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DISCRETE CHOICES IN A CONTINUOUS TIME MODEL:

LIFECYCLE TIME ALLOCATION AND FERTILITY DECISIONS

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Abstract:

This paper, a summary of my dissertation, describes an application of the analysis of discrete choices in continuous time models. A lifecycle time allocation model is integrated with fertility decisions, in particular the timing of children over the lifecycle. A derived empirical model is estimated with longitudinal data, and indicates support for the theory offered.

1. INTRODUCTION

During the last two decades it has gradually been recognized that labor supply decisions are made within a dynamic framework. While traditional one-period models assume the absence of savings and, by their very nature, abstract from human capital accumulation, a dynamic model implies that labor supply at any moment in time is a function of current and future wages, wealth, rates of discount and of time preference, and other variables (current and future) as dictated by the problem under consideration.

One of the variables playing an important role in the lifecycle labor supply of married women is the demand for their time within the family. This is evident in the U-shaped pattern of female labor force participation rates across age groups. Starting with Mincer (21,1962) and Becker (2,1965) the issue of the demand for home time has extensively been examined within the static framework.[1/] In these studies the number of children, especially the younger ones, in the family often affected labor supply in a negative way.

This paper proposes and tests a dynamic model that contains time allocation decisions and fertility choices. The latter consist of timing of children and completed family size. Section 2 argues the need to integrate both types of choices in order to arrive at a meaningful empirical analysis, and shows that, with exceptions, research on female labor supply has largely ignored the progress made in studies on fertility. This paper attempts to fill, at least partially, this gap.

Sections 2 and 3 describe the basic structure of the model and its implications. Fertility choices are modeled as a series of 0-1 choices in continuous time. This type of analysis could be applied to a wide variety of problems in consumer theory (decisions on when to replace a car or any consumer durable, or when to replace the tires of a car; decision on when to enter a life insurance arrangement), in labor economics (decision on when to change jobs, if jobs are heterogeneous in training offered), and in production theory (decision of car industry on when to change car models; decisions to update technology used). All these examples describe choices made at a certain point in time within a continuous-time problem, but other dynamic programming problems with 0-1 choices could be analyzed. An example would be the degree of vertical integration by firms in their production range, or by countries in their range of specialization in production of intermediate inputs with varying capital - labor ratios (see Dixit and Grossman (8,1981)).

In section 5 we derive a set of estimation equations by specifying two of the functions of the theoretical model. To test a dynamic model one preferably uses a longitudinal data set. The estimation results, obtained by the maximum likelihood method from the Panel Study of Income Dynamics, are reported in section 6, and indicate support for the model as well as the need for further research. Section 7 contains concluding remarks.

2. THE INTERACTION OF FEMALE LABOR SUPPLY AND FERTILITY DECISIONS

A successful analysis of lifecycle labor supply of females has to be more than an extension of the three-tier approach of leisure - home time - labor supply utilized in static models. To illustrate this we ask two questions. First of all, is the demand for the wife's time in the family exogenous? Within a static framework, Mincer (22,1963) showed that the wife's wage is an important price variable in the demand for children. Willis (43,1973) and others argued, in the spirit of Becker's time allocation model (2,1965), that since time is an input in the "production" of children, time allocation choices naturally affect fertility decisions. The demand for hometime is determined, to a significant extent, by the number of children and thus becomes an endogenous variable. Therefore, using the number of children in a labor supply equation implies biased parameter estimates. In the context of static models, two solutions are offered: omit variables indicating the presence and age of children from the labor supply equation in order to avoid simultaneity bias,[2/] or estimate a multi-equation model that incorporates the mutual effects between demand for children and labor supply.[3/]

Table 1 gives empirical evidence of the interaction between these two variables. It compares the number of children born per 1000 women for different cohorts in 1970, differentiated according to employment status. Women in the labor force at a particular time tend to have fewer children. Also, table 1 shows an interesting pattern in that between older age groups the difference is markedly smaller but does not disappear.

The second question we ask is whether, given the number of children, there is systematic variation over the lifecycle in the demand for the wife's hometime, and perhaps leisure.[4/] Typical static labor supply estimates show that older children affect the wife's labor supply less than younger children. So the demand for home time also depends on the children's age structure, and therefore on their birth intervals. Table 2 shows how birth intervals changed since the Second World War. Birth intervals shortened prior to the sixties, especially the intervals before the first and second births, but during the last two decades they lengthened significantly. The interval from the time of marriage to first birth almost doubled. In addition, there is evidence, not presented here, that these trends are similar across income groups,[5/] and that higher income families tend to have larger birth intervals.[6/] Unfortunately, the interaction of birth intervals and work history (or work situation) is not documented by tables in the public realm, but clearly the timing of children over the parents' lifecycle is subject to change over time. Economic variables that systematically affect timing patterns have an indirect impact on labor supply decisions.

Current analyses of the dynamics of female labor supply generally do not explore the home sector in great depth. Frequently the demand for home time is assumed exogenous (viz. Smith (36,1977 and 37,1977), Hill (15,1977), and Heckman and MaCurdy (14,1980)), but as we have seen, this assumption must be relaxed. The studies of Hotz (17,1980) and Moffitt (24,1980 and 25,1980) offer a good starting point. They make the number of children in the family at every point in the lifecycle an endogenous variable, though one of a real type (i.e. non-integer); and they assume that child care time is related to the number and age structure of the children, but is not related to other economic variables. The studies show the difficulty of putting to test a dynamic model that represents only little more than the simplest possible ideas.[7/]

Rosenzweig and Wolpin (33,1980) examine a dynamic model that analyzes labor supply, fertility decisions and human capital accumulation simultaneously. They do not model the home sector explicitly. Rather they focus on the interpretation of the estimated effect of fertility on labor supply under alternative assumptions on the (interaction of) components of the underlying model. From their twins-first methodology,[8/] applied to cross-sectional data, they conclude that purely exogenous fertility variations have stronger intertemporal substitution effects on female labor supply than what estimates using actual (endogenous) fertility measures suggest.[9/] In other words, the labor supply - fertility interaction is an important empirical issue.

Within the realm of economic theories of fertility the issue of child spacing has been largely ignored. A notable exception is Razin (32,1978) who, under rather strong assumptions, reduces this inherently dynamic problem to a static one, in which spacing, completed family size and labor force participation are decision variables. In order to generate a typical U-shaped lifecycle labor supply profile, he assumes that "child quality" is produced with the wife's time. It is not straightforward how this production process translates into a dynamic process; the empirical study based on this model by Nerlove and Razin (28,1979), although confirming to some degree its predictions, does not appear to solve this problem.[10/]

The model proposed in this paper attempts to fill the need for a dynamic model of labor supply and child spacing with endogenous home sector. Issues of human capital accumulation and child quality production can be built into it, although at cost of considerable complexity. Completed family size choices can be examined intuitively, but not analytically. The model is tested with longitudinal data. As such the empirical investigation has a scope that was lacking in previous research.

3. THE MODEL

3.1 The Basic Assumptions

In this section we will set out a lifecycle model of a household making decisions on the allocation of the wife's time, and the spacing of children.

The basic ideas behind the model are as follows. At the beginning of the planning period the family maximizes a lifecycle utility function, which contains as its arguments the consumption goods of the family, leisure of the wife, and number of children in the family at any time during the planning period. The time horizon of the family is assumed to be the death of either parent, and is known with certainty.[11/] The planning period of our model starts at the time of marriage of the parents.

The restrictions on the maximization process are three: a budget relation, which by permitting (dis-)savings to take place is dynamic in nature; a time contraint; and a relation indicating the amount of time and commodities spent on rearing children. This relation will be called the production function of children, in analogy to Becker (2,1965) and Gronau (11,1977).

The production function of children illustrates the idea that when parents decide to have a child, they take the obligation upon themselves to take care of it. That is, having a child means that parents have to spend money and time in specified ways. This assumption is in contrast to the quantity - quality trade-off analyzed by Willis (43,1973), Becker and Lewis (3,1973), and others. Since this trade-off is absent, the production process involves a known output of fixed magnitude; the choice of inputs (commodities and time) depends on the particular production technology of the family.[12/] We assume also, that the family plans to have a total of I children at the end of the lifecycle, determined "outside" the model.[13/] Thus the model analyzes the spacing of these I children.

Another fundamental feature of the model is the way fertility decisions enter. Previous research neglected the fact that the variable for children is inherently an integer (Moffitt (24,1980 and 25,1980), Hotz (17,1980), Rosenzweig and Wolpin (33,1980)). As a result, the analysis concentrated on infinitesimal changes in number of children in relation to wage and price changes. One cannot treat children as a continuous variable, when the object of study is the individual response of the family. How, for example, should a dynamic path of optimal number of children be interpreted, when it rises continuously from 0.3 at the beginning of the planning period to 2.8 at the end? Does or doesn't the individual have three children at the end of his (her) life? Moreover, at which point in the lifecycle are the first and second child born? These questions cannot be answered by a model containing a continuous variable for the number of children, and therefore estimation of such a model on individual data is necessarily imprecise.

Our model circumvents this problem. It represents the number of children in the family as an integer variable. Instead of concentrating on the optimal number of children at each point in time, which would create problems in the maximization procedure, it emphasizes the choice of points in time, at which the family desires to have the next child.

Thus the lifecycle is broken down in smaller time periods, each one of which is characterized by a number of children (0,1,...).

While this formulation is new in this area of economic science, elsewhere, for example in operations research, the same technique is in use already. A parallel problem in inventory management is one of placing orders at discrete points in time in order to keep inventories at such a level that costs of ordering and holding are minimized. See for example Hillier and Lieberman (16,1967, chapter 12, p. 394.)

3.2 The Mathematical Formulation

The planning horizon of the family extends from $t=0$ to $t=T$. Let us define t_i as the date of birth of the i^{th} child or, as we call it, the i^{th} switchpoint. As was mentioned in section 3.1, parents choose inputs to produce children; for the i^{th} child this process starts at $t=t_i$ and ends at $t=t_i'$, where $t_i'-t_i$ is a fixed time span of, say, 20 years. For convenience we assume that the last child (I) is born before the responsibilities for the first one end, or $t_I < t_1'$. T is far enough in the future so that all child-caring responsibilities are fulfilled, so $t_I' < T$.

We define the i^{th} period as the time interval during which the family has i children, running from t_i to t_{i+1}. The time interval from t_I to T, during which the family has I children, is divided into I+1 periods: period (I,j) is the time interval during which I children are present and the responsibilities for j of them has ended, with $j=0,\dots,I$. These periods run from t_j' to t_{j+1}'. Figure 1 summarizes these concepts for I=2.

We assume that lifecycle utility is a discounted sum of instantaneous utility functions. The instantaneous utility attained at time t (U(t)) is a function of the family's consumption (Z(t)), the wife's leisure time (L(t)), and the number of children (i). Therefore, if $t_0=0$ and $t_{I+1}=T$, the instantaneous utility function is defined as:

$$U(t) = U(Z(t), L(t), i) \quad \text{for} \quad t_i \leq t < t_{i+1} \quad \text{and} \quad i = 0,1,\ldots,I \tag{3.1}$$

Note that periods (i,j), j=0,...,I, are captured in equation (3.1) by setting i=I, so that $t_I \leq t < T$. Marginal utilities are assumed positive and decreasing. Moreover, we assume for simplicity separability between leisure and consumption through $U_{12}=0$.[14/] The sum of instantaneous utility over period i, discounted with rate ρ to t=0, is equal to:

$$\int_{t_i}^{t_{i+1}} e^{-\rho t} U(Z(t), L(t), i)\, dt \tag{3.2}$$

Then, lifecycle utility LCU is a sum over all periods i:

$$LCU = \sum_{i=0}^{I} \int_{t_i}^{t_{i+1}} e^{-\rho t} U(Z(t), L(t), i)\, dt + e^{-\rho T} B(A(T)) \tag{3.3}$$

A bequest function B(.) is added, with the argument assets at the end of life, A(T), to indicate the utility derived by the parents in leaving some assets for the children after death.[15/] This function is assumed to have a positive but decreasing first derivative.

Next we formulate the restrictions. The first of them is the production function of children. In the following, $C_k(t)$ stands for commodities used for the production (care) of child k, and $B_k(t)$ is the amount of child care time devoted to child k. The production obligation of the parents concerning child k extends from time $t=t_k$, the k^{th} switchpoint, to time $t=t_k'$, where $t_k'-t_k$ is a fixed time span. The production function of child k is defined by:

$$f^k(B_k, C_k, t-t_k) = 1 \qquad \text{for} \quad t_k \leq t \leq t_k' \tag{3.4}$$

and consequently outside the time span from t_k to t_k':

$$B_k(t) = 0 \tag{3.5}$$
$$C_k(t) = 0 \qquad \text{for} \quad t < t_k \quad \text{and} \quad t > t_k' \tag{3.6}$$

The lefthand side of (3.4) indicates how the inputs B_k and C_k relate to output, while the righthand side shows the magnitude of that output, i.e. one child, or equivalently in our formulation, one unit of child services.

The function f^k is assumed to be strictly concave in B_k and C_k. The third argument of the function f^k measures the age of child k, which could have an influence on the mix of inputs chosen. For example, if f^k_{13} is negative, the productivity of child care time decreases with the child's age, so with constant input prices one would expect the family to reduce child care time. We will analyze this in more depth later on. The superscript k on f^k indicates the possibility of different production technologies for each child, including (dis-)economies of scale in production. These effects are relatively unimportant for the analysis, and are more interesting empirically.

The second restriction is the time constraint. At time t in period i total child care time sums up to $\sum_{k=1}^{i} B_k(t)$. When N(t) is the wife's labor supply, and all time variables are dimensioned as fractions, the time constraint states:

$$L(t) + N(t) + \sum_{k=1}^{i} B_k(t) = 1 \quad \text{for} \quad t_i \leq t < t_{i+1} \tag{3.7}$$

In period (I,j) the wife does not spend any child care time on the j oldest children, so the time constraint can be written as:

$$L(t) + N(t) + \sum_{k=j+1}^{I} B_k(t) = 1 \quad \text{for} \quad t'_j \leq t < t'_{j+1} \tag{3.8}$$

The third restriction deals with the family's budget. It states that savings $\dot{A}(t)$, being added to assets A(t), is the difference between income and expenditures.16/ Sources of income are interest income (rA(t)), outside income including husband's earnings (V(t)), and wife's earnings (W(t)N(t)). Money is spent on family's consumption and commodities for children, at prices P_Z and P_C respectively. So the budget constraint in the i^{th} period is:

$$\dot{A}(t) = rA(t) + V(t) + W(t)N(t) - P_Z Z(t) - P_C \sum_{k=1}^{i} C_k(t)$$

$$\text{for} \quad t_i \leq t < t_{i+1} \tag{3.9}$$

The budget relation for period (I,j) is similarly defined, with a modification in the sum of commodities C_k. The path of assets A(t) starts at some exogenously given initial level A_0 at t=0.

3.3 Maximization in Two Stages

The family is assumed to maximize the objective function (3.3) subject to the restrictions (3.4) through (3.9). This is a typical problem for analysis with dynamic programming techniques,[17/] but there is an additional wrinkle in the form of the determination of the optimal optimal switchpoints. An often used technique, the Pontryagin Maximum Principle,[18/] has to be modified in order to accommodate the switchpoint determination. Basically the Pontryagin Maximum Principle uses a so-called Hamiltonian function, which is a composition of the objective function and the restrictions for one particular point in time t, and states the conditions to maximize the objective function. Since in our case the restrictions change between periods i and (I,j), it is not possible to set up the same Hamiltonian at every point of the lifecycle. Moreover, the Hamiltonian function does not lead to maximization conditions on the switchpoints.

The adaptation of the Pontryagin Maximum Principle to our model leads to a two-stage procedure. In the first stage the values of the switchpoints are taken as given, and the Maximum Principle is applied to each of the (2I+1) periods separately. This is allowed by virtue of Bellman's Principle.[19/] In other words, given the switchpoints, which affect restrictions (3.4) to (3.9), the Maximum Principle can be used on a certain period, say i, and gives the optimal solution of control and state variables, conditional upon the value of the state variables (in our case only A(t)) at the beginning of period i, $t=t_i$.

In the second stage the objective function (3.3) is evaluated at the values of the control and state variables that are optimal given the switchpoints. So the first-stage maximum, call it LCU^*, is still a function of the switchpoints. Optimal switchpoints are determined when LCU^* is maximized.

In a more intuitive sense we can see that the model also solves for completed family size I. The value of lifecycle utility after the second stage is at a maximum, given I. Call this function $LCU^{**}(I)$. Parents will choose that (integer) value of I that maximizes LCU^{**}. This is not a straightforward analytical problem. The function LCU^{**} consists of a number of separate points, as is illustrated in figure 2, where the maximum is found at I equal to 3. In the subsequent analysis I is kept constant.

4. EXAMINATION OF THE MODEL

When we analyze the solution of the model, we also consider one corner solution, namely that the wife withdraws temporarily from the labor market, since this is a characteristic of female labor force participation patterns. Initially, however, we will assume that the wife supplies some positive amount of labor throughout the lifecycle. In section 4.2 we look at the corner solution.

4.1 The Results under the Positive-Labor-Supply Assumption

A crucial variable throughout the solutions in $\lambda_1(t)$, which can be interpreted as the "marginal utility of money" at time t: it is the discounted value of the marginal utility of bequests:[20/]

$$\lambda_1(t) = B'(A(T))\ e^{-\rho T+rT-rt} \tag{4.1}$$

The λ-constant demand functions in period i for commodities Z_i and leisure L_i are given by:

$$Z_i(t) = Z(\lambda_1(t)P_Z e^{\rho t}, i) \qquad Z_{1(i)} = U_{11}^{-1} < 0 \tag{4.2}$$

$$Z_{2(i)} = -U_{11}^{-1}\ U_{13} \gtrless 0 \text{ as } U_{13} \gtrless 0$$

$$L_i(t) = L(\lambda_1(t)W(t)e^{\rho t}, i) \qquad L_{1(i)} = U_{22}^{-1} < 0 \tag{4.3}$$

$$L_{2(i)} = -U_{22}^{-1}\ U_{23} \gtrless 0 \text{ as } U_{23} \gtrless 0$$

where $Z_{j(i)}$ is the derivative of $Z_i(t)$ with respect to the j^{th} argument (j=1,2), and similarly for $L_{j(i)}$.

Equation (4.3) shows that the lifecycle leisure path is a composition of four effects. First of all, the number of children rises over the lifecycle, and with it the demand for leisure if $U_{23} > 0$. A second effect originates from the wage profile; leisure varies inversely with wages. The third effect is the time effect. The wage rate is discounted by the difference between the interest rate and the rate of time preference. This discount factor causes the full price of time to decrease over the lifecycle when $r > \rho$. We will assume that this is the case; it implies that households work harder at the earlier stages of the lifecycle, and also that they tend to postpone spending their assets on consumption Z. A rising trend results for the leisure (and consumption) profiles. A fourth variable in (4.3), due to (4.1), is A(T), bequests. Any exogenous variable affecting A(T) influences the whole leisure profile indirectly.

In static models the effect on the demand for leisure of a change in the wage rate can be decomposed into an income effect and a substitution effect. A similar decomposition is obtained in the dynamic model. Let the wage profile be varied by an amount dW from $t=t'$ to $t=t''$, i.e.:

$$\frac{dW(t)}{dW} = 1 \qquad t' \leq t \leq t'' \tag{4.4}$$
$$= 0 \qquad \text{otherwise}$$

The response in the leisure profile can be written as the sum of a negative direct effect and a positive bequest effect, working through A(T):

$$\frac{dL_i(t)}{dW} = L_{1(i)}\lambda_1(t)e^{\rho t}\,\frac{dW(t)}{dW} + L_{1(i)}W(t)e^{\rho t}\,\frac{d\lambda_1(t)}{dA(T)}\,\frac{dA(T)}{dW} \tag{4.5}$$

Later on we shall see an interesting parallel with the optimal choice of switchpoints.

The child production process is one of cost minimization, given output. The optimal amounts of child care time and child care commodities spent on child k depends only on input prices and the child's age $(t-t_k)$:

$$B_k(t) = B_k(W(t),P_C,t-t_k) \qquad B_{1(k)} < 0 \qquad B_{2(k)} > 0 \tag{4.6}$$

$$C_k(t) = C_k(W(t),P_C,t-t_k) \qquad C_{1(k)} = B_{2(k)} > 0 \qquad C_{2(k)} < 0 \tag{4.7}$$

These demand functions are defined from $t=t_k$ to $t=t_k'$.

If the time intensity of child production decreases with the child's age, $(B_{3(k)}/B_k)$ is less than $(C_{3(k)}/C_k)$. At this point we make a stronger assumption, namely that $B_{3(k)}$ is negative. Because of the cost-minimizing nature, the marginal utility of money, and thus A(T), does not affect child production inputs directly; as in production theory, only the input price ratio is relevant to the choice of inputs. On the other hand, switchpoints are endogenous variables, so any variable, e.g. outside income, affecting t_k does have an indirect impact on the optimal $B_k(t)$ and $C_k(t)$.

We want to emphasize the relevance of equation (4.6) to the labor supply profile. Labor supply is the mirror image of the sum of leisure and total child care time. Equation (4.6) shows that the labor supply profile depends partly on child timing decisions (the interval (t_k,t_k')), the age structure of the children $(t-t_k)$, and the wage elasticity of child care time. Having children to be taken care of at time t will make labor supply at that moment more elastic.[21/]

The remaining endogenous variables in the model are bequests and the switchpoints. No explicit solutions exist, but we find (I+1) implicit functions for these (I+1) variables. We call these g^0 and $g^{1,i}$ with $i=1,\ldots,I$:[22/]

$$g^{0}(A(T),t_0,\ldots,t_i,\ldots,t_I;\ A_0,V,W,P_Z,P_C,r,\rho) = 0 \qquad (4.8)$$

$$g^{1,i}(A(T),\quad t_i \quad ; \quad W,P_Z,P_C,r,\rho) = 0 \qquad (4.9)$$

For notational convenience we define the derivatives of these functions as follows:

$$X_{00} = \frac{\partial g^0}{\partial A(T)} \qquad X_{0i} = \frac{\partial g^0}{\partial t_i} \qquad X_{0W} = \frac{\partial g^0}{\partial W} \quad \text{etc.}$$

$$X_{i0} = \frac{\partial g^{1,i}}{\partial A(T)} \qquad X_{ij} = \frac{\partial g^{1,i}}{\partial t_j} \qquad X_{iW} = \frac{\partial g^{1,i}}{\partial W} \quad \text{etc.}$$

Table B.1 in appendix B gives the signs of the derivatives and the conditions to find these signs.

Using Cramer's rule one finds easily the total effects of exogenous variables on bequests. The signs of these are given in table 3, column 1. To demonstrate the importance of the endogeneity of the switchpoints, column 2 of table 3 shows the signs of the same derivatives when switchpoints are kept (exogenously) fixed, and column 3 compares column 1 relative to column 2. The importance of this comparison relates to the bequest effect found in (4.5) for the leisure profile and which also exists for the consumption profile.23,24/

Applying Cramer's rule to (4.8) and (4.9) generally does not yield results on the total effect of exogenous variables on the switchpoints that are as straightforward as in the case of bequests. The total effect, in the case of a changing wage rate, can be written as:

$$\frac{dt_i}{dW} = -\frac{X_{iW}}{X_{ii}} - \frac{X_{i0}}{X_{ii}}\frac{dA(T)}{dW} \qquad (4.10)$$

Here we see the parallel with other, more common consumption decisions, such as the choices of L and Z. Compare (4.10) with (4.5); the first term of the righthand side of (4.10) is a direct effect, while the second operates through a change in bequests and is appropriately called a bequest effect. Table 4 gives the signs of these two effects for each of the exogenous variables. A positive sign means that t_i goes up, and thus a decrease in the consumption of child services. So the positive direct effect of the price variables indicates that because of increased costs of children the consumption of child services falls.

4.2 The Results with Temporary Labor Force Withdrawal

This section examines a fairly specific case of labor force withdrawal, namely the case in which labor supply N(t) equals zero from the time of marriage $t=0$ to some time $t=\bar{t}$ between the birth of the last child (t_I) and the time that production responsibilities end for the first one (t_1'). Other more general patterns of zero and positive labor supply, as well as permanent retirement (i.e., zero labor supply from some point to the end of the lifecycle) can be analyzed in a similar fashion.

As long as N(t) equals zero, the value of the wife's time, called $\mu_1(t)$, exceeds the value available in the labor market, $\lambda_1(t)W(t)$. We will call $\mu_1(t)$ the reservation wage, even though it is measured in utils.[25/] Quite predictably, the formulas of the optimal leisure and child care time choices change into:

$$L_i(t) = L(\mu_1(t)e^{\rho t}, i) \tag{4.11}$$

$$B_k(t) = B_k(\lambda_1(t)/\mu_1(t), P_C, t-t_k) \tag{4.12}$$

The value of the reservation wage at any time t is determined in the interaction of the demand for time (leisure and child care time of all then existing children) and the supply of time (equals 1). This is valuable information at the empirical stage of this study.

The profile of $\mu_1(t)$ over time shows a discontinuity at each switchpoint, a jump upward, since the demand for time rises. Moreover, when the individual is due to return to the labor market, the reservation wage has to decrease faster than the value of time in the market, i.e. $\lambda_1(t)W(t)$. While this need not be so for every t, it certainly has to be true for $t=\bar{t}$.

At any time t, $t_1 < t < \bar{t}$, the reservation wage depends on all previous switchpoints. If t_k rises, then for $t > t_k$ the demand for time (and so the reservation wage) increases due to our assumption that $B_{3(k)}$ is negative. But the value of time is one of the arguments in the choice of each switchpoint. So switchpoint k affects the choice of switchpoint i for $i > k$. On the other hand, a similar reasoning shows that switchpoint i affects switchpoint k. Therefore, the I switchpoints are interrelated to a larger degree than was the case with positive labor supply.

This interaction complicates the analysis. The implicit function $g^{1,i}$ is now a function of all switchpoints:

$$g^{1,i}(A(T),t_1,\ldots,t_i,\ldots,t_I;\ W,P_Z,P_C,r,\rho) = 0 \qquad (4.13)$$

The signs of the derivatives of g^0 and $g^{1,i}$ under the temporary labor force withdrawal assumption are given in table B.2 in appendix B. A number of derivatives become unsigned, although two derivatives (X_{ij}

and X_{iP_C}) turn positive, when a fixed-coefficient child production process is assumed.

Due to the interaction of the switchpoints, total effects cannot be signed. But let us, as before, decompose the total effect of, say, wages on switchpoint t_i:

$$\frac{dt_i}{dW} = -\frac{X_{iW}}{X_{ii}} - \frac{X_{i0}}{X_{ii}}\frac{dA(T)}{dW} - \sum_{\substack{j=1 \\ j\neq i}}^{I} \frac{X_{ij}}{X_{ii}}\frac{dt_j}{dW} \quad . \tag{4.14}$$

The first and second term are familiar from section 4.1 as the direct and bequest effect. The third term represents the interaction between the switchpoints and could properly be called an indirect effect. The sum of the parameters $(-X_{ij}/X_{ii})$ would be positive if X_{ij} is positive, and less than 1 if $(X_{ii}+\sum X_{ij})$ is negative. We cannot show that either is necessarily true, but both results would be intuitively reasonable: t_j pushes t_i in the same direction, up or down, but all t_j's together push t_i only part of the way in that direction.

5. THE EMPIRICAL SPECIFICATION

The theoretical model described in the previous sections is dynamic in nature. In this section we proceed to set up an empirical model based on specific functional forms inserted in the theoretical model. This preserves, in a natural way, the dynamic and interactive aspects of fertility and time allocation choices. A set of four equations results, analyzing wages, leisure, hometime and switchpoints.

The utility function is specified as a variant of the addilog form:[26/]

$$U(t) = \alpha_1 Z(t)^{\beta_{11}}(i+i_1)^{\beta_{12}} + \alpha_2 L(t)^{\beta_{21}}(i+i_2)^{\beta_{22}} + \alpha_3 i^{\beta_{31}} \qquad (5.1)$$

where $0 < \beta_{11}, \beta_{21} < 1$ for decreasing marginal utility of consumption goods and leisure; i_1 and i_2 are positive parameters. Note that complementarity between children on the one hand, and leisure and consumption goods on the other implies that $\beta_{12} > 0$ and $\beta_{22} > 0$.

The child production function is specified as a Cobb-Douglas type with time-varying coefficients:

$$f^k(B_k, C_k, t-t_k) = \delta_{3k}\, B_k(t)^{\delta_1}\, C_k(t)^{\delta_2} \qquad (5.2)$$

where $\delta_1 = \varepsilon_1 + \varepsilon_2(t-t_k)$ and $\delta_2 = 1-\delta_1$. If ε_2 is negative, child care time decreases with children's age. The parameter δ_{3k} would increase for $k=1,2,\ldots,$ if there exist economies of scale in the child production process.

The first part of the empirical model analyzes time allocation decisions. The wage equation is specified in a semi-log linear fashion, as is customary in labor supply studies, based on human-capital considerations. The derivation of the other equations is a lengthy process, on which a few comments are made here.[27/]

While one can find an analytical solution for childcare time from (5.2), viz. (4.6), childcare time for each child is an unobservable variable. Instead, data on total hometime are collected, which is the sum of all childcare time and "other hometime." We assume that this "other hometime" could be written as a linear combination of a vector of exogenous characteristics $X_3'(t)$, a person-specific constant τ_{31}, and a random factor $u_h(t)$. In the hometime equation thus formulated, the price of childcare commodities, P_C, is assumed to be constant and therefore can be treated as an estimated parameter.

The wage, leisure and hometime equations are written as:

$$\ell nW(t) = X_1(t)\pi_1 + \tau_1 + u_1(t) \tag{5.3}$$

$$L(t) = \gamma_2 \ell nW(t) + X_2(t)\pi_2 + \pi_{21}\ell n(i_L+i_2) + \tau_2 + u_2(t) \tag{5.4}$$

$$B(t) = (\gamma_{31}i_B+\gamma_{32}X_{31}(t))\ell nW(t) + X_3(t)\pi_3 + \tau_{31} + \tau_{32}i_B + u_3(t) \tag{5.5}$$

The variable i_L refers to the total number of children the family has, living anywhere and of any age, while i_B measure the number of children for whom the parents have financial responsibilities.[28/] This parallels the distinction made in the theoretical model between periods i and (I,j). The variable $X_{31}(t)$ stands for the sum of the ages of those children counted in i_B, i.e. $\sum(t-t_k)$. The vector $X_3(t)$ includes the vector $X_3'(t)$, while $u_h(t)$ is absorbed in $u_3(t)$. τ_1, τ_2, τ_{31}, and τ_{32} are person-specific constants, or as commonly called, fixed effects. Variation in τ_2 among individuals is an index of variation in bequests, so τ_2 measures the bequest effect of section 4.

Equations (5.3) tot (5.5) are the basic structure of the time allocation part of the empirical model. In years that the woman does not work in the labor market, her wage rate is unobserved, while the sum of leisure and hometime equals the maximum yearly hours available to her, which we take to be 8760 hours. In such years the following condition on the error terms is obtained by substituting (5.3) into (5.4) and (5.5):

$$(\gamma_2+\gamma_{31}i_B+\gamma_{32}X_{31}(t))u_1(t) + u_2(t) + u_3(t) \geq 8760 -$$
$$- (\gamma_2+\gamma_{31}i_B+\gamma_{32}X_{31}(t))(X_1(t)\pi_1+\tau_1) - X_2(t)\pi_2 -$$
$$- \pi_{21}\ell n(i_L+i_2) - X_3(t)\pi_3 - \tau_2 - \tau_{31} - \tau_{32}i_B \qquad (5.6)$$

Moreover, since $L(t)+B(t)=8760$, the (log of) reservation wage in monetary units could be expressed, by means of (5.4) and (5.5), in terms of exogenous variables, fixed effects and random variables. Substituting this for $\ell nW(t)$ into (5.5), a so-called restricted hometime equation is obtained:

$$B(t) = b\cdot 8760 - b\cdot X_2(t)\pi_2 - b\cdot\pi_{21}\ell n(i_L+i_2) + (1-b)\cdot X_3(t)\pi_3 -$$
$$- b\cdot\tau_2 + (1-b)\cdot(\tau_{31} + \tau_{32}i_B) - b\cdot u_2(t) + (1-b)\cdot u_3(t)$$
$$(5.7)$$

where $b = (\gamma_{31}i_B+\gamma_{32}X_{31}(t))/\{\gamma_2+\gamma_{31}i_B+\gamma_{32}X_{31}(t)\}$. Equation (5.7) enables us to utilize information on actual hometime used during years, in which the woman does not work. This forms a major difference with currently existing studies on the dynamics of labor supply.

The second part of the empirical model deals with the timing of children. The theoretical switchpoint equation, for working women, is $g^{1,i}(t)=0$ for $t=t_i$ (see (4.9), or (A.1) in appendix A for the explicit formula). This relation is the first order condition on the choice of t_i. If t_i is optimal, than at any time $t < t_i$ it is optimal for the family to delay switchpoint i until $t=t_i$, since lifecycle utility rises in that way. Therefore $g^{1,i}(t) > 0$ for $t < t_i$, and similarly $g^{1,i}(t) < 0$ for $t > t_i$. This information is used in the estimation procedure: while $g^{1,i}(t)$ is unobservable, we know whether it is greater or less than zero because of the (non)occurrence of a switch in a certain year. These ideas introduce a probit element into the empirical investigation.

No specification of utility and child production function yields a manageable empirical specification of $g^{1,i}(t)$ directly; equation (A.1) appears to be too complex. Instead, the following relation is estimated:

$$\begin{aligned} g^{1,i}(t) = {} & (\phi_0+\phi_1 t+\phi_2\tau_2)\cdot\{(i_S-1+i_1)^{\pi_{40}} - (i_S+i_1)^{\pi_{40}}\} + \\ & + \alpha_2 L(t)^{\beta_{21}}\cdot\{(i_S-1+i_2)^{\beta_{22}} - (i_S+i_2)^{\beta_{22}}\} + \\ & + \tau_4\{(i_S-1)^{\beta_{31}} - i_S^{\beta_{31}}\} + X_4(t)(\pi_{41}+\pi_{42}\tau_2) + u_4(t) \end{aligned} \tag{5.8}$$

where ϕ_0, ϕ_1, and ϕ_2 are scalars, π_{41} and π_{42} are vectors, and $\pi_{40}=\beta_{12}/(1-\beta_{11})$. The following cross-equation parameter restrictions apply: $\gamma_2=1/(\beta_{21}-1)$ and $\pi_{21}=\beta_{22}/(1-\beta_{21})$. The first three terms of (5.8) approximate the utility difference $U(Z_{i-1},L_{i-1},i-1)-U(Z_i,L_i,i)$ in equation (A.1), while the term $X_4(t)(\pi_{41}+\pi_{42}\tau_2)$ approximates the the integral. The second line of (A.1) disappears in the approximation of the utility difference. The variable i_S indicates the order of the next child to be born.

The row-vector $X_4(t)$ contains the following five terms: t, t^2, and the predicted values of wage rate, hometime, and probability of working in year $t+4$. These predictions are based on an estimation of the time allocation model separately. The choice of year $t+4$ as the "future" is arbitrary, of course, but these predicted variables allow us to test some implications concerning the effect of the cost of rearing children on the choice of switchpoints.

The variable $\lambda_1(t)$, the discounted marginal utility of bequests, appears in the theoretical switchpoint equation. The fixed effect τ_2, used in the leisure equation (5.4), approximates $\lambda_1(0)$ closely. Since $\lambda_1(t)=\lambda_1(0)e^{-rt}$, see equation (4.1), τ_2 enters the empirical switchpoint equation (5.8) as well.

The distributional assumption of the error terms $u(t)=(u_1(t),u_2(t),u_3(t),u_4(t))$ completes the model. By assumption $u(t)$ follows a multivariate normal distribution with mean 0 and covariance matrix Σ_i, where Σ_i is defined as:

$$\Sigma_i = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma^i_{13} & \sigma_{14} \\ \sigma_{12} & \sigma_{22} & \sigma^i_{23} & \sigma_{24} \\ \sigma^i_{13} & \sigma^i_{23} & \sigma^i_{33} & \sigma^i_{34} \\ \sigma_{14} & \sigma_{24} & \sigma^i_{34} & \sigma_{44} \end{pmatrix} \tag{5.9}$$

where $\sigma^i_{k3}=\sigma_{k31}+i_B\sigma_{k32}$ for $k=1,2,4$; and $\sigma^i_{33}=\sigma_{331}+i_B^2\sigma_{332}$. The reason behind the heteroskedastic variance-covariance matrix lies in the composition of $u_3(t)$, which is the sum of error terms from "other hometime," $u_h(t)$, and childcare time of each child, called $u_{cj}(t)$: $u_3(t)=u_h(t)+\sum u_{cj}(t)$. This summation runs over the i_B children for whom parents have financial responsibilities at time t. Behind the definition of σ^i_{33} is the assumption of independence of u_{cj} mutually and between u_{cj} and u_h; there is no systematic relation between the allocation of childcare and "other" time apart from what is explained by economic variables or the fixed effects. Thus one may interpret σ_{331} and σ_{332} as the variance of u_h and u_{cj} respectively.[29,30]

6. DATA AND RESULTS

6.1 The Data

The data used for the empirical analysis comes from the Panel Study of Income Dynamics (PSID), conducted at the Survey Research Center of the Institute for Social Research at the University of Michigan.[31] The study is longitudinal in set-up; in each year data are gathered about families' activities of the previous year (e.g., hours worked and income received over the whole year), and about the situation of the families at the time of the interview (e.g., county unemployment rate, family size). So the twelve waves used for this study describe each family fully for eleven consecutive years, from 1968 to 1978, which forms the length of the sample period for our purposes.

One can divide the families of the PSID study into four categories, according to whether the wife worked at least once during the sample period or never, and whether the family had at least one child during the sample period or none. Only for families with working wives, who had one or more children during the sample period, are all five fixed effects estimable.[32] On the other hand, taking into account that for the other three categories of families some fixed effects take on an "optimal" value of $\pm\infty$, one could write down a reduced joint likelihood function over all four categories, and estimate the parameters of the model with the maximum amount of information. This could not be done, however, on the computer facilities used, due to memory restrictions. This paper reports results obtained from a sample of wives, who worked at least once and had one or more children during the sample period.[33]

Further selection criteria, described more fully in appendix C, restrict the sample to white married women, aged between 20 and 45 in 1968, and living with the same husband during all years of the sample period. Thus the sample size of usable responses was 162.

Table 5 contains the definition of the variables used in this study, as well as the sample means and standard deviations over the eleven-year sample period. The construction of the variables related to the number of children and their ages needs elaboration. While i_S always takes on an integer value, in i_L and i_B account is taken of the proportion of the year that each child is part of the family; a child born in March of a year counts as .75 child for that year. The dummy variables DUMCH2, DUMCH3 and DUMCH4 follow the same rules. As for children's ages, those are defined as the age they have at the midpoint of the part of the year that they are part of the family. A child born in March has age .375, while a child with age 7.3 on January 1st has age 7.8 in our data set. When children disappear between two interviews, say 1974 and 1975, or become 18 years of age, they are assumed to be part of the family until the end of 1974.

6.2 The Estimation Results[34/]

The model is estimated by the method of maximum likelihood, iteratively performed on the set of parameters and the set of fixed effects until overall convergence is achieved.[35/] Heckman and MaCurdy (14,1980) reported that this procedure went pretty rapid. Our experience with a larger scale model is not as encouraging. On the other hand, substantial savings in computer time were realized when we included a constant parameter parallel to each fixed effect in the parameter stage. Such constants allow the average of the fixed effects to shift in the parameter stage, but do not have an empirical interpretation, and as such are absorbed in the reported fixed effects.

Table 6 contains the estimation results for both the time allocation model (TAM), which according to the theory offered in this paper should suffer from simultaneity bias, and the full model (FM) of fertility and time allocation decisions. To shorten the discussion, the role of each variable is indicated as a control or a model-induced one, as well as the expected sign of the parameter estimate. The results of table 6 imply values of the parameters of the utility function (5.1) and child production function (5.2), which are given in table 7.

In both sets of estimates the parameters of the experience variables in the wage equation are as expected, while the direction of the labor market condition variables is somewhat uncertain.

In the leisure equation we find that higher wages decrease leisure, but not to the extent that the addilog specification of the utility function is supported; the parameter β_{21} is quite negative, while it should fall between 0 and 1. This is in contrast to the finding of β_{21}=.0014 by Heckman and MaCurdy (14,1980), who apparently restricted the parameter to its required range (see their footnote 27). The result is consistent with another specification of the utility function, which is of interest in analysis of uncertainty, as it exhibits a constant rate of risk aversion:

$$U(t) \approx - K_1 h(i) e^{-K_2 L(t)} \tag{6.1}$$

with K_1 and K_2 positive parameters if $\partial U/\partial L > 0$, and $\partial h/\partial i$ negative if $\partial^2 U/\partial L \partial i > 0$. The estimated K_2 would equal .1124 (TAM) or .523 (FM).

Children appear to be complementary with leisure (viz. the positive π_{21} and β_{22}), a result similar to Hotz (17,1980). A working woman with two children would have 458 (TAM) or 110 (FM) hours of leisure extra, when she would have one child more. The coefficient of YRSMA is negative, implying that the rate of time preference ρ exceeds the rate of interest r by 4.6 (TAM) or 1.3 (FM) percentage points, contrary to our assumptions in section 4 and results found frequently in other research.[36/]

In the hometime equation large economies of scale in rearing children are found; the second child needs 697 (TAM) or 559 (FM) hours less than the first one. The parameters of AGECH and AGECH2 support the general perception that younger children are more time-intensive; a minimum occurs for each child when it reaches age 15. The parameters γ_{31} and γ_{32} indicate that hometime is more wage - elastic with rising number of children (i_B) and their ages. The implied parameters ε_1 and ε_2 take on reasonable values although $\delta_1=\varepsilon_1+\varepsilon_2(t-t_k)$ turns negative for $t-t_k=7.5$ (see equation (4.2)), which happens to be the mean age of children at home in the sample (=average AGECH/average i_B).

The variable FAMINC enters the leisure, hometime (twice) and switch-point equations in order to test the hypothesis that "exogenous income" does not have any impact in all four cases; only fixed effects are supposed to be affected by exogenous income. Two of the three TAM estimates are insignificant, as well as those in the hometime equation of the full model. However, in the full model FAMINC appears to affect leisure and switchpoints directly in the direction it was expected to affect them indirectly through the fixed effects.[37/] This tends to suggest that the fixed effect τ_2 is not a good proxy of the bequest effects, and may explain the large change in the estimate of $\hat{\gamma}_2$ between the two models TAM and FM.

The reader can verify, that of the remaining 14 predictions made concerning the parameters of the switchpoint equation six are supported (evaluate the expressions like $(\pi_{413}+\pi_{423}\tau_2)$ at the average value of τ_2). Most noteworthy is the sign of π_{40}, indicating complementarity between the consumption of commodities Z and children (equation (4.2)); the positive sign of $(\pi_{414}+\pi_{424}\tau_2)$ supporting the notion that women make a choice between a career and raising children; the negative sign of $(\pi_{413}+\pi_{423}\tau_2)$ indicating that higher predicted wages lead to having children earlier, and thus to having more children, opposite to our expectation as well as to Mincer's result (22,1963); and the positive estimate of $(\pi_{415}+\pi_{425}\tau_2)$ implying that higher predicted childcare discourages parents from having children.

The large size of the switchpoint parameters relative to the variance σ_{44} causes concern. We analyzed the predictive power of the equation. Suppose a birth is predicted to occur, if its probability exceeds 50 percent. For the 101 women who had one birth during the sample period, the prediction was correct in 99.3 percent of the cases (i.e., 101 individuals times 11 periods). For 61 women who had two or more additional children during the sample period, 93.1 percent of the predictions were correct. Further diagnostics showed light on the role of τ_4 in the estimated equation. Note that i_S increases by unity in the year after a birth. The expression $\tau_4(i_S-1)^{\beta_{31}} - \tau_4 i_S^{\beta_{31}}$ increases with rising i_S by so much, that it dominates the effect of other explanatory variables (equation (5.8)). The problem could be one of misspecification of the switchpoint equation or one of lack in variation in the dependent variable, i.e. the occurrence of births. One may find a solution by developing a way to incorporate the information on switchpoints that occurred before the start of the sample period, or to use the information on family size at the start of the sample period, recognizing the problem of initial conditions (Heckman (13,1981)). However, one cannot freely experiment, as the estimation of the model is quite expensive.

7. CONCLUDING REMARKS

The model in this paper analyzes fertility decisions as discrete choices within a continuous time model of time allocation. The methodology employed is applicable to a wide variety of dynamic programming problems with discrete choices. The approach proves quite fruitful, as it highlights the parallel between leisure and commodity choices on the one hand, and fertility decisions on the other. In both types of choices we can distinguish a direct effect and a bequest effect, that are similar to substitution and income effects in static models.

The dynamics of the theoretical model are as far as possible preserved in the empirical model, which is estimated with longitudinal data. The results indicate, that intertemporal substitution effects of wages on leisure are smaller than generally thought, in contrast to assumptions behind static labor supply models. The addilog utility function used in other studies on the dynamics of female labor supply appears to be rejected. Furthermore the substitution effect of wages on hometime increases with number of children at home and their ages. Significant economies of scale are estimated.

Estimates of the switchpoint equation indicate qualified support for the model. We find that women appear to choose between a career and raising a family. With a 50 percent rule of predicted occurrence versus non-occurrence the estimates give an almost perfect "prediction" of births within the sample. This may be due to the lack of variation in the dependent variable (births) and perhaps to a certain extent to the specification of the switchpoint equation. These and other issues on the dynamics within the household remain for future research.

FOOTNOTES

1/ Interesting discussions are found in contributions by Michael and Becker (19,1973), Pollak and Wachter (29,1975 and 30,1977), Barnett (1,1977), Gronau (11,1977), Nerlove (27,1974).

2/ See Schultz (34,1975)

3/ Cain and Dooley (5,1976), Fleisher and Rhodes (9,1979), and Conger and Campbell (6,1979) estimate such multi-equation models, with varying degrees of success. See also Schultz (35,1978).

4/ We exclude human capital arguments from the analysis. Although present labor market hours may be a significant determinant of future wages in the case of males (Heckman (12,1976), Blinder and Weiss (4,1976)), and therefore may be a source of intertemporal systematic variation in the demand for market time (or time of investment in human capital), such phenomena are observed to be much less important in the case of females. See Mincer and Polachek (23,1974), Smith (36,1977).

5/ See for first births Current Population Reports (39,1978), table 43, and for higher order births ibid., table 52.

6/ In the tables mentioned in the previous footnote, we found only three entries out of the relevant 48 entries, for which birth intervals were slightly shorter for higher income groups.

7/ Hotz derives a two-equation model of market work hours and probability of a birth, which is estimated using cross-sectional data. Moffitt (25,1980) estimates a probability-of-a-birth relation on longitudinal data; this relation is derived from a full dynamic model (24,1980).

In both cases results are mixed and not very susceptible to easy interpretation in the light of the underlying dynamic models.

8/ While children are, presumably, planned, it is unlikely that twins are anything more than random phenomena, impossibly predicted. Rosenzweig and Wolpin use twins at first birth as a random, purely exogenous variable, which is therefore unrelated to preferences. Its effect on labor supply can be interpreted in relation to the effect of the price of children.

9/ The same finding is reported by Schultz (35,1978) in a distinction between actual fertility and instrumental variable measures.

10/ Due to its assumptions the model leads to corner solutions, in which child quality is proportional to child spacing (i.e., the number of years between births or to the end of the fecund period). For couples with only one child, the child spacing variable is defined as (45 - age at first birth). For couples with two children, it is the mean of (age at second birth - age at first birth) and (45 - age at second birth). It is only the model that suggests such large implied child spacing (and child quality) differences, not the data.

11/ The conclusions of the model do not change, when the time horizon of the parents is uncertain, as long as it extends with certainty beyond the end of the childbearing period, called t_I' later on.

12/ In order to analyze child quality in a dynamic model, one has to define quite precisely what is meant by it: The child's earnings capacity at some age (e.g., age 20); or a stock of something to be built up, from which parents derive a flow of services at each point in time; or, more in the spirit of consumer durables, an inherently unchangeable characteristic of children. Each interpretation has its own theoretical implications and is beset by its own empirical problems; static theories of fertility do not offer a clear direction in this respect.

13/ In section 3.3 we elaborate on the choice of I.

14/ U_{12} stands for $\partial^2 U(t)/\partial Z(t)\partial L(t)$.

15/ The number of children benefitting from the bequests, I, could be made an argument of the bequest function without affecting the results of the model.

16/ A dot (·) above a variable denotes its time derivative. So $\dot{A}(t)=\frac{dA(t)}{dt}$

17/ Descriptions of dynamic programming techniques are found in Intriligator (18,1971), Miller (20,1979), and Takayama (38,1974).

18/ The technique originated with Pontryagin et.al. (30,1962).

19/ Consider a problem, in which one needs to obtain an optimal path of a variable X(t) from $t=0$ to $t=T$. Consider an intermediate point in time τ. Bellman's Principle states that the optimal path from $t=\tau$ to $t=T$, given the value $X(\tau)$ of the variable X at $t=\tau$, does not depend on the path of X between $t=0$ and $t=\tau$.

20/ This variable is introduced in the maximization procedure as the costate variable assigned to the state variable assets.

21/ This result, in static models, goes back to Mincer (21,1962) and others more recently. Note that in this statement we have controlled for effects of child spacing, which in itself will also vary with wages.

22/ The function g^0 is found by integrating (3.9) using the previously found solution of the control variables. The second-stage first order conditions yield the function $g^{1,i}$. This function is given in appendix A, equation (A.1), for the case of positive labor supply, for reference when the empirical model is set up.

23/ Interesting conclusions can be drawn from these results related to cross-sectional studies. The bequest effect falls for outside income; this may be one explanation for the fact that in cross-sectional analyses the wife's leisure (or labor supply) is often found to be relatively unresponsive to husband's income. As far as it concerns W, the bequest effect becomes smaller when X_{iW} is small relative to X_{ii}. While we cannot compare these derivatives analytically, if this is the case, the introduction of endogenous switchpoints provides another explanation for the fact that in cross-sectional studies the wage effect dominates the income effect.

24/ The model also has some interesting implications for the lifecycle profile of assets and savings (Vijverberg (42,1981)). The conclusion of Smith (36,1977) that savings peak in the middle stage of the life-cycle must be qualified if child production takes up large amounts of resources (time B_k and commodities C_k). E.g., sending one's children to college may lead to a savings profile with two peaks.

25/ Usually in the literature, the reservation wage is a term in monetary units, equal to $\mu_1(t)/\lambda_1(t)$.

26/ This specification is commonly used in empirical analyses, e.g., Hotz (17,1980), and Heckman and MaCurdy (14,1980).

27/ For more detail, see Vijverberg (42,1981). In the process of derivation, first order Taylor expansions are taken in order to linearize the demand relations of leisure around the sample mean (L_0) and child care time around an unknown estimable mean value (B_0). This is necessary due to the restriction that the sum of leisure and hometime cannot exceed a certain maximum number of hours, in our case equal to 8760

(= 365 x 24). Other adjustments reduce the nonlinearity of the age variable $(t-t_k)$ in the child care equation.

28/ In the empirical investigations we had to draw a somewhat arbitrary line in this respect, since the data did not indicate, whether children living outside the parents' home were financially independent. See section 6 for the exact definition.

29/ Without the independence assumption σ^i_{33} consists of three parts (instead of two as is the case here), made up of four parameters, from which σ_{332} is not identifiable.

30/ The variable i_B depends on whether or not a switch has occurred, and therefore Σ_i depends on the value of $u_4(t)$ relative to some fixed point (fixed at time t; derivable from equation (5.8)). It is easy to show that such a distribution is still proper in the sense that the integral of the density function over all values of the error terms equals unity.

31/ See Morgan (26,1974) for documentation.

32/ Heckman and MaCurdy (14,1980) experienced a similar problem; their distinction was whether or not the wife worked at least once.

33/ In Vijverberg (42,1981) estimates are reported also for the sample of women who worked at least once but did not have additional children during the sample period. The combined sample of the remaining two categories was too small to warrant estimation.

34/ I gratefully acknowledge the assistance of the University of Pittsburgh in providing sufficient resources to perform the estimation.

35/ In each of the sub-stages we use a method developed by Davidon (7,1959) and Fletcher and Powell (10,1963). The final results reported here are checked by reestimating the parameters from their initial starting values, using the final fixed effects. The reported standard deviations are obtained in this checking procedure.

36/ On the other hand, estimation results on the sample of 315 women, who did not have additional children but worked at least once during the sample period (not reported here), indicated that r exceeds ρ by 2.1 percentage points. Customarily, these two samples are pooled. Our results could indicate a "stress" situation for women in their child-bearing years.

37/ See the sign of $\partial A(T)/\partial V$ in table 3, the effect of $A(T)$ on leisure in equation (4.5) and the bequest effect on switchpoints, and thus on $g^{1,i}(t)$, in table 4.

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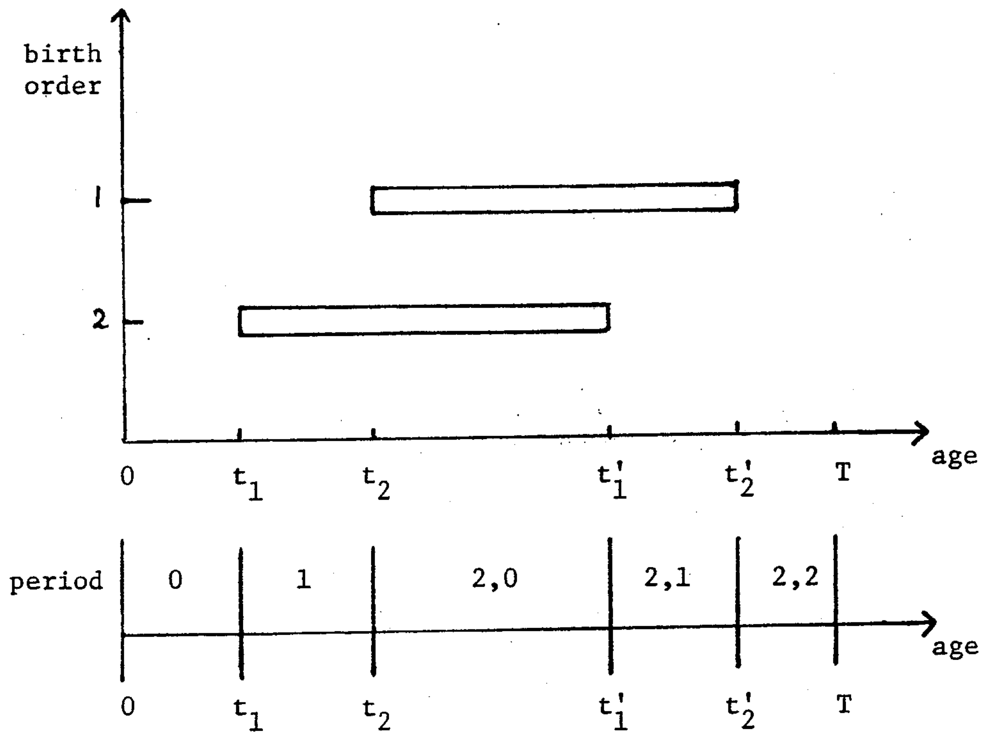

Figure 1

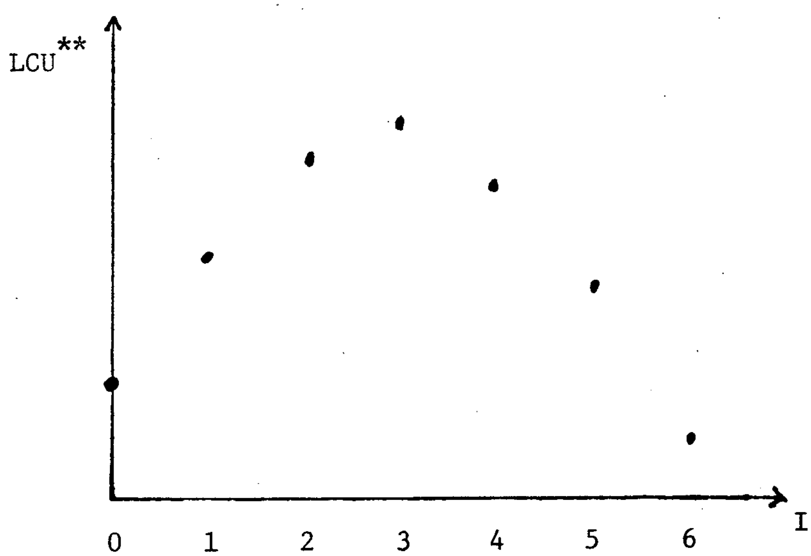

Figure 2

Table 1

Number of children ever born per 1000 women ever married, white, by employment status, 1970

Age group	(1) In the labor force	(2) Not in the labor force	(3) Ratio of (2) to (1)
20 to 24 years	628	1343	2.139
25 to 29 years	1466	2210	1.508
30 to 34 years	2392	2959	1.237
35 to 39 years	2796	3326	1.190
40 to 44 years	2740	3273	1.195

Source: Current Population Reports (41,1978), table 4-3

Table 2

Median birth intervals in months for first to fourth order births since the Second World War

Order	1975 – June 1978	1970 – 1974	1965 – 1969	1960 – 1964	1955 – 1959	1950 – 1954	1945 – 1949
Time of marriage to first birth	24.7	18.5	15.5	14.5	15.9	17.6	
First birth to second birth		31.7	28.1	24.7	26.2	28.8	30.3
Second birth to third birth		35.4	32.0	29.0	29.8	30.4	30.4
Third birth to fourth birth		35.0	32.5	29.2	28.9	29.8	30.5

Source: Line 1: Current Population Reports (40,1979), table 20
Line 2-4: Current Population Reports (39,1978), table 46

Table 3

Effects of exogenous variables on A(T)

		(1) with endogenous switchpoints	(2) with exogenous switchpoints	(3) size of column (1) relative to column (2)
$\frac{dA(T)}{dA_0}$		+	+	smaller
$\frac{dA(T)}{dV}$		+	+	smaller
$\frac{dA(T)}{dW}$		+	+	undetermined
$\frac{dA(T)}{dP_Z}$	$\varepsilon_1^Z < -1$ a/	+	+	undetermined
	$\varepsilon_1^Z > -1$	undetermined	-	larger b/
$\frac{dA(T)}{dP_C}$		undetermined	-	larger b/

Notes: a/ ε_1^Z is the elasticity of Z with respect to $\lambda_1(t)P_Z e^{\rho t}$, see equation (4.3).

b/ Column (1) is larger then column (2), since the effect of column (1) is closer to zero or even positive.

Table 4

The signs of the direct and bequest effects of changes in exogenous variables on the switchpoints

		Direct effect	Bequest effect
$\frac{dt_i}{dA_0}$		0	-
$\frac{dt_i}{dV}$		0	-
$\frac{dt_i}{dW}$		+	-
$\frac{dt_i}{dP_Z}$	$\varepsilon_1^Z < -1$	+	-
	$\varepsilon_1^Z > -1$	+	undetermined
$\frac{dt_i}{dP_C}$		+	undetermined

Table 5

Definition of variables, sample means, and standard deviations

Name	Mean	St. Dev.	Definition
EXPER	12.6	6.52	experience, defined as age-schooling-6
EXPER2	200.	192.	square of EXPER
URATE	5.54	2.55	county's unemployment rate
URAFEB	.224	.415	dummy, =1 if the market situation for unskilled females is better than for unskilled males
URAFEW	.354	.476	dummy, =1 if the market situation for unskilled females is worse than for unskilled males
i_L	2.60	1.66	number of children of parents, living anywhere, of any age
i_B	2.42	1.40	number of children of parents, living at home, younger than 18 years
i_S	3.53	1.68	the order of the next child to be born
FAMINC	97.0	73.2	family income, excluding wife's earnings in hundreds of 1967 dollars; this variable corresponds to "V(t)" in our model
LIMIT	.013	.115	dummy, =1 if husband is limited in his ability to work
YRSMA	12.1	5.91	number of years since marriage; this variable corresponds to "t" in our model
YRSMA2	181.	168.	square of YRSMA
NADULT	.056	.252	number of adults in the family, except for parents
AGECH	18.1	16.6	sum of ages of children younger than 18 living at home; this variable corresponds to "$\sum(t-t_k)$"
AGECH2	185.	226.	sum of squared ages of children younger than 18, living at home : "$\sum(t-t_k)^2$"

Table 5 continued

Name	Mean	St. Dev.	Definition
DUMCH2	.719	.441	dummy, =1 if second child is living at home and younger than 18
DUMCH3	.425	.489	dummy, =1 if third child is living at home and younger than 18
DUMCH4	.439	1.03	number of children from fourth to eighth, who are living at home and younger than 18
WAGEP4	2.34	1.30	wage rate, predicted 4 years ahead
PRWOP4	.529	.363	probability that the individual works, predicted 4 years ahead
KCAREP4	2.86	10.1	child care time needed for the next child born when it would be 4 years old, predicted 4 years ahead (100's of hours)
LNWAGE	.368	.632	log of hourly wage rate, in 1967 dollars
LEISURE	62.2	9.32	hours of leisure, defined as (8760 - labor supply - home time) / 100
HOMETIME	19.2	9.69	hours of home time / 100

Table 6

Maximum likelihood estimates of the time allocation equations and of the full model[a]

Equation	Variable/ parameter	C/M[b]	Expected sign	Time allocation model	Full model
Wage equation	EXPER	C	+	.0334 (.0057)*	.0201 (.0042)*
	EXPER2	C	-	-.0008 (.0002)*	-.0005 (.0081)*
	URATE	C	-	.0034 (.0040)	.0060 (.0026)*
	URAFEB	C	+	.0124 (.0260)	.0015 (.0156)
	URAFEW	C	-	-.0565 (.0231)*	-.0250 (.0160)
Leisure equation	γ_2	M	-	-8.8926 (.3482)*	-19.1035 (.4229)*
	π_{21}	M	+	52.8021 (6.380)*	2.7859 (.5224)*
	i_2	M	+	9.0935 (1.558)*	.0711 (.0463)
	FAMINC	M	0	.0007 (.0010)	.0115 (.0034)*
	YRSMA	M	+[c]	-.4104 (.0495)*	-.2532 (.0644)*
	NADULT	C	+	-1.0795 (.6983)	.7127 (.5187)
	LIMIT	C	+?	-.0008 (.0010)	.0084)(.0034)*
Hometime equation	γ_{31}	M	-	-.1964 (.1683)	-.6101 (.2276)*
	γ_{32}	M	-	-.0449 (.0205)*	-.0623 (.0251)*
	AGECH	M	...	-.8569 (.0764)*	-.7277 (.0820)*
	AGECH2	M	-	.0284 (.0043)*	.0237 (.0096)*
	DUMCH2	M	-[d]	-6.9743 (.6288)*	-5.5937 (.6910)*
	DUMCH3	M	-[d]	-4.6118 (.5028)*	-3.8388 (.5746)*
	DUMCH4	M	-[d]	-4.9451 (.4217)*	-4.1458 (.4825)*
	FAMINC$\cdot i_B$	M	0	-.0004 (.0019)	-.0006 (.0017)
	LIMIT	C	+	.3706 (.6609)	.9695 (1.056)
	FAMINC	M	0	.0800 (.0032)*	.0045 (.0048)
	YRSMA	C	+?	.2705 (.0324)	.1181 (.0366)
Switch-point equation	ϕ_0	M	+		795.9120 (91.40)*
	ϕ_1	M	+[e]		3.8433 (2.348)
	ϕ_2	M	+		-3.5749 (.0898)*
	π_{40}	M	+		1.0154 (.0027)*
	i_1	M	+		.0000 (.0000)

Table 6 continued

Equation	Variable/ parameter	C/M	Expected sign	Time allocation model	Full model
Switch-point equation	β_{31}	M	+		2.6028 (.0136)*
	YRSMA	M	... f/		3.0735 (2.773)
	YRSMA2	M	... f/		-.0382 (.0421)
	WAGEP4	M	+ g/		-14.6767 (1.038)*
	PRWOP4	M	...		11.7386 (1.237)*
	KCAREP4	M	+ h/		-2.1785 (.1985)*
	YRSMA$\cdot\tau_2$	M	... f/		-.0758 (.0131)*
	YRSMA2$\cdot\tau_2$	M	... f/		.0019 (.0005)*
	WAGEP4$\cdot\tau_2$	M	- g/		-.0469 (.0099)*
	PRWOP4$\cdot\tau_2$	M	...		.0419 (.0109)*
	KCAREP4$\cdot\tau_2$	M	- h/		.0437 (.0026)*
	FAMINC	M	0		-.0330 (.0025)*
	CONSTANT	C	...		549.6952 (92.17)*
Averages/standard deviations of fixed effects					
	τ_1	M	...	.40 (.58)	.49 (.54)
	τ_2	M	...	-53.21 (11.55)	77.99 (12.56)
	τ_{31}	M	...	11.78 (22.37)	12.83 (22.84)
	τ_{32}	M	+?	9.33 (9.71)	9.02 (9.99)
	τ_4	M	+		-8.58 (7.51)
Covariance matrix	σ_{11}			.2204 (.0101)*	.2469 (.0113)*
	σ_{12}			2.6373 (.1984)*	5.6715 (.3232)*
	σ_{22}			106.0535 (4.992)*	207.2551 (10.59)*
	σ_{131}			.2800 (.1743)	.0168 (.0306)
	σ_{231}			-35.4306 (3.181)*	-39.6798 (2.554)*
	σ_{331}			36.7146 (1.780)*	36.5189 (1.868)*
	σ_{14}				.1244 (.0487)*
	σ_{24}				-.0002 (.0002)
	σ_{34}				-.1525 (.0448)*
	σ_{44}				5.1452 (.7535)*
	σ_{132}			.0453 (.0675)	.2525 (.0493)*

Table 6 continued

Parameter		Time allocation model	Full model
Covariance matrix	σ_{232}	1.3197 (1.113)	6.6251 (1.275)*
	σ_{332}	.6873 (.2062)*	.9577 (.2308)*
	σ_{342}		-.7682 (.0995)*
Value of likelihood function		-10119.2	-10359.8
Number of observations		162	162

Notes:

a/ Asymptotic standard deviations in parentheses; * indicates significant at 5 percent level or better.

b/ C indicates control variable; M indicates model-induced variable.

c/ Based on the conjecture that the interest rate r exceeds the rate of time preference ρ (section 4).

d/ If economies of scale in rearing children exist.

e/ ϕ_1 should have the same sign as $(r-\rho)$.

f/ It is expected that, if $(\pi_{411}+\pi_{421}\tau_2)$YRSMA + $(\pi_{412}+\pi_{422}\tau_2)$YRSMA2 is positive (negative), $(\pi_{421}+\pi_{422}$YRSMA$)$ will be negative (positive).

g/ In addition, $(\pi_{413}+\pi_{423}\tau_2)$, the coefficient of WAGEP4, should be positive.

h/ In addition, $(\pi_{415}+\pi_{425}\tau_2)$, the coefficient of KCAREP4, should be positive.

<u>Table 7</u>

Implied parameters

Parameters	Time allocation model	Full model
β_{21}	-5.994	-2.256
β_{22}	5.938	.150
$r-\rho$	-.046	-.013
ε_1	.629	.406
ε_2	-.085	-.061
B_0	.529	1.027

APPENDIX A THE IMPLICIT FUNCTION $g^{1,i}$

The explicit form of the implicit function $g^{1,i}$ is:

$$g^{1,i} = e^{-\rho t_i} \{U(Z_{i-1},L_{i-1},i-1) - U(Z_i,L_i,i)\} + \\ + \lambda_1 P_Z (Z_i-Z_{i-1}) + \lambda_1 W (L_i-L_{i-1}) + \\ + \int_{t_i}^{t_i'} \{r\lambda_1(WB_i+P_C C_i) - \lambda_1 \frac{dW}{dt} B_i\} dt \qquad (A.1)$$

where the variables of the first two lines are evaluated at $t=t_i$. Equation (A.1) shows how costs and benefits of delaying child i are balanced at time $t=t_i$. The first line of (A.1) shows the utility loss of delay. The second line measures, in utils, the savings of reduced consumption of Z and L, if child i would be delayed. The third line involves the entire expenditure profile of child production. The term $r(WB_i(t)+P_C C_i(t))$ indicates interest earned on the money spent at stage $t-t_i$ of the production proccess: if t_i shifts to t_i+dt_i, this stage shifts to $t-t_i+dt_i$. Since the wage rate is not constant, this new stage may be faced with a different price of time, the effect of which is measured by $B_i(t)(dW/dt)$.

APPENDIX B

Table B.1

Signs, with their conditions, of the derivatives of the implicit functions g^0 and $g^{1,i}$, when N is positive over the whole lifecycle.

Derivative	Sign	Condition a/
X_{00}	+	
X_{0i}	-	$U_{13} > 0$, $U_{23} > 0$,
		$\dot{W} < rW$ for $t_i \leq t \leq t_i'$
X_{0A_0}	-	
X_{0V}	-	
X_{0W}	-	
X_{0P_Z}	+	if $\varepsilon_1^Z > -1$ b/
	-	if $\varepsilon_1^Z < -1$
X_{0P_C}	+	
X_{i0}	-	$U_{13} > 0$, $U_{23} > 0$,
		$\dot{W} < rW$ for $t_i \leq t \leq t_i'$
X_{ii}	-	due to second order conditions
X_{ij}	0	for $i \neq j$
X_{iA_0}	0	
X_{iV}	0	
X_{iW}	+	$U_{23} > 0$ if $t' \leq t_i \leq t''$,
		$\dot{W} > rW/\varepsilon_1^B$ if $t_i < t_i'$ c/
X_{iP_Z}	+	$U_{13} > 0$
X_{iP_C}	+	$\dot{W} < rW$, $\varepsilon_2^C > -1$ d/

Notes: a/ All conditions are sufficient.

b/ ε_1^Z is the elasticity of Z with respect to $\lambda_1(t)P_Z e^{\rho t}$

c/ ε_1^B is the elasticity of B_k with respect to W

d/ ε_2^C is the elasticity of C_k with respect to P_C

Table B.2

Signs, with their conditions, of the derivatives of the implicit functions g^0 and $g^{1,i}$, in the case of temporary labor force withdrawal

Derivative	Sign	Conditions/Remarks a/
X_{00}	+	
X_{0i}	undetermined	
X_{0A_0}	-	
X_{0V}	-	
X_{0W}	-	
X_{0P_Z}	+	if $\varepsilon_1^Z < -1$
	-	if $\varepsilon_1^Z < -1$
X_{0P_C}	+	if $\varepsilon_2^C > -1$
X_{i0}	undetermined	
X_{ii}	-	sufficient but not necessary condition in second order conditions
X_{ij}	undetermined	It is positive when child production follows a fixed-coefficient technology, for $i \neq j$
X_{iA_0}	0	
X_{iV}	0	
X_{iW}	undetermined	if $t' \leq \bar{t} < t''$
	+	if $\bar{t} < t'$ and $\dot{W} < rW/\varepsilon_1^B$
	0	else
X_{iP_Z}	+	$U_{13} > 0$
X_{iP_C}	undetermined	It is positive when child production follows a fixed-coefficient technology

Notes: a/ All conditions are sufficient.

APPENDIX C THE SAMPLE SELECTION CRITERIA

This appendix lists the criteria we used to determine the usable responses for our sample. After twelve waves, in 1979, the PSID study supplied data on 6373 observations (i.e. household units). Our criteria cut into this set in the following way (each "loss" represents the number of observations that did not satisfy the additional criterion):

1. White women, married in 1979. . . . . loss: 3792
2. No change in marital status from 1968 to 1979 . . . . loss: 824
3. Sex of head of family is male in every year. . . . . loss: 16
4. No change in husband or wife. . . . . loss: 585
5. Education of wife is known . . . . loss: 3
6. County unemployment rate is not missing for two or more consecutive years. . . . . loss: 55
7. Variable that indicates whether the market situation for unskilled females is better, same or worse than for unskilled males is not missing for two or more consecutive years. . . . . loss: 102
8. Consistent reports of wages and hours worked (i.e. not wages zero and hours worked positive or vice versa) . . loss: 55
9. Age of wife in 1968 is between 20 and 45. . . . . . . loss: 301
10. Year in which wife married is known . . . . loss: 2
11. Reported ages of children are usable . . . . loss: 2
12. Wife worked at least once during the sample period . . loss: 84
13. Wife bore a child when older than 45 . . . . loss: 30
14. Wife did not bear a child during the sample period . . loss: 320
15. Changes in family composition were tractable . . . . loss: 36
16. Estimation of fixed effects converged. . . . . loss: 4

The total number that failed to satisfy one or more of the criteria is 6211, leaving 162 usable observations.

A note should be made about criteria 6 and 7. In the event that the county unemployment rate and the market situation variable were unknown for one year in a row only, the unemployment rate was taken to be the average of that of the year before and after. Moreover, if the unemployment rate was unknown in 1978, the last year, it was set equal to that of 1977, if that was not missing. A similar rule was used for the market situation variable, on which we based the dummy variables URAFEB and URAFEW. Due to the averaging rule, these dummy variables can take on the value of 0.5. The first two years, 1968 and 1969, of the PSID study did not contain this variable, and therefore URAFEB and URAFEW are given the 1970 value in those two years. These variables occur only in the wage equation.

In explanation of criterion 15, in some cases reported family composition was inconsistent with the 1976 report, which we used as benchmark to establish the size and age structure of the family. When discrepancies could not be obviously reconciled, the observation was rejected.