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TIME STRUCTURE OF PRODUCTION
AND THE THEORY OF INTERNATIONAL TRADE*

Richard A. Brecher and Ian C. Parker

1. Introduction

In the theory of international trade, little attention has been paid to the time required for transforming inputs into outputs. Nevertheless, this neglected requirement -- which is central to capital theory -- has been recognized as deserving attention in the literature on international trade[1]. Departing from usual practice in trade theory, the present paper focuses explicitly upon the time structure of production, as characterized by Hicksian (1973) profiles of inputs and outputs over time. As shown by the analysis below, inter-sectoral differences in this structure have important implications for the impact of world trade on a country's employment of labor, accumulation of capital and level of income. These implications are considered for a country subject to a rigid but uniform real wage, which is the type of constraint that Bhagwati (1968) , Brecher (1974a, 1974b), Haberler (1950) and Johnson (1965) introduced in the context of a different technology.

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Section 2 develops a model of a large open economy, in which the time structure of production may differ between sectors. Primary homogeneous labor works with flow-input flow-output technology to produce (within each sector) heterogeneous capital goods and final output, as in the work of Hicks (1973), whose one-sector (or composite-commodity) analysis of a closed economy is extended here to deal with two consumer tradables using sector-specific capital.[2] To abstract from complications due to changing prices, the economy's uniform wage is institutionally fixed in real terms, as in the unemployment models (with homogeneous capital) in the rigid-wage literature cited at the end of the previous paragraph. Under these assumptions, the country's quantity of aggregate employment and value of total capital both depend upon the commodity composition of production , as affected by the pattern and volume of international trade.

Characteristics of the model, in situations of long-run equilibrium and of inter-equilibria transition, are examined in Section 3. The discussion begins by concentrating on stationary states, in order to facilitate comparison with standard literature on comparative statics. As established by two of this section's stationary-state propositions, both of which emphasize departures from previous rigid-wage results of conventional trade models, the replacement of autarchy by free trade may cause a country to experience (1) a reduction in the national levels of employment and income even when the country's capital/labor ratio is smaller for the exportable than for the importable and (2) a change in the value of the national stock of capital at constant prices. Then, brief treatments of steady-state growth and of transition further highlight departures from conventional results. While the analysis reaffirms the importance of relative input intensities, the traditional role of capital/labor ratios is eclipsed by the analogous roles of investment/labor and investment/capital ratios, which all depend crucially on time structures of production in the present model.

Section 4 concludes the paper with a brief summary.

2. The Model

Section 2.1. outlines the production aspects of the home country. Then, home and foreign demand are introduced in Section 2.2.

2.1. Production

Consider an economy in which sectors one and two respectively produce consumer commodities one and two, using homogeneous primary labor, with flow-input flow-output technology exhibiting constant returns to scale. Each sector uses only one (unchanging) technique[3], whose time profiles of inputs and final outputs are described by the following equations:

$$L_i^t = \sum_{j=0}^{\Omega_i} \alpha_{ij} N_{ij}^t \quad , \qquad i = 1,2, \qquad (1)$$

and

$$X_i^t = \sum_{j=0}^{\Omega_i} \beta_{ij} N_{ij}^t \quad , \qquad i = 1,2, \qquad (2)$$

where L_i^t is the labor required by sector i during time t, time t being a unit interval of calendar time; X_i^t is the output produced by sector i during time t; α_{ij} is the labor required by a unit process in its j-th period of life in sector i; β_{ij} is the output produced by a unit process in its j-th period of life in sector i; N_{ij}^t is the number of unit processes in their j-th period of life in sector i during time t; and the unit process (started at the outset of its 0-th period) is terminated (with neither input nor output) at the beginning of its (Ω_i+1)-th period in sector i. It is assumed that the quantities α_{ij} and β_{ij} physically occur at the end of period j, but that all contracts for them are made at the start of the period. Also, let each period j (of process life) coincide with some time t (as a unit interval of calendar time). Throughout the analysis, it is assumed that the economy remains incompletely specialized in production, in the sense that both sectors operate at positive levels (as illustrated diagrammatically in Section 3 below).

All home markets are perfectly competitive, except for the wage rigidity yet to be introduced. Also, profits are maximised everywhere, subject to wage-rate, profit-rate and interest-rate equality between sectors. Under these circumstances, it is well known that

$$\sum_{j=0}^{\Omega_i}(w_i\alpha_{ij} - \beta_{ij})(1+r)^{-j} = 0\,, \qquad i = 1,2, \qquad (3)$$

where w_i is the economy's (uniform) real wage, in terms of the i-th good; r is the country's (uniform) rate of profit and interest, as a pure number; and each of these rates is assumed to be constant over time.[3a] According to equation (3), the present discounted value of net inputs (or net outputs) for the entire lifetime of each unit process must be zero.

From equations (3), it follows readily that

$$w_i = \left[\sum_{j=0}^{\Omega_i}\beta_{ij}(1+r)^{-j}\right] \Big/ \sum_{j=0}^{\Omega_i}\alpha_{ij}(1+r)^{-j}\,, \qquad i = 1,2. \qquad (4)$$

The right-hand side of equations (4) may be denoted by the function $h_i(r)$ of r, where $h_i(r)$ has several properties worth noting. First, $h_i(0) > 0$ -- assuming that (while $\beta_{ij} \geqq 0$ and $\alpha_{ij} \geqq 0$) $\sum_{j=0}^{\Omega_i}\beta_{ij} > 0$ and $\sum_{j=0}^{\Omega_i}\alpha_{ij} > 0$, which simply rules out the uninteresting case of a unit process with no inputs or no outputs throughout its entire life. Second, $\lim_{r\to\infty} h_i(r) = 0$ -- assuming that $\beta_{i0} = 0 < \alpha_{i0}$, which means that the first inputs must precede the first outputs in the life of a unit process.[4] Also, the derivative of $h_i(r)$ with respect to r may be taken to be negative for all[4a] finite $r \geq 0$. Equations (4) then may be represented diagrammatically in the well-known way by a negatively sloped wage-interest curve (asymptotically approaching the interest-rate axis) for each sector i, as shown in Figure 1, whose left-hand and right-hand quadrants correspond respectively to sectors one and two. Each wage-interest curve illustrates the one-to-one relationship between the real wage and the rate of interest (and profit) in equilibrium.

Now let w_2 be institutionally fixed at some constant level, denoted $\bar{w}_2$, so that

$$w_2 = \bar{w}_2. \tag{5}$$

Equations (4) and (5) can be solved for unique values of r and w_1, with these values respectively denoted $\bar{r}$ and $\bar{w}_1$, as illustrated in Figure 1. Since $\bar{w}_i$ equals the nominal wage divided by the nominal price of commodity i (i = 1, 2), the relative price of the first good in terms of the second equals $\bar{w}_2/\bar{w}_1$, which is a constant denoted by $\bar{p}$. Thus, the chosen real wage in terms of the second good uniquely determines the rate of interest (and profit), as well as the real wage in terms of the first good and the product-price ratio which prevail under incomplete specialization.

During time t, the economy is endowed with a perfectly inelastic supply of homogeneous primary labor , in the amount $\bar{L}^t$. This endowment may not be exceeded by L^t, which denotes the total employment of labor during time t, so that $L_1^t + L_2^t = L^t \leq \bar{L}^t$. By assumption, $\bar{w}_2$ is sufficiently

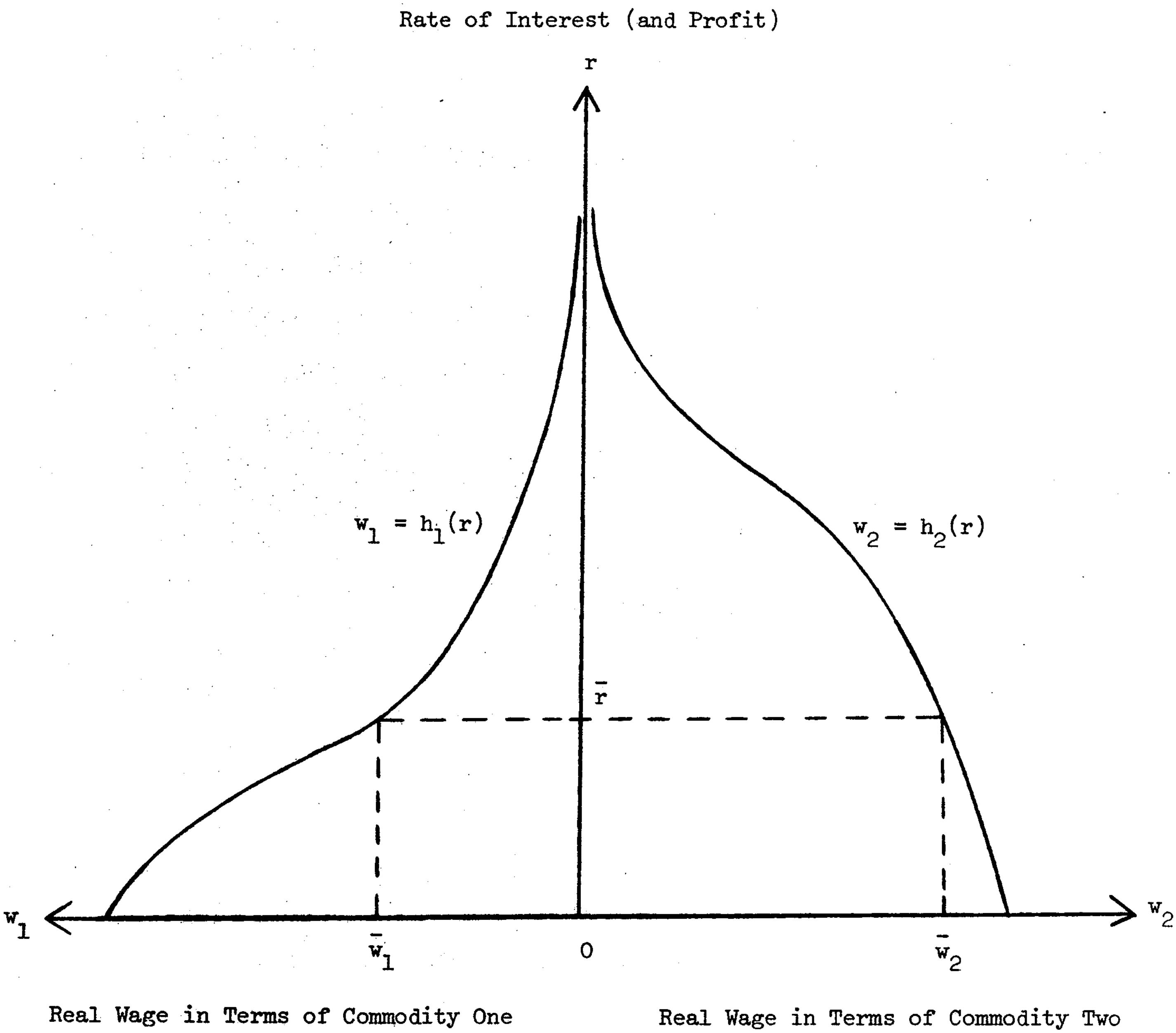

Rate of Interest (and Profit)
r
$w_1 = h_1(r)$
$w_2 = h_2(r)$
$\bar{r}$
w_1
$\bar{w}_1$
0
$\bar{w}_2$
w_2
Real Wage in Terms of Commodity One
Real Wage in Terms of Commodity Two

Figure 1

great to ensure continuous unemployment, with the aggregate labor constraint never binding:

$$L_1^t + L_2^t = L^t < \bar{L}^t. \qquad (6)$$

Labor is assumed to be perfectly mobile between home sectors.

The real value (in terms of commodity i) of gross investment in sector i during time t is denoted I_i^t, given by

$$I_i^t = \sum_{j \in J_i} (\bar{w}_i \alpha_{ij} - \beta_{ij}) N_{ij}^t, \qquad i = 1,2, \qquad (7)$$

where J_i is a subset of $(0,1,\ldots,\Omega_i)$, and $j \in J_i$ if and only if $\bar{w}_i \alpha_{ij} - \beta_{ij} > 0$.[5] In other words, I_i^t is the total of current net inputs into all of the i-th sector's processes having positive net inputs during time t. Alternatively stated, I_i^t is the i-th sector's current wages for those workers who cannot be paid from their own current output and who therefore must be paid from gross national savings during time t (see below).

The real value (in terms of the second good) of total gross investment during time t is denoted I^t and may not exceed $\bar{I}^t$, which is the real value (in terms of the second good) of the economy's supply of investable funds during time t, so that $\bar{p}I_1^t + I_2^t = I^t \leqq \bar{I}^t$. The level of $\bar{I}^t$ is determined by and equal to gross national savings during time t. (Further discussion of saving assumptions is deferred to Section 3). Perfect competition in the market for investable funds ensures that these funds at all times are fully utilized, with the aggregate investment constraint always binding:

$$\bar{p}I_1^t + I_2^t = I^t = \bar{I}^t. \qquad (8)$$

Investable funds are assumed to be perfectly mobile between home sectors. The subsequent analysis shows that constraint (8) is analogous to, but clearly different from, the standard constraint specifying a fully utilized stock of homogeneous and perfectly shiftable capital in traditional models of international trade.

In contrast with these conventional models, the value of each sector's capital stock here depends upon the stock's age composition, as determined historically by the intersectoral allocations of labor and investable funds. In the present context,

$$K_i^t = \sum_{k=0}^{\Omega_i+1} \sum_{j=0}^{k-1} (\bar{w}_i \alpha_{ij} - \beta_{ij})(1+\bar{r})^{k-j-1} N_{ik}^t \,, \qquad i = 1,2, \qquad (9)$$

where K_i^t is the real value (in terms of the commodity i) of the i-th sector's capital stock at the beginning of time t, and $N_{i\Omega_i+1}^t$ denotes the number of unit processes terminating in sector i during time t. In other words, K_i^t is the value of past net inputs accumulated forward to the beginning of time t -- using a rate of interest (and profit) equal to $\bar{r}$ -- for all of the i-th sector's processes existing (or just terminating) during time t.[6] In equations (9), the inner summation is the total capitalized value of all of the i-th sector's processes which are k periods old at the beginning of time t, and the outer summation adds together these values for processes of all ages.

Treating capital in this way -- rather than treating it as a homogeneous, perfectly shiftable, primary (or instantaneously produced) factor of production -- emphasizes the following two points. First, the "transformation" of one sector's capital goods (say, for concreteness, "machines" or "trees") into the other sector's capital goods requires an intersectoral reallocation of both labor and investable funds over a number of time periods. Second, this reallocation may change the value of the economy's stock of capital.

The real value (in terms of the second good) of the total capital stock at the beginning of time t is denoted K^t, where

$$\bar{p}K_1^t + K_2^t = K^t. \qquad (10)$$

The value K^t has the following important property. By reasoning analogous to that of Hicks (1973,pp.25-35), $\bar{r}K^t$ is a real measure (in terms of the second good) of net national profits during time t, because

$$\bar{w}_2L^t + \bar{r}K^t = Y^t , \qquad (11)$$

where Y^t denotes the real value (in terms of the second good) of net national income during time t.[6a]

2.2. Demand

The country's consumption of commodity i during time t is denoted C_i^t, which depends upon relative commodity prices and upon aggregate expenditure on final goods. Given the constant product-price ratio $\bar{p}$, the relationship can be expressed simply by

$$C_i^t = D_i(C^t), \qquad i = 1,2, \qquad (12)$$

where $\bar{p}C_1^t + C_2^t = C^t$,with C^t denoting the real value (in terms of the second good) of aggregate consumption during time t; and $D_i(C^t)$ is a function of C^t. Assuming that neither good is inferior in aggregate consumption, each derivative $dD_i(C^t)/dC^t > 0$, so that (given $\bar{p}C_1^t + C_2^t = C^t$) each $dD_i(C^t)/dC^t < 1$.

Without loss of generality, let the foreign(home) exportable be the first (second) good. Then, foreign demand may be given by the function $Z_1^t(E_2^t)$ of E_2^t; where E_2^t denotes foreign imports (and home exports) of the second good, exchanging during time t for $Z_1^t(E_2^t)$ of foreign exports (and home imports) of the first good, at an international terms of trade equalling $Z_1^t(E_2^t)/E_2^t$ (as the relative price of the second good in terms of the first); and $E_2^t \gtreqless 0 \lesseqgtr Z_1^t(E_2^t)$. It is assumed that the function $Z_1^t(E_2^t)$ is well behaved, in the usual sense that $Z_1^t(0) = 0$ and the derivative $d[Z_1^t(E_2^t)/E_2^t]/dE_2^t < 0$, indicating that an increase in foreign imports requires a decrease of their relative price in world markets. Throughout the present paper, only consumer goods are traded internationally.

Since each country's imports equal the other country's exports,

$$E_2^t = X_2^t - C_2^t \quad , \tag{13}$$

and $Z_1^t(E_2^t) = C_1^t - X_1^t$. Also, free trade requires equality between the world price ratio and the domestic price ratio:

$$E_2^t/Z_1^t(E_2^t) = \bar{p} \ . \tag{14}$$

Under autarchy, on the other hand, $Z_1^t(E_2^t) = E_2^t = 0$.

3. Comparison of Equilibria in the Model.

Section 3.1. compares the stationary states corresponding to autarchy and free trade. This comparison highlights the importance of relative investment intensities, which depend crucially upon intersectoral differences in the time structure of production. As suggested by stationary-state analysis, these intensities (which have related consequences in non-stationary situations) are analogous to the traditional factor intensities of conventional trade models. The boundaries of this analogy are suggested, however, by briefly discussing variations in the rate of steady-state growth in Section 3.2 and situations of transition in Section3.3. These two sections further emphasize the importance of time structures of production within different sectors of the economy. The complex problems concerning stability of and convergence to equilibrium are not addressed in the present paper[7], which (except for Section 3.3) is restricted entirely to comparing steady-state equilibria.

3.1. Stationary States

In stationary state, superscript t may be dropped from all variables of the model and

$$N_{ij} = N_{i0}\ , \qquad i = 1,\ 2;\ \ j = 0,\ 1, \ldots,\ \Omega_i+1. \qquad (15)$$

Equation (15) simply states that, in each sector, the number of unit processes is the same for all ages. Also, it is assumed that $\bar{I}$ has the same value in all stationary states compared. Thus, the economy has a constant level of gross savings (equalling $\bar{I}$), no matter what the national levels of income or profits. Although the analysis could be extended readily to allow for more complicated assumptions regarding savings behavior[8], the present fixed-investment (fixed-savings) assumption provides the clearest contrast with the fixed-capital assumption of standard trade models.

First consider the economy's production possibilities. Equation (8) can be manipulated simply to yield

$$X_2 = \bar{I}\theta_2/\delta_2 - \bar{p}(\delta_1\theta_2/\delta_2\theta_1)X_1, \qquad (16)$$

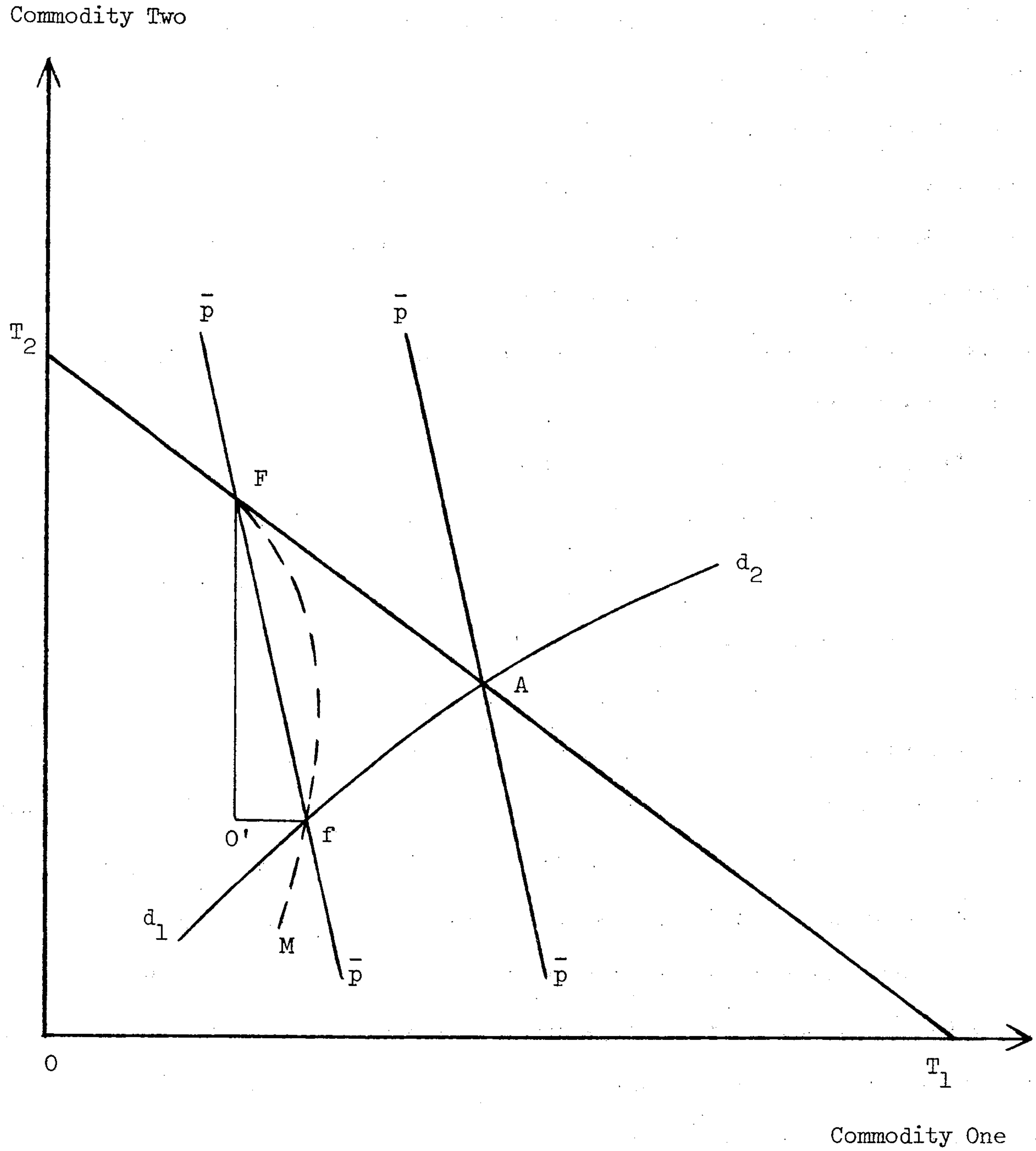
where $\theta_i \equiv X_i/K_i$ and $\delta_i \equiv I_i/K_i$, $i = 1,2$. From equations (2),(7),(9) and (15), it follows readily that each θ_i and δ_i is a positive constant, which depends upon the time flows of inputs and outputs in sector i. Because of this constancy, equation (16) may be represented in Figure 2 by a negatively sloped straight line, such as T_2T_1, which is the stationary-state locus of production possibilities. Although line T_2T_1 is drawn flatter than the price line for $\bar{p}$ in Figure 2, the analysis of this paper applies without modification also to the opposite situation and to the case in which these two lines are parallel. The actual divergence (if any) between the two lines depends upon conditions discussed later in the paper[9].

The relationship between the home levels of production and consumption may now be considered. Since the net national levels of savings and investment are both zero in stationary state,

$$C = Y = \bar{p}X_1 + X_2 \ , \tag{17}$$

where $\bar{p}X_1 + X_2$ is the real value (in terms of the second good) of aggregate production of final goods. Thus, in Figure 2, each production point (say F) and its corresponding consumption point (say f) both lie on the same price line for $\bar{p}$. Curve d_2d_1 in Figure 2 is the positively sloped income-consumption curve corresponding to equations (12) and (17). Throughout this stationary-state section, in view of equation (17), anything said about net national income applies also to the national levels of consumption and production of final goods.

Full international equilibrium may now be determined. In autarchy, $E_2 = 0$. When free trade occurs , E_2 rises from 0 to $\bar{E}_2$ (>0), where $\bar{E}_2$ is the unique solution to equation (14). This rise in E_2 implies an increase in X_2, since the derivative $dX_2/dE_2 = [1-dD_2(C)/dC + (\delta_2\theta_1/\delta_1\theta_2)dD_2(C)/dC]^{-1}>0$. This result is obtained easily by using equations (12), (13), (16) ad (17), and by recalling that $dD_2(C)/dC < 1$ because of the non-inferiority



Commodity Two
T_2
$\bar{p}$
$\bar{p}$
F
d_2
A
O'
f
d_1
M
$\bar{p}$
$\bar{p}$
O
T_1
Commodity One

Figure 2

assumption. In Figure 2, autarchy occurs with the home levels of production and consumption at point A; whereas with free-trade, home production is at F, home consumption is at f, home exports (and foreign imports) in the amount $\overline{E}_2$ are represented by line segment O'F, home imports (and foreign exports) in the amount $Z_1(\overline{E}_2)$ are represented by line segment O'f, exports equal imports at the (world and domestic) price ratio $\overline{E}_2/Z_1(\overline{E}_2) = \overline{p}$, and the foreign offer curve is the dashed curve FfM with origin at point F. Because X_2 increases with E_2 when autarchy is replaced by free trade, the effects of this replacement[10] may be studied by concentrating upon the implications of a rise in X_2, as in the following analysis of employment, capital and income.

Letting $\overline{p} = 1$ (without loss of generality) by appropriate choice of units of output, simple manipulation of equations (6) and (8) yields

$$L = \overline{I}/\lambda_1 - [(\lambda_2 - \lambda_1)/\lambda_1\gamma_2]X_2 , \qquad (18)$$

where $\lambda_i \equiv I_i/L_i$ and $\gamma_i \equiv X_i/L_i$, $i = 1,2$. From equations (1), (7) and (15), it follows easily that each λ_i and γ_i is a positive constant, which depends on the time profiles of inputs and outputs in sector i. Differentiation of equation (18) shows that $dL/dX_2 \lesseqgtr 0$ as $\lambda_2 \gtreqless \lambda_1$. In other words, when there is a home expansion of the export sector, total employment decreases (increases) if the investment/labor ratio is higher (lower) for this sector than for the other sector[11]. Thus, opening the economy to free trade affects aggregate employment in a way that depends upon relative investment intensities, as determined by intersectoral differences in the time structure of production.

Evidently, the role of λ_i in determining dL/dX_2 within the present model is analogous to the familiar role of traditional capital/labor ratios in determining the total employment change within conventional trade models[12]. (Although this analogy may be heuristically helpful, it should not be allowed to mask important differences between present investment/labor ratios and conventional capital/labor ratios, as shown below in Sections 3.2 and 3.3). It is not difficult to understand why the relevant intensities in the present context are given by I_i/L_i -- rather than K_i/L_i, which is relevant in conventional trade models. The explanation lies in the fact that the fixed and binding input constraint here is $\bar{I}$ -- rather than K, which is an endogenous variable in the present model but an exogenous parameter in conventinal trade models.[12a] It is important to observe that having $\lambda_2 > \lambda_1$ is not inconsistent with $K_2/L_2 < K_1/L_1$[13]. Thus an expansion of the sector with the smaller capital/labor ratio may reduce total employment in the present model, even though the opposite result would occur in conventional trade models with a rigid wage[14].

From equations (8) and (10), it easily follows that

$$K = \bar{I}/\delta_1 - [(\delta_2 - \delta_1)/\delta_1\theta_2]X_2 \qquad (19)$$

Thus, $dK/dX_2 \lesseqgtr 0$ as $\delta_2 \gtreqless \delta_1$. In other words, in the event of home expansion of the export sector, the value of the aggregate capital stock decreases (increases) if the investment/capital ratio is greater(smaller) for this sector than for the other sector.[15] Thus, the opening of free trade affects the value of the capital stock in a way that depends upon relative investment intensities and , therefore, upon intersectoral differences in the time structure of production.

It is possible to go further in interpreting the δ_i coefficients. Since gross investment equals depreciation in stationary state, δ_i is the i-th sector's overall rate of depreciation. Also, appealing to what Dorfman (1959) calls the "bathtub theorem", $1/\delta_i$ is (in one sense) the "average period of investment" in sector i. Thus, if $\delta_2 > \delta_1$ and X_2 increases, investable funds are reallocated to the sector with the greater rate of depreciation and the shorter average period of investment. Not surprisingly, therefore, the value of the capital stock decreases under these circumstances, as shown above. The ability of trade to affect the value of the capital stock at constant prices runs contrary to results of conventional trade models, in which there is a fixed stock of homogeneous capital.

From equations (11), (18) and (19), it immediately follows that $dY/dX_2 \lesseqgtr 0$ as $\bar{w}_2(\lambda_2 - \lambda_1)/\lambda_1\gamma_2 + \bar{r}(\delta_2 - \delta_1)/\delta_1\theta_2 \gtreqless 0$. This general condition implies the following special conditions (sufficient but not necessary) for determining the sign of dY/dX_2: $dY/dX_2 < 0$ if both $\lambda_2 \geqq \lambda_1$ and $\delta_2 \geqq \delta_1$ with not more than one equality; $dY/dX_2 = 0$ if both $\lambda_2 = \lambda_1$ and $\delta_2 = \delta_1$; and $dY/dX_2 > 0$ if both $\lambda_2 \leqq \lambda_1$ and $\delta_2 \leqq \delta_1$ with not more than one equality. In other words, a rise in production of the home exportable may imply a fall (rise) in net national income, as for example in the case where the export sector is more (less) intensive than the other sector in using investable funds per unit of both labor employment and capital value. Thus, opening the economy to free trade affects the level of net national income in a way that depends upon relative investment/labor and investment/capital ratios, which themselves are dependent upon intersectoral differences in the time structure of production.

It is clear, from equations (16) and (17), that $\bar{p} \gtreqless \bar{p}\delta_1\theta_2/\delta_2\theta_1$ as $dY/dX_2 \lesseqgtr 0$; where $-\bar{p}$ is the slope of the equilibrium price line and $-\bar{p}\delta_1\theta_2/\delta_2\theta_1$ is the slope of line T_2T_1 in Figure 2. Thus, depending on the values of the parameters which determine dY/dX_2, the price line for $\bar{p}$ may be steeper than T_2T_1 (the case when $dY/dX_2 < 0$ as in Figure 2), parallel to T_2T_1 (the case when $dY/dX_2 = 0$) or flatter than T_2T_1 (the case when $dY/dX_2 > 0$). It is important to note that the first of these three cases may occur even when the capital/labor ratio at home is smaller in the second sector than in the first sector, whereas this capital/labor ranking guarantees the third case in the rigid-wage models of conventional trade theory[16].

The foregoing analysis reaffirms, as might be expected, the importance of relative input intensities in an open economy. In the present context these are the λ_i and δ_i -- rather than the traditional K_i/L_i important in conventional trade models -- and depend crucially upon the time structure of production in sector i. Although the investment intensities of the present model are somewhat analogous to the traditional factor intensities of conventional models, the boundaries of this analogy are suggested by the following discussinns of growth and transitions.

3.2. Investment Intensities and the Rate of Steady-State Growth.

Consider a more general steady-state situation, in which all sectors, the labor force and the stock of investable funds grow at the same constant rate (positive, zero or negative), consistent with the growth of home and foreign demand at constant prices. In this case — whereas the traditional factor intensities in conventional trade models are uniquely determined by the real wage — the present investment intensities depend also upon the rate of growth, even with the real wage and choice of techniques unchanged.

This result may be obtained by considering an economy in steady-state growth at rate $g(-1 < g < \infty)$, in which case

$$N^t_{ij} = N^t_{i0}(1+g)^{-j}, \qquad i = 1,2;\ j = 0,1,\dots,\Omega_i+1, \qquad (20)$$

which is a generalization of equation (15). From equations (1), (7), (9) and (20), it follows readily that λ_i and δ_i are functions of g[17], confirming that the investment intensities depend upon the rate of steady-state growth.

3.3. The Transition Between Long-Run Equilibria

Although the investment intensities are constant over time for any particular rate of steady-state growth, they will generally vary from period to period when the economy is not in steady state, since equations (20) are not generally satisfied in the latter situation. Thus, during the transition between steady states, the economy's time paths of labor employment and capital stock (as well as other aggregates) may be quite complicated, exhibiting many peaks and troughs. The gains or losses during the transition must be taken into account, if there is to be a complete evaluation of the switch from autarchy to free trade.

To illustrate this last point, the present section compares aggregate employment levels for the following two paths. Along the first path, the economy would be forever in autarchy with steady-state growth at rate g. Along the second path, which coincides with the first until free trade is introduced at the beginning of time $t = 0$, the economy would make the transition to and then remain forever after in free-trade steady state growing at the initial rate g.

The full-employment analysis of Solow (1967) may be extended straightforwardly for the present case of unemployment, thereby yielding the following result (assuming $\bar{r} > g > -1$):[18]

$$\lim_{\tau\to\infty} \sum_{t=0}^{\tau} (C_f^t - C_a^t)(1+\bar{r})^{-t-1} = \lim_{\tau\to\infty} \sum_{t=0}^{\tau} \bar{w}_2(L_f^t - L_a^t)(1+\bar{r})^{-t-1} \quad (21)$$

where C_a^t and L_a^t are the aggregate levels of consumption (valued in terms of the second good) and employment, respectively, during time t along the autarchic (first) path; C_f^t and L_f^t are these variables along the free-trade (second) path (including the transition); and for all accounting purposes, $\bar{w}_1$, $\bar{w}_2$, $\bar{p}$ and $\bar{r}$ are (by assumption) used exclusively, even during the transition when there may be changes in the market (but not in the accounting) prices and interest rate. The left-hand and right-hand sides of equation (21) are the present discounted values (at the beginning of time $t = 0$) of all differences in aggregate consumption and total wages, respectively, between the two paths. According to equation (21), if the introduction of free trade leads to an overall employment loss or gain in terms of wages, there is an equal overall loss or gain in consumption. It is worth noting that these overall losses (gains) may occur even when steady-state employment is not lower (higher) in free trade than in autarchy.[19]

The foregoing discussion supports the well-known caveat that policy conclusions, when based upon steady-state comparisons, must also be assessed in terms of the costs or benefits occuring during the transition from one steady state to another. It is not, however, the intention of the present paper to analyse these issues in detail. They are mentioned, rather, as avenues for further research on the time structure of production as an important determinant of employment, capital and consumption in a trading economy.

4. Summary

The present paper develops a model of employment, capital and income in an open economy, to emphasize the significance of the time structure of production as a determinant of the domestic impact of international trade. The analysis first concentrates upon stationary-state properties of the model, to facilitate comparison with the previous literature, and to underline the role of investment intensities in determining equilibrium levels of employment, capital and income. Finally, the discussion indicates how the model can be extended

in subsequent work to deal more fully with situations of steady-state growth and periods of transition, which highlight further departures from conventional results.

FOOTNOTES

1. See, for example, the work of Mainwaring (1974), Metcalfe and Steedman (1973), Smith (1974a, 1974b) and Steedman and Metcalfe (1973). Also see Corden's (1974, p. 276) comments on the importance of a long installation and gestation period, for distinguishing research and development from ordinary investment. This period is relatively pronounced also in production of the following goods, among others: tree-based cash crops, such as cocoa [studied by Brecher and Parker (1975)], coconuts (and other palm tree products), coffee, tea and rubber; forestry products; minerals, such as metals and petroleum products; and ships. The concept "gestation period" is discussed somewhat more technically in footnote 5 below.

2. The flow-input and flow-output specifications imply the presence of circulating and fixed capital, respectively, as suggested by the discussions of Hicks (1973,p.8) and von Weizsäcker (1971, especially p.67). Burmeister (1974) has discussed the relationship between the Hicksian (1973) and other approaches to capital theory. For a model related to Hicks' (1973), see also Nuti (1970).

3. There is no incentive in either sector to do otherwise (except perhaps temporarily during the transition in Section 3.3) under the fixed-wage assumption made below, according to the choice-of-technique analyses of Hicks (1973) and Nuti (1970). (In Section 3.3, the discussion assumes that there is no occurrence of temporary technical substitution during the transition between steady-state equilibria, thereby simplifying the analysis without detracting from the main thrust of the argument.)

3a. As shown by Nuti (1974), however, the standard wage-interest analysis generalizes to the case in which the time structures of r and w_i are given.

4. The analysis can be extended readily to allow for $\beta_{i0} > 0$, in which case $\lim_{r\to\infty} h_i(r) = \beta_{i0}/\alpha_{i0} > 0$.

4a. This restriction -- whose purpose is to ensure that w_i is a strictly monotic function of r -- is innocuous (but simplifies the exposition) in the present paper, assuming that each unit process could be terminated (or truncated) freely at any time before the beginning of period $\Omega_i + 1$. By the reasoning of Hicks (1973) and Nuti (1973) under this truncation assumption, the duration of a process is determined optimally and w_i is a strictly monotonic (decreasing) function of r, even when this strict monotonicity fails to hold for any fixed-duration wage-interest function like $h_i(r)$.

5. The intuitive concept of a "gestation period" can be related to I_i^t in several ways. It would be possible, for example, to define a gestation period as any period j for which $j \in J_i$; whence total gestation-period investment in sector i would be equivalent to I_i^t. Alternatively, the gestation period in sector i could be defined as ranging from the inception of the sector's unit process to the point at which the capital value of the process (see below) first reaches its global maximum. In the special case corresponding to Hicks' (1973) "simple profile", the two definitions give equivalent results.

6. This sentence would remain true if the phrase "past net inputs accumulated forward" were replaced by the phrase "future net outputs discounted backward", by reasoning analogous to that of Hicks (1973, pp. 27-35).

6a. By assuming above that inputs and outputs occur at the end of each period, the present paper avoids the problems encountered by Hicks (1973,pp.27-35) with discrete time.

7. As shown by Hicks (1973), to guarantee that the economy converges to its new equilibrium, strong assumptions concerning disequilibrium behavior (in response to temporary windfall gains or losses) are necessary even for the one-sector case. Additional types of assumptions are required for convergence in the present two-sector case, as suggested implicitly by the discussion in footnote 19 below. For treatments of stability and convergence in closed-

economy models with heterogenous capital goods, see also Burmeister (1974,pp.447-448), Ross (1975) and refeences cited by them.

8. For alternative savings behavior in a related modd, see Brecher and Parker (1975).

9. In the meantime, it may be noted that any such divergence in the present model is similar to a divergence between the ratio of labor coefficients and the product-price ratio in Ricardo's (1821, Chapter I, Sections IV and V) model.

10. The results of this analysis hold also for an expansion of the volume of the free trade, due to a shift of foreign demand in favor of home exports.

11. The following example provides a case where $\lambda_2 > \lambda_1$. Let the vectors of inputs and outputs be $(\alpha_{10}\ \alpha_{11}\ \alpha_{12}) = (2\ 3\ 0)$, $(\alpha_{20}\ \alpha_{21}\ \alpha_{22}) = (1\ 2\ 1)$, $(\beta_{10}\ \beta_{11}\ \beta_{12}) = (0\ 3\ 8)$, $(\beta_{20}\ \beta_{21}\ \beta_{22}) = (0\ 1\ 7)$; and set $\bar{w}_1 = 1, \bar{w}_2 = 1$, $\bar{r} = 1$ and $\bar{p} = 1$. In this case, $\lambda_2 = 1/2 > 2/5 = \lambda_1$.

12. In Brecher's (1974b) treatment, for example, consider the role of $\bar{k}_i$ $(i = 1,2)$ in determining dL/dX_2.

12a. Further to footnote 8 and related discussion above, each I_i/L_i would retain its present relevance if (under some alternative assumptions) the value of $\bar{I}$ (and gross national savings) were not the same for all stationary states compared.

13. In the example of footnote 11, where $\lambda_2 > \lambda_1$, $K_2/L_2 = 1 < 6/5 = K_1L_1$.

14. See footnote 12.

15. In the example of footnote 11, $\delta_2 = 1/2 > 1/3 = \delta_1$.

16. In the example of footnote 11, where $\lambda_2 > \lambda_1$ and (recalling footnote 15) $\delta_2 > \delta_1$, $\bar{p} = 1 > 8/11 = \bar{p}\delta_1\theta_2/\delta_2\theta_1$ and $dY/dX_2 = -3/8$ -- even though $K_2/L_2 < K_1/L_1$ (recalling footnote 13). Compare this result with the corresponding one in, for example, Brecher (1974a).

17. It should be noted that these functions are not necessarily monotonic, as can be demonstrated by considering their first derivatives with respect to g. Similarly, the present average products (θ_i and γ_i) are not-necessarily-monotonic functions of g, although the traditional average products and capital/labor ratios in conventional trade models depend only upon the real wage. Concerning the influence of g on average products of labor and capital/labor ratios, see also Hicks (1973, pp.63-67) and Nuti (1970).

18. To obtain this extension, simply let employment in Solow's (1967) last paragraph of section 5 be (in his notation) L_t on one path and L_t^1 on the other path (instead of L_t on both paths), and repeat the reasoning of his paragraph with the obvious modifications now necessary. Note that proof by induction leads from his last equation of p.34 to his first equation of p.35. (Because of what seems to be a misprint, subscript t appears to need replacing by subscript j on the right-hand side of the latter equation.) Also observe (in his notation) that $(1+r)^{-t}(V_t-V_t^1) = (1+r)^{-t}(V_0-V_0^1)(1+m)^t$, which $\rightarrow 0$ as $t \rightarrow \infty$ if $r > m > -1$, where m is defined in his section 7. Finally, because the occurrence of technical substitution is absent even during the transition (recalling the assumption of footnote 3 above), no process is truncated prematurely (i.e., before the beginning of period $\Omega_i + 1$), since premature truncation would constitute a type of technical substitution (which is absent). If there were premature truncation, the right-hand side of equation (21) could exceed this equation's left-hand side by an amount corresponding to the "discrepancy" of Hicks (1973, pp.30 and 34), as could be shown without difficulty.

19. For instance, let g = 0, and modify the example of footnote 11, by setting $\alpha_{22} = 2$ (instead of 1) and $\beta_{22} = 8$ (instead of 7). Then, $\lambda_2 = 2/5 = \lambda_1$ in stationary state, and (by the reasoning of Section 3.1) the stationary-state level of L is the same with free trade as with autarchy. Nevertheless, both sides of equation (21) equal -1/4, assuming that the introduction of free trade causes two units of investable funds to be transferred from the first sector to the second by the following two

steps. First, during time t=0, investment on new starts is decreased permanently by one unit in sector one and increased permanently by one unit in sector two. Second, during time t=1, one unit of investable funds is transferred permanently from new starts in sector one to previous starts in sector two. If the transfer did not follow similar types of stepwise procedures in other examples, investment required by previously started processes could exceed the economy's current supply of investable funds -- a possibility closely related to the "impasse" discussed by Hicks (1973, p.132) in his one-sector model of a closed economy -- and therefore unit processes could be subject to premature truncation (but recall footnote 18 above).

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