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### Growth Epochs and Compensatory Fiscal Policy

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GROWTH EPOCHS AND COMPENSATORY FISCAL POLICY

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and

John C. H. Fei

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## Growth Epochs and Compensatory Fiscal Policy

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### Abstract

The purpose of this paper is to reexamine compensatory fiscal policy --a basically short-run tool--from the long-run perspective of changing growth epochs, with particular reference to a shift from a fast-growth epoch to a slow-growth one, such as occurred in the early 1970's. It is shown that if we treat a secular decline in economic growth as if it is a cyclical decline, and attempt to combat it with compensatory fiscal policy, undesirable side effects are likely to arise. These effects include an upward shift in the cyclical path of the government budget deficit, which then means a higher secular level of interest rate and more crowding out of private investment--precisely the problems that seem to have plagued the United States in recent years. More specifically, we show that the size of the deficit, the level of the interest rate, and the extent of crowding out, are all dependent on the outcome of a "race" between the rate of growth of the economy and another rate which we refer to as "the rate of command intensification." When the rate of growth falls in the secular context, the odds of the race inevitably shift in favor of higher long-run levels of deficit and interest rate, as well as more crowding out of private investment. Inasmuch as investment ought to be given positive stimulation during a slow-growth epoch, the appropriate policy is to scale down the rate of command intensification in step with the fall in the rate of growth. Such a policy prescription would imply the exact opposite of what is called for under the principle of compensatory fiscal policy.

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Growth Epochs and Compensatory Fiscal Policy\*

The Keynesian notion of compensatory fiscal policy has had a long reign as a key instrument of economic stabilization. Not only has it been hailed for its countercyclical faculty; it has even been considered to be a major contributing factor to the rapid economic growth in the United States after World War II (e.g., Okun, 1970). In view of its apparent success over a long span of time, no serious doctrinal challenges emerged until Monetarism came onto the scene, followed later by the so-called "New Classical Economics" (see Stein, 1982, for a comparative study). The attack of the challengers is frontal in nature: they raise the fundamental question of whether any governmental attempts at stabilization are indeed beneficial as alleged, or even at all potentially effective (Modigliani, 1977). The bases of these challenges consist of such considerations as time lags, the natural rate of unemployment, and rational expectations, some of which are still controversial.

The present paper also questions the appropriateness of compensatory fiscal policy, but from a vastly different perspective. That perspective involves a change of growth epoch, with particular reference to the shift from a fast-growth epoch to a slow-growth one. It is well-known that the "engine of growth" experienced a sudden slowdown in the early 1970's, thus bringing to an end a long era of unprecedented growth that lasted for two full decades (Lewis, 1980). Indeed, Lewis warns us that the earlier fast-growth epoch is unlikely to repeat itself in the future.<sup>1</sup> Whether or not one is inclined to concur with this pessimistic prediction, there is no doubt about the need to study the implications of a slowdown in growth. The purpose of this paper is to reexamine compensatory fiscal policy in the light of such a change in growth epoch.

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\* University of Connecticut and Yale University, respectively. We gratefully acknowledge the computational assistance of Chu-Ping C. Vijverberg.

Our main finding is that if compensatory fiscal policy--a basically short-run instrument--is used as a tool to combat a long-run slowdown in growth, there will inevitably arise problems of large government budget deficits, high interest rates, and crowding out of private investment. These are, of course, precisely the problems that have plagued the United States in recent years. The underlying economic reason is simple: compensatory fiscal policy produces widely different consequences in different growth epochs. In a fast-growth epoch, with the productive capacity expanding rapidly, government spending can serve as a desirable supplement to private demand to help absorb the growing capacity. But this is all changed in a slow-growth epoch. When resource availability is curtailed, government spending tends to become instead a rival claimant on the now more limited goods and services. If we continue to apply compensatory fiscal policy in such a setting, the above-enumerated problems will emerge as the inevitable symptoms. The diagnosis of these symptoms, however, is not easy until we look at them from the perspective of growth epochs.

More specifically, the problems of large deficits, high interest rates and crowding out of investment can be shown to arise whenever the government exercises its political power to command the use of more resources than what is warranted by the rate of growth in the economy. Let us refer to the proportion of resources commanded by the government sector as the command ratio, and denote it by  $g \equiv G/Q$ , where  $G$  is government expenditure and  $Q$  is national product; and refer to an increase in the command ratio over time as command intensification. Then, given a progressive tax system (to be explained later), the emergence of large deficits and the related problems is contingent upon a "race" between the economy's rate of growth and its rate

of command intensification. In the fast-growth epoch, the odds are in favor of the rate of growth, and no problems arise; but with the advent of a slow-growth epoch, the odds swing the other way, making large deficits and the related problems likely. In this light, we may note, the belief of the Keynesians that the rapid growth after World War II is the result of the faithful adherence to compensatory finance may be illusory. Rather, the causation should run the other way: it is the rapid growth during that period that made compensatory fiscal policy appear successful by neutralizing and masking the side effects that would otherwise surface in a slow-growth epoch.

An essential policy objective during a slow-growth epoch is to stimulate investment, which calls for lowering the interest rate. In the long run, we cannot rely on monetary policy to influence the real interest rate (Friedman, 1968). As to fiscal policy, the objective of interest-rate reduction would call for curbing budget deficits and government borrowing in the loanable-funds market. However, to reduce the deficit in the face of an economic slowdown means the abnegation of the very principle of compensatory finance. Since this is a radical departure from a universally recognized economic doctrine, a thorough examination of the underlying issues is in order. The present paper attempts that task with special reference to the United States fiscal experience, 1950-80. But our analytical assumptions are not specific to the United States. To maintain a sharp focus on fiscal policy, we shall abstract from monetary factors; in our long-run context, this simplification should not pose any serious problem because of the long-run neutrality of money.

Section I outlines the fundamental relationships between government finance and resource allocation in our model. The experience of the United States with compensatory finance during 1950-80 is then reviewed in Section II in the light of the analytical structure of this model. In Section III, we derive

on the basis of a given tax system two formal propositions on the behavior of the budget deficit and the deficit ratio, respectively. By extending the discussion to include a saving function (Section IV) and an investment function (Section V), we then derive two more formal propositions on the crowding-out effect and the real interest rate. Section VI presents some concluding remarks.

### I. The Model

Our model involves the following macroeconomic variables, all defined in real terms and denoted by capital letters:

- C = consumption,
- D = disposable income,
- G = government expenditure,
- I = investment,
- P = private expenditures,
- Q = national product (income),
- S = saving,
- T = taxes,
- $\Delta$  = government budget deficit,
- $\Sigma$  = government budget surplus.

Lower-case letters are used to denote variables expressed as ratios to Q:

- $c = C/Q =$  consumption ratio,
- $d = D/Q =$  disposable-income ratio,
- $g = G/Q =$  command ratio,
- $i = I/Q =$  investment ratio,
- $p = P/Q =$  private-expenditure ratio,
- $s = S/Q =$  saving ratio,

$t_a = T/Q =$  average tax ratio,

$\delta = \Delta/Q =$  deficit ratio,

$\sigma = \Sigma/Q =$  surplus ratio.

Other symbols used in this paper include:

$\pi =$  fiscal prudence index,

$\lambda =$  rate of employment,

$t_m = T'(Q) =$  marginal tax ratio,

$\eta_x = (dx/dt)/x =$  rate of growth of variable  $x$ ,

$\epsilon_{yx} = (dy/dx)/(y/x) =$  elasticity of  $y$  with respect to  $x$ ,

$I^* = I/S =$  investment-saving ratio,

$\Delta^* = \Delta/S =$  deficit-saving ratio, or crowding ratio,

$\Sigma^* = \Sigma/S =$  surplus-saving ratio,

$r =$  real interest rate.

#### A. Accounting Equations

The key variables of our model are related to one another through the following five accounting equations:

$$(1.1) \quad Q = P + G, \quad (l = p + g);$$

$$(1.2) \quad P = C + I, \quad (p = c + i);$$

$$(1.3) \quad Q = T + D, \quad (l = t_a + d);$$

$$(1.4) \quad D = C + S, \quad (d = c + s);$$

$$(1.5) \quad G = T + \Delta, \quad (g = t_a + \delta).$$

From these equations, we can also see that<sup>2</sup>

$$(1.6) \quad \delta = g - t_a = d - p,$$

$$(1.7) \quad \Delta + I = S.$$



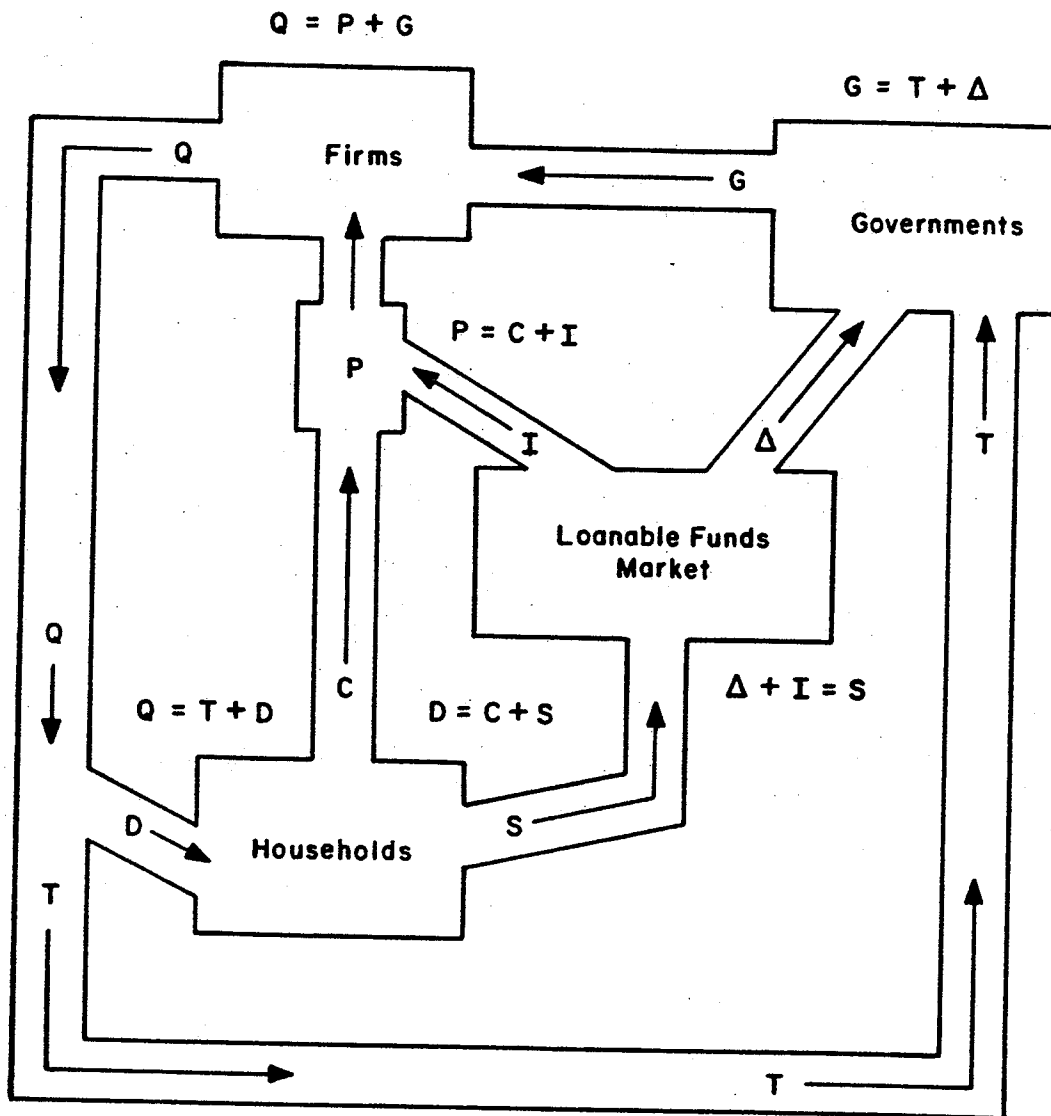


Fig.1

For clarity, we present an overall schematic view of these relationships in Fig. 1.

While these equations are largely self-explanatory, two observations may prove useful in view of the specific context of this paper. First, in (1.1),  $P$  and  $G$  (private and government expenditures) can be, and are often, interpreted as two complementary sources of income generation. This view, based on demand considerations, is of course the very foundation for the notion of compensatory fiscal policy. Alternatively,  $P$  and  $G$  may be considered as two competing claimants on the output  $Q$  of the economy. When a slowdown in economic growth results in greater scarcity in resource supply, this alternative view becomes more relevant and significant.

Second, compensatory fiscal policy calls for the periodic occurrence of budget deficits, with implications shown in (1.6) and (1.7). From (1.6), we see that a deficit ( $\delta > 0$ ) implies that the command ratio will exceed the average tax ratio ( $g > t_a$ ) and that the private share of resource usage will fall short of the disposable-income share of national product ( $p < d$ ). In terms of (1.7), the implication of a deficit ( $\Delta > 0$ ) is that investment will fall short of saving ( $I < S$ ). The latter phenomenon, reflected in the displacement of private investment in the loanable-funds market, has been referred to "crowding out"--a process believed to be brought about via the interest rate. In light of (1.6), however, it may be more revealing to view the crowding out in the loanable-funds market as a mere manifestation of another, more fundamental type of crowding out, namely, the crowding out of private expenditures by government expenditure in the goods market, which is unrelated to the interest rate.

## B. Exogenous Variables and Behavioral Equations

A total of nine real variables appear in Fig. 1, but there are only five independent equations linking them, (1.1) through (1.5). Thus four more assumed relations are needed to close the system.

First, we assume that the national product  $Q$ --in fact the time path of  $Q$ --is exogenously determined. This assumption would be out of place in the usual macro models where the determination of  $Q$  is the subject of study. But it is appropriate here because our purpose is to analyze the implications of a given change in growth epoch.

Second, we also assume that  $G$  is exogenous. Strictly speaking, if an economy adheres faithfully to a policy of compensatory fiscal policy,  $G$  will become a function of  $Q$ . However, since government spending tends to be decided by ad hoc fiscal actions even under such a policy, we can still treat  $G$  as an exogenous variable.

In contrast, the tax variable  $T$  will be assumed to be a function of  $Q$ :

$$(1.8) \quad T = T(Q),$$

and this gives us the third relation. Such a function would be needed in the first place to show the built-in stabilization feature that stems from progressive income tax rates. But we shall interpret (1.8) to include the discretionary aspect of tax policy as well. To the extent that tax legislators are elected officials who perceive their constituents to be generally averse to tax increases, the scope of prospective use of discretionary tax policy is quite limited. Even if not, the effect of discretionary tax policy would be in the same direction as the built-in stabilization aspect. Thus it can be subsumed under (1.8) without undue distortion of the analytical results. To emphasize that (1.8) transcends the concept of a legislated tax schedule,

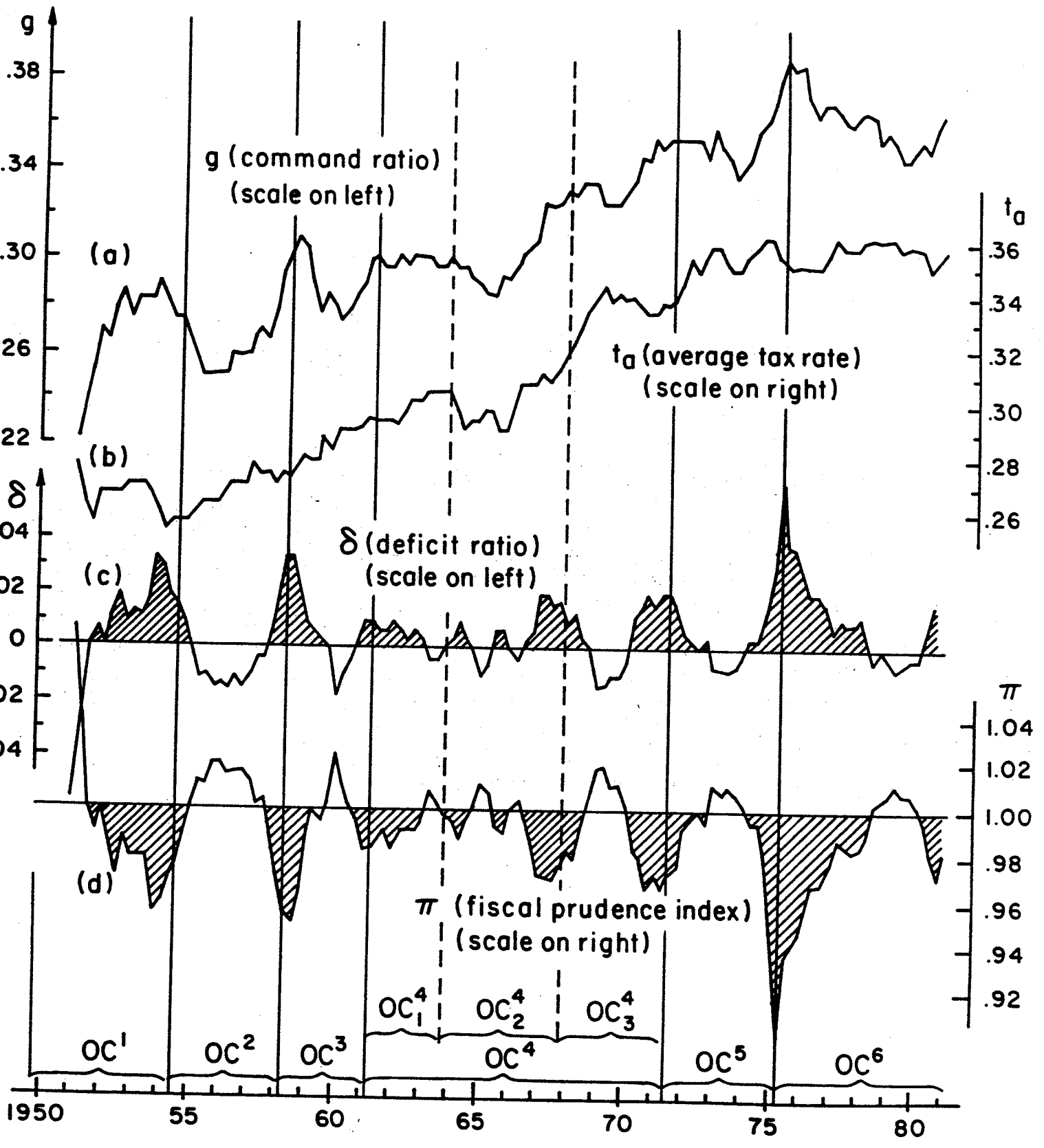


Fig. 2

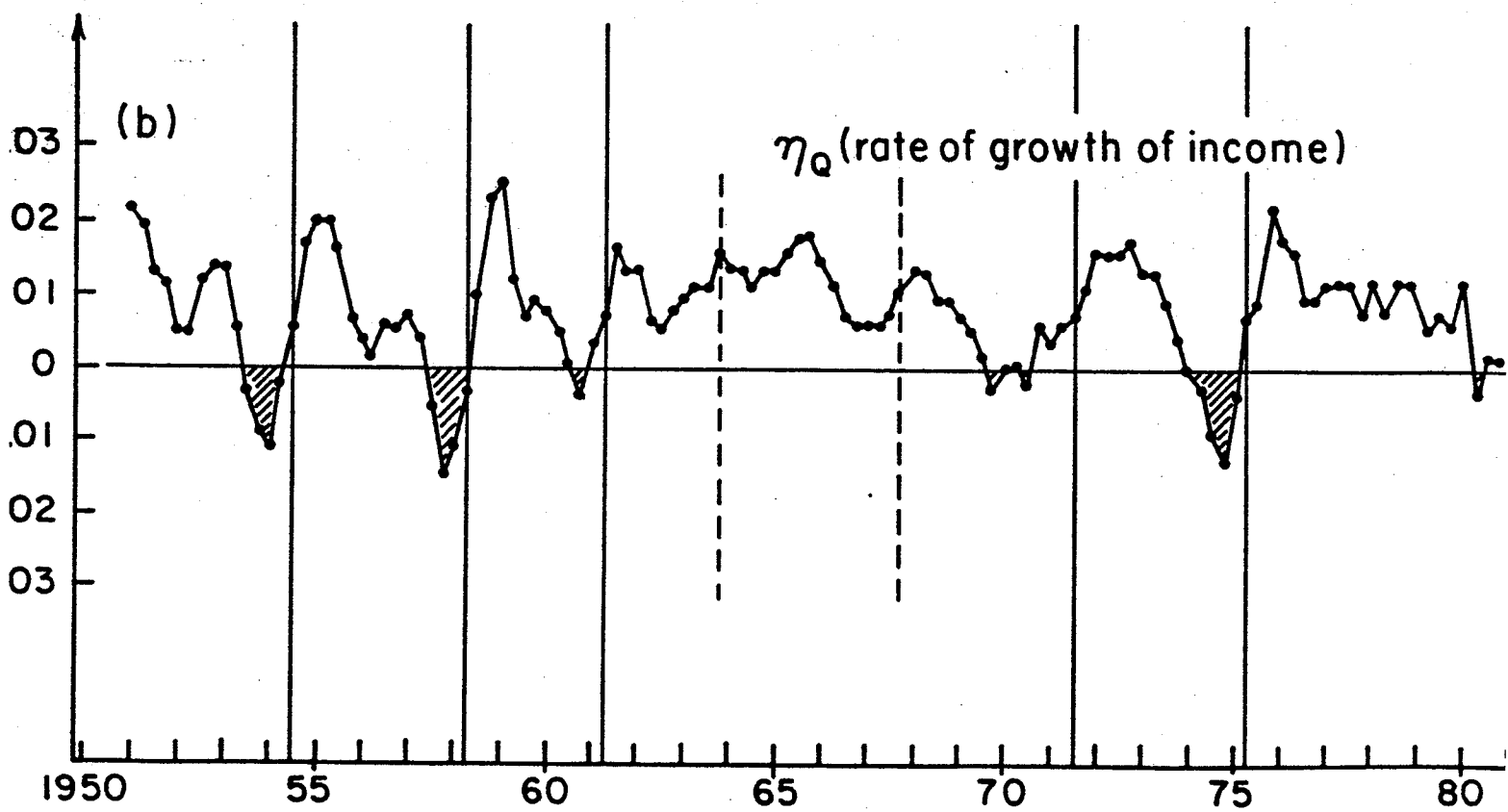
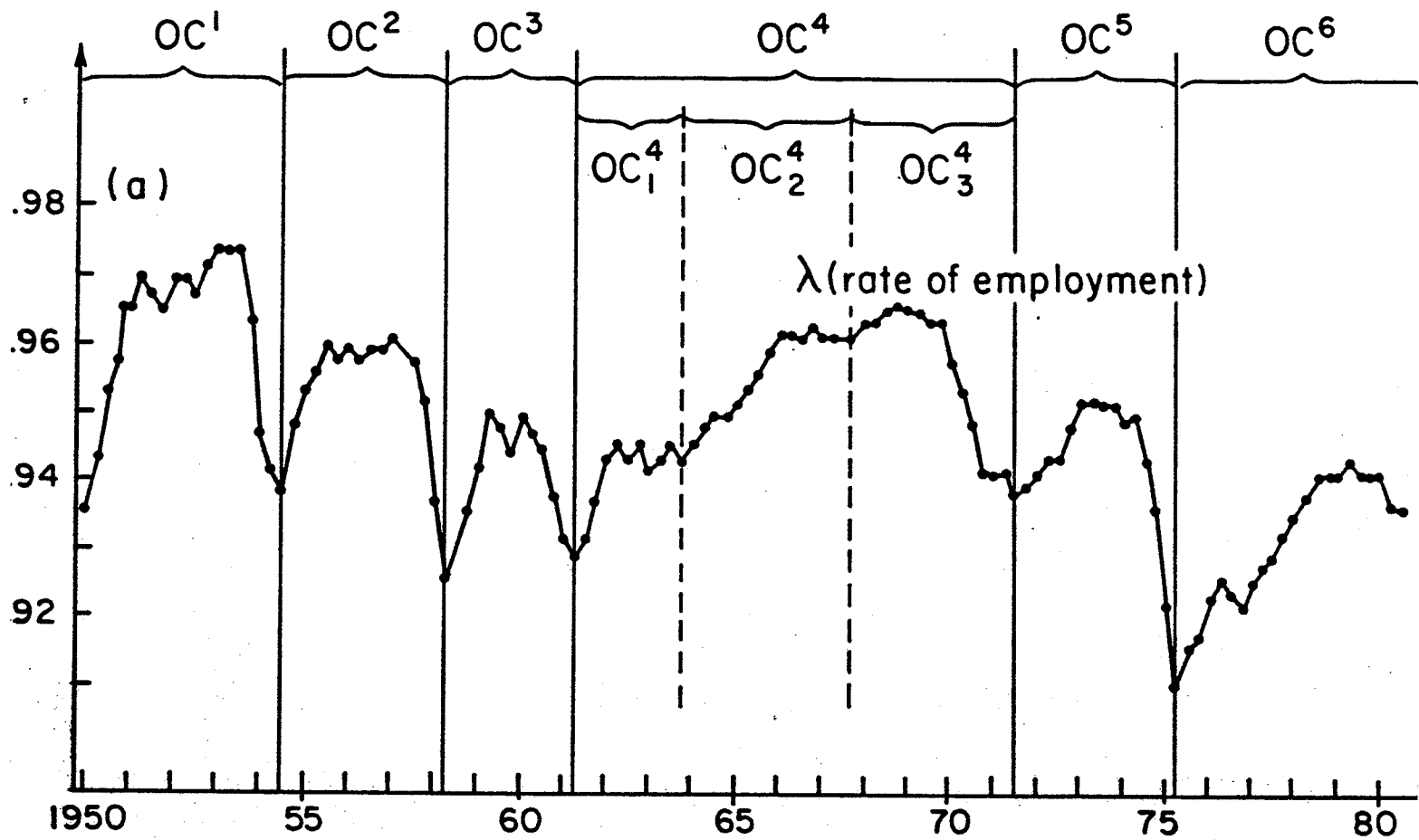


Fig. 3

and contains a measure of the political pliability of the economy regarding the application of discretionary tax policy, we shall refer to it as a tax system rather than a tax function.

The fourth relation to be used is the standard saving function

$$(1.9) \quad S = S(D).$$

These relations make the system determinate. At a later point, we shall introduce the real interest rate as an additional endogenous variable. To accommodate the latter, an investment function will be added to the system:

$$(1.10) \quad I = I(r, Q).$$

Equations (1.8), (1.9) and (1.10) embody the only behavioral assumptions of the model. Since they will be assigned only the "normal" general properties, our analysis should have general applicability.

## II. The U. S. Fiscal Experience, 1950-80

Before proceeding to the analysis, we shall first review briefly the fiscal experience of the United States, and let that serve as a backdrop for the ensuing discussion. Presented under four headings, this review is based on Figs. 2 and 3.<sup>3</sup>

### A. The Command Ratio and the Average Tax Ratio

One striking characteristic of the three-decade period is a persistent trend of command intensification, as depicted in Fig. 2a. The value of the command ratio  $g$  had a low of about 23 percent in the early 1950's, and a high of almost 40 percent in 1975. This uptrend, traceable to the proliferation (and self-perpetuation) of public spending programs under the long espousal of the Keynesian philosophy, was in part made possible by the rapid economic growth during the early part of the period. As growth slowed

down in the 1970's, the government apparently decided to combat the slow growth by raising the command ratio at an even faster pace.<sup>4</sup> Only after 1975 did an opposite tendency appear.

Along with the general uptrend in  $g$ , the average tax ratio  $t_a$  also underwent a secular rise. As Fig. 2b shows, the value of  $t_a$  had a low of about 25-26 percent in the mid-1950's, but rose to a high of 36 percent around the middle of the 1970's. This pattern reflects in part the growth of  $Q$ , and in part the nature of the tax system. The average tax ratio rose because the growth in  $Q$  not only raised  $T$ , but did it more than proportionately as a result of the progressivity of that system.

#### B. The Deficit Ratio and the Fiscal Prudence Index

Because of the simultaneous uptrends in  $g$  and  $t_a$ , the deficit ratio  $\delta$  turned out to be more or less trendless, as shown in Fig. 2c. But it has a clear cyclical pattern attributable to the implementation of compensatory fiscal policy as well as the operation of built-in stabilizers in the economy. We shall return to this pattern after examining the cycles in the rate of employment. A more significant feature of the  $\delta$  curve for the growth-epoch context is that the early (fast-growth) part of the period had fewer years with large deficit ratios (shaded), whereas the 1974-78 period (after the "slowing down of the engine of growth") showed deficit ratios of unprecedented magnitude and persistence. This is the very phenomenon that we intend to analyze.

To describe the state of government finance, it is sometimes convenient to use a fiscal prudence index  $\pi$ , defined as follows:<sup>5</sup>

$$(2.1) \quad \pi = \frac{P}{D} = \frac{P}{d} = \frac{1-g}{1-t_a} > 0.$$

This index increases when the command ratio  $g$  is reduced while the average tax ratio  $t_a$  remains unchanged, or when  $t_a$  is raised while  $g$  remains unchanged. Since the budget deficit  $\Delta$  and the deficit ratio  $\delta$  are related to the fiscal prudence index by the equations<sup>6</sup>

$$(2.2) \quad \Delta = D(1 - \pi) \quad \text{and} \quad \delta = d(1 - \pi),$$

it follows that

$$(2.3) \quad \Delta \text{ (and } \delta) \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if and only if} \quad 0 < \pi \begin{matrix} < \\ > \end{matrix} 1.$$

Thus, fiscal prudence (budget surplus) is associated with  $\pi > 1$ , and fiscal imprudence (budget deficit), with  $0 < \pi < 1$ .

The time path of  $\pi$  for the United States is given in Fig. 2d, where the shaded portions ( $0 < \pi < 1$ ) correspond timewise to the shaded portions of the  $\delta$  path in Fig. 2c, in line with (2.3). Accordingly, the  $\pi$  path also displays a cyclical pattern, but opposite in direction to  $\delta$ .

### C. Employment Cycles and Compensatory Fiscal Policy

Compensatory fiscal actions are triggered by the observed changes in various economic indicators. One important indicator is the rate of unemployment. In this paper, however, we shall use its complement--the rate of employment--instead. The time path of this rate,  $\lambda$ , is shown in Fig. 3a. On the basis of the fluctuation in this rate, we can divide the thirty-year period under study into five and a half observed cycles ( $OC^1$  through  $OC^6$ ). The demarcation lines of these cycles occur at 1949:IV, 1954:III, 1958:II, 1961:II, 1971:III and 1975:II. For a finer classification, we may further divide  $OC^4$  into three subcycles ( $OC^4_1$  through  $OC^4_3$ ). When we superimpose these  $\lambda$  demarcation lines upon the curves in Fig. 2, there emerges a clear view of the relationship between employment cycles and government fiscal operations.



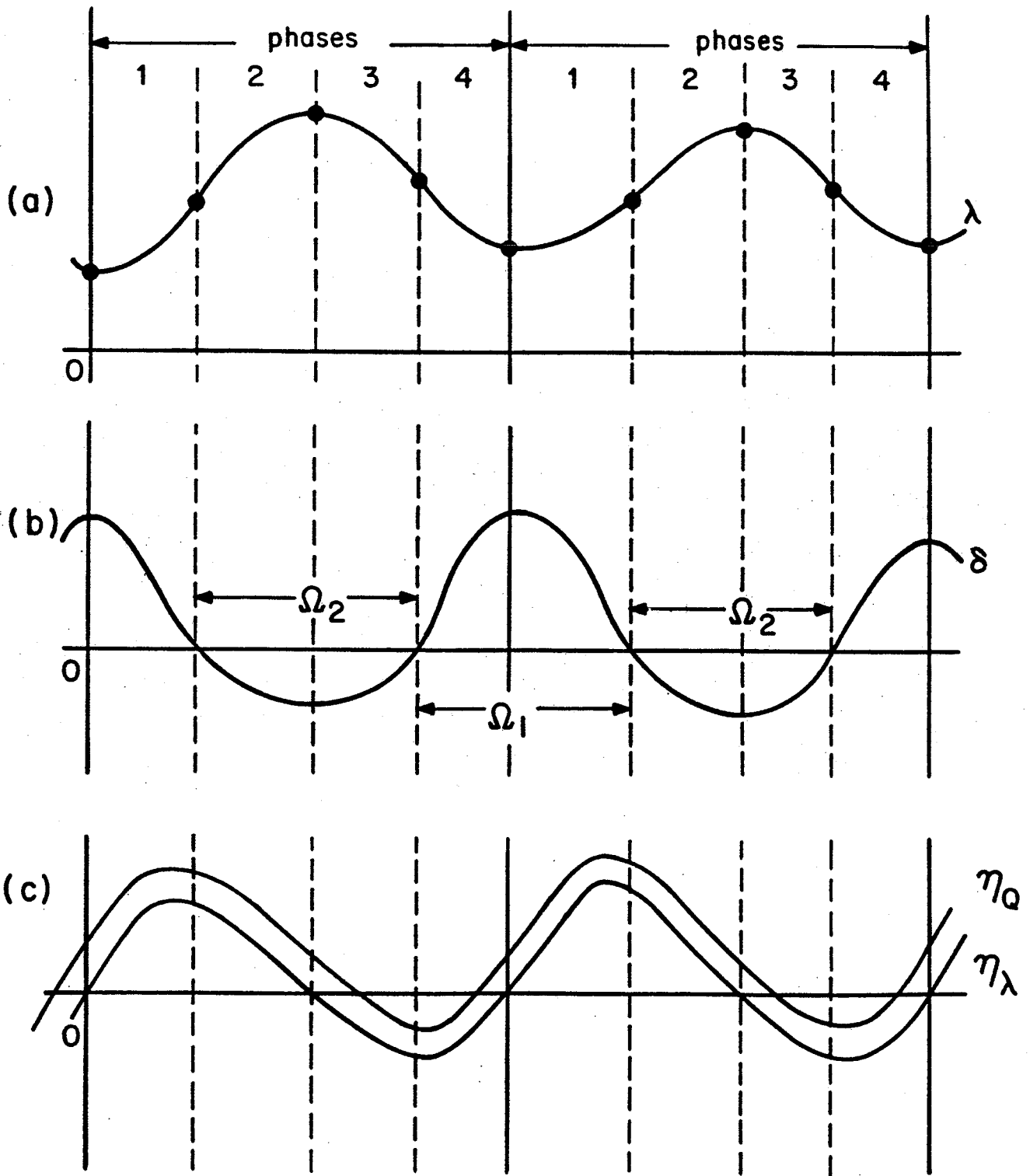


Fig. 4

Starting from  $OC^2$  (when compensatory fiscal policy was already firmly established), Fig. 2a shows a general tendency for the command ratio  $g$  to be low in the middle of each cycle when employment is high. The opposite tendency for the average tax ratio  $t_a$  to be high during high employment is discernible from Fig. 2b. As a result, Figs. 2c and 2d reveal that, beginning with  $OC^2$ , budget surpluses tend to occur in the middle portion of a cycle (and subcycle), whereas deficits tend to appear at the two tails of a cycle. There is thus a harmonic movement between the rate of employment  $\lambda$  and the fiscal prudence index  $\pi$ , and a counterharmonic movement between  $\lambda$  and the deficit ratio  $\delta$ . In fact, the  $\lambda$ -cycle demarcation lines are capable of marking off the turning points in the  $\delta$  and  $\pi$  cycles in Fig. 2 with almost perfect precision.

The empirical relationship between the cyclical patterns of  $\lambda$  and  $\delta$  is stylized in Fig. 4. Following a standard practice, we divide a typical employment cycle into four phases such that phases 1 and 4 (2 and 3) correspond to the bottom (upper) half of the cycle. The counterharmonic movement of the  $(\lambda, \delta)$  pair then implies that a government budget deficit (surplus) generally occurs in the bottom (upper) half of a  $\lambda$ -cycle.<sup>7</sup> Moreover, in the bottom (upper) half, the budget deficit (surplus) increases during the 4th (2nd) phase, to be reversed only after business turns upward (downward), i.e., as we move into the 1st (3rd) phase of the next (same) employment cycle.

#### D. The Rate of Growth and Growth Epochs

The rate of growth of the national product  $\eta_Q$ --also interpretable as the rate of resource augmentation--obviously is important to our analysis, too. There are two aspects to this factor, cyclical and epochal, which will

be discussed in turn.

At the cyclical level,  $\eta_Q$  can serve as an economic indicator for guiding the conduct of compensatory fiscal policy. Indeed,  $\eta_Q$  should follow a pattern of fluctuation closely linked to  $\lambda$  in Fig. 4a, and the associated rate of growth of  $\lambda$  as shown by the  $\eta_\lambda$  curve in Fig. 4c. Assuming that the full-employment output grows at a constant rate, the time path of  $\eta_Q$  can be obtained by a constant upward shift of the  $\eta_\lambda$  path.<sup>8</sup> Hence the stylized  $\eta_Q$  curve in Fig. 4c displays the same cyclical configuration as  $\eta_\lambda$ . It is noteworthy that this stylized  $\eta_Q$  path indeed conforms generally to the observed  $\eta_Q$  path in Fig. 3b.

As to the epochal aspect of  $\eta_Q$ , we note from Fig. 3b that the period under study was, on the whole, one of positive growth. Periods of negative growth (shaded) occurred with very low frequency. However, if we calculate an average  $\eta_Q$  for each separate decade, it becomes clear that the average  $\eta_Q$  declined significantly in the last decade, accompanied by a decline in the average value of  $\lambda$  as well:

	<u>1950's</u>	<u>1960's</u>	<u>1970's</u>
average $\eta_Q$	4.0%	4.2%	3.2%
average $\lambda$	95.5%	95.3%	93.7%

These declines are of course the manifestations of the change in growth epoch pointed out earlier. In the face of such a change, the continued adherence to compensatory finance is apt to create the same type of budget consequences as a cyclical recession--deficits born of a rise in government spending coupled with a decline in tax revenue. Only, the deficits will be long-term ones.

### III. The Behavior of Deficit and Deficit Ratio

The determination of the magnitude of the deficit and the deficit ratio will depend importantly on the structure of the tax system. In line with the U. S. data, we shall assume that the tax system is progressive, i.e., an increase in  $Q$  raises  $T$  more than proportionately.

#### A. Progressive Tax System

Given any tax system  $T = T(Q)$ , the average and marginal tax ratios can be written as

$$(3.1) \quad t_a = \frac{T(Q)}{Q} \quad \text{and} \quad t_m = T'(Q).$$

By the progressivity assumption, the elasticity of  $T$  with respect to  $Q$  must exceed unity:

$$(3.2) \quad \epsilon_{TQ} = \frac{t_m}{t_a} > 1, \quad (0 < t_a < t_m < 1).$$

Thus the marginal tax ratio must exceed the average tax ratio. An important property of such a tax system is that, as long as  $\eta_Q > 0$  (the usual case for a growing economy), the rise in tax revenue can allow the government to increase its resource command without having to unbalance the budget. That is, command intensification can occur in a growing economy even when the government remains prudent.

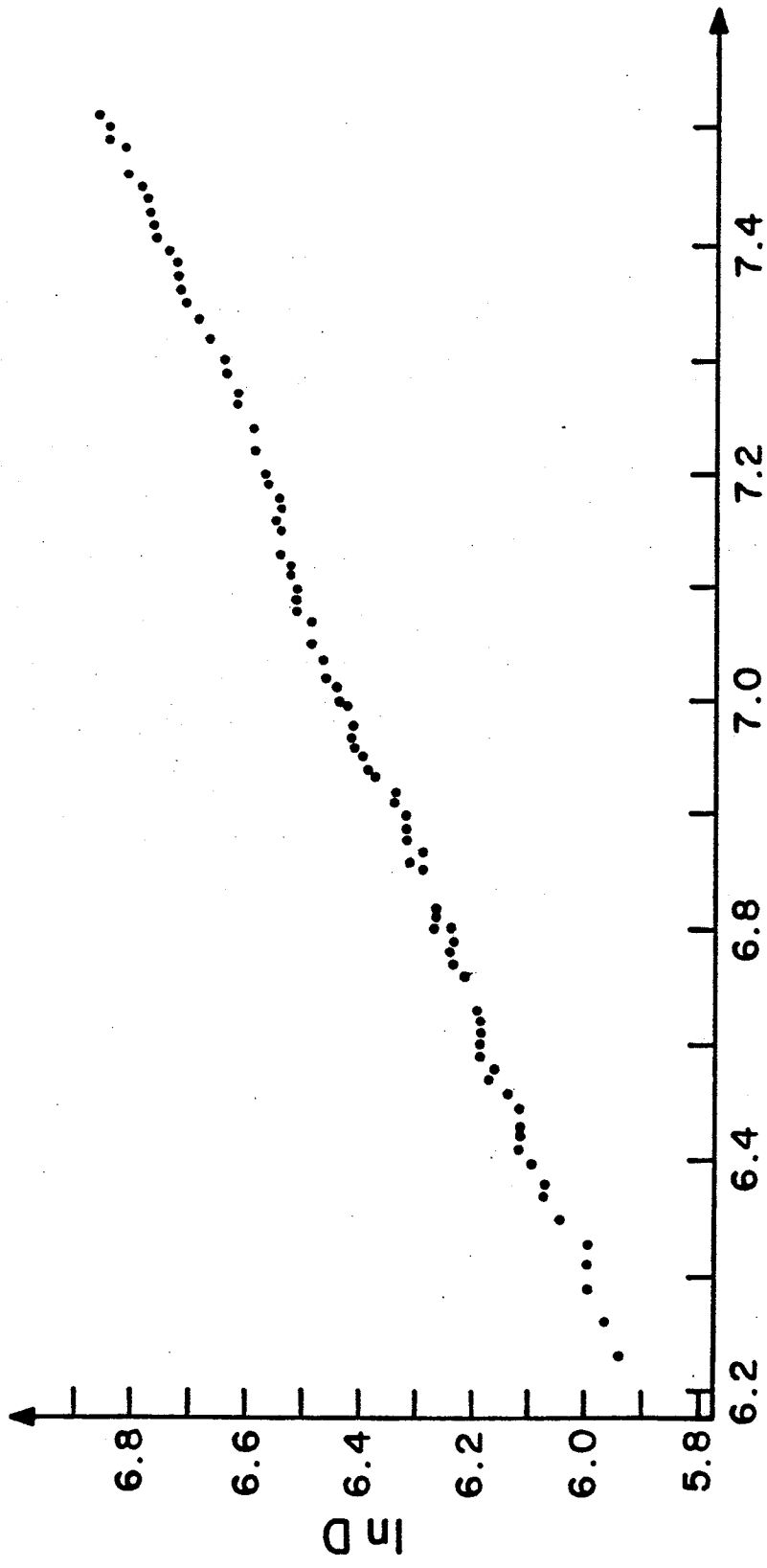
The untaxed portion of the national product remains as disposable income:

$$(3.3) \quad D = D(Q) = Q - T(Q).$$

Since the disposable-income function has its marginal and average functions as follows:

$$(3.4) \quad D'(Q) = 1 - T'(Q) = 1 - t_m,$$

$$(3.5) \quad \frac{D}{Q} = 1 - \frac{T(Q)}{Q} = 1 - t_a,$$



$\ln Q$   
Fig.5

it follows that the elasticity of D with respect to Q is

$$(3.6) \quad \epsilon_{DQ} = \frac{D'(Q)}{D/Q} = \frac{1 - t_m}{1 - t_a}, \quad (0 < \epsilon_{DQ} < 1).$$

The magnitude of this elasticity will be of use below. For the United States, we estimate  $\epsilon_{DQ}$  to be 0.8325 by the following procedure. First, we plot the scatter diagram of  $\ln D$  against  $\ln Q$ , as in Fig. 5, using quarterly data of the thirty-year period. Since the plot is almost perfectly linear, we deem it reasonable to adopt the constant-elasticity <sup>form</sup> ~~for~~  $D = aQ^b$  for the disposable-income function. This then gives us the linear regression equation

$$(3.7) \quad \ln D = 0.7679 + 0.8325 \ln Q, \quad R^2 = 0.9974,$$

(29.14) (215.07)

where the figures in parentheses are t-statistics. The coefficient of the  $\ln Q$  term is our estimate of  $\epsilon_{DQ}$ . Note that its value does fall into the range specified in (3.6).

### B. Fiscal Prudence Regimes

The fiscal prudence index  $\pi$  makes possible a distinction between regimes of fiscal prudence ( $\pi > 1$ ) and imprudence ( $\pi < 1$ ). Heuristically, by comparing the magnitudes of  $\pi$  and  $\epsilon_{DQ}$ , we can further distinguish between "imprudence" and "hyperimprudence," giving a total of three regimes ( $\Omega$ ):

$$(3.8) \quad \begin{array}{ll} \Omega_0: & 0 < \pi < \epsilon_{DQ} < 1 \quad (\text{hyperimprudence}), \\ \Omega_1: & 0 < \epsilon_{DQ} < \pi < 1 \quad (\text{imprudence}), \\ \Omega_2: & 0 < \epsilon_{DQ} < 1 < \pi \quad (\text{prudence}). \end{array}$$

According to (2.3), the first two regimes are characterized by budget deficits, and the third, by surplus. But  $\Omega_0$  further differs from  $\Omega_1$  in that

the  $\epsilon_{DQ}$  value dominates  $\pi$ . Consequently, in  $\Omega_0$ , a growing trend in  $Q$  (say) increases disposable income more (and tax revenue less) than what is regarded as "prudent," thereby aggravating the deficit already implied by  $\pi < 1$ .

The distinction between the three regimes can also be expressed in terms of the command ratio and the average and marginal tax ratios:<sup>9</sup>

$$(3.9) \quad \begin{aligned} \Omega_0: & \quad t_a < t_m < g, \\ \Omega_1: & \quad t_a < g < t_m, \\ \Omega_2: & \quad g < t_a < t_m. \end{aligned}$$

In this alternative characterization,  $\Omega_0$  differs from the other two regimes in that  $g$  exceeds  $t_m$ ; i.e., the government's share in resource usage exceeds the percentage represented by the marginal tax ratio.

Empirically, we note from Fig. 2d that although  $\pi$  frequently fell below one, it never fell below  $\epsilon_{DQ} = 0.8325$  in the period under study. Thus the  $\Omega_0$  regime is of theoretical interest only here. Hereafter, we shall thus confine the discussion to  $\Omega_1$  (imprudence) and  $\Omega_2$  (prudence) only.

### C. A Proposition on the Budget Deficit

Given a progressive tax system, an upward trend in national income always leads<sup>to</sup> an increasing share of national income going into the government coffer, especially in a fast-growth epoch. However, if the government is overly ambitious in its spendings, the command ratio will rise excessively fast. In that event, budget deficits will occur or worsen, unless offset by a sufficiently high  $\eta_Q$ . The rate of growth of the deficit is accordingly dependent on the relative magnitudes of  $\eta_Q$  and  $\eta_g$ .

PROPOSITION 1: The rate of growth of the budget deficit  $\Delta$  is the following weighted sum of  $\eta_Q$  and  $\eta_g$ :

$$(3.10) \quad \eta_{\Delta} = J_Q \eta_Q + J_g \eta_g,$$

where  $J_Q$  and  $J_g$  are defined, respectively, as follows:

$$(3.11) \quad J_Q = \frac{\epsilon_{DQ} - \pi}{1 - \pi} = \frac{g - t_m}{\delta} = \frac{g - t_m}{g - t_a},$$

$$(3.12) \quad J_g = \frac{\pi g}{(1 - \pi)(1 - g)} = \frac{g}{\delta} = \frac{g}{g - t_a}.$$

PROOF:

From (2.2), we have  $\Delta = D(1 - \pi)$ . Taking the rates of growth of both sides, we get

$$(3.13) \quad \eta_{\Delta} = \eta_D + \eta(1 - \pi).$$

From  $D = D(Q)$  in (3.3), we can deduce that

$$(3.14) \quad \eta_D = \epsilon_{DQ} \eta_Q, \quad [\epsilon_{DQ} \text{ assumed constant}],$$

and the last term in (3.13) can be written as

$$(3.15) \quad \eta(1 - \pi) = -\frac{\pi}{1 - \pi} \eta_{\pi}.$$

Since, from (2.1),  $\pi = (1 - g)/(1 - t_a) = (1 - g)Q/D$  [by (3.5)], we have

$$(3.16) \quad \eta_{\pi} = \eta(1 - g) + \eta_Q - \eta_D.$$

Similarly to (3.15), we can write

$$(3.17) \quad \eta(1 - g) = -\frac{g}{1 - g} \eta_g.$$

Substituting (3.17) into (3.16), then (3.16) into (3.15), and finally (3.14)

and (3.15) into (3.13), we obtain upon simplification

$$(3.18) \quad \eta_{\Delta} = \frac{\epsilon_{DQ} - \pi}{1 - \pi} \eta_Q + \frac{\pi g}{(1 - \pi)(1 - g)} \eta_g = J_Q \eta_Q + J_g \eta_g.$$

Moreover, the two coefficients  $J_Q$  and  $J_g$  can be given various alternative expressions as follows:

$$\begin{aligned} J_Q &= \frac{\epsilon_{DQ} - \pi}{1 - \pi} = \frac{(g - t_m)/(1 - t_a)}{\delta/d} \quad [\text{by (3.6), (2.1) and (2.2)}] \\ &= \frac{g - t_m}{\delta} \quad [\text{by (1.3)}] = \frac{g - t_m}{g - t_a} \quad [\text{by (1.5)}]; \end{aligned}$$



$$\begin{aligned}
 J_g &= \frac{\pi g}{(1 - \pi)(1 - g)} = \frac{g/(1 - t_a)}{\delta/d} \quad [\text{by (2.1) and (2.2)}] \\
 &= \frac{g'}{\delta} \quad [\text{by (1.3)}] = \frac{g}{g - t_a} \quad [\text{by (1.5)}]. \quad \text{Q.E.D.}
 \end{aligned}$$

For the two regimes  $\Omega_1$  (imprudence) and  $\Omega_2$  (prudence), we find

$$\begin{aligned}
 (3.19) \quad &\text{In } \Omega_1: J_Q < 0, \quad J_g > 0, \\
 &\text{In } \Omega_2: J_Q > 0, \quad J_g < 0.
 \end{aligned}$$

That is,  $J_Q$  and  $J_g$  always take opposite signs in any given regime, and they always switch signs when one regime is succeeded by another. Together with  $\eta_Q$  and  $\eta_g$ , these two coefficients determine the rate of growth of the budget deficit in the short-run (cyclical) as well as the long-run (epochal) contexts.

At the cyclical level, the  $\Omega_1$  regime typically occurs at the lower half of an employment cycle, as shown in Fig. 4b. In the last phase of any cycle, the deficit tends to worsen because the  $J_g \eta_g$  term in (3.10) is likely to have a large positive value in phase 4 (as  $\eta_g$  is boosted by compensatory fiscal policy), which is either reinforced by a positive  $J_Q \eta_Q$  term (if  $\eta_Q$  happens to be negative in phase 4) or be only partially offset by a negative  $J_Q \eta_Q$  term (if  $\eta_Q$  succeeds in maintaining a low positive value in phase 4). In phase 1 of the next cycle, on the other hand,  $\eta_Q$  takes on a larger positive value (as business turns upward) while  $\eta_g$  decreases or even turns negative (as the need for compensatory finance recedes). Thus the negative  $J_Q \eta_Q$  term in (3.10) becomes dominant, resulting in a negative  $\eta_\Delta$ , or a reduction in the deficit. The analysis of  $\Omega_2$  is similar.

In the long-run epochal context, a secular decline in  $\eta_Q$  means a long-term shift in the level of the  $J_Q \eta_Q$  term in (3.10). For the above-examined  $\Omega_1$  regime, where  $J_Q < 0$ , such a decline in  $\eta_Q$  would render the  $J_Q \eta_Q$  term less effective as an offset to the positive  $J_g \eta_g$  term in phase 4, and also less likely to become a dominant negative term in phase 1 of an employment cycle. (In

fact, the secular ~~rise~~<sup>decline</sup> in  $\eta_Q$  may even induce a secular rise in  $\eta_g$  as the government becomes more anxious to "compensate" for the slowdown in growth.) The net result is therefore a secular rise (fall) in the frequency of occurrence of positive (negative)  $\eta_\Delta$ --in short, a secular upward shift of the deficit path. Similarly, for the  $\Omega_2$  regime, a secular decline in  $\eta_Q$  implies a downward shift of the surplus path.

Instead of considering the coefficients  $J_Q$  and  $J_g$  separately, we can combine them into a single critical multiplier  $\tau$ :

$$(3.20) \quad \tau = -\frac{J_Q}{J_g} = \frac{t_m - g}{g} > 0.$$

Then we may state:

COROLLARY 1: The direction of change of the budget deficit  $\Delta$  (in  $\Omega_1$ ) and the surplus  $\Sigma$  (in  $\Omega_2$ ) depends on  $\eta_g$ ,  $\eta_Q$  and  $\tau$  as follows:

$$(3.21) \quad \left. \begin{array}{l} \text{In } \Omega_1: \eta_\Delta \begin{array}{l} > \\ < \\ = \end{array} 0 \\ \text{In } \Omega_2: \eta_\Sigma \begin{array}{l} > \\ < \\ = \end{array} 0 \end{array} \right\} \text{if and only if } \eta_g \begin{array}{l} > \\ < \\ = \end{array} \tau \eta_Q.$$

PROOF:

Dividing (3.10) through by  $J_g$ , and using (3.20), we get  $\eta_\Delta/J_g = -\tau\eta_Q + \eta_g$ . For  $\Omega_1$ , with  $J_g > 0$ , the first line of (3.21) follows directly. For  $\Omega_2$ , with  $J_g < 0$ , we have instead

$$(3.22) \quad \eta_\Delta \begin{array}{l} < \\ > \end{array} 0 \text{ as } \eta_g \begin{array}{l} > \\ < \end{array} \tau \eta_Q.$$

However,  $\Omega_2$  is characterized by budget surplus  $\Sigma = -\Delta$ , whose rate of growth is the same as that of  $\Delta$ :  $\eta_\Sigma = \eta_{(-1)} + \eta_\Delta = \eta_\Delta$ . Thus we can replace  $\eta_\Delta$  in (3.22) by  $\eta_\Sigma$  to derive the second line of (3.21). Q.E.D.

The economic message of this collary is straightforward: Given a progressive tax system  $T(Q)$ , the direction of change of the budget deficit (or surplus) hinges on a "race" between the rate of resource augmentation  $\eta_Q$  and

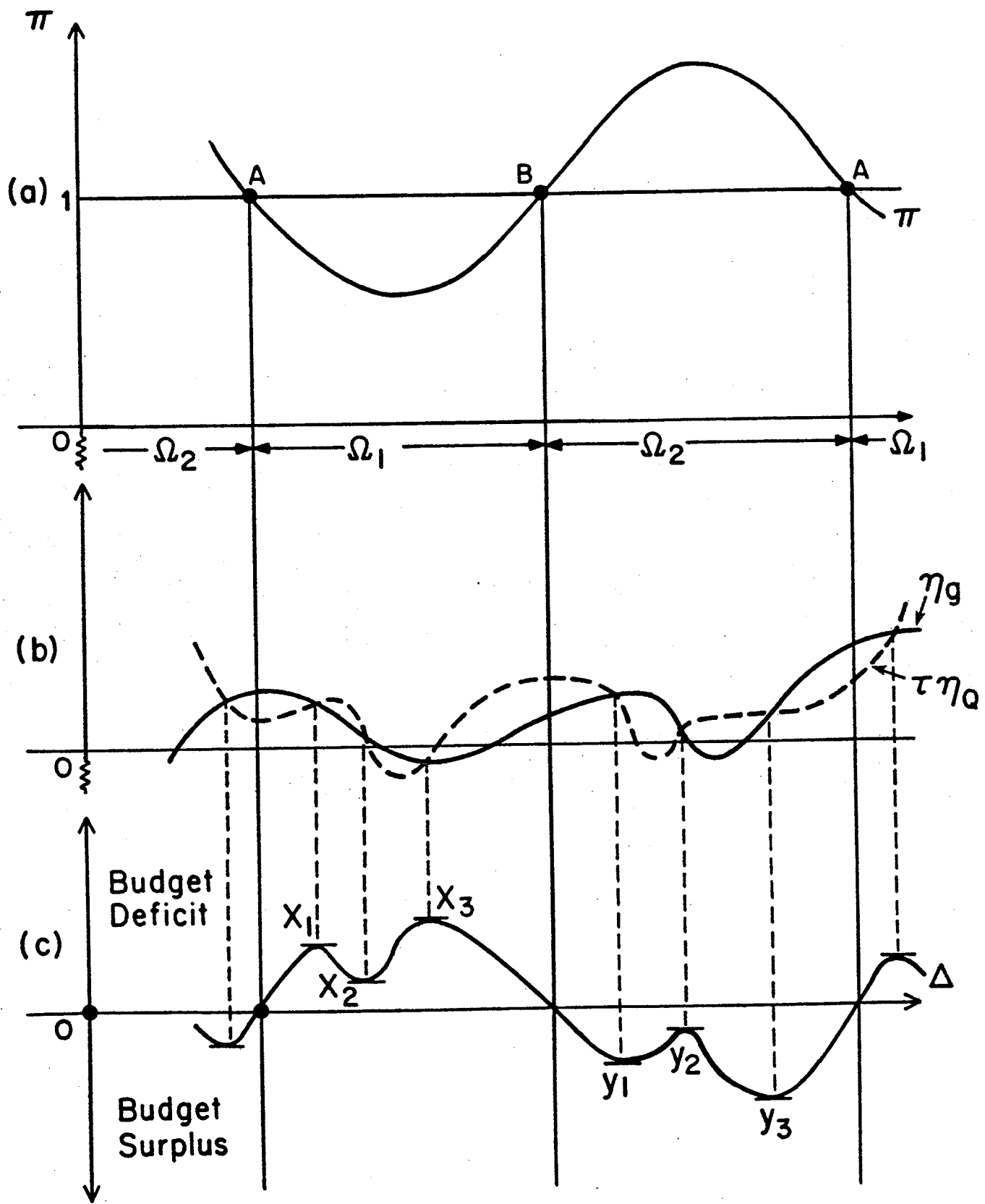


Fig. 6

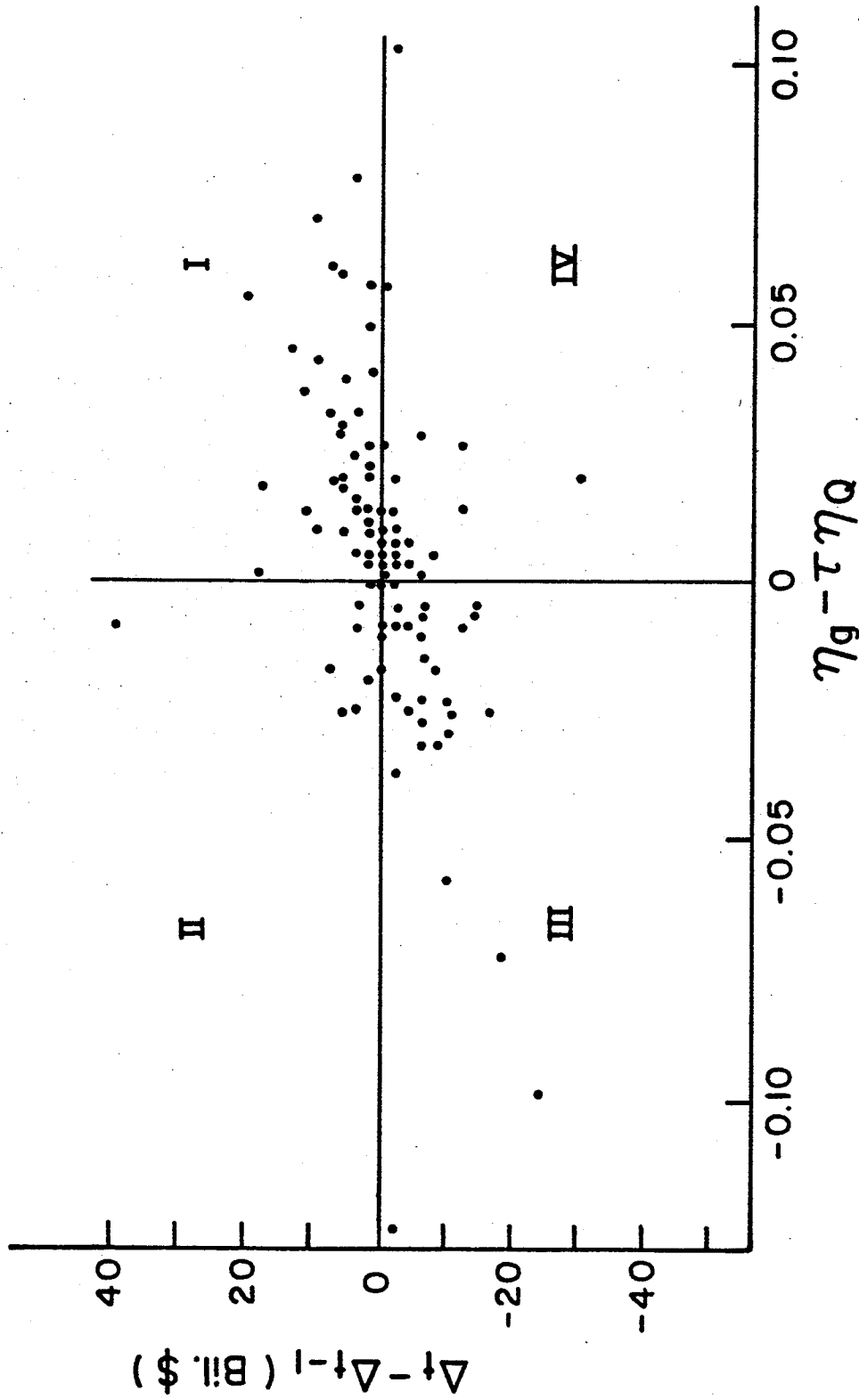


Fig. 7

the rate of command intensification  $\eta_g$ . The deficit increases (or surplus decreases) whenever (a) at a given rate of resource augmentation, the government becomes too ambitious in its command of resources, or (b) given the rate of command intensification (as necessitated by, say, exogenously imposed defense needs or internally committed spending programs), the rate of resource augmentation drops. The role of the multiplier  $\tau$  is crucial here, since its value is what draws the boundary between harmless and detrimental governmental spending ambition, and between tolerable and deleterious slowdown in economic growth.

To illustrate this corollary, we have drawn in Fig. 6a a hypothetical time path of the fiscal prudence index  $\pi$ , exhibiting alternating  $\Omega_1$  and  $\Omega_2$  regimes in accordance with (3.8). In each  $\Omega_1$  ( $\Omega_2$ ) regime, the deficit curve  $\Delta$  in Fig. 6c lies above (below) the horizontal axis. Within a given regime, the direction of the  $\Delta$  curve depends on the height of the solid  $\eta_g$  curve relative to that of the broken  $\tau\eta_Q$  curve in Fig. 6b. According to Corollary 1, whenever the solid curve lies above (below) the broken curve, the  $\Delta$  curve must slope upward (downward). Moreover, whenever the solid and the broken curves intersect, the  $\Delta$  curve attains a local extremum.

To test the analytical results against the U. S. data, we plot  $(\Delta_t - \Delta_{t-1})$  against  $\eta_g - \tau\eta_Q$  in Fig. 7. The scatter points located in quadrants I and III are the ones that would confirm our theoretical predictions. Since the large majority of points are indeed found in the said quadrants, our analytical results seem well borne out. The regression equation for Fig. 7 is

$$(\Delta_t - \Delta_{t-1}) = -0.418792 + 120.232851 (\eta_g - \tau\eta_Q), \quad R^2 = 0.1796,$$

$$(-0.6161) \quad (5.1462)$$

where the figures in parentheses are t-statistics. The fact that the coefficient of  $(\eta_g - \tau\eta_Q)$  is significant confirms that its impact on  $\eta_\Delta$  as specified

in (3.21) is in the right direction. The fact that the constant term is insignificant suggests that the scatter points in quadrants II and IV are statistical "accidents."

#### D. A Proposition on the Deficit Ratio

Sometimes we may want information on the deficit ratio instead of the deficit per se. The following proposition then becomes relevant.

PROPOSITION 2: The rate of growth of the deficit ratio  $\delta$  is the following weighted sum of  $\eta_Q$  and  $\eta_g$ :

$$(3.23) \quad \eta_\delta = (J_Q - 1)\eta_Q + J_g\eta_g,$$

where

$$(3.24) \quad J_Q - 1 = \frac{g - t_m}{g - t_a} - 1 = \frac{t_a - t_m}{g - t_a}, \quad [\text{by (3.11)}],$$

and  $J_g$  is defined in (3.12).

PROOF:

The rate of growth of  $\delta \equiv \Delta/Q$  is  $\eta_\delta = \eta_\Delta - \eta_Q$ . Substitution of (3.10) into this last result yields (3.23). Q.E.D.

For the two regimes  $\Omega_1$  and  $\Omega_2$ , we have:

$$(3.25) \quad \begin{aligned} \text{In } \Omega_1: & \quad J_Q - 1 < 0 \text{ and } J_g > 0, \\ \text{In } \Omega_2: & \quad J_Q - 1 > 0 \text{ and } J_g < 0. \end{aligned}$$

If we define a new critical multiplier

$$(3.26) \quad \tau' = -\frac{J_Q - 1}{J_g} = \frac{t_m - t_a}{g} > 0, \quad [\text{by (3.24) and (3.12)}]$$

we can state a corollary similar to Corollary 1.

COROLLARY 2: The direction of change of the deficit ratio  $\delta$  (in  $\Omega_1$ ) and the surplus ratio  $\sigma$  (in  $\Omega_2$ ) depends on  $\eta_g$ ,  $\eta_Q$  and  $\tau'$  as follows:

$$(3.27) \quad \left. \begin{array}{l} \text{In } \Omega_1: \quad \eta_\delta \begin{array}{l} > \\ \equiv \\ < \end{array} 0 \\ \text{In } \Omega_2: \quad \eta_\sigma \begin{array}{l} < \\ \equiv \\ > \end{array} 0 \end{array} \right\} \text{ if and only if } \eta_g \begin{array}{l} > \\ \equiv \\ < \end{array} \tau' \eta_Q.$$

PROOF: Similar to Corollary 1; hence omitted.

The economic message of Corollary 2 is not much different from Corollary 1. If the rate of command intensification wins the "race" against the rate of resource augmentation (taking into account the multiplier  $\tau'$ ), the deficit ratio (surplus ratio) will increase (decrease). And this is true in both the short-run (cyclical) and the long-run (epochal) contexts.

#### IV. The Loanable-Funds Market and the Crowding Ratio

##### A. The Demand for and Supply of Loanable Funds

When a deficit occurs, the government has to resort to borrowing in the loanable-funds market, competing against private investors on the demand side of that market. This fact has been stated in (1.7):  $\Delta + I = S$ . Dividing this equation by  $S$ , we have

$$(4.1) \quad \Delta^* + I^* = 1.$$

Since  $\Delta^*$  ( $= \Delta/S$ ) and  $I^*$  ( $= I/S$ ) represent two competing shares of the loanable funds acquired by the government and the private investors, respectively, the deficit-saving ratio  $\Delta^*$  can be considered as a measure of the "crowding out effect" of deficit finance. We shall therefore refer to  $\Delta^*$  as the crowding ratio, and an increase in  $\Delta^*$  as crowding intensification.

The supply of loanable funds consists of private saving as a function of disposable income,

$$(4.2) \quad S = S(D).$$

Since we are abstracting from money, saving is the only source of supply (see Fig. 1). This saving function has elasticity

$$(4.3) \quad \epsilon_{SD} = \frac{S'(D)D}{S} > 1.$$

For the United States,  $\epsilon_{SD}$  is approximately constant at 1.3119. This estimate is obtained similarly to  $\epsilon_{DQ}$ . The scatter diagram of  $\ln S$  against  $\ln D$  is found to have an almost perfectly linear configuration (not shown); thus we take  $\epsilon_{SD}$  to be approximately constant. And inasmuch as the regression result is

$$(4.4) \quad \ln S = -3.4330 + 1.3119 \ln D, \quad R^2 = 0.9769, \\ (-29.2174) \quad (71.8483)$$

we obtain the estimate  $\epsilon_{SD} = 1.3119$ .

When (4.2) is combined with the disposable-income function (3.3), a composite function  $S = S[D(Q)]$  results. From this, it follows that

$$(4.5) \quad \epsilon_{SQ} = \epsilon_{SD} \epsilon_{DQ} = \frac{S'(D)D}{S} \frac{D'(Q)Q}{D} = \frac{S'(D)D'(Q)}{s},$$

and

$$(4.6) \quad \eta_S = \epsilon_{SD} \epsilon_{DQ} \eta_Q = \epsilon_{SQ} \eta_Q.$$

Given the saving function  $S(D)$ , we can readily infer the consumption function and the marginal propensity to consume:

$$(4.7) \quad C(D) = D - S(D),$$

$$(4.8) \quad C'(D) = 1 - S'(D).$$

Also, from the composite function  $C = C[D(Q)]$ , it can be deduced similarly to (4.5) that

$$(4.9) \quad \epsilon_{CQ} = \epsilon_{CD} \epsilon_{DQ} = \frac{C'(D)D'(Q)}{c} > 0.$$

These will prove useful in the analysis below.

#### B. A Proposition on the Crowding Out Effect

Analogously to the discussion of the deficit and deficit ratio, we can now state



PROPOSITION 3: The rate of growth of the crowding ratio  $\Delta^*$  is the following weighted sum of  $\eta_Q$  and  $\eta_g$ :

$$(4.10) \quad \eta_{\Delta^*} = (J_Q - \epsilon_{SD} \epsilon_{DQ}) \eta_Q + J_g \eta_g.$$

PROOF:

By definition,  $\Delta^* = \Delta/S$ . Thus  $\eta_{\Delta^*} = \eta_{\Delta} - \eta_S$ . Substitution of (3.18) and (4.6) into the last result leads directly to (4.10). Q.E.D.

If we define yet another critical multiplier

$$(4.11) \quad \tau'' = - \frac{J_Q - \epsilon_{SD} \epsilon_{DQ}}{J_g} = \tau + \frac{\epsilon_{SD} \epsilon_{DQ} \delta}{g}, \quad [\text{by (3.20) and (3.12)}]$$

then we can state another corollary.

COROLLARY 3: The direction of change of the crowding ratio  $\Delta^*$  (in  $\Omega_1$ ) and its negative  $\Sigma^*$  (in  $\Omega_2$ ), depends on  $\eta_g$ ,  $\eta_Q$  and  $\tau''$  as follows:

$$(4.12) \quad \left. \begin{array}{l} \text{In } \Omega_1: \eta_{\Delta^*} \begin{array}{l} > \\ = \\ < \end{array} 0 \\ \text{In } \Omega_2: \eta_{\Sigma^*} \begin{array}{l} < \\ = \\ > \end{array} 0 \end{array} \right\} \text{if and only if } \eta_g \begin{array}{l} \geq \\ < \end{array} \tau'' \eta_Q.$$

PROOF: Similar to Corollary 1; hence omitted.

From the cyclical standpoint, crowding out occurs only in the lower half of an employment cycle ( $\Omega_1$ , or phases 1 and 4). In  $\Omega_1$ ,  $\tau''$  is always positive [by (3.19), (4.3) and (3.6)]; thus the inequalities on the right of (4.12) again describes a "race" between  $\eta_g$  and  $\eta_Q$ . Whenever  $\eta_g$  is large (small) relative to  $\eta_Q$  by more than a critical multiple, crowding intensification will occur (abate). This implies that crowding out tends to increase in phase 4 of an employment cycle (where  $\eta_Q$  is low because of recession and  $\eta_g$  tends to be high on account of compensatory finance), and decrease in phase 1 of the next cycle (as  $\eta_Q$  becomes larger with economic recovery and  $\eta_g$  recedes as a result).

For the  $\Omega_2$  regime (upper half of a cycle), there is budget surplus and

the crowding effect is negative. This would mean the use of the surplus to retire some public debt, with the same effect as an augmentation of private saving. But in  $\Omega_2$  the sign of the critical multiplier  $\tau$  happens to be indeterminate. Nevertheless, an examination of the observed ranges of value for the various component terms of  $\tau$  reveals that  $\tau$  tends to be positive even in the  $\Omega_2$  regime.<sup>10</sup> Consequently, the "race" interpretation remains applicable in  $\Omega_2$ .

More germane to our central theme, however, is the epochal implication of Corollary 3. As we move into a slow-growth epoch, the secular decline in  $\eta_Q$  would depress the long-term level of the  $\tau\eta_Q$  term in (4.12), making it more likely to have crowding intensification in any imprudence regime. And this tendency will be further amplified if the government faces political pressure to compensate for the decline in growth (smaller  $\eta_Q$ ) by boosting public expenditures (larger  $\eta_g$ ). Similarly, in the prudence regime, the consequence of a secular decline in  $\eta_Q$  is to make it more likely to have a decreasing  $\Sigma^*$  (less retirement of public debt). Thus a secular decline in  $\eta_Q$  would result in an upward (downward) shift of the entire cyclical  $\Delta^*$  ( $\Sigma^*$ ) path.

In an attempt to verify (4.12) empirically, we have plotted  $(\Delta^*_t - \Delta^*_{t-1})$  against  $(\eta_g - \tau\eta_Q)$  using U. S. data. The scatter diagram is very similar to Fig. 7, but, to save space, we shall omit it, and merely present the related regression result:

$$\Delta^*_t - \Delta^*_{t-1} = -0.00488 + 1.050579 (\eta_g - \tau\eta_Q), \quad R^2 = 0.2964,$$

$$(-0.9788) \quad (7.1398)$$

The fact that the constant term is insignificant, whereas the coefficient of the  $(\eta_g - \tau\eta_Q)$  term is, serves to support the validity of (4.12).

### C. Policy Implications

As the three propositions above have made clear, when the long-run  $\eta_Q$  is reduced by an epochal change, a policy of maintaining the original long-run  $\eta_g$  will result in higher deficit and deficit-ratio paths, and more crowding out of private investment. If the policy is instead to raise  $\eta_g$  (compensatory fiscal policy), the above-cited effects will be correspondingly intensified.

In Keynesian thinking, budget deficits are not objectionable. Indeed, they are a sine qua non of compensatory finance. And, in the short-run context, deficits are only temporary, to be offset by surpluses that will emerge at other times. However, in the epochal context of a slowdown in growth, the situation is different. For we now have an upward shift of the entire cyclical path of deficit, so that the over-the-cycle offsetting of deficit and surplus cannot be expected to work as before. The consequent accumulation of deficits will then operate to crowd out private investment, just at the juncture when investment is very much needed to raise the productive capacity and to counter the slowdown that has caused the problem in the first place.

The remedy suggested by Corollary 3 is to have  $\eta_g$  fall along with  $\eta_Q$ . That is, we should follow the exact opposite of compensatory fiscal policy. This sharp contrast arises because compensatory fiscal policy is meant for the problem of short-run unemployment, whereas our prescription views the encouragement of investment as the overriding objective in the long-run context. It is not implied here that unemployment can be disregarded. But that problem should, in the long-run context, be tackled through investment rather than government expenditure.

Note that the crowding ratio has been discussed without any reference to the rate of interest. This is in line with our earlier statement that crowding out in the loanable-funds market is a manifestation of the more fundamental type of crowding out in the goods market which is unrelated to the real interest rate. Nevertheless, a slowdown in growth does affect the interest rate, too. We shall now turn to this effect.

## V. The Real Interest Rate

### A. The Investment Function

To discuss the determination of the real interest rate, we introduce the following investment function to interact with the saving function (4.2):

$$(5.1) \quad I = I(r, Q), \quad (I_r < 0, I_Q \geq 0).$$

This function includes  $I = f(r)$  as a special case with  $I_Q = 0$ . There are two partial elasticities to (5.1):

$$(5.2) \quad \epsilon_{Ir} = \frac{I_r r}{I} < 0 \quad \text{and} \quad \epsilon_{IQ} = \frac{I_Q Q}{I} \geq 0.$$

Using these, we can write the rate of growth of  $I$  as

$$(5.3) \quad \eta_I = \epsilon_{Ir} \eta_r + \epsilon_{IQ} \eta_Q.$$

### B. A Proposition on the Real Interest Rate

The inclusion of the investment function enables us to derive:

**PROPOSITION 4:** The rate of growth of the real interest rate  $r$  is the following weighted sum of  $\eta_Q$  and  $\eta_g$ :

$$(5.4) \quad \eta_r = A\eta_Q + B\eta_g,$$

where  $A$  and  $B$  are defined, respectively, by

$$(5.5) \quad A = \frac{1}{\epsilon_{Ir} i} [(1 - \epsilon_{CQ})c + (1 - \epsilon_{IQ})i], \quad (i = \frac{I}{Q}, c = \frac{C}{Q}),$$

$$(5.6) \quad B = -\frac{g}{\epsilon_{Ir} i} > 0.$$

PROOF:

From (1.7), we have  $I = S - \Delta$ , which implies that

$$(5.7) \quad \begin{aligned} \eta_I &= \frac{S}{S - \Delta} \eta_S - \frac{\Delta}{S - \Delta} \eta_\Delta = \frac{S}{I} \eta_S - \frac{\Delta}{I} \eta_\Delta \\ &= \frac{S}{I} \epsilon_{SD} \epsilon_{DQ} \eta_Q - \frac{\Delta}{I} (J_Q \eta_Q + J_g \eta_g) \quad [\text{by (4.6) and (3.18)}]. \end{aligned}$$

Equating (5.7) and (5.3), we get

$$(5.8) \quad \eta_r = \frac{1}{\epsilon_{Ir}} \left( \frac{S}{I} \epsilon_{SD} \epsilon_{DQ} - \frac{\Delta}{I} J_Q - \epsilon_{IQ} \right) \eta_Q - \left( \frac{\Delta}{\epsilon_{Ir} I} J_g \right) \eta_g.$$

The coefficients of  $\eta_Q$  and  $\eta_g$  in (5.8) are, respectively, equal to A and B in (5.5) and (5.6). To show the former, apply (3.11) to the coefficient of  $\eta_Q$  to get

$$\begin{aligned} & \frac{Q}{\epsilon_{Ir} I} \left( \frac{S}{Q} \epsilon_{SD} \epsilon_{DQ} - \frac{\Delta}{Q} \frac{g - t_m}{g - t_a} - \frac{I}{Q} \epsilon_{IQ} \right) \\ &= \frac{1}{\epsilon_{Ir} i} [s \epsilon_{SD} \epsilon_{DQ} - (g - t_m) - i \epsilon_{IQ}] \quad [\text{by (1.6)}] \\ &= \frac{1}{\epsilon_{Ir} i} [S'(D)D'(Q) - g + 1 - D'(Q) - i \epsilon_{IQ}] \quad [\text{by (4.5) and (3.4)}] \\ &= \frac{1}{\epsilon_{Ir} i} [p - C'(D)D'(Q) - i \epsilon_{IQ}] \quad [\text{by (1.1) and (4.8)}] \\ &= \frac{1}{\epsilon_{Ir} i} [c + i - c \epsilon_{CQ} - i \epsilon_{IQ}] \quad [\text{by (1.2) and (4.9)}] \\ &= \frac{1}{\epsilon_{Ir} i} [(1 - \epsilon_{CQ})c + (1 - \epsilon_{IQ})i] = A. \end{aligned}$$

To show that the coefficient of  $\eta_g$  in (5.8) is equal to B, we only need to recall from (3.12) that  $J_g = g/\delta$ . Q.E.D.

Since  $B > 0$ , an increase in  $\eta_g$  in (5.4) always raises  $\eta_r$ . This provides a theoretical basis for the popular belief that larger deficits tend to cause

higher interest rates. In contrast,  $A$  can take either sign. The case of a negative  $A$  occurs if and only if  $(1 - \epsilon_{CQ})c + (1 - \epsilon_{IQ})i$  is positive.<sup>11</sup> Since  $\epsilon_{CQ}$  is a positive fraction,<sup>12</sup> and  $\epsilon_{IQ}$  is nonnegative,  $A$  will be negative if  $\epsilon_{IQ}$  is sufficiently small. In particular,  $\epsilon_{IQ} < 1$ --which is automatically satisfied by the function  $I = f(r)$ --is sufficient for  $A < 0$ . In general,  $A$  is more likely to be negative, the smaller the two elasticities  $\epsilon_{CQ}$  and  $\epsilon_{IQ}$ , i.e., if the slowdown in growth does not substantially discourage investment and consumption, but, by implication, does substantially reduce saving. We can therefore refer to <sup>the</sup>  $A$  case of negative  $A$  as the saving-sensitive case. In contrast, the case of positive  $A$  is characterized by large values of  $\epsilon_{CQ}$  and  $\epsilon_{IQ}$ , and represents the investment-sensitive case.

In the saving-sensitive case (negative  $A$ ),  $\eta_r$  can again be viewed as the outcome of a "race" between  $\eta_Q$  and  $\eta_g$ . Defining a new critical multiplier

$$(5.9) \quad \tau''' = -\frac{A}{B} > 0, \quad (A < 0),$$

we can state on the basis of (5.4):

**COROLLARY 4:** The direction of change of the real interest rate  $r$  depends on  $\eta_g$ ,  $\eta_Q$  and  $\tau'''$  as follows:

$$(5.10) \quad \eta_r \begin{matrix} > \\ \equiv \\ < \end{matrix} 0 \quad \text{if and only if} \quad \eta_g \begin{matrix} > \\ \equiv \\ < \end{matrix} \tau''' \eta_Q.$$

The economic meaning of (5.10) is that, if  $A < 0$  (if saving is more sensitive than investment to national output), then a slowdown in growth will produce an upward pressure on the real interest rate, because it reduces the supply of loanable funds more than the demand for them. To offset that pressure, the rate of command intensification ought to be appropriately reduced.

The only circumstance in which such a reduction in  $\eta_g$  is not mandatory for preventing a rise in  $r$  is when the coefficient  $A$  is positive, i.e., when

we have the investment-sensitive case. Here the slowdown in growth reduces the demand for loanable funds more than the supply, and thus exerts a downward influence on the real interest rate without any help from a reduction in  $\eta_g$ . Note, however, that this does not alter the fact that a larger  $\eta_g$  will mean a larger  $\eta_r$ , other things being equal.

## VI. Conclusion

When compensatory fiscal policy is reconsidered in the light of a slow-growth epoch, whatever merits it may possess as a short-run tool of economic stabilization quickly becomes doubtful. Instead, its "compensatory" feature is seen to be conducive to greater government budget deficits, increasing crowding out of private investment, and higher real interest rates. These effects are undesirable in a slow-growth epoch, when private investment should be stimulated, not discouraged. In order to avoid these undesirable effects, we should refrain from using the time-honored but essentially short-run-oriented compensatory fiscal policy as an instrument for tackling the problem of a secular slowdown in economic growth. On the contrary, the appropriate policy is, according to our analysis, to adjust the rate of command intensification downward, in harmony with the direction of the rate of growth of the economy.

APPENDIX

The data used in this paper are from the following sources:

- (a) Government expenditure: Business Conditions Digest, May 1981, p. 104 (federal) and p. 105 (state and local).
- (b) Implicit price deflator: Business Conditions Digest, May 1981, p. 99.
- (c) Unemployment rate: Business Conditions Digest, February 1981, p. 99.
- (d) Government budget surplus or deficit: Business Conditions Digest, May 1981, p. 104.
- (e) Saving: Business Conditions Digest, May 1981, p. 103.
- (f) Gross national product: Federal Reserve Bulletin, 1951-1981.
- (g) Tax revenue: calculated from (a) and (d) above.

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FOOTNOTES

<sup>1</sup> Among other reasons, he points out that the prospect of future innovations does not seem capable of matching the magnitudes of the period 1950-73, and that there is on the horizon a serious impending shortage of minerals (including, but not confined to, OPEC oil policy).

<sup>2</sup> To prove (1.6), we have:  $\delta = g - t_a$  [by (1.5)] =  $(1 - p) - (1 - d)$  [by (1.1) and (1.3)] =  $d - p$ . Multiplying the equation  $\delta = d - p$  by  $Q$ , we get  $\Delta = D - P = S - I$  [by (1.4) and (1.2)], which leads directly to (1.7).

<sup>3</sup> See Appendix for the sources of the data used. In the revenue and expenditure data, all levels of government are included.

<sup>4</sup> The command ratio went up by a total of 0.13 (from 0.23 to 0.36) over the twenty-one years 1951-72. But it had a gain of as much as 0.03 (from 0.36 to 0.39) in the two-year period 1973-75 alone.

<sup>5</sup> This index, introduced heuristically here, will be used analytically in Section III below, when we discuss the tax system.

<sup>6</sup> Proof:  $\Delta = G - T$  [by (1.5)] =  $Q - P - (Q - D)$  [by (1.1) and (1.3)] =  $D - P = D - D\pi$  [by (1.8)] =  $D(1 - \pi)$ . This is the first equation in (2.2). Dividing the latter through by  $Q$  yields the second equation.

<sup>7</sup> The  $\Omega_1$  and  $\Omega_2$  markings in Fig. 4 refer to the "fiscal imprudence regime" and the "fiscal prudence regime," respectively. These will be discussed further in Section IIIB.

<sup>8</sup> Let the full-employment output,  $Q_f$ , be growing at the rate  $\gamma$ . Then, since  $Q = \lambda Q_f$ , we have  $\eta_Q = \eta_\lambda + \gamma$ .

<sup>9</sup> The transition from (3.8) to (3.9) is based on the following three pieces of information. First,  $\pi \geq 1$  if and only if  $t_a \geq g$  [by (2.1)]. Second,  $t_m > t_a$  in all cases [by (3.2)]. Third, inasmuch as  $\epsilon_{DQ} - \pi = (1 - t_m)/(1 - t_a) - (1 - g)/(1 - t_a)$  [by (3.6) and (2.1)], we know that  $(1 - t_a)(\epsilon_{DQ} - \pi) = g - t_m$ . Given that  $t_a < 1$ , it follows that  $\epsilon_{DQ} \geq \pi$  if and only if  $g \geq t_m$ .

<sup>10</sup> This is due to the fact that the minimum possible value for the (positive)  $\tau$  term in the thirty-year period in fact exceeds the maximum possible absolute value for the (negative)  $\epsilon_{SD} \epsilon_{DQ} \delta/g$  term in (4.11). By applying (3.11), (3.12) and (2.1) to the definition of  $\tau$  in (3.20), we have  $\tau = (\pi - \epsilon_{DQ})(1 - t_a)/g$ , which must obviously be no less than  $[\min(\pi - \epsilon_{DQ}) \cdot \min(1 - t_a)]/\max g$ . In  $\Omega_2$ ,  $\pi > 1$ ; thus  $\min(\pi - \epsilon_{DQ}) = 1 - 0.8325 = 0.1675$ . From Fig. 1b, we have  $\min(1 - t_a) = 0.64$ . Similarly, Fig. 1a shows that  $\max g = 0.39$ . Thus  $\tau$  is not less than  $(0.1675)(0.64)/0.39 = 0.2749$ .

As for the other term, we note that  $\epsilon_{SD} \epsilon_{DQ} |\delta|/g$  must be less than or equal to  $\epsilon_{SD} \epsilon_{DQ} \max|\delta|/\min g$ . We already know from (4.4) and (3.7) that  $\epsilon_{SD} \epsilon_{DQ} = 1.3119(0.8325) = 1.0922$ . Figure 2c shows that, ignoring 1951 for the moment, the negative range of  $\delta$  (for  $\Omega_2$ ) ran from 0 to -0.02. Thus we may take  $\max|\delta|$  to be 0.02. From Fig. 2a, we get  $\min g = 0.22$ . Hence,  $\epsilon_{SD} \epsilon_{DQ} |\delta|/g \leq 1.0922(0.02)/0.22 = 0.0993$ . Comparing this to  $\tau$ , we can conclude that, ignoring 1951,  $\tau$  had been positive even in the  $\Omega_2$  regime.

For 1951,  $|\delta|$  was as high as 0.06, which would greatly boost the absolute value of the negative term in (4.11). But there was a correspondingly high  $\tau$  value (0.8213), because  $\pi$  was high (at 1.08), and so was

$(1 - t_a)$  (at 0.73). Thus  $\tau''$  was still positive.

<sup>11</sup> It is also possible to express A in terms of  $(1 - \epsilon_{SQ})^s$  instead of  $(1 - \epsilon_{CQ})^c$ . But then some extra terms will appear, thereby destroying the symmetry in (5.5).

<sup>12</sup> To see this, recall that  $\epsilon_{CQ} = \epsilon_{CD} \epsilon_{DQ}$ . From (3.7),  $\epsilon_{DQ} = 0.8325$ . We now show that  $\epsilon_{CD}$  is also a positive fraction, which would then make  $\epsilon_{CQ}$  a positive fraction as well. First, the elasticity

$$\epsilon_{CD} = \frac{C'(D)}{C/D} = \frac{1 - S'(D)}{1 - S/D}$$

is clearly greater than zero. From the estimate  $\epsilon_{SD} = 1.3119$  in (4.4), it follows that  $S'(D) > S/D$ , and  $\epsilon_{CD} < 1$ . Thus  $\epsilon_{CD}$  lies between 0 and 1.

Q.E.D.