epsilon_t^*$ ,  $\eta_t^*$  and  $\mu_t^*$ ) are mutually and time independent. On the other hand, we do allow positive or negative correlation between  $\epsilon_t$  and  $\epsilon_t^*$ ,  $\eta_t$  and  $\eta_t^*$ , or  $\mu_t$  and  $\mu_t^*$ . Let us denote  $E(\epsilon_t^2) = \sigma_\epsilon^2$ ,  $E(\eta_t^2) = \sigma_\eta^2$  and  $E(\mu_t^2) = \sigma_\mu^2$ , and so forth.

The model can be described as follows:

$$(1a) \quad m_t - p_t = -\alpha i_t + y_t + \epsilon_t \quad (1b) \quad m_t^* - p_t^* = -\alpha i_t^* + y_t^* + \epsilon_t^*$$

$$(2a) \quad y_t = -\beta(i_t - E_t p_{t+1} + p_t) - \gamma(p_t - s_t - p_t^*) + \eta_t \quad (2b) \quad y_t^* = -\beta(i_t^* - E_t p_{t+1}^* - p_t^*) - \gamma(p_t^* + s_t - p_t) + \eta_t^*$$

$$(3a) \quad p_{t+1} - p_t = \delta(y_t - \bar{y}) + \mu_t \quad (3b) \quad p_{t+1}^* - p_t^* = \delta(y_t^* - \bar{y}^*) + \mu_t^*$$

$$(4) \quad E_t s_{t+1} - s_t = i_t - i_t^*$$

Here  $E_t p_{t+1}$ , for example, indicates the price in period  $t+1$  forecasted in period  $t$ .

Equations (1a), (1b) and (2a), (2b) are respectively the standard LM, IS curves with the simplifying assumption of the unitary income elasticity of the demand for money. (3a) and (3b) indicate that price levels adjust only slowly, that is,  $\delta$  is small, ... the crucial



assumption in the Dornbusch overshooting model. We do not dare to defend this formulation ourselves, but we will rather contrast the characteristics of the optimal feedback rules in this sluggish price model with those in a neoclassical flex-price model in Section IV. Equation (4) indicates the uncovered interest parity through perfect capital mobility.

Under the normalization that  $\bar{y} = \bar{y}^* = 0$ , the price equation becomes

$$(5a) \quad p_{t+1} - p_t = \frac{\delta}{\alpha + \beta} \left\{ -\beta(1 + \alpha)p_t + \alpha\beta E_t p_{t+1} - \alpha\gamma z_t + \beta(m_t - \varepsilon_t) + \alpha\gamma\eta_t \right\} + \mu_t$$

and

$$(5b) \quad p_{t+1}^* - p_t^* = \frac{\delta}{\alpha + \beta} \left\{ -\beta(1 + \alpha)p_t^* + \alpha\beta E_t p_{t+1}^* + \alpha\gamma z_t + \beta(m_t^* - \varepsilon_t^*) + \alpha\gamma\eta_t^* \right\} + \mu_t^*$$

where  $z_t \equiv p_t - s_t - p_t^*$  denotes the real exchange rate. Notice that there is no counterpart with an asterisk to  $z_t$  but it enters with different signs in (5b). There is an element of arbitrariness in what one assumes about the information set on the basis of which private agents make decisions (see Weber (1981) Canzoneri, Henderson, and Rogoff (1983)). For simplicity, we assume that private agents at time  $t$  possess full information on all current variables  $y_t$ ,  $p_t$ ,  $i_t$ ,  $s_t$  and on the value of expected exchange rate  $E_t s_{t+1}$  that is embodied in the forward exchange rate. This means that private agents can infer exactly the values of  $\varepsilon_t$  and  $\eta_t$ . However, they cannot observe  $p_{t+1}$  at time  $t$  and accordingly not the value of  $\mu_t$ . Therefore,

$$(6a) \quad p_{t+1} = E_t p_{t+1} + \mu_t^*, \quad (6b) \quad p_{t+1}^* = E_t p_{t+1}^* + \mu_t^*.$$

### III. Optimal Feedback Rules in Decomposed Systems

Utilizing the method of Aoki (1981), we denote the additive variables with the superscript  $a$  and the difference variables with the superscript  $d$ .<sup>2</sup> That is, for any variable (or disturbance)  $X$ , define

$$X_t^a = X_t + X_t^*$$

$$X_t^d = X_t - X_t^*$$

Then the real exchange rate can be written  $z_t = p_t^d - s_t$ , and the interest parity becomes  $E_t s_{t+1} - s_t = i_t^d$ .

As the objective of the system, we will take the minimization of the unconditional output variances in the two countries. By the following argument, we will show that the objectives of minimizing unconditional output variances can be reduced in this symmetrical world, to the minimization of both the unconditional variance of average output and the unconditional variance of difference in output.

Consider the following two country linear systems of variables (or vectors)  $X$  and  $X^*$  that depend on (vectors of) disturbances  $\epsilon$  and  $\epsilon^*$  as well as policy parameters  $\pi$  and  $\pi^*$ .

$$(7) \quad X = F(\epsilon, \epsilon^*; \pi, \pi^*)$$

$$(8) X^* = F^*(\epsilon, \epsilon^*; \pi, \pi^*)$$

Suppose they are decomposed into two separate systems such that

$$(9) X^a = F^a(\epsilon^a, \pi^a)$$

$$(10) X^d = F^d(\epsilon^d, \pi^d)$$

where as usual  $X^a = X + X^*$ ,  $X^d = X - X^*$ , and where  $\pi^a$  and  $\pi^d$  can be independently chosen. Then we can state:

Proposition 1. If the variances of  $X^a$  and  $X^d$  are minimized with respect to  $\pi^a$  and  $\pi^d$ , then the sum of the variances,  $\text{Var}(X) + \text{Var}(X^*)$ , will be minimized. In particular, if the two countries are completely symmetric such that  $\text{Var}(x, \hat{\pi}^a, \hat{\pi}^d) = \text{Var}(x^*, \hat{\pi}^a, \hat{\pi}^d)$ , where  $\hat{\pi}^a$  and  $\hat{\pi}^d$  are respectively the maximizer of  $\text{Var}(x^a)$  and  $\text{Var}(x^d)$  then the minimization with respect to  $\pi^a$  and  $\pi^d$  will achieve both the minimization of  $\text{Var}(x, \pi)$  and  $\text{Var}(x^*, \pi^*)$ ..

Proof Since

$$\text{Var}(X^a) = \text{Var}(X) + \text{Var}(X^*) + 2\text{Cov}(X, X^*),$$

$$\text{Var}(X^d) = \text{Var}(X) + \text{Var}(X^*) - 2\text{Cov}(X, X^*),$$

it follows

$$\text{Var}(X) + \text{Var}(X^*) = (\text{Var}(X^a) + \text{Var}(X^d))/2.$$

By assumption  $\text{Var}(X^a; \pi^a) \geq \text{Var}(X^a; \hat{\pi}^a)$

$$\text{Var}(X^d; \pi^d) \geq \text{Var}(X^d; \hat{\pi}^d)$$

Therefore  $\text{Var}(X) + \text{Var}(X^*)$  are minimized when  $\pi^a$  and  $\pi^d$  are chosen in such a way as to minimize  $X^a$  and  $X^d$  respectively. Thus the minimization of  $\text{Var}(X^a)$  and  $\text{Var}(X^d)$  amounts to minimizing the sum of variances of  $X$  and  $X^*$ . By assumption  $\pi^a$  and  $\pi^d$  can be chosen independently. The second half of the proposition automatically follows from the hypothesis that  $\text{Var}(X; \hat{\pi}^a, \hat{\pi}^d) = \text{Var}(X^*; \hat{\pi}^a, \hat{\pi}^d)$ .<sup>11</sup>

Thus, we can consider separately the minimization of the variance of the sum of outputs in the additive system and the variance of the difference of outputs below.

#### A. The Analysis of the Additive System

Under assumption (6a) and (6b), equations (5a) and (5b) lead to the following difference equation:

$$(11) \quad p_{t+1}^a = \frac{\alpha + \beta - (1 + \alpha)\beta\delta}{\alpha + \beta - \alpha\beta\delta} p_t^a + \frac{\beta\delta}{\alpha + \beta - \alpha\beta\delta} (m_t^a - \epsilon_t^a) + \frac{\alpha\delta}{\alpha + \beta - \alpha\beta\delta} \eta_t^a + \mu_t^a$$

Thus the additive system is completely separated from all endogenous and exogenous difference variables such as nominal and real exchange rates. Equation (11) indicates that the average price level of the world depends on the aggregate money supply, and is independent of the exchange rate in this symmetric setting. This may be viewed as giving a supporting ground to McKinnon's proposal that the countries of the world should agree to provide a stable growth rate for the global money stock (McKinnon, 1984).<sup>3</sup> In this symmetric system neither the variation of aggregate output nor the optimal monetary feedback rule depends on the

exchange rate. That is, the average unemployment rate in the world does not depend on exchange rates or the rule of intervention.

Based on our previous discussions of the decomposition of objectives, we take the target of minimizing the unconditional variance of average output fluctuations for the objective function for the average system:

$$\text{Var}(y_t^a) = \text{Var}(E_t p_{t+1}^a - p_t^a) / \delta^2,$$

That is, we are interested in minimizing

$$(12) \quad V^a = E[(E_t p_{t+1}^a - p_t^a)^2].$$

If monetary authorities were able to observe all the current variables, and thus to infer exactly  $\varepsilon_t^a$  and  $\eta_t^a$ , and if they were able to react without lags, then they could reduce the expression in (12) to zero except for the influence of  $\mu_t^a$ . For example, from equation (11) the  $V^a$  is minimized if they set

$$m_t^a = \varepsilon_t^a - \frac{\alpha}{\beta} \eta_t^a.$$

However, it would be unrealistic to assume that the monetary authorities can not only observe but also react to all the current variables and disturbances inferred from them. For example, price index data usually come with some lags while interest rates are known more readily. Moreover, the spirit of our approach is to find out some simple feedback rule that can be implemented by monetary authorities, without extensive

information requirements. Thus we follow Poole (1970) and consider the case where the monetary authorities observe and jointly react to the sum of the interest rates or, equivalently, to the world average interest rate. In particular, we focus on the following feedback rule:

$$(13) \quad m_t^a = \pi i_t^a$$

The aggregate money supply is assumed to be controlled jointly through a McKinnon type scheme.

Then, one obtains:

$$(14) \quad p_{t+1}^a = \frac{\psi(\pi)}{f(\pi)} p_t^a - \frac{\beta\delta}{f(\pi)} \epsilon_t^a + \frac{(1+\alpha)\delta}{f(\pi)} \eta_t^a + \mu_t^a$$

where  $\psi(\pi) = \alpha + \beta - \beta(1+\alpha)\delta + (1-\beta\delta)\pi$ ,  $f(\pi) = (\alpha + \beta) - \alpha\beta\delta + (1-\beta\delta)\pi$ .

In this sluggish price model, (14) has a nonexplosive (backward) solution if and only if

$$(15) \quad \left| \frac{\psi(\pi)}{f(\pi)} \right| < 1.$$

If the price movement in the goods market is sluggish,  $\delta$  takes a small value. Accordingly, let us assume that

$$(16) \quad \delta < \min. \left[ \frac{\alpha + \beta}{\beta(1 + 2\alpha)}, \frac{1}{\beta} \right].$$

Then one can easily check that  $|k| < 1$  and accordingly that the system is

stable in the absence of a feedback rule ( $\pi = 0$ ). Also, the stability condition is satisfied if and only if  $f(\pi) > 0$ .

It is rather difficult to consider optimal policies encountering various kinds of shocks at the same time. Therefore we will consider one by one the optimal feedback rules that will be appropriate for each type of shock individually. Define:

$$\phi_1(\pi) = \psi(\pi)/f(\pi), \quad \phi_2(\pi) = \beta\delta/f(\pi), \quad \phi_3(\pi) = (1+\alpha)\delta/f(\pi)$$

then

$$(17) \quad p_{t+1}^a = \phi_1(\pi)p_t^a + u_t^a + \mu_t^a,$$

$$\text{where } u_t^a \equiv -\phi_2(\pi)\epsilon_t^a + \phi_3(\pi)\eta_t^a.$$

By using a lag operator  $L$ , (17) will read

$$\begin{aligned} (18) \quad E_t p_{t+1}^a - p_t^a &= (1-L)p_{t+1}^a - \mu_t^a \\ &= (1-L)(1+\phi_1 L + \phi_1^2 L^2 + \dots)u_t^a \\ &\quad + (\phi_1 - 1)(1+\phi_1 L + \phi_1^2 L^2 + \dots)\mu_{t-1}^a \end{aligned}$$

so that given the stability condition  $f(\pi) > 0$  and given the mutually independent white-noise property of  $\epsilon_t$ ,  $\eta_t$ , and  $\mu_t$ ,

$$\begin{aligned} (19) \quad \text{Var}(E_t p_{t+1}^a - p_t^a) \\ = \frac{2}{1+\phi_1(\pi)} \left\{ [\phi_2(\pi)]^2 \sigma_{\epsilon a}^2 + [\phi_3(\pi)]^2 \sigma_{\eta a}^2 + \frac{1-\phi_1(\pi)}{2} \sigma_{\mu a}^2 \right\} \end{aligned}$$

where  $\sigma_{\epsilon_a}^2$  is defined as  $E[(\epsilon_t^a)^2]$  and so forth. We will consider consecutively the following cases:

(i) (LM disturbances)  $\sigma_{\epsilon_a}^2 > 0$ ,  $\sigma_{\eta_a}^2 = \sigma_{\mu_a}^2 = 0$

To minimize  $V^a$  implies

$$\text{Min}_{\pi} \frac{(\phi_2(\pi))^2}{1+\phi_1(\pi)} = \text{Min}_{\pi} \left[ \frac{(\beta\delta)^2}{(\alpha+\beta-\alpha\beta\delta+(1-\beta\delta)\pi) \{2(\alpha+\beta)-\beta(1+2\alpha)\delta+(1-\beta\delta)\pi\}} \right]$$

Under the assumption (16), the optimal feedback rule that retains the stability should be  $\pi \rightarrow +\infty$ . Thus the optimal monetary policy is to fix the world nominal interest rates.

(ii) (IS disturbances)  $\sigma_{\eta_a}^2 > 0$   $\sigma_{\epsilon_a}^2 = \sigma_{\mu_a}^2 = 0$

Min  $V^a$  implies

$$\text{Min}_{\pi} \frac{(\phi_3(\pi))^2}{1+\phi_1(\pi)} = \text{Min}_{\pi} \frac{\delta^2(\alpha+\pi)^2}{[(\alpha+\beta-\alpha\beta\delta)+(1-\beta\delta)\pi][(2-\beta-2\alpha\beta)+(1-\beta\delta)\pi]}$$

Therefore  $\pi = -\alpha$  will give the solution. We also obtain  $f(-\alpha) > 0$  if  $\delta < 1$ , so that the stability condition is satisfied. Thus this feedback rule is effective if the price movement is sufficiently slow in this Dornbusch model. If the interest elasticity of money demand  $\alpha$  is small, then the global monetarism that proposes the constant world money supply will be a proper prescription.

(iii) (Price disturbances)  $\sigma_{\mu_a}^2 > 0$ ,  $\sigma_{\epsilon_a}^2 = \sigma_{\eta_a}^2 = 0$

Min  $V^a$  implies

$$\text{Min}_{\pi} \frac{1-\phi_1(\pi)}{1+\phi_1(\pi)} = \text{Min}_{\pi} \frac{\beta\delta}{2(\alpha+\beta)-\beta(1+2\alpha)\delta+(1-\beta\delta)\pi}$$

Since  $\lim_{\pi \rightarrow \infty} f(\pi) > 0$ , the optimal rule is  $\pi \rightarrow +\infty$  as in the case of (i).

What happens if all kinds of the disturbances coexist? Figure 1 roughly illustrates how the three components of output variances due to



LM, IS and price equation disturbances vary with respect to the value of  $\pi$ . The relative heights of these curves, of course, depends on relative magnitudes of various disturbances. LM and price disturbances will be eliminated if  $\pi$  approaches plus infinity. The effect of IS disturbances increases and approaches  $\delta\sigma_{ra}^2 / (1-\beta\delta)^2$  if  $\pi$  approaches infinity. Whether the system should adopt a rule close to that of fixing the average interest rate ( $\pi \rightarrow \infty$ ) or that of the global monetarist proposal depends on the relative magnitude of three kinds of shocks.

Thus the additive system behaves almost exactly as the single country Keynesian system analyzed by Poole (1970). If aggregate disturbances in the LM equation dominate, then it will be desirable to try to stabilize the average world interest rate. On the other hand, if aggregate disturbances in the IS equation dominate, monetary policy should make the average world interest rate fluctuate slightly. In particular, if the interest elasticity of money demand is negligible, it will be desirable to stabilize the aggregate world money supply. The McKinnon (1979) proposal naturally makes more sense if money demand in each country is relatively more stable and interest rate inelastic.

#### B. The Analysis of the Difference System

Recalling the definition of difference variables  $X_t^d = X_t - X_t^*$ , we obtain the following pair of dynamic equations in terms of  $p_t^d$  and  $s_t$  from (5a)-(5b) and (4) by recalling  $z_t = p_t^d - s_t$ , and (6a)-(6b):

$$(20) (\alpha + \beta - \alpha\beta\delta)p_{t+1}^d = [\alpha + \beta - (\alpha\beta + \beta + 2\alpha\gamma)\delta]p_t^d + 2\alpha\gamma\delta s_t + \beta\delta(m_t^d - \epsilon_t^d) + \alpha\beta\eta_t^d + (\alpha + \beta - \alpha\beta\delta)\mu_t^d$$

$$(21) -\beta p_{t+1}^d + (\alpha + \beta)E_t s_{t+1} = (1 - \beta - 2\gamma)p_t^d + (\alpha + \beta + 2\gamma)s_t - (m_t^d - \epsilon_t^d) + \eta_t^d - \beta\mu_t^d$$

At first we will consider the feedback rule  $m_t^d = -\tau s_t$ . An increase in  $m_t^d$  is a relative increase of money supply in the home country. Our feedback rule can be implemented easily by an unsterilized intervention in the foreign exchange market in such a way as to stabilize the spot exchange rate. Unsterilized interventions in the form of purchase of foreign currencies, for example, increase the home money supply and reduce the foreign money supply. The change in  $m^d$  is twice as large as the amount of foreign currency purchased by the home country. Needless to say  $m^d$  increases when the home country expands its money supply by domestic credit creation without any action on the part of the foreign country.

Under this feedback rule (20) and (21) become

$$(22) \begin{bmatrix} p_{t+1}^d \\ E_t s_{t+1} \end{bmatrix} = A \begin{bmatrix} p_t^d \\ s_t \end{bmatrix} + \Lambda \begin{bmatrix} \epsilon_t^d \\ \eta_t^d \\ \mu_t^d \end{bmatrix}$$

where

$$A = \begin{bmatrix} \alpha + \beta - (1 + \alpha)\beta\delta & 0 \\ -\beta & \alpha + \beta \end{bmatrix}^{-1} \begin{bmatrix} \alpha + \beta - (\alpha\beta + \beta + 2\alpha\gamma)\delta & (2\gamma - \beta\tau)\delta \\ 1 - \beta - 2\gamma & \alpha + \beta + 2\gamma + \tau \end{bmatrix},$$

and

$$A = \begin{bmatrix} -\beta\delta & \alpha\delta & \alpha + \beta - \alpha\beta\delta \\ 1 & 1 & -\beta \end{bmatrix}.$$

Let us denote the characteristic roots of  $A$  as  $\lambda_1$  and  $\lambda_2$  such that  $|\lambda_1| \leq |\lambda_2|$ . The characterization of a system like (24) was given by Blanchard-Kahn (1980). The system has (i) a unique saddle path solution if

$|\lambda_1| < 1 < |\lambda_2|$  (ii) no solution that satisfied non-explosion condition if  $1 < |\lambda_1| < |\lambda_2|$ , and (iii) an infinite number of convergent solutions if  $|\lambda_1| \leq |\lambda_2| < 1$ . Moreover, if  $|\lambda_1| < 1 < |\lambda_2|$ , the unique convergent solution of (22) can be written in the following form where  $\lambda_1$  is the characteristic root of  $A$  with the smaller norm.

$$(23) \quad p_t^d = \lambda_1 p_{t-1}^d + \phi_1 \epsilon_{t-1}^d + \phi_2 \eta_{t-1}^d + \phi_3 \mu_{t-1}^d,$$

$$(24) \quad s_t = \psi_1 p_t^d + \psi_2 \epsilon_t^d + \psi_3 \eta_t^d + \psi_4 \mu_t^d.$$

Either by appealing to the expressions near the end of Blanchard-Kahn (1980), or by the method of undetermined coefficients, one obtains the following expression for  $\phi_1$ ,  $\phi_2$  and  $\phi_3$ .

$$(25) \quad \phi_1 = \lambda_1 \frac{\delta(\beta + 2\gamma)}{H\tau + J}, \quad \phi_2 = \lambda_1 \frac{\delta(\tau + \alpha)}{H\tau + J}, \quad \phi_3 = 1$$

where  $H = -[1 - \delta(\beta + 2\gamma)]$

$$J = -[\alpha + \beta + 2\gamma - \delta(\beta + 2\gamma)(1 + \alpha)].$$

If the price is sufficiently sluggish i.e. if  $\delta$  is small,  $H$  and  $J$  are likely to be negative. We assume that  $H$  and  $J$  are negative.

Define the characteristic polynomial of  $A$  as:

$$(26) \quad g(\lambda) = \lambda^2 - (\text{Trace } A) \lambda + |A|.$$

Then, the necessary and sufficient conditions for saddle point stability are either (a)  $g(1) < 0$  and  $g(-1) > 0$ , or (b)  $g(1) > 0$  and  $g(-1) < 0$ . (See for example Sargent (1979)).

If  $\tau = 0$ ,  $g(1) < 0$  and  $g(-1) > 0$  for a small value of  $\delta$ . Thus for a sufficiently small value of  $\delta$ , the system has the saddle point property in the absence of any feedback. Now we shall turn to the minimization of the objective function through policy reactions to various kinds of shock, considering the optimization of the following objective in terms of differences:

$$\text{Minimize } V^d = \text{Var}(E_t p_{t+1}^d - p_t^d),$$

which can be written by calculations similar to (19) as

$$V^d = \frac{2}{1 + \lambda_1(\tau)} \left\{ \left[ \phi_1(\tau) \right]^2 \sigma_{\epsilon d}^2 + \left[ \phi_2(\tau) \right]^2 \sigma_{\eta d}^2 \right\} + \frac{1 - \lambda_1(\tau)}{1 + \lambda_1(\tau)} \sigma_{\mu d}^2$$

where  $\sigma_{\epsilon d}^2 = E[(\epsilon^d)^2]$  and so forth.

Again consider the following cases of individual differential shocks.

$$(1) \quad (\text{LM disturbances}) \quad \sigma_{\epsilon d}^2 > 0 \quad \sigma_{\eta d}^2 = \sigma_{\mu d}^2 = 0$$

Minimization of  $V^d$  implies

$$\text{Min}_{\tau} \frac{\phi_1^2}{1+\lambda_1} = \text{Min}_{\tau} \frac{\lambda_1^2}{1+\lambda_1} \left[ \frac{\delta(\beta+2\gamma)}{H\tau + J} \right]^2 .$$

$\tau \rightarrow \pm \infty$  are the solutions. It is also possible to show that

$$\lim_{\tau \rightarrow \pm \infty} g(1) = \pm \infty \text{ and } \lim_{\tau \rightarrow \pm \infty} g(-1) = \pm \infty,$$

provided that  $\delta$  is sufficiently small so that  $H$  is negative. Thus if price levels are sluggish enough, this feedback rule keeps the saddle point property intact. However,  $\tau$  makes economic sense only if it approaches positive infinity. Moreover, during the process in which the monetary authority decreases the value of  $\tau$  from zero to  $-\infty$ , explosive solutions are easily produced as is illustrated in Figure 2. The best way to cope with differential LM disturbances is to fix exchange rates.

$$(ii) \text{ (IS disturbances) } \sigma_{\gamma d}^2 > 0, \sigma_{\epsilon d}^2 = \sigma_{\mu d}^2 = 0$$

Minimization of  $V^d$  implies

$$\text{Min}_{\tau} \frac{\phi_2^2}{1+\lambda_1} = \text{Min}_{\tau} \left[ \frac{\lambda_1}{1+\lambda_1} \left[ \frac{\lambda_1 \delta (\tau + \alpha)^2}{H\tau + J} \right]^2 \right]$$

Thus the best policy is again  $\tau = -\alpha$ .

When  $\tau = -\alpha$ ,  $g(1) < 0$  if  $\alpha < 1$  and  $g(-1) > 0$  for a small value of  $\delta$ .

Accordingly, this policy keeps the saddle point property intact. Thus the best way to cope with differential IS disturbance is to intervene slightly in such a way as to lean with the wind. In particular, if the interest elasticity of the demand for money is negligible ( $\alpha=0$ ), then the best regime will be the flexible exchange rate system without any intervention, i.e. the clean float.

$$(iii) \text{ (Price disturbances) } \sigma_{\mu d}^2 > 0, \sigma_{\epsilon d}^2 = \sigma_{\gamma d}^2 = 0$$

Min.  $V^d$  implies

$$\text{Min}_{\tau} \left[ \frac{1-\lambda_1}{1+\lambda_1} \right].$$

Apparently the value of  $\tau$  that generates  $\lambda_1 = 1$  would be the solution. However, this definitely violates the stability condition of price dynamics because  $\lambda_1$  is  $A$ 's characteristic root with the smaller norm. Thus there is no well defined optimal solution.

Figure 2 roughly illustrates how the two components of output variances due to LM and IS disturbances vary, in a typical case, with respect to the value of  $\pi$ .<sup>4</sup> Again the relative heights between  $h_1$  and  $h_2$  depends on the relative magnitudes of differential disturbances in LM and IS curve. (The effect of price equation disturbances is omitted because the optimal policy is undefined.) Thus to cope with differential shocks to money demand or money supply functions, the best regime is the fixed exchange rate regime. To cope with differential shocks in IS curves, interventions leaning slightly with the wind are desirable. In particular if the interest elasticity of money demand is negligible, clean floating is optimal.

Similarly one could work with interventions responding to the forward exchange market, namely

$$(27) \quad m_t^d = \tau' E_t s_{t+1}$$

which would give the following  $A$  in equation (22):

$$A = \begin{bmatrix} \alpha + \beta - \alpha\beta\delta & -\beta\delta\tau' \\ -\beta & \alpha + \beta + \tau' \end{bmatrix}^{-1} \begin{bmatrix} \alpha + \beta - (\alpha\beta + \beta + 2\alpha\gamma)\delta & 2\alpha\gamma\delta \\ 1 - \beta - 2\gamma & \alpha + \beta + 2\gamma \end{bmatrix}$$

The solution to the price equation is again written as equation (23), where  $\lambda_1$  is the characteristic root of A with a smaller norm and

$$(28) \quad \phi_1 = -\lambda_1 \frac{(\beta+2\gamma)\delta}{(\alpha+\beta+2\gamma) - (\beta+2\gamma)(1+\alpha)\delta}, \quad \phi_2 = \lambda_1 \frac{\alpha\delta}{(\alpha+\beta+2\gamma) - (\beta+2\gamma)(1+\alpha)\delta}, \quad \phi_3 = 1$$

Then, in order to stabilize  $V^d$  against both kinds of shocks  $\epsilon_t^d$  and  $\eta_t^d$ , the best policy is to minimize the absolute value of  $\lambda_1$ . This would be achieved by setting  $\tau' \rightarrow \pm \infty$  because the product of the two characteristic roots, i.e. the determinant of A, approaches zero.

However,

$$\lim_{\tau' \rightarrow \pm \infty} g(1) = \frac{2\gamma\delta}{\alpha+\beta-\beta\delta} > 0$$

$$\lim_{\tau' \rightarrow \pm \infty} g(-1) = \frac{2(1-(\beta+\gamma)\delta)}{\alpha+\beta-\beta\delta} > 0.$$

for a reasonably small value of  $\delta$ . Thus the saddle point property is not satisfied by this feedback rule (27). Moreover, the economic meaning of the feedback rule is not clear. In order to stabilize the forward exchange rate by intervening in the spot market,  $\tau'$  seems to be only meaningful when negative. But under the assumption of  $\alpha+\beta-\beta\delta > 0$ , if one reduces the value of  $\tau'$  from zero to minus infinity, at one time  $|A|$  and Trace A will explode. For these reasons interventions in response to the forward exchange rate market cannot necessarily be recommended even though the same therapy seems to apply to both kinds of shocks in this feedback scheme.

Finally, let us consider the following intervention scheme responding to interest rate differences

$$(29) m_t^d = \theta (E_t s_{t+1} - s_t) = \theta i_t^d.$$

Then A in equation (22) can be written as

$$A = \begin{bmatrix} \alpha + \beta - \alpha\beta\delta & \beta\delta \\ -\beta & \alpha + \beta + \theta \end{bmatrix}^{-1} \begin{bmatrix} \alpha + \beta - (\alpha\beta + \beta + 2\alpha\gamma)\delta & (2\alpha\gamma - \beta\theta)\delta \\ 1 - \beta - 2\gamma & \alpha + \beta + 2\gamma + \theta \end{bmatrix}$$

The solution to the price equation is again written as:

$$\phi_1 = \lambda_1 \frac{\delta(\beta + \gamma)}{H\delta + J}, \quad \phi_2 = \lambda_1 \frac{-\delta(\alpha + \theta)}{H\delta + J}, \quad \phi_3 = 1$$

where H and J are defined as before.

$$(i) \text{ (LM disturbances) } \sigma_{\epsilon d}^2 > 0, \quad \sigma_{\eta d}^2 = \sigma_{\mu d}^2 = 0$$

Minimization of  $V^d$  is achieved by setting  $\theta \rightarrow \pm \infty$ . But when  $1 > (\beta + \gamma)\delta$ ,

$$\lim_{\theta \rightarrow \pm \infty} g(1) = \mp 0 \quad (\text{meaning } g(1) \text{ approach zero from}$$

negative or positive side)

$$\lim_{\theta \rightarrow \pm \infty} g(-1) = \frac{2\{1 - (\beta + \gamma)\delta\}}{1 - \beta\delta} > 0.$$

Thus, only the feedback rule  $\theta \rightarrow + \infty$  satisfies the saddle point property for a small value of  $\delta$ .

$$(ii) \text{ (IS disturbances) } \sigma_{\eta d}^2 > 0, \quad \sigma_{\epsilon d}^2 = \sigma_{\mu d}^2 = 0$$

$\theta = -\alpha$  is the solution which minimizes  $V^d$ . When  $\theta = -\alpha$ ,

$$g(1) = -\frac{2\gamma\delta}{\beta} < 0 \text{ and } g(-1) = \frac{1}{\beta} (\beta + \gamma)(2 - \delta) > 0 \text{ if } \delta < 2.$$

Thus, this feedback rule keeps the saddle point property.

This feedback rule based on interest rate differentials has the attractive property that the product of the two characteristic roots remains finite in any case. Again, the intervention policy that keeps



interest rate differentials from changing is desirable against LM shocks, leaning with the wind, or a little intervention when money demand is interest inelastic, is desirable against IS shocks specific to a particular country.

#### IV. The Comparison with the "Eclectic" Neoclassical Model.

In this section we will consider optimal monetary rules in the "eclectic" neoclassical model where output fluctuates and compare them with our previous results. In fact, the analysis is much easier in this model, and the analogy carries through with a few reservations. Let us start from the macroeconomic model where prices clear in any period, but output decisions depend on the discrepancies between the actual and expected price level, for example, due to the nature of wage contracts lagging behind for a period (see, e.g. Fischer (1977) and Gray (1976)). Then the system of symmetric two country economies can be written exactly as in the 7 equations (1a) - (4) in the outset of Section II, except that the price equations (3a) and (3b) are replaced by supply functions (and implicitly by goods market clearing equations that aggregate demand equals the aggregate supply), as follows.

$$(3'a) \ y_t = \bar{y} + \hat{\delta} (p_t - E_{t-1} p_t) + \mu_t, \quad (3'b) \ y_t^* = \bar{y}^* + \hat{\delta} (p_t^* - E_{t-1} p_t^*) + \mu_t^*$$

The economic meaning of  $\hat{\delta}$  is different from  $\delta$  we used in the previous sections, but for the economy of notations, we will use the same symbol.

Here again, we can decompose these equations into the additive system and the difference system. In the additive system, if the

feedback rule of money supply is  $m_t^a = \pi i_t^a$ , the dynamic equation of the sum of prices is written

$$(30) \quad -(\alpha+\pi)\beta E_t p_{t+1}^a + (\beta(1+\delta) + (\beta+\delta)(\alpha+\pi))p_t^a - (\alpha+\beta+\pi)\delta E_{t-1} p_t^a = u_t^a$$

$$\text{where } u_t^a = -\beta\epsilon_t^a + (\alpha+\pi)\eta_t^a - (\alpha+\beta+\pi)\mu_t^a$$

It is easy to show (see Taylor 1977) that (30) has (i) a unique solution if  $\pi > -\alpha$  or  $\pi < -\alpha-2$ , and, (ii) multiple solutions if  $-\alpha-2 < \pi < -\alpha$ . If (30) has a unique solution, (30) can be solved as

$$(31) \quad p_t^a = \frac{1}{\beta(1+\delta) + (\alpha+\pi)(\beta+\delta)} u_t^a$$

Since

$$V^a = E \left[ \delta(p_t^a - E_{t-1} p_t^a) + \mu_t^a \right]^2 = \frac{1}{\{\beta(1+\delta) + (\alpha+\pi)(\beta+\delta)\}^2} \left\{ -\beta\delta\sigma_{\epsilon a}^2 + (\alpha+\pi)\delta\sigma_{\eta a}^2 + \beta(\alpha+1+\pi)\sigma_{\mu}^2 \right\}$$

we obtain the feedback rule that minimizes  $V^a$  as follows:<sup>5</sup>

- (i) (LM disturbances)  $\sigma_{\epsilon a}^2 > 0, \sigma_{\eta a}^2 = \sigma_{\mu a}^2 = 0, \pi \rightarrow +\infty$
- (ii) (IS disturbances)  $\sigma_{\eta a}^2 > 0, \sigma_{\epsilon a}^2 = \sigma_{\mu a}^2 = 0, \pi = -\alpha$
- (iii) (Price disturbances)  $\sigma_{\mu a}^2 > 0, \sigma_{\epsilon a}^2 = \sigma_{\eta a}^2 = 0, \pi = -\alpha - 1$

Not surprisingly, these feedback rules are exactly the same as those of Section III-A, except for the case of (iii) where a finite value of  $\pi$  is prescribed.

However, in the case of (iii),  $\pi = -\alpha - 1$  will induce multiple solutions. Even in the case of (ii),  $\pi = -\alpha$  is a critical value which might also induce multiple solutions.<sup>6</sup>

In the difference system, if the feedback rule is in the form  $m_t^d = -\tau s_t$ , the difference equations of  $p_t$  and  $s_t$  are derived as follows;

$$(32) \beta E_t p_{t+1}^d - (\beta + 2\gamma + \delta) p_t^d + \delta E_{t-1} p_t^d - \beta E_t s_{t+1} + (\beta + 2\gamma) s_t = -\eta_t^d + M_t^d$$

$$(33) (1 + \delta) p_t^d - \delta E_{t-1} p_t^d - \alpha E_t s_{t+1} + (\alpha + \tau) s_t = -\epsilon_t^d - M_t^d$$

These difference equations have:

- (i) a unique solution if  $\tau < -(2\alpha + 1)$  or  $\tau > -1$ , and, (ii) multiple solutions if  $-(2\alpha + 1) < \tau < -1$ .

When the above difference equations have a unique solution, we can obtain  $E_{t-1} p_t^d = E_t p_{t+1}^d = E_t s_{t+1}^d = 0$ , and therefore,

$$(34) p_t^d = \frac{1}{\Gamma} \{ -(\beta + 2\gamma) \epsilon_t^d + (\alpha + \tau) \eta_t^d - (\alpha + \beta + 2\gamma + \tau) M_t^d \}$$

where  $\Gamma = (\beta + 2\gamma + \delta)(\alpha + \tau) + (1 + \delta)(\beta + 2\gamma)$

Thus, the feedback rule which minimizes  $v^d$  is

- (i) (LM disturbances)  $(\sigma_{\epsilon d}^2 > 0, \sigma_{\eta d}^2 = \sigma_{\mu d}^2 = 0)$ ,  $\tau \rightarrow \pm \infty$   
(ii) (IS disturbances)  $(\sigma_{\eta d}^2 > 0, \sigma_{\epsilon d}^2 = \sigma_{\mu d}^2 = 0)$ ,  $\tau = -\alpha$   
(iii) (Price disturbances)  $(\sigma_{\mu d}^2 > 0, \sigma_{\epsilon d}^2 = \sigma_{\eta d}^2 = 0)$ ,  $\tau = -(\alpha + 1)$

These results are also exactly the same as those of Section III - B except for those in case (iii). Since  $\alpha$  is usually less than 1,  $\tau \rightarrow \pm \infty$  and  $\tau = -\alpha$  satisfy the uniqueness conditions. Here again, setting the value of  $\tau$  at minus infinity does not make much economic sense. Also in the process the variance of  $p_t^d$  explodes because  $\Gamma$  becomes zero.  $\tau = -(\alpha + 1)$  will produce multiple solutions as in the additive system.

Similarly, other feedback rules such that:

$$m_t^d = \tau' E_t s_{t+1}$$

$$m_t^d = \theta i_t^d = \theta (E_t s_{t+1} - s_t)$$

could be considered. But, since  $E_t s_{t+1} = 0$  if there exists a unique solution, there is no optimal value of  $\tau'$  and the optimal values of  $\theta$  are exactly the same as the optimal  $\tau$ .

Thus we find that the nature of the optimal feedback rules is quite robust regardless of whether the structure of two economies are either Keynesian or neoclassical. Interest rate targets are desirable if LM curves are more unstable, the combination of global monetarism and little intervention is desirable if IS curves are more unstable.

#### V. Concluding Remarks

The main findings are summarized in Table 1. By distinguishing the additive from the difference system as well as additive from difference disturbances, one can clarify the relationship between the global monetarist proposal and the discussion of misalignment of exchange rates. It is quite natural that the analogy of the discussion of Poole (1970) applies to the additive system as a whole. More interesting is the fact that a similar analogy prevails in the difference system and the fact that the feedback to interest rate differentials can be equally effective as the feedback to exchange rates.

In the appendix, we discussed the possible extension of our results to n-country cases. The analogy is not perfect where scales of national economies differ, but there is a case for decomposing the system into the world average variable and the divergence of a country variable from the world average.

These results in our text are derived in a simplified model where economic structures of the two countries are symmetric and disturbances are time-independent white noises. Therefore we should recognize the gap between such theoretical exercise and the appropriate policy proposal to implement some desirable regimes. Among many necessary modifications and reservations, we may just mention relatively important ones.

First, in the actual world, disturbances are not necessarily time independent but serially correlated. Moreover, we do not know exactly

which disturbances are permanent and which are transient. Economic agents and monetary authorities engage in guessing games as to which are permanent or transient. The simple policy recommendation obtained under the assumption of white noises may be modified. Presumably, more adaptive rules should be implemented when there are serial correlations in disturbances, and less rigid or drastic rules should be applied when we cannot identify the permanent or transient nature of disturbances. Secondly, the optimal rule will depend on the lag structure of the system and the information set on which private agents and monetary authorities take actions and formulate expectations on future variables.

Finally, if we are to apply the feedback process to the current world situation, we need to modify our conclusions taking account of the ongoing process of international credit accumulation due to differences in saving behaviors among countries, which could be based on the possible difference in the pattern of time preferences, the technological development and the given historical datum of resource endowment. The two-country model developed in our paper does not take account of these long-run factors so that the stationary state of the model corresponds to the equilibrium where the current account will be equated to zero. In reality, however, equilibrium may consist of a path that allows some trend in the current account of the balance of payments.

#### Appendix

In this appendix we will consider the generalization of the 2 country model in the previous sections into a  $n$ -country setting. First

suppose the world consists of  $n$  symmetric countries that have identical parameters. Taking the Dornbusch model as an example, we have for  $j^{\text{th}}$  country (omitting time subscript  $t$ ) with subscript  $j$ .

$$m_j - p_j = -\alpha i_j + y_j + \epsilon_j,$$

$$y_j = -\beta(i_j - E p_{+1,j} + p_j) - \sum_{k \neq j} \gamma(p_j - s_{jk} - p_k) + \eta_j,$$

$$p_{+1,j} - p_j = \delta y_j + \mu_j,$$

and for  $k \neq j$

$$E s_{+1,jk} - s_{jk} = i_j - i_k$$

where  $s_{jk}$  indicates the exchange rate of  $k^{\text{th}}$  currency measured in terms of  $j^{\text{th}}$  currency.

Define the average variable such that  $\bar{x}^a = \sum_{j=1}^n x_j / n$ . Then the

average system is written as

$$m^a - p^a = -\alpha i^a + y^a + \epsilon^a,$$

$$y^a = -\beta(i^a - E p_{+1}^a + p^a) + \eta^a.$$

Note that  $\sum_{j=1}^n \sum_{k \neq j} s_{jk} = 0$  because  $s_{jk} = -s_{kj}$ . Also

$$p_{+1}^a - p^a = \delta y^a + \mu^a.$$

Thus the analysis for the average system just goes through as for the two-country case.

Define also the difference (from the average) variable for country  $j$  as

$$x_j^d = x_j - \bar{x}^a.$$

Then the system for difference variables can be written

$$m_j^d - p_j^d = -\alpha_j^d + y_j^d + \epsilon_j^d$$

$$y_j^d = -\beta(i_j^d - EP_{+1,j}^d + p_j^d) - n\gamma(p_j^d - s_j) + \eta_j^d$$

where  $s_j = \sum_{k \neq j} s_{jk} / n$  ( $= \sum_{k=1}^n s_{jk} / n$  since  $s_{jj} = 0$  by definition) indicates the effective exchange rate for country  $j$  relative to all the other countries. Similarly, interest parity will read,

$$Es_{+1,j} - s_j = i_j^d.$$

Thus discussion of the different system carries through just as in that of the 2-country model.

Next we will turn to the case where there is a difference of scale among countries even though the parameters are still symmetric. We modify the IS curve taking account of the effect of different scales



$$y_j = -\beta(i_j - Ep_{+1,j} + p_j) - \sum_{k \neq j} \gamma_{jk} (p_j - s_{jk} - p_k) + \eta_j$$

where  $\gamma_{jk} = \bar{\gamma} w_k$ ,  $w_k$  being the relative scale of country  $k$  in the world

such that  $\sum_{k=1}^n w_k = 1$ . Then noting  $\sum_{k \neq j} w_k (p_j - s_{jk} - p_k) =$

$\sum_{k=1}^n w_k (p_j - s_{jk} - p_k)$ , the average system and the divergence system can

similarly be defined in terms of

$$x^a = \sum_{k=1}^n w_k x_k, \quad x_j^d = x_j - x^a.$$

For example, IS equations are

$$y^a = -\beta(i^a - Ep_{+1}^a - p^a) + \eta^a$$

and

$$y_j^d = -\beta(i_j^d - Ep_{+1,j}^d - p_j^d) - \bar{\gamma}(p_j^d - s_j) + \eta_j^d$$

The interest parity will also hold if we define

$$s_j \equiv \sum_{k=1}^n w_k s_{jk} = \sum_{k \neq j} w_k s_{jk}.$$

Then how can Proposition 1 be generalized to a  $n$ -country case? We can state the following proposition for the case of identical scale.

Proposition 2: Suppose the world system of  $n$  symmetric countries with an identical scale can be decomposed into an average variable that depends only on an average disturbance and  $(n-1)$  difference variables that depend only on difference disturbances. Moreover, suppose that the average as well as all of these difference variables can be controlled by the parameters that can be independently chosen. Then the minimization of variances  $\text{Var}(X^a)$  and  $\text{Var}(X_j^d)$   $j = 2, \dots, n$  is equivalent to the minimization of the sum of variances,

$$\text{i.e. } \sum_{j=1}^n \text{Var}(X_j).$$

Proof Define:

$$N_0 = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & -1 & \frac{1}{n} & \dots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} & -1 \end{bmatrix}$$

and

$$N = \begin{bmatrix} \sqrt{\frac{n}{n-1}} & 0 & 0 & \dots & 0 \\ 0 & \sqrt{\frac{n}{n-1}} & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & \sqrt{\frac{n}{n-1}} \end{bmatrix} N_0$$

$N_0$  and  $N$  are orthogonal and normalized orthogonal matrix. Define

$$X = [X_1, X_2, \dots, X_n]^T, \text{ and}$$

$$Y = NX = \left[ \sqrt{n} X^a, \sqrt{\frac{n}{n-1}} X_2^d, \dots, \sqrt{\frac{n}{n-1}} X_n^d \right]^T$$

Then

$$Y^T Y = X^T X.$$

Therefore

$$\text{Var}(\sqrt{n} X^a) + \sum_{j=2}^n \text{Var}\left[\sqrt{\frac{n}{n-1}} X_j^d\right] = \sum_{j=1}^n \text{Var}(X_j)$$

By the same reasoning in the proof of Proposition 1, this proposition follows.  $\square$

Unfortunately, when there are differences in the relative scales of national economies, the same reasoning does not apply because

$$N = \begin{bmatrix} w_1 & w_2 & w_3 & \dots & w_n \\ w_1 & w_2^{-1} & w_3 & \dots & w_n \\ w_1 & w_2 & w_3^{-1} & \dots & w_n \\ w_1 & w_2 & w_3 & & w_n^{-1} \end{bmatrix}$$

is no longer an orthogonal matrix. Only when  $\text{Cov}(X_i, X_j)$  is zero for any  $i \neq j$ , can a result similar to Proposition 2 be obtained.

Thus in the real world where scales of countries differ, the average rule combined with the difference rule conducted by  $n-1$  countries falls a little short of the overall optimization of the system. However, this exercise shows that the division of labor involved in the average policy rule and the difference policy rule could have the advantage of simplicity and informational economy.

Let us now come to the question of how to implement this decomposition rule. To implement the average rule, it is natural to consider a coordination body for the aggregate or average money supply. Some international arrangement or institution should be made to coordinate the control of the global money stock according to a principle similar to the McKinnon plan. The proposal to stabilize the aggregate money supply should be adopted if disturbances in IS curves dominate; the rules aimed at stabilizing the average interest rate should be adopted if disturbances in LM curves dominate. In order to implement the difference rule, unsterilized interventions in the spot exchange market in response to exchange rates or to interest differentials are the natural choice. By intervening, keeping domestic monetary policy intact, in the exchange market in such a way as to purchase the  $k$ 'th currency with the  $j$ 'th currency, the  $j$ 'th country increases its money supply and reduces the money supply of the  $k$ 'th country without changing the total world money supply. Thus the combination of the coordinated money supply reacting to aggregate

disturbances and the non-sterilized market intervention policies reacting to differentiated (or country specific) disturbances is the right assignment of policy instruments. Needless to say, coordination and interventions are called for only when the nature of disturbances, for example IS or LM, requires them. One can compute back from the values of  $m^a$  and  $m^d$  the appropriate values of  $m$  and  $m^*$ . An international organization or a surveillance body consisting of finance ministers may compute those assignments of monetary policies for participating countries by specifying the average interest rate and the divergences from it.

Some readers will notice here the classical policy assignment proposal by Mundell (1971). If difference monetary disturbances dominate, and accordingly the optimal management of exchange rates should come close to fixed exchange rates, why not let a leader country such as the United States take care of the aggregate price level, and let the other country, say Europe, adjust their exchange rates? In a more generalized framework for  $n$  countries, this kind of proposal would work. A large country like the United States ... a hegemon if we use the favorite word in political science ... would be the natural choice for such a country. Suppose an aggregate disturbance without any differential disturbance strikes the system. If only a single country, say country 1, expands its money supply, then  $m^a$  will increase all right. However, at the same time the divergence of money supply from the world average will increase by the same amount even in the absence of differential shock. Then other countries will have to adjust their

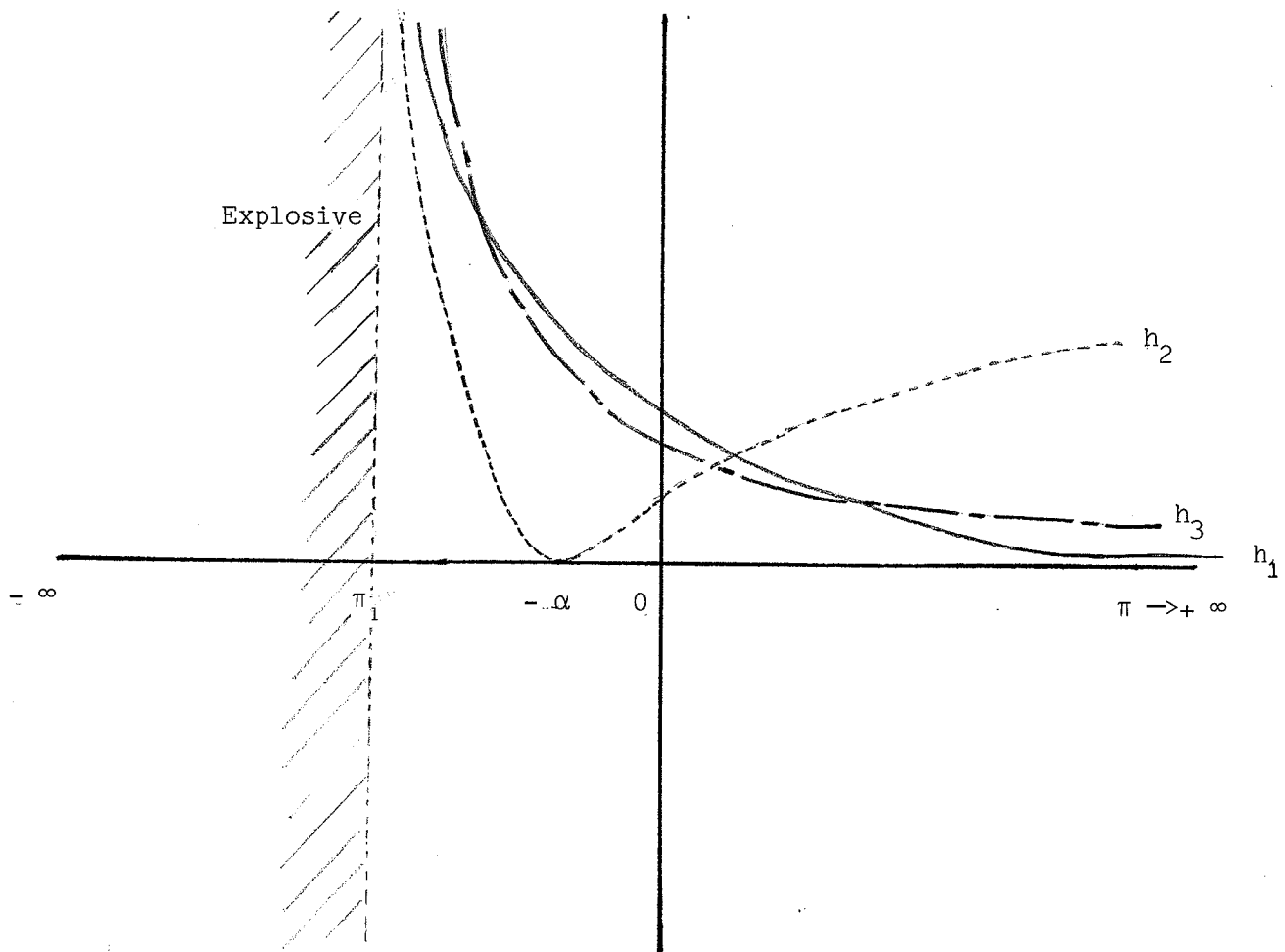
money supplies to stabilize their exchange rates or interest rates through intervention by purchasing the currency of country 1 with its own currency. As long as the differential rule reacting to exchange rates or interest rates functions well, the optimal rule will be achieved.

Table 1

	Dominant Disturbances	Closed Economy Keynesian (Poole '70)	Two-Country	
			Dornbusch	Neoclassical
Additive (Co-ordination rule)	LM	Interest rate target	Average interest rate target	
	IS	Money target (if $\alpha=0$ ), slightly leaning with the interest rate (otherwise)	global money target (if $\alpha=0$ ), slightly leaning with the interest rate (if $\alpha > 0$ )	
	Price		Average interest rate target	
Difference (Intervention rule)	LM		Feedback to exchange rates, or feedback to interest rate differential (both leaning against the wind)	
	IS		Flexible exchange rates (if $\alpha=0$ ), feedback to exchange rate or interest differential, both slightly leaning with the wind (if $\alpha > 0$ )	
	Price		undefined	leaning with the difference in interest rate

$\alpha$  is the interest elasticity of money demand

FIGURE 1



$$h_1: \text{ LM generated disturbance} = \left\{ \phi_2(\pi) \right\}^2 \sigma_{\varepsilon a}^2 / \left\{ 1 + \phi_1(\pi) \right\}$$

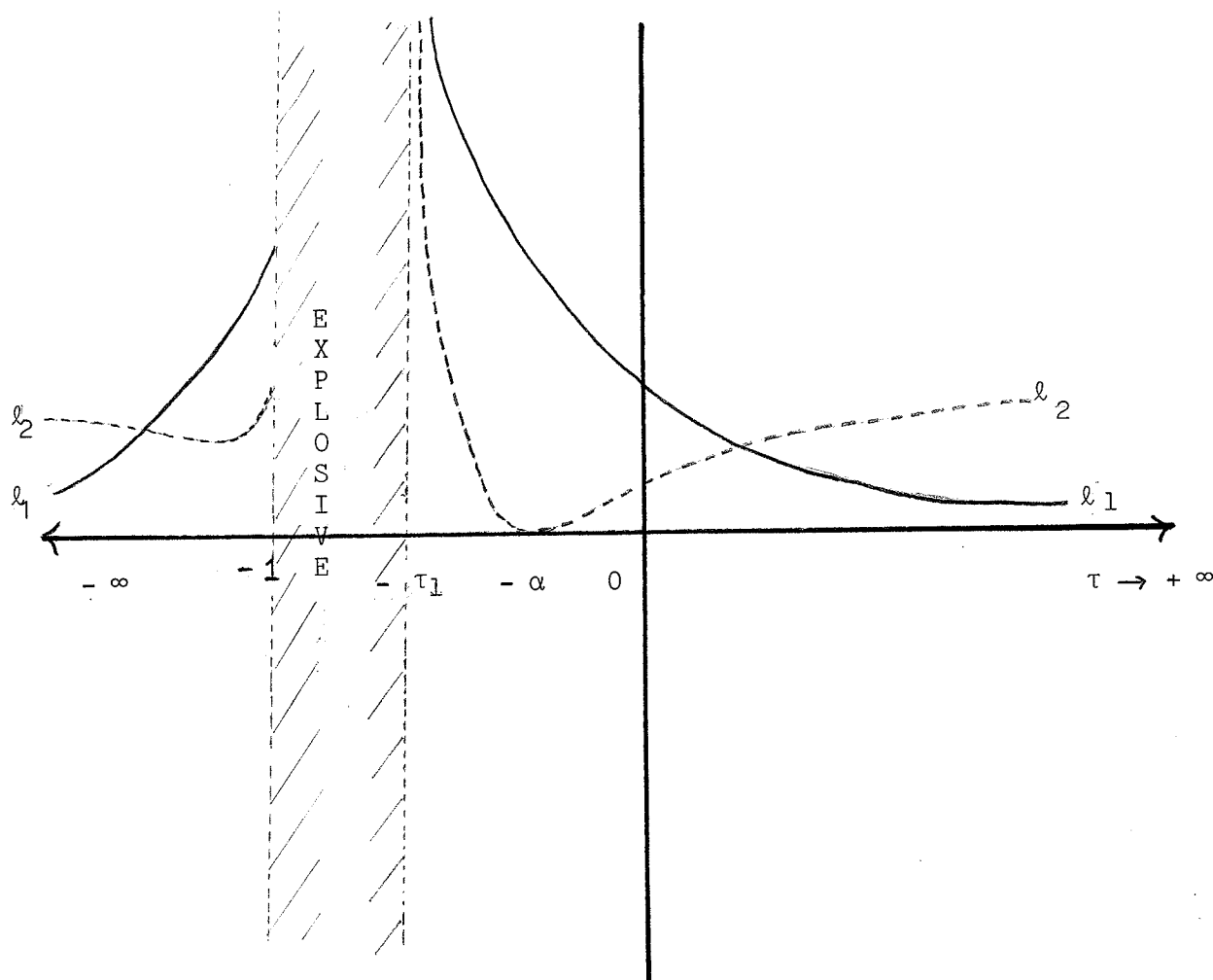
$$h_2: \text{ IS generated disturbance} = \left\{ \phi_3(\pi) \right\}^2 \sigma_{\eta a}^2 / \left\{ 1 + \phi_1(\pi) \right\}$$

$$h_3: \text{ Price equation generated disturbance} = \left\{ 1 - \phi_1(\pi) \right\} \sigma_{\mu a}^2 / \left\{ 1 + \phi_1(\pi) \right\}$$

$\pi_1$  is given by  $1 + \phi_1(\pi_1) = 0$



FIGURE 2



$$e_1: \text{LM generated disturbance} = (\phi_1)^2 \sigma_{Ed}^2 / (1 + \lambda_1(\tau))$$

$$e_2: \text{IS generated disturbance} = (\phi_2)^2 \sigma_{nd}^2 / (1 + \lambda_1(\tau))$$

$$\tau_1 \text{ is given by } 1 + \lambda_1(\tau_1) = 0$$

Footnotes

<sup>1</sup> See also Jones (1985) and Ueda (1983) for different uses of similar models.

<sup>2</sup> The structure of our model is almost identical with Buiter (1985), except that our model does not contain the expected current growth rate of the money stock as an augmentation term in the price equation and is formulated in discrete time.

<sup>3</sup> For a similar result in a flex-price model with many countries, see also Hamada 1985, Ch. 7.

<sup>4</sup> Depending on values of parameter value, multiple solutions may appear adjacent to the explosive region. It can be easily seen that  $\tau$  must pass at least one explosive region corresponding to  $\tau = -J/H$  if  $\tau$  decreases from zero to minus infinity.

<sup>5</sup> In a similar type of model, Aizenman and Frenkel (1985) proposed the objective function which minimizes the dead weight loss in the labor market. In the absence of supply shocks, our criterion in this neoclassical model would be justified by their microeconomic consideration. (See also Kawai (1986)).

<sup>6</sup> Of course, this statement can be relaxed to some extent if the economy chooses a minimum variance solution when  $|\lambda_1| < |\lambda_2| < 1$ . See Taylor (1977), McCallum (1983).

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