4-1-1983

The Dynamics of Agricultural Supply: A Reconsideration

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CENTER DISCUSSION PAPER NO. 440

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April 1983

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1. Introduction

Since the beginning of this century economists have been using aggregate time series data and farm surveys to analyze empirically the characteristics of agricultural supply. Cyclical movements of outputs, inputs and prices were recognized, analyzed and debated in many studies (for example, Coase and Fowler, 1935, Ezekiel, 1938, Schultz and Brownlee, 1941-42, and Heady and Kaldor, 1954). Particular attention had been given to the estimation of agricultural supply elasticities (for example, Nerlove, 1958; Muth, 1961; Behrman, 1968; and the survey by Askari and Cummings, 1976).[1]

This paper focuses on the dynamics of output, land allocations and output price movements for an annual agricultural commodity. The optimal land allocations become a complicated dynamic programming problem when the marginal product of land for a particular crop depends on the cultivation history of the plot. There may be at least two distinct aspects for the dependence of current land decisions on past cultivation; (i) the plot preparation for the crop is costly and can be done once for several seasons of the same crop on the same plot; (ii) For some crops (corn and cotton, in particular) there is a severe soil fertility deterioration due to nitrate depletion from the land. The farmer may build up the land productivity by the application of fertilizers. The first aspect, (i), suggests that the marginal costs are decreasing due to past cultivation, while the second aspect, (ii), suggests the opposite. In both cases the total area that is allocated currently to a given crop affects the cultivation costs in the future.

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*Partial support from the General Services Foundation is gratefully acknowledged. I would like to thank Jon Eaton, Bob Evenson and Ken Wolpin for useful discussions and comments on a previous draft of this paper.

In such an environment, current input decisions depend on the expected output price movements in the entire horizon of the optimization problem. Using a simple framework for production and costs I derive the optimal dynamic land allocation demand equation. The costs of land preparation give rise to a dynamic path of land allocations that gradually converges toward the steady state as is the standard result in models with adjustment costs. The soil fertility deterioration gives rise to oscillatory fluctuations in land allocations that can be interpreted as crop rotations (Eckstein, 1981). Using the simple model I define long run and short run supply elasticities with respect to expected and unexpected changes in prices. I show that the expected supply elasticities are determined by the cost function parameters and they are sensitive to the particular dynamic aspect of the crop production. Hence, the analyses of deterministic policy changes require only the identification of the parameters in the agent's objective function. I show that the farmer optimization problem provides a simple regression equation that exactly identifies consistent estimators of these cost function parameters. On the other hand, the unexpected elasticities are determined not only by the parameters of the cost function but by the parameters of the stochastic process of prices as well. Analysis of shocks to prices requires estimation of the entire system, but it does not necessarily require the identification of the underlying parameters of the model. Finally, the analysis of changes in the price process or an interpretation of the observed correlations require complete identification of the model's parameters.

It turns out that the basic supply equation of the Nerlovian (1958) supply response (NSR) model is compatible with a supply equation that I derive from the farmer optimization problem. However, the adaptive expectations formula does not seem to be acceptable and I suggest rational expectations as the modeling strategy for solving the expectations part of the dynamic land demand equation. The idea that the data and the empirical work would be able to tell us whether farmers form expectations using a conditional expectations operator on the true process of prices (rational expectations) or an ad-hoc weighting scheme on past prices (adaptive expectations), proved here to be wrong. I show that the two extremely different methodologies yield different interpretations of the same correlations and different policy conclusions (Lucas, 1976), but give rise to observationally equivalent equations (Sargent, 1976).

For many years agriculture economists suggested that cyclical oscillations in output are due to farmer's static expectations (the cobweb model).[2] Here I show that the type of dynamics in the cost function which reflect deterioration in soil fertility, can give rise to an equilibrium movements in prices and output that have exactly the same form as in the simple cobweb model. However, here the price-output sequence is stable, and always converges to the steady state, the farmer's price expectations are rational and the market allocation of resources is optimal.

The remainder of the paper is organized as follows. In section 2 I outline the model. The farmer optimization problem is solved and the supply elasticities are defined and analyzed in sector 3. In section 4

I compare the model to the traditional agricultural supply model and in section 5 I discuss the issues of observational equivalence of these models. In section 6 I discuss some estimation methods and section 7 presents an equilibrium for the model.

2. The Model

The output sector for the commodity x consists of N farmers indexed by i, i = 1, 2, ..., N. The production function of each farmer has the form

$$(2.1)\quad x^i_{t+1} = (f + V^i_{t+1})\, a^i_t + \frac{e_{t+1}}{N}, \qquad f > 0$$

where x^i_{t+1} is the output of the (representative) farmer at time t+1 and a^i_t is the land allocated at time t by the farmer for production of x at time t+1.[3] e_{t+1} and V^i_{t+1} are shocks to production and land productivity as of time t+1, where e_{t+1} is a persistent economy-wide shock to production that has the form of

$$(2.2)\quad e_{t+1} = \delta_e\, e_t + U^e_{t+1} \qquad |\delta_e| < 1$$

and V^i_{t+1} is a completely transitory farm specific shock. Furthermore, V^i_{t+1} and U^e_{t+1} have zero mean, constant variance and are serially uncorrelated.[4]

Each farmer has the following total costs of production at time t+1

$$(2.3)\quad TC_{t+1} = c^i_t\, a^i_t + F^i_{t+1}\, a^i_t + \frac{g}{2}(a^i_t)^2 + d\, a^i_t\, a^i_{t-1}, \; g > 0, \quad d \gtrless 0$$

where c_t^i is the sum of the per-acre costs of production that are known at the cultivation time and F_{t+1}^i are the sum of the per-acre costs of production that are known only at the harvest time. Assuming that each farmer is endowed with a fixed amount of land ($\bar{a}^i$) and that the farmer produces an alternative commodity $\tilde{x}$, then, the total cost function (2.3) includes the revenues from the alternative crop as a cost per acre allocated for the main crop x. In this case, F_{t+1}^i is a linear (negative) function of the price of the alternative commodity $\tilde{x}$. The term $\frac{g}{2}a_t^2$ induces decreasing returns to scale over the "long run" and may represent existing rent on the fixed amount of capital (land). The term $d\ a_t\ a_{t-1}$ represents the dynamics in production decisions and costs of adjusting the cultivation area. In Eckstein (1981) it is shown that in infinite horizon problems, $d < 0$ implies that the term $da_t a_{t-1}$ is equivalent to the conventional Lucas (1967), Gould (1968) and Sargent (1979) adjustment costs function, while $d > 0$ implies that the marginal cost of producing the crop x at time t+1 is an increasing function of the land allocated to that crop in the previous period. Adjustment costs ($d < 0$) could be justified by the costs involved in land preparation and plot arrangement that are required for the particular crop and are done for each crop on the same plot for several seasons. An increase in marginal costs ($d > 0$) could be due to deterioration in land fertility (Eckstein, 1981). These costs can be reduced by applications of fertilizer and rotation of crops on the plot.[5]

The market is confronted with an exogenous, linear demand schedule for the commodity, where under market clearing conditions

$$(2.3)\quad P_t = b_0 - b_1 D_t + b_2 Y_t \qquad b_0, b_1, b_2 > 0$$

where P_t is the price of the produced commodity at time t, D_t is the aggregate consumption of the commodity at time t and Y_t is aggregate income ("demand shifter") at time t. Income, Y_t, is exogenous to this market and is assumed to follow a second-order autoregressive process

$$(2.4)\quad Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + U_t^y$$

where $\alpha(z) = 1 - \frac{\alpha_1}{\alpha_0} z - \frac{\alpha_2}{\alpha_0} z^2$ has roots inside the unit circle and U_t^y is a "white noise" with zero mean and a constant variance. The second-order process of Y_t is sufficient to capture an economy wide business-cycle type activities that I assume to be exogenous to the agricultural commodity market.[6]

3. The Farmer's Supply

The decision problem confronting each farmer at time zero is to choose a sequence of contingent plans for land allocations in order to maximize discounted expected profits, that is

$$(3.1)\quad \max_{\{a_t^i\}_{t=0}^{\infty}} E_0^i \sum_{t=0}^{\infty} \beta^t \left\{ (P_{t+1} + S_{t+1}^i) \left[(f + V_{t+1}^i)\, a_t^i + \frac{e_{t+1}}{N} \right] - (c_t^i + F_{t+1}^i)\, a_t^i - \frac{g}{2} a_t^{i^2} - d\, a_t^i\, a_{t-1}^i \right\}.$$

where E_0^i denotes the expectations of future variables conditioned on information available to the farmer i at time $t = 0$, Ω_0^i;

β is the discount factor, $0 < \beta < 1$; S_t^i is the subsidy to former i at time t. $\{P_t, S_t^i, C_t^i, F_t^i\}_{t=0}^{\infty}$ are taken parametrically and each is bound in the mean. The contingent plan for a_t^i is a function of the information set at time t, Ω_t^i. I assume that the farmer information at time t includes all realizations of all the variables in the market at time $t, t-1, t-2, \ldots$ etc..

The first order necessary conditions for this problem consist of the following Euler equation as well as the associated transversality conditions.

$$(3.2)\quad d\left[a_{t-1}^i + \frac{g}{d} a_t^i + \beta E_t^i a_{t+1}^i \right] = E_t^i \left[(P_{t+1} + S_{t+1}^i)(f + V_{t+1}^i) - c_t^i - F_{t+1}^i \right]$$

Assuming that $\left| \frac{g}{d} \right| > 1 + \beta$, the unique optimal solution for (3.2) that obeys the transversality condition, is[7]

$$(3.3) \quad (1 - \lambda_1 L)\, a_t^i = -\frac{\lambda_1}{d} E_t^i \sum_{j=0}^{\infty} \Big\{ (\lambda_1 \beta)^j \Big[(P_{t+1+j} + S_{t+1+j}^i)(f + V_{t+1+j}^i) - c_{t+j}^i - F_{t+1+j}^i \Big] \Big\}$$

where L is the lag operator that is defined by the property that $L^k X_t = X_{t-k}$, and λ_1 is the smaller root, in absolute value, that solves

$$(3.4) \quad \frac{1}{\lambda_1} = -\frac{g}{d} - \beta\lambda_1 .$$

From (3.4) it is immediate that $|\lambda_1| < 1$, $\frac{\lambda_1}{d} < 0$ and $\lambda_1 \lessgtr 0$ if $d \gtrless 0$. In order to find the land allocation decision rule, the conditional expectations into the infinite horizon (the right hand side of (3.3)) have to be solved in terms of variables in the farmer's information set, Ω_t^i. Equation (3.3) can be viewed as a general demand for acreage for the particular crop, or by substituting (2.1), the farmer supply equation of the commodity x for a general form of expectation formation. Observe that the aggregate shock, e_t, does not enter directly into the land demand equation since it is separable from the area in the production function. However, it enters indirectly into the land equation through its expected effect on the price process and directly into the supply equation. Hence, the farmer views the price process as random even if the aggregate demand for x is nonstochastic.

The existence of a multiplicative aggregate shock to production, V_t^i, may violate the certainty equivalence (linearity) of the model.[8] **In particular, the covariance of V_t^i and P_t as well as V_t^i and S_t^i should be considered. If P_t is endogenously determined in the model, then with aggregate V_t^i I cannot seek an analytical solution to the model. If the price process is exogenously given**, one can easily redescribe the model in terms of **the stochastic process of the product $V_t^i P_t$ which, for some cases,** may lead to a solvable model. As such, it seems that unless one believes that the aggregate shock is multiplicative and calculations of the covariance between the commodity price and the shock to production are of central interest, then using the above assumptions about V_t^i (or even assuming that $V_t^i \equiv 0$) I lose almost no insight into the model.[9]

The Rational Expectations Decision Rule

The solution for the land demand equation (3.3) is called the decision rule. The solution requires postulating a way in which the farmer solves his infinite horizon conditional expectations problem.[10] Assuming that each right hand side variable in (3.3) has a Wold moving average representation, I can use the results in Hansen and Sargent (1980, lemma 1 in appendix A) to solve for the land allocation decision rule.[11] If prices are determined endogenously then the parameters in the moving averages of prices are related to the underlying parameters of aggregate supply and demand. However, it is still valid that in general the price process would have a Wold moving average representation and it is not necessary to impose the equilibrium constraints in order to solve for the particular farmer's land allocation decision rule (see section 7).

Suppose that each of the right hand side variables in (3.3) have a finite order autoregressive representation,[12] then the land allocation decision rule could be written as

$$a_t^i = \lambda_1 a_{t-1}^i + \mu_0 + \mu_1(L)P_t + \mu_2(L) S_t^i + \mu_3(L) C_t^i + \mu_4(L) F_t^i + \mu_5(L) I_t^i \tag{3.5}$$

where $\mu(L)$ is a finite order polynomial in the lag operator (j = 1, 2, 3, 4, 5) which depends on the order of the autoregressive process of the uncontrolled variables, and I_t^i is a vector of information variables that helps to predict future values of prices, subsidies and cost terms and is part of the autoregressive process of these variables.[13] The μ's are non-linear functions of the cost and production functions parameters as well as the laws of motion for the uncontrollable variables. Hence, changes in the price process, for example, affect the structure of the correlations between the right hand side variable in (3.5) and the land allocations (Lucas, 1976). In order to see this point as well as to analyze the effect of changes in the prices on land allocations, I consider the following simple example:

Let the aggregate price process be

$$P_t = \delta_0 + \delta_1 P_{t-1} + \delta_2 P_{t-2} + U_t^P$$

where the δ's are scalars, $\delta_0 > 0$, $|1 - \frac{\delta_1}{\delta_0} z - \frac{\delta_2}{\delta_0} z^2| = 0$ has no roots less than one, U_t^P is i.i.d. with zero mean and constant variance. There is a fixed specific subsidy for each farmer of $S_t^i = S^i$, $F_t^i = V_t^i = 0$ and

$$C_t^i = \bar{C}^i + U_t^{ci} \tag{3.6}$$

where U_t^{ci} is i.i.d. zero mean and constant variance. Define the farmer's price as $P_t^i = P_t + S^i$ where the farmer's price process is

$$P_t^i = \delta_0^i + \delta_1 P_{t-1}^i + \delta_2 P_{t-2}^i + U_t^{Pi} \tag{3.7}$$

where $\delta_0^i = \delta_0 + S^i(1 - \delta_1 - \delta_2)$, and U_t^{Pi} is i.i.d. with zero mean and constant variance.

In this case the land allocation decision rule has the following form.[14]

$$a_t^i = \lambda_1 a_{t-1}^i + \mu_0^i + \mu_1 P_t^i + \mu_2 P_{t-1}^i + U_t^{ai} \tag{3.8}$$

where

$$\left\{\begin{aligned} \mu_0^i &= \frac{\lambda_1}{d} \bar{C}^i \\ \mu_1 &= \frac{-\lambda_1(\delta_1 + \delta_2\lambda)\cdot f}{d(1 - \delta_1\lambda - \delta_2\lambda^2)} \\ \mu_2 &= -\frac{\lambda_1 \delta_2 \cdot f}{d(1 - \delta_1\lambda - \delta_2\lambda^2)} \\ \lambda &= \lambda_1\beta \\ U_t^{ai} &= \frac{\lambda_1}{d} U_t^{ci} \end{aligned}\right. \tag{3.9}$$

Equation (3.9) shows the restrictions across equations (3.8) and (3.7) as well as the restrictions within equation (3.8). These restrictions are called by Sargent the hallmark of the rational expectations hypothesis. Equations (3.5) and (3.8) analytically characterize the land allocation decision rule in the general case and for a specific example, respectively. Next I define supply elasticities with respect to expected and unexpected changes in the right hand side variable in (3.3) and (3.5).

The Supply Elasticities

An expected or unexpected change in one of the uncontrolled variables alters the demand for a_t and, therefore, through the linear production function (2.1), affects the value of actual production. Let Z be one of the uncontrollable variables on the right hand side of (3.3). I define two types of dynamic elasticities with respect to changes in Z. The first is concerned with the change in the expected land demand (output supply) due to a change in the expected value of Z, while the second concerns the actual change in land demand due to an unexpected change in Z. Both elasticities are computed with respect to the unconditional means of land and Z.

Definition 1: The long run (expected) elasticity of land demand (output supply) with respect to Z is

$$\eta_Z \equiv \frac{\partial E(a\ )}{\partial E(Z)} \cdot \frac{E(Z)}{E(a)} \qquad (3.10)$$

Definition 2: The short-run (expected elasticity) of land demand (output supply) with respect to Z is

$$\eta_Z^j \equiv \frac{\partial E_t(a_t)}{\partial E_t(Z_{t+j})} \cdot \frac{E(Z)}{E(a)} \qquad (3.11)$$

The long run elasticity, η_Z, measures the effect of the expected mean change in Z on the mean change in area (output), while the short run elasticity, η_Z^j, measures the effect of the expected change in Z, j period ahead, conditional on current information on the <u>current</u> change in area (output). Observe that one may be interested in considering also a "medium run" elasticity which can be defined as

$$\eta_Z^j(s) \equiv \frac{\partial E_t(a_{t+s})}{\partial E_t(Z_{t+j})} \cdot \frac{E(Z)}{E(a)} \quad \text{for } j > s$$

and measures the effect on area (output), s period ahead, from a change in conditional expected Z, j periods ahead.

An unexpected change in Z at time t is defined as $\varepsilon_t^Z \equiv Z_t - E_{t-1}(Z_t)$, which is serially uncorrelated. Define $\hat{a}_{t+s}$ as the value of the land allocations at time t+s for the case where $a_s = E(a)$ for $s < t$, and $\varepsilon_t^z = \sqrt{\text{var}(\varepsilon_t^z)} = \sigma^z$ and $\varepsilon_{t+s}^z = 0$ for all $s \neq 0$.

<u>Definition 3:</u> The (unexpected) elasticity response of area (output) s periods ahead with respect to a once-but-not-for-all one standard deviation shock in Z is

$$(3.12) \quad \rho(s) \equiv \frac{\hat{a}_{t+s} - E(a)}{\sigma^z - E(Z)} \quad \frac{E(Z)}{E(a)}$$

Since the a_t process is stationary by the unique solution to the optimization problem, the result is that in the long run $\hat{a}_{t+s}$ converges to $E(a_t)$. Therefore, the long run effect of an unexpected change in Z on area is zero. One may also be interested in the cumulative effect of the shock,

$\sum_{j=0}^{s} \hat{a}_{t+j} - E(a)$, as well as the cumulative effect of the unexpected shock on the variance, i.e., $\sum_{j=0}^{s} [\hat{a}_{t+j} - E(a)]^2$.

In order to calculate the above elasticities of land demand (output supply) with respect to an expected change in the output price I ignore, without loss of generality, the other terms on the right hand side of (3.3). Furthermore, let f = 1 and $V_t^i = e_t = 0$, so that there is a complete equivalence between output and area. Equation (3.3) can be rewritten as

$$(3.13) \qquad (1 - \lambda_1 L)a_t = -\frac{\lambda_1}{d} \sum_{j=0}^{\infty} (\lambda_1\beta)^j \, E_t(P_{t+j+1}) \quad .$$

Taking unconditional expectations in both sides we get

$$(1 - \lambda_1)\, E(a) = -\frac{\lambda_1}{d}\, E(P)\, \frac{1}{1 - \lambda_1\beta}$$

so that the long run elasticity is

$$(3.14) \qquad \eta_P = \frac{-\lambda_1}{d(1 - \lambda_1)(1 - \lambda_1\beta)} \cdot \frac{E(P)}{E(a)}$$

and the short run elasticity is

$$(3.15) \qquad \eta_P^{j+1} = -\frac{\lambda_1}{d} (\lambda_1\beta)^j \frac{E(P)}{E(a)}$$

The interesting aspect of the above elasticities is their different magnitudes with respect to the value of the dynamic element in the production, i.e., the parameter d. The absolute value of d is bounded between zero and $\frac{g}{1+\beta}$. From (3.4) it can be proved that $|\lambda_1| \to 0$ (1) as $|d| \to 0(\frac{g}{1+\beta})$

and $-\frac{\lambda_1}{d} \rightarrow \frac{1}{g}$ as $d \rightarrow 0$. Given that $\lambda_1 \gtrless 0$ as $d \lessgtr 0$, it is clear that the long run elasticity is higher for negative d (adjustment costs) vis-a-vis positive d (soil fertility deterioration). When d = 0, the long run elasticity is $\eta_p = \frac{1}{g} \frac{E(P)}{E(a)}$ and the model is static. The long run elasticity is greater (lower) than $\frac{1}{g}$ as d is negative (positive).

Unlike static models and dynamic models with adjustment costs, here η_p^{j+1} is negative if $d > 0$ (land fertility deterioration) and j is odd, while η_p^{j+1} is positive if $d < 0$ (adjustment costs) for all t or $d > 0$ and t is even. The intuitive reason for the negative short run elasticity is that if the farmer expects next year's output price to increase he would 'save' the land productivity for that year's production and farmers smooth income by oscillating land.[15] Observe that the magnitude of the response gradually declines as the expected change lies further into the future. Recall that F_{t+1}^i includes, as one additive variable, the proportion of the price of an alternative crop and therefore the negative value of the elasticities above is proportionate to the elasticities with respect to the price of the alternative crop. The size of the expected elasticities is fully determined by the values of g, d and β and the means of Z(P) and area (output). Finally, these elasticities can be analyzed directly from (3.3) without the calculation of the land allocation decision rule.

Consider now the response (elasticity) of area with respect to an unexpected change in P_t^i, i.e., a shock in U_t^{Pi}. In order to do that I ignore the existence of other variables in the model and I use the example of the land allocation decision rule that is summarized by the bivariate autoregressive processes (3.7) and (3.8) . Consider the following 'experiment' for an unexpected change in P_t^i;[16]

$$(3.16)\quad \begin{cases} U_t^{Pi} = \sigma_p,\ U_s^{Pi} = 0 \text{ for all } s \neq t, \\ U_t^{ai} = 0 \text{ for all} \\ a_{t-1}^i = \bar{a}^i \text{ (mean area)} \\ P_{t-s}^i = \bar{P}^i \text{ for } s = 1,2 \text{ (mean price)} \end{cases}$$

As a result of this transitory (unexpected) change in price by the level of standard deviation in the innovation in prices, the crop area follows the sequence

$$(3.17)\quad \begin{cases} a_t^i = \bar{a}^i + \mu_1\sigma_P \\ a_{t+1}^i = \bar{a}^i + \lambda_1\mu_1\sigma_P + \mu_1\delta_1\sigma_P + \mu_2\sigma_P \\ \text{etc., } \ldots \end{cases}$$

where, if the system is stable $a_{t+s} \rightarrow \bar{a}$ as $s \rightarrow \infty$. Given the triangular form of the bivariate autoregressive equations (3.7) and (3.8), it is straightforward to show that the system is stable given that $|\lambda_1| < 1$ and the assumption above on the price process. **Observe that by using** (3.16) **one can easily compute** $\rho(s)$ **for** $s = 0, 1, 2$, **and that both the values of the cost function parameters,** (g, d, β), **and the parameters of the stochastic process of the price,** $(\delta_o, \delta_1, \delta_2)$, **play an important role in the determination of the response of land allocations to shock in prices. It is easy to use numerical values for the parameters to show cyclical movements of areas** in response to shock in prices. This result can be attributed either to cyclical movements in prices (δ's) or to a negative value of $\lambda_1 (d > 0)$. Only the estimation of the structure of the economy can reveal the source for a cobweb type phenomenon in a model where faremrs are rational profit maximizers.

The estimation of the elasticities with respect to an unexpected change in price can be calculated from an unrestricted (reduced form) specification and the estimation of the land allocation equation and the price processes. On the other hand, the estimation of elasticities with respect to expected changes require only the identification of the cost function parameters. A complete economic interpretation of the patterns of output responses to some changes in prices requires the identification of the entire structure of the model. Therefore, estimation strategies are not independent of the particular questions that the researcher seeks to answer (see section 6).

4. A Comparison with the Nerlovian Supply Response (NSR) Model

In this section I first present the Nerlovian model and show its properties. I argue that the basic supply equation of the NSR model can be justified by using problem (3.1), but the adaptive expectations formula cannot be justified. An example illustrates the qualitative differences between the models.

The literature on agricultural supply considers an annual crop that is planted in a period before the output price is realized. Output (x) in period t is assumed to be a linear function of the time t-1 expectations of the output price at period t(P_t^e). In discussing this assumption Nerlove (1958) stated that:

> "...a principal reason why low estimates of the elasticities of supply of corn, cotton, and wheat have previously been obtained is that insufficient attention has been devoted to the problem of identifying the price variable to which farmers react."

To the simple linear supply equation Nerlove added a partial adjustment equation that related desired (long run) and actual (short run) output. The supply equation of the Nerlovian Supply Response (NSR) model can be written as[17]

(4.1) $a_t = \gamma a_{t-1} + \alpha_0 + \alpha_1 P_t^e + \alpha_1 Z_t \qquad \alpha_0, \alpha_1, \alpha_2 > 0$

where $0 \leq 1 - \gamma \leq 1$ is the partial adjustment coefficient, and $\gamma = 0$ implies that 'desired' and actual production are the same. P_t^e is called "the expected 'normal' price" and the (adaptive) expectations formula is given by

(4.2) $P_t^e - P_{t-1}^e = \delta(P_{t-1} - P_{t-1}^e)$ $0 < \delta \leq 1$

or the "generalized adaptive expectations" equation[18]

(4.3) $P_t^e = \sum_{j=0}^{\infty} \delta_j P_{t-1-j}$, $\sum_{j=0}^{\infty} \delta_j = 1$

Finally, Z_t is a vector of some exogenous factors affecting supply at time t.

The main objective of the NSR model has been to estimate the short run and the long run price elasticity using equations (4.1) and (4.2). These elasticities were defined (Nerlove, 1958) as the immediate and the mean response, respectively, of area (or output) to a once-and-for-all change in the expected price, P_t^e. They are equivalent to the expected price elasticities that I have defined above. To understand Nerlove's remark on low supply elasticities, I consider the case of $\gamma = 0$ where the supply equation (4.1) equals

(4.4) $a_t = a_0\delta + (1-\delta)a_{t-1} + \alpha_1\delta P_{t-1} + \alpha_2 Z_t + \alpha_2(1-\delta)Z_{t-1}$

The reduced form coefficient of P_{t-1} is the product of the supply slope and the expectations coefficient which lies between zero and one. Observe that a permanent increase of one unit in P_t^e in the supply equation (4.1) ($\gamma \neq 0$) has an immediate effect of α_1 on a_t, while $\frac{\alpha_1}{1-\gamma}$ on the steady state level of a_t - the desired area. Hence, the early studies where $P_t^e = P_{t-1} (\delta=1)$ and $\gamma = 0$ had a significant bias toward low supply elasticities.

Here I claim that the farmer's dynamic optimization problem (3.1) can be viewed as a microeconomic justification for the ad-hoc supply equation (4.1) of the NSR model. I simplify the model of section 3 as follows: $f \equiv 1$, $V^i_{t+j} = S^i_{t+j} = F^i_{t+j} = 0$ for all j. Define

(4.5) $$P^e_t = \frac{-\lambda_1}{d} E_t\left\{\sum_{j=0}^{\infty} (\lambda_1\beta)^j P_{t+1+j}\right\}$$ and

(4.6) $$\alpha_2 Z_t = -\frac{\lambda_1}{d} E_t\left\{\sum_{j=0}^{\infty} (\lambda_1\beta)^j C_{t+j}\right\}$$

and ignoring the i superscript for the farmer i, equation (3.3) is just a somewhat more restrictive form of (4.1). Nerlove's (1979) view that P^e_t is a 'normal' future price is consistent with (4.5). Furthermore, γ should not be restricted to be positive, and the explicit optimization problem (3.1) imposes some additional constraints within the supply equation.

Given that problem (3.1) is accepted as a microeconomic justification for the supply equation (4.1), it should be emphasized that using the formula (4.2) or (4.3), and the definition (4.5) for P^e_t, the resulting land allocation decision rule is not the optimal rule that maximizes (3.1) given any information set, Ω_t. To prove this statement observe that for any Ω_t there exists, in general, a unique solution for problem (3.1), as it is described in section 3.1 (see also Hansen and Sargent, 1981). Here formula (4.2) may be the solution to the model only if the price process is consistent with

$P_t^e = E_t \{ \sum_{j=0}^{\infty} (\lambda_1 \beta)^j P_{t+1+j} \} = \frac{\delta}{1 - (1-\delta)L} P_{t-1}$. To see that (4.2) is not compatible with the model, I consider a counter example where $d \equiv \lambda \equiv 0$, so that $P_t^e = E_t(P_{t+1})$. Then, the price process is required to be $P_t = \frac{\delta}{1 - (1-\delta)L} P_{t-1} + U_t$ or $P_t - P_{t-1} = [1 - (1-\delta)L]U_t$ (U_t is a 'white noise') in order for (4.1) to hold. This price process is not stationary since P_t does not have a finite variance and, therefore, (4.2) is not consistent with a stationary model that is described here (see section 7 for the equilibrium solution). Muth (1961, p. 541) proved that formula (4.2) is compatible with rational expectations if the shocks to supply, in his model, follow a random walk process.

To illustrate the qualitative differences between models with rational vis-a-vis adaptive expectations, I consider the case in which P_t is serially uncorrelated, e.g., $\delta_1 = \delta_2 = 0$ in equation (3.7). Rational output decisions of farmers that observe this statistical property of prices imply that production does not respond to past movements in prices which contain no information on future prices. This is implied also by the rational expectations land allocation rule (3.8), while NSR equation (4.4), for example, stays <u>independent</u> of the actual price process. The independence between the adaptive expectations equation and the price process that is derived by the model or is given by the data, may lead to misleading estimates of supply elasticities and the predicted response of farmers to governmental policies (Lucas, 1976).

5. On the Observational Equivalence of Agricultural Supply Models

Sargent (1976) discussed the equivalence of the reduced form of models that differ only in their expectations specification and, hence, in their policy implications. One model (Keynesian) called for active monetary policy while the other (Classical) implied that any non stochastic money rule is optimal. Here I show the equivalence between the NSR model with adaptive expectations, the cobweb model where $P_t^e = P_{t-1}$ and a simple rational expectations model. First, I define the meaning I use for the observational equivalence of different models. If a single specification of each model is chosen independently from the other model, there is no reason to expect that the two models would be equivalent. The possible specifications of each model are so large that a 'random' comparison would yield, almost surely, no similarity between the models. The number of variables, lags, definition of prices, period of estimation (quarter, year, etc.), all or some may differ substantially. Therefore, the definition of observational equivalence should be a conditional statement.

Definition: Two models (A and B) are said to be observationally equivalent if for a given specification of model A there exists a specification of model B such that the reduced forms of both models are identical. The two models are strictly observationally equivalent if both models are just or under identified.

This definition implies that even if A is the true model, model B can fit the data equally well. Even if both models are over identified, it is not necessary that the wrong model will be rejected by the data. The equivalence between the models is demonstrated here by examples.

Consider the example of the rational expectations decision rule where P_t is given by equation (3.7) and where the superscript i is omitted. The solution for (3.3) is given by

$$(5.1) \qquad a_t = \frac{\lambda\delta_0}{d(1-\lambda_1\beta)} + \lambda_1 a_{t-1} - \frac{\lambda_1\delta_1}{d(1-\lambda_1\beta\delta_1)} P_{t-1} + U_t^{ai}$$

Consider the NSR model where $\gamma = a_2 = 0$ and equation (4.2) for expectations. The area equation of the NSR model is

$$(5.2) \qquad a_t = \alpha_o\delta + (1-\delta)\, a_{t-1} + \alpha_1\delta P_{t-1} + \varepsilon_t$$

where ε_t is an additive 'white noise' error. Finally, consider equation (4.1) where $\alpha_2 = 0$ and the expectations are given by the naive assumption that $P_t^e = P_{t-1}$ (cobweb model). The land equation is given by

$$(5.2) \quad a_t = \alpha_0 + \gamma a_{t-1} + \alpha_1 P_{t+1} + \varepsilon_t$$

Given the standard assumption that U_t^{ai} and ε_t are i.i.d., for some values of $\{\lambda_1, d, \beta, \delta_0, \delta_1\}$ in (3.7) and (5.1), there exist $\{\delta,\alpha_0,\alpha\}$ for equation (5.2) as well as $\{\alpha_0,\alpha_1,\gamma\}$ for equation (5.3) where the three equations, (5.1)-(5.3), are identical. Estimating (5.1) jointly with (3.7) implies that the rational expectations model is just identified. The NSR model (5.2) and the cobweb model (5.3) are just identified as well. Since the rational expectations model is the only framework here that fully characterizes simultaneously the laws of motion for the area and the price it is natural to use a specification of that model as the benchmark for comparison. The above example shows the <u>strict observational equivalence</u> between a model where farmers have full information on the price stochastic process vis-a-vis models that farmers' expectations are independent from the actual price process. Data which have been generated from the rational expectations model could fit well a NSR model with adaptive expectations and the cobweb model.

The above results imply that econometric methods using data sets that have been assumed to come from a single structure would not be able to reject one of the above models in favor of another model. However, particular over identified specifications of each model can be tested against one maintained alternative and non nested tests can be used to test the models against each other. These would be tests of particular specifications, while the general observational equivalence among the models could hold if I use the flexible form of the adaptive expectations formula (4.3).

On the other hand, the three models differ in their predictions on the implications of an alteration in the price process. Since we analyze these models primarily because we are interested in the affect of a permanent change in prices (supply elasticities) the choice of the model is of crucial importance (see section 4 above).

6. Estimation

The main objective of the agricultural supply literature and of this paper is to develop an acceptable methodology for estimating supply elasticities and for interpreting serial and cross-correlations of outputs, yields and prices. Here I wish to distinguish between three different research objectives which yield different estimation strategies;

(i) The most general goal is to estimate all the models' parameters subject to most general goal is to estimate all the models' parameters subject to all the models' restrictions in order to test the models' interpretation of the data. This objective requires a simultaneous non-linear estimation methods. Examples of full information maximum likelihood (FIML) that use a single time series data set exist in Sargent (1978), Eckstein (1981) and Eichenbaum (1981). Hansen (1982) and Hansen and Sargent (1982) developed a nonlinear instrumental variables (NLIV) method to achieve this goal. The particular choice of the method for estimation is determined by the particular model of the error term. Naturally the complete identification of the model's parameters provides estimators for the supply elasticities and a test for the overidentifying restrictions of the model.

(ii) The main objective of the NSR model is to estimate the supply elasticities with respect to an <u>expected</u> change in prices. In the model here this objective requires the identification of the parameters of the cost function and it is not necessary to identify the entire decision rule. Kennan (1979) showed how the cost function parameters can be estimated directly from equation (3.2).[19] In order to develop this method for the agricultural supply model of section 3, I use the properties of the conditional expectations operator[20] to rewrite equation (3.2) as

$$(6.1) \quad a_{t+1} + \frac{g}{\beta d} a_t + \frac{1}{\beta} a_{t-1} - \frac{f}{d}(P_{t+1} + S_{t+1}) + \frac{1}{d}(c_t + F_{t+1}) = \phi_{t+1}$$

where $E_t \phi_{t+1} = 0$. Hence, if Z_{jt} is a variable that belongs to the farmer's information set Ω_t, I get the orthogonality condition

$$(6.2) \quad E[\phi_{t+1} \cdot Z_{it}\} = 0 \text{ for all } Z_{jt} \varepsilon \Omega_t$$

The orthogonality condition (6.2) provides us instruments in order to estimate equation (6.1). The need for instruments arises since P_{t+1} and F_{t+1} do not belong to the time t information set and therefore (6.1) is not a regression equation ($E(\phi_{t+1} | F_{t+1}, P_{t+1}) \neq 0$). Using $[a_t, a_{t-1}, P_t + S_t, c_{t-1} + F_t]$ as a vector of four instruments for the four regression coefficients in (6.1), the standard instrumental variable regression method yields consistent and unique estimators of the parameters g, d, β and f. Hansen (1982) proved the consistency and he provided a method for an efficient estimator using general methods of moments (GMM). Using the time average of prices and area as estimators for their means the long run and short run supply elasticities can be estimated in a fairly simple way.

(iii) The elasticities with respect to an unexpected change in prices can be estimated by an unrestricted reduced form of the model. In general, these linear quadratic models give rise to a restricted vector ARMA specification (see the example in section 3). Assuming that the model is not rejected by the data, the unrestricted estimated specification is "close" to the true model and one can use this specification to analyze the response of area to one standard deviation shock in prices. This response is equivalent to tracing out the moving averages of an estimated simultaneous dynamic system. Sims (1980) recommended using this method to interpret simultaneous dynamic models when a vector autoregression (VAR) is estimated and Sargent (1978) compared the estimated moving averages from an unrestricted model to the estimated moving averages of the restricted model.

Each of the agricultural supply models could be written as a vector ARMA that is subject to restrictions across and within equations and is not linear in the underlying parameters. The restrictions on the model are the main source for identification of the structural parameters. Hansen (1982), Hansen and Sargent (1980, 1982), Hyashi and Sims (1983), Wallis (1980) and Wilson (1973) discuss methods for estimating ARMA models from a single time series data set.

The existence of panel data from farm surveys brings new hopes to the task of estimating supply elasticities. Methods for estimating ARMA models are discussed by MaCurdy (1983). The model in section 3 is written in such a way that it is straightforward to estimate the cost function parameters using panel data. Most studies on the agricultural supply used aggregate data for the estimation of supply elasticities. Using aggregate data one should carefully consider the market interaction issues which may affect the permissible way of specifying the price process for estimating the farmer land allocation decision rule. The next section provides a framework for these considerations as well as an insight into the possible implications of the market equilibrium on the dynamics of supply.

7. The Market Equilibrium and the Dynamics of Supply

What is the effect of the market on the dynamics of supply? Sargent (1981) argues that all the movements in demand and inventory behavior affect the production decision rule through the producer's expectations of the future movements of prices. It is well known that speculative inventories induce a positive serial correlation in prices (Muth, 1961), which makes the output to be positively serially correlated as well. Here I abstract from the dynamic effect of inventory speculation and focus the discussion on the effect of the dynamics in the production process on the equilibrium movements of prices and output. A particular attention would be given to the 'rationality' of price and output oscillation and the 'cobweb theorem'.

The market equilibrium is defined as a stochastic process for $\{a^i_{t-1}, D_t = \sum_{i=1}^{N} x^i_t, P_t\}_{t=1}^{\infty}$ which satisfies the necessary conditions for the maximum problem of the farmer (3.1), the demand euqation (2.3), the production function (2.1) given a^i_{-1} and the given stochastic processes of S^i_t, c^i_t, F^i_t, e_t, V^i_t and Y_t.

I simplify the algebra, without loss of generality, by assuming that c_t follows the process (3.6), $F^i_t = 0$ and $S^i_t = S^i$. Furthermore, I assume that all farmers have the same information so that $\Omega^i_t = \Omega_t$. Summing both sides of (3.2) over all i gives

$$(4.1)\quad d[A_{t-1} + \frac{g}{d}A_t + \beta E_t A_{t+1}] = NfE_t P_{t+1} + E_t P_{t+1} \sum_i V^i_{t+1}$$

$$+ fS + S \sum_i V^i_{t+1} - \overline{C} + \sum_i U^{ci}_t$$

where $A_t = \sum_i a^i_t$, $S = \sum_i S^i$, $\overline{C} = \sum_i \overline{C}^i$ and I assume that $\sum_i V^i_t \doteq 0$

and $\sum_i U_t^{ci} = 0$, i.e., the farmer's specific shock disappears in the aggregation. Summing up over the production function (2.1) over all i, gives $\sum_i x_{t+1}^i = fA_t + e_{t+1}$, since it assumed that $\sum_i V_{t+1}^i a_t^i = 0$. In equilibrium, where demand is equal to supply, it is true that

(7.2) $D_{t+1} = fA_t + e_{t+1}$.

Substituting (7.2) into the demand equation (2.3) and the result for P_{t+1} into (7.1) I get

$$(7.3)\quad A_{t-1} + \frac{g + f^2 b_1 N}{d} A_t + \beta E_t A_{t+1} = \frac{1}{d}[N\, fb_0 - Nf\, b_1 E_t\, e_{t+1} + Nf\, b_2 E_t\, Y_{t+1} + fS - \overline{C}]$$

Now the equilibrium is a solution for (7.3) that satisfies the transversality condition of the farmer's problem (3.1). The unique solution can be found equivalently to the way (3.2) is solved. First, factorize(7.3) to get

$$(7.4)\quad (1 - \tilde{\lambda}_1 L)A_t = -\frac{\tilde{\lambda}_1}{d} E_t \sum_{j=0}^{\infty} \{(\tilde{\lambda}_1 \beta)^j [Nf\, b_0 + fS - \overline{C} - Nfb_1 e_{t+1} + Nfb_2 Y_{t+1}]\}$$

where $\tilde{\lambda}_1$ is the smaller root, in absolute value, that solves

$$\frac{1}{\tilde{\lambda}_1} = -\frac{\tilde{g}}{d} - \beta\tilde{\lambda}_1 \tag{7.5}$$

where $\tilde{g} = g + Nf^2b_1$, $|\tilde{\lambda}_1| < 1$ and $\tilde{\lambda}_1 \lessgtr 0$ if $d \gtrless 0$. Observe that $\tilde{\lambda}_1 \to 0$ as $N \to \infty$ and if $b_1 = 0$ then $\tilde{\lambda}_1 = \lambda_1$ i.e., if the demand is perfectly elastic the solution of the equilibrium is identical with the solution of the supply equation in section 3. Now I can solve the mathematical expectations (7.4) using the prediction formula (see footnote 10) to get the equilibrium laws of motion of the aggregate land allocations in the market, i.e.,

$$A_t = \tilde{\lambda}_1 A_{t-1} + \tilde{\mu}_0 + \tilde{\mu}_1 Y_t + \tilde{\mu}_2 Y_{t-1} + \mu_3 e_t \tag{7.6}$$

where

$$(7.7)\begin{cases} \tilde{\mu}_0 = -\dfrac{\tilde{\lambda}_1}{d}\,\dfrac{Nf\ b_0 + fs - \overline{C}}{1 - \tilde{\lambda}_1\beta} \\ \tilde{\lambda} = \tilde{\lambda}_1\beta \\ \tilde{\mu}_1 = -\dfrac{\tilde{\lambda}_1}{d}\,Nfb_2\,\dfrac{(\alpha_1 + \alpha_2\tilde{\lambda})}{(1 - \alpha_1\tilde{\lambda} - \alpha_2\tilde{\lambda}^2)} \\ \tilde{\mu}_2 = -\dfrac{\tilde{\lambda}_1}{d}Nfb_2\,\dfrac{\alpha_2}{1 - \alpha_1\tilde{\lambda} - \alpha_2\tilde{\lambda}^2} \\ \tilde{\mu}_3 = -\dfrac{\tilde{\lambda}_1}{d}\,Nfb_1\,\dfrac{\delta_e}{1 - \delta_e\tilde{\lambda}} \end{cases}$$

The equilibrium law of motion for aggregate consumption is computed by using (2.1) and $D_{t+1} = \sum_i x^i_{t+1}$, hence

(7.8) $$D_{t+1} = \tilde{\lambda}_1 D_t + f\tilde{\mu}_0 + f\,\tilde{\mu}_1 Y_t + f\tilde{\mu}_2 Y_{t-1} + (f\tilde{\mu}_3 - \tilde{\lambda}_1)\, e_t + e_{t+1}\,.$$

Substituting (7.8) into the demand equation, the equilibrium law of motion of prices is:

(7.9) $$P_{t+1} = \tilde{\lambda}_1 P_t + b_0(1 - \tilde{\lambda}_1) - b_1 f\tilde{\mu}_0 + b_2 Y_{t+1} + (-b_1 f\tilde{\mu}_1 - \tilde{\lambda}_1 b_2) Y_t$$

$$+ (-b_1 f\,\tilde{\mu}_2) Y_{t-1} - b_1(f\tilde{\mu}_3 - \tilde{\lambda}_1) e_t - b_1 e_{t+1}$$

Consider a simple case of the above equilibrium where $b_2 \equiv 0$, so that by using (2.2) the price process can be written as

(7.10) $$P_t = \delta_0 + \delta_1 P_{t-1} + \delta_2 P_{t-2} + U^P_t$$

where $$\delta_0 = (1 - \delta_e)[b_0(1 - \tilde{\lambda}_1) - b_1 f\,\hat{\mu}_0]$$

$$\delta_1 = \tilde{\lambda}_1 + \delta_e$$

$$\delta_2 = -\tilde{\lambda}_1\,\delta_e$$

$$U^P_t = -b_1[(f\,\tilde{\mu}_3 - \tilde{\lambda}_1)L + 1]\,U^e_t$$

Given that $P_t^i = P_t + S^i$ for farmer i the only difference between (3.7) and (7.10) is that U_t^P is not i.i.d. and has a first order moving averages representation. Yet $\frac{1}{\delta_0}[1 - \delta_1 z - \delta_2 z^2]$ has roots outside the unit circle and P_t has a Wold moving average representation in terms of the innovations U_t^e's. However, (7.10) does not necessarily imply that P_t has an autoregressive representation, since the root of U_t^P process is not necessarily greater than one.[21] Given the (7.10) process for P_t, it is straightforward to use the prediction formula (footnote 10) in order to solve the land allocation decision rule. Hence, Nerlove et al.'s (1979a) estimation method of "quasi-rational expectations" is consistent with the rational expectations equilibrium, but their method ignores some of the model's restrictions, therefore, some statistical efficiency is "lost".

The Cobweb Cycles

As observed above the sign of $\tilde{\lambda}_1$ depends on the sign of the dynamic element in the cost function. If current marginal costs are higher (lower) due to last year increase in production, then $\tilde{\lambda}_1$ is negative (positive). The model predicts that the path of A_t, D_t and P_t, from an arbitrary initial allocation toward the steady state, are all characterized by the same dynamic properties. In particular, the dynamic effect in costs determines whether prices and quantities follow a cobweb oscillator path ($\tilde{\lambda}_1 < 0$) or a smooth gradual adjustment costs style path ($\tilde{\lambda}_1 > 0$). The above rational expectation equilibrium model can therefore exhibit the two period cyclical aspects of the cobweb model. The equilibrium and the aggregation over farmers do not necessarily elimiante from the price process the effect of the dynamics in the production function.

Discussion:

Observe that even a simple equilibrium model, without any income effect on demand ($b_2 \equiv 0$), provides an interesting dynamic structure for the price process. Furthermore, the serial correlation in prices is not solely determined by the dynamics in supply but also by the dynamics of related variables. Least squares estimates of δ_0, δ_1 and δ_2 in equation (7.10) are consistent and can provide some insight into the dynamics of the price and the land allocations processes, through the identification of $\tilde{\lambda}_1$. If δ_1 is positive and δ_2 negative, it is evident that $\tilde{\lambda}_I > 0$ and therefore the supply seems to be subject to adjustment costs ($d < 0$) in production. Is it necessarily true that in this case there is not a significant effect for the land fertility deterioration? The answer is no, since some alternative explanations are possible. One case is given in my (1981) paper where it is shown that if an input such as fertilizers is omitted, then the sign of the root ($\tilde{\lambda}$) may be reversed even if d is positive. If income (Y) and the shocks to production (e) are sources of disturbances to the market, then one can easily generate long as well as short cycles in output (consumption) and prices using numerical values for the underlying parameters of the model. Taking into consideration the price of the alternative crop (which is in F),it is possible to imitate the alleged cross correlations between prices of different crops. The corn-hog cycle is a natural candidate for this analysis. The argument here is that the regular cycles in the corn-hog industries and the cattle industries could be explained by the dynamic aspects of the production process.

These industries require inventory of the output for the reproductive activity and, therefore, the production process includes enough dynamic elements that can explain the cyclical movements of these industries. Nerlove et al. (1979a) analyzed some aspects of these issues within a quadratic producer behavior model. However, they did not identify the effect of the equilibrium on the joint dynamic movements of all the variables.

8. Conclusion

The linear rational expectations models which have been recently developed in the macroeconomic literature proved here to provide a useful methodology for analyzing and estimating agricultural supply elasticities as well as for interpreting the cyclical behavior of agricultural markets. A simple model provides a supply equation that is conceptually consistent with the traditional basic agricultural supply equation. I argue against using adaptive expectations methods, but I show that empirically the different expectations models give rise to observationally equivalent models. Although the very simple model here gives rise to complicated non-linear restrictions on the land allocation decision rule, I provide a simple estimation method for the supply elasticities with respect to expected changes in prices. Finally, I show that this model provides a simple rationale for cobweb cycles that previously have been alleged to be explained by farmers' stupidity.

Given the available econometric methods the estimable dynamic models and the consistent definitions of supply elasticities, the real objective is to explore the available aggregate and, in particular, farm level data, to provide evidence and to shed more light on the actual facts of the dynamics of agricultural supply.

Footnotes

[1]The above list of papers is a small selection of important articles in this line of research. Other significant contributions could be found in the reference list of those papers I do mention.

[2] The cobweb model was designed originally by Ezekiel (1938) to explain the so-called stylized facts on agricultural markets, e.g., the corn-hog cycle. Muth (1961) in his classical paper on rational expectations discussed the 'Rationality and Cobweb Theorems'. He cited two objections to the cobweb model: (i) the model assumes that farmers do not learn from experience; (ii) the observed hog cycles in the 30's were too long in order to be explained by the cobweb theorem (Coase and Fowler, 1935). Muth was concerned with the cobweb model since it introduced a negative characteristic root into the moving average of quantities and, therefore, was considered as a successful explanation of cycles, while his equilibrium model with inventories had a positive root. Muth correctly claimed that by consideration of serial correlation in the shocks to supply his model could account equally well for the observed phenomenon.

[3]In Nerlove (1958) and throughout the agricultural supply response literature a production function such as (2.1) (without shocks) has been assumed implicitly or explicitly.

[4]Obvious candidates are weather variables such as rain, wind, flood, etc. Here the production relation (2.1) implies that output decisions are done entirely during planting time. However, there are large differences in output due to differences between cultivated and harvested area. The shocks here may capture some of this element but not in a fully satisfactory way.

[5]Additional discussion of these issues exist in Eckstein (1981). One can easily specify an explicit production function that yields a cost function like (2.3), given that some other input prices do not change.

[6]Inventories play an important role in the determination of the dynamics of prices, output and consumption. However, in this paper I focus on the dynamics that emerge from production and,therefore, I abstract from the role of inventories. Muth (1961), Aiyagari et al. (1980), Eichenbaum (1981), and Wright and Williams (1982) analyze the dynamic effects of inventory in models that are closely associated with the model that I present here.

[7]See Sargent (1979) and Hansen and Sargent (1981) for detailed proof and description of this result.

[8]See Sargent(1979) for discussion and definition of the concept of certainty equivalence. As of now there does not exist a close form solution for nonlinear dynamic models.

[9]For a study that emphasizes the correlations between the shock and the price, see Wright and Williams (1982). These and other studies that calculate producers' and consumers' surpluses show that quantitatively a shock to the slope of the supply equation may have a significant difference in the relative gains of producers and consumers from stabilization programs vis-a-vis an additive shock to supply.

[10]Formally problem (3.1) is not well defined unless the distribution on the uncontrollable stochastic processes is specified.

[11]Let the autoregressive representation of Z be $A(L)Z_t = \varepsilon_t$ where ε_t is (1 x n) vector white noise, $A(L) = I - A_1L - A_2L .. - A_rL^r$ then Hansen and Sargent (1980) extension of Wiener-Kolmogorove prediction formula is

$$\sum_{j=0}^{\infty} \lambda^j E_t(Z_{t+j}) = \frac{L^{-1}[I-A(\lambda\beta)^{-1}A(L)]}{1-\lambda\beta L^{-1}} = A(\lambda\beta)^{-1} \{\sum_{j=0}^{r-1} \sum_{k=j+1}^{r} (\lambda\beta)^{k-j-1} A_k]L^j\}$$

where $|\lambda| < 1$ **and the moving averages of** Z_t **is given by** $Z_t = A(L)^{-1}\epsilon_t$.

[12]Note that not every moving averages has an autoregressive representation. Only fundamental processes have an invertible moving averages representation. The class of exact finite order autoregressive processes is even smaller. In section 7 I show an example in which the equilibrium law of motion of prices can be written as a univariate ARMA (2,1) which may not be fundamental. Nerlove at al. (1979a) introduced the term "quasi-rational expectations"for the method for solving the decision rule by using a univariate ARMA process for prices which is not computed from the equilibrium.

[13]**To verify (3.5). one should use footnote** 11 **and specify the vector** ARMA **process for** $Z_t' = [P_t, S_t^i, C_t^i, F_t^i, I_t^i]$.

[14]Observe that using Hansen and Sargent's formula, I get

$$E_t[\sum_{j=0}^{\infty} \lambda^j P_{t+1+j}] = \frac{1}{\lambda}\{E_t[\sum_{j=0}^{\infty} \lambda^j P_{t+j}] - P_t\}$$

$$= \frac{1}{\lambda}\{\frac{P_t}{(1-\delta_1\lambda-\delta_2\lambda^2)} + \frac{\lambda\delta_2 P_{t-1}}{1-\delta_1\lambda-\delta_2\lambda^2} - P_t\}, \text{ where } \lambda = \lambda_1\beta .$$

[15]This result can be interpreted as crop rotation and may give rise to a cobweb cycle in production (see section 7).

[16]This assignment is equivalent to tracing out the moving averages of the bivariate stochastic process (3.8) and (3.9).

[17]The exposition here follows Nerlove (1958, 1979). I omit the explicit presentation of the adjustment equation and the error term in order to simplify the discussion.

[18]Ezekiel's (1938) cobweb model is equivalent to the case where $\delta = 1$ and $\gamma = 0$. Nerlove (1958) showed how one can use his model to test the 'naive' expectations hypothesis where $\delta = 1$ in equation (4.2).

[19]Hansen and Sargent (1982) discussed Kennan's (1979) method versus their NLIV method. Both papers were concerned with estimating Euler equations that come from a linear-quadratic optimization problem.

[20]If $\{Z_t\}_{t=0}^{\infty}$ is a stochastic process with a finite mean and if $E_t Z_{t+1} = 0$ then

(i) $Z_{t+1} = \phi_{t+1}$

(ii) $E_t \phi_{t+1} = 0$

(iii) $E(Z_{t+1} \phi_{t+1}) = 0$

This is a standard result in statistics that is used here.

Footnotes to section 7

[21]P_t is an ARMA (2.1) process. It does not have an autoregressive of representation if

$$z = \left| - \frac{1}{f\tilde{\mu}_3 - \tilde{\lambda}_1} \right| < 1$$

This is a possible outcome of the model that should be seriously considered in estimating land allocation decision rules or price processes. Standard estimation methods of ARMA models (Box and Jenkins) and VAR's (Sims) require the existence of an autoregressive representation for the estimated process.

REFERENCES

Aiyagari, S., R. Eckstein, Z., and M. Eichenbaum, "Rational Expectations, Inventories, and Price Fluctuations," Economic Growth Center, Yale University, Discussion Paper No. 363, (1980).

Askari, Hosseni and John T. Cummings, Agricultural Supply Response: A Survey of the Econometric Evidence (New York: Praeger Publishers, 1976).

Behrman, Jere R., Supply Response in Underdeveloped Agriculture: A Case Study of Four Major Annual Crops in Thailand, 1937-1963 (Amsterdam: North-Holland, 1968).

Brennen, J. M., "The Supply of Storage," American Economic Review, (1958), 49-71.

Coase, R. H. and R. F. Fowler, "The Pig-Cycle in Great Britain: An Explanation," Econometrica, 2 (1935), pp. 143-167.

Eckstein, Zvi, "Rational Expectations Modeling of Agricultural Supply," Economic Growth Center, Yale University, Discussion Paper No. 381, (1981).

Eichenbaum, Martin, "A Rational Expectations Equilibrium Model of the Cyclical Behavior of Inventories and Employment," unpublished Ph.D. Dissertation, University of Minnesota (1981).

Ezekiel, M., "The Cobweb Theorem," Quarterly Journal of Economics, 52 (February 1938), pp. 255-280.

Gould, J.P., "Adjustment Costs in the Theory of Investment of the Firm", Review of Economic Studies, 35 (1968): pp. 47-55.

Hansen, Lars P., "Large Sample Properties of Generalized Method of Moments Estimators," Econometrica, 50 (1982): 1029-55.

______________, and T.J. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models," Journal of Economic Dynamics and Control, 2:1 (February 1980), pp. 7-46.

______________, "Linear Rational Expectations Models for Dynamically Interrelated Variables," in R.E. Lucas Jr. and T.J. Sargent (eds.), Rational Expectations and Econometric Practice (Minneapolis: University of Minnesota Press, 1981).

______________, and Thomas J. Sargent , "Instrumental Variables Procedures for Estimating Linear Rational Expectations Models", Journal of Monetary Economics, 9 (1982), 263-296.

Hayashi, Fumio and Christopher A. Sims, "Nearly Efficient Estimation of Time Series Models with Predetermined, but not Exogenous, Instruments." Econometrica (1983) forthcoming.

Heady, Earl O., and D.R. Kaldor , "Expectations and Errors in Forecasting Agricultural Prices", Journal of Political Economy, Vol. 62 No. 1 (1954), 34-47.

Kennan, John, "The Estimation of Partial Adjustment Models with Rational Expectations," Econometrica, 47, (November 1979).

Kydland, F. E. and E. C. Prescott, "Rules Rather Than Discretion: The Inconsistency of Optimal Plans," Journal of Political Economy, 85, (1977).

Lucas, Robert E., Jr., "Economic Policy Evaluation: A Critique," in K. Brunner and A. H. Meltzer (eds.), The Phillips Curve and Labor Markets, Carnegie-Rochester Conference Series on Public Policy (Amsterdam: North-Holland, 1976).

____________________, "Adjustment Costs and the Theory of Supply", Journal of Political Economy, 75 (1967): pp. 321-334.

MaCurdy, Thomas G., "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," Journal of Econometrics, (1983, forthcoming).

Muth, J. F., "Rational Expectations and the Theory of Price Movements," Econometrica, 29:3, (1961), pp. 315-335.

Nerlove, Marc, "Estimates of the Elasticities of Supply of Selected Agricultural Commodities," Journal of Farm Economics (May 1956).

____________, The Dynamics of Supply: Estimation of Farmers' Response to Price (Baltimore: Johns Hopkins University Press, 1958).

____________, "Lags in Economic Behavior" Econometrica, 40:2, (March 1972), pp. 221-251.

____________, "The Dynamics of Supply: Retrospect and Prospect," American Journal of Agricultural Economics, 61:5 (December 1979), pp. 874-888.

____________, D. M. Grether and J. L. Carvalho, Analysis of Economic Time Series: A Synthesis (New York: Academic Press, 1979a).

Sargent, Thomas J., Macroeconomic Theory (New York: Academic Press, 1979).

____________________, "Interpreting Economic Time Series," Journal of Political Economy, 89 (1981): pp. 213-248.

____________________, "Estimation of Dynamic Labor Demand Schedules under Rational Expectations," Journal of Political Economy, 86 (1978), pp. 1009-1044.

________________, "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics, Journal of Political Economy, 84, 3 (1976): pp. 631-639.

Schultz, T.W. and O.H. Brownlee, "Two Trails to Determine Expectation Models Applicable to Agriculture", Quarterly Journal of Economics, Vol. 56 (1941-2), 487-96.

Sims. Christopher A., "Macroeconomics and Reality," Econometrica, 48:1 (1980), pp. 1-48.

Wallis, Kenneth, F., "Econometric Implications of The Rational Expectations Hypothesis," Econometrica, 48, 1 (January 1980): pp. 49-73.

Wilson, G. T., "The Estimation of Parameters in Multivariate Time Series Models", Journal of Royal Statistics Society, Series B, No. 1, (1973).

Wright, B. D. and Williams, J. C., "The Economic Role of Commodity Storage", The Economic Journal, (1982), 595-614.