8-1-1981

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Box 1987, Yale Station
New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 381

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August 1981

Notes: This paper is based on the author's 1981 Ph.D. Dissertation at the University of Minnesota. He is grateful to Anne O. Krueger, Lars Hansen, Thomas Sargent, Christopher Sims, Vernon Ruttan, Rao Aiyagari, Martin Eichenbaum, Wallace Huffman, Dan Peled, T.N. Srinivasan and Ken Wolpin for useful comments on previous drafts of the paper.

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I. Introduction

The issues concerning the determinants of agricultural production, food supply, and their growth are currently of great interest in developing and developed countries. This in turn has led to extensive research into the effectiveness of various price intervention schemes and other incentives that can be offered within the agricultural sector. Basic to the entire analysis is a qualitative and quantitative understanding of the determinants of the dynamics of supply and its responses to altered incentives in agriculture.

The land allocation decision could be regarded as an example of a discrete process over time within a competitive market for the output. Using annual average prices, economists have suggested different theoretical and empirical ways to evaluate farmers' responses to changes in crop prices. The existence of consistent patterns of serial and cross-serial correlations between land allocations, production and prices has been observed and debated in the economic literature for many years. The best known were the Cobweb theory (Ezekiel [1933]) and the observations on the Corn-Hog Cycle as discussed in Coase and Fowler [1935, 1937]. The fact that output selling price is not observed at the time when input decisions are made and the necessity for farmers to form expectations on the future price have been suggested as the main reasons for the cyclical movements of output.

Early single equation estimates, with current output as a function only of one past price, showed small link between prices and output. Then came the pioneering work of Nerlove [1956, 1958], who showed that a distributed lag model could explain much of the supply response to output price changes. Using static microeconomic theory Nerlove [1958] justified an econometric framework for interpreting farmers' responses to prices by estimating a single distributed lag equation. This equation describes the current area as a linear function of lagged areas, the lagged price and other current and lagged exogenous variables. The coefficients are non-linear functions of the parameter of a linear supply equation, an adjustment parameter for desired area versus actual area and an adaptive expectations parameter.[1] Askari and Cummings [1976] report on more than 600 estimates of different versions of Nerlove's model for many crops and countries. Muth [1961] criticized the adaptive expectation formulation of Nerlove and suggested the rational expectations hypothesis. More recently, Nerlove [1979] analyzed the traditional supply response model in light of recent developments in economic time series models (e.g., Nerlove et al. [1979a]). In my view, the main drawbacks of the Nerlovian [1958] model are that it did not analyze the specific dynamics of the crops production functions and that the model's structural parameters are independent of the crops price processes (see Eckstein [1981]). Hence, the Nerlovian [1958] model is subject to Lucas's [1976] general critique on economic policy evaluation.

In this study, an empirical model of agricultural supply is derived from a dynamic and stochastic framework where farmers are assumed to maximize the expected present value of profit subject to dynamic and stochastic technology and their information.[2] Farmers are assumed to form rational expectations, i.e., they are assumed to know the actual distributions of exogenous variables, as well as land productivity which is assumed to be endogenous. The analysis focuses on the dynamics of the crop production technology and the simultaneous determination of aggregate land productivity, land allocation and crop prices. Hence, a farmer's input decision rules depend on the parameters of the actual dynamic process of prices which are subject to governmental control. In this context, it is straightforward to show that rational farmers are unlikely to interpret price fluctuations that are serially uncorrelated as signalling permanent alteration in the incentives confronting them. Furthermore, any permanent or temporary changes in taxes, subsidies and tariffs policies affect the dynamic response of the cropped area, such that the structural form of the land allocation equation varies with the policy rule. Consequently predictions with respect to changes in policy require complete identification of the economic relations. We show that this model may give rise to dynamic land allocation that exhibit the "Cobweb Phenomenon" of frequent fluctuations. The main causes for the fluctuations in land allocations and production are the inherent dynamics of land productivity in the production function (i.e. depletion of land fertility), the stochastic movement of international crop prices and the shocks to productivity from some uncontrolled events (e.g., weather and water supply). The model is implemented by investigating data on the Egyptian agricultural sector, including cropped acres, crop yields and prices. The farmers produce an export crop (cotton) and an import crop (wheat) so they respond to prices and to governmental policies in an open economy.

The plan of the paper is as follows. In section II we discuss the technology of annual crops production. In section III we solve and analyze a dynamic land allocation model for two crops where output prices are exogenously given. In section IV we discuss the effects of other inputs on the dynamics of supply. Time series analysis of the Egyptian data and estimation of the land allocations model from section III are reported in section V.

II. The Technology

When land is continuously cultivated, the issue of substitution and complementary effects in production of alternative crops becomes important. Cotton and corn are high nitrogen using crops. Soybeans, clover and alfalfa (leguminous plants) supplement the nitrate content of soil. The depletion of nitrate from the soil is an important direct constraint on the development of land fertility and the production of all crops. Furthermore, monoculture cause an accumulation of crop specific insects and worms which have an important indirect effect on the actual crop yield from the land. Hence, the current productivity of land for a given crop depends on the cropping history of a plot of land.

Crop rotation is the well known method to prevent the direct and the indirect deterioration in land productivity under continuous cultivation. Fertilizer and pesticides are the main inputs which control directly land productivity by building up the content of the soil and eliminating the insects and the worms.

The existance of deterioration in land productivity introduces a non-trivial dynamic element in the allocation of land between different crops. In general, the above technological characteristics of crop production imply that the current marginal product of past land allocations for a specific crop is negative. Furthermore, farm production is identified with the fact that almost all input decisions are made before output prices are known, and the final output is subject to unknown shocks from water supply and weather conditions. Both the prices and the shocks to production are uncontrolled stochastic processes that affect farmers' income. Hence, the practice of crop rotation and the application of fertilizers and pesticides are outcomes of a stochastic dynamic optimization problem that farmers have to solve. Thus, crop rotation, that is defined by the sequence of land allocations, is a function of the past land allocations as well as the stochastic processes of the uncontrolled variables.

In what follows, we analyze the effects of deterioration in land productivity on the dynamics of crop supply, land allocation, farmers response to price and the observed serial and cross-correlations between area, yield and prices. The analysis is done by using explicit approximations for a production process that includes almost all of the technological components that have been described above. The explicit functional forms enable us to derive analytical solutions for the farmers optimization problem which simplify the exposition of the results and provide regression equations for estimation.

III Dynamic Land Allocation for Two crops

In this section we analyze a stochastic dynamic optimization problem of a farmer endowed with land that can be allocated between two different crops (e.g., cotton and wheat). We show that if the cultivation of at least one crop (e.g., cotton) results in deterioration of land productivity, due to successive use of the land for that crop, the optimization yields a dynamic land allocation process. The optimal decision can be interpreted as a crop rotation with the property that current land allocation depends on past land allocations, expectations of future crop prices, and other variables that are part of the objective function or part of the constraint functions.

It is assumed that crop prices are exogenously determined such that aggregate land allocations do not affect the movement of the prices over time. For simplicity, the model considers a representative farmer whose only variable factor of production is land.

Consider the definitions of the following variables:

X_{it} is the production of crop i at time t,

P_{it} is the price that farmers receive for the production of crop i at time t,

A_{it} is the land allocated to crop i at time t,

$\overline{A}$ is the total available cultivated land at time t,

$0 < \beta < 1$ is the objective discount factor,

a_{it} is the shock to production of crop i at time t,

S_t is a vector of n-2 exogenous variables at time t, such as taxes, tariffs and other variables that contain information on P_{it}'s, and a_{it}'s,

f_1, f_2, g_1, d_1 are positive parameters of the production functions,

E is the mathematical expectation operator, where $E_t(X) = E(X|\Omega_t)$ and Ω_t is the information set at time t + 1,

L is the lag operator which is defined by the property

$$L^k X_t = X_{t-k}.$$

The farmer is assumed to maximize his discounted expected profit in terms of the price of crop 1 (cotton). Hence, the farmer's objective is to maximize

$$E_{-1} \lim_{N\to\infty} \sum_{t=0}^{N} \beta^t (X_{1t} + \frac{P_{2t}}{P_{1t}} X_{2t}) . \tag{3.1}$$

The maximization is subject to three technological constraints ,

Land Constraint

$$A_{1t} + A_{2t} = \overline{A} \tag{3.2}$$

The production function of crop 1

$$(3.3) \qquad X_{1t} = \{(f_1 + a_{1t}) - \frac{g_1}{2} A_{1t} + d_1(1 - \frac{A_{1t-1}}{\bar{A}} - \frac{A_{1t}}{\bar{A}})\} A_{1t}$$

The production function of crop 2

$$(3.4) \qquad X_{2t} = (f_2 + a_{2t})A_{2t} \cdot$$

The production function of crop 1 is quadratic, strictly concave in A_{1t} and is subject to shocks, a_{1t}. The last term in (3.3), $d_1(1 - \frac{A_{1t-1}}{\bar{A}} - \frac{A_{1t}}{\bar{A}})$, is meant to approximate the deterioration in land productivity. For $d_1 > 0$, our particular approximation suggests that if the summation of the fractions of land from last and current periods is greater than one, then the current average productivity of land reduced. Furthermore, if the summation of $A_{1t}/\bar{A}$, and $A_{1t-1}/\bar{A}$ is less than one, the current cultivation of crop 1 is on land that has been used for crop 1 for only the current year. Hence, the average productivity is increased. If the sum of $A_{1t}/\bar{A}$ and $A_{1t-1}/\bar{A}$ is equal to one, there is no linkage between the current average productivity of land and past cultivations. Notice that this term introduces a dynamic element into the production function. Only if it turns out that $\frac{A_{1t}}{\bar{A}} = \frac{1}{2}$ for all $t \geq 0$, would the farmer's problem seem to be static. In what follows, we show that a positive d_1 gives rise to a land allocation process that can be regarded as crop rotation, which is a well known practice in agriculture when land deteriorates under continuous cropping.

If we substitute (3.2) - (3.4) into (3.1), the farmer's problem becomes:

Maximize

$$(3.5) \qquad J = E_{-1} \lim_{N\to 0} \sum_{t=0}^{N} \beta^t\{(f_1 + a_{1t})A_{1t} - \frac{g_1}{2} A_{1t}^2 + \frac{d_1}{\overline{A}} (\overline{A} - A_{1t-1} - A_{1t})A_{1t}$$

$$- R_t A_{1t} + R_t \overline{A}\}$$

by choice of A_{10}, A_{11}, A_{12}, ..., where $R_t = \frac{1}{P_{1t}} \{P_{2t}(F_2 + a_{2t})\}$ is the "real shadow price" for crop 1 land allocations, and Ω_{t-1} is the farmer's information set at time t which assumed to be

$$\Omega_{t-1} = \{A_{1t-1}, A_{1t-2}, \ldots, a_{1t-1}, a_{1t-2}, \ldots, R_{t-1}, R_{t-2}, \ldots$$
$$S_{t-1}, S_{t-2}, \ldots\}.$$

The optimization is subject to a given level of $A_{1}, _{-1}$ and a given law of motion for the stochastic processes of a_{1t}, R_t and S_t, i.e.,

$$(3.6) \qquad \delta(L)Z_t = U_t$$

where $Z_t' = [a_{1t}, R_t, S_t]$ and

$$\delta(L) = I - \delta_1 L - \delta_2 L^2 - \ldots - \delta_k L^k ,$$

where δ_j is an n x n matrix for j = 1, ..., k, U_t is an n x 1 vector, where $E(U_t | \Omega_{t-1}) = 0$ and $E[U_t U_t'] = \Sigma_t$, and where Σ_t is a positive semi-definite matrix. Further, it is assumed that the vector stochastic process (3.6) is of mean exponential order less than $1/\sqrt{\beta}$, so that a constant and a trend can be part of the vector S_t. It is assumed that the variables in the vector Z_t are uncontrollable and unaffected by the farmer's decisions, i.e., prices are assumed to be exogenously given to the representative farmer.

In appendix A we derive the optimal decision rule for problem (3.5) and we show that the unique solution can be written as (see A.8):

$$(3.7)\quad A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1 \overline{A}}{d_1} \sum_{j=0}^{\infty} (\beta\lambda_1)^j \left[f_1 + \frac{d_1}{2} + E_{t-1}(a_{1t+j}) - E_{t-1}(R_{t+j})\right]$$

for all $t = 0, 1, 2, \ldots$. Where $-1 < \lambda_1 < 0$ and λ_1 is a function of g_1, d_1, $\overline{A}$ and β.[3]

Note that A_{1t} depends on current expectations of <u>all</u> future values of the exogenous variables weighted by a factor that depends on the parameters of the production function. Further, land allocation at time t depends on the last period decision which is known at time t. In general, if we include more than a one year deterioration effect, the number of lags of land allocations in (3.7) will be equal to the number of years in the cumulative dynamic factor in the production function.[4]

For any arbitrary set of expectations, (3.7) implies that:

$$(3.8)\qquad \frac{\partial A_{1t}}{\partial E_{t-1}(R_t)} < 0 \qquad \text{and} \qquad \frac{\partial A_{1t}}{\partial E_{t-1}(R_{t+1})} > 0 .$$

Hence, if farmers expect that the <u>current</u> output price of crop 2 relative to the price of crop 1 is going to decrease, they will <u>increase</u> the current land allocated to crop 1. But, if farmers expect that in the following year the price of crop 2 relative to the price of crop 1 is going to decrease, they will <u>decrease</u> the quantity of current land allocated to crop 1. The first result is exactly as any static model would predict. However, the second result is different from that of any static model or the usual dynamic model with costs of adjustment in land allocations.[5] In a static model the second term in (3.8) is zero. Dynamic models with adjustment costs in land allocations, imply that the one-year ahead output prices affect current decisions. In Appendix B we show that the adjustment costs model is equivalent to our model if d_1 is <u>negative</u>. In this case, λ_1 is positive and less than one, and we have the same result for the first term in (3.8) but the opposite result with respect to the second term.

The assumption of rational expectations implies that farmers maximize (3.5) subject to the true stochastic process of the exogenous variables. Therefore, the conditional mathematical expectations of the exogenous variables depend on their stochastic process (3.6) and the information farmers are assumed to have at time t, which includes $Z_{t-1}, Z_{t-2}, \ldots$.

Assuming rational expectations in the certainty case, (3.7) is the optimal decision rule for land allocations to crop 1, where $E_{t-1}(a_{1t+j}) = a_{1t+j}$ and $E_{t-1}(R_{t+j}) = R_{t+j}$ for all $j = 0,1,2, \ldots$, <u>i.e.</u>, perfect foresight. In the uncertainty case the optimal decision rule can be written as (see Appendix A, (A.11)) a function of variables that are known to the farmer at time t, i.e.,

$$(3.9) \quad A_{1t} = \lambda_1 A_{1t-1} + \gamma + \mu_1(L)\, a_{1t-1} + \mu_2(L)\, R_{t-1} + \mu_3(L)\, S_{1t-1} + \mu_4(L)\, S_{2t-1} + \cdots + \mu_n(L)\, S_{n-2t-1} \; .$$

for all $t = 0, 1, 2, \ldots$. Where $\gamma = \dfrac{\lambda_1 \bar{A}(f_1 + \frac{d_1}{2})}{d_1 \beta(1 - \beta\lambda_1)}$

and $\mu_i(L) = \mu_{i0} + \mu_{i1}L + \ldots + \mu_{iJ}L^J$ for all $i = 1, 2, \ , n$,

where $J \leq k$.

Equation (3.9) is an exact closed form analytical solution for the farmer's optimal land allocation decision rule at time t.

Observe that μ_i's coefficients are some non-linear function of λ_1, β, d_1 and δ_s's coefficients, which expresses the restriction imposed across the decision rule and the parameters of the stochastic processes for variables in Z_t. Further, notice that all the variables that are in the information set which help to predict future values of prices (R's) and technological shocks (a_1's) are in the decision rule. Hence, the lagged Z's are instruments for the farmer's solution of his prediction problem and they turn out to be instruments for the econometrician's estimation problem. Note that the constants in the vector stochastic process Z_t are part of the decision rule, therefore, one of the μ's is a constant containing the constants of the processes. For example, a once-and-for-all deterministic shift in prices will immediately affect the current land allocation through a change in the constant of the R_t process. The magnitude of the immediate and the long run response depend on the values of λ_1, β, d_1 and δ_s's. Hence, predictions with respect to a permanent change in relative prices require a complete identification of the model's parameters, eventhough prices are exogenous (see Lucas [1976]). As long as the uncontrolled variable are stochastic, land allocations do not necessarily move toward a static allocation. However, the mean of A_{1t} is deterministic and can be regarded as the long run land allocation. From (3.9) it is clear that a negative λ_1 ($d_1 > 0$) implies a lower mean for A_{1t}, versus a positive ($d_1 < 0$) or zero ($d_1 = 0$) degree of serial correlation in land allocations. Hence, the deterioration in land productivity decreases the average land allocations for crop one and implies a particular pattern of cyclical movements in the areas planted to different crops.

Suppose we consider the following case: the shocks to production (a_1's) and the price (R's) are serially uncorrelated and are independent of variables that are in the information set, a_{1t} has zero mean and R_t has a positive mean. The equation (3.9) can be written as:

(3.10)

$$A_{1t} = \lambda_1 A_{1t-1} + \gamma + \frac{\lambda_1 \bar{A}}{d_1 (1 - \lambda_1 \beta)} \cdot \text{(mean of R)}$$

and the mean of A_{1t}, A_1^*, is

$$A_1^* = \frac{\gamma}{1 - \lambda_1} + \frac{\lambda_1 \bar{A}}{d_1 (1 - \lambda_1 \beta)(1 - \lambda_1)} \cdot \text{(mean of R)}$$

For the relevant domain of d_1, we obtain $\frac{\partial A_{1t}}{\partial d_1} < 0$ and $\frac{\partial A_1^*}{\partial d_1} < 0$. Thus increasing the rate of land deterioration decreases the area allocated to crop 1. Equation (3.10) shows that farmers would not interpret price fluctuations and shocks to production that are serially uncorrelated as signalling permanent alteration in the incentives confronting them.[6]

Consider the experiment of a once-and-for-all increase in the mean of the relative price, R. Using equation (3.10) the immediate response for A_{1t} is a decrease below the (lower) new level of A_1^*, and by frequent fluctuations to converge toward the new mean of A_{1t}. Hence, the 'short run' effect is greater than the 'long run' and the "Cobweb Phenomenon" is, in this model, an optimal response and has nothing to do with price expectations.

In the general case, the first equation in (3.10) is only part of (3.9), where the mean of R is replaced by the constant in the stochastic difference equation for R in (3.6). The second equation in (3.10) is the unconditional mean of A_{1t} ignoring the effects of variables which are in Ω_{t-1} besides the relative prices (R's). Observe that a once-and-for-all increase in the mean of R is equivalent to an increase in the constant in R's stochastic equation. Hence, the qualitative implication of the above experiment holds in the general case as well.

It is straightforward to see that in the case of adjustment cost ($\lambda_1 > 0$ and $d_1 < 0$), the sign of both the immediate and the long run effects of the above experiment are retained, but the magnitude of both <u>increases</u>. However, the short run effect is <u>lower</u> than the long run (see Nerlove [1958]) and the convergence toward the mean is a downward smooth path, rather than the frequent fluctuations as in the case where $d_1 > 0$. In general, the structure of the stochastic process of the relative price has an important effect on the predicted movements of land allocations due to changes in prices or/and other variables that affect prices. This includes the magnitude of the difference between the immediate response (short run) and the average change (long run) in land allocations due to changes in prices.

In order to see the difference between a cost of adjustment model and a model where land productivity deteriorate, consider the following numerical example:[7]

Case 1: Land productivity deteriorates such that $\frac{d_1}{\bar{A}} = d = .1$, and the land allocation decision rule is:

$$A_{1t} = -.48A_{1t-1} + 79.0 - 1.97R_{t-1} + 1.63a_{1t-1}$$

Case 2: An adjustment costs model where $d = -.1$ and

the land allocation decision rule is:

$$A_{1t} = .48A_{1t-1} + 49.0 - 3.08R_{t-1} + 2.33a_{1t-1}$$

Assuming that the innovations in R_t and a_{1t} processes are distributed as normal with mean zero and variance of one, we simulated the model for 100 observations. The means and the variances for land allocations are 40.2 and 4.8 for case 1 and 39.6 and 29.0 for case 2. The wide difference in the variance of land allocations between case one and two is due to the strong responses (high elasticity) to changes in prices and shocks to productivity in the adjustment costs model vis-a-vis moderate responses (low elasticity) in the case of deterioration in land productivity.[8] Figure 1 depicts the difference in the area responses to a once-but-not-for-all shock in productivity - - the "Cobweb Phenomenon" in case one and the conventional adjustment process in case two.

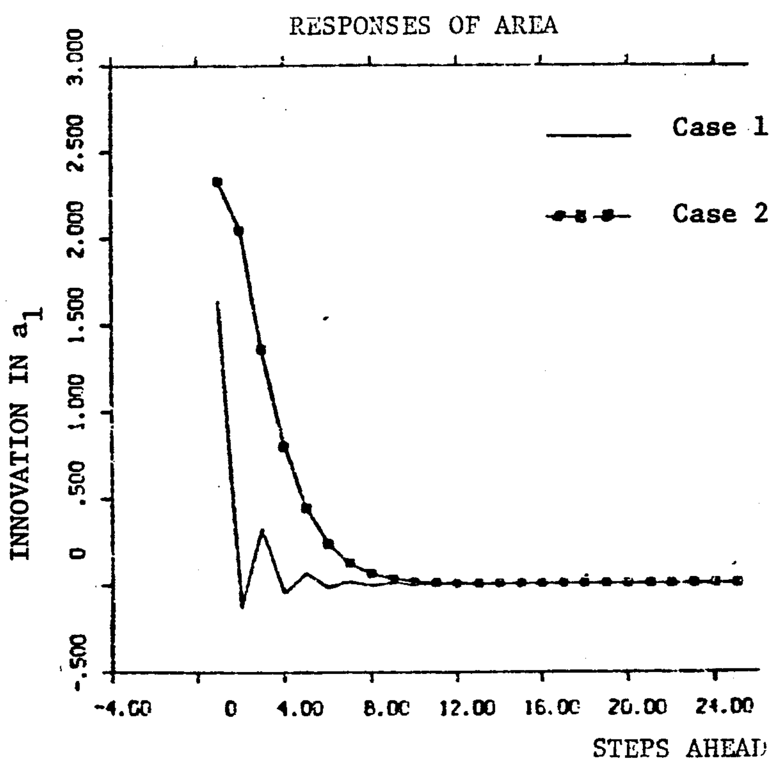

FIGURE 1

This line of reasoning emphasizes the important role of the dynamic structure of the production technology, the information farmers have at the time inputs are committed to production and the way relative prices are moving over time, in the determination of farmers' response to changes in crop prices. In order to understand the dynamics of supply, to evaluate and to predict farmers' responses to changes in incentives, we should investigate jointly the dynamics of the production process and the dynamics of the actual crop prices that farmers observe. Note that the traditional supply response model ignores both of them.

Estimating the underlying parameters of the model is one of the main objectives in the process of understanding supply responses and the land allocation decision process. Equation (3.9) is almost a regression equation. If we do not observe some of the variables that are part of the farmers' information set, we can construct an error term for (3.9) that has the properties of a regression equation. This equation has a distributed lag form where the coefficients are some non-linear functions of the parameters in the objective function (3.5) and the stochastic processes (3.6). Furthermore, the reduced form of this equation is observationally equivalent to the traditional supply response model (Nerlove [1958, 1979]), but the model of this work has a completely different interpretation of the observed pattern of serial and cross-correlations between crop areas and crop prices.[9] In particular, the correlations that we may find by estimating the reduced form distributed lag equation from (3.9) reveals almost nothing regarding the response to the traditional experiment of a one-and-for-all change in the relative prices. Moreover, we do not restrict the sum of the coefficients on the lagged R_t's to be less or equal to one and their values have no particular economic or econometric meaning. Finally the existance of any pattern of serial and cross-correlations between areas and prices can be due to controlled technological constraint (e.g. depletion of nitrate or costs of adjustment) or/and uncontrolled variables (e.g. shocks to productivity) that are not observed by the econometrician. Hence, the interpretation of any observation is entirely an empirical question that can be partially resolved by estimating the above model.

IV. Land Allocations and Other Inputs

What are the effects of fertilizer, labor and pesticides on the land allocation decision rule? In general, if the production function of crop one is separable between land and any other inputs, the decision rule (3.9) stays the same. The average product of land may change due to labor and fertilizer decisions and the separability does not rule out substitution between factors of production.

Theoretically, we can specify a production function that exhibits a complicated interactions between factors of production which includes both static and dynamic elements. Hansen and Sargent [1981] discuss methods for solving these types of models. The main problems in attempting to do this are more practical. First, we usually do not have observations on inputs (aside from land) according to their allocation for the different produced crops. Second, the number of series and parameters increases such that we are not able to estimate the system. However, the interaction between inputs may affect the main dynamic properties of the land allocation decision rule. To see that, we consider a simple example with fertilizer. Let F_{1t} be the fertilizer that is allocated to crop 1 at time t, and let the production function for crop 1 be,

$$(3.3)' \quad X_{1t} = \left\{ f_1 + a_{1t} - \frac{g_1}{2} A_{1t} + d_1 \left(1 - \frac{A_{1t-1}}{\bar{A}} - \frac{A_{1t}}{\bar{A}}\right) + w_1 F_{1t} \right\} A_{1t} - \frac{w_2}{2} F_{1t}^2$$

where w_1 and w_2 are positive scalars. Then, substituting (3.3)' rather than (3.3) into (3.1) and subtracting the cost of fertilizer from the farmer's problem, we can find the first order necessary conditions of the farmer's problem with respect to F_{1t} and A_{1t}. Hence, we can solve for F_{1t} in terms of A_{1t} and the current price of fertilizer, since F_{1t} has no dynamic interaction with A_{1t}.[10] Then,

$$(4.1) \qquad F_{1t} = \frac{w_1}{w_2} A_{1t} - \frac{PF_t}{w_2} \quad \text{for } t = 0, 1, 2, \ldots$$

where PF_t is the price of fertilizer at time t divided by the price of crop one. Using (4.1), the first order condition with respect to A_{1t} can be transformed to the following equation:

$$(4.2) \qquad \beta d\left(1 + \frac{g}{\beta d} L + \frac{1}{\beta} L^2\right) A_{1t+1} = f + E_{t-1}(c_t) \text{ for } t = 0, 1, 2, \ldots$$

where

$$d = \frac{d_1}{\bar{A}}, \quad f = f_1 + \frac{d_1}{2}, \quad g = g_1 + 2\frac{d_1}{\bar{A}} - \frac{w_1^2}{w_2} \quad \text{and} \quad c_t = a_{1t} - R_t - \frac{w_1}{w_2} PF_t .$$

Solving (4.2) using the methods in Appendix A, the land allocation decision rule has exactly the same form as the solution for the original problem (3.5). Here we have the price of fertilizer, PF, as an additional element in the optimal decision rule and in the uncontrolled vector stochastic process of Z_t. However, we may have one important difference between the two solutions. If $\frac{w_1^2}{w_2} > g_1 + 2\frac{d_1}{\bar{A}}$ the coefficient g is negative and if $|\frac{g}{d}| > 1 + \beta$ we have a real solution with $0 < \lambda_1 < 1$. Hence, the serial correlation in land allocation is positive such as in the costs of adjustment case. The economic interpretation of the above result is very simple. If the production of crop one is very responsive to fertilizer applications (large w_1 and small w_2), the rotation element in land allocation may completely disappear. In the above example the predicted effects of changes in the expected price of fertilizer are exactly as of the relative crop price (R) and crop 2 can be viewed as taking the role of fertilizer in the land allocation for crop one.

The above example shows that direct interaction of different factors of production with land productivity may strongly affect the dynamic properties of the optimal land allocations and the supply responses to changes in the relevant prices.

V. Time Series Analysis and Estimation

Econometric analysis of observed data is central to the understanding of the dynamics of crop supply and land allocations. The main objective is to evaluate whether a particular qualitative interpretation of a general phenomena is supported by the data. Furthermore, quantitative evaluations of supply responses to changes in incentives improve our ability to measure and to forecast the effects of policies and distortions in agriculture. In sections III and IV we showed that the dynamic properties of the technology may have important implications for production responses to changes in prices. Hence, the goal is to estimate the model's parameter and to test the model's assumption using all the restrictions and information that are included in the model and the available data.

An important virtue of models such as in section III and IV is that the solution provides a system of linear equations by which we can estimate the model's parameters and test the model's assumptions. The reduced form equations of almost any model based on a linear-quadratic optimization problem, is a system of stochastic difference equations which consist of exogenous and endogenous stochastic variables. The equations of the endogenous variables are linear transformations of the decision rules and the additive errors are due to unobserved exogenous variables. The exogenous variables equations are part of the optimization problem (e.g., (3.6) in section III). In general, the reduced form equations can be written as a vector ARMA model that is subject to cross equation and within equation restrictions. Thus, the reduced form coefficients are non-linear functions of the underlying parameters of the model. Furthermore, the model's parameters are usually over identified and efficient estimation methods require the joint estimation of all equations.[11] If the unobserved variables are assumed to have a low order (e.g. first order) serial correlation we usually can write the reduced form as a finite order vector autoregression (VAR) or a system of stochastic linear difference equations. The exogenous stochastic variables have the assumed property that they are not Granger [1969] caused by the endogenous variables. This property holds only if the observed variables are not Granger caused by unobserved variables.[12] Then, the reduced form VAR has a triangular form.

The models in the previous sections exhibit the property that different specifications of farmers' objective functions and constraints as well as different market structures give rise to almost identical reduced form equations. Hence, the a priori choice of a particular specification of a model for estimation is not a well defined problem that can rigorously be solved.

In what follows, we first introduce the data set from our case study - -

Egyptian agriculture. Then, we analyze and summarize the dynamic properties of the data by estimating and simulating a finite order unrestricted VAR. Finally, we present estimation results of a particular specification of the land allocation model for two crops.

V.1. Cotton and Wheat in Egyptian Agriculture

The motivation for this study comes largely from the important role of cotton and wheat in agricultural production and the balance of trade of the Egyptian economy, as well as the fairly good time series data available on them.[13] We used fifty-seven annual observations on crop areas, prices and output for the period 1913-1969.

The reasons for selecting cotton and wheat for our analysis of the Egyptian case can be summarized as follow:[14]

(1) Cotton is the main crop in production and both the lint and the seeds have been the main sources of export earnings for many years (since 1880).

(2) Wheat is second to cotton in production; its growing period overlaps with that of cotton and it is a part of the crop rotation system that Egyptian farmers follow. Furthermore, wheat became an important imported commodity and substitution between wheat and cotton in production has a direct effect on the trade balance.

(3) Soil deterioration and insect accumulation in soils under continuous cotton production are the main reasons for crop rotation in Egyptian agriculture.

(4) Since both wheat and cotton are traded it is reasonable to assume that their prices are determined in the world markets and are unaffected by Egyptian production. The average cotton area and the average cotton

land productivity show almost no trend over the entire century. However, we observe frequent and sharp fluctuations in cotton as well as in wheat total acreage after 1912. The average wheat area has also stayed the same but productivity has been increasing since about 1960.

V.2 Estimating and Simulating Unrestricted VAR's[15]

We estimated a finite order VAR of the following vector of variables; Cotton lint price (COT-P) over wheat price (WT-P), cotton area (COT-AR), wheat area (WT-AR), cotton lint yield (COT-YLD) and wheat yield (WT-YLD) over the period 1913-1969 with a constant, a linear trend and a dummy for the Second World War period. Each variable is regressed on its own lags and lags of the other variables such that the error is a serially uncorrelated innovation for that variable. We do not impose any linear, non-linear or zero restrictions on the system.[16] Then Zellner's seemingly unrelated regressions method is used in estimating the coefficients and the variance-covariance matrix of the vector of innovations.

The asymptotic likelihood ratio tests (χ^2 test) for lag length rejected specifications with less than five lags. In order to test for non-Granger causality from areas and yields to the relative prices, we use F-tests for the separate equations. The test for exclusions of lagged COT-AR, WT-AR, COT-YLD and WT-YLD from the relative price equation have F values of .94, 1.17, 1.33 and 2.16 with significance levels of .47, .35, .28 and .09, respectively. Hence, we do not reject the hypothesis of non-Granger causality from areas and yields on prices and we support the hypothesis that crop prices are not affected by farmers' decisions on land allocation.

The estimated unrestricted VAR summarizes the dynamic properties of the data. Following Sims [1978, 1980] and Sargent [1978] we interpret the results by looking at the moving average representation (MAR) of the model. It turns out that the MAR is equivalent to the simulated responses of the variables to a once-but-not-for-all one standard deviation change in the innovations. In order to do so we imposed a triangularized linear transformation on the system of estimated equations, such that the variance-covariance matrix of the transformed vector of innovations is the identity matrix.

Table 1 summarizes the results of 15 years ahead decomposition of the forecast error variance that is produced by each innovation.

TABLE 1

PERCENTAGE OF FORECAST ERROR VARIANCE 15 YEARS AHEAD
PRODUCED BY EACH INNOVATION

	Triangularized Innovation in:*				
	COT-P / WT-P	COT-AR	WT-AR	COT-YLD	WT-YLD
COT-P / WT-P	45	9	21	17	8
COT-AR	17	52	9	14	7
WT-AR	15	14	48	9	14
COT-YLD	9	10	18	57	8
WT-YLD	15	18	14	8	46

*The order of the triangularization is according to the above order of the variables.

The innovation in any variable accounts for most of the variance error in the same variable. The innovations in prices are the second-most important factor in accounting for the variance error of land allocations. These results support the claim that farmers in Egypt do respond to prices in making their decision. However, a low response to prices cannot determine whether farmers do not optimize with rational expectations. To illustrate this claim, we can consider the simple example of the land allocation model from section III. The resulting forecast error in area accounted for by innovation in the price is 60 percent for Case 1 where there is a deterioration of land productivity ($d_1 > 0$), and 70 percent for Case 2 where there is a cost of adjusting land ($d_1 < 0$). Therefore, the fact that innovations in prices account for a low proportion of the variance error can be attributed to technological constraints. The results of the estimated forecast error do not support the exogeneity of prices, and indicate that the F-test support of the null hypothesis is due to a high variance of the estimated coefficients. In addition, the simulated responses of all variables converged to numbers that are close to zero. Thus, the system seems to be stationary.

The interesting phenomenon that has been observed from the computed MAR is that COT-AR and WT-AR respond to innovations in any variable in opposite ways; that is, when COT-AR increases, WT-AR decreases and both frequently fluctuate. Figure 2 shows this result for innovations in COT-P over WT-P. The positive (negative) one-step-ahead response of COT-AR (WT-AR) to an innovation in the relative price is as we can expect for almost any product. However, most adjustment-type theories predict a smooth gradual return to the mean. Notice that this is not the case here. The second step is a sharp decrease (increase) in COT-AR (WT-AR), and the third is an increase, etc. Then the fluctuations become less frequent. It turns out that this phenomenon exists in all of the estimated VAR's and in response to innovation in almost any variable.

These fluctuations in cotton and wheat areas are the same as the responses of the land allocation decision rules in the model of section III. In particular, figure 1 shows that deterioration in land productivity may account for this type of "Cobweb phenomenon". Hence, the main dynamic phenomenon in the data is consistent with a model of dynamic technology, optimization and rational expectations.

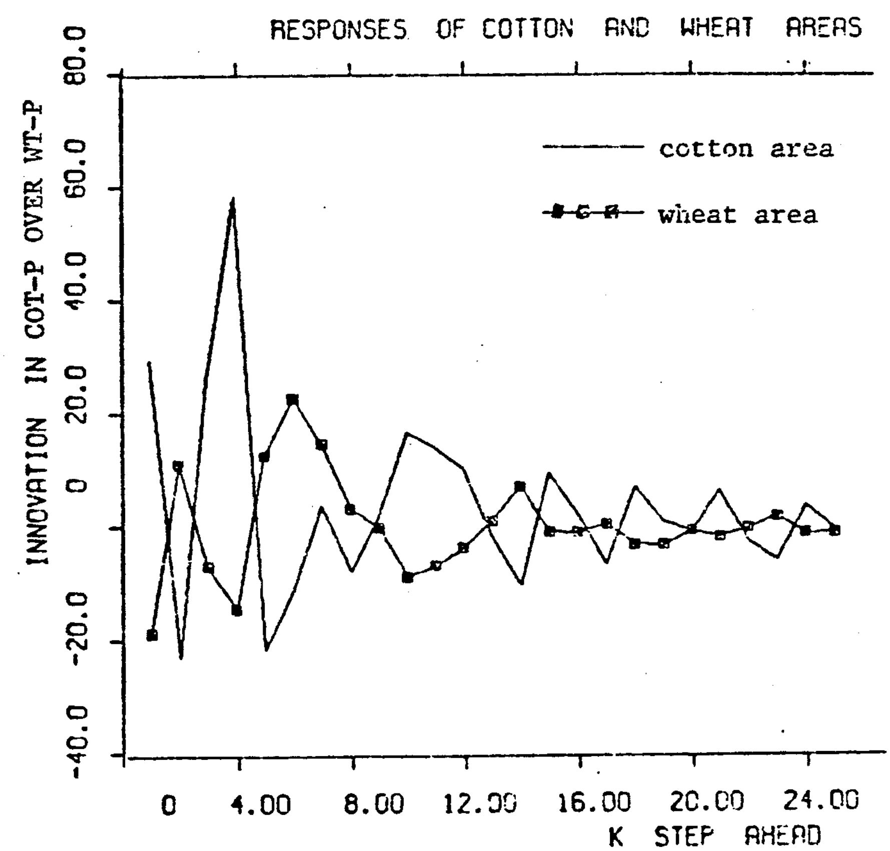

FIGURE 3

V.3 Estimating the Dynamic Land Allocation Model

In this section we present results from maximum likelihood estimator of a simple bivariate specification of the land allocation model using Egyptian annual data on cotton area, cotton lint price, wheat price and wheat yield from 1913 to 1969.[17] Following the traditional agricultural supply response models, the two variables are the cotton crop area (A_{1t}) and the relative price (R_t).[18] We assume that R_t and the shocks to productivity (a_{1t}) have the following autoregressive processes.[19]

$$(5.1)\quad \begin{cases} R_t = \alpha_o + \alpha_1 R_{t-1} + \alpha_2 R_{t-2} + u_t^R \\ a_{1t} = \rho\, a_{1t-1} + u_t^q \qquad |\rho| < 1 \end{cases}$$

where we assume that $|\rho| < 1$ and the roots of $|1 - \alpha_1 z - \alpha_2 z^2| = 0$ are outside the unit circle.

Using the farmer's land allocation decision rule (3.9), and since we do not observe a_{1t}, we can write the VAR for A_{1t} and R_t as

$$(5.2)\quad \begin{cases} \begin{bmatrix} A_{1t} \\ R_t \end{bmatrix} = \begin{bmatrix} \mu_o \\ \alpha_o \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \text{WAR} + \begin{bmatrix} \rho+\lambda_1 & \mu_1 \\ 0 & \alpha_1 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ R_t \end{bmatrix} \\ + \begin{bmatrix} -\rho\lambda_1 & \mu_2-\rho\mu_1 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ R_{1t-2} \end{bmatrix} + \begin{bmatrix} 0 & -\rho\mu_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-3} \\ R_{1t-3} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \end{cases}$$

where WAR represents dummies for the second World War period (1941-45), μ_o contains several deterministic (time independent) elements from the decision rule (3.9) which can not be identified separately, $\varepsilon_{1t} = \mu_3 u^a_{t-1}$ and $\varepsilon_{2t} = u^R_t$. μ_1, μ_2 and μ_3, as they are defined in (5.3), are the restrictions across the equations in (5.2) and they represent the implications of the rational expectations hypothesis and were obtained from the forecasting formula in Appendix A.[20]

$$(5.3)\quad \begin{cases} \mu_1 = \dfrac{\lambda_1}{d}\left[\dfrac{\alpha_1 + \alpha_2\lambda}{1-\alpha_1\lambda - \alpha_2\lambda}\right] \\[2ex] \mu_2 = \dfrac{\lambda_1}{d}\left[\dfrac{\alpha_2}{1-\alpha_1\lambda - \alpha_2\lambda^2}\right] \\[2ex] \mu_3 = \dfrac{\lambda_1}{d}\left[\dfrac{\rho}{1-\rho\lambda}\right] \end{cases}$$

Here $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the vector of innovations that is assumed to have a bivariate normal distribution with $E(\varepsilon_t \varepsilon_t') = V$. Hence, estimators of the free parameters $\theta = \{\lambda_1, d, \rho, \alpha_1, \alpha_2, \alpha_o, \mu_o, w_1, w_2\}$[21] are obtained by maximizing the likelihood function with respect to θ. Let $\ell_t = (\ell_{1t}, \ell_{2t})'$ be the sample residual vector for given value of the parameter vector θ. Then the log likelihood function of the sample of observations on the residuals over $t = 1, \ldots, T$ is

$$(5.4)\quad \mathcal{L}(\theta) = -T \log (2\Pi) - T/2 \log |V| - \frac{1}{2} \sum_{t=1}^{T} \ell_t(\theta)' V^{-1} \ell_t(\theta)$$

where the number of variables (equations) is two. For a given θ, with V unknown, the maximum likelihood estimator of V can be found by setting (see Bard [1974]):

$$(5.5) \qquad \hat{V}(\theta) = \frac{1}{T} \sum_{t=1}^{T} \ell_t(\theta)\, \ell_t(\theta)'$$

Substituting (5.5) into (5.4) we obtain the concentrated likelihood function as,

$$(5.6) \quad \mathcal{L}(\theta) = -T(\log(2\Pi) + 1 - \log T) - \frac{1}{2} T \log \left| \sum_{t=1}^{T} \ell_t(\theta)\, \ell_t(\theta)' \right|$$

(5.6) was maximized with respect to θ where $\ell_t(\theta)$ is defined by (5.2) and (5.3) for each observation.[22] Observe that (5.2) has eleven non-zero regressors while the vector θ has only nine free parameters. Hence, there are two over-identifying restrictions that are due to the theory which imposed the restrictions in (5.3). These restrictions as well as the a priori zero restrictions will be tested using conventional likelihood ratio tests.

Table 2

Estimated Parameters of the Land Allocation Model*

λ_1 = .081	μ_o = 1551.03
d = -.008	α_o = 3.79
α_1 = .524	w_1 = -719.13
α_2 = .250	w_2 = .06
ρ = .081	

The log likelihood = $\mathcal{L}(\theta)$ = -506.088

* β = discount factor = .95 , imposed a priori.

The estimated parameters (see table 2) of the model satisfy the restrictions that we imposed on the farmer's problem in section III, i.e., $|\lambda_1| < 1$, $|\rho| < 1$ the roots of $|1 - \alpha_1 z - \alpha_2 z^2| = 0$ are outside the unit circle and the sign of d is opposite to the sign of λ_1. However, the hypothesis that Egyptian cotton production exhibits significant deterioration in land productivity is not supported by the point estimators of λ_1 and d. In particular, the values of λ_1 and d are consistent with costs of adjustment effect in production and are not compatible with our simple specification of the soil deterioration in cotton production. In section IV we showed how interaction between land and fertilizer may affect the dynamics of land allocations such that if we omit the data on fertilizer, λ_1 may be positive. Thus, the traditional omitted variable argument may explain the "wrong" signs of λ_1 and d. Using the estimated parameters we can calculate the response of land allocations to a permanent or temporary change in prices. It turns out that the long run supply elasticity, i.e., the percent of change in the mean of A_1 divided by the percent of change in the mean of R, is equal to -.13.[23]

Under the null hypothesis that the model is correct, the estimators in table 2 are consistent and the inverse of the Hessian at the maximum is the asymptotic variance-covariance matrix of the estimators. Let $\mathcal{L}(\theta)$ be the value of the log likehood of the model and let $\mathcal{L}_u$ be the value of the log likelihood of an estimated unrestricted version of the VAR (5.2). Then, $-2(\mathcal{L}(\theta) - \mathcal{L}_u)$ is distributed $\chi^2(q)$, where q is the number of restrictions that are tested. Table 3 reports the estimated VAR for the land allocation model and two unrestricted alternatives. From testing the restrictions that are imposed by the theory (not the a priori zero restrictions) the

TABLE 3

THE REDUCED FORM ESTIMATES OF (5.2)

The Model's VAR, $\ell(\theta) = -506.088$

$$\begin{bmatrix} A_{1t} \\ R_t \end{bmatrix} = \begin{bmatrix} 1551.0 \\ 3.8 \end{bmatrix} + \begin{bmatrix} -719.1 \\ .06 \end{bmatrix} \text{WAR} + \begin{bmatrix} .16 & 5.7 \\ 0 & .52 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} -.006 & 2.1 \\ 0 & .25 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ R_{t-2} \end{bmatrix} + \begin{bmatrix} 0 & -.21 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-3} \\ R_{t-3} \end{bmatrix}$$

The Unrestricted VAR (with zero restrictions), $\ell_{u1} = -505.489$

$$\begin{bmatrix} A_{1t} \\ R_t \end{bmatrix} = \begin{bmatrix} 1563.5 \\ 3.9 \end{bmatrix} + \begin{bmatrix} -724.5 \\ .81 \end{bmatrix} \text{WAR} + \begin{bmatrix} .19 & 3.8 \\ 0 & .56 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} -.06 & 7.2 \\ 0 & .21 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ R_{t-2} \end{bmatrix} + \begin{bmatrix} 0 & -4.4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-3} \\ R_{t-3} \end{bmatrix}$$

The Unrestricted VAR (without zero restrictions), $\ell_{u2} = -495.9$

$$\begin{bmatrix} A_{1t} \\ R_t \end{bmatrix} = \begin{bmatrix} 1197.6 \\ -14.2 \end{bmatrix} + \begin{bmatrix} -661.4 \\ 4.6 \end{bmatrix} \text{WAR} + \begin{bmatrix} .27 & -7.6 \\ .005 & .36 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} -.11 & -8.7 \\ .002 & .11 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ R_{t-2} \end{bmatrix} \begin{bmatrix} .21 & 9.7 \\ .004 & .36 \end{bmatrix} \begin{bmatrix} A_{1t-3} \\ A_{t-3} \end{bmatrix}$$

marginal confidence level is less than .5 ($\chi^2(2) = 1.2$). Hence, the theoretical restrictions have not been rejected by the data. Furthermore, since the VAR's parameters of the two models turn out to be very close, there is high confidence in the model's interpretation of the reduced form parameters. However, the likelihood ratio test of our model versus a complete three lags unrestricted VAR, rejects (at 5% significant level) the null hypothesis with marginal confidence level of .995 ($\chi^2(7) = 20.3$). Likelihood ratio tests of lag length for the complete unrestricted (symmetric lags) rejected 2 vs. 4 lags (marginal confidence level = .92), but did not reject 3 vs. 4 lags (marginal confidence level = .44).

These results suggest that a naive specification of the model such as in section III can successfully interpret a bivariate simultaneous, dynamic and stochastic system. However, the Egyptian data require a more complete specification of the environment that should consider higher lag orders (e.g., higher order of productivity deterioration) as well as some existance of feedback from lagged areas (production) on current prices (e.g., local demand for cotton.)[24]

VI. Concluding Remarks

This work is best viewed as an attempt to construct an economic theory that is stochastic, dynamic and simultaneous and that can interpret observed data on land allocations, crop yields and prices. By introducing an explicit approximation to a well known characteristic of the crop production process (depletion of soil productivity), we demonstrate how the dynamic properties of the land allocations and their interaction with crop prices depend on the production technology. Thus, the model's parameters can interpret the dynamics of land allocations as a result of different technologies: the depletion effect in land productivity; costs of adjusting crop areas; due to omitted inputs that interact with land (e.g., fertilizer). In such a model the supply response elasticities are functions of the technology and the parameters of the price processes. It turns out that our model received slim support from the data. Structural estimates conform to a cost of adjustment framework, even though the estimated VAR's (Figure 3) exhibit a dynamic phenomenon that seems to be compatible with the depletion effect. Analysis of changes in the economic structure due to exogenous intervention (e.g., policy) requires an underlying model that is not rejected by the data. That might be achieved by considering additional dynamic components of the crop market. In particular, land allocation decisions are made annually but other inputs can be applied throughout the growing and harvesting seasons; the demand for crops is relatively stable over time, but output is produced over only a short interval during the year; most crops are storable, homogenous and are usually traded in future markets. Each of the above characteristics of crop markets contains a non-trivial dynamic element which our economic theory and the econometric framework should consider for a meaningful interpretation of the observed economic data on farmers' production activities - - the agricultural supply.

Appendix A: Solution to quadratic Optimal Control Problem Under Uncertainty

Consider the discrete time control problem, to maximize

$$(A.1)\quad E_{-1} \lim_{N\to\infty} \sum_{t=0}^{N} \beta^t \{(f + c_t)A_t - \frac{g}{2} A_t^2 - dA_{t-1}A_t\}$$

where $\{c_t\}_{t=0}^{\infty}$ is a stochastic process with mean exponential order less than $1/\sqrt{\beta}$, the discount factor β, satisfied $0 < \beta < 1$, f and g are positive and g/d satisfies $+\infty > |g/d| > 1 + \beta$. The maximization in (A.1) is subject to the initial condition A_{-1} given, and is over $A_0, A_1, A_2, \ldots$.[25]

The quadratic form of (A.1) implies that we can use the certainty equivalence or separation theorem by first solving (A.1) for the certainty case.[26] In particular, we may regard the sequence of $\{c_t\}_{t=0}^{\infty}$ as known and of exponential order less than $1/\sqrt{\beta}$.

To obtain the first order necessary conditions for maximization of (A.1), let fix $N >> 1$ in (A.1), differentiate with respect to $A_0, A_1, \ldots, A_N$, and then set the derivatives to zero.

$$(A.2)\quad \beta^t [f + c_t - gA_t - dA_{t-1}] - \beta^{t+1} dA_{t+1} = 0 \quad t = 0, \ldots, N-1$$

$$(A.3)\quad \beta^N [f + c_N - gA_N - dA_{N-1}] = 0.$$

(A.2) are the N Euler equations, and (A.3) is the terminal condition. For the infinite time problem (A.1), the Euler equations are the same, but the transversality condition is found by taking the limit of (A.3) as $N \to \infty$. Further, we impose the condition that $A_t \leq \bar{A} < +\infty$ for all t = 0, 1,..., where $\bar{A}$ is a positive and a finite scalar. Thus, the solution for $\{A_t\}_{t=0}^{\infty}$ should satisfy the condition

$$\text{(A.4)} \quad \lim_{N \to \infty} \sum_{t=0}^{N} \beta^t A_t^2 < \frac{\bar{A}^2}{1-\beta} < +\infty$$

Given that $d \neq 0$, we can rewrite (A.2) as:

$$\text{(A.5)} \quad \beta d \left(1 + \frac{g}{\beta d} L + \frac{1}{\beta} L^2\right) A_{t+1} = f + c_t$$

(A.5) can be solved uniquely for a given A_{-1}, the transversality condition and (A.4). First, we seek a factorization of (A.5) such that:

$$\left(1 + \frac{g}{\beta d} L + \frac{1}{\beta} L^2\right) = (1 - \lambda_1 L) \quad (1 - \lambda_2 L).$$

$$= 1 - (\lambda_1 + \lambda_2)L + \lambda_1 \lambda_2 L^2$$

Given $0 < \beta < 1$, the sign and the values of λ_1 and λ_2 are determined by the sign and the value of g/d. Furthermore, if $|g/d| > 1 + \beta$ and if we choose $|\lambda_1| < |\lambda_2|$, then, $|\lambda_1| < 1$ and $|\lambda_2| > 1$.

We can rewrite equation (A.5) as:

$$\text{(A.6)} \quad \beta d\,(1 - \lambda_1 L)\,(1 - \lambda_2 L)\,A_{t+1} = f + c_t$$

where $\frac{1}{\lambda_1} = -\frac{g}{d} - \beta\lambda_1$

and $\lambda_2 = \frac{1}{\beta\lambda_1}$

Take the non-stable part $(1-\lambda_2 L)$ to the right-hand-side of the equation and solve it "forward" in order to satisfy the transversality condition and (A.4). Hence,

$$A_{t+1} = \lambda_1 A_{t-1} - \frac{\lambda_2^{-1}}{\beta d} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j L^{-j}\,(f + c_t)$$

As a result the unique solution for the Euler equations (A.2) for all $t = 0,1,2,\ldots$, and the given A_{-1}, is:

$$\text{(A.7)} \quad A_t = \lambda_1 A_{t-1} - \frac{\lambda_1}{d} \sum_{j=0}^{\infty} (\lambda_1 \beta)^j\,(f + c_{t+j}) \text{ for all } t=0,1,2\ldots$$

In the certainty case (A.7) is the optimal decision rule for the infinite horizon problem (A.1). Now we add uncertainty by assuming that the sequence $\{c_t\}_{t=0}^{\infty}$ is a stochastic process. Then the optimal rule for the uncertainty case is obtained by replacing $(f + c_{t+j})$ with $E_{t-1}(f + c_{t+j})$ in (A.7), since the certainty equivalence principle applies to (A.1). Therefore, the unique solution, if it exists, has the following form:

$$\text{(A.8)} \quad A_t = \lambda_1 A_{t-1} - \frac{\lambda_1}{d} \sum_{j=0}^{\infty} (\beta \lambda_1)^j [f + E_{t-1}(c_{t+j})]$$

for all $t = 0,1,2,\ldots$

In order to find the optimal decision rule for A_t, the terms $E_t(c_{t+j})$ must be eliminated by expressing them as functions of variables known by agents at time t. Hence, we should specify the agents' information set at time t and the form of the stochastic process $\{c_t\}_{t=0}^{\infty}$ that the optimization problem (A.1) is subject to.

Suppose $c_t = c_{1t} + c_{2t} + \ldots + c_{rt}$ and let

$$C_t = \begin{bmatrix} c_{1t} \\ c_{2t} \\ \vdots \\ c_{rt} \end{bmatrix} \quad \text{and} \quad S_t = \begin{bmatrix} s_{1t} \\ s_{2t} \\ \vdots \\ s_{n-rt} \end{bmatrix}$$

where $n \geq r$ and S_t is a vector of n-r variables. Furthermore, let $Z_t = \begin{bmatrix} C_t \\ S_t \end{bmatrix}$

and we assume that the stochastic process of Z_t is of mean exponential order less than $1/\sqrt{\beta}$ and can be approximated by a finite order Markov process, i.e.,

$$\text{(A.9)} \quad \delta(L)\, Z_t = U_t$$

where $\delta(L) = I - \delta_1 L - \delta_2 L^2 - \ldots - \delta_k L^k$, and δ_j

is an n x n matrix for j = 1,..., k, U_t is an n x 1 vector, where $E(U_t \mid \Omega_{t-1}) = 0$, $E[U_t U_t'] = \Sigma_t$, Σ_t is a positive semi-definite matrix and $\Omega_{t-1} = \{Z_{t-1}, Z_{t-2}, \ldots\}$ is the agent's information set at time t-1, when the decision on A_t is made.

(A.9) and the above information set complete the specification of the stochastic optimal control problem (A.1) and provide sufficient conditions for existance and uniqueness for the analytical solution of the decision rule for A_t (A.11, below).[19] Following Hansen and Sargent [1980] and Eckstein [1981], the optimal projection for (A.8) given (A.9) and the information set, Ω_{t-1}, is:

$$\text{(A.10)} \quad \sum_{j=0}^{\infty} \lambda^j E_{t-1}(c_{t+j}) = v \cdot \{\lambda^{-1} \delta^{-1}(\lambda) [I + \sum_{j=1}^{k-1} (\sum_{s=j+1}^{k} \lambda^{s-j} \delta_s) L^j] - \lambda^{-1} I\} . Z_{t-1}$$

where v = [1, 1, ..., 1, 0, 0, ..., 0] is a row vector with ones in the first r positions and zeros in the next n-r positions, and where $v \cdot Z_t = c_t$, $\lambda = \lambda_1 \beta$ and I is an nxn identity matrix.

The optimal decision rule for A_t is:

$$\text{(A.11)} \quad A_t = \lambda_1 A_{1t-1} + \gamma + \mu(L) . Z_{t-1} \quad \text{for } t = 0,1,2, \ldots$$

where

$$\gamma = - \frac{\lambda_1 \, f}{d\beta \, (1-\lambda)} \quad \text{is a scalar}$$

$$\text{and} \quad \mu(L) = -\frac{\lambda_1}{\beta} v . \{ \lambda^{-1} \delta^{-1}(\lambda) [I + \sum_{j=1}^{k-1} (\sum_{s=j+1}^{k} \lambda^{s-j} \delta_s) L^j]$$

$$- \lambda^{-1} I \}.$$

such that $\mu(L) = \mu_1 + \mu_2 L + \ldots + \mu_k L^{k-1}$

and μ_i is a $1 \times n$ row vector for $i=1,\ldots,k$.

In order to solve the different problems in this paper one may use the following definitions:

Problem (3.5)

$$f = f_1 + d_1 \quad , c_{1t} = a_{1t} \quad , c_{2t} = -R_t \quad , d = \frac{d_1}{\bar{A}} \quad , r = 2 \text{ and } g = g_1 + 2\frac{d_1}{\bar{A}}$$

Observe that the condition for real solution, i.e. $|g/d| > 1 + \beta$, is satisfied.

Problem (3.5) with (3.3)' as the production function

$$f = f_1 + d_1 \ , \ d = \frac{d_1}{\bar{A}} \quad , c_{1t} = a_{1t} \ , c_{2t} = -R_t , \ c_{3t} = -\frac{w_1}{w_2} PF_t$$

and $r = 3$.

Appendix B: Adjustment Costs, the sign of the parameter d and the roots λ_1 and λ_2.

Suppose we consider a quadratic objective function with adjustment costs, such as these that were considered by Sargent [1979], (among others) for firms and households decisions on capital, labor and consumption. Then, the objective function includes the following typical term:

$$J = \sum_{t=0}^{\infty} \beta^t \{- \frac{h}{2} A_t^2 - \frac{h_1}{2} (A_t - A_{t-1})^2\}$$

where h and h_1 are postive scalars. Observe that

$$J = \sum_{t=0}^{\infty} \beta^t \{- \frac{h}{2} A_t^2 - \frac{h_1}{2} A_t^2 + h_1 A_t A_{t-1} - \frac{h_1}{2} A_{t-1}^2 \}$$

and that: $\sum_{t=0}^{\infty} \beta^t (- \frac{h_1}{2} A_{t-1}^2) = - \frac{h_1}{2} A_{-1}^2 - \beta \sum_{t=0}^{\infty} \frac{h_1 \beta^t}{2} A_t^2$

Then let

$$J' = \sum_{t=0}^{\infty} \beta^t \{ - \frac{h+h_1(1+\beta)}{2} A_t^2 + h_1 A_t A_{t-1}\} = J + \frac{h_1}{2} A_{-1}^2$$

Since A_{-1} is given, the optimization of J' is identical to the optimization of J:

In order to compare J' with the dynamic term in (A.1) let,

$$T = \sum_{t=0}^{\infty} \beta^t \{ - \frac{g}{2} A_t^2 - d A_t A_{t-1}\}$$

It is clear that if $-d = h_1 > 0$ and $g = h + h_1 (1+\beta)$ then $T = J'$ and the condition $|g/d| > (1+\beta)$ is satisfied, since

$$\left| \frac{g}{d} \right| = \frac{g}{h_1} = \frac{h \pm h_1 (1 + \beta)}{h_1} > 1 + \beta$$

Hence, the problem (A.1) is equivalent to the adjustment costs problem that has been considered in the literature if and only if d is negative. However, if d is positive, then we have $d = -h_1 > 0$ and $g = h-d\ (1+\beta) > 0$ which implies that $\frac{h}{d} > (1+\beta)$. Then the requirement for a real solution for (A.1) is equivalent to viewing g in T as equal to h in J. From (A.6), it is clear that for a given $g > 0$, sign (d) = - sign (λ_1) and that the value of $|\lambda_1|$ is dependent of the sign of d. If $g < 0$ the above solution for (A.1) is a minimum and not a maximum. Finally, we can say that for the difference equations (A.5) the sign of the roots λ_1 and λ_2 is determined by the sign of the parameter that multiply A_t. If this parameter is greater than $(1+\beta)$ in absolute value, the roots are real and $|\lambda_1|$ is less than one.

FOOTNOTES

[1]See Behrman [1968] for a detailed discussion of the issues and a complete country work that follows the Nerlovian model. See Eckstein [1981] for a critical review of the Nerlovian model.

[2]This approach follows Sargent [1979, 1981] and is consistent with T.W. Schultz'[1978, p. 4] view:

> Farmers the world over, in dealing with costs, returns and risks, are calculating economic agents. Within their small, individual, allocation domain they are fine-tuning entrepreneurs, tuning so subtly that many experts fail to see how efficient they are, ...

[3]See (A.6) and the definitions at the end of Appendix A.

[4]See Hansen and Sargent [1981]

[5]The Nerlovian supply response model uses the costs of adjustment argument to justify adjustment in actual area vis-a-vis desired land allocations. (See Nerlove [1953, 1979]).

[6]Tobin [1972] put it:

"Price movements observed and experienced do not necessarily convey information on the basis of which a rational man should alter his view of the future. When a blight destroys half the midwestern corn crop and corn prices subsequently rise, the information conveyed is that blights raise prices. No trader or farmer under these circunstances would change his view on the future of corn prices, much less of their rate of change, unless he is led to reconsider his estimated of the likelihood of blights."

[7]The underlying parameter that we hold fixed in both models are:

$g_1 = .25$, $\beta = .9$, $f_1 = 20$, $\bar{A} = 80$, $R_t = 5 + .5R_{t-1} + U_t^R$ and

$a_{1t} = .4\, a_{1t-1} + U_t^a$.

[8]Observe that if R_t and a_{1t} were fixed, but A_{1t} still random, the variance of A_{1t} would have been the same for both cases.

[9]See Sargent [1976] for a similar result with respect to macroeconomic models.

[10]The production function (3.3) explicitly rules out any carry over effects that are usually exist in fertilizers applications.

[11]Specification and estimation of linear rational expectations models are discussed in Hansen [1980], Hansen and Sargent [1980a], Sargent [1978] and Wallis [1980]. The joint estimation of even a specific small model turns out to be complicated and expensive computationally. (e.g. see Sargent [1978], Eckstein [1981] and Eichenbaum [1981]).

[12]The properties of Granger causality, econometric exogeneity and omitted variables are discussed in detail in Granger [1969], Sims [1972], Hansen and Sargent [1980] and Sargent [1979a].

[13]Almost the same data have been used by Hansen and Nashashibi [1974, 1975] and is available also in Eckstein [1981].

[14]Detailed discussions are available in Owen [1969], Hansen and Marzouk [1965], Hansen and Nashashibi [1975], Hansen [1964] and Eckstein [1981].

[15]It should be emphasized that the results from estimating several unrestricted VAR's have preceded the formulation of the models in section III and IV. Detailed information on the results and the methodology exists in Eckstein [1981]. We estimated several different vectors of variables

and the results turn out to be almost the same for all systems of equations. Here we report on only one system.

[16]The methodology for estimating and interpreting VAR's models was developed by Sims [1978, 1980] who used it to analyze macroeconomic questions. T. Doan and R. Litterman's package of Regression Analysis of Time Series (RATS) has been used for computations.

[17]Stategies for estimating this type of model are discussed in Hansen and Sargent [1980], Sargent [1978] and Wallis [1980]. The time domain full-information maximum likelihood is the most conventional method for multivariate non-linear models. It turns out that for our model this method is also computationally efficient versus frequency domain approximations of the likelihood function. In the author's [1981] work a four variate model has been estimated, using frequency domain approximations to the likelihood function.

[18] R_t = Wheat Price x $\frac{\text{Wheat Production}}{\text{Wheat Area}}$ x ^{1}/Cotton Lint Price and is equivalent to R_t in section III.

[19](5.1) is a particular specification for equation (3.6). Since a_{1t} is not observed we assume the lowest autoregressive process. The lag order in R_t process is supported by estimating univariate autoregressive process.

[20]We define λ_1, λ and d in section III.

[21]We fixed the discount factor at $\beta = .95$.

[22]The maximization has been done using DFP algorithm from the GQOPT Package of Princeton University. The complicated non-linear structure of the model implies no gain from writing the analytical first and second derivatives, hence, we used the derivatives-free method. We held 10 digit

accuracy level and checked that we don't have in "the neighborhood" another maximum. We do not report the asymptotic standard errors of our estimators since the Hessian, at the maximum, had not been negative definite. The computer program had been tested using a Monte-Carlo experiment of the same model that we estimated.

[23] The mean of R ≅ 16.8

The mean of A ≅ 1530.0

$$\text{The elasticity} = \frac{\lambda_1}{d\,(1-\lambda_1\beta)(1-\lambda_1)} \cdot \frac{16.8}{1530.} = -.13$$

[24] A brief discussion of models for land allocation that incorporates demand for cotton and wheat exists in Eckstein [1981].

[25] Problem (A.1) is a special case of the general type of problems that are considered by Hansen and Sargent [1981].

[26] See Simon [1956], Theil [1959] and Sargent [1979].

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