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The Aggregation of Controls in the Autonomy of Subordinates

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I. The Unique-Supervisee Problem:

In contemporary economic systems, the great majority of people spend their working hours in a relation of subordination to one or more of the fellow-members of the organizations to which they belong. When their goals or preferences with respect to the outcomes of their activities in the organization diverge from those their superiors would like to see them pursue, their decisions are normally constrained by orders or rules limiting their "autonomy." How constraining these orders or rules may be will depend, in part, on the supervisors' perception of their performance--on the information that reaches supervisors about it. Supervisors, in general, have a limited capacity to collect, process, and act upon the detailed information available to their subordinates. They must make their decisions, therefore, on the basis of aggregated information, whether collected in consolidated form from their subordinates or summarized from the more detailed reports they receive "from the field."

My purpose here is to analyze precisely an intuitively obvious notion: that the more aggregated the information available to a supervisor happens to be, the more autonomy the individuals he supervises will enjoy.

Due to the aggregation of controls, multi-level hierarchies, including Soviet-type economic administrations, may be far less "centralized" than they appear to be from a cursory examination of their command structure. Indeed, there is a parallel between decentralization via the aggregation of controls and decentralization via the parceling out of linear objective functions that subordinates are instructed to maximize.[1]

In this first section, I analyze the relation between a supervisor and a single supervisee. In the second, I consider controls over several supervisees. In the third, I use an elementary game-theoretic formulation to analyze the strategies open to a supervisor and his supervisee, where the former may either accept aggregated information or "inspect" (at a cost) and the latter may either take advantage of the possibilities opened up by aggregation or behave as if the supervisor were actually going to inspect.

The model about to be described relies heavily on an approach to information theory developed by Jacob Marschak[2] and later by Roy Radner,[3] which may be summarized as follows in the context of our problem.

Let $Y \equiv \{y\}$ denote the set of possible outcomes of supervisee k's actions. We assume all vectors in Y to be in R^n. (The elements of a vector y in Y may, for example, be quantities or numbers of n different goods or expenditures corresponding to n budget items.) It is taken that no other individual in the system can have more detailed information about these outcomes.

A partition of Y is a set of possible descriptions of the outcomes of k's actions. We have just seen that k's information about the outcomes of his own actions corresponds to the finest partition $\{y\}$ of Y, where every element is represented separately. Suppose the n elements of every vector y are partitioned into q subvectors where q is smaller than n. Consider now subvector y^i of a vector y in Y, defined by this partition denoted ζ. Let z be the description of k's outcomes in a certain period communicated to a supervisor h. All the n_i elements of y^i defined by the finest partition $\{y\}$ are mapped into a single element z_i of z, where z_i is our "aggregated vector" corresponding to partition ζ.

In this paper, the mapping is assumed to be linear. That is, $z_i = \sum_{j=1}^{n_i} \pi_j^{io} y_j^i$, where π_j^{io} is element j of subvector π^{io}, which is defined by the same partition ζ of the n elements in a vector of "aggregation prices" π^o; and y_j^i is the j'th element of subvector y^i ($j = 1, \ldots, n_i$; $i = 1, \ldots, q$). Aggregation may be by tonnage or number of items, in which case the elements of any subvector π^{io} will all be unity; by some indicator of quality (yarn count for cloth, calorific value for fuels); or by any conventional price system.

Any vector y in Y is thus aggregated to a vector z with the aid of an aggregation matrix Π^o, the rows of which are obtained by partitioning the set of the elements of π^o according to the same ζ already used to define the subvectors of y.

$$\Pi^o \equiv \begin{bmatrix} \pi^{1o} & \ldots & \pi^{1o}_{n_1} & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 \\ 0 & \ldots & 0 & \pi_1^{2o} & \ldots & \pi^{2o}_{n_2} & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & \pi_1^{qo} & \ldots & \pi^{qo}_{n_q} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \pi_1^o & & & 0 \\ & \pi_2^o & & \\ & & \ddots & \\ 0 & & & \pi_q^o \end{bmatrix}$$

We have therefore:

$$z = \Pi^o y$$

where z is an aggregated vector in R^q ($q < n$).

The main idea used in this model of the supervision relation is that once the elements of y^i have been "scrambled" in z_i, they cannot be retrieved individually. Supervisor h cannot distinguish between z_i and z_i', as long as they are equal, even though they may have been aggregated from distinct subvectors in y.

Two examples: (1) y^i is a vector of the quantities produced of n_i different kinds of steels by a subordinate mill; z_i is the combined tonnage of these steels; the supervisor of the mill (main office of a corporation or a ministry in a centrally managed economic system) receives the message z_i from the mill, which it cannot unscramble to differentiate the quantities produced of the various kinds of steel. (2) y_j^i is the expenditure on a specific item in a school budget; z_i is the combined expenditure on n_i such items. The school superintendent can control z_i but not the expenditures on the individual items y_j^i.

A supervisor may receive very detailed information from a subordinate but be incapable of using it in this form. If he has to aggregate y^i to z_i and if he loses the individual elements y_j^i in so doing, one would be tempted to conclude that he might just as well have received the message in already aggregated form, although it is conceivable that, in situations where supervisees were uncertain as to how the information might be employed, a supervisor might still wish to collect information in more detailed form than he could use.

We begin with the analysis of the behavior of a supervisor controlling the performance of a single supervisee.

In the present model, it is assumed that a supervisor has preferences P (a complete preordering) over all possible outcomes of the activities of his supervisee, where these outcomes are described according to the finest partition of these outcomes {y}. For reasons that will be spelled out later (which have to do with his uncertainty regarding the capabilities of the supervisee), he will accept any performance yielding him a minimum U^M of satisfaction. To be precise, he will accept any vector y in Y at least as desirable as some vector y' in an indifference class M.[4] A performance during a given period is "acceptable" when the supervisor imposes no penalty on his supervisee and/or takes no action to correct his future behavior (in case the environment is expected to stay the same in subsequent periods). The "acceptable set" of the supervisor, the set of vectors y at least as desirable as any vector in M, is denoted G. This set is assumed to be closed. It must also be bounded from below: there always exists some vector $\bar{y}$ with the property $y \geq \bar{y}$ for all y in G. Finally, I assume non-satiety: if y' is in G, then any vector y such that $y \geq y'$ must also be in G. These three assumptions seem natural and realistic for many situations that might be analyzed.

What criteria of acceptability will the supervisor apply if the information he receives about his supervisee's performance is aggregated? Consider a q-dimensional vector z^e obtained by aggregating an n-dimensional vector y^e in the acceptable set G. The supervisor will not be able to distinguish z^e from some other vector $\bar{z}^e$ with identical elements but aggregated from a vector $\bar{y}^e$ not in G. The basic assumption of this paper is that the supervisor will accept $\bar{z}^e$ if he exercises what may be called "aggregated controls," even though he would not have accepted $\bar{y}^e$ if he had been able to check on his subordinate's disaggregated performance.[5] As we shall see presently, the supervisor may find, if he exercises such aggregated controls, that the performance of his "sector" has deteriorated. We will consider this possible deterioration from the supervisor's point of view in section 3. For the time being, our concern is with the possibilities opened up for the supervisee as a result of the aggregation of controls.

Example: a manufacturer of shoes instructs one of its plants to produce at least 50,000 pairs of shoes in the next quarter with an assortment by sizes corresponding to last quarter's actual sales. The plant produces 50,000 pairs but discreetly violates the assortment order. It is too expensive for the manufacturer to check on the assortment. The retail stores receiving the unbalanced assortment make no complaint because there is a seller's market and it is hard to get good shoes. The supervisor cannot act otherwise than accept the plant's performance.

It may be presumed that if the costs of obtaining information are high, a supervisor will not check on the detailed performance of a supervisee, unless he believes that the losses he <u>may</u> be suffering from the latter's failure to comply to his detailed orders are great enough to justify the expense of finding out what is really going on "below deck" (through an audit, a census, or a random sample).[6]

The formal consequences of the assumption made above is this. Where the information reaching a supervisor is aggregated, his acceptable set G^R will include, in addition to G, any vector y which, upon aggregation to z, will be identical with a vector z^e aggregated from a vector y^e in G. Formally:

$$G^R \equiv \{y | \Pi^o y = \Pi^o y^e \text{ , all } y^e \text{ in } G\}.$$

To fix these ideas, we suppose the supervisee transforms inputs (negative elements of y) into outputs (positive elements of y) per period according to a routinized technology known to the supervisee but not necessarily, in such detail at least, to the supervisor. The set Y contains every feasible vector y.

In addition to assuming that this set is closed, I shall also confine my analysis to cases where it may be assumed to be convex, thus excluding important instances where economies of scale are powerful enough to "deconvexify" the production set.

The inputs used in these transformation activities are subject to various constraints (on the availability of physical factors of production, of borrowable funds, etc.). Some of these constraints are determined by states of the environment, the probability distribution of which is assumed to be at least approximately known to the supervisee but not to the supervisor. Given a vector of constraints ω in Ω, the set of all possible constraint vectors, the vectors y are limited to a set $\hat{Y}_{\omega}$ in a particular period t where ω has occurred ($t = 1, \ldots, T$). This set is obviously bounded from above, in the sense that there exists a vector $\bar{\bar{y}}$ such that $y \leq \bar{\bar{y}}$ for all y in Y. The set $\hat{Y}_{\omega}$ is called "the attainable set, given ω."

The supervisee is either subject to an incentive plan or pursues one or more self-assigned goals. In either case, we assume that he seeks to maximize a linear functional πy where π is a vector of non-negative price weights. The "vector of incentive prices" π in general will not be identical with the vector of aggregation prices π^o.

The elements of π may correspond to market prices if the supervisee is operating in a market setting and is maximizing profits; but he may also minimize his expenditures (i.e. maximize $\pi^- y^-$ where y^- is the subvector of inputs in y and π^- are their corresponding prices) or maximize the value of his output, irrespective of costs.[7] Any linear objective function with weights π will do.

That there must exist at least one maximizer y^* for π on $\hat{Y}_\omega$ in all such cases is guaranteed by the assumption that Y is closed, which extends to $\hat{Y}_\omega$, and by the imposition of constraints on Y. These two conditions together ensure the compactness of $\hat{Y}_\omega$.

Even though ω is a vector of random variables at the time the supervisor issues his orders (e.g. sets a "plan"), we take it that the constraints are known to the supervisee during the period in which the vectors y are observed. Hence he should be capable of finding a maximizer for π on $\hat{Y}_\omega$ (or at least a vector in $\hat{Y}_\omega$ which comes "close" to maximizing πy in this set).

The intersection of G with $\hat{Y}_\omega$ is denoted E_ω. It too is obviously compact. Any maximizer for π on E_ω is denoted $y^{\bar{a}}$. Thus $\pi y^{\bar{a}}$ is the maximum value that the supervisee's objective function would assume if he were compelled to produce in the set G.

We can now offer a reason why E_ω, the intersection of $\hat{Y}_\omega$ and G, is likely to occupy a significant subset of $\hat{Y}_\omega$, except in rare occurrences where the vector of constraints is extremely disadvantageous to the supervisee (in case of floods, unusual cold or hot spells, etc.). If it is costly for the supervisor to interfere directly in his supervisee's affairs or to levy penalties on him for non-compliance (fines, dismissal, etc.),[8] then he must "set" M and hence G in such a way that, given almost any constraint vector in Ω, the supervisee must be capable of producing in G. This means that, for a "typical" constraint vector ω, some vectors in G will be interior to $\hat{Y}_\omega$.[9]

But how can the supervisor set G if he does not have detailed knowledge of his supervisee's production capabilities in the latter's most detailed nomenclature?

It has to be assumed that, on the basis of occasional inspection, the supervisor has a sufficiently good idea of the capabilities of his supervisee, under typical constraints, to determine what he should accept and what he should reject if he did decide to pay the cost of obtaining disaggregated information. I assume also that either G has been set before the vector of constraints ω has been revealed (i.e. in a planned economy before the plan has gone into effect) or that so little information is available to the supervisor about the impact of a given ω on his supervisee's capacities that it will have no effect on his minimum requirements.

From now on, we shall dispense with the subscript ω, although it should be kept in mind that every set $\hat{Y}$ and E is contingent on the occurrence of a random vector of constraints ω, assumed to remain fixed for the period under consideration.

The autonomy of the supervisee is now defined as the ratio of the value of his objective function constrained by the necessity of producing a vector y acceptable to his supervisor to the value of his objective function in the absense of this organizational constraint.[10] If the supervisee were compelled to produce in G, his autonomy would be the ratio of $\pi y^{\bar{a}}$ to πy^{*}, where y^{*} is a maximizer for π on $\hat{Y}$. We will confine our attention to cases where every maximizer y^{*} for π on $\hat{Y}$ lies outside E (otherwise the relation between supervisors and supervisees would be so harmonious as to be totally devoid of interest).[11]

The intersection of G^R with $\hat{Y}$ is denoted E^R. It too is obviously compact (since $\hat{Y}$ and G^R are themselves compact) and hence contains at least one maximizer for any price system π.

Let us denote by y^o a maximizer for π on E^R. πy^o is then the maximum value of his objective function that a supervisee can attain (given ω) when controls are exercised in aggregated form.

Our first task is to figure out whether πy^o is greater or just equal to $\pi y^{\bar{a}}$. (It cannot be smaller, since E, and hence $y^{\bar{a}}$, is contained in E^R.)

This problem is tackled in two stages. First we find out what the effect on enterprise autonomy would be if the supervisor could not distinguish vectors $\bar{y}$ and y^e in $\hat{Y}$, where y^e is in E and $\bar{y}$ is not, as long as they both aggregated to the same value $\pi^o\bar{y}$ ($\equiv \pi^o y^e$). I call this case "aggregation from n dimensions to scalars." We then go on to partial aggregation of vectors, such that a supervisor cannot distinguish $\bar{y}$ and y^e as long as $\Pi^o\bar{y} = \Pi^o y^e$, where Π^o has more than one row. It turns out that these two cases are linked in a significant way.

What can be said about E^R in the case of aggregation from n dimensions to scalars? Consider y^m, a minimizer for π^o on E. (Such a minimizer must exist since E is closed and bounded from below.) We now define E^m as the set of vectors y in Y such that $\pi^o y \geq \pi^o y^m$. It is easy to prove that E^m is identical with E^R.[1] From now on, therefore, if we wish to find a maximizer y^o for π on E^R, we will seek it among the vectors y satisfying the condition $\pi^o y \geq \pi^o y^m$.

We first note that if π is identical with π^o and, as already assumed, $\pi y^* > \pi y^{\bar{a}}$, then πy^o must equal πy^*. Or, to put the point in another way, y^* must be in E^R and be a maximizer for π on this set.[13] The autonomy of the supervisee is complete. This obvious result may be interpreted as follows. If the production vectors of the supervisee are aggregated with the help of incentive prices or if supervisees are subject to an incentive system geared to their aggregate output expressed in terms of quasi-prices (e.g. if they receive a bonus based on aggregate tonnage produced), then E^R must include all the maximizers y^*, whether or not any of these y^* are included in the set E of points that h would consider acceptable under inspection.

More analytically interesting are the cases where π and π^o differ.

A proposition, formally proved in a previous paper,[14] is that if (1) $\hat{Y}$ is compact and convex, (2) $\pi y^* > \pi y^{\bar{a}}$, and (3) $\pi^o y^{\bar{a}} > \pi^o y^m$, then $\pi y^o > \pi y^{\bar{a}}$. The new, and fundamental assumption, is that $\pi^o y^{\bar{a}} > \pi^o y^m$. To put the assumption differently, the vector $y^{\bar{a}}$ is an interior point of the half space generated by the hyperplane $\pi^o y$ through y^m. This ensures that some convex combinations of $y^{\bar{a}}$ and y^* with a positive weight on y^* will be in E^R. Since all such combinations are worth more at prices π than $y^{\bar{a}}$, $y^{\bar{a}}$ cannot be a maximizer for π on E^R. Hence any maximizer for π on E^R must be worth more than $y^{\bar{a}}$. The autonomy of the supervisee has increased as a result of the aggregation ζ.[15]

Diagram 1 illustrates, in two dimensions, the nature and extent of the increase in the supervisee's autonomy.

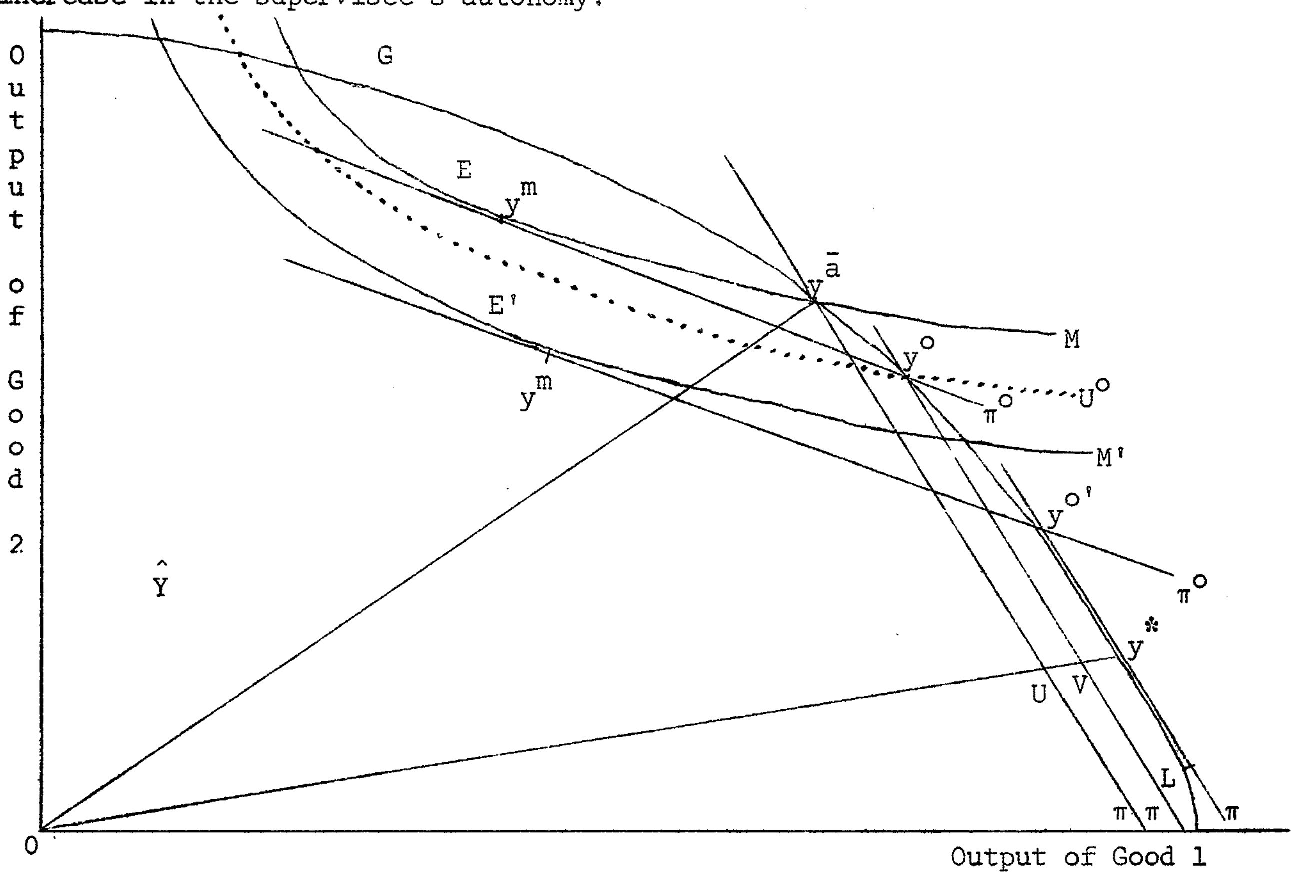

Diagram 1

In Diagram 1 above, $\hat{Y}$ is represented in the supervisee's disaggregated nomenclature. It should be observed that y^* is located on a straight-line segment (from $y^{o'}$ to L) with a slope equal to that of π.

In the absence of any restraints imposed by the supervisor on his output mix, the supervisee would maximize his payoff πy at y^* or at any point on the straight-line segment on which y^* is located. If he were obliged to produce an output mix in G, he could do no better than produce at point $y^{\bar{a}}$. His autonomy could then be measured as the ratio OU to Oy^*. If the supervisor were willing to accept any point in E worth as much at aggregation prices π^o as any arbitrary point in E, the supervisee would maximize his payoff at y^o. His autonomy would have increased from $\frac{OU}{Oy^*}$ to $\frac{OV}{Oy^*}$. Such an increase is evidently possible since the basic condition of our first proposition is satisfied--$\pi^o y^{\bar{a}}$ exceeds $\pi^o y^m$ (e.g. $y^{\bar{a}}$ is an interior point of the half space generated by the line $\pi^o y$ through y^m).

Suppose that G, instead of being bounded from below by the unbroken curve M going through y^m and $y^{\bar{a}}$ were bounded by the curve M' through $y^{m'}$. The payoff of the supervisee could then be increased from $\pi y^{\bar{a}'}$ to $\pi y^{o'}$. At $y^{o'}$ he would enjoy 100 per cent autonomy, inasmuch as $\pi y^o = \pi y^*$. From this we conclude that there is a strictly monotonic (inverse) relation between the ratio $\pi^o y^m$ to $\pi^o y^{\bar{a}}$ and the ratio $\pi y^{\bar{a}}$ to πy^o, up to the point where $\pi y^o = \pi y^*$.[16] In other words, the smaller the ratio of the minimum value at aggregation prices of points in E to the value at these prices of $y^{\bar{a}}$, the greater will be the autonomy enjoyed by the supervisee.

For given $y^{\bar{a}}$, y^*, and y^m, this model also allows us to predict the minimum increase in autonomy that would result from the aggregation of controls with the aid of prices π^o. The following proposition is demonstrated geometrically for two goods, but it is easy to prove it algebraically for any number.[17]

We denote by Δ the maximum potential increment in the payoff of the supervisee, starting from a maximizer on E such as $y^{\bar{a}}$, if all controls were removed. Δ is equal to $\pi y^* - \pi y^{\bar{a}}$. Next we compute $\pi^o y^*$ and the differences $\pi^o y^{\bar{a}} - \pi^o y^*$ and $\pi^o y^{\bar{a}} - \pi^o y^m$, which are denoted γ and δ respectively. The proposition is that the minimum increment in the supervisee's payoff, irrespective of the curvature of (convex) $\hat{Y}$, equals $\frac{\delta}{\gamma}\Delta$.[18] In other words the fraction of the maximum potential increment in payoff Δ resulting from the aggregation of controls is at least $\frac{\delta}{\gamma}$, the ratio of the difference in the value at aggregation prices of $y^{\bar{a}}$ and y^m to the difference in the value at aggregation prices of $y^{\bar{a}}$ and y^*.

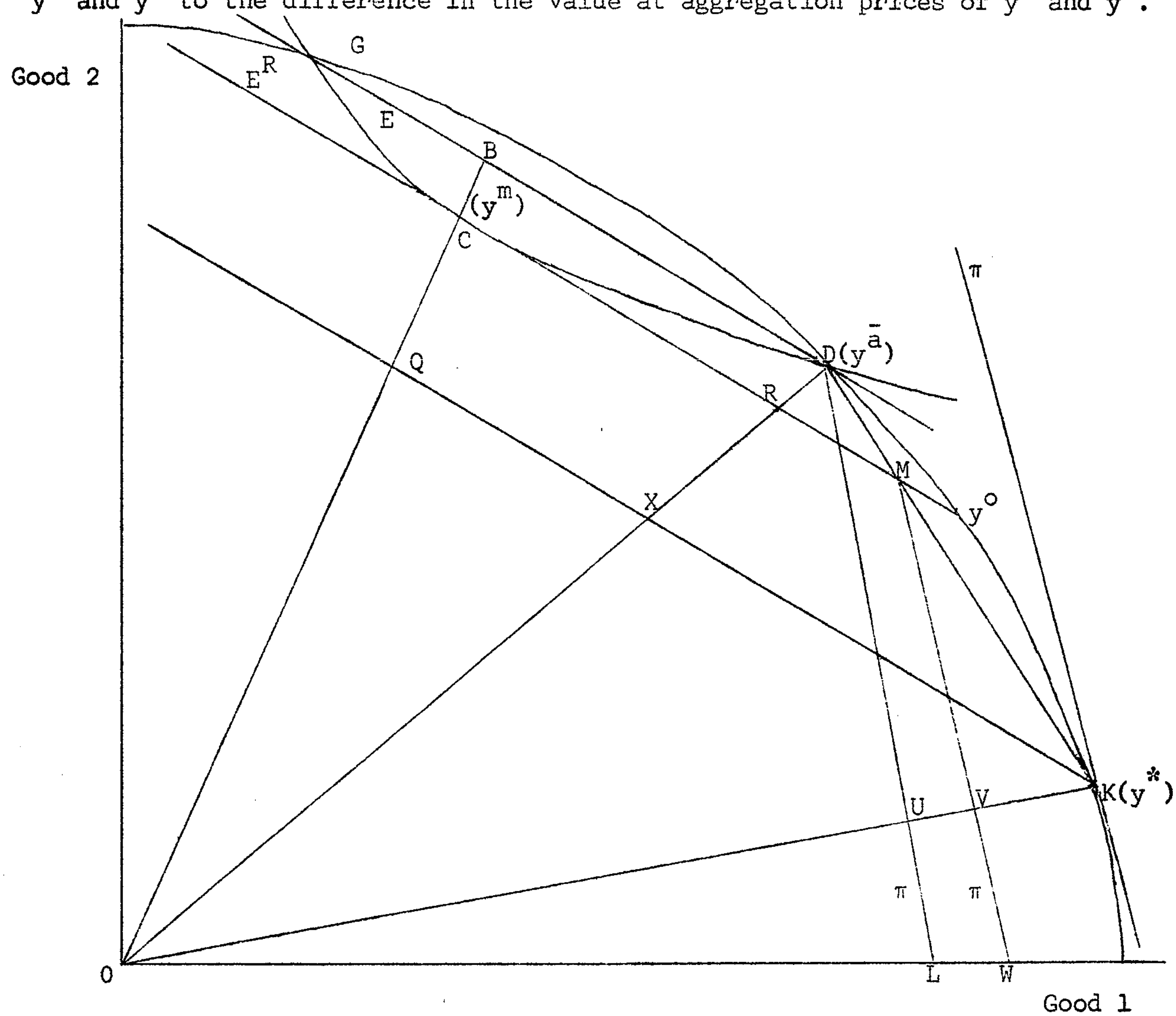

Diagram 2

In Diagram 2 above a line has been drawn between points D ($y^{\bar{a}}$) and K (y^{*}). Since $\hat{Y}$ is convex, all the points on this line must be in $\hat{Y}$ including M, the point where this line intersects with the line $\pi^{o}y$ through y^{m}.

We first observe that B has the same value at prices π^{o} as D ($y^{\bar{a}}$), C (y^{m}) as R, and Q as X. The ratio $\frac{\delta}{\gamma}$ is equal to $\frac{BC}{BQ}$, which in turn equals $\frac{DR}{DX}$ (proportional segments of lines crossed by parallel lines). Consider now the triangle DXK. Clearly $\frac{DR}{DX} = \frac{DM}{DK}$ (since RM is parallel to the base of the triangle XK). Shifting to triangle DKU, we observe that $\frac{DM}{DK} = \frac{UV}{UK}$ (since MV and DU both have the same slope π). Thus $\frac{\delta}{\gamma} = \frac{UV}{UK} = \frac{UV}{\Delta}$, and $UV = \Delta \frac{\delta}{\gamma}$, which was what we were trying to prove.

That UV is only a minimum is apparent from Diagram 2. If we were to draw a line with slope π^{o} through y^{o}, it would intersect OK considerably to the right of V. But we cannot predict the precise location of y^{o}. About all we can say is that, <u>given the location of $y^{\bar{a}}$ and y^{*} on the efficiency frontier of $\hat{Y}$</u>, the increment in payoff beyond $\frac{\delta}{\gamma}\Delta$ will be greater, the greater the curvature of the curve linking $y^{\bar{a}}$ and y^{*}.[18]

It is worth noting that the increase in autonomy due to the possibility of producing an aggregated vector in G^{R}, as compared to the obligation of producing a disaggregated vector in G, is related to the elasticity of substitution among the different goods in the supervisor's preference function: the smaller this elasticity for any pair of goods, the greater will be the increase in autonomy resulting from the supervisor's aggregation of controls. Conversely, if goods 1 and 2 in Diagram 1 had been related at y^{m} by an infinite elasticity of substitution in the supervisor's preferences, y^{o} would have been in G, and there would have been no increase in autonomy resulting from aggregation. (Autonomy would of course have been greater to begin with.)

We now turn to the more difficult (and more realistic) problems raised by partial aggregation.

Sufficient conditions for an increase in autonomy are fairly self-evident. We begin with a partitioning of π^o into subvectors π^{1o} to π^{qo} and a corresponding partitioning of y and π. As we have already seen, the i'th row of matrix Π^o is a vector of n elements, the i'th subvector of which equals π^{io}, consisting of n_i elements, and all other elements are equal to zero, and the matrix Π^o maps production vectors in n-dimensional space into vectors in q-dimensional space ($1 < q < n$). The acceptable and attainable set under partial aggregation is defined as:

$$E^R \equiv \{y | \Pi^o y = \Pi^o y^e;\ y^e \in E\}$$

The procedure we are about to follow is this. After locating a given minimizer $y^{\bar{a}}$ in E we pick out a subspace defined by a row of Π^o--say the i'th--and construct a set containing all the vectors in $\hat{Y}$ that are identical with $y^{\bar{a}}$ except in the i'th subspace. We then look for a vector in this set whose i'th subvector is worth more at the prices listed in the i'th subvector of π^o than the i'th subvector of $y^{\bar{a}}$ itself. If this vector exists, we see immediately that it must be worth more at prices than $y^{\bar{a}}$.[19] If it does not, we go on to the next subspace and eventually investigate every one of the q subspaces to find at least one vector with the desired properties. Eventually the procedure may be repeated with other maximizers for π on E if $y^{\bar{a}}$ is not unique. A sufficient condition for finding a vector y^s with the desired properties in some subspace i of the set there would have been no increase in autonomy resulting from aggregation. (Autonomy would of course have been greater to begin with.)

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The procedure we are about to follow is this. After locating a given minimizer $y^{\bar{a}}$ in E we pick out a subspace defined by a row of Π^o--say the i'th--and construct a set containing all the vectors in $\hat{Y}$ that are identical with $y^{\bar{a}}$ except in the i'th subspace. We then look for a vector in this set whose i'th subvector is worth more at the prices listed in the i'th subvector of π^o than the i'th subvector of $y^{\bar{a}}$ itself. If this vector exists, we see immediately that it must be worth more at prices than $y^{\bar{a}}$.[19] If it does not, we go on to the next subspace and eventually investigate every one of the q subspaces to find at least one vector with the desired properties. Eventually the procedure may be repeated with other maximizers for π on E if $y^{\bar{a}}$ is not unique. A sufficient condition for finding a vector y^s with the desired properties in some subspace i of the set previously defined (i.e. of the vectors y identical with $y^{\bar{a}}$, or any other maximizer for π on E, in every subspace but the i'th) is that $\pi^{io}y^{i\bar{a}}$ should exceed $\pi^{io}y^{im}$, where the superscript i refers to the i'th subvector of a vector of prices or of production. In other words, the same condition that allowed us to find a vector y^o worth more at prices π than $y^{\bar{a}}$ where Π^o consisted of a single row π^o allows us to find a subvector y^{io} such that $\pi^i y^{io} > \pi^i y^{i\bar{a}}$. This is not surprising considering that the elements in every subvector defined by a partition aggregate to a scalar (a single element in the aggregated vector).[20]

Necessary conditions for πy^o to exceed $\pi y^{\bar{a}}$ are even harder to spell out in the case of partial than in the case of aggregation from n goods to a scalar. We do know, however, that if there does not exist a vector y^o such that $\pi y^o > \pi y^{\bar{a}}$ when controls are aggregated from n goods to a scalar, there cannot exist a vector $y^{o'}$ such that $\pi y^{o'} > \pi y^{\bar{a}}$ where E^R is defined as the set of points in $\hat{Y}$ such that $\Pi^o y > \Pi^o y^e$, for all y^e in E, and Π^o has more than one row built up from the partition of the same π^o (partial aggregation). This (weak) necessary condition for a gain in autonomy under partially aggregated controls follows from the familiar proposition that if a single-constraint maximum problem is broken up into two or more separate constraints, the value of the maximand to the new problem cannot be greater than what it was for the single-constraint problem. Thus if $y^{\bar{a}}$ is a maximizer for π on E^R generated from a single acceptability constraint, there cannot exist a vector worth more than $y^{\bar{a}}$ at prices π if this single constraint is broken up into separate constraints. But if sufficient conditions for a gain in autonomy under complete aggregation are satisfied, how can we be sure to generate an improvement under partial aggregation?

Our first problem is that, even though $y^{i\bar{a}}$, for some subspace i, is located on the hyperplane $\pi^{io}y^{i}$ through y^{im}, and hence our sufficiency condition does not obtain, there may still be points on this hyperplane worth more at prices π^{i} than $y^{i\bar{a}}$. In this case the analysis in the appendix applies with appropriate modifications. But even if we cannot find a y^{o} such that $\pi y^{o} > \pi y^{\bar{a}}$ in any of the q subspaces that we have constructed starting from $y^{\bar{a}}$ or some other maximizer for π on E, we may still be able to discover such a vector by starting from some vector in E other than a maximizer. Suppose that for some vector y^{e} in E and y^{s} in $\hat{Y}$, it happened that $\pi^{i}y^{is} > \pi^{i}y^{ie}$ in the i'th subspace (where $y^{de} = y^{ds}$ ($d \neq i$; $i = 1, \ldots , q$) and y^{is} is such that y^{s} is in $\hat{Y}$). Now we cannot mechanically claim that πy^{s} necessarily exceeds $\pi y^{\bar{a}}$, since $\pi y^{e} < \pi y^{\bar{a}}$, and hence, πy^{s} may be smaller or larger than $\pi y^{\bar{a}}$. If we write out the component subvectors of y^{e}, y^{s}, and $y^{\bar{a}}$, however, we immediately see that, for an improvement to occur, the difference between $\pi^{i}y^{is}$ and $\pi^{i}y^{ie}$ must exceed the difference between $\pi y^{\bar{a}}$ and πy^{e}.

I could not find the precise conditions that would permit one to predict whether there existed vectors such as y^{e} in E that were superior to $y^{\bar{a}}$ as starting points.

II. A Supervisor and K Supervisees

The extension of the analysis in the previous section to a situation where a supervisor h has more than one supervisee is fairly straightforward.

Denoting a supervisee by the subscript k ($k = 1, \ldots , K$), we first add the constrained production sets of all K supervisees to obtain the joint-production set of h (i.e. $\hat{Y}_h \equiv \sum_{k=1}^{K} \hat{Y}_k$). This set must evidently be compact and convex if the component sets have these properties. The acceptable set G_h intersects with Y_h to form the attainable and acceptable set E_h. The set E_h^R is now defined as:

$$E_h^R \equiv \{y_h | \Pi^o y_h \geq \Pi^o y_h^e \ ; \ y_h \in Y_h \quad \text{and} \quad y_h^e \in E_h\}.$$

For πy_h^o to exceed $\pi y_h^{\bar{a}}$, where $y_h^{\bar{a}}$ and y_h^o are maximizers for π on E_h and E_h^R respectively, the same sufficient conditions apply as in the case of a unique supervisee. The question to be analyzed now is the relation of the autonomy of the individual supervisee to any putative increase in the ratio of πy_h^o to $\pi y_h^{\bar{a}}$ for all K supervisees together.

By the non-satiety assumption only points on the efficiency frontier of $\hat{Y}_h$ qualify as possible maximizers for any set of positive prices on E_h and E_h^R respectively. We first proceed to decompose these efficient subsets, denoted $\hat{E}_h$ and $\hat{E}_h^R$ respectively, into subsets of $\hat{Y}_1$ to $\hat{Y}_K$. According to a basic mathematical theorem, if $\bar{y}_h$ is an efficient point on a compact set then there must exist some semi-positive vector of prices ρ such that $\rho\bar{y}_h \geq \rho y_h$ for all y_h in $\hat{Y}_h$. By another theorem on sets formed as the sum of compact sets, if $\bar{y}_h = \sum_{k=1}^{K} \bar{y}_k$, then $\rho\bar{y}_k \geq \rho y_k$ for all y_k in $\hat{Y}_k$ $(k = 1, \ldots , K)$.[21] Thus, by the choice of appropriate price vectors, every efficient point of E_h and E_h^R can be decomposed into K efficient points of $\hat{Y}_1$ to $\hat{Y}_K$. Let $\hat{E}_k^D$ and $\hat{E}_k^{DR}$ stand for the efficient sets for k obtained, respectively, from the decomposition of $\hat{E}_h$ and $\hat{E}_h^R$. It follows also from the theorem just cited that if $y^{\bar{a}}$ and y^o are maximizers for π on $\hat{E}_h$ and $\hat{E}_h^R$ then $y_k^{\bar{a}}$ and y_k^o must be maximizers for π on $\hat{E}_k^D$ and $\hat{E}_k^{DR}$ respectively $(k = 1, \ldots , K)$.[22]

The fact that $\pi y_h^* > \pi y_h^o > \pi y_h^{\bar{a}}$ implies that at least one supervisee must gain autonomy from the aggregation of controls. But it is easy to construct an example where h has two supervisees k and ℓ such that $\pi y_k^o = \pi y_k^{\bar{a}}$, even though $\pi y_h^o > \pi y_h^{\bar{a}}$ and $\pi y_\ell^* > \pi y_\ell^o$ -- such, in other words, that the entire gain would accrue to ℓ. This will be the case if ℓ can shift from $y_\ell^{\bar{a}}$ toward a more advantageous product mix (presumably in the direction of the mix represented by y_ℓ^*) without incurring substantially higher marginal costs, whereas any attempt by k to increase its payoff would face steeply rising marginal costs.

This example is illustrated in Diagram 3 below.

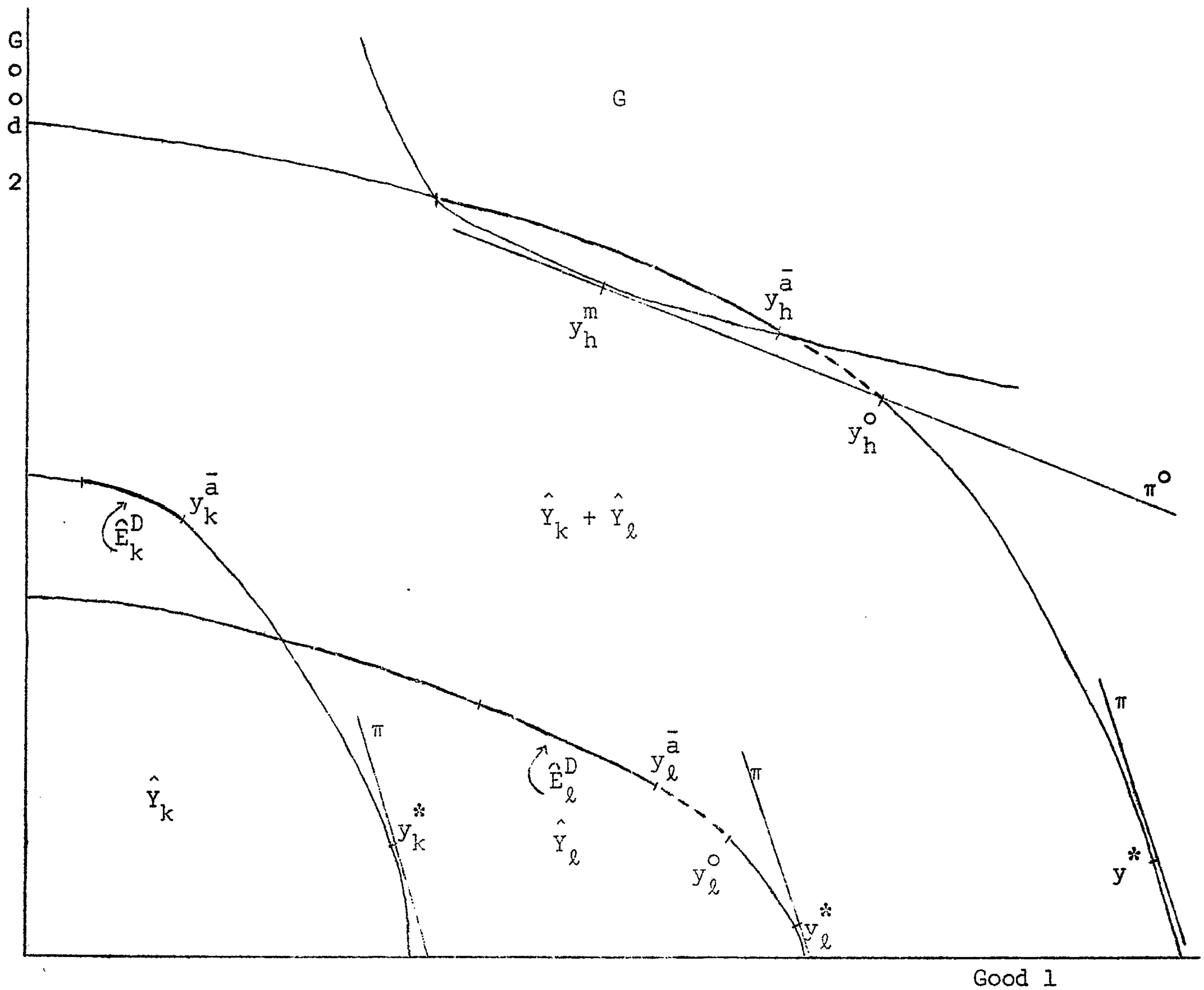

Diagram 3

In this diagram $\hat{E}_k^D$ and $\hat{E}_\ell^D$ are indicated by heavy lines on the efficiency frontiers of $\hat{Y}_k$ and $\hat{Y}_\ell$ respectively. As a result of the aggregation of controls, the point y_h^o becomes admissible for the joint production of k and ℓ. The area opened up between $y_h^{\bar{a}}$ and y_h^o makes it possible for ℓ to improve its payoff by moving from $y_\ell^{\bar{a}}$ to y_ℓ^o. Supervisee k, on the other hand, cannot increase his autonomy--his maximizer for π remains at $y_k^{\bar{a}}$. The reason is clearly that, because of the sharply increasing relative marginal cost of producing good 1 for k, any combination of a point to the right of $y_k^{\bar{a}}$ and a point between $y_\ell^{\bar{a}}$ and y_ℓ^o would yield a joint production y_h that would fall inside the efficiency frontier of h.

While it does not strain the imagination to assume that a unique supervisee might, after some experimentation, figure out the general pattern of his supervisor's preferences, it is hard to believe that several supervisees would not only have this knowledge but be able to locate points such as $y_k^{\bar{a}}$ and y_k^o for various possible degrees of aggregation of controls (of which more will be said presently). Even if a plan or target were set by h for each of its supervisees and all the individual targets were known to each supervisee, this would only give k a first approximation to the location of these critical points. He would also have to have information on how orders or "plans" were being fulfilled by each co-supervisee. Indeed, if the penalties for a failure on the part of all K supervisees to produce a joint vector in G--which presumably contains the aggregate target as well as tolerable deviations therefrom--were very heavy, there would be a strong inducement for the supervisees to exchange information and collude to avoid this transgression. It might even pay ℓ in the situation depicted in Diagram 3 to transfer to k a part of its increased payoff in exchange for k's agreement to "stay put" at $y_k^{\bar{a}}$. This strategy would only make sense of course if ℓ had reason to believe that h would not impose detailed controls on the output mix and would be content to assess the performance of k and ℓ in its aggregated nomenclature.

III. An Inspection Game

In this concluding section, we shall ignore the special information problems arising from a multiplicity of supervisees and analyze the relation between a supervisor and his supervisees as if the latter behaved as a single decision-making entity k. The problem I wish to consider is the following. Suppose that the supervisor has a choice of the degree of aggregation of controls that it can impose on its supervisees. The finer the controls--the greater the disaggregation of production vectors--which the supervisor wishes to impose, the higher the information cost he must pay. On the other hand, the supervisees are aware that for each degree of aggregation, there is a different joint payoff; they may report their performance according to any of the degrees of aggregation or "channels" both they and the supervisor can communicate in, but they risk the chance of a penalty if they choose production vectors on the assumption that the supervisor will exercise controls in a more aggregated form than the one he will actually opt for (if he decides to "inspect"). Given the production and aggregation strategies available to the "players"--the supervisor and the colluding supervisees--we seek to throw light on the general character of the solutions that may be expected to games with this format.

But first I have a preliminary observation to make on the supervisor's strategy of aggregation. We have seen that, _ceteris paribus_, the smaller the elasticity of substitution between pairs of commodities in G, aggregated in G^R, the greater will be the increase in payoff open to the supervisee(s) upon aggregation. Suppose that we could measure the supervisor's loss, corresponding to the supervisee's potential gain in autonomy resulting from the admissibility of a vector y_h^o in G^R compared to $y_h^{\bar{a}}$ in G, as the difference in the supervisor's utility associated with the indifference classes containing $y_h^{\bar{a}}$ and y_h^o respectively. This loss will clearly be smaller if the supervisor can partition the set of n goods in such a way that pairs of goods related by a high elasticity appear in the same subvector and pairs with a lower elasticity in different subvectors. The headquarters of the shoe plant already cited in an earlier example might want to include in a list of goods for potential aggregation (if detailed controls are not exercised) men's shoes of the same size though of different patterns and styles, on the presumption that the elasticity of substitution among shoes of different styles but of the same size would be higher than between shoes of the same style but of different size. Similarly, if the head of a trust or Soviet-type association supervising different coal mines had preferences reflecting the specialized demands of its customers, it would aggregate coals by calorific value or other demand indicator rather than according to geological or mineral characteristics that would affect primarily production costs. If a higher degree of aggregation were desired, any two subvectors v and w resulting from the first partition might be aggregated, provided the goods entering into v had a relatively higher elasticity of substitution with goods entering w than in the case of pairs of goods in subvectors that would be aggregated in different aggregates of subvectors.

Consider again two partitions of the n goods in a production vector, one obtained as a coarsening of the other. As in the first section, the supervisee produces at $y^{\bar{a}}$ if he seeks to maximize his payoff and satisfy his supervisor in terms of his disaggregated preferences (i.e. $y^{\bar{a}}$ is a maximizer for π in G) and at y^{o} if he wishes only to satisfy his supervisor's aggregated preferences (i.e. y^{o} is a maximizer for π on G^{R}). Assume that sufficient conditions for an increase in autonomy are met and that $\pi y^{o} > \pi y^{\bar{a}}$. Clearly, in terms of the supervisor's disaggregated preferences, his level of satisfaction at y^{o}, denoted U^{-}, is lower than that attained at $y^{\bar{a}}$, denoted U^{m}. (In Diagram 1, the lower level corresponding to U^{o} is shown as a dotted indifference curve passing through y^{o}.)

Now for the game to be of any interest (and to make any sense), supervisor h must pay a cost, expressible in utility foregone, to obtain the detailed information corresponding to the finer of the two partitions (for if the additional information were free, he would always "inspect."). Call the decrement in utility level associated with this cost of information I. We now introduce a reward R,[23] which will be conferred on the supervisee if the supervisor finds on inspection that his supervisee k has produced a vector in G, and a penalty T, which will be imposed on k if he finds that k has produced in G^{R} but not in G (as if controls were in fact aggregated).

We assume that R and T are fixed, irrespective of the extent to which compliance may have been easy or difficult to achieve for k or of the extent of transgression by k of h's directives. To begin with, we also assume that there is a unique vector of resource constraints ω.

The payoff matrix of the game is given below.

Supervisee k's production decision		Supervisor's choice of channel: Aggregated (no inspection) (β_1)	Supervisor's choice of channel: Disaggregated (inspection) (β_2)
	As if controls were to be aggregated (α_1)	πy^o / U^-	$\pi y^o - T$ / $U^- - I$
	As if controls were to be disaggregated (α_2)	$\pi y^{\bar{a}}$ / U^m	$\pi y^{\bar{a}} + R$ / $U^m - I$

Let us first assume that the game is going to be played only once. Then it is evident that h's second strategy β_2 is dominated by the first β_1: unless information costs are negligible, it would not pay h to inspect, since, irrespective of k's decision to produce at $y^{\bar{a}}$ or at y^o, h's utility level would be lower than if he had chosen not to inspect. But if k knows that I is non-negligible in terms of h's utility level, he will guess that h will play β_1; he should therefore play α_1. His payoff will be πy^o and h's will be U^-.

But a supervisor-supervisee relation is not a one-shot affair. It is embedded in the rules of organizations, which are normally endowed with a certain degree of continuity, if not of permanence. Let us therefore suppose that the same game is repeated a number of times. If h is concerned with his total (or average) utility over time, it may now be rational for him to play β_2 on some trials, in order to induce k to play α_2 at least part of the time. Clearly, the larger T and R and the smaller I, the more likely h is going to be successful in "teaching" k that his first pure strategy will not pay.

At this point we may ask what mixed strategy k can adopt that will guarantee him the highest "security level"--the largest minimum-payoff he can expect if h tried to punish him by adopting the strategy that would minimize his (k's) payoff.[24]

That v, the highest "security level" for k, will be the same whether h adopts β_1 or β_2 is shown in the diagram below.[25]

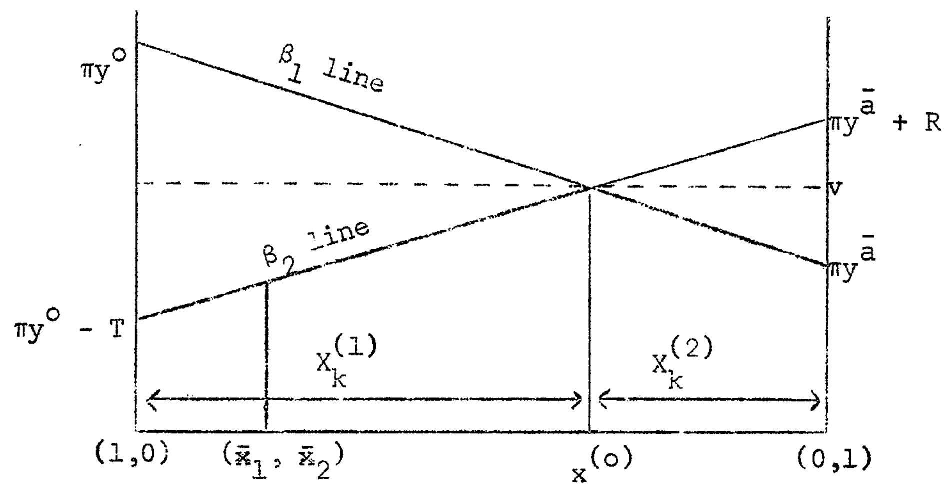

Diagram 4

The payoffs associated with each pure strategy pair are shown on the vertical lines above (1,0) and (0,1). The β_1 line exhibits all the convex combinations of the payoffs πy^o and $\pi y^{\bar{a}}$ associated with different combinations of k's strategies α_1 and α_2 when h plays β_1. The β_2 line is similarly defined for combinations of α_1 and α_2 when h plays β_2. The lines cross because R and T are positive.

The coordinates (x_1, x_2) of any point x are the weights attached to α_1 and α_2 in a convex combination of these two strategies.

If k chooses a mixed strategy (x_1, x_2) in the interval $X_k^{(1)}$, then h's best response, on the assumptions made above, is β_2. The vertical distance from any point $(\bar{x}_1, \bar{x}_2)$ (such as the one shown in the diagram) to the β_2 line represents k's security level. The interval of the β_2 line corresponding to points in $X_k^{(1)}$ is marked as a heavy line. On the other hand, if (x_1, x_2) is in $X_k^{(2)}$, then h's best response is β_1. The interval of the β_1 line corresponding to points in $X_k^{(2)}$ also appears in heavy type. The highest security level v is clearly attained at the point $x^{(o)}$ where the two lines cross.

Using this observation to solve for the optimal mixed strategy (x_1^{opt}, x_2^{opt}), we find that $x_1^{opt} = \frac{R}{R+T}$ and $x_2^{opt} = \frac{T}{R+T}$. As we would have expected, the greater T relative to R, the more often k should play α_2 if he wishes to protect himself from h's punitive strategy. If R were zero, this maximin strategy would induce k to play α_2 exclusively, thus limiting his payoff to $\pi y^{\bar{a}}$.

v, the minimum payoff k can expect if he adopts the mixed strategy (x_1^{opt}, x_2^{opt}), equals:

$$\frac{(\pi y^o)R + (\pi y^{\bar{a}})T}{R + T}$$

It is easily verified that $\frac{\partial v}{\partial R} > 0$ and $\frac{\partial v}{\partial T} < 0$, as long as $\pi y^o > \pi y^{\bar{a}}$.

What should h do to maximize his utility if he were to observe that k actually employed the mixed strategy (x_1^{opt}, x_2^{opt})? Again he can do no better than play β_1. But then k should revert to the pure strategy x_1 and h will be worse off. The only hope for h in an extended game is to play β_2 often enough for k to form the impression that h is more interested in punishing him for transgressions than in immediately maximizing his (h's) utility. This impression will be fortified if T is relatively low and weakened if it is high, for h will not be willing to adopt a strategy dominated in a game limited to a single play if he is likely to incur a heavy immediate loss in so doing (unless the penalty for transgression is so heavy that k need only be reminded on rare occasions that h is capable of playing β_2).

Let (y_1, y_2) denote any of h's mixed strategies. Suppose $\bar{y}_2$ to be the minimum level of β_2 required to induce k to seek cover and play (x_1^{opt}, x_2^{opt}). Then h's average payoff will be:

$$w = (1-\bar{y}_2)(\frac{R}{R+T})U^- + (1-\bar{y}_2)(\frac{T}{R+T})U^m + \bar{y}_2(\frac{R}{R+T})(U^- - I) + \bar{y}_2(\frac{T}{R+T})(U^m - I) =$$

$$\frac{R}{R+T}U^- + \frac{T}{R+T}U^m - \bar{y}_2 I$$

Thus h's average payoff will be a convex combination of U^- and U^m with weights $\frac{R}{R+T}$ and $\frac{T}{R+T}$, minus a fraction of inspection costs I equal to the proportion of the plays where h has resorted to his second strategy.

That T should be as great as possible to maximize h's utility is self-evident.[26] It is perhaps not so intuitive that the reward for (revealed) compliance should be as low as possible if this goal is to be achieved.

A stochastic vector of input constraints may now be re-introduced. As in the first sections, we assume that (1) M is invariant to the particular input constraintsthat k is actually faced with in a given period, (2) G is known to k with certainty, and (3) R and T are fixed scalars, invariant to any of the variables in the problem. Given the first of these assumptions, it does not seem to matter, in playing his game, how much or how little h may know about k's capabilities.[27]

A simple example, involving aggregation from n goods to a scalar, is illustrated below.

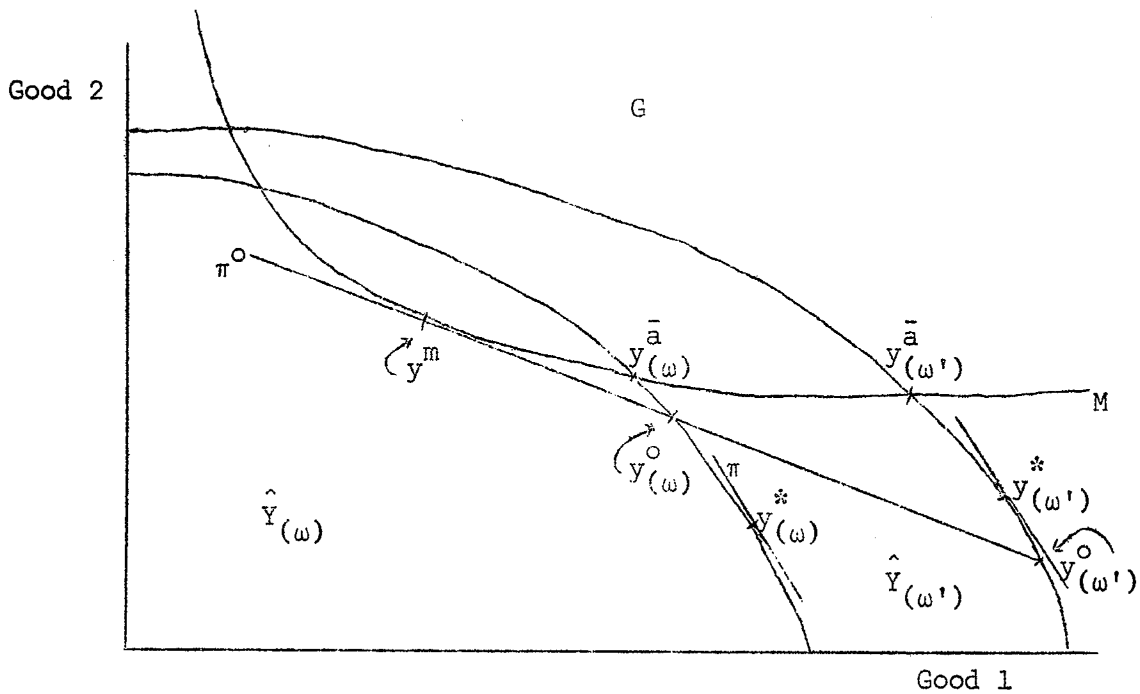

Diagram 5

In the case illustrated, ω' is a more favorable configuration of input constraints than ω, allowing an expansion from $Y_{(\omega)}$ to $Y_{(\omega')}$. It happens here that $y^*_{(\omega')}$ is included in G^R so that aggregation of controls, in case ω' crops up, enables k to attain 100 per cent autonomy. This is of course accidental. While the payoff of k will necessarily increase if ω' occurs rather than ω, the autonomy ratio may actually decrease (if, for example, the slope of the efficiency frontier of $Y_{(\omega')}$ were appreciably <u>smaller</u> above $\pi^{o}y$ through y^{m} and <u>greater</u> below that line). Neither is there any way of ascertaining a priori whether $\frac{\pi y^{\bar{a}}_{(\omega)}}{\pi y^{o}_{(\omega)}}$ is larger or smaller than $\frac{\pi y^{\bar{a}}_{(\omega')}}{\pi y^{o}_{(\omega')}}$.

Whichever is larger, however, should make little or no difference to the choice of strategies that k and h are likely to adopt, since, whether ω occurs or ω', β_2 will be dominated by β_1 and the maximizing strategy of k will still be $(\frac{R}{R+T}, \frac{T}{R+T})$. The latter point is evident from the fact that neither πy^{o} nor $\pi y^{\bar{a}}$ appear in the solution to the problem of maximizing v. Since R, T, and I are fixed, there are no new factors to affect the behavior in a repeated game of either k or h. I conjecture, therefore, that under these assumptions the relative frequency of inspection (β_2) in an optional strategy for h over an indefinite number of periods will be the same for any vector of constraints to which k may be subject.[28]

I shall not dwell on the case, already alluded to in the beginning of this paper, where h decides on his minimum acceptable level of utility U^{m} after observing ω or, more generally, after observing k's capabilities. The supervisor may, for example, set U^{m} in such a way that k can only produce in G if his vector of constraints is exceptionally favorable; then when a typical constraint vector such as crops up, k is forced to cite "objective difficulties" for his failure to comply with h's orders. If h is not willing to accept just about <u>any</u> performance backed up by such "excuses," he must pay additional costs to obtain the information necessary to check on these objective difficulties. This may be extremely costly, especially for a highly disaggregated product mix.

That contingent controls of this sort are at times applied in command economies of the Soviet type may have to do with the fact that much internal information about the production capacities of producing units are routinely passed on to their superiors at frequent intervals so that the incremental costs of obtaining information to check on excuses for transgressing directions may be fairly low. It is generally recognized that pressure on subordinates to coax out maximal performance--feasible only under unusually favorable circumstances--is diminishing in these economies.[29] If so, the assumption of an invariant G may become increasingly realistic in analyzing supervisor-supervisee relations in such a setting.

APPENDIX

The linear programming formulation of the maximizing and minimizing problems considered on pages 11-42 can be represented as follows:

$$\max \pi y \quad \text{or} \quad \min \pi^{o} y$$

subject to:

$$-Cy \leq -c$$

$$By \leq b$$

$$y_j \geq 0 \quad (j = 1, \ldots, n)$$

where C is an $m \times n$ matrix and c an m-dimensional vector of non-negative coefficients, and B is an $r \times n$ matrix and b an r-dimensional vector of non-negative coefficients $(n > 2)$.

Let $y^{\bar{a}}$ and y^{m} both satisfy the $(m + r)$ constraints; $y^{\bar{a}}$ is a maximizer for π and y^{m} a minimizer for π^{o}. Suppose $\pi^{o} y^{\bar{a}} = \pi^{o} y^{m}$.

The secondary problem (after aggregation of the constraints defining G) is:

$$\max \pi y$$

subject to:

$$By \leq b$$

$$-\pi^{o} y \leq -\pi^{o} y^{e}$$

where y^{e} is any vector satisfying the $(m + r)$ constraints of the first problem.

y^{o} is a maximizer for this problem. We reintroduce the assumption made earlier that E consists of more than one point. Consider the hyperplane $\pi^{o} y$

through y^m and denote by $\hat{y}$ the set of all y on this hyperplane. If $y^{\bar{a}}$ and y^m are distinct (if they are not, see below), then the condition $\pi^o y > \pi^o \hat{y}$ for all y in E (except $y = y^m$) cannot hold (since not only $\pi^o y^{\bar{a}} = \pi^o y^m = \pi^o \hat{y}$ but all convex combinations of $y^{\bar{a}}$ and y^m are on this hyperplane). It follows that either 1) one of the n inequations defining G, say the j'th, has the same or proportional coefficients as the elements in π^o or 2) y^m and $y^{\bar{a}}$ lie in the intersection of two or more of the constraint equations defining G, neither of which have coefficients equal or proportional to the elements in π^o. If 1), then a necessary and sufficient condition for $\pi y^o > \pi y^{\bar{a}}$ is that at least one of the n inequations defining G other than the j'th should have a positive shadow price for the basis corresponding to $y^{\bar{a}}$ (i.e. should be just satisfied). Proof: If there is no other binding constraint in G, every vector in Y located on $\pi^o \hat{y}$ must have been in E. Hence $y^{\bar{a}}$ must be a maximizer for π on E^R as well as on E, and there cannot exist a y^o in E^R such that $\pi y^o > \pi y^{\bar{a}}$. If, however, there was another binding constraint in G at the point $y^{\bar{a}}$, then clearly an increase in the value of the maximand πy would have been possible if this constraint had been removed. This constraint is, by definition, absent in E^R. Hence at least one vector worth more at prices π than $y^{\bar{a}}$ can be found in E^R. All maximizers y^o for π on E^R must be worth at least as much at prices π as this vector. Hence $\pi y^o > \pi y^{\bar{a}}$.

If 2), we have at least two of the inequations defining G strictly satisfied at $y^{\bar{a}}$. All the points $\hat{y}$ on $\pi^o y$ through y^m and $y^{\bar{a}}$ are either such that $a\hat{y} \leq ay^{\bar{a}}$ or such that $d\hat{y} \leq dy^{\bar{a}}$, where a and d are respectively row-vectors of coefficients of the j'th and d'th inequations that are just satisfied at $y^{\bar{a}}$.

The above inequations follow from the fact that the only points such that $ay > ay^{\bar{a}}$ <u>and</u> $dy > dy^{\bar{a}}$ are in the interior of G and therefore cannot be on the minimizing hyperplane $\pi^{o}\hat{y}$. Hence if either of the inequations have a positive shadow price and the constraint to that inequation is relaxed, points on $\pi^{o}\hat{y}$ satisfying the remaining constraints will be opened up that will be worth more at prices than $y^{\bar{a}}$.

Finally, if $y^{\bar{a}}$ and y^{m} are identical and E consists of more than one point, then $y^{\bar{a}}$ must be located at the intersection of two or more hyperplanes defined by the inequations defining G. It suffices that these inequations should have positive shadow prices for the proposition in 2) above to go through.

Returning to case 1), we note that it is "almost but not quite" necessary for two of the inequations defining G to have positive shadow prices. We could have a situation where 1) y^{*} was identical with y^{o} and 2) the inequation with the coefficients proportional to the elements of π^{o} had a zero shadow price, but πy^{o} could still exceed $\pi y^{\bar{a}}$, even though only one other constraint in G had a positive shadow price.

FOOTNOTES

*I am grateful to Geoffrey Heale, Yoav Kislev, Richard R. Nelson, and Martin Weitzman for their perceptive comments on a preliminary draft of this paper.

[1]On this latter type of decentralization, see Kenneth Arrow, "Control in Large Organizations," Management Science, Vol. 10, April 1964, pp. 399-401.

[2]Jacob Marschak, "Problems in Information Economics in Management Controls," in New Directions in Basic Research, New York, Toronto, and London, 1964.

[3]Roy Radner, "Competitive Equilibrium under Uncertainty," Econometrica, Vol. 36, 1968.

[4]This satisficing approach was first developed in J. Kornai, Anti-equilibrium; On Economic Systems and the Tasks of Research, Amsterdam and London, 1971, pp. 96-97 and, independently, in J. M. Montias, "A Framework for Theoretical Analysis of Economic Reforms in Soviet-type Economies," in M. Bornstein, ed., Plans and Market, New Haven, 1973.

[5]An alternative way of modeling this supervision relation would be to assume that the supervisor would insist on a higher level of performance if he received performance reports in an aggregated than in a disaggregated form. In this way the supervisor might compensate for the greater risk he incurred of accepting an aggregated vector that would turn out to be inferior for the system as a whole. This strategy would have the disadvantage of causing the supervisor to reject certain vectors that were actually aggregated from vectors

in G. More frequent inspection might be required than under the behavior conditions spelled out in the model developed in the text. To determine whether it would pay the supervisor to adopt the strategy of restricting the autonomy space of supervisees when his controls were aggregated, however, would demand a more detailed specification of the costs of inspection and of interference than the present model provides.

[6]On the supervisor's possible strategies for maximizing his utility, see below, section 3.

[7]If the supervisee is rewarded for 100 per cent fulfillment of his targets but gets nothing more for overfulfillment, his behavior cannot be represented by a linear objective function. It also strains belief to assume, in case his performance should depend on his exertions, that his effort can be incorporated as an input with a fixed weight in his objective function. For a model incorporating alternative assumptions, see Michael Keren, "On the Tautness of Plans," Review of Economic Studies, forthcoming.

[8]On the "power" of the supervisor over his supervisee and its relation to the cost to the former of imposing penalties, see below, section 3.

[9]If E consisted of a single point on the efficiency frontier of $\hat{Y}_{\omega}$ then a less favorable constraint vector ω' would be likely to generate a production set $\hat{Y}_{\omega'}$ that did not intersect with G at all.

[10]I will not dwell on the interesting question, raised by Geoffrey Heale in a recent discussion, as to whether a definition of autonomy should not include some notion of increased choice, irrespective of whether the new options available enable a supervisee to increase the value of his maximand. My own

feeling is that the widening of the set of options in order to be valued by the supervisee, must, under certain circumstances at least, lead to a higher value of his maximand. It would not, in any case, be an easy matter to define autonomy in terms of opportunity sets in such a way that two situations could be compared and their relative degree of autonomy cardinally measured.

[11]The reader may wonder whether it is realistic to represent a situation where a supervisor can decide what a supervisee should do, at least in aggregated terms, but cannot set up an incentive system that will induce him automatically to perform according to his desires. For one thing, immediate supervisors in many organizations are not free to determine either the incentive system according to which their supervisees are rewarded nor the price systems that are used to evaluate their performance. The elaboration of an incentive system that will be equitable when applied to a significant number of subordinates in an organization and that will induce all of these subordinates to perform in a desirable manner is a formidable task which goes much beyond the narrow range of problems discussed in this paper. Another consideration is that, if the supervisor does have control over the incentive system for his supervisees, the performance of the latter may be subject to appreciable fluctuations (e.g. in the volume and composition of his output) due to random exogenous constraints, so that it may be very difficult for the supervisor to set incentives in such a way that supervisees will be induced to produce in an acceptable set at all times.

[12]Every vector $\bar{y}$ in E^R is in E^m, since (1) by the definition of E^R, $\pi^o\bar{y} = \pi^o y^a$ where y^a is some vector in E, and (2) $\pi^o y^a \geq \pi^o y^m$ by the definition of y^m, and hence $\pi^o\bar{y} \geq \pi^o y^m$, which satisfies the definition of vectors in E^m.

I now show that an arbitrary vector y^+ in E^m must also be in E^R. By the definition of E^m, $\pi^o y^+ \geq \pi^o y^m$. Suppose $\pi^o y^+ = \pi^o y^m$. Then y^+ is in E^R, since y^m is in G. Alternatively, suppose $\pi^o y^+ > \pi^o y^m$. Since both sides of the inequality are scalars, this can be written $\pi^o y^+ = d\pi^o y^m$ where d is a constant larger than unity or $\pi^o y^+ = \pi^o (dy^m)$. But dy^m, by the non-satiety assumption, must be in G. Thus if we substitute dy^m for y^a and π^o for Π^o in the definition of G^R above, we see that y^+ must be in G^R. Since y^+, by assumption, is in E^m, it must be in $\hat{Y}_\omega$. If y^+ is in G^R and in $\hat{Y}_\omega$, it must be in E^R, the intersection of these two sets.

[13]Proof: $\pi y^* \geq \pi y^m$ by the definition of y^*. Then $\pi^o y^* \geq \pi^o y^m$, since, by assumption, $\pi = \pi^o$. Therefore, by the definition of E^m, y^* is in that set, and hence in E^R. But if y^* is a maximizer for π on $\hat{Y}$, it must also be a maximizer on E^R, a subset of $\hat{Y}$. This result does not depend on the convexity of $\hat{Y}$. Note, however, that it does not necessarily hold for aggregation from n to q, where $q > 1$ (see footnote 20).

[14]J. M. Montias, op. cit., Appendix B.

[15]The condition $\pi^o y^{\bar{a}} > \pi^o y^m$ is necessary when $n = 2$, but, when n is equal to three or more goods, vectors in E^R located on the plane $\pi^o y$ through y^m (or $y^{\bar{a}}$) may be found that are worth more at prices π than $y^{\bar{a}}$. I was only able to find necessary conditions for πy^o to exceed $\pi y^{\bar{a}}$ in case $n > 2$ under conditions where Y can be taken to conform to a linear technology and G can be defined as the set of points $Cy \geq c$ where C is an $m \times n$ matrix and c is an m-dimensional vector of non-negative coefficients (see Appendix).

[16] As long as $y^{\bar{a}}$ (respectively $y^{\bar{a}'}$) is an interior point of the half space generated by $\pi^o y$ through y^m (respectively through $y^{m'}$), the basic condition of proposition 1 is satisfied ($\pi^o y^{\bar{a}} > \pi^o y^m$ and $\pi^o y^{\bar{a}'} > \pi^o y^{m'}$) so that $\pi y^o > \pi y^{\bar{a}}$ (respectively $\pi y^{o'} > \pi y^{\bar{a}'}$). The upper limit to this improvement is of course set by the condition where the maximizer for π on E^R is worth as much at prices π as y^* (e.g. $y^{o'}$ in the diagram is the upper limit in this case).

[17] By definition $\frac{\delta}{\gamma}$, defined in the next paragraph in the text, equals $\frac{\pi^o y^{\bar{a}} - \pi^o y^m}{\pi^o y^{\bar{a}} - \pi^o y^*}$. Since $\hat{Y}$ is convex and $\pi^o y^{\bar{a}} > \pi^o y^m > \pi^o y^*$, there must exist a convex combination y^b of $y^{\bar{a}}$ and y^* on $\pi^o y$ through y^m. Substituting $\pi^o y^b$ ($= \pi^o y^m$) in the above expression:

$$\frac{\delta}{\gamma} = \frac{\pi^o y^{\bar{a}} - \pi^o y^b}{\pi^o y^{\bar{a}} - \pi^o y^*}$$

By definition $\Delta = \pi y^* - \pi y^{\bar{a}}$. The payoff corresponding to y^b is πy^b. The increment in payoff due to aggregation is $\pi y^b - \pi y^{\bar{a}}$. The critical ratio of the increase in payoff from aggregation to the maximum possible increment is $\frac{\pi y^b - \pi y^{\bar{a}}}{\pi y^* - \pi y^{\bar{a}}}$. To show that this ratio is equal to $\frac{\pi^o y^{\bar{a}} - \pi^o y^b}{\pi^o y^{\bar{a}} - \pi^o y^*}$, we need only note that we have here the ratios of two aggregates at different prices of the same vectors with elements $(y_j^{\bar{a}} - y_j^b)$ and $(y_j^{\bar{a}} - y_j^*)$ respectively ($j = 1, \ldots, n$). A familiar proposition from index-number theory tells us that the ratio of all such aggregates must be equal, irrespective of price weights. The equality of the two ratios allows us to conclude that $\pi y^b - \pi y^{\bar{a}}$ equals $\frac{\delta}{\gamma}\Delta$.

[18] If there is a set of maximizers for π on $\hat{Y}$ rather than a unique maximizer (as in Diagrams 1 and 2), then the measure $\frac{\delta}{\gamma}$ will depend on which of the

maximizers are chosen on this set of maximizers. The largest value of $\frac{\delta}{\gamma}\Delta$, and the one making the most sense, will be obtained by choosing any maximizer for π on $\hat{Y}$ that maximizes the value $\pi^o y$.

[19] Call this vector y^s and its subvectors in the i'th and j'th subspaces y^{is} and y^{js}. Since $\pi^i y^{is} > \pi^i y^{i\bar{a}}$ and $\pi^j y^{js} = \pi^j y^{j\bar{a}}$ ($j \neq i$; $j = 1, \ldots, q$), it follows that $\pi y^s > \pi y^{\bar{a}}$.

[20] For details, see Montias, op. cit. It should be observed that if π and π^o are identical, it is not necessarily true that y^* will be in E^R when $q > 1$. For this proposition to hold, we must have:

$$\pi^i y^{i*} \geq \pi^i y^{im}$$

for every subvector i from 1 to q. (Any vector y^e in E other than y^m satisfying the set of q relations will also do.) If we substitute π^{io} on both sides of the above inequation ($i = 1, \ldots, q$), we see that $\Pi^o y^* \geq \Pi^o y^m$ and that y^* must be in E^R.

[21] This follows from Theorem I.2 in T. C. Koopmans's, Three Essays on the State of Economic Science, New York, Toronto and London, 1957, p. 12.

[22] Consider a price system ρ and a vector $y_h^{\bar{a}}$ such that $\rho y_h^{\bar{a}} \geq \rho y_h$ for all y in E and $\rho y_k^{\bar{a}} \geq \rho y_k$ for all y_k in E_k ($k = 1, \ldots, K$), where $\sum_{k=1}^{K} y_k^{\bar{a}} = y_h^{\bar{a}}$. By the definition of $\hat{E}_k^D$, $y_k^{\bar{a}}$ must be in this set for every k ($k = 1, \ldots, K$). Clearly, $\pi y_k^{\bar{a}} \leq \pi y_k^{\bar{\bar{a}}}$ for all k, where $y_k^{\bar{\bar{a}}}$ is a maximizer for π on $\hat{E}_k^D$. Suppose that for some supervisee ℓ, $\pi y_\ell^{\bar{a}} < \pi y_\ell^{\bar{\bar{a}}}$. Then, summing over all supervisees, $\pi y_h^{\bar{a}} < \pi y_h^{\bar{\bar{a}}}$, where $y_h^{\bar{\bar{a}}}$ equals $\sum_{k=1}^{K} y_k^{\bar{\bar{a}}}$. By the theorem already cited, $y_h^{\bar{\bar{a}}}$ must be a maximizer for π on $\sum_{k=1}^{K} \hat{E}_k^D$ and hence on $\hat{E}_h^D$, a subset of this last sum of sets.

But the last inequality contradicts the assumption that $y_h^{\bar{a}}$ is a maximizer for π on E_h^D. This proves that $\pi y_k^{\bar{a}} = \pi y_k^{\bar{\bar{a}}}$ for all k and hence that $y_k^{\bar{a}}$ is a maximizer for π on E_k^D for all k. The same reasoning can be applied to prove that y_k^o must be a maximizer for π on E_k^{DR} for all k.

[23] This reward may be quite small relative to $\pi y^{\bar{a}}$ and to T, but it would be unrealistic to suppose that k would derive no advantage from an inspection that had uncovered no serious deviation of performance from orders.

[24] In this non-zero-sum game, such a strategy would not necessarily be optimal for h. But is is relevant because it places limits on h's ability to punish k for adopting α_1 at least part of the time and thereby to "teach" him not to transgress his orders.

[25] The diagram is adapted from R. D. Luce and H. Raiffa, Games and Decisions; Introduction and Critical Survey, New York and London, 1957, Appendix 3, pp. 394-97.

[26] A more complex and realistic model would consider the cost to h of imposing a penalty on k (e.g. the manager of k might resign and it might be costly to replace him). Such a consideration, *ceteris paribus*, would tend to limit T. For a more general model of power in hierarchies incorporating the cost to h of imposing penalties on k, see John Harsanyi, "Measurement of Social Power" in Game Theory and Related Approaches to Social Behavior, M. Shubik, ed., New London and Sidney, 1964, pp. 183-206. While this is the only reference to Harsanyi's paper, I should acknowledge its general influence over the conceptual approach adopted in this paper.

[27]We recall that h needs to know something about $\Omega \equiv \{\omega\}$ and about k's technology in order to set M at a level that will permit k to produce in G "most of the time." The greater the cost to h of imposing a penalty on k for not producing in G, the greater the likelihood that M will be set in such a way as to enable k to produce in G, for almost any ω in Ω, if he wishes to do so.

[28]The possibility of an exception to this general principle should be recognized in case ω' changed the constrained production set in such a way as to cause a relatively large difference in the ratios

$$\frac{\pi y^{\bar{\bar{a}}}_{(\omega)}}{\pi y^{o}_{(\omega)}} \quad \text{and} \quad \frac{\pi y^{\bar{a}}_{(\omega')}}{\pi y^{o}_{(\omega')}}$$

For, while k's maximin strategy would not be affected, the desirability of adopting it against an occasional play of h's second strategy might differ in the two situations: k might take more chances--i.e. play α_1 more often than the maximum strategy would dictate--in situations where differences between πy^{o} and $\pi y^{\bar{\bar{a}}}$ were exceptionally large.

[29]See the discussion in Michael Keren, <u>op. cit.</u>