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Queues, Rations and Market: Comparisons of Outcomes for the Poor and the Rich

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ABSTRACT

This paper compares the outcomes of several basic types of allocation systems which are commonly employed in developing countries and centrally planned economies to distribute certain goods among individuals. The allocation systems (to distribute the limited supply of a deficit good) that we compare are: convertible and non-convertible rations, the queue system with and without secondary trade, the bundling of goods (in which the deficit good is bundled with some other good), and non-intervention (that is, the unhindered market). Our analysis focusses on obtaining positive results: for each pair of allocation systems, we attempt to ascertain whether a specific group of individuals (particularly the rich and the poor) is better-off under one allocation system or another. The resulting insights and conclusions are valid and informative, regardless of the social criterion (or political reasons) based on which a government might choose an allocation system.

Among the results we obtain are that, for the poor, the ranking of allocation systems (from better to worse) is: convertible rations, non-convertible rations, the queue system without secondary trade, and non-intervention. The queue system, thus, does not turn out to be relatively as beneficial to the poor as it is often thought to be. The bundling system is shown to be inferior for the poor than either convertible or non-convertible rations. The rich are found to be better-off under non-intervention than under most other allocation systems. Also, contrary to the common belief, we show that a rationing system with convertibility is not weakly Pareto superior to the one without convertibility. These and other results are notably robust not only to many of the parameters of the economy, but also to certain types of commodity taxes (and subsidies) and administrative costs.

QUEUES, RATIONS AND MARKET:
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Raaj Kumar Sah*

Governments in less developed countries and centrally planned economies employ a variety of 'non-market' systems to allocate certain goods among individuals. Among the most common systems are the rationing and the queue systems.[1] Also, there are differences in how a particular system functions; in some rationing and queue systems, the rationed good is not convertible (that is, individuals can not exchange this good in secondary markets) whereas it is partly or fully convertible in others. Such differences, as we shall see, have important economic implications.

Each of the above allocation systems leads to a markedly different distribution of welfare among various individuals in the economy, and these welfare distributions are quite different, in turn, from the one that would emerge if the government were not intervening. The primary objective of this paper is to compare the welfare of specific groups of individuals (particularly the poor and the rich) when the limited supply of a good (the deficit good) is allocated through alternative allocation systems, including non-intervention. We do this in two steps: (i) we ascertain the utilities of various groups of individuals under each of a number of allocation systems, and then (ii) we take each pair of allocation systems and attempt to determine whether a specific group of individuals is better-off under one allocation system or another.

This analysis is strictly <u>positive</u> and, therefore, the results and insights we obtain are valid and informative, regardless of the social criterion (or political reasons such as the unwillingness to allow an increase in the market price) based on which a government might choose an allocation system. Furthermore, our comparisons of individuals' welfare, particularly of the poor, under alternative allocation systems are central to typical policy debates. For instance, a main argument often given in favor of the queue or the ration system is that (since direct income subsidies to the poor are not feasible) these allocation systems might be effective ways of helping the poor. Our analysis helps to recognize some of the circumstances when such arguments are useful. We should stress, however, that it is not the objective of this paper to analyze the societal desirability of alternative allocation systems; such an analysis must necessarily be based on some normative criterion.

The allocation systems which we compare are: non-intervention, convertible and non-convertible rations, and the queue system with and without secondary trade.[2] Another allocation system that we examine is the bundling system (in which the deficit good is bundled with some other good). We use relatively simple models to depict each of these allocation systems. Among the results we obtain are the following.

(i) For the poor, *the ranking of allocation systems (from better to worse) is: convertible rations, non-convertible rations, the queue system without secondary trade, and non-intervention*. The queue system, thus, does not turn out to be relatively as beneficial to the poor as it is often thought to be. Also, governments frequently attempt to enforce non-convertibility of rations. Such an emphasis is potentially harmful to the poor.

(ii) The rich are better-off under non-intervention than they are under rations (convertible or non-convertible) or the queue system (with or without secondary trade). Also, the rich are better-off under convertible rations or the queue system with secondary trade than they are under the queue system without secondary trade. These results, as we shall see, are understandable consequences of the high wages and large endowments that the rich typically have.

(iii) It is often believed that no one can be worse-off, and some individuals would be better-off, under convertible rations than under non-convertible rations, because there are gains to trade in the former system. We show this view to be incorrect; that is: Convertible rations are not weakly Pareto superior to non-convertible rations. The reason behind this counter-intuitive result is that the convertibility of rations influences individuals' endowments by affecting their incentives to buy the deficit good from ration shops and, therefore, the standard gains to trade arguments do not apply here.

(iv) Secondary trade in a queue system can generate additional employment opportunities for the poor because, under this system, they are the ones who typically stand in queues, not only for themselves but also for others. This has sometimes prompted suggestions that the poor are better-off in a queue system with secondary trade than in the one without it. We show that such a view is not always correct because, though the poor might get a higher wage when there is secondary trade, they also may face a higher opportunity price for the deficit good.

(v) The poor are better-off under either convertible or non-convertible rations than they are under the bundling system, if the income elasticities of the deficit and the bundled goods are constant and close to one another, and if there is secondary trade in at least one of the two goods. Under the same conditions, the rich are better-off under non-intervention than they are under the bundling system.

A methodological aspect of this paper is that the standard tools of marginal analysis are not usable here because alternative allocation systems result in equilibria which can not be assumed to be in the neighborhood of one another. Yet, as we shall see, our results are robust not only to many of the parameters of the economy but also to certain types of commodity taxes and administrative costs. Moreover, an obvious strength of our pairwise comparisons among alternative systems is that the comparison between any two systems does not depend on whether some other system is considered feasible or not. For instance, non-intervention may not be a realistic alternative in certain contexts; particularly, in centrally planned economies. In these contexts, the relevant comparisons are those which we conduct among alternative government managed systems (that is, among ration, queue, and bundling systems).

The comparison of outcomes of alternative allocation systems has not received as much attention in the literature as it deserves. A central contribution is that by Weitzman (1977) in which he compared, based on a specific social criterion, the allocation of a fixed quantity of the deficit good through non-convertible rations versus a 'price system.'[3] The present paper differs from Weitzman's in not only the scope (we compare many important allocation systems in addition to the two that he does) and the emphasis (ours is on obtaining positive results, whereas his is on normative analysis based on a specific social criterion), but also in the underlying model of the 'price system.' The last point concerns the fact that there are profits in the economy if the market clearing price of the deficit good is higher than its unit cost. The distribution of these profits among individuals, no matter what it is, affects not only the welfare and the consumption of individuals but also the market clearing price. Though Weitzman notes the critical role that the distribution of these profits plays in determining the outcome of the 'price system,' his model assumes that the profits disappear altogether. In our analysis, we take into account the distribution of profits; for instance, under non-intervention, the profits accrue to individuals in proportion to their ownership of the firms which own the deficit good.

This paper is not related to the important literature which has extended parts of the theory of second-best and the theory of optimal commodity taxation to instruments such as rations and queues. For instance, Bucovetsky (1984) shows that, starting from a second-best situation, a government can do better under certain circumstances if a queue system (without secondary trade) is partly introduced into an economy. Guesnerie and Roberts (1984) show that the same is possible if non-convertible rations are partly introduced into an economy. The underlying economic reason is simple: the government can not do worse by having additional policy instruments (whatever the instruments might be, provided it is assumed that there are no administrative costs) and, under some circumstances, it may do strictly better, regardless of what the criterion might be (for example, whether the government wants an improvement in the Pareto sense, or whether it wants an increase in some social welfare function). This literature also addresses the issue of optimal rations and queues, given a social welfare function.[4]

The present paper has a different aim. Our motivation here is not to study rations or queues as <u>additional</u> policy instruments through which the government can do better, based on some criterion. Instead, our motivation is to examine rations, queues and other mechanisms as alternative allocation systems to distribute the limited supply of a good. Further, our focus is on comparative analysis. Therefore, we are not interested in showing that a combination of two policy instruments can do better than any one of them. Instead, we specify a number of basic types of allocation systems, none of which is a special case of another system under consideration, and compare their outcomes on specific groups of individuals.[5]

It is perhaps useful to point out another difference between our comparative approach and that based on the theory of second-best. In the latter, the administrative costs of policy instruments are ignored (though these costs are important in practice); in part, because of the difficulties in formulating generalizable relationships between the administrative cost and the nature of a policy instrument. Specifically, the second-best type results mentioned above, that the government can potentially do better by employing additional instruments, are based on an assumption that additional instruments do not entail any administrative costs. The comparisons undertaken in the present paper, on the other hand, are based on an assumption that different allocation systems under consideration entail approximately the same administrative cost (that is, the administrative cost of an allocation system depends on the quantity of the deficit good distributed). Furthermore, we show that many of our results can be extended to those cases where administrative costs of alternative systems are different. Thus, though the present analysis also abstracts from an explicit modelling of administrative costs, our treatment of these costs can be viewed as a step in the right direction.[6]

In Section I, we derive the expressions for individuals' utilities under four alternative systems (non-intervention, convertible and non-convertible rations, and the queue system without secondary trade). The method for comparing an individual's utility is summarized in Section II. The four systems described above are then compared to one another in Section III. Section IV contains extensions and generalizations; specifically we (i) examine two other allocation systems (the queue system with secondary trade and the bundling system), (ii) describe the extensions or modifications of our results when commodity taxes and administrative costs are taken into account, and (iii) point out certain assumptions one might have to make in attempting to use our positive analysis as a basis for societal (normative) comparisons among alternative systems. Concluding remarks are presented at the end.

I. INDIVIDUALS' UTILITIES UNDER ALTERNATIVE ALLOCATION SYSTEMS

Each allocation system implies a different combination of income and opportunity prices that an individual faces and, thus, a different level of utility that he (she) has. In this section, we derive expressions for the utility levels of different individuals under four allocation systems: non-intervention (market), non-convertible rations, convertible rations, and the queue system without secondary trade. These systems are respectively denoted by $I = M, R, C$ and Q. Individuals are denoted by superscript h, and n^h is the proportion of individuals of type h in the economy. $n^h > 0$, and $\sum n^h = 1$. A summation sign without index means, throughout the paper, that the sum is being taken over all h.

Denote the available supply (per capita) of the deficit good by X, and its unit cost by p. The deficit good is a normal consumption good; that is, an individual's demand for this good is increasing in his income, and decreasing in the price he faces. Also, individuals' tastes are sufficiently similar (though a homogeneity of tastes is not required for much of our analysis), so that the demand for the deficit good (at any given price) is larger for a person with higher income. For individual h, x^h and V^h respectively denote the demand function for the deficit good, and the indirect utility function; these functions are defined over the opportunity prices this person faces and his full income. We assume that the market demand for the deficit good would exceed the available quantity if its market price was to be set equal to its unit cost. That is

$$(1) \qquad \sum n^h x^h(p, m^h) > X$$

where m^h is the full income (value of endowment) of individual h if the market price of the deficit good is p.[7]

Under non-intervention, therefore, private firms (owners of the deficit good) adjust the consumer price of the deficit good to equate its demand and supply. Under a government managed system (that is, under allocation systems R, C, and Q; and the systems to be considered later), the government procures the available quantity of the deficit good at its unit cost p, and distributes it through one or another allocation system.[8] We assume at present that the price of the deficit good that the government charges at its shops is p (of course, the opportunity price of the deficit good would be different under different allocation systems, as we shall see below). That is, there is no commodity tax (or subsidy) on the deficit good and, correspondingly, there is no public surplus (or deficit) under the allocation systems under consideration. Issues concerning commodity taxes and administrative costs are discussed later.

For individual h, let x^{hI} and V^{hI} denote the quantity of the deficit good consumed, and the utility obtained, under the allocation system I. The economy-wide consumption of the deficit good equals its available quantity under the allocation system I; that is

$$(2) \qquad \sum n^h x^{hI} = X .$$

We now obtain the expressions for V^{hI} for various systems, which are needed for later comparisons.

<u>Non-Intervention</u>: The individual h owns (through partial ownership of firms) $\alpha^h X$ units of the deficit good. Naturally, $\alpha^h \geq 0$, and $\sum n^h \alpha^h = 1$. If the market clearing price is p^M, then the full income of individual h is $m^h + \alpha^h(p^M - p)X$. Thus

$$(3) \qquad V^{hM} = V^h(p^M, m^h + \alpha^h(p^M - p)X)$$

and $x^{hM} = x^h(p^M, m^h + \alpha^h(p^M - p)X)$. The market price p^M is obtained by substituting the expression for x^{hM} into (2); that is, from

$$\sum n^h x^h(p^M, m^h + \alpha^h(p^M - p)X) = X .$$

We restrict our analysis to those situations where the aggregate demand curve for the deficit good is downward sloping in its price.[9] The relevant implication of this restriction, in combination with (1) and the above expression for determining p^M, is that the market price p^M

is higher than p. This implication is consistent with the intuition that systems such as rationing are typically employed in those situations where the market allocation would entail a significant rise in the price of the deficit good.

<u>Non-convertible Rations</u>: In this case, individuals can buy (at government shops) up to a fixed quantity of the deficit good, but no more, and resale is not permitted. The superscript R denotes this allocation system, and X^R denotes the maximum quantity of ration. Naturally, the population self selects itself into two groups. The first group consists of those who wish to buy the deficit good in quantities smaller than or equal to X^R. These individuals are not constrained by rationing. For them, $x^{hR} = x^h(p, m^h) \leq X^R$, and

$$(4) \qquad V^{hR} = V^h(p, m^h) .$$

The second group consists of those who want to consume more deficit good than X^R, but are constrained to consume only X^R. A convenient representation of an individual's utility under a rationing constraint is as follows [see Neary and Roberts (1980) for details]. Define the shadow price of the deficit good for person h to be p^{hR}, which is obtained from: $x^h(p^{hR}, m^h + (p^{hR} - p)X^R) = X^R$. Then, this person's consumption behavior under rationing is the same as that in the hypothetical case when he faces price p^{hR}, receives an income transfer $(p^{hR} - p)X^R$, and faces no rationing. Therefore, the utility level of person h can be expressed as

$$(5) \qquad V^{hR} = V^h(p^{hR}, m^h + (p^{hR} - p)X^R) ,$$

where $p^{hR} > p$.[10] The maximum ration quantity is obtained from

(6) $$\sum_{h \varepsilon L} n^h x^{hR} + X^R \sum_{h \varepsilon U} n^h = X ,$$

where the first summation is over those individuals who are not constrained by rationing (group L), and the second summation is over those who are (group U).

We assume that there are at least some individuals (the poorest persons are among them) who do not (or can not) buy the maximum ration quantity X^R. This, we believe, is a more accurate representation in most situations (particularly in developing countries) than to assume that everyone buys the maximum ration quantity. From (6), therefore

(7) $$X^R > X .$$

<u>Convertible Rations</u>: If rations purchased from the government shops can be subsequently traded, and if the resulting equilibrium price of the deficit good is higher than p, then everyone would buy the full quantity of available ration. The ration per person is thus X. If p^C denotes the equilibrium price, then the full income of person h is $m^h + (p^C - p)X$, and his utility level is

(8) $$V^{hC} = V^h(p^C, m^h + (p^C - p)X) .$$

The price p^C is obtained by substituting $x^{hC} = x^h(p^C, m^h + (p^C - p)X)$ into (2). Comparison of (8) with (3) shows, as one might expect, that the key difference between non-intervention and convertible rations is that, in the latter system, the government intervention has equalized the virtual ownership of the deficit good. Since the income distribution in these two cases is different, p^C and p^M are not the same, in general. But $p^C > p$, given our earlier restriction that the aggregate demand curve is downward sloping in price.

Queues without Secondary Trade: In this case, consumers wait in queues to purchase the deficit good. The waiting time is assumed to be proportional to the quantity purchased. This representation approximates those cases where individuals make several purchases in small lots within a single decision period; for instance, because private storage of the deficit good is expensive. If the waiting time per unit purchase is t, then the opportunity price of the deficit good is $p + tw^h$. Thus, the utility level of the individual h is

(9) $$V^{hQ} = V^h(p + tw^h, m^h) .$$

The waiting time per unit, t, is determined from $x^{hQ} = x^h(p + tw^h, m^h)$ and (2).

We assume that the prices of the non-deficit goods (that is, of goods other than the deficit good) and the wage of any given individual are not significantly different under the four allocation systems described above. What it means is that if the economy were to switch (hypothetically) from non-convertible rations to convertible rations (for the deficit good), for instance, then the induced adjustments in the aggregate demands and supplies of the non-deficit goods and of different types of labor are such that the market price of these goods and labor types are not significantly affected. This would be the outcome if, for example, the supply elasticities of the non-deficit goods and the demand elasticities for different types of labor are large.

II. METHOD FOR COMPARING AN INDIVIDUAL'S UTILITY

The method to compare the utilities of specific individuals under alternative allocation systems is summarized in this section. Specifically, if I and J represent two different allocation systems, then we want to ascertain whether the individual h is better-off or worse-off under I ; that is whether V^{hI} is larger or smaller than V^{hJ} . In some cases, such a comparison is straightforward. For notational brevity, let p^{hI} and m^{hI} denote the price of the deficit good and the income, corresponding to the individual h , under the system I . Let p^{hJ} and m^{hJ} denote the respective variables under the system J . Then the individual is better-off under the system I if: $m^{hI} \geq m^{hJ}$ and $p^{hI} \leq p^{hJ}$, with at least one strict inequality. This is because a higher income or a lower price (or both) yield a higher utility.

To deal with the remaining cases, in which one of the two allocation systems entails a higher price but also a higher income for an individual, define the following metric

$$(10) \qquad \Delta^h(I, J) = (m^{hI} - m^{hJ}) + (p^{hJ} - p^{hI})x^{hJ}$$

where we recall that x^{hJ} is the quantity of the deficit good consumed by the individual h under the system J . Then it can be shown that

$$(11) \qquad V^{hI} > V^{hJ} , \text{ if } \Delta^h(I, J) \geq 0 .$$

A derivation for (11) is provided in Appendix 1, but it can also be established through the following revealed preference argument. If $\Delta^h > 0$, then (10) implies that this individual could have purchased, in allocation system I , the same bundle of goods as he did in the allocation system

J . The individual's actual purchase under the allocation system I , however, was different. Therefore, the individual h must be better-off under I .

Note that this method does not yield a verdict when the metric (10) is negative or when its sign is unclear, but it is the best available method for comparing an individual's utility under two different situations, without imposing restrictions on his preferences. In the analysis below, therefore, we compare as many pairs of allocation systems as are possible based on the above general method.

III. COMPARISONS AMONG ALTERNATIVE ALLOCATION SYSTEMS

In this section, we compare the outcomes of the allocation systems described in Section I. We do this first for the poor, then for the rich. In addition, we point out certain important aspects of the comparison between convertible and non-convertible rations.

Comparisons for the Poor: The poor are denoted by $h = 1$. Since the poor belong to the lower tail of the distribution of incomes and wages, their demand for the deficit good is relatively low. In particular, we expect a poor person's demand for the deficit good under non-convertible rations to be smaller than the per capita available quantity. That is

$$x^{1R} < X . \tag{12}$$

No special assumption is needed for the poor to behave this way; the budget constraint itself will generate such a demand behavior at sufficiently low incomes. Next, we assume that the poor do not get any part of the profit under non-intervention; this is a reasonable assumption because the poor do not typically possess ownership of firms. That is, $\alpha^1 = 0$, and from (3)

(13) $$V^{1M} = V^1(p^M, m^1) .$$

The last assumption, as will become clear below, is relevant only for the comparison of non-intervention with other systems. We now derive the following result: The ranking of allocation systems for the poor (from better to worse) is convertible rations, non-convertible rations, the queue system without secondary trade, and non-intervention.

Begin by comparing convertible rations to non-convertible rations. Expressions (4), (8) and (10) yield

(14) $$\Delta^1(C, R) = (p^C - p)(X - x^{1R}) .$$

Using (12) and recalling that $p^C > p$, it follows that (14) is positive. Therefore, the poor are better-off under the ration system with convertibility than they are if rations are non-convertible. The reason for this is as follows. Convertibility of rations brings an income gain to the poor, but it also entails a higher price for the deficit good. On the whole, the poor are better-off with convertibility because the (income producing) ration quantity they can get under this system exceeds the quantity of the deficit good they consume under non-convertible rations.

Next, the comparison between non-convertible rations and the queue system without secondary trade is straightforward since, from (4) and (9), the poor have the same income under these two systems, but they face a higher price of the deficit good under the latter. This is because the queue system entails an <u>extra</u> cost of waiting, small though this extra cost may be for the poor. Thus, $V^{1R} > V^{1Q}$. Finally, compare (13) and

(9). The poor have the same income under the queue system and non-intervention, but the respective prices for the deficit good are $p + tw^1$ and p^M. Now recall that $p^M > p$. It follows then that a person with sufficiently low wage is better-off under the queue system than under non-intervention.

Comparisons for the Rich: The rich are denoted by $h = r$, and they belong to the upper tail of the distribution of incomes and wages. As one would expect, the comparisons between non-intervention and other systems depend, in part, on the ownership of the deficit good that the rich have under non-intervention. We show here that: The rich are better-off under non-intervention than under other allocation systems (that is, under convertible or non-convertible rations, or under the queue system without secondary trade), if their ownership of the deficit good under non-intervention is large; specifically if

$$(15) \qquad \alpha^r X \geq x^{rI}, \quad \text{for } I = R, C \text{ and } Q .$$

That is, if the rich own more deficit good under non-intervention than what they consume under other systems.

The condition (15) is automatically satisfied in a two-class economy because, in this case, the rich own all of the deficit good under non-intervention, but (regardless of the allocation system) the poor consume at least some of the deficit good. To see this, recall that $\sum n^h \alpha^h = 1$. This, in the two-class case, implies $\alpha^r = 1/n^r$, because $\alpha^1 = 0$. Further, (2) implies $x^{rI} = (X - n^1 x^{1I})/n^r$. The last two expressions, along with the fact that $x^{1I} > 0$, yield (15). In fact, we expect the condition (15) to be satisfied even in a multi-class economy, because the rich typically own proportions of firms' shares which are far in excess of the proportions of the outputs (of firms) that they consume.

To confirm that the rich are better-off under non-intervention than under other systems (when (15) holds), we obtain the following from (3), (5), (8), (9) and (10).[11]

$$(16)\qquad \Delta^r(M, R) = (p^M - p)(\alpha^r X - X^R)$$

$$(17)\qquad \Delta^r(M, C) = (p^C - p)(x^{rC} - X) + (p^M - p)(\alpha^r X - x^{rC})$$

$$(18)\qquad \Delta^r(M, Q) = (p^M - p)(\alpha^r X - x^{rQ}) + tw^r x^{rQ}$$

Recall that $p^M > p$, and $p^C > p$. Using (15), thus, (16) and (18) are non-negative. Also, the rich have more than the (economy-wide) average income under convertible rations. Therefore, their consumption is more than average; that is $x^{rC} > X$. Hence, (17) is positive.

We can also show that those with very high wages (which includes the rich) are better-off under convertible rations than under the queue system without secondary trade. Specifically, expressions (8), (9) and (10) yield

$$(19)\qquad \Delta^h(C, Q) = (p^C - p)X + [tw^h - (p^C - p)]x^{hQ} .$$

Since $p^C > p$, the above expression is positive if $w^h \geq (p^C - p)/t$.

<u>Convertible versus Non-convertible Rations</u>: Often it is thought that a rationing system with convertibility must be weakly Pareto superior to the one without convertibility; after all, it could be argued that the gains from trade can not harm anyone and should help at least some individuals. Such an argument overlooks the fact that the convertibility of rations can alter individuals' endowments and, therefore, the gains to

trade argument can not always be applied. In fact, we show that: Convertible rations are not weakly Pareto superior to non-convertible rations; that is, certain individuals are better-off under non-convertible rations.

In particular, consider individuals whose consumption of the deficit good under convertible rations is between X and X^R ; that is: $X^R \geq x^{hC} \geq X$. Among these individuals, there could be two types: those whose consumption is not constrained under non-convertible rations, and those whose consumption is constrained. First take up the former type; for them, expressions (4), (8) and (10) yield

$$(20) \qquad \Delta^h(R, C) = (p^C - p)(X^{hC} - X) .$$

Next, take up those whose consumption is constrained under the non-convertible ration system. For them, expressions (5), (8) and (10) yield

$$(21) \qquad \Delta^h(R, C) = (p^{hR} - p)(X^R - x^{hC}) + (p^C - p)(x^{hC} - X) .$$

Both (20) and (21) are non-negative because $p^C > p$, and $p^{hR} > p$. Thus, this entire group of individuals is better-off under non-convertible rations than under convertible rations.

The intuition behind this result can be seen in two steps. First, under convertible rations, everyone has an incentive to buy the maximum quantity of rations available; consequently, this quantity equals X . Under non-convertible rations, there is no such incentive and, further, there are individuals who do not buy the maximum ration quantity; correspondingly, the maximum ration quantity, X^R , is larger than X . Second, recall that the convertibility of rations implies a higher price of the deficit good, but also an income gain $(p^C - p)X$. Thus, for those individuals whose consumption under convertible rations is larger than X but smaller than X^R, the loss due to higher price exceeds the income gain from convertibility.[12]

It should be emphasized that the above result is based on our assumption that some individuals in the economy (the poorest are among them) do not (or can not) buy the maximum ration quantity under the non-convertible ration system. This assumption, as argued earlier, is more realistic than to assume that everybody buys the maximum quantity under the non-convertible ration system.[13]

IV. EXTENSIONS

In this section, we first examine two other allocation systems (the queue system with secondary trade, and the bundling system), and briefly compare them to some of the systems discussed in the preceding sections. Next, we describe extensions or modifications of our results when commodity taxes and administrative costs are taken into account. Finally, we point out certain assumptions one might have to make in using our positive analysis as a basis for normative comparisons of alternative systems.

Queues with Secondary Trade: In the queue system examined earlier, an individual must himself stand in the queue to be able to consume the deficit good. In some developing countries' cases, it is observed that individuals hire others (or use domestic help) to stand in queues. A polar representation of this type of queue system is the one in which there is secondary trade in the deficit good; in which case, standing in queues becomes a separate economic activity undertaken by only those with the lowest wage. Consequently, the opportunity price of the deficit good is the same for all individuals; unlike in the queue system without secondary trade where the opportunity price is higher for those with higher wages. Clearly, therefore: Those with very high wages (which includes the rich) are better-off under the queue system with secondary trade than under the one without secondary trade.

Also, introduction of secondary trade in a queue system may raise the wage of the group of workers with the lowest wage, because now there is additional demand for their labor.[14] This effect has sometimes prompted suggestions that the introduction of secondary trade in a queue system is helpful to the poor. This view may not, however, be correct under certain conditions; the reason for this can be qualitatively understood as follows. One of the possible consequences of introducing secondary trade in a queue system is that the waiting time per unit of the deficit good increases to balance the demand and the available supply of the deficit good. In this case, the poor face not only a higher wage but also a higher opportunity price of the deficit good. If the increase in their wage is sufficiently small (for instance, if the elasticity of their labor supply with respect to the wage is sufficiently large) then the poor would be better-off under a queue system without secondary trade than in the one with secondary trade.

Bundling of the Deficit Good: One of the allocation systems which has sometimes been employed in developing countries entails bundling of goods; for instance, the quantity of the deficit good that an individual can buy from a government shop is proportional to the quantity of some other good (the 'bundled good') he buys. To understand some of the consequences of such a system, we begin with the case in which there is secondary trade in both goods. Let (p, q) denote the unit prices of the deficit and the bundled good at government shops, and let b denote the units of the deficit good which an individual can buy when he buys a unit of the bundled good at these shops. If (p^B, q^B) are the equilibrium prices for the respective goods at which individuals exchange them, then the absence of arbitrage requires

$$(22) \qquad q^B = q - (p^B - p)b .$$

Therefore, the utility level of the individual h can be represented as

$$(23) \qquad V^{hB} = V^h(p^B, q - (p^B - p)b, m^h) .$$

If x^{hB} and z^{hB} respectively denote the quantities of the deficit and the bundled good consumed by the individual h under this system, then p^B and b are obtained from

$$(24) \qquad \sum n^h x^{hB} = X , \quad \text{and} \quad b\sum n^h z^{hB} = X .$$

An intuitive property of the above system is that the consequence of bundling is the same whether secondary trade is possible in both the deficit and the bundled goods, or whether secondary trade is possible in only one of the two goods. This is because tradability of either of the two goods, or of both, leads to exactly the same relationship between the opportunity prices of the two goods. Specifically, if only the deficit good can be traded and if its exchange price is p^B, then the opportunity price of the bundled good is given by q^B in (22). Similarly, if only the bundled good can be traded and if its exchange price is q^B, then the opportunity price of the deficit good is p^B, given by (22).

To compare the outcome (for the poor) of the bundling system to that of non-convertible rations, we obtain the following expression from (4), (10) and (23).[15]

$$(25) \qquad \Delta^1(R, B) = (p^B - p)(x^{1B} - bz^{1B}) .$$

It is obvious that the sign of the above expression can be determined only for certain types of individuals' demand behavior. We consider here the case in which the income elasticities of the demand for the deficit and the bundled goods are constant and equal. If η denotes the common income elasticity, then $x^{hB} = k_1(m^h)^\eta$, and $z^{hB} = k_2(m^h)^\eta$, where k_1 and k_2 are positive numbers which depend on prices. Substitution of these expressions into (24) yields $k_1 = k_2 b$. Thus, $x^{hB} = bz^{hB}$, and (25) equals zero. Therefore, $V^{1R} > V^{1B}$.

Combining this conclusion with an earlier result (that convertible rations are better for the poor than non-convertible rations), it follows that: The poor are better-off under either non-convertible or convertible rations than they are under the bundling system. It can also be ascertained, by comparing (3) to (23), that: The rich are better-off under non-intervention than they are under the bundling system, provided the condition (15) is satisfied for $I = B$.

Commodity Taxes and Subsidies: We have abstracted in this paper from issues concerning commodity taxation. This is not because we view commodity taxes to be playing an unimportant role (particularly in developing countries) but because many aspects of such taxes are relatively well understood in the literature, whereas the questions examined in this paper have not received adequate attention. An important generalization of the analysis presented earlier is, however, noteworthy. Specifically, our results remain unchanged if there is a tax (or subsidy) on the deficit good, provided the same tax exists under all allocation systems.

To see this, let s denote the tax per unit of the deficit good. That is: (i) the price of the deficit good at government shops is $p + s$, under the ration, queue, or bundling system; (ii) under non-intervention, s is the difference between the equilibrium price of the deficit good and the price which firms owning this good receive;[16] (iii) the resulting budget surplus (or deficit) to the government, in each case, is sX per capita. Then, it can be verified that our comparisons among the alternative allocation systems are unaffected, regardless of what s is; this is because s cancels out when an individual's utility under alternative systems is compared.[17] In the more general case where commodity taxes differ under different allocation systems (leading to different government surpluses or deficits), it is obvious that the comparisons among systems would combine the implications of the allocative properties of alternative systems as well as those of differential tax policies.

Administrative Costs: It can be ascertained that our results are unchanged if alternative allocation systems entail the same administrative cost (that is, personnel, storage and similar other costs depend only on the total quantity of the deficit good), and if this cost is passed on to the consumers through the price of the deficit good. This is because the effect of administrative cost, in this case, is analogous to that of a tax on the deficit good.

Additional generalizations of the following kind are, therefore, also possible: Suppose we find that $V^{hI} > V^{hJ}$ when systems I and J have the same administrative cost, then the same conclusion holds even if the system J has a higher administrative cost than that of I. To see a specific example, recall the result that convertible rations are better for the poor than non-convertible rations. Though this result was obtained in the context without administrative costs, it holds not only when the two allocation systems entail the same administrative cost, but also when the administrative cost of non-convertible rations is larger (for instance, if the cost of enforcing non-convertibility exceeds the cost of transacting secondary trade). An explicit modelling of administrative costs is, however, not attempted in this paper (or in much of the literature), because there appears to be an inadequate conceptual or empirical basis, at present, to formalize generalizable relationships between the administrative cost and the detailed nature of an allocation system.

Normative Comparisons: It is possible, in principle, to use our positive analysis as a basis for conducting societal comparisons of alternative allocation systems, given any normative criterion. For instance, if the social comparisons were to be based on a Bergson-Samuelson welfare function, then such an analysis would require a calculation of the value of the social welfare function under each system, and a comparison of these values across systems. In practice, however, such comparisons face limitations.

Specifically, the standard tools of marginal analysis are not usable in comparing alternative systems because the resulting equilibria are not in the neighborhood of one another. Therefore, to conduct normative comparisons across systems, one would need to posit specific functional forms for the social welfare function as well as for individuals' utility functions. Even then, analytical comparisons may not always be possible; for example, because of the discontinuity in the non-convertible ration system.[18] Social comparisons, thus, may require considerably more detailed assumptions concerning the parameters of the economy than what we found to be necessary for our positive comparisons.[19]

V. CONCLUDING REMARKS

Allocation systems such as rationing and queues are extensively employed in many developing countries and centrally planned economies. In this paper, we have compared the consequences of several basic types of such systems with one another, and with that of unhindered market (non-intervention). Our analysis has concentrated on positive comparisons: we have attempted to ascertain, for each pair of allocation systems, whether a specific group of individuals (particularly the poor and the rich) is better-off under one system or another. The results and insights obtained from these comparisons are valid and informative, regardless of the social criterion or political reasons based on which a government might choose an allocation system.

We recognize that there is a great diversity in the structures and the economic outcomes of the allocation systems that are employed in different contexts. In this paper, we have used relatively simple models to depict alternative allocation systems and have focussed on the comparisons of their outcomes within an important class of circumstances when the supply of a good is limited. Within this class, our results are robust not only to parameters such as the quantity of the deficit good available in the economy, and its unit cost, but also to certain types of commodity taxes and administrative costs. Moreover, the results concerning the comparisons among various government managed systems (that is, among rationing, queues, and the bundling system) hold even when the quantity of the deficit good to be distributed among individuals is a policy choice, rather than a datum for the economy. The corresponding comparison between a government managed system and non-intervention would, of course, be affected by the nature of supply response; the present paper has not analyzed this important case.

Also, for both the queue and the ration system, we have considered two polar specifications: one in which there is no secondary trade and the other in which there is full secondary trade. In some countries, intermediate cases are observed in which partial secondary trade is conducted in underground markets, in contravention of the formal law. In such cases, different individuals participate in these underground markets to various degrees depending, in part, on their incomes and risk-aversion, on the difference between the prices at the government shops and in the underground markets, and on the nature of the legal enforcement system. We hope that comparisons of the outcomes of these and other specifications of alternative allocation systems would be undertaken in the future research work.

FOOTNOTES

*I thank Martin Weitzman for comments on an earlier draft of this paper.

1. These systems have been employed (and have been a source of important controversies) in developed countries as well, particularly during external hostilities and embargoes.

2. Intermediate cases of rationing and queues in which the deficit good is partially convertible in underground markets are briefly discussed at the end of the paper.

3. The social criterion used is as follows. An ideal distribution of the consumption of the deficit good is posited and, then, the social loss under an allocation system is defined to be the sum (over the individuals) of the square of the deviation of the actual distribution of consumption (under the system) from the ideal distribution. The two allocation systems are then compared on the basis of the respective social losses. Rivera-Batiz (1981) extends this analysis by adding a cubic term to the definition of the social loss.

4. Specifically, Bucovetsky (1984) derives the optimal (multi-person Ramsey-like) rule when the government uses queues, in addition to commodity taxes. Younes (1984) derives the optimal rule when the additional instrument is rationing. Guesnerie and Roberts (1984) contrast the commodity structure of optimal rations and taxes.

5. There are many economic reasons (such as the unavailability of information, and the limitations on third-party enforceability) why only simple allocation systems, such as those considered in this paper, are typically feasible. Specifically, we do not consider mechanisms such

as nonlinear pricing schemes (with arbitrary nonlinearities), because such schemes are never feasible for consumption goods. For a discussion of some of these economic reasons in the context of taxation in developing countries, see Sah and Stiglitz (1985).

6. The present paper is also unrelated to the work of Kornai and others [see Kornai (1980), Kornai and Martos (1981), and Hare (1982)] which addresses issues such as control, communication and the endogeneity of shortages in models of centrally planned economies. To the extent this work addresses the effects of non-price allocation systems on consumers, its emphasis is on describing these effects for specific allocation systems rather than on comparing the consequences of alternative systems.

7. The expression (1) captures the notion that there is a 'shortage' of the deficit good at the 'desired' price level. In fact, it is under these conditions that governments typically intervene by employing allocation systems such as rations or queues. Also, unless explicitly needed, we suppress some of the arguments of the demand function and the indirect utility function; in particular, the prices of non-deficit goods, and the individual's wage rate are suppressed.

8. In those contexts where non-intervention is not a feasible alternative (for instance, when the deficit good is produced in the public sector), p is the unit cost to the government.

9. That is, if D_p denotes the price derivative of the aggregate demand, $\sum n^h x^h(p^M, m^h + \alpha^h(p^M - p)X)$, then we are assuming that $D_p < 0$, regardless of how α's are distributed among individuals. What this assumption means, in more elementary terms, can be seen as follows. First, note that $D_p = \sum n^h(x_p^h + \alpha^h X x_m^h)$, where

$x_p^h = \partial x^h/\partial p^M$ and $x_m^h = \partial x^h/\partial m^h$. Using a Slutsky relationship, then, $D_p = \sum n^h x_p^{hu} + \sum n^h x_m^h(\alpha^h X - x^{hM})$, where x_p^{hu} is the price response of the compensated demand of an individual. For brevity, denote the two terms in the right hand side of the last expression as D_{p1} and D_{p2} . Clearly, $D_{p1} < 0$, since $x_p^{hu} < 0$ from a standard property of the compensated demand (we assume that there is some possibility of substitution in an individual's choices). Thus, our restriction that D_p is negative means that either (i) D_{p2} is non-positive, or (ii) D_{p2} is positive but it is dominated by D_{p1} . An example where D_{p2} is zero is when individuals have linear Engel curves with identical slopes. For this example, it is easy to verify that D_p is automatically negative.

10. To see that $p^{hR} > p$, note from (5) that $\partial V^{hR}/\partial X^R = \mu^h(p^{hR} - p)$, where μ^h is the (positive) marginal utility of income for this person. Also $\partial V^{hR}/\partial X^R$ is positive because this person wants to consume more of the deficit good. Hence, $p^{hR} > p$.

11. Note that in the derivation of (16), the utility level of the rich under non-convertible rations is given by (5) because their consumption of the deficit good is constrained under this system.

12. This analysis is based on a different logic than that in Baumol (1982). In the latter, salable and non-salable ration points are examined under the assumption that individuals have envy towards each others' consumption bundles, and that the social criterion is that of fairness.

13. Under the latter assumption, it is easily verified that convertible rations are weakly Pareto superior to non-convertible rations.

14. For simplicity, we assume that the same group of individuals remains at the bottom of the wage distribution even after an increase in their wage.

15. Note that q is the market price of the bundled good under allocation systems other than the bundling system. Also, since the opportunity price of the bundled good differs under the two allocation systems presently being compared, a slight extension of (10) is required to derive (25). Specifically, if q^I and q^J represent the opportunity prices of the bundled good under allocation systems I and J, and if z^{hJ} is the quantity of this good consumed by person h under the system J, then the term $(q^J - q^I)z^{hJ}$ is added to the right side of (10).

16. The expression 'non-intervention' is somewhat awkward here, but the economic meaning should be apparent.

17. This generalization assumes that the relationship (1) is satisfied at the consumer price $p + s$; that is, the market demand at price $p + s$ exceeds the supply of the deficit good. Also, note that if the price of the deficit good at government shops is very low (due to a large subsidy) then everyone would buy the maximum ration quantity under the non-convertible ration system. The consequence of such a possibility has already been discussed in the preceding analysis.

18. The operational problem in this case is analogous to the one faced by Blinder and Rosen (1985) in analyzing notches (jumps) in social policy. Also note that the limitations on social comparisons that are being pointed out here exist even if the social criterion is something other than a Bergson-Samuelson social welfare function.

19. There may be exceptions, however. For instance, if the social comparisons were to be conducted on the basis of the Rawlsian criterion, then the results would be the same as those we have obtained for the poorest group of individuals.

APPENDIX 1

Let e^h denote the expenditure function for person h ; that is, $m^{hJ} = e^h(p^{hJ}, V^{hJ})$. Define $\emptyset^h = e^h(p^{hI}, V^{hI}) - e^h(p^{hI}, V^{hJ})$. Clearly: $V^{hI} > V^{hJ}$, if $\emptyset^h > 0$. This is because higher utility costs more at any given prices. Now using the definition of the expenditure function, one can reexpress $\emptyset^h$ as

$$(26) \qquad \emptyset^h = (m^{hI} - m^{hJ}) + [e^h(p^{hJ}, V^{hJ}) - e^h(p^{hI}, V^{hJ})] .$$

Next, among the standard properties of an expenditure function are that it is concave in prices, and that its derivatives with respect to prices equal an individual's consumption quantities. Further, if we assume that there is some possibility of substitution in the consumption choice of an individual, then

$$(27) \qquad e^h(p^{hJ}, V^{hJ}) - e^h(p^{hI}, V^{hJ}) > (p^{hJ} - p^{hI})x^{hJ} .$$

Substituting (27) into (26) and using the definition (10), one obtains: $\emptyset^h > \Delta^h(I, J)$. Finally, recall from above that: $V^{hI} > V^{hJ}$, if $\emptyset^h > 0$. It follows then that

$$(28) \qquad V^{hI} > V^{hJ} , \text{ if } \Delta^h(I, J) \geq 0 .$$

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