

Yale University

EliScholar – A Digital Platform for Scholarly Publishing at Yale

Discussion Papers

Economic Growth Center

6-1-1980

The Resource Allocation Problem in Research and Development

Brian D. Wright

Follow this and additional works at: <https://elischolar.library.yale.edu/egcenter-discussion-paper-series>

Recommended Citation

Wright, Brian D., "The Resource Allocation Problem in Research and Development" (1980). *Discussion Papers*. 361.

<https://elischolar.library.yale.edu/egcenter-discussion-paper-series/361>

This Discussion Paper is brought to you for free and open access by the Economic Growth Center at EliScholar – A Digital Platform for Scholarly Publishing at Yale. It has been accepted for inclusion in Discussion Papers by an authorized administrator of EliScholar – A Digital Platform for Scholarly Publishing at Yale. For more information, please contact elischolar@yale.edu.

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

Box 1987, Yale Station
New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 353

THE RESOURCE ALLOCATION PROBLEM IN RESEARCH AND DEVELOPMENT

Brian D. Wright

June 1980

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

THE RESOURCE ALLOCATION PROBLEM IN RESEARCH AND DEVELOPMENT*

Without some kind of government intervention, a competitive market does not produce sufficient inventions and innovations. The reason is that except in some cases where secrets can be protected without legal support, the producer of new ideas and processes receives an insufficient reward for his efforts in such a market. Not only was this problem understood by policy-makers long before economists were around to write about public goods, but the modern solutions had been located before the aid of my profession could be invoked.

Among these solutions, the patent system has attracted the most interest on the part of economists. However a large literature on the subject (see the survey by Machlup 1958) has added little that is useful as a guide to policy. On the one hand the essentially agnostic conclusions of Machlup are by his own admission hardly the stuff on which policy shifts are based. The detailed investigation of Nordhaus (1969), on the other, resulted in a general endorsement of the system, while contributing an additional analytical justification for the widespread standard patent life limitations of around fourteen or seventeen years as a minor improvement in the effectiveness of the system, relative to an infinite patent life. And the analysis of patents in non-competitive environments has been predominantly descriptive rather than prescriptive, and inevitably plagued by lack of generality. (See Kamien and Schwartz 1975 for an excellent survey.)

* This paper is being prepared for inclusion in a book, Research and Development Policy, edited by George Tolley. I would like to thank with the usual caveat, Cindy Arfken, Martin Baily, Steven Englander, Robert Evenson, Richard Levin, Richard Nelson, John Quigley, and Marguerite Alejandro-Wright for assistance of various kinds.

In this paper I present a new view of the problem of resource allocation which connects the old obsession, the tradeoff between the excess burden of patents and the appropriability which they confer, with other sources of market failure which have attracted increased attention in the past decade. This new analysis shows the resource allocation problem to be much more serious and less tractable than has been traditionally believed. Only by charting some of the dimensions of the problem in this way, can we hope to move towards policy adjustments which will enable research and development to make the fullest possible contribution to the processes of growth and of adjustment to rapid changes in resource prices.

The paper is organized as follows: After making necessary distinctions between different types of inventions and innovations in Section I, I present a very simple research model in Section II. This is used to contrast the old economics of patents, outlined in Section III, with the new view of the patent instrument, presented in Section IV. This new approach casts the ancient appreciation of the role of disclosure in a new light (Section V). Then several alternatives to the current patent system are considered in Section VI, which draws on some results derived elsewhere (Wright 1979). The final section contains the conclusions of this study.

This volume is, very appropriately, aimed at an interdisciplinary audience of engineers and policy-makers as well as economists. With this in mind I have tried to make this chapter intelligible to those who perhaps have some training in economics, but are currently engaged in a field other than economic research.

I. Four Categories of Inventions and Innovations

Research and development programs aim to produce new useful information. Since the same item of information can be used jointly by any number of persons without depleting its factual content, its marginal cost is the cost of communication. The output of research and development can be divided into four categories, related to the possibility of financing its production through in effect selling it at a price in excess of marginal cost:

Category I: Included in this group are discoveries like a new assembly tool, which could be exploited in productive use without divulging its nature, or else patented and sold to others.

Category II: This category includes inventions like a design modification to render a product more efficient, which can be copied at little cost by anyone who wishes to inspect the product in which the discovery is incorporated. Appropriation of the benefits of this type of invention for private gain can be made possible by legal attribution of rights or by non-competitive "barriers to entry."

Category III: In this category are discoveries like a new method for analyzing petroleum exploration data, which need not be divulged by attempts to exploit them, but which could not be patented. Thus the inventor can conceivably obtain his reward for the embodied invention through the free market, but only as long as secrecy is maintained.¹

Category IV: This contains discoveries which cannot be exploited by embodiment in a specific process or marketable product. An example is the theory of relativity, or, on a more mundane level, many theoretical results published in scholarly journals.

If these four categories of invention were always produced by separate and distinct processes, the advantages of direct government allocation of the fourth is rather clear, while the private sector is equally clearly a feasible candidate for allocating the first. The second category can also be allocated "privately," but the necessary role of government extends beyond the "normal" legal protections accorded to decentralized production.

Thus the governmental role in private research and development most generally takes the form of public attribution and enforcement of exploitation rights to private persons. Patents in essentially modern form antedate the beginnings of modern economics. The economic analysis of optimum research and development incentives is dominated by the study of patents, which seem to hold a special fascination for our profession.

Section III below is devoted entirely to patents because, despite the mass of economic research which has been devoted to their study, no adequately clear and comprehensive analysis of their general economic effects is available. But this paper considers patents as just one of several means of optimizing research and development. To accommodate treatment of this wider set of means of controlling research in the confines of this paper, I restrict my attention to the starkly simplistic invention model outlined in Section II, which fortunately incorporates at least some of the characteristics which determine the relative efficiency of research and development activity in different allocative systems.

II. A Simple Research Model

The purposes of this study are served by a very simple model of research toward a well-defined goal. This goal is discrete, and one might think of it as knowledge of a new technique which, on discovery, reduces the marginal cost of production of some non-public good from MC_0 to MC_1 , as in Figure 1. One can think of this new technique as, for example, a more efficient means of energy extraction or conversion, which lowers the cost of the energy produced. The demand curve for the output (energy) is AR , the marginal revenue curve is MR .

Imagine initially that the new discovery is a category II invention as defined above. Research imparts no knowledge which could be used for any other purposes, including further research. To simplify the exposition, I rule out distributional considerations by assuming all consumers are identical. If the cost-reducing information is discovered and freely available, its social value is the increase in consumer surplus, approximated² by the area P_0ABP_1 in Figure 1, which is defined as equal to B dollars. Though these benefits occur over time, the analysis presented below loses little of importance and gains considerably in simplicity by assuming that B accrues in a single period.

The total amount of research effort is comprised of m units of research activity supplied by competitive researchers who are of similar ability and, in common with society at large, assumed to be risk-neutral. The cost of research $C(m)$ and the probability that at least one re-

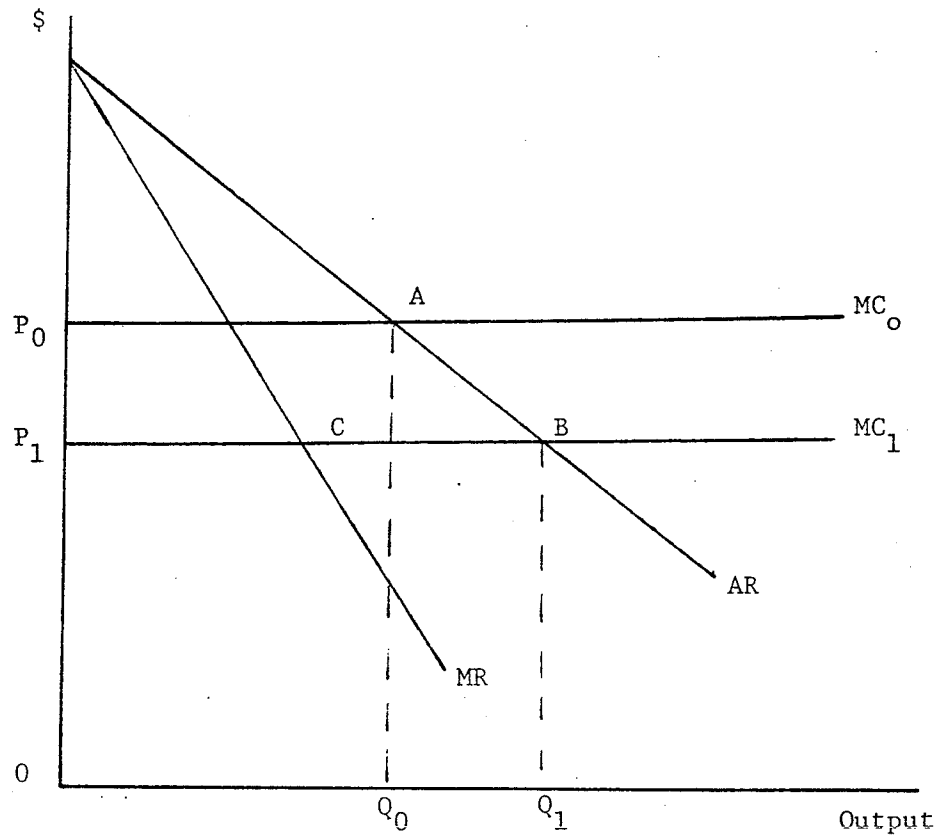


Figure 1
The Welfare Cost of a "Run-of-the-Mill"
Patent Caused by Restriction of Use

searcher will succeed $P(m)$ are assumed to be continuous and twice-differentiable with $C'(m) > 0$, $C''(m) \geq 0$, $P'(m) > 0$, $P''(m) < 0$. The expected benefits of research are $R(m)$, where

$$(1) \quad R(m) \equiv BP(m)$$

The optimal allocation of resources to research in a model of this type is illustrated as m^* in Figure 2, where

$$(2) \quad R'(m^*) = B \cdot P'(m^*) = C'(m^*),$$

and the expected net social benefit is equal to the distance AF .

III. The Old Economics of Patents

There is only one problem with the above scenario. All the economic benefits are passed on to the users of the product (e.g. energy) and none are reaped by the producer of the invention which made the cost saving possible. In this situation an invention bestows its maximum economic benefit on society, but not many inventions are likely to occur without some specific encouragement, if the process of invention requires scarce resources as inputs.

This problem can be solved by the acquisition of an exclusive patent which enables the discoverer to appropriate most of the social benefit B . The holder of an unlimited patent maximizes profits by limiting use of the discovery to production of OQ_0 units of the good.³ Profits are then represented in Figure 1 by area P_0ACP_1 . This is the entire net social benefit of

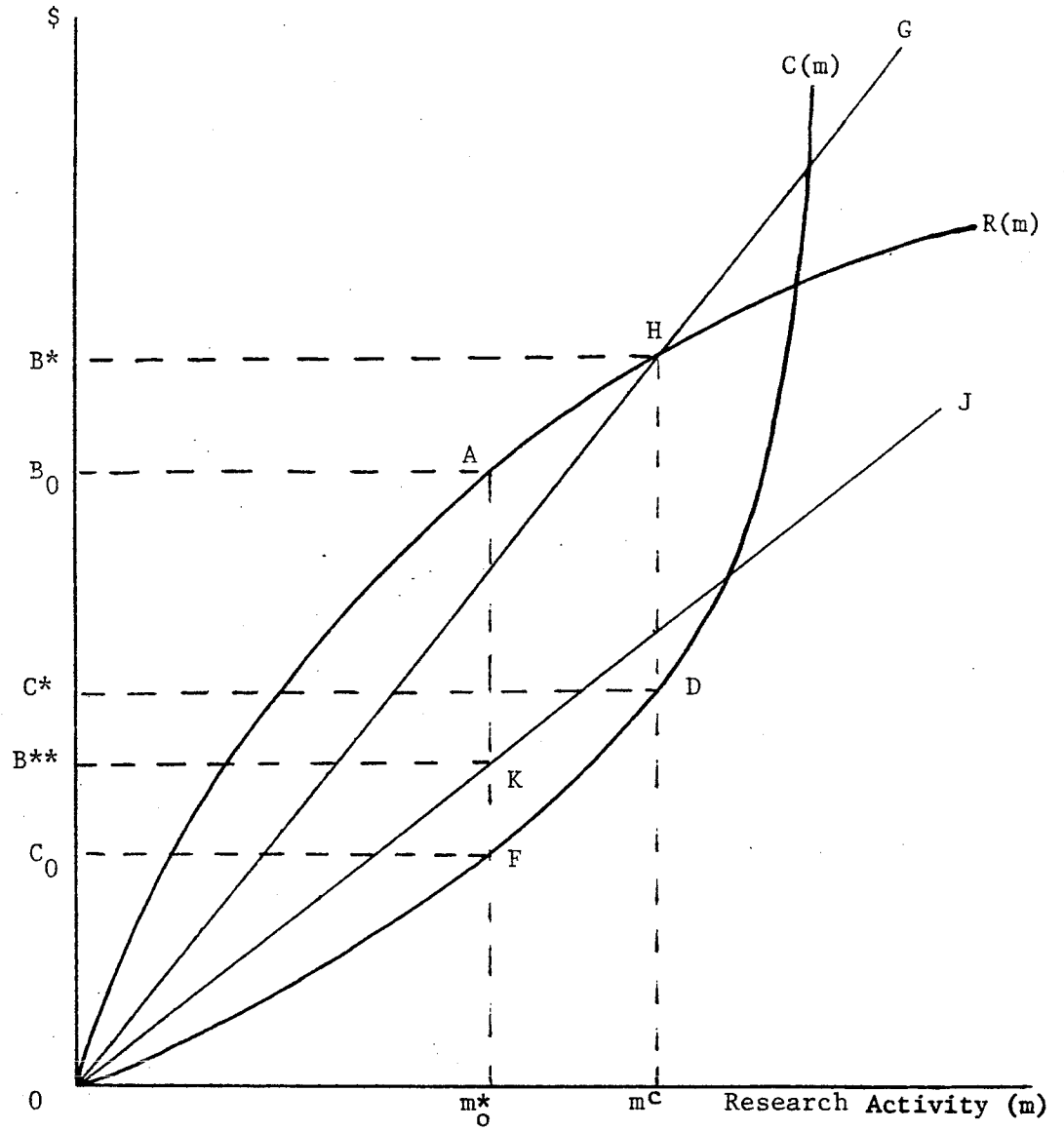


Figure 2

The Resource Allocation Problem in the Competitive Search for Inventions

the invention under the patent system, since price remains at P_0 and consumers are no better off than if the invention had not occurred. Therefore the invention has less social value if it is exploited by a patent holder than if it is freely available for use, the difference being represented by $L \equiv \text{area } P_0ABP_1 - \text{area } P_0ACP_1 = \text{area } ABC$. L is called the "excess burden" or "welfare cost" of the patent.

In practice patent lives are limited, in the United States to seventeen years. Although Posner (1977, pp. 54-55) suggests that a life limitation is essential for administrative reasons, an alternative rationale has been developed by Nordhaus (1969). The essentials of his analysis are represented in Figure 3, using the simple model developed above. (Linearity of $C'(m)$ and $P'(m)$ is assumed in order to make the diagram easier to draw.) The social optimum when knowledge is freely available is m^* , as in Figure 2. According to this analysis, under an unlimited patent the equilibrium research effort is m_1 , where

$$(3) \quad (B - L)P'(m_1) = C'(m_1)$$

When the patent life is limited to T years, the present value of profits and excess burden are both reduced proportionately by a multiplicative factor α . Assuming the annual returns would have been constant forever,

$$(4) \quad \alpha(T, r) = 1 - \int_0^T e^{-rt} dt$$

where r is the rate of interest. The equilibrium under the finite-life patent, given $\alpha(T^0, r)$, can be represented in our atemporal model as occurring at m_2 , where

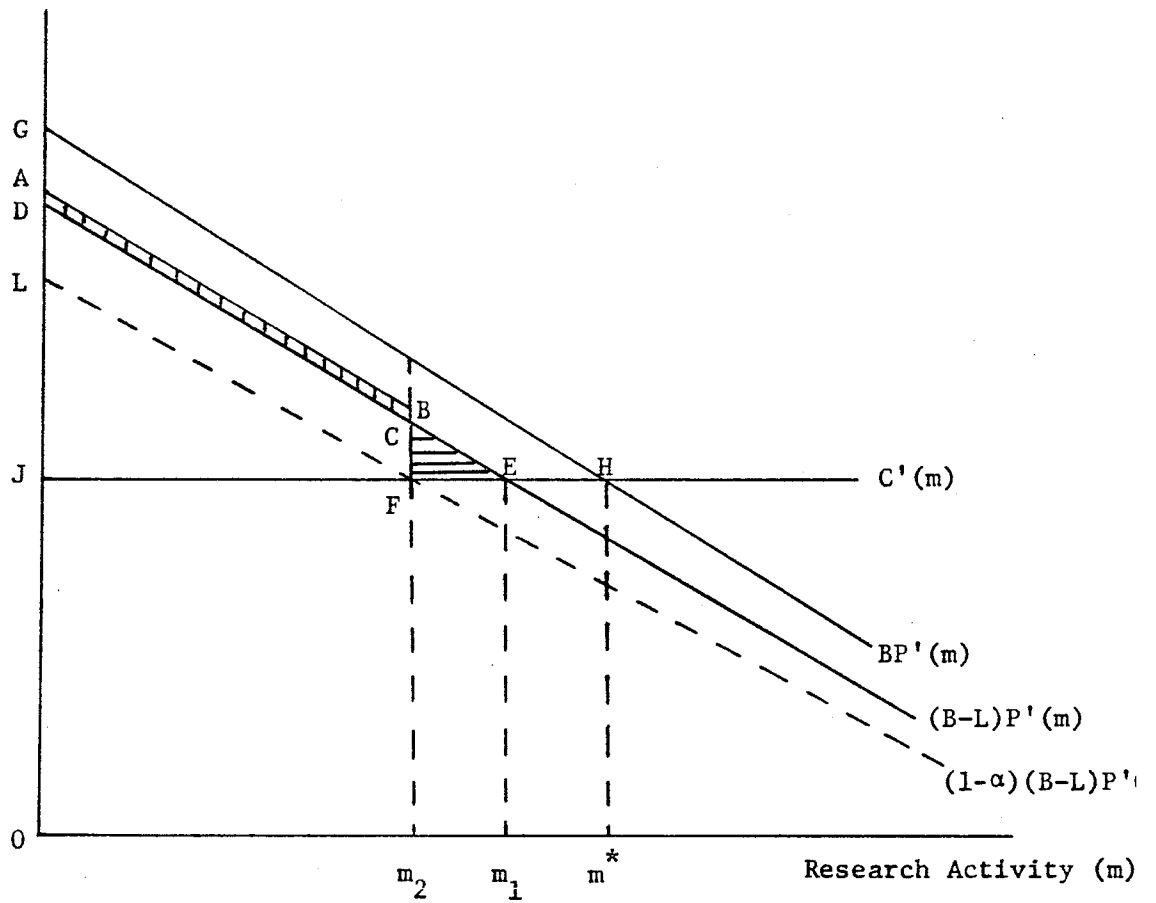


Figure 3

The Old Economics of Patents:
Determination of Patent Life

$$(5) \quad (1 - \alpha)(B - L)P'(m_2) = C'(m_2)$$

The marginal social value of research, given α , is $(B - (1 - \alpha)L)P'(m)$. This is the equation for the straight line through A and B in Figure 3. The difference between the social value of the patent with a life of T^0 years and the social value of the infinite patent is represented in Figure 3 as the difference between area ABCD and area CEF. Assuming L is positive, as α increases from 0 (i.e. as T decreases from infinity) this difference is initially positive, and a maximum occurs at some life T^* where the increase in area of triangle CEF equals the increase in area ABCD.

The magnitude of the potential returns from reduction of T from infinity is limited to some fraction of $P(m_1).L$, which is itself a fraction of the gross expected benefits at m_1 , $P(m_1).B$. In Figure 1 L/B equals the ratio of area ABC to area P_0ACP_1 . More generally, the following approximation holds:

$$(6) \quad \begin{aligned} L/B &\approx \frac{\frac{1}{2} dP \cdot dQ}{dP \cdot Q} \\ &= \frac{1}{2} \eta^D \frac{dP}{P} \end{aligned}$$

Thus the value of L/B for a category II (cost-reducing) invention depends upon the elasticity of demand η^D for the product which has its production cost lowered, as well as on the degree of cost reduction dP . To take an example considered typical of run-of-the-mill inventions by Nordhaus (1969, p. 81), if the demand elasticity is minus one, and the cost reduction is five percent, L/B equals 0.025. Hence the potential gain from adjusting T is typically a small percentage of the gross expected benefits of research.

The relation between patent life and the social value of the patent, under the old view of patents, is illustrated for several cases in Figure 4. Cases IA and IB show the relation between net expected social gain from invention and the life of the patent using the example illustrated in Figure 3⁴, with $r = 0.15$ and values of L/B of 0 and 0.1 respectively, the latter being chosen as a relatively "high" value for L/B . Note that the optimal patent life T^* is 17 years for $L/B = 0.1$ but infinity for $L/B = 0$. (For $L/B = 0.025$ (not illustrated) the optimal life is 23 years.) Cases IIA and IIB illustrate the relation when the (constant) marginal cost of research is fifty percent higher than in the previous example, for $L = 0$ and $L = 0.1$ respectively.⁵ This case represents a line of research of lower expected productivity.⁶ Optimal patent life for $L = 0.1$ is 23 years.

Though optimal patent life is sensitive to L/B in both cases shown, cases IA through IIB show that social welfare is remarkably insensitive to T over a large range. Other examples yield similar results. (See Nordhaus (1969, p. 84) for an examination of numerous parameter combinations in a similar model.)

These results imply that choosing a statutory patent life is a relatively simple problem. Because precision in choice of T is unimportant, a standard statutory life can be chosen for all patents without significant diminution of the expected social value of each invention. The administrative economy (if not absolute necessity) of such a broad rule is obvious to anyone remotely familiar with the patenting process. Further, the loss function for errors in T has a happy asymmetry--large losses can occur only if a fairly short patent life is chosen.

Thus the prescription of this analysis is straightforward. The fixed patent lives of seventeen years in the USA (and fourteen in the

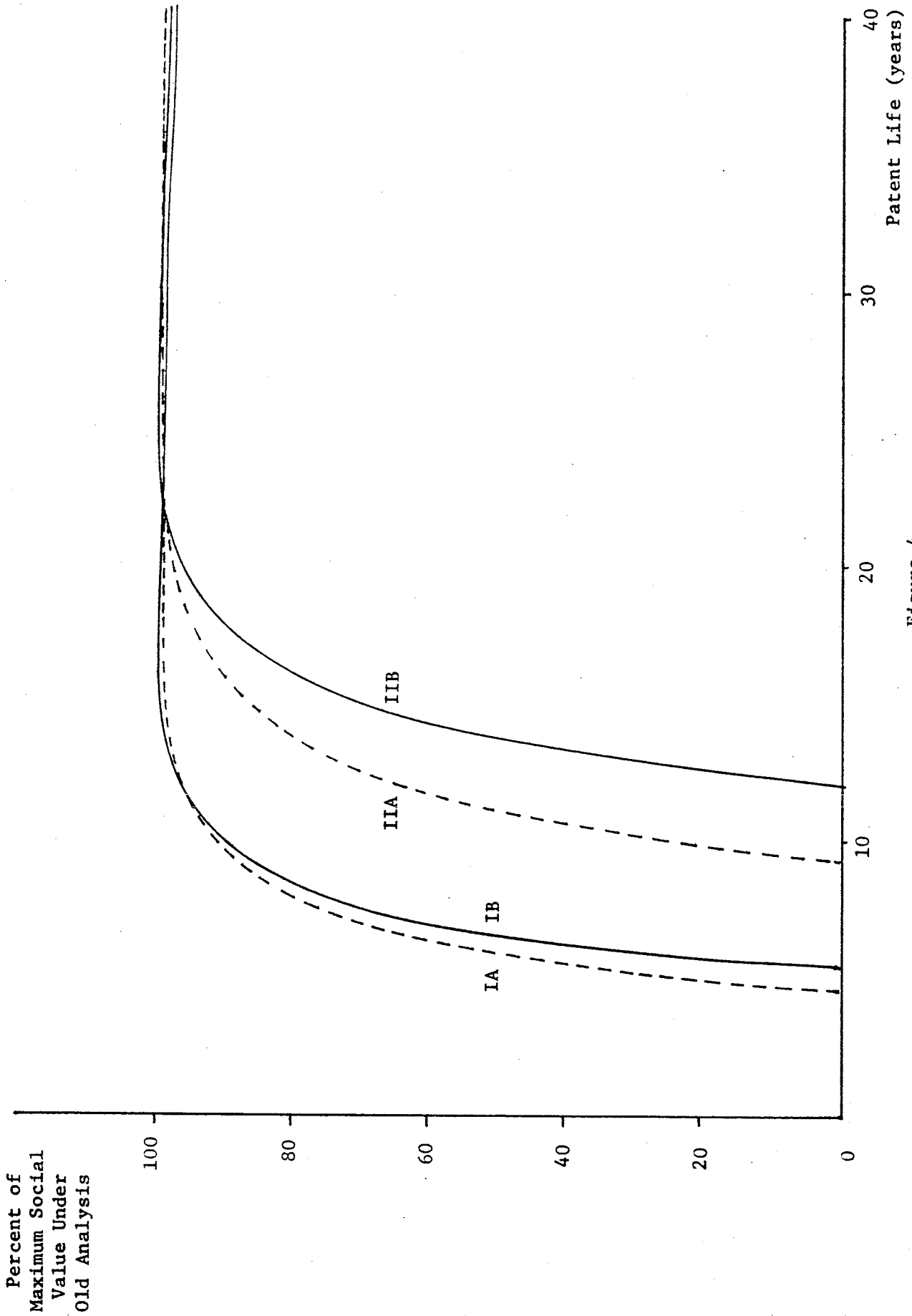


Figure 4

The Effect of Patent Life on the Social Value of Research: The Old Patent Economics

United Kingdom and many other countries) are both quite acceptable, since they both come close to maximizing the social value of competitive research processes over a wide range of the relevant parameters. The above analysis obviously indicates that the standardization of patent lives, which brings enormous administrative benefits, comes at negligible cost. Further, attempts to determine the optimum standardized life appear unlikely to be worth the effort involved. But we shall see in Section IV that the above analysis, and its unusually happy public policy implications, are unfortunately incorrect.

IV. The "Fishing Problem" and the New Economics of Patents

The old economics of patents assumes that the production of inventions is like other productive activities which use private inputs under competitive profit maximization. But using the model developed above, it is easy to show that this is not the case. Since all of the competitive researchers are equally proficient in this model, each has an equal chance of success. Hence each will perceive his marginal and average return for each unit of effort (each "ticket" in the "invention lottery") as $\frac{(B-L)P(m)}{m}$, the expected average return. The researchers, being rational profit-maximizers, will set marginal cost equal to this expected return.

$$(7) \quad C'(m) = \frac{(B-L)P(m)}{m}$$

In Figure 2, this equilibrium occurs at m^c , where the slope of $C'(m)$ equals the slope of the ray OG . Net social benefit is HD , which is by construction less than the optimal net social benefit, AF . If the total cost curve $C(m)$ were linear (i.e. if research effort were

supplied at constant cost) it is easy to see that research would yield no net social benefit at all under this scheme!

This misallocation, which was noted by Usher (1964) is similar to the problem of over-exploitation of an unregulated fishery (Gordon 1954), which is why I have called it the "fishing problem." It is also akin to what is understood in the popular literature as the "tragedy of the commons" (Hardin 1968), and to other "rent-seeking behavior" (Krueger 1974). The dissipation of the benefits of research by competitive production of inventions before the socially optimal time (Barzel 1968, Kitti 1973) is a dynamic intertemporal version of the same type of market failure.

Attention to this type of resource misallocation may have been reduced by empirical evidence showing private rates of return to research and development in the 1950's and 1960's of thirty percent or more.⁷ But a recent study (Pakes and Shankerman 1979) concludes that the private return on research and development is only between three and five percent higher than the return on other capital. They explain this differential as a modest risk premium. The contrast with earlier results arises mainly from their estimate of at least 18 percent for the depreciation rate of the private value of a discovery. Previous studies assumed that the depreciation rate was equal to the much lower rate of decay of physical productivity of other capital. Though Pakes and Shankerman attribute the low rate of return to the increasing difficulty of preventing appropriation of inventions by competitors, the model presented here shows why one might expect their result regardless of the significance of that problem.

There are two ways of correcting the misallocation of research resources in the model developed above. The first (and less practical) solution is to change the rules so that the patent is awarded only when

just one successful outcome is achieved for only then is the successful effort marginally productive. If several researchers discover the same discrete objective considered here, the marginal social value of each of their efforts is zero. In this static context, all researchers would submit their results to the Patent Office; competition should prevent collaboration between researchers to mask multiple successes, since verification would require communication of the result with no guarantee that the receiving party has really made the same discovery.⁸

In the real world, this system might be much more difficult to administer than for example the current patent system, which is itself perceived by some legal authorities as being in serious trouble due to the cost and inefficiency of legal enforcement. In continuous time, distinguishing independent achievements from copying, and preventing collusive behavior, might prove to be insurmountable difficulties. Because of these questions about its administrative feasibility, this alternative is not considered further here, though market incentives awarded under this criterion otherwise dominate conventional patents, as shown in the more technical discussion presented in Wright (1979).

A second method of correcting this problem is to reduce the award sufficiently to obtain the optimal resource allocation m^* . The necessary reduction factor is a fraction α_0 , the elasticity of $P(m)$ with respect to m :

$$(8) \quad \alpha_0 = h(P(m)) \equiv \frac{mP'(m)}{P(m)}$$

In Figure 2 the value of the patent should be reduced by a fraction

$(B_0^{**} - C_0)/(B_0 - C_0)$ of the value of the discovery. If several researchers are successful, this reduced reward is bestowed randomly to one of

them or is shared by all. In practice this would be achieved by limitation of patent life, which is usually justified by economists (e.g. Nordhaus 1969, Scherer 1972; but see Kitti 1973 for a notable exception) only as a means of achieving an optimal tradeoff between the excess burden of the patent restriction and the supposed advantages of awarding successful researchers the full social value of a discovery, as discussed above.

The relation between patent life and the social contribution of research and development, according to this new view of the patent incentive, is illustrated in Figure 5, where Cases IA through IIB illustrate the same cases used in Figure 4. Note that optimal patent life increases with L in this case, in contrast to the old patent economics analysis of Figure 4. Clearly the patent welfare cost L has much less significance for the optimal patent life T^* in this figure than in Figure 4. But the level of L/B , and the length of patent life are both much more crucial for patent system efficiency than suggested in that figure.

In contrast to Figure 4, large losses can result when too long a patent life is chosen. For cases IA and IB, a patent life of seventeen years (as in the U.S.A.) wastes over 60 percent of the maximum expected net social value! (If the current practice of providing tax incentives to research and development through expensing of capital inputs were also taken into account, the discrepancy would be even greater.) For case IB, the loss incurred at optimally adjusted T due to the welfare cost L of the patent system, which has been the focus of much of the literature, is much smaller than the loss caused by a seventeen year patent life - it represents only about 30 percent of the net social value of research. (This

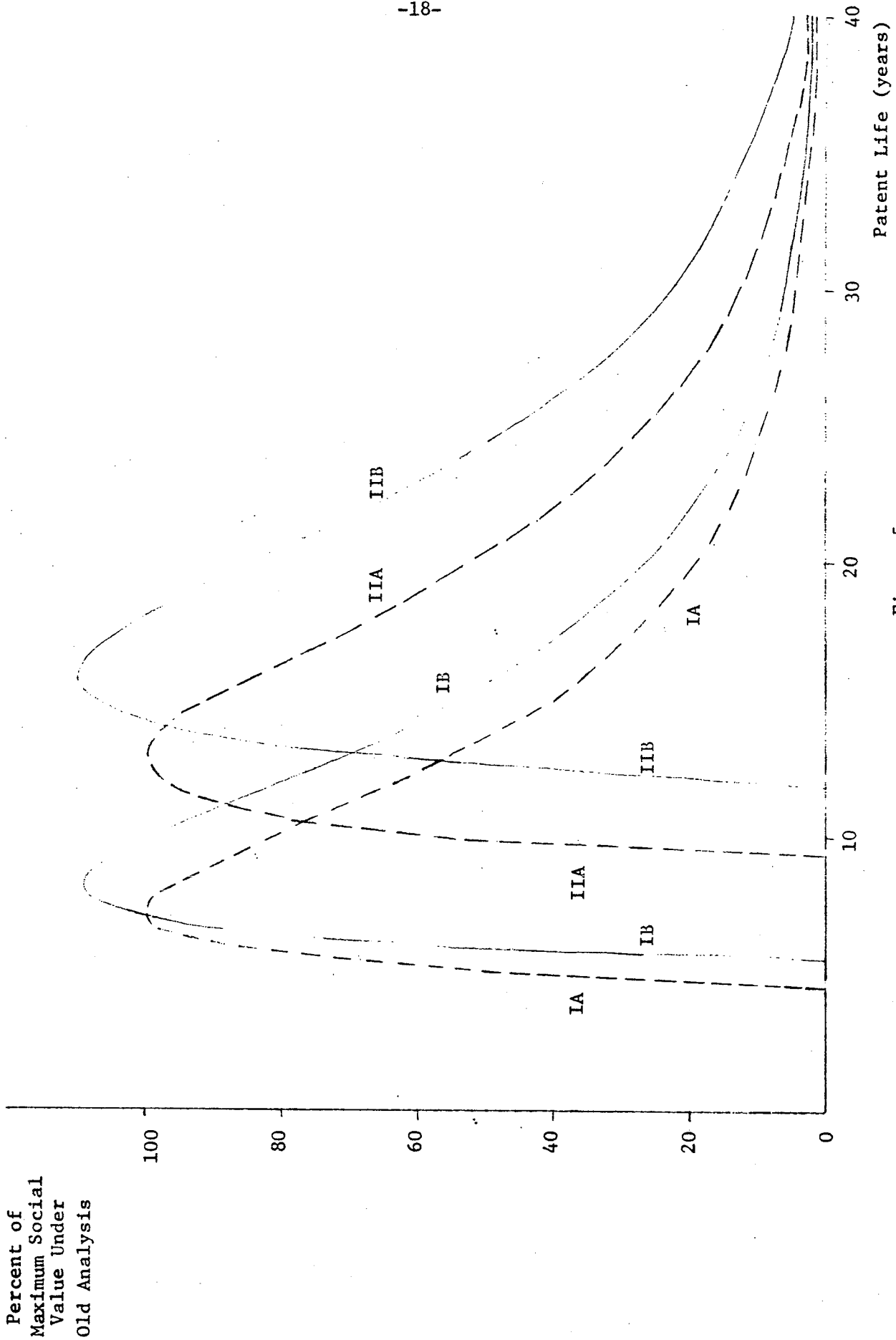


Figure 5

The Effect of Patent Life on the Social Value of Research: The New Patent Economics

number cannot be read from the figure, because of the proportional scaling of the vertical axis.) For case IIB, on the other hand, a seventeen year life limit is almost optimal, whereas the welfare cost of $L/B = 0.1$ causes a 55 percent loss in the (lower) potential social value of research in this higher-cost case.

Although T is less sensitive to L/B than in Figure 4, accurate estimation of L is more important in this new analysis. In Figure 4, mistakenly assuming that L/B equals zero when it is really 0.1 means a large error in T , but a trivial loss in potential social benefit of research. In Figure 5, the same mistake means a much smaller error in T . But an error of only a few years in either direction in choosing T can destroy a large portion of the expected net social value of the invention process. If the optimal T is chosen using Case IIA, when Case IIB should be used, over 40 percent of the maximum social value is lost.

Most serious of all, the efficiency of a given choice of T is also very sensitive to the parameters of the invention model. If T is chosen with Case II in mind, the greater part of the potential social value expected from the invention process is lost if costs have been over-estimated by 50 percent, for then Case I applies. If the reverse mistake is made, all of the potential contribution of the invention process is wasted, since no research takes place.

For a given success function $P(m)$, the sensitivity of the expected social benefit to patent life is highest when the supply elasticity of research effort is infinite, as in the above examples. For example, if Case I is altered so that the supply curve is unit elastic (i.e. a straight line through 0 and H in Figure 3) then the relation between patent life and

expected social benefit is shown, for a "typical" $L/B = 0.025$, in Figure 6. This case, denoted Case III, looks much more like the relation derived for Cases I and II using the old patent economics in Figure 4, and accuracy in the choice of patent life is less crucial than for Case I and II.

The three cases examined are sufficient to show that if research supply is elastic, the cost of a uniform patent life is much greater than the old patent economics indicated. Even though Cases I and II differ only in the level of costs, a standard patent life to cover both would waste at least twenty percent of the expected benefits of one or the other. More extreme examples would only reinforce the conclusion that the administrative advantage of a uniform-life patent system is a potentially costly convenience.

Of course if the relevant parameters were known to the Patent Office, individual optimal patent lives would be tailor-made for each definable area of research. If the optimal lives were correctly calculated, the achievable welfare for $L > 0$ could conceivably be higher than in the conventional analysis outlined in the previous section. For example, the optimum for Case IIB illustrated in Figure 5 yields a net expected welfare gain nearly ten percent higher than the maximum gain reported for Case IIB in Figure 4. Hence given that patents were the chosen means of encouraging research, the "fishing problem" would arguably be a (small) blessing if all pertinent information were freely available and administration were costless. But would any kind of patent system be the best means of encouraging research under these conditions? I will return to this question below.

Percent of
Maximum Value
Under
Old Analysis

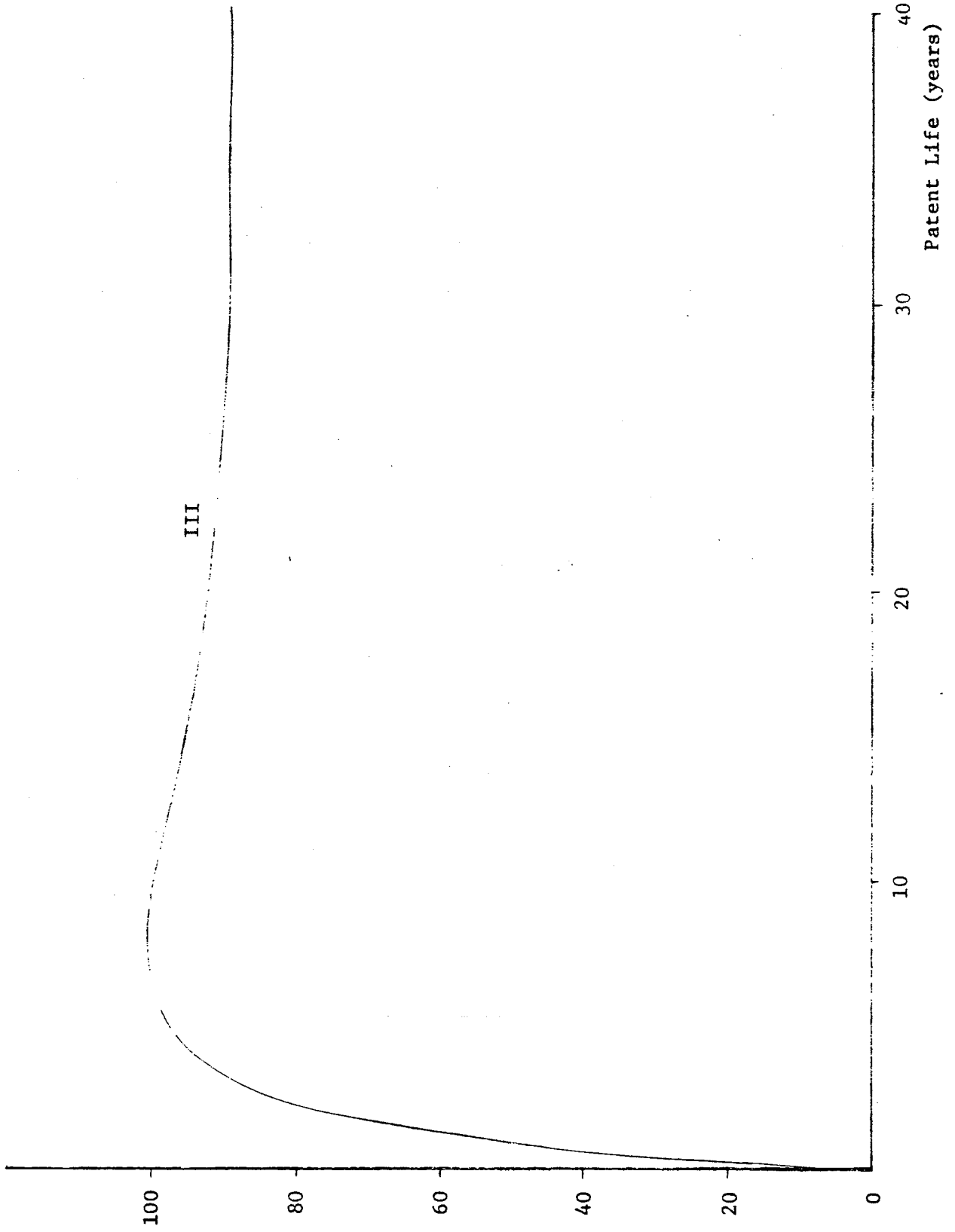


Figure 6

The Effect of Patent Life When Supply Is Unit Elastic

V. A Re-evaluation of the Role of Disclosure

The above analysis of the existing patent system refers exclusively to Category II inventions. But the system does not appear to have been adopted with inventions of this type in mind. Rather, the classical analysis of patents implicitly assumes that most discoveries are a composite of Category III with Category II and/or Category I inventions. Discovery of a specific process or product (a Category I or II inventor) is often accompanied by information about the prospects for similar lines of investigation (a Category III discovery). When a patent is awarded, the value of the Category II or Category I element of the discovery is (in the "run-of-the-mill" case—see above) awarded to the inventor under government protection. But the Category III element of the discovery may be made freely available to all others by the description of the invention, filed for public inspection, from which the patent system derives its name. Traditionally defenders of patents have stressed this role of the patent system as a means of disclosure, given that a discovery has been made, whereas I have up to now concentrated on its role in the allocation of research resources.

When the "fishing problem" is taken into account, these two roles of the patent system can be highly complementary. Disclosure of the Category III element of the discovery means that the holder of a patent does not receive all of the social benefits of his discovery. But in order to prevent over-allocation of resources to research some reductions in the realized benefits is generally desirable, as the examples given in Section IV showed. If, as is likely to be the case, the optimal reduction is greater than that effected by the seventeen-year patent life limitation, then some loss of patent value through disclosure

would be socially beneficial because of its effects on the allocation of research resources.⁹

Of course the required reduction varies with the relevant parameters of the research model; there is no reason to expect that disclosure leads to the optimal adjustment of research benefits discussed below in Section VI. However if the supply curve of research is horizontal for all positive levels of research, disclosure will always improve the allocation of research resources relative to patents without disclosure, since in the latter case expected net social benefits are zero. More generally, if supply has positive elasticity, some degree of disclosure is always beneficial.

Before concluding this discussion of disclosure, I should point out that though I have recognized administrative constraints in this section by assuming that all patents have a standard length of life, the characterization of the patent system remains rather idealized. In existing patent systems, protection of Category I and II inventions is incomplete, because litigation to prevent violations is costly, and competitors can often "invent around" a patent, reducing its value. On the other hand the disclosure provisions are also imperfect--in fact some writers argue that they have little practical value (see Kitch 1977). Though these qualifications must be recognized no evaluation of their importance will be attempted here.

Despite the possible positive contribution of the disclosure function, a patent system with fixed patent life cannot be expected to be nearly as efficient as the old economics of patents had indicated. In the next section, I consider the relative merits of possible alternatives to the current patent system.

VI. Alternatives to the Current Patent System

A. Choices Requiring Greater Administrative Control

In Section IV I noted that if patent life can be "tailor-made" at no cost for each type of research process, and if all relevant information were freely available, then the social value of research under patents might be much higher than under the current system. But under these conditions the patent is clearly a non-optimal choice. For example, direct control through public research agencies, or by contracts to private individuals, could ensure performance of research tasks identical to those which would have taken place under patents, but financed from tax revenues, presumably at a lower welfare cost than the welfare cost¹⁰ of the patent restriction.

To justify choice of patents over all other measures, it is necessary to change one of the assumptions of the model. Consider the case where administration is costless but the public authority in control of research does not have all the information available to (all) researchers about the parameters of the research process. Direct acquisition of the researchers' information by the public authority is ruled out as too costly, or just downright infeasible. An informational imbalance between researchers and the government authority is incorporated in the equations for the cost of research and the benefits of inventions as follows:

$$(9) \quad \hat{C}(m) = (1 + \theta) C(m),$$

and

$$(10) \quad \hat{B} = B(1 + \zeta)$$

where θ and ζ are random shifters and

$$(11) \quad E[\tilde{\theta}] = E[\tilde{\zeta}] = 0, E[\tilde{\theta}^2] = \sigma_{\theta}^2, E[\tilde{\zeta}^2] = \sigma_{\zeta}^2, \text{ and } E[\tilde{\theta}\tilde{\zeta}] = 0.$$

For now the welfare cost L of the patent restriction, and the welfare costs of financing contracts, are assumed away.

This is a two-period model. In period 1 the government authority must choose the means it will use to encourage research, knowing σ_{θ}^2 and σ_{ζ}^2 but not θ and ζ , and researchers make their production commitments, given their knowledge of the relevant values of θ and ζ . The simple rational expectations assumption is made that all parties know the equations of the model (Muth 1961), and all are assumed risk-neutral.

Consider the simple case where $\theta = 0$ and $\zeta = \begin{matrix} + \\ - \end{matrix} \zeta^0$, with probabilities $P(\zeta^0) = P(-\zeta^0) = 0.5$. Then the expected value of the patent instrument, relative to a direct central allocation of research, is illustrated in Figure 7. Since the government authority does not know ζ , the best it can do is to set m equal to m_0 by, for example, contracting for m_0 units of research. If there were no "fishing problem", then when $\zeta = +\zeta^0$, the new equilibrium value of m under (infinite-life) patents would be at m_1 . In this case the value of the added flexibility of the patent system is the increased economic surplus represented by area GHE, relative to the contractual allocation of m_0 . Similarly when

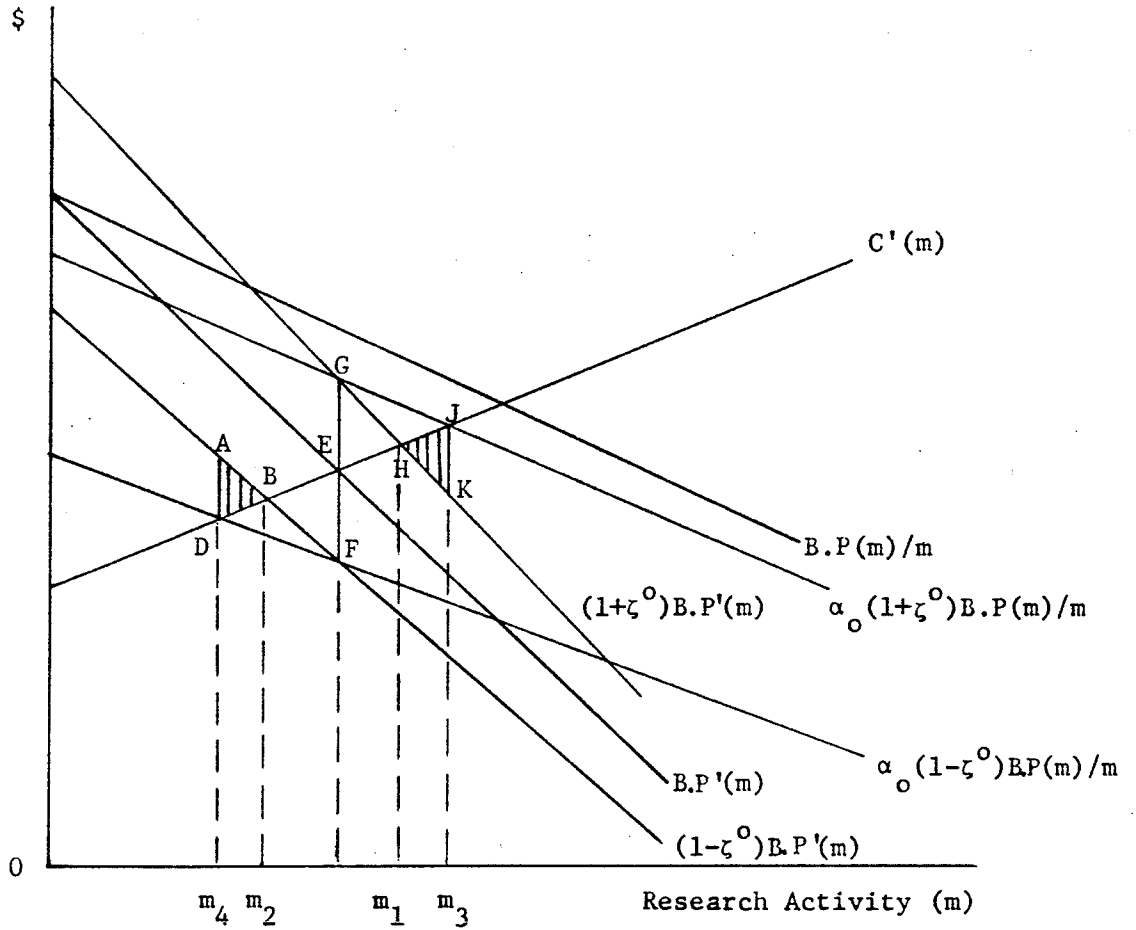


Figure 7

The Social Value of Research Under Patents Relative to Prizes or Contracts When Researchers Have Exclusive Information about Discovery Value

$\zeta = -\zeta^0$, the equilibrium level of research in m_2 , and the differential value of the patent system is represented by the area EFB. Since both areas represent gains, if there were no "fishing problem" and if L were zero, because of its informational advantages patents would always dominate contracts for Category I and Category II inventions.¹¹

(1) Variable-Life Patents

But in a competitive model the "fishing problem" does exist. If the correction factor $\alpha_0 = h(P) \equiv \frac{mP'(m_0)}{P(m_0)}$ is applied to the perceived incentive $(1 + \zeta) \frac{BP(m)}{m}$ (through "tailor-made" patent life limitation in a multi-period model) then equilibrium occurs at either m_3 or m_4 in Figure 7. At $\zeta = +\zeta^0$, the reduction in social benefit from the patent is represented by area HJK, the increase in costs net of the increase in expected benefits. Similarly if $\zeta = -\zeta^0$, the reduction in expected net social benefit of the patent is shown in area ABD. Depending on the relative slopes of $C'(m_0)$ and $B.P'(m_0)$, the loss areas HJK and ABD might completely exceed the gain areas GHE and BEF. If that happens then contracts are the superior instrument for getting the research done.

The case where $\zeta = 0$ and $\theta = \pm\theta^0$, with $P(\theta^0) = P(-\theta^0) = 0.5$, is illustrated in Figure 8. The expected net gain from the patent system relative to the contract alternative is represented by half of the sum of areas FHG and EFB minus half the sum of areas HJK and

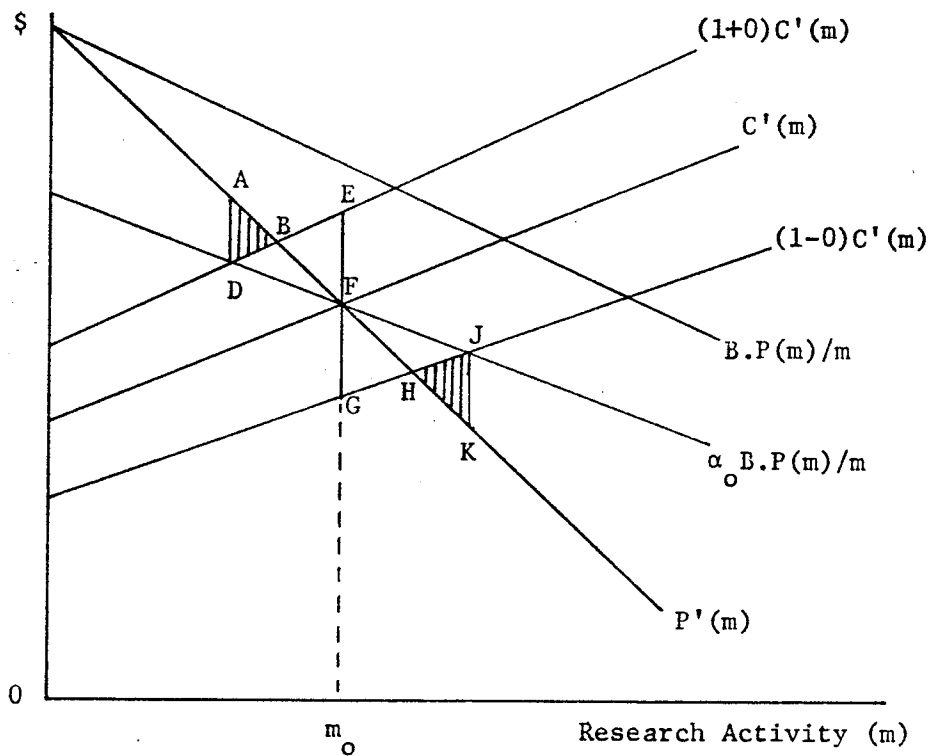


Figure 8

The Social Value of Patents or Prizes Relative to Contracts
When Researchers Have Exclusive Cost Information

ABD . More generally, assume $\tilde{\zeta}$ and $\tilde{\theta}$ are independent and have "compact" distributions (Samuelson 1970), and have "small" variances σ_{ζ}^2 and σ_{θ}^2 . If contracts can be financed with negligible excess burden, the expected net difference between the social value of using patents and the social value of contracts is approximately (see Appendix):

$$(12) \quad E[D_1] = m_o C'(m_o) \left\{ \left[\frac{\sigma_{\zeta}^2 + \sigma_{\theta}^2}{\frac{1}{\eta} + 1 - h(P)} \right] \left[1 - \frac{\frac{1}{\eta} - h(MP)}{2(\frac{1}{\eta} + 1 - h(P))} \right] + \frac{\left(\frac{L}{B}\right)^2}{2[\frac{1}{\eta} - h(MP)]} - \frac{L}{B} \right\}$$

where η is the elasticity of supply of research inputs and $h(MP)$ is the elasticity of marginal probability of success with respect to m . This expression is, in the absence of further restrictions, of ambiguous sign, even if L equals zero. Thus more sophisticated, variable-life patents are not necessarily better than contracts, in this model.

(2) Prizes

By assumption the entire welfare cost L can be avoided if the government authority offers prizes instead of patents--but the cost is that information on ζ is lost, since the reward for success is set by the central authority. The expected net difference in social welfare from using prizes rather than contracts is derived analogously to (12):

$$(13) \quad E[D_2] = m_o C'(m_o) \left\{ \left[\frac{\sigma_\theta^2}{\frac{1}{\eta} + 1 - h(P)} \right] \left[1 - \frac{\frac{1}{\eta} - h(MP)}{2(\frac{1}{\eta} + 1 - h(P))} \right] \right\}$$

The sign of this expression is also in general ambiguous. Thus if administration is costless, it is necessary to know $h(MP)$, $h(P)$, η , L/B , σ_θ^2 and σ_ζ^2 to optimally choose between patents, prizes and contracts. Since none of these parameters is very easy to observe, the choice of the best means of controlling research is no simple task in this context. Actually the superiority of direct control (i.e. contracts) over the other options can be determined from just the three elasticities $h(MP)$, $h(P)$ and η .¹² But estimation of these elasticities appears to be a real challenge, given the present state of data on research and development. Perhaps close cooperation between economists and those directly involved in research and development could help generate the data needed to inform our intuition about these parameters, at least for some of the better-understood types of research processes.

In the absence of such information, I must resort to a rather arbitrarily specified functional form for $P(m)$ to provide an example of the optimal choice between the three instruments. If each of m units of research effort can be expended on a subset of $n > m$ possible research possibilities, each with the same probability of success π , and if each researcher knows which of these are being pursued by others, then

$$(14) \quad P(m) = 1 - (1 - \pi)^m$$

In this case the optimal choice between patents, prizes (both limited in value to eliminate the fishing problem) and contracts can be determined from Table 1 (see Wright (1979) p. 24 and Table 2). As the table shows, prizes are most attractive when the elasticity of supply of research, η , and the probability of success $P(m_0)$, are both low. Contracts are best when the elasticity of supply is very high, and a high probability of success also favors contracts. For a plausible value of $L/B/\sigma_c^2$ like 0.025, patents are best only for the cluster of entries at the middle of the top of the table.

Other forms of the function $P(m)$ will give somewhat different results. Most of the reasonable alternative specifications which come easily to mind would probably imply a smaller range of dominance for patents. For example, if the number of promising research alternatives is rather small, and duplication of research is more likely the divergence between marginal and average rewards which lies at the heart of the "fishing problem" might be much greater than in the above example, in which case the relative attractiveness of contracts or other means of direct central control would be greater.

B. Choices Requiring Less Administrative Attention

(3) Secrecy

The set of feasible alternatives for controlling research activity depends on the administrative capacity of the system and on the nature of the anticipated discoveries. If any system requiring a greater degree of public involvement (legal and bureaucratic) than the current system

Table 1

Choosing the Best Invention Incentive (Ignoring Administrative

Costs): Critical Values^a of $(L/B)/\sigma_{\zeta}^2$

Probability that Success Is Achieved ^a	Elasticity of Supply of Research					
	.1	.5	1	2	10	
.1	0.05 ^b	0.24	0.45	0.82	2.11	-
.3	0.05	0.21	0.36	0.56	0.55	-
.6	0.05	0.16	0.22	0.23	-	-
.9	0.04	0.08	0.03	-	-	-

^aThe probability that at least one research effort will be successful is assumed to be $P(m) = 1 - (1 - \pi)^m$, evaluated at the non-stochastic optimum m_0 , with $\pi = 0.01$.

^bIf $(L/B)/\sigma_{\zeta}^2$ exceeds the figure in the table, a prize is best. If not, the patent system is best. A dash indicates cases where the patent and prize options are inferior to the contract alternative.

of awarding patents is ruled out as administratively infeasible (or unacceptably costly, which amounts to the same thing) then the only real alternative is maintenance of secrecy.

Secrecy has been discussed frequently in the literature, but has attracted little analytical attention. Though I shall not attempt a full economic analysis of the secrecy alternative here, important aspects of this alternative can be understood within the analytical approach developed above.

Obviously secrecy cannot usefully protect Category II or Category IV inventions, which cannot be exploited without revealing the discovery to others. For other inventions, secrecy implies a welfare cost L incurred due to the profit-maximizing restriction of the use of the knowledge generated, like the patent welfare cost illustrated in Figure 1 above. The size of this cost depends on the duration of the successful defence of the secret, which depends on the extent of public protection of secrecy, on the private resources devoted to attacking and defending secrecy, and on the effectiveness of both kinds of resources.

In the two-period stochastic model used above, the efficiency of secrecy probably is bounded by the following alternative cases. If two or more can keep (and exploit) a secret as easily as one, then the "fishing problem" will tend to dissipate the social value of the research performed, just like a patent with unlimited life. If two or more cannot keep a secret at all, then the secrecy alternative is approximately as socially valuable as the alternative patent system mentioned above, that would award a patent only when just one researcher makes a discovery (This patent system which has been ruled out as infeasible in this paper.)

Under this assumption the expected advantage of secrecy over contracts can be obtained using the same general approach outlined in the derivation of $E[D_1]$ in the Appendix (see Wright (1979, pp. 12-15)):

$$(15) \quad E[D_3] = m_o C'(m_o) \left[\frac{(\sigma_\zeta^2 + \sigma_\theta^2 + (\frac{L}{B})^2)}{2[1/\eta - h(MP)]} - L/B \right]$$

Since $E[D_3] \geq E[D_1]$, secrecy can be socially attractive if "two can't keep a secret," especially when the administrative economies are taken into account. Unfortunately I do not possess the data necessary to form a judgment on the extent of joint exploitation of research secrets independently discovered by different researchers. Since evidence of such activity might imply criminal collusion in the United States, it would be surprising if it would be easy to come by.

Unfortunately secrecy may exacerbate another source of waste of resources, namely, duplication of research which has already been performed by others (see Kitch 1977, p. 276). Under a patent system, the publicity of the patent award helps reduce this type of waste, by informing potential researchers that certain lines of investigation have already been covered.

Throughout this discussion, I have assumed that the potential for exploiting Category I or III discoveries under secrecy is not hampered by the need for the discoverer to provide the other complementary inputs (or to find a partner or backer who will keep the secret). In practice this may be unrealistic especially for small, competitive research firms. To the extent that this is true, the advantages of secrecy considered in this section have been overstated.

For Category I and Category III inventions, secrecy is always available as an alternative to the patent system. Reduction of patent lives for the allocative reasons indicated in Section IV will encourage abandonment of the patent system in favor of this alternative. Whether this is a socially desirable outcome depends on the net effect of the advantages and disadvantages of secrecy outlined above. It may be that no general answer is possible. If secrecy is viewed as socially undesirable, despite its administrative advantages, a natural conclusion might be that governments should not protect corporate secrets. But this apparently obvious conclusion is not generally correct. If government protection is withdrawn, it is reasonable to expect that more inventions will be patented, which is the desired result. But it is equally reasonable to expect that private resources will be devoted to attacking and defending the Category I secrets which remain, and the Category III discoveries which by definition have no alternative protection besides secrecy. The desirability of government protection depends on the net result of these two effects. The resources expended in this way represent additional costs of secrecy, which should be taken into account in a more detailed analysis.

VII. Conclusion

The benefits which society reaps from research and development may well fall far below the theoretical optimum which could be attained with perfect public knowledge of the costs and expected benefits of different research activities. Economists and policy makers have not fully appreciated the magnitude of this gap between potential and performance. Research and development activity is subject to several types of market failure. Though plausible solutions can be found for each, separately, their interaction presents problems which are much more difficult to handle, and which justify a full reappraisal of the current set of public policy instruments for controlling research and development.

Currently patents, where feasible, are the favored means of encouraging research because of the administrative economies and informational efficiency associated with their relative decentralization. Their most widely recognized drawback, the welfare cost associated with the restriction of use of the patented invention, is considered to be of minor significance, though it has helped to justify the limitation of patent life to seventeen years or thereabouts. Since the social benefits in the old patent economics are remarkably insensitive to patent life for lives above a dozen years or so, the uniformity of the life limitation, which is such a great administrative economy, appears to come at a trivial welfare cost.

But according to recent theoretical arguments, supported by current

empirical results, such patents should encourage competitive over-allocation of resources for patentable research, unless the unpatentable "spillover" associated with discoveries is sufficiently large. Indeed, if the supply of research is elastic, this spillover, which is commonly viewed as a source of resource misallocation, may constitute the major net economic benefit of the research, a large part of the remainder of the social benefit being dissipated through wasteful duplication of results and/or research tasks. The subsidy implicit in the tax treatment of research and development may well exacerbate this problem.

It is possible to improve the allocation of research resources by adjusting patent life. In this new analysis, the social benefit of research can be highly sensitive to the accuracy of this adjustment; a mistake of a few years in either direction can lead to a heavy loss of new research benefits. And optimal patent life is a function of several parameters which are difficult to observe, and of the welfare cost of the patent restriction. (In contrast to the old patent economics, optimal patent life is a positive function of the latter.) Unfortunately much of the gap between current and optimal patent performance may remain unless different patent lives were offered for different areas of research.

Given sufficient information about the relevant parameters, each area of research could have a different patent life. But I have shown that if some information about the costs or rewards of research remains hidden from the public authority controlling research, direct government control might be superior to patents. This is more likely when the supply

of research effort is elastic. Paradoxically, high economic responsiveness can penalize the more decentralized alternative! And in this situation the administrative advantages of the current uniform-life patents would be largely lost. Further, if this type of patent system were administratively feasible, prizes, which are now offered mainly for nonpatentable discoveries, might be the best type of incentive, especially if the supply of research effort is inelastic.

Of course the administrative economy of a fixed-life patent system may well outweigh any theoretical advantages of the above alternatives, for most types of applied research. If that is the case, further work to determine the best patent life, and the optimal tax treatment of research and development, certainly appears to be justified. And the above analysis has suggested that the alternative of protecting discoveries through secrecy, where practicable, might in fact be the socially desirable alternative in certain situations.

This appraisal of resource allocation in research and development should be understood as tentative and even speculative. Important facts of life, like risk aversion and monopoly, have been assumed away. The main point is that getting the greatest net return from this sector is much more tricky than we have hitherto believed. In fact it may be one of the most difficult allocative problems of modern society. The problem is clearly important enough to justify further investigation by economists and others concerned with public policy--encouraged, presumably, by explicit or implicit prizes, and/or research contracts.

FOOTNOTES

¹Hirschleifer (1971) points out that the value of inventions could in principle be appropriated using futures markets. But the additional gains from speculation are limited when the discovery is protected by a patent or secrecy. For example under a "run-of-the mill" patent of the type considered below, the discovery does not cause a change in product prices, and no opportunity for additional gain through the futures market exists.

²The value of the prospective invention is assumed to be small relative to total income; hence the problem of compensation in economic surplus calculations can be neglected. (For a rigorous justification, see Willig (1976).

³Only "run-of-the-mill" inventions are considered here. This means that cases where optimal exploitation of the patent right involves an increase in Q are excluded. Minor cost-reducing inventions will generally be of the run-of-the-mill type. (Nordhaus (1969) pp. 70-73 provides a clear discussion of the distinction between these cases.)

⁴The actual functions are $C(m) = 0.005 m$, $P(m) = .01m - .0001m^2$, and $B = 1$. At equilibrium with $L = 0$, the latter curve has unit elasticity with respect to m .

⁵In Figure 4, curves IA and IIA converge on the single dotted line to the top right of the figure.

⁶At $L = 0$, and unlimited patent life, expected benefits exceed costs by $1/6$ in this case, whereas the equivalent figure for the previous case was $1/2$.

⁷See for example Mansfield (1965), or Griliches (forthcoming).

⁸The interested reader can compare this solution to the "Clarke Tax" proposed as a mechanism for obtaining honest revelation of preferences for public goods (Clarke 1971 ; Tideman and Tullock 1976); Susan Rose-Ackerman drew my attention to an analogy between the two schemes.

⁹This benefit of disclosure is recognized by Barzel (1968).

¹⁰It is of course possible that a sufficiently irrational tax scheme would incur a higher welfare cost than L , but one needs only minimal confidence in the fiscal structure to rule out this possibility.

¹¹The method of analysis used here is an extension of the approach of Weitzman (1974). The material in this section follows the more detailed and comprehensive treatment found in Wright (1979), to which those readers with a special interest in this analytical approach may wish to refer.

¹²A sufficient condition for the inferiority of contracts relative to prizes depends on just η and $h(MP)$. Contracts are inferior if the expected marginal cost curve $C'(m)$ is steeper than the expected marginal return curve $BP'(m)$, i.e. if $\eta \cdot h(MP) \leq -1$. This condition can be verified by constructing alternative cases similar to that illustrated in Figure 8. It is analogous to the condition derived by Weitzman (1974) in evaluating the simpler choice between price and quantity controls in his model.

APPENDIX

The Social Value of a Variable-Life Patent

This patent is awarded to a successful inventor, or shared by all successful inventors, in period 2. The fishing problem is avoided by reducing the value of the patent by a factor α , (the atemporal version of the "patent life limitation") announced in period 1. The analysis which follows is presented in more detail in Wright (1979) to which the interested reader is referred. The cost of research is:

$$(a.1) \quad C(m) = (1 + \theta)[\bar{C} + a(m - m_0)] + \frac{1}{2}b(m - m_0)^2$$

where \bar{C} is the cost of research when m equals m_0 and $\theta = 0$, the optimal allocation if ζ and θ are fixed at zero. The social value of an invention, \hat{B} , is

$$(a.2) \quad \hat{B} = B(1 + \zeta)$$

where B is assumed independent of m . The probability that at least one research effort will succeed is

$$(a.3) \quad P(m) = \bar{P} + e(m - m_0) + \frac{1}{2}f(m - m_0)^2$$

where \bar{P} is the probability of success at m_0 .

Assume that there are no administrative costs, and that revenue can be raised with no excess burden. Then in the non-stochastic case, the equilibrium allocation m_0 can be achieved by direct government purchases at the price

$$(a.4) \quad R'(m_0) = \alpha_0 \frac{B\bar{P}}{m_0}$$

where

$$(a.5) \quad \alpha_o = h(P) \equiv \frac{em_o}{\bar{P}}$$

If the patent instrument is adopted in the stochastic model, and $\frac{L}{B}$ is small, the first order difference in the incentive from (a.4) is

$$(a.6) \quad E_c \left[\alpha_o \frac{R(m)}{m} - R'(m_o) \right] = E_c \left[\alpha_o \frac{B}{m_o} \left(P'(m_o) - \frac{\bar{P}}{m_o} \right) (m - m_o) \right. \\ \left. + \alpha_o \frac{B\bar{P}}{m_o} \left(\zeta - \frac{L}{B} \right) + \frac{B\bar{P}}{m_o} (Y - Y_o) \right].$$

Inventors determine m in period 1 based on their knowledge of the stochastic terms and the announced value of α :

$$(a.7) \quad m = t(\zeta, \theta, \alpha).$$

The equilibrium change in marginal cost is

$$(a.8) \quad E_c [C'(m) - C'(m_o)] = \theta a + b(m - m_o)$$

where E_c denotes the expectation of researchers as of period 1.

Equating (a.6) to (a.8), discarding higher order terms, and using (a.7)

$$(a.9) \quad t(\zeta, \theta, \alpha) - m_o = \frac{a\theta - \frac{\zeta\alpha_o B\bar{P}}{m_o} + \frac{\alpha_o \bar{P}L}{m_o} - \frac{B\bar{P}(\alpha - \alpha_o)}{m_o}}{\alpha_o \frac{Be}{m_o} - \alpha_o \frac{B\bar{P}}{2m_o} - b}.$$

The Patent Office chooses α in period 1 to maximize expected net social benefits, given that it knows the parameters of the model and σ_ζ^2 and σ_θ^2 , but not ζ and θ . Assuming an interior maximum and that the second order conditions are satisfied, expected consumer and producer surplus is maximized at

$$(a.10) \quad E_G[(1 + \tilde{\zeta})(1 - L/B)B[e + f(t(\zeta, \theta, \alpha) - m_o)]] \\ = E_G[(1 + \tilde{\theta})a + b(t(\zeta, \theta, \alpha^*) - m_o)]$$

where E_G denotes the expectation of the Patent Office as of period 1.

Since, in the non-stochastic case, equilibrium by definition occurs at m_o ,

$$(a.11) \quad Be = a$$

Substituting (a.9) in (a.10) using (a.11) and eliminating higher-order terms,

$$(a.12) \quad \frac{B\bar{P}(\alpha - \alpha_o)}{m_o} = \frac{aL/B}{b - fB} \left[\frac{\alpha_o Be}{m_o} - \frac{\alpha_o B\bar{P}}{m_o^2} - b \right] + \frac{\alpha_o \bar{P}L}{m_o}$$

Substituting (a.12) in (a.9) using (a.11) and (a.5)

$$(a.13) \quad t(\zeta, \theta, \alpha) - m_o = \frac{\zeta - \theta}{\frac{b}{a} + \frac{1}{m_o} - \frac{e}{\bar{P}}} - \frac{aL/B}{b - fB}$$

The expected cost of the patent distortion is reduced by the same fraction α as the expected value of the patent. So the expected social gain from using the patent incentive rather than a direct allocation of m_o units to research is, using (a.7):

$$(a.14) \quad E_G[D_1] = E_G[R(t) - R(m_0) - (C(t) - C(m_0))] - \alpha P(m)L/B$$

Substituting (a.13) in (a.14), using (a.5), and taking a second-order approximation,

$$(a.15) \quad E[D_1] = m_0 C'(m_0) \left\{ \left[\frac{\sigma_\zeta^2 + \sigma_\theta^2}{1/\eta + 1 - h(P)} \right] \left[1 - \frac{1/\eta - h(MP)}{2(1/\eta + 1 - h(P))} \right] + \frac{(L/B)^2}{2[1/\eta - h(MP)]} - L/B \right\}$$

where $C'(m_0) = a$ from (a.1), and η is defined as the elasticity of supply of inventive effort at m_0 :

$$(a.16) \quad \eta = \frac{a}{bm_0}$$

and $h(MP)$ is defined as the elasticity of marginal probability of success

$$(a.17) \quad h(MP) = \frac{fm_0}{e}$$

REFERENCES

- Barzel, Yoram, "Optimal Timing of Innovations," Review of Economics and Statistics (August 1968), 348-55.
- Clarke, E. H., "Multipart Pricing of Public Goods," Public Choice, 11 (Fall, 1971), 17-33.
- Gordon, H. Scott, "The Economic Theory of A Common-Property Resource: The Fishery," Journal of Political Economy, Vol. 62, April 1954, 124-142.
- Griliches, Zvi, "Returns to Research and Development in the Private Sector," in J.W. Kendrick and B. Vaccara, eds., New Developments in Productivity Measurement, Studies in Income and Wealth, forthcoming.
- Hardin, Garrett, "The Tragedy of the Commons," Science, 13, Dec. 1968, pp. 1243-1248.
- Kamien, Morton I. and Nancy L. Schwartz, "Market Structure and Innovation: A Survey," Journal of Economic Literature, 13, No. 1 (March 1975), 1-37.
- Kitch, Edmund W., "The Nature and Function of the Patent System," Journal of Law and Economics, XX, 2 (October 1977), 265-290.
- Kitti, Carole, "Patent Policy and the Optimal Timing of Innovations," Ph.D. dissertation, University of Chicago, 1973.
- Krueger, A. O., "The Political Economy of the Rent-Seeking Society," American Economic Review, June 1974.
- Machlup, Fritz, An Economic Review of the Patent System, Study No. 15 of the Subcommittee on Patents, Trademarks, and Copyrights of the Committee on the Judiciary, U.S. Senate, 85th Congress, 2nd Session (Washington, D.C., 1958).
- Mansfield, Edwin, "Rates of Return From Industrial Research and Development," American Economic Review Papers and Proceedings LV, May 1965, 310-22.
- Nordhaus, William D., Invention, Growth and Welfare, Cambridge, Mass: MITI Press, 1969.

Pakes, Ariel and Mark Schankerman, "The Rate of Obsolescence of Knowledge, Research Gestation Lags, and the Private Rate of Return to Research Resources," National Bureau of Economic Research Working Paper No. 346, 1979.

Plant, Arnold, "The Economic Theory Concerning Patents and Inventions," Economica (New Series), Vol. 1 (1934), 30-51.

Posner, R. A., Economic Analysis of Law, Boston, Mass.: Little, Brown and Co., Second Edition, 1977.

Scherer, F. M., The Economic Effects of Compulsory Patent Licensing, New York University Graduate School of Business Administration, Center for the Study of Financial Institutions, Monograph 1977-2, New York University, 1977.

_____. "Nordhaus' Theory of Optimal Patent Life: A Geometric Reinterpretation," American Economic Review (June 1972), 422-427.

Tideman, T. Nicolaus and Gordon Tullock, "A New and Superior Process for Making Social Choices," Journal of Political Economy, 84, No. 6 (December 1976), 1145-1159.

Usher, Dan., "The Welfare Economics of Invention," Economica, 31 (1964), 279-287.

Weitzman, Martin L., "Prices vs. Quantities," Review of Economic Studies, XLI, No. 4 (1974).

Willig, Robert D., "Consumer's Surplus Without Apology," American Economic Review 66, No. 4 (September 1976): 589-97.

Wright, Brian D., "The Economics of Invention Incentives: Patents, Prizes, and Research Contracts," Yale University, December 1979.