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Optimal Savings and Foreign Debt Policy with Export Revenue Uncertainty

James McCabe
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It has been argued that, in determining the optimal rate foreign debt accumulation, one should consider the variability of export earnings, in addition to such parameters as the domestic saving rate, the marginal propensity to import, and the projected mean export growth rate.[1] This is obvious from the point of view of lending countries in that the possibility of defaulting on debt service payments increases with the variance of export revenues. However, the uncertainty associated with variability in export revenues may also have an impact on optimal debt policy in the case where maximization of the borrowing country's welfare is the sole consideration.

While heuristic arguments have been presented in the foreign debt literature sponsored by the World Bank, for a rigorous demonstration of this point we must refer to an analogous result in household savings theory. The fluctuation in export revenue may be broken down into two components: one attributable to random variation in sales and one to random variation in the world price of the exportable. In the case where export sales are unaffected by government policy, even though they are random it can be shown that, under certain assumptions about risk aversion, the optimal rate of debt accumulation will be reduced as export revenues become more uncertain and the path of total gross investment remains unchanged. This is true for the same reason that the household can be shown to have a precautionary demand for savings as a result of uncertainty in wage income. An increase

[1] See, for example, Avromovic, et al. [1].

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in net foreign capital inflow allows the borrowing country to consume more in the current period (with total gross investment fixed) but also results in greater debt service payments and lower consumption in subsequent time periods, which are associated with uncertainty in the level of export revenues. Since the variance of consumption is an increasing function of the variance of the export revenues, there may well be a precautionary motive for decreasing the rate of debt accumulation when export revenues become more uncertain (although their projected mean rate of increase remains constant). Debt accumulation is simply dis-savings, and a rise in export uncertainty in an aggregate model creates an effect on optimal net foreign inflow which is exactly opposite to the increase in the household's precautionary savings demand induced by future wage income becoming more uncertain. By means of a two-period optimization mode, Leland has shown that precautionary savings increase with uncertainty when the household utility function has the property of diminishing absolute risk aversion [5].

Aside from presenting the aggregate debt accumulation version of this model in detail, we shall demonstrate in this paper that the dampening effect of export-revenue uncertainty on foreign borrowing is critically dependent on the assumed exogenous and policy variables, and the parameterization of the utility function. For example, one important variable assumed to be exogenous in the preceding discussion, in addition to export revenues, is investment.

Debt accumulation has a definite bearing on the optimal domestic saving and investment policies of a developing country, and the optimal trajectories for these two variables over time should be determined simultaneously. In a system in which both net foreign capital inflow and domestic savings are policy variables, and hence total investment is allowed to vary, it is possible for domestic savings to act on an inter-temporal averaging device which offsets the consumption impact of variation in export earnings. We shall test whether or not this type of savings behavior influences the sensitivity of optimal debt accumulation to changes in the variance of export earnings. Further, we use a dynamic programming model to derive a closed-form solution for the optimal feedback relationship between domestic savings and export earnings.

Another objective of this paper is to investigate the effect of making export output a variable which is influenced at least indirectly by economic policy. We consider cases in which both the price of exports and their production and/or sales level are uncertain. Our main results are based upon the assumption that the interest rate on new debt depends on the level of total debt and GNP is a linear function of capital. We demonstrate that, given these assumptions, making both domestic savings and net foreign capital inflow policy variables will cause optimal debt accumulation to be independent of uncertainty provided that the volume of exports is exogenous. When export production is endogenous the dampening effect of revenue uncertainty on foreign borrowing depends on parameterization of the utility function.

In Part I, static systems are presented which relate consumption to real exports, the terms of trade, debt service payments, and domestic savings in two cases: (1) where the volume of exports is exogenous; and (2) where it is endogenous. In Parts II and III dynamic models based on these static systems are developed and both two-period and T-period solutions are analyzed. In Part IV, we use existing empirical studies to test the real-world applicability of the assumptions on which the two models are based. We also cite evidence indicating that the direction of response to export-revenue fluctuation in the case of several countries is consistent with that obtained from our optimizing model with exogenous exports. In the concluding section the results are summarized.

I. The Static Model

The static model is based on the assumption that there are two commodities. Good 1 is a primary product export, not domestically consumed. Good 2 is an import-competing commodity used for final investment and consumption purposes. Suppose that the real wage expressed in terms of Good 2, $w(2)$ is subject to the constraint

$$w(2) \geq \bar{W}(2) \tag{1.1}$$

where $\bar{W}(2)$ is an exogenously determined real minimum wage, and that the economy is in competitive equilibrium. In addition, assume that there are two factors of production, capital and labor, and that there is constant returns to scale in production in the two sectors. Then, as Brecher [3] has shown, with constraint (1.1) binding, labor will be openly unemployed and the transformation surface will be linear in the region of incomplete specialization. The transformation surface in this region, corresponding to a given real minimum wage and aggregate capital stock $\bar{K}$ is depicted as T_1T_2 in Figure 1. The variables measured along each of the axes in this case represent the real value added of the commodity produced in a particular sector.

Given that good 2 is the numeraire commodity and the home country is a price taker in world trade with a variable export tax, both gross domestic product at internal prices and gross domestic product at world prices may be determined in the following manner. The ratio of the internal price of commodity 1 to that of commodity 2, $\hat{p}$, is uniquely determined by the specified real minimum wage as a result of the Samuelson relationship between factor and relative commodity prices [5]. Hence there are an infinite number of commodity price lines, each having the slope $-1/\hat{p}$. Denote the quantity produced and sold of the exported good by E . The commodity price line corresponding to the specified value of E , whose slope is determined by the real minimum wage, is depicted as $\hat{p}\hat{p}$ in Figure 1. It is clear that, under these conditions, GDP at internal prices, Q , is given by the distance oa , on the horizontal axes.[1] In algebraic terms this variable is a linear function of E and the aggregate capital stock K ;

$$Q = aE + bK . \tag{1.2}$$

Denote the capital output ratio in the sector producing commodity i by z(i) . Then it can be shown that $a = (\hat{p} - z(1)/z(2))$ and $b = 1/z(2)$, where $z(1)/z(2)$ is the inverse of the slope of the transformation surface. Brecher has shown that when the commodity measured along the vertical axes is relatively labor intense the absolute value of the slope transformation surface will be greater than that of the commodity price line. This implies

[1]An increase in total capital stock from $\bar{K}$ to $\bar{\bar{K}}$ will cause a parallel outward movement in the transformation surface from T_1T_2 to $T_1'T_2'$.

that the export coefficient a will be positive. The export coefficient will be negative in the case where the import-competing commodity, good 2, is relatively labor intense.

It is assumed that the home country is a price taker in world trade, and that the export tax or subsidy on good 1 determined by the difference between the constant internal price ratio, $\hat{p}$, and the external commodity price ratio, P. Gross domestic product at world prices, Y, may be depicted as a distance ob along the horizontal axes in Figure 1 when good 2 is the numeraire commodity.

This variable is determined by the relationship

$$Y = bK + [a + (p - \hat{p})]E = Q + (p - \hat{p})E \ . \tag{1.3}$$

Denote the average interest rate on debt by r and total debt (deflated by the price of good 2) by D. Then gross national product at world prices GNP is given by the identity

$$GNP = Q + (p - \hat{p})E - rD \ . \tag{1.4}$$

Consumption C, may be determined as difference between gross national product at world prices and gross domestic saving S, or by the identity

$$C = GNP + F - I = Q + (p - \hat{p})E - rD + F - I \tag{1.5}$$

where F is net foreign capital inflow and I is gross investment (all expressed in terms of the numeraire commodity, good 2).

This approach is reasonable in the case where the price of the export commodity is random and the real output of good 1 can be viewed as exogenously

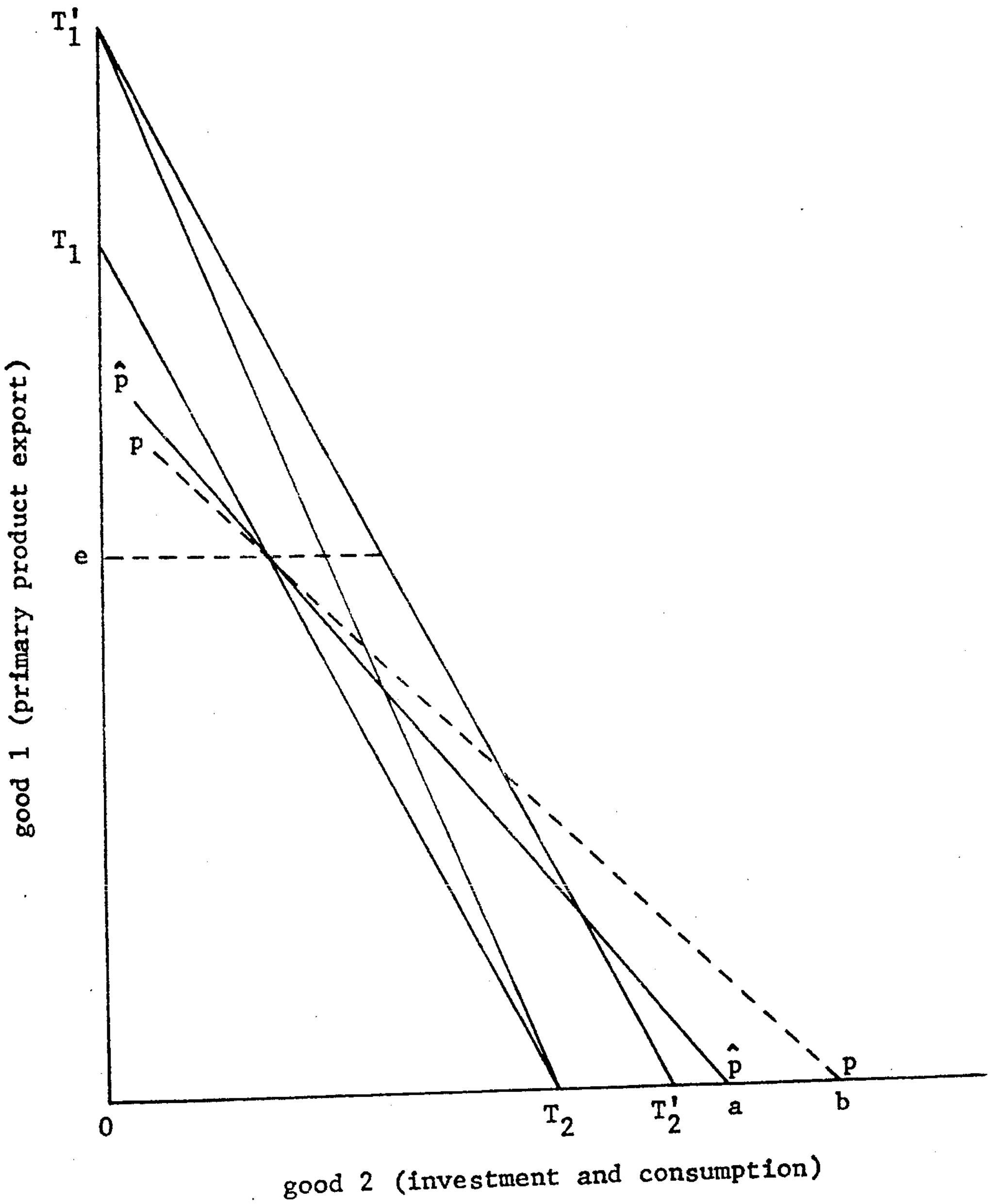

FIGURE 1

Transformation Surface and Commodity Price Lines given. This is not true when control variables affect real output of good 1. To formulate this latter case let the production function of sector 1 take the form

$$Q(1) = \hat{B}F(K(1), L(1)) \tag{1.6}$$

where $Q(1)$ is output, $K(1)$ is capital stock, $L(1)$ is employed labor in sector 1, and $\hat{B}$ is a multiplicative factor which could be random, e.g., reflecting the influence of weather on export output. Denote the wage rental ratio in both sectors by ω and the capital-labor ratio in sector 1 by $k(1)$. It can be shown that, given constant returns to scale and internal price certainty for commodity 1, the capital-labor ratio of the profit maximizing firm in sector 1 will still be determined by the relation

$$\frac{f(k(1))}{f'(k(1))} - k(1) = \omega \tag{1.7}$$

where $f(k(1)) = F(K(1)/L(1), 1)$ and $f'(k(1)) = f' = \partial F/\partial k(1)$.[1] Since

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[1]In this case, the firm's objective function takes the form

$$EU(P\hat{B}F - wL - rK)$$

where E is the expected value operator, U is the well-behaved utility function, p is the exogenously given price and w and r are the labor wage and capital rental rates respectively. The first order-conditions yield

$$EU'\left(P\hat{B}\frac{\partial F}{\partial K} - r\right) = 0$$

$$EU'\left(P\hat{B}\frac{\partial F}{\partial L} - w\right) = 0 .$$

From these relationships, (1.7) may be obtained by assuming constant returns to scale.

with diminishing returns to scale, $f''(k(1)) < 0$, it can be shown that al-

$$\frac{d\omega}{dk} > 0$$

ways holds and that $k(1)$ is uniquely determined by the wage rental ratio associated with the specified real wage (expressed in terms of good 2). Consequently the capital labor ratio is independent of random parameter $\hat{B}$. However, the capital-output ratio is given by the expression

$$\frac{k(1)}{\hat{B}f(k(1))} = \hat{Z}(1) \tag{1.8}$$

which depends on the random variable. Since the expression for the transformation surface relating output of good 2, $Q(2)$, to output of good 1, $Q(1)$, takes the form

$$Q(2) = \frac{1}{Z(2)} K - \frac{\hat{Z}(1)}{Z(2)} Q(1)$$

the problem may be viewed as one in which the intercept of the transformation surface on the vertical axis changes but not on the horizontal axis as result of random variation in $\hat{B}$. When capital is allocated completely to sector 2, $Q(2) = \frac{1}{Z(2)} K$ which is independent of $\hat{B}$. This type of production variability is illustrated by the shift from T_1T_2 to $T_1'T_2$ in Figure 1.

Under these conditions, it seems reasonable to assume that output of good 2 is specified but not good 1, since no uncertainty is associated with the production of the former commodity and it is dubious that resources move instantaneously into or out of sector 2 to compensate for changes in $\hat{B}$.

Let $\hat{\theta} = 1/\hat{Z}(1)$, then the output of good 1 $Q(1)$ is given by the formula

$$Q(1) = K\hat{\theta} - Z(2)\hat{\theta}\overline{Q}(2)$$

where $\overline{Q}(2)$ is specified output level of good 2. This formulation implies an expression for aggregate consumption of the form

$$C = p\hat{\theta}(K - Z(2)\overline{Q}(2)) + \overline{Q}(2) - S \; .$$

The expression capital in sector 1, $K(1)$, is

$$K - Z(2)\overline{Q}(2) = K(1)$$

and $K(1)$ is assumed to be positive. Therefore, a random increase in the output capital ratio in sector 1 will cause consumption to increase, with world relative commodity prices (P) fixed, and vice versa.

II. Dynamic Models with Exogenous Exports

The optimal debt accumulation model is precisely analogous to Leland's optimal savings model for the household in the case where gross investment is held constant, net foreign capital inflow is controlled, and uncertainty is confined to export revenues. Assume that E is exogenous, possibly random, and that P is a random variable with a fixed mean and a constant standard deviation σ_p . Then we may derive a new random variable, $\widetilde{E}$, which is given by the function

$$\tilde{E} = (p - \hat{p})E + aE . \tag{2.1}$$

Since this function is linear in p with E constant, the standard deviation of $\tilde{E}$ is simply $E\sigma_p$.[1]

The social welfare function for the two-period model takes the form

$$L = U(C_1) + \frac{1}{(1+\delta)} \mathcal{E} U(C_2) \tag{2.2}$$

where δ is the rate of social discount (assumed constant), $\mathcal{E}$ is the expected value operator, and the subscripts 1 and 2 refer to consumption in the first and second time periods. The second utility function for each time period, U, is always restricted to meet the regularity conditions

$$U'(C) > 0 \text{ and } U''(C) < 0 . \tag{2.3}$$

In the initial time period, P is known and E_1 is equal to a specified value, whereas the value of the expression in the second time period, $\tilde{E}_2$, is uncertain. We assume not only that gross investment per laborer in the initial time period (I_1) is fixed but also, without loss of generality, that gross investment is zero in the terminal period. The final assumption upon which our model is based is that the interest rate on new debt is an increasing function of total debt D.[2] Assuming that world capital markets are imperfect in this sense is a way of capturing risk aversion on the part

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[1] If E were random,

$$Var(E) = \bar{p}^2\sigma_E^2 + \bar{E}^2\sigma_p^2 + \bar{P}\bar{E}\,cov(P,E) + [\bar{E}\bar{P} + cov(P,E)]^2$$
$$+ 2(a - \hat{p})(\sigma^2_{PE^2} - \bar{P}(\bar{E}^2 + \sigma_E^2)) + (a - \hat{p})^2(\bar{E}^2 + \sigma_E^2)$$

where $\bar{P}$ is the mean of P and $\bar{E}$ is the mean of E.

[2] This assumed relationship was originally imployed by Bardhan [2].

of lendors. This assumption also implies that the average interest rate on total debt, r , is an increasing function of the total debt ratio, i.e.,

$$\begin{aligned} r &= r(D) \\ r' &> 0 \ . \end{aligned} \tag{2.4}$$

Consequently, total interest payments are given by the function

$$rD = r(D)\cdot D = \pi(D) \ , \tag{2.5}$$

the first derivative of which takes the form

$$\emptyset = r + r'(D)D > 0 \ . \tag{2.6}$$

The value of this expression represents the amount by which an extra unit of debt raises the total cost of borrowing.

The optimization problem[1] is one of choosing a value of net foreign capital inflow in the initial time period (F_1) which maximizes

$$L = U(GNP_1 - r_1D_1 + F_1 - I_1) + \frac{1}{1+\delta}\mathcal{E}U(b(K_1 + I_1) - r_2D_2 + \tilde{E}_2) \ . \tag{2.7}$$

The first order condition for the maximum in this case is simply

$$\frac{\partial L}{\partial F_1} = U' - \emptyset\,\frac{\mathcal{E}U'}{1+\delta} = 0 \tag{2.8}$$

[1] We assume no labor force growth, at zero cost in generality.

where $\emptyset = \pi'(d)$. Denote the variance of $\tilde{E}$ by $\sigma^2_{\tilde{E}}$. Then from (2.8) it follows that $\partial F/\partial\sigma^2_{\tilde{E}} < 0$ provided that the utility function exhibits decreasing absolute risk aversion.[1]

This result breaks down in the 2-period case when domestic savings rather than total investment is held fixed, and in the T as well as the 2-period case when both domestic savings and net foreign capital inflow are allowed to vary. The 2-period model with initial domestic saving, S_1 , fixed and no savings in the terminal period takes the form

$$\text{Max } L = U(GNP_1 - S_1) + \frac{1}{1+\delta} \mathcal{E}U(b(K_1 + S_1 + F_1) + \tilde{E}_2 - r_2 D_2) . \quad (2.9)$$

The first order condition in this case is

$$\frac{\partial L}{\partial F_1} = \frac{1}{1+\delta} \mathcal{E}U'(b - \emptyset) = 0 \quad (2.10)$$

which implies that $b = \emptyset$. In other words, one will accumulate debt until the incremental cost of borrowing is equal to the constant social marginal product of capital, b . Here the debt accumulation is independent of export

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[1] Absolute risk aversion is defined as $-U''/U'$, and can be interpreted as reflecting the individual's willingness to undertake a bet of fixed size. It is generally agreed that absolute risk aversion should decline with wealth; a sufficient condition for this is that the third derivative of the utility function, U''' , be positive. In the present instance, this also guarantees the convexity of U' in $\tilde{E}$, so that

$$\frac{\partial EU'}{\partial \sigma^2_{\tilde{E}}} > 0 .$$

revenue uncertainty as measured by $\sigma_{\tilde{E}}$. This result also holds when S_1 is allowed to vary in the two-period example. In that case, differentating the first-order conditions $\partial L/\partial S_1 = 0$ and $\partial L/\partial F_1 = 0$ with respect to $\sigma^2_{\tilde{E}}$, with decreasing absolute risk aversion yields

$$\frac{\partial F_1}{\partial \sigma^2_{\tilde{E}}} = 0 \ , \quad \frac{\partial S_1}{\partial \sigma^2_{\tilde{E}}} > 0 \ . \tag{2.11}$$

Thus, when uncertainty increases, optimal savings policy takes the form of shifting the mean of C_2 to the right in order to compensate for its higher variance.

The T-period case may be analyzed by means of dynamic programming. For the sake of simplicity, we assume that total capital stock financed by foreign debt (K_f) is controlled, rather than net foreign capital inflow. This variable should be distinguished from the stock of imported capital and in the case where the depreciation rate is zero, it is identically equal to foreign debt, D . Making this simplification affects nothing.

We assume that capital financed by foreign debt (K_f) and capital financed by domestic savings (K_d) are homogenous factors of production. Given that the return on total capital is b , consumption in period t is given by the expression:

$$C_t = b(Kd_t + Kf_t) + E_t - r_t D_t - S_t \ . \tag{2.12}$$

Assume that welfare is intemporally additive with a constant social discount rate, and that the utility function is one with constant absolute risk aversion of the form

$$U(C) = \frac{e^{\eta C}}{\eta}\ , \quad \eta < 0\ , \quad e = \text{base of natural logarithms.} \qquad (2.13)$$

In this case utility is always negative; but this is irrelevant since all we are concerned with is the ordering of preferences and the function is well behaved in the sense that the following regularity conditions are met:

$$U' > 0\ , \quad U'' < 0\ .$$

The assumption of constant absolute risk aversion is objectionable in a portfolio choice model. However, since the value of export revenues is exogenous, this problem is not relevant here. In addition, for getting a closed solution to our problem, the constant absolute risk aversion formulation is convenient.

The planning authority's objective is to

$$\max_{\{\vec{k}_F,\ \vec{S}\}} \mathcal{E}\left[\sum_{t=0}^{T} \frac{U(C_t)}{(1+\delta)^t}\right]$$

$$\vec{k}_F = \langle k_{F_T},\ k_{F_{T-1}},\ k_{F_{T-2}},\ \ldots,\ k_{F_0} \rangle$$

$$\vec{S} = \langle S_T,\ S_{T-1},\ \ldots,\ S_0 \rangle\ .$$

The dynamic programming problem may be solved by starting first with the terminal period and then working back in time. Assuming that there is no domestic saving in this period, the end-period maximization is simply

$$\max_{\{k_{T_T}\}} \frac{e}{\eta}^{\eta[b(k_{D_T}+S_{T-1}+k_{F_{T-1}})+\tilde{E}_T - r_T k_{F_T}]} \quad . \tag{2.14}$$

The first-order condition is simply $\phi = b$; denote $\phi^{-1}(b)$ by k_F^*, and $r(k_F^*)\cdot k_F^*$ by π^*. The maximized value of utility in the end-period we denote by J_0, so

$$J_0 = \frac{e}{\eta}^{\eta[b(k_{D_{T-1}}+S_{T-1}+k_F^*)+\tilde{E}_T - \pi^*]} \quad .$$

We notice that S_{T-1} appears as a parameter in the end-period problem; S_{T-1} is chosen to

$$\max_{\{k_{F_{T-1}}, S_{T-1}\}} \frac{e}{\eta}^{\eta[b(k_{D_{T-1}}+k_{F_{T-1}})+\tilde{E}_{T-1} - r_{T-1}k_{F_{T-1}} - S_{T-1}]}$$

$$+ \frac{1}{1+\delta} \mathcal{E} \frac{e}{\eta}^{\eta[b(k_{D_{T-1}}+k_F^*+S_{T-1})+\tilde{E}_T - \pi^*]} \quad . \tag{2.15}$$

The first-order condition for $k_{F_{T-1}}$ implies, as before that $b = \phi$, so $k_{F_{T-1}} = k_F^*$, independently of savings. The first order condition for S_{T-1} can be solved for S_{T-1}, giving us

$$S_{T-1}^* = \frac{\tilde{E}_{T-1}}{1+b} - \frac{1}{\eta} \ln b \frac{V_1}{1+\delta} \quad , \quad V_1 = \mathcal{E} e^{\eta\tilde{E}} \, . \tag{2.16}$$

Henceforth $*$ will denote an optimal value. Since $e^{\eta\tilde{E}}$ is convex in $\tilde{E}$ and $\eta < 0$,

$$\frac{\partial S^*_{T-1}}{\partial \sigma^2_{\tilde{E}}} = -\frac{1}{\eta}\,\frac{\partial \ln[bV_1|1+\delta]}{\partial \sigma^2_{\tilde{E}}} > 0 \ .$$

Also, we have a marginal propensity to save out of the period-to-period random draws $\tilde{E}_t$ of $1/1+b$, or about .25 for most LDC's. Having S^*_{T-1}, we may write the maximized value of expected utility from periods T-1 to T as

$$J_1 = \frac{e}{\eta}^{\eta[b(k_{D_{T-1}} + k^*_F) - \pi^* + \tilde{E}_{T-1} - S^*_{T-1}]} + \frac{1}{1+\delta}\,\frac{e}{\eta}^{\eta[b(k_{D_{T-1}} + k^*_F + S^*_{T-1}) - \pi^*]} \cdot V_1$$

$$= \beta_1\,\frac{e}{\eta}^{\eta[b(k_{D_{T-1}} + k^*_F) + \frac{b}{1+b}\tilde{E}_{T-1}]} \ . \tag{2.17}$$

Where β_1 can be determined from (2.16).

Having J_1, we can now go back a period and

$$\max_{\{S_{T-2},\ k_{F_{T-2}}\}} \frac{e}{\eta}^{\eta[b(k_{D_{T-2}} + k_{F_{T-2}}) - r_{T-2}k_{F_{T-2}} - S_{T-2} + \tilde{E}_{T-2}]} + \beta_1 \mathcal{E}\,\frac{1}{1+\delta}\,\frac{e}{\eta}^{\eta[b(k_{D_{T-2}} + k^*_F + S_{T-2}) + \frac{b}{1+b}\tilde{E}_{T-1}]} \ .$$

Again $k_{F_{T-2}} = k^*_F$ and the savings rule takes the same form as (2.16), with slope equal to $1/1+b$, but a different intercept. In the infinite horizon case

$$J = \bar{\beta}\,\frac{e^{\eta[b(k_D + k_F^*) + \frac{b\tilde{E}}{1+b}]}}{\eta} \tag{2.18}$$

$$\bar{\beta} = \text{constant} = \left(\frac{\mathcal{E} e^{\frac{b\tilde{E}}{1+b}}}{1+\delta}\right)^{b} \left[b^{\frac{1}{1+b}} + b^{-\frac{b}{1+b}} \right]^{\frac{b+1}{b}} .$$

The dynamic programming problem is to find S^* such that

$$\bar{\beta}\,\frac{e^{\eta[b(k_D + k_F^*) + \frac{bE}{1+b}]}}{\eta} = \frac{e^{\eta[b(k_D + k_F^*) - \pi^* + \underline{\tilde{E}} - S^*]}}{\eta} + \frac{1}{1+\delta}\mathcal{E}\frac{e^{\eta[\bar{B}(k_D + k_F^* + S^*) + \frac{b\tilde{E}}{1+b}]}}{\eta}$$

where $\underline{\tilde{E}}$ denotes the current realized value of $\tilde{E}$. Solving for S^* we find

$$S^* = \frac{\tilde{E}}{1+b} - \frac{1}{\eta}\ln\left(\frac{\mathcal{E} e^{\frac{\eta b\tilde{E}}{1+b}}}{1+\delta}\right)^{b+1} \cdot \frac{b\left(b^{1+b} + b^{-\frac{b}{1+b}}\right)^{\frac{b+1}{b}}}{1+\delta} . \tag{2.19}$$

Again, $\partial S^*/\partial\sigma_E^2 > 0$.

Starting from the end period, when we specified zero savings, the savings function drifts up over time with a constant slope until the infinite-horizon savings function (2.19) is reached. Throughout the planning period foreign-financed capital, K_F , is at a constant level K_F^* , independent of savings behavior and uncertainty.

We can sum up the influence of export revenue uncertainty as follows:

(a) F controlled, I fixed: $\partial F/\partial\sigma_{\tilde{E}}^2 < 0$

(b) F controlled, S fixed: $\partial F/\partial\sigma_{\tilde{E}}^2 = 0$

(c) F , S both controlled: $\partial F/\partial\sigma_{\tilde{E}}^2 = 0$, $\partial S/\partial\sigma_{\tilde{E}}^2 > 0$

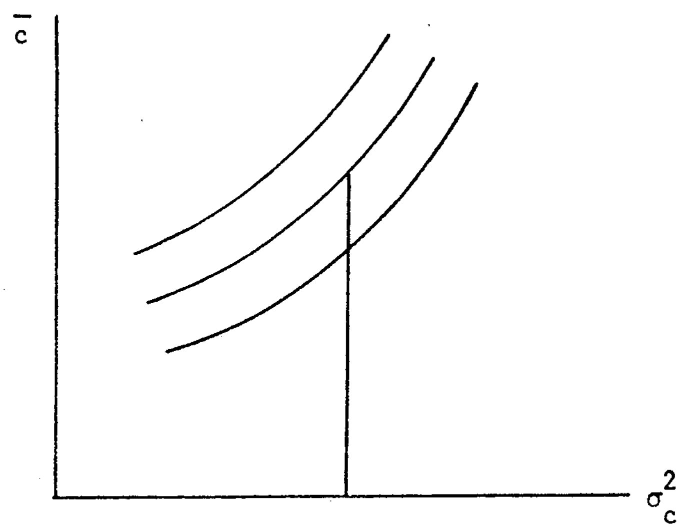
(d) F , I controlled: $\partial F/\partial\sigma_{\tilde{E}}^2 \neq 0$, $\partial I/\partial\sigma_{\tilde{E}}^2 \neq 0$ signs unclear.

In cases (a)-(c) the effect of policy response to increased export uncertainty is to shift over the mean of the distribution of future consumption in order to compensate for its increased variance; case (d) is ambiguous. In addition, it can be shown, in (a), that $\partial F/\partial \underline{\tilde{E}} < 0$ and $\partial S/\partial \underline{\tilde{E}} > 0$; in other words as the current realized value of $\underline{\tilde{E}}$ rises, optimal savings rises, and foreign borrowing falls.

We should emphasize what features of the models lead to the result that with S fixed or controlled, foreign borrowing is independent of uncertainty; recall that

$$C_2 = b(K_1 + S_1 + F_1) - r_2 D_{E_2} + \tilde{E}_2 .$$

If we draw the "opportunity locus" in mean-variance space facing the country for different values of F , we see that it is vertical; varying F affects only its height (which is maximized when $b = \emptyset$). Since, therefore, foreign capital inflow leaves σ_c^2 unaffected, $dF/d\sigma_{\tilde{E}}^2$ is zero. <u>Thus, it was crucial that the volume of exports be exogenous</u>.



<u>Another essential element is the constancy of</u> b , <u>the marginal product of capital</u>. With decreasing returns to capital, the equilibrium level of debt is not uniquely determined by D independent of K . In the latter case the effects on F_1 on C_2 are intertwined with those of S , and net foreign capital inflow is affected by $\sigma^2_{\tilde{E}}$; here the fact that both F_1 and S_1 constitute additions to capital stock is relevant.

Finally, the assumption that $r'(D) > 0$ allows $b = \emptyset$; with the interest rate constant, despite $\tilde{E}$ being exogenous and b mixed, uncertainty will affect foreign borrowing. <u>Therefore imperfect capital markets and/or risk aversion by lending countries is crucial</u>.

We should note very briefly that although with investment fixed increased export revenue uncertainty does cause less foreign borrowing, this does not imply that multilateral commodity price stabilization would raise foreign borrowing; as we defined $\tilde{E}$,

$$\tilde{E} = E(p - \hat{p}) + aE .$$

So

$$\begin{aligned} \mathrm{Var}(\tilde{E}) = {} & (a - \hat{p})^2(\bar{E}^2 + \sigma_E^2) + \{\bar{p}^2\sigma_E^2 + \bar{E}^2\sigma_P^2 + \sigma_P\sigma_E\rho_{P\cdot E}(2\bar{E}\,\bar{P} + 1) + \bar{P}\,\bar{E}\}^2 \\ & + (a - \hat{p})\mathcal{E}(E^2P) + [\bar{E}\,\bar{P} + \sigma_P\sigma_E\rho_{P\cdot E} + (a - \hat{p})\bar{E}]^2 \qquad (2.21) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial\,\mathrm{Var}(\tilde{E})}{\partial\,\sigma_P} = {} & 2\{\bar{p}^2\sigma_E^2 + \bar{E}^2\sigma_P^2 + \sigma_P\sigma_E\rho_{P\cdot E}(2\bar{E}\,\bar{P} + 1)\}\{2\bar{E}^2\sigma_P + \sigma_E\rho_{P\cdot E}(2\bar{E}\,\bar{P} + 1)\} + (a - \hat{p})\frac{\partial\mathcal{E}(E^2}{\partial\,c} \\ & + 2[\bar{E}\,\bar{P} + \sigma_P\sigma_E\rho_{P\cdot E} + (a - \hat{p})\bar{E}]\sigma_E\rho_{P\cdot E} . \qquad (2.22) \end{aligned}$$

Clearly, with highly negatively correlations between the international price and output of the export good, a rise in σ_p can <u>stabilize</u> income, since fluctuations in prices would offset fluctuations in output. Quite conceivably, then, price stabilization, brought about by an international agreement, could lower the optimal level of foreign borrowing. This highly negative correlation between international price and domestic output would be most likely to occur in a country whose normal output of the export commodity is large relative to total world production. Therefore, <u>the response of optimal borrowing to price stabilization will be less positive</u>, <u>the greater control the country has over the price of its export</u>.

III. <u>Variable Export Volume</u>

We recall the alternative definition of consumption given in Part I:

$$C = P\hat{\theta}(k - z(2)\overline{q(2)}) + \overline{q(2)} - s - rd .$$

Throughout this section we fix $\overline{q(2)}$, the domestic production of the importable, so that exports will vary according to the random variable $\hat{\theta}$ and the capital stock K.

Let us take the simple cases where savings are fixed and net foreign capital inflow, f, is controlled; when the volume of real exports was fixed, we got $\partial F/\partial\sigma^2_{\tilde{E}} = 0$ in this case. Again,

$$\max_{\{f_1\}} L = U(GNP_1 - S_1) + \frac{1}{1+\delta}\mathcal{E}U(P\hat{\theta}((k_1 + S_1 + f_1) - z(2)\overline{q(2)} + \overline{q(2)} - r_2d_2) .$$

The first order condition is

$$\phi = \frac{\mathcal{E}U'P\hat{\theta}}{\mathcal{E}U'} .$$

Clearly foreign borrowing is now affected by uncertainty. Specifically, treating F as an implicit function of $\sigma^2_{P\hat{\theta}}$

$$\frac{dF}{d\sigma^2_{P\hat{\theta}}} = \frac{\frac{\partial}{\partial\sigma^2_{P\hat{\theta}}}(EU'P\theta - \emptyset EU')}{\emptyset EU' - EU''(P\hat{\theta})^2} .$$

Looking at the numerator, U' is convex in $P\hat{\theta}$ given decreasing absolute risk aversion, so $\partial EU'/\partial\sigma^2_{P\hat{\theta}} > 0$. The derivative of $EU'P\hat{\theta}$ with respect to $\sigma^2_{P\hat{\theta}}$ is harder; to get a handle on the convexity or concavity of $U'P\hat{\theta}$ in $P\hat{\theta}$, we assume that the utility functions constant relative risk aversion:[1]

$$U(c) = \frac{c^\eta}{\eta}, \quad \eta < 1 .$$

If we define $U'P\hat{\theta} \equiv H(P\hat{\theta})$,

$$H''(P\hat{\theta}) = (\eta-1)(k_1 + s_1 + f_1)\cdot(C)^{\eta-3}\{\eta(k_1 + s_1 + f_1 - z_2\overline{q(2)}) + \overline{q(2)} - rd\} .$$

We have then, a sufficient condition for convexity or concave of $H(P\hat{\theta})$:

$$H''(P\hat{\theta}) \leq 0 \quad \text{if} \quad \begin{cases} \eta \geq 0 \\ \overline{q(2)} - rd \geq 0 \end{cases}$$

$$H''(P\hat{\theta}) > 0 \quad \text{if} \quad \begin{cases} \eta < 0 \\ \overline{q(2)} - rd < 0 . \end{cases}$$

[1] Relative risk aversion defined as $-U''\cdot C/U'$, can be interpreted as reflecting the individual's willingness to undertake a bet involving a fixed proportion of his wealth.

The sign of the derivative of $EU'P\hat{\theta}$ depends, then, not only on a specific type of utility function, but on the parameterization of that utility function; and on the sign $\overline{q(2)} - rd$, which one would generally assume to be positive, since $C = P\hat{\theta}(\cdot) + \overline{q(2)} - rd$, and even with an extensive export shortfall, countries show positive consumption.

Assuming $\eta \geq 0$ and $\overline{q(2)} - rd \geq 0$, we have

$$\frac{\partial EU'(P\hat{\theta})}{\partial \sigma^2_{P\hat{\theta}}} \leq 0$$

and

$$\frac{dF}{d\sigma^2_{P\hat{\theta}}} = \frac{\frac{\partial EU'P\hat{\theta}}{\partial \sigma^2_{P\hat{\theta}}} - \emptyset \quad \partial \sigma^2_{P\hat{\theta}}}{\emptyset EU' - EU''(P\hat{\theta})^2} < 0 \ .$$

It is possible, then, to get the dampening effect traditionally imputed to export revenue uncertainty; but had we let $\eta < 0$, the sign of $dF/d\sigma^2_{P\hat{\theta}}$ would be ambiguous; we could easily have a case where export revenue uncertainty <u>increased</u> foreign borrowing.

If we control investment, our problem is

$$\max_{\{I, f_1\}} L = U(GNP_1 + f_1 - I) + \frac{1}{1+\delta} \mathcal{E} U(P\hat{\theta}(k_1 + I_1 - z(2)\overline{q(2)} - \overline{q(2)} - r_2 d_2) \ .$$

Implying:

$$\frac{\partial L}{\partial U} = -U' + \frac{1}{1+\delta} \mathcal{E} U'P\hat{\theta} = 0$$

$$\frac{\partial L}{\partial f_1} = U' - \emptyset \, \frac{1}{1+\delta} \mathcal{E} U' = 0 \ .$$

Differentiating the first-order conditions with respect to $\sigma^2_{P\hat{\theta}}$ and solving the resulting system, we find that

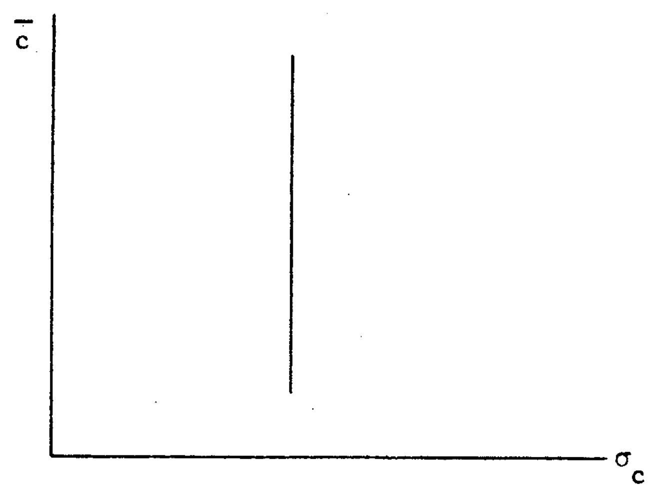
$$\frac{dI}{d\sigma^2_{P\hat{\theta}}} = \frac{\begin{vmatrix} -\overset{(+)}{\dfrac{\partial EU'P\hat{\theta}}{\partial\sigma^2_{P\hat{\theta}}}} & \overset{(+)}{\dfrac{\partial^2 L}{\partial I \partial F_1}} \\ \phi\,\overset{(+)}{\dfrac{\partial EU'}{\partial\sigma^2_{P\hat{\theta}}}} & \overset{(-)}{\dfrac{\partial^2 L}{\partial F_1^2}} \end{vmatrix}}{|J|} < 0$$

$$\frac{dF}{d\sigma^2_{P\hat{\theta}}} = \frac{\begin{vmatrix} \overset{(-)}{\dfrac{\partial^2 L}{\partial I^2}} & -\overset{(+)}{\dfrac{\partial EU'P\hat{\theta}}{\partial\sigma^2_{P\hat{\theta}}}} \\ \overset{(+)}{\dfrac{\partial^2 L}{\partial I \partial F_1}} & \phi\,\overset{(+)}{\dfrac{\partial EU'}{\partial\sigma^2_{P\hat{\theta}}}} \end{vmatrix}}{|J|} < 0$$

$|J|$ = determinant of the Jacobian; $|J| > 0$. This, again, assumes constant relative risk aversion with $\eta \geq 0$ and $\overline{q(2)} - rd > 0$. Clearly with $\eta < 0$ we could get the opposite result.

We have seen, then, that the dampening effect of export revenue uncertainty is critically dependent on the specific parameterization of the utility function, and is certainly not a general result.

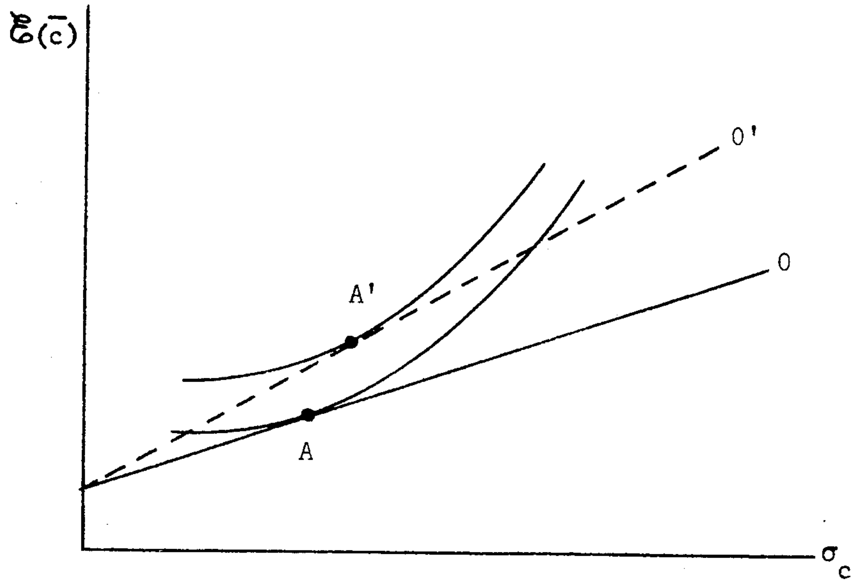
There is, however, a more fundamental question involved here: why does uncertainty matter at all? In these previous sections we found that for two of the four tableaux considered, $\phi = b$ in dependently of uncertainty; now in this section the equivalent symbol to b, $\hat{\theta}$ is random, but even if it weren't, our results would be unchanged, even with savings controlled, which led to $dF/d\sigma_{\tilde{E}} = 0$ previously. If we simplify by considering this a mean-variance problem, the opportunity locus facing the country if the value of exports is exogenous is a vertical line; borrowing, as we said above, affects only the height of the line, leaving the variance of consumption unchanged. Therefore one's decision about F will be made independent of uncertainty.



If $\overline{q(2)}$ is controlled or exogenous, changing f <u>does</u> affect the probability distribution of export revenues, since borrowing augments capital stock and

$$\mathcal{E}(\text{value of exports}) = \mathcal{E}(P\hat{\theta})(K - z(2)\overline{q(2)})$$
$$\mathrm{Var}(\text{value of exports}) = \sigma^2_{P\hat{\theta}}(K - z(2)\overline{q(2)})^2 \ .$$

If we draw the opportunity locus for this case we have the line $\overline{00}$ in the figure below; the equilibrium point is A , from which can be deduced the optimal values of I and F . Changing F rotates the locus to $\overline{00'}$, leading to a new equilibrium point A' . Net foreign capital inflow affects the slope and height of the opportunity locus, so uncertainty should be considered in choosing F .



A further point to be made is that price stabilization does not necessarily increase foreign borrowing in this model, either. It can be shown that

$$\frac{\partial \sigma^2_{P\hat{\theta}}}{\partial \sigma_P} \gtreqless 0 \quad \text{as} \quad \frac{\sigma_P}{\bar{P}} \gtreqless -\rho \frac{\sigma_{\hat{\theta}}}{\bar{\hat{\theta}}}$$

(ρ = correlation coefficient between P and $\hat{\theta}$). Thus,

$$\frac{dF}{d\sigma_P} = \frac{dF}{d\sigma_{P\hat{\theta}}} \frac{d\sigma^2_{P\hat{\theta}}}{d\sigma_P} \lesseqgtr 0 \quad \text{as} \quad \frac{\sigma_P}{\bar{P}} \gtreqless -\rho \frac{\sigma_{\hat{\theta}}}{\bar{\hat{\theta}}} .$$

Again, one would expect to find price stabilization dampening foreign borrowing, where the country is important in world production of the export commodity.

IV. Preliminary Empirical Testing

The basic difference between the system analyzed in Part II and that analyzed in Part III concerns whether or not the volume of exports is influenced by economic policy. Massel [8] in his empirical investigation of export instability assumes that economic policy does not influence the size of the export sector. This may well be true in a large number of countries where food and raw material based exports sectors are financed through foreign direct investment. There is evidence, however, that in a few developing countries, export promotion schemes have been employed successfully [4]. One of these schemes has been the extension of long-term loans to the export sector at subsidized rates. In this way $\bar{Q}_2$ is reduced by increasing the absolute amount of total capital allocated to the export sector.[1]

In most LDC's, therefore, the model analyzed in Section II where the volume of exports is exogenous, though random, seems to be more applicable than that presented in Section III where the proportion of total capital allocated to the export sector is controlled. But certain other critical assumptions applicable to both systems should be analyzed in the light of the available empirical evidence. The first is the assumption that the exported commodity is not domestically consumed. This assumption seems to be most valid in the case of raw materials primary product exports such as copper and oil. Linder has argued that industrial exports involve for the most part only those commodities which are domestically consumed;

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[1] A precise formulation involving policy instruments which determine the size of the export sector will be considered in a separate paper.

however, in most developing countries the industrial export share of total exports is very small. It is true, of course, that a significant proportion of the total production of certain food commodities, used for export purposes, is consumed domestically. But less than 50 percent of the developing countries in Massel's sample domestically consumed more than 50 percent of the domestic production of these commodities [8]. Moreover, the share of food exports domestically consumed did not prove to be significant explanatory variable determining the degree of export revenue variation in the countries. Matheson and McKinnon [9] have collected data which indicates that the variance of GDP at domestic prices, adjusted for a time trend, is greater in the developing than it is in the developed countries. Further, their regression analysis indicates that there is a negative correlation between the real export GDP ratio and the degree of export variability. However, these results are not inconsistent with those implied by the formulations developed in this paper; Matheson and McKinnon do not attempt to explain the variance of GDP at world prices, and there is evidence that a great deal of export revenue variability is attributable to fluctuations in the terms of trade, arising from shifts in the demand schedule, rather than production instability in the export sector [8]. Consequently, our simplifying assumptions, along with presumed association of consumption with export-revenue variability, are for the most part supported by the available empirical studies.

The closed form solution derived from the dynamic programming model indicates that level of domestic savings is an increasing function of export revenues. In other words, along the optimal trajectory, declines in domestic savings serve to neutralize the consumption impact of export revenue shortfalls and increases in domestic savings are associated with rises in export revenues above normal levels. This is consistent with Vanek's empirical results for Columbia which uses the terms of trade, not export revenues, to explain savings and Maizels' study of 12 countries, 8 developing and 4 developed, [7] and [13]. The regression equation for domestic savings involves two independent variables, the volume of exports and GDP at constant prices. In Vanek's study, there appears to be a significant positive association between domestic savings and the terms of trade (defined as the ratio of the world price of exports to the world price of imports). Maizels' study indicates that domestic savings are for the most part positively correlated with the volume of exports. Although their specification is not identical to equation (2.10), the estimated regression equations do imply a direction of association which is consistent with the closed-form solution.

Weisskopf's time series regressions for 17 developing countries include GDP, exports, and net foreign capital inflow, all in real terms, as independent variables [14]. Since a common GDP price deflator is used to derive these variables from the expenditure components of GDP measured in current terms, changes in the export variable are to a degree attributable to changes in the world price of the commodity. In 10 of the 17 countries, the export coefficient is significantly positive at the five percent level for the post-war periods examined. Aside from this result, there is in most cases a significant negative association between net foreign capital inflow and domestic savings. The sign of this, as well as the export, coefficient is consistent with the optimization model presented in Part I when gross domestic investment is fixed. Under these conditions, net foreign capital inflow and domestic savings react in opposite directions to period-to-period fluctuations in export revenues (which are not completely captured by Weisskopf's export variable). If, on the other hand, net foreign capital inflow is exogenous along with export revenues and gross investment endogenous, as Weisskopf assumes, the partial derivative of saving with respect to net foreign capital inflow will be negative according to our model provided that $b > \emptyset$. In this case of exogenous net foreign capital inflow, the partial derivative of domestic savings with respect to export revenues will again be positive at least in the two-period model. In sum, then, under two alternative sets of assumptions about exogenous variables, Weisskopf's savings function estimates qualitatively conform to the optimal behavior implied by our models.

<u>Conclusions</u>

1. The usual assumption that export revenue uncertainty should dampen foreign borrowing does not hold generally. Indeed, if exports are exogenous, as they probably are in a number of LDC's, in our system foreign borrowing should proceed to the point where the marginal cost of borrowing ($\emptyset$) is equal to the social marginal product of capital (b), independently of uncertainty.

2. To amplify this last point, if exports are exogenous, foreign borrowing affects only the expected value of consumption, and none of its higher moments. Thus, uncertainty will not affect foreign borrowing.

3. Even in the cases where export revenue uncertainty can be shown to have a dampening effect on foreign borrowing, if the country is a large exporter, it can happen that international price stabilization will <u>destablize</u> export revenues if output uncertainty is present, and <u>dampen</u> optimal net foreign capital inflow.

4. Having derived a closed solution for optimal savings behavior using dynamic programming, we noted that it was consistent with observed behavior in several developing countries, at least as regards the direction of response of this variable to export fluctuation. To the extent that this is a consequence of the use of export taxes as major source of government revenues, reliance on such an indirect tax regime will beeconomically efficient according to the exogenous-export model presented in Part II. However, it must be noted that this conclusion is critically dependent on the assumption that random fluctuation in the level of investment is costless.

5. The level of foreign exchange reserves is not included as policy variable in the models. Working out the case where the average interest rate on debt is a decreasing function of this variable, as well as an increasing function of debt, would be tedious. It can be easily shown that such a modification will not affect the qualitative response to parameter changes in the case of the model presented in Part II, provided that interest return on exchange reserves is constant and less than the marginal product of capital in the borrowing country.

6. In the first models it is true (a) that savings and net foreign capital inflow react in opposite directions to period-to-period fluctuations in the value of exports with investment fixed; and (b) that, with investment allowed to vary, optimal domestic saving is a decreasing function of a given level of net foreign capital inflow. This is consistent with cross-sectional findings by Weisskopf [14]. With exports affected by government policy, as is the case in Part III, this result may not hold.

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