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CENTER DISCUSSION PAPER NO. 91

GROWTH AND TECHNICAL PROGRESS IN THE SOCIALIST ENTERPRISES OF YUGOSLAVIA:
A COBb -DOUGLAS ANALYSIS USING EXTRANEGS ESTIMATORS

Charles S. Rockwell

July 22, 1970

Note: Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

## This paper estimates the coefficients of a Solow type Cobb-Douglas

 function: the regression equation relates ral value added to real capital, labor and a technological proxy variable, time. The model is applied to nineteen productive industries of the social sector of the Yugos lav economy, cross classified by five geographic regions. The estimates are to be used in two companion pieces that analyze the behavior of enterprises and sources of growth in Yugos lavia.Econometric research of the past decade has made the statistical estimation of production functions less, not more credible. Much of the discussion here is concerned with two issues raised by these writings: simultaneous equation bias; and the instability of the estimates for different samples and estimators. The conclusion is reached that the amount of simultaneous equation bias present in the estimates is small, and that the estimates are highly stable with respect to the estimators but less stable with respect to the grouping basis and time period of the sample. The estimates themselves are judged to be economically meaningful measures of the Cobb-Douglas model that is assumed.

- Threa econometric innovations are employed. One is to use the multitable method of Yoel Haitovsky to obtain estimates of the capital and labor output elasticities. This is possible because fer 1963 and 1964, cross-section data is available for the nineteen industries. The tables are for Yugoslavia, but not for the four sub-regions. The data groups all firms in each industry into twelve cells according to their size; separate tables are published for size as measured by fixed assets and by employment. Haitovsky's method uses the capital table to estimate the capital coefficient and the labor table to estimate the labor coefficient, and then corrects these estimates to remove the bias due to mis-specification.

Another innovation is to use a "reverse covariance" estimator and Haitovsky's method to demonstrate the unimportance of the simultaneous equation blas that arises from a correlation between labor and the stochastic term. A "reverse covariance" estimator reverses the table subscripts in Haitovsky's method so that the capital table is used to estimate the labor coefficient and vice versa. It is an inefficient estimator, but one that is bias-free. Its counterpart, the "ordinary covariance" estimator that results from a standard application of Haitovsky's method, is efficient but subject to bias. A collation of the ordinary and reverse covariance estimates reveals that the estimates for the capital and labor coefficients are identical for both estimators for the aggregate economy and for its largest sub-sector, industry and miaing. The common capital estimate for both industries is . 13, the labor estimate is .89. It is argued that differences between the estimators for the seventeen remaining industries can be explained by sampling variation. The conclusion is reached that simultaneous equation bias is not of practical ime portance, and therefore, on the basis of efficiency the ordinary covariance


The third innovation is to use the cross-section capital and labor estimates as extraneous estimators in the 1952-1964 time series analysis. This leaves only the coefficient of neutral technical progress to be estimated from the time series. To extend the analysis to the five regions it is necessary to assume no regional variability in the capital and labor coefficients, thus permitting use of the Yugoslav cross-section capital and labor coefficients for all regions. Formally, this is not permissisle Statistical tests using data available only for industry and mining indicate that these coefficients do differ between regions. However, the differences are less important
because of the manner in which the majority of the estimates cluster about the values . 13 and . 89 mentioned above. The stability and magnitude of the regional coefficients of technical progress support the contention that extraneous estimators give meaningful results. For example, the regional technical progress coefficients for industry and mining are:-

## Yugos lavia

## 3.8\%

## North <br> South

3.7\%

| Serbia Proper | $3.7 \%$ |
| :--- | :--- |
| South less Serbia Proper | $2.7 \%$ |

Although not an innovation, the paper does derive and present, in the Appendix, production data not heretofore available. For five regions, for nineteen industries, for the years 1952 to 1966 , four variables are given: employment, total fixed assets, equipment, and value added (social product). The last three are in constant 1966 prices and therefore benefit from the price rationalizations of the 1965 Reform. The most important new contribution of this data is the creation of constant price, regional series on value added for twelve branches of industry and mining. The capital series is unique in that empirically obtained estimates of length of life for plant and for employment are used as durability weights in the manner advocated by Haave 1 mo.
 A. COBB-DOUGLAS ANALYSIS USING ERTRANEOUS ESITHATORS
$\qquad$
PART I
PROBLEMS Ci' SPECITICAIICN AMO IDENTATICATION

## Introduction

This paper ${ }^{1}$ provides a formal statistical analysis of the growth of real output among the socialist enterpises of Yugoslavia. According to the Cobb-Douglas model used, growth is explainad by thren factors: the mobilization of capital and labor, increasirg yeturns to scale at the industry level, and disembodied technicョl progress. Temporarily, no cognizaree is given to the changing quality of $1 a b o z$ o capital, io moz-neutral technical progress, or to structural shifts between the branches of the social sector. The objective is to see hots succesofully a statistical analysis of inputs and outputs can explain differences in cutput between regions, between industries, and overtime. Attention is restricted to the tine period between the establishment of the New Economic Policy in 1952 and the Reforn of 1965. Since this paper serves as a foundation for more economic and policy-oriented works under preparation, concentration cenvers on the statistical methedology and results rather than their conoric interpretation.

Already we can imagine a scowl facm conometricians, and a yawn from development economists. A quick summaty of tio majcr problems and our proposed solution is necessaty to rolax these coumeannes and proserve readers.
${ }^{1}$ Work is currently under way on too ecmpenion pieces. The first is a Denison type analysis of the determirants of agregate growith for all sectors. Since wages and pioces camot be relied upon to reflect rarginal products, the productivities derived in this raper axa a crucial input. The second is a theoretical and empirical microanalysic of eatcrprises behavior. How has the system of Workers lamgement ecmexibuted to the ropid growth of the Yugoslav economy? Again, this pape: providas the foundation for the analysis.

Only a very brief search of the literature is needed to find eminently qualified critics of statistical production functions. Professor Edmund Malinvaud writes:
...the calculated regression is not a satisfactory estimate of the production function. It constitutes a purely artificial relation which depends on the correlations among the...error terms... just as much as on and - Statistical Methods of Econometrics (Chicago: Rand McNally, 1966), p. 519.
or, Professor Murray Brown:
The impossibility of identifying the estimates because of multicollinearity when using cross-section data has been touched on, with the conclusion that cross-section data is useless except for very limited purposes in the present context. However, there is also an identification problem because of multi-collinearity usinc \&ime-series data. On the Theory and Measurement of Technological Change (Cambridge: Cambridge University Press, 1966), p. 126.
or finally, Sir John R. Hicks:
I cannot myself perceive that there is any economic sense in such a physical measure of the capital stock. It is futile to erect great edifices of theory, and of econometrics, upon it. The estimation of production functions-..involving a distinction between accumulation of capital (in some such sense as this) and technical progress (residual technical progress)--seems therefore to me to be a vain endeavor. "The Measurement of Capital," a paper delivered at the International Statistical Conference, London, Summer of 1969, p. 11.

These criticisms are selected not only because of the excellent credentials of the authors but also because they describe the three problem areas that are most relevant to this study: (1) lack of identification due to simultaneous equation bias; (2) or to multi-collinearity; and (3) difficulties in the definition and estimation of the capital stock.

The greatest hurdle in making production function estimates credible to econometricians is the lack of identification due to simultaneous equation blas. One tour de force that can be performed is to incorporate simultaneous equation bias into one's theory thereby making it an effect we wish to measure rather than a "bias." Granted the purpose of our estimates, institutional
realities in Yugos lavia make it possible, even essential, to incorporate certain mechanisms of resource allocation into the aggregate parameters. Specifically, the distribution of management ability and the intra-industry investment allocation mechanism are effects which are built into our estimates of the capital and labor coefficients. Effects of this type that are included in our estimates of the coefficients are consequently excluded from the measure of technical progress. The rationale for not including management and investment effects under the technical progress rubric are explained later in this section.

Even if the reader agrees to go along with us and like some of the things which cannot be changed, the problem of correcting what isn't liked remains: A model and an estimator are needed that will eliminate the unwanted portion of the bias. Our approach is to first specify a model which is appropriate to the Yugoslav economy, and define six different statistical estimators of the parameters of the model. Next, on a_priori grounds these six estimators are crudely ranked in two ways: according to the possible biases that might affect them; and according to their expected efficiency. Finally, after the estimates are computed, select the most bias free estimator that meets a minimum efficiency standard. Anticipating the conclusion, the estimator which rants highest (under a favored assumption it is completely bias free) and the estimator which ranks lowest on our bias scale but has maximum efficiency, give nearly identical results for aggregate sectors. Consequently, we conclude that simultaneous equation bias is not an important problem with the model used, and that considerations of efficiency may be allowed to determine the best overall estimator. We will treat the other two problems of production function estimation more briefly since, with respect to multi-collinearity, there is not much that can be said, and with respect to the capital stock a Fmore detailed disucssion is given in the Appendix.

In a properly specified model, the deleterious effects of multi-collinearity reveal themselves in large standard errors for the coefficients. ${ }^{2}$ However, Brown's concern (and that of the myriad scholars he cites) ${ }^{3}$ is that the true values of capital, labor and output prescribed by our theory are so highly correlated in the data sample that the parameter estimates are really being fitted to perturbations in the data arising from short run disequilibria, monopoly imperfections, and so forth. Not being able to observe short run disequilibria, monopoly imperfections and similar phenomena, no real test of this assertion is possible. We would expect, however, that if the estimates were princlpally determined by such perturbations, the parameter estimates for different, independent, cross-section samples would be highly unstable. We do not feel our estimates show this degree of instability, but the reader may reserve judgment until the estimates are presented. There is no question but that muleiecollinearity in the data is high. For example, from the Employment grouping in Table 2, the capital-labor correlation is .906, the capital-output -991. and labor-output. 295.4 These high correlations are typical of the cressesection data and yet they do not cause destructive increases in the stane dard errors of the coefficients. Another statistic from Table 2 suggests the reason for this: while multi-collinearity is large, so too is the range of the capital-labor ratio (from a minimum value of 1.2 to a maximum of 5.2 ).
${ }^{2}$ "Thus the standard errors should give ample warning of the imprecision attoching to the estimates of the separate effects of $X_{2}$ and $X_{3}$, when the two varlables are highly correlated" J. Johnston, Econometric Methods (New York: McGraw Hill, 1560), p. 204.
$3_{\text {Brown, op. cit., p. }} 3_{n}{ }_{n}$
$\cdot \quad$ 4The measure presented is computed from unveighted, per-firm data for the twelve size categories.

This great range of the ratio of the independent variables provides adequate Information for the estimation of statistically significant coefficients. Hopefully, the range is also sufficient to overcome the distorting effects of any systematic perturbations of the type mentioned by Brown. Like the cross-section data, the time-series also exhibits high multi-collinearity. In this case, however, the range is much smaller, and consequently we place as little emphasis as possible on the use of time-series to unscramble the competing effects of capital and labor.

While identification is the statistical hurdle most prominently hindering creditable estimates, the theoretical problem of greatest difficulty is how to measure capital's contribution to production. It is this difficulty that leads Professor Hicks to question the validity of any attempt to production function estimation similar to the type we propose. The more detailed questions of deflation and measurements of capital stock are relegated to Appendix $C$. At this point we are only concerned with the more overriding question of whether or not theoretical problems in the definition of capital and in the contribution of capital to production make it a "vain endeavor to construct statistical production functions." In a recent review of this literature, Israel M. Kerzner ${ }^{5}$ convincingly concludes that whether capital is to be treated as a flow of services or as a stock of goods whose very existence contributes to production with no diminishment of the stock's capability, depends on the time period of the analysis. Where the relevant time period is the planning horizon of the firm, all inputs must be considered variable so that a flow approach is the proper one. On the other hand, as we consider shorter and shorter time periods, more variables become fixed for the purpose of analysis and it
${ }^{5}$ An Essay on Capital (New York: August M. Kelley, 1966), particularly Chapter Two.
becomes appropriate to treat them as a stock which contributes to production aimply by its presence. This latter approach is espoused by Trygue Haavelmo ${ }^{6}$ and adopted by us. In adopting the position that capital contributes to production simply by its presence rather than by providing a stream of services, we subject ourselves to Kerzner's criticism of this approach. Essentially it is that we neglect the question of multi-period planning which both generates the capital stock at the beginning of the year and which receives it at the termination of each year.

One of the principal difficulties in the Havelmo model is the necessity of adjusting for differing durabilities of capital goods, a problem which is discussed in the capital stock Appendix C. It will suffice here to mention that we make no such attempt at adjustment in the cross-section data and consequently make the implicit assumption that the durability mix for the capital stock of firms in different size categories is all equal. In the time series data we make an explicit adjustment for the varying durabilities of equipment as opposed to structures.

Buttressed by these comments; we hope the reader will hold his skepticism in abeyance while the model and its statistical estimators are discussed in detail. Those more interested in results than method may skip the folloving section without great loss.

## Data, Mode 1, and Estimators

It is assumed that the real output of the enterprise depends on five inputs, three measureable and two not measureable: the former are the input of labor in man years, the input of capital goods measured in constant price

[^0]dollars (and adjusted for differing durabilities), and intermediate inputs; the latter are the skill of management in combining the productive factors, and the state of technological knowledge. A visual introduction to these variables is given in equation (1.1) where $Y, L, K$ and $G$ denote the quantitatively observable variables--output, labor, capital and intermediate goods; and $M$ and T represent the non-observable variables--management and technology. This overly abstract statement is intended to serve only as a peg for discussing some of the more general problems of production function estimation.
$$
\text { (1.1) } Y=f(K, L, G: M, T)
$$

Our first problem is aggregation. We begin with a description of the data generated by the disaggregate firm and discuss, step by step, the aggregations made by ourselves and the Federal Statistical Bureau of Yugos lavia (SZS). This somewhat round-about process serves to emphasize that the underlying data collection is done on an exhaustive basis covering all firms each year. Although the published variables and aggregates vary from year to year, they are generated by the same censal process. At times we are forced to splice together various series because the data for the entire population is not published annually. The underlying continuity of the censal process is important since it means we do not have such serious problems in comparing data from different time periods and different sectors as we would have if they were generated by differing sets of surveys and samples. What we have are various windows looking into the population of firms, the windows change their location through time, but they always continue to observe the complete population of firms without distortion.
. Since 1958, individual firm data covering a multitude of variables including K., $K$ and $G$ are available to the SZS on an annual basis.: For a few Ers.
years this data is also available outside of Yugos lavia and can serve as the basis for making a completely disaggregate study. For reasons of cost and availability, our study does not utilize such data but instead relies on publicly available aggregates. The aggregation of firms into industries is an obvious first step. In this direction it is possible to obtain much of our data for a 41-sector breakdown of the economy. However, even this level of aggregation is too burdensome.

Table 1 describes how we aggregate the nine basic sectors of the econony into six, and how the twenty-two branches of industry and of mining are aggregated into twelve. This aggregation of firms into industries is not as destructive to information as it might appear since after 1262 we have available cross-sectional data on each of the industries. The cross-section data, described in more detail below, groups firms in each industry according to their size so that our aggregation ultimately produces the ovservable variables of (1.1) for each of ninsteen industries (two aggregates and seventeen independent branches) cross-classified by 12 size categories. In the dimensions of geography, we use a 5 -region aggregate. ${ }^{7}$ With respect to the temporal unit, although some of the data is available on a monthly basis, we are not sufficiently interested in short-tern dynamics to attempt to utilize this information: the basic unit of analysis is the year. In summary, the first step in simplifying the data is to aggregate into 18 industrial branches, 12 size categories, 5 regions, and all in all, some 15 years. Obviously, this still leaves us with a need for much further simplification.

[^1]- The greatest contribution to data simplicity, and the greatest loss to information occurs because the cross-section data does not become publicly available until 1962. At the time of this writing, a time series of the crosssection data by our nineteen sectors is available for 1962 through 1966. However, we will only be concerned with two years of this data: 1963 and 1964 . The year 1962 was one of mini-recessions and the existence of excess capacity in many plants makes it ill-suited for supply analysis. The years 1965 and 1966 are beyond our temporal focus and, particularly in the later years also suffer from the fact that severe cut-backs in the rate of growth and transition problems associated with the reform of 1965 again cause low capacity and labor utilization to distort production relationships. A pilot study described below shows that the incorporation of years subsequent to 1964 does not improve the estimates. The lack of availability of size-classified data further restricts our attention to Yugos lavia as a whole. Only for the sector industry and mining is data available by size category and by republics. This breakdown for industry and mining does enable us to make trial tests of parameter stability over regions, but an extensive analysis of stability for all sectors is not possible.

2. What we are left with by these aggregations and data black-out are three basic sets of data: first, time-series data for the years 1952 to 1966 according to 19 economic sectors and 5 regions; second, for the 19 sectors, for Yugos lavia only, for the years 1963 and 1564 we have cross-section data where the cross-section grouping is according to the size of the firm with 12 -levels being presented; third, for industry and mining alone, for 1563 and 1964, and also for 1965 through 1967 the same aforementioned cross-section data further presented according to Republics.

## TABLE I

AGGREGATION OF PRODUCTIVE SOCIAL SECTOR ACTIVITIES: ECONOHIC GROWTH CENTER AND RELATED TWO-DIGIT

## YUGOSLAV CLÁSSIFICATIONS



We initially focus attention on the terminal years 1963 and 1964 where the best data is available, analyze this period in detail, then use the results obtained from this benchmark to investigate the time path which brought the economy to this terminal point. A crucial step in the statistical analysis is to use the output elasticities obtained from the 1963-64 cross-section analysis as extraneous estimators for our analysis of technological change in the broader 1952 to 1962 period.

Equation (1.1) postulates a relationship between gross output and a set of inputs which include intermediate products. A significant simplification of the analysis is achieved by deleting intermediate products from the inputs and relating value added to capital, labor, and the non-observable variables. Table 1 presents evidence that suggests this constriction of the analysis does not have any serious effects on our appraisal of the sources of growth. This table presents for the total economy (social plus private sectors), the social sector, and industry and mining, the ratio of intermediate products consumed to value added. For each of these three sectors of the econony, but particularly for the first two, the change in this ratio between 1962 and 1964 is unimportant. In a nore practical vein, although we do have current price time series data on intermediate goods (the variable G), no deflated series are currently available and the possible gain from creating such a series does not seem to be worth the work required.

The question of whether or not to include intermediate goods also arises in our analysis of the cross-section data. Since we mean to use this data to obtain extraneous estimators of output elasticities, there is the possibility that the omission of intermediate goods from the production relationship will be a mis-specification of the true model and consequently lead to

RATIO OF MATERIAL EXPENDITUR'E TO VALUE ADDED (SOCIAL PRODUCT)*

| SECTOR | 1952 | 1959 | 1964 |
| :---: | :---: | :---: | :---: |
| Total Economy | . 95 | 1.05 | . 96 |
| Social Sector | . 95 | 1.05 | . 96 |
| Industry and Mining | 1.15 | . 73 | 1.24 |

* All underlying measures are in curnent prices and taken from $S B 228$ and $S G 1966$.
biased estimates of the capital and labor output coefficients. When using value added as a dependent variable, the inclusion of intermediate goods as an independent variable implies that these goods can be substituted for either capital or labor to obtain increases in value added. ${ }^{8}$ To our knowledge no empirical evidence on this question is available. In the Yugoslav cross-section data there is a tendency for the larger firms to have relatively high capital/ labor, output/labor, and intermediate-good/labor ratios. This could mean that larger firms tend to substitute intermediate goods for labor thus biasing the coefficients of a model which excludes intermediate coods. Unfortunately, we do not have adequate data for making a rigorous test of this possibility. In all the work that follows we assume that the input of intermediate products does not influence the output of value added.

The next variable, one particularly important to the cross-section analysis, is management ability as denoted by the variable $M$ in equation (1.1). Distinguishing technology, as represented by $T$, from the ability of management is an awkward definitional problem. For our purposes it will suffice to define managerial input as a class of decisions: specifically, those dealing with pricing, organization, finance, and product line decisions. These decisions are to be distinguished from the more purely technological ones concerning plant layout, production processes, etc. that relate machines and labor to output. While "management decisions" are made at all levels, they are concentrated in the Director and Workers' Council. This distinction is important because we argue that in under-developed countries the absence of a large stock of professional managers or an annual crop of business achool graduates means that the principal determinant of manacement capability is
${ }^{B_{A}}$ brief survey of this literature is available in Murray Brown, op. cit., pp. 120-127.
management experience, and this experience is gained by operating the plant where that management is currently employed. Not only is formal education without experience a relatively unimportant determinant of management capability, but also there is a small amount of management switching between enterprises. ${ }^{9}$ Certainly, in the case where management is selected on the basis of political rather than economic considerations, we may attribute superior performance by management in the larger firms to the experience they get from running such firms.

But it is Workers' Management in Yugoslavia that is a more overriding reason for feeling that management capability is a non-transferable input. Since the top policy-making boards of. the enterprise, the Workers' Council and the Board of Management, are elected on a rotational basis from among the workers, it can be argued that a correlation between the efficiency of manarement and the size of the firm is a direct consequence of that scale. Formally, we may express this association between management skill and the scale of operations by the functioning in (1.2). That is, we measure the scale of operations by the inputs capital and labor,

$$
\text { (1.2) } \quad M=g(K, L)
$$

The consequence of this definition is that we attribute to the capital and labor inputs their role in improving management as well as their direct productive uses; therefore, it is implied that largeness is itself the source of management improvement, so that increases in scale provoke automatic increases in efficiency.

We do not know of any surveys that present data on the extent to which the recruiting of management is done internally. The ILO describes the formal requirements for "open competition," but also notes that these were oiten not successful because of the lack of qualified candidates. Workers Management in Yugos lavia (Geneva: 1962), p. 102, fr. 3. In the one relevant example cited by the ILO, a new director was internally promoted. Ibid., p. 115.

A related problem with a similar solution is posed by investment policies. Central planning of investment may result in the most efficient firms getting the largest allocation of investment funds so that efficient firms are large and inefficient firms small. This intra-industry efficiency of investment allocation is an effect that will be embodied in our production.fruition estimates. ${ }^{10}$ It is a bias if the sole objective is to estimate parameters for a representative individual firm, However, where we wish to measure sources of growth, it is permissible to consider the incra-industry investment allocation mechanism as an unchanging, "invisible hand." Consequently, parametex estimates incorporate the activities of both those economic agents who allocate intra-industry investment as well as those agents' management who determine production given the set of available resources. ${ }^{11}$ For the 1952-1964 period, this former set of agents would include members of the National Bank, the Investment Bank. The effects of inter-industry allocation, or "investment strategy" and typically practices by a planning bureau are absent except in estimates for aggregate sectors.

A modified production ralationship incorporating value added rather than gross output as the independent variable and removing intermediate goods management skill as inputs is given by equation (1.3) where $Y$ denotes value added. The companion piece mentioned earlier adjusts for changes in the

$$
(1.3) \quad \dot{Y}=h(K, L ; T)
$$

${ }^{1}$ Where data or the individual firm is availaile Yair Mundlak describes "management bias" may be removed by covariance analysis. Sec his "Estimation of Production and Behavioral Functions from a Combination of Cross-Section and Time-Series Data" Measurement in Economic: Studies in Mathematical Economics - Econonetrics in Memory of Yegura Grunfeld (Stanford: Stanford University Press, 1963), p. 143. Since our cross-section data is grouped, this approach is not available.
${ }^{11}$ This distinction between agents is advocated by Thomas Marschal., "On the Comparison of Centralized and Decentralized Economics," American Economic Review: Papers and Proceedings, May 1969, Vol. 5 5, No. 2.
length of the work week, the participation ratio for women, education, and other variables influencing labor input, but at this point we rely on a crude man-year definition of labor input. The capital variable is based upon the purchase cost to the enterprise, or accounting value before depreciation. The cross-section studies: in 1963 and 1964 benefit from a revalorization of all capital goods in Yugos lavia in 1962 which sought to adjust their book value to current market prices, but nc attempt is made to deflate the 1963 and $1: 04$ increments in the capital stock in constanc doliars, nor is there any attempt to weigh the various equipment and structural components according to durabilities. However, as discussed in the data appendix, the time series of capital stock does correct for durabilitie and price change. We now turn to the question of functional forms.

While a great variety of functional forms are potentially available for this analysis we consider only two as serious contenders: a conventional Cobb-Douglas type function with disembodied technological progress as introduced by Solow; and a CES production function of the form fitted by Martin L. Weitzman to the Soviet $e=$ onony. ${ }^{12}$ We conclude in favor of a Cobb-Douglas function.

This is important since Weitzman's objective is similar to ours, and centers its focus on the same time period, The most important factor leading Weitzman to fit a CES rather than a Cobb-Douglac function is the rapid increase in the Soviet capital/labor ratic during the period from 1950 to 1966: it increased from a base of 100 in 1952 , to 150 by 1559 , and 286 by 1964. Clearly, capital/labor substitution is an important part of Soviet grovth so that if the elasticity of substitution is mistakenly assumed to be unity, this ${ }^{12}$ Martin L. Weitzman, "Soviet Poscwar Economic Growth and Capital Labor Substitution," Cowles Foundation Discuesion Paper No. 256, October 30, $1: 63$.
specification error may have an important effect upon results. The situation in Yugos lavia is quite different. For the social sector the same capital/labor ratio with a base 1952 value of 100 actually declines to .94 by 1559, and increases only moderately to 1.20 by $1966 .^{13}$ Therefore, due to the absence of capital/labor substitution the implicit assumption of the Cobb-Douglas function that the elasticity of substitution is unity cannot be of great importance to the analysis. For the briefer period 1952 to 1864 , the unimportance of substitution becomes still clearer-the 1064 value is only 106. This does show, however, that between 1564 and 1566 the capital/labor ratio grew by 13 percentage points so that a model of the post-reform econony may require a CES function performed by Heitzman.

Equation (1.4) summarizes our description of the available data and our decision to incorporate it into a Cobb-Douglas type function. Data limitations impose that the cross-section variables referenced by the subscripts.are available only for 1963 and 1964; and with the exception of industry and mining, we do not have these cross-sections available by regions. Two additional variables included in the data appendix but not included in relationship (1.4) are provided by a breakdown of the capital stock into its structures and equipment components. Since this subdivision is not available for the cross-section data it is simpler to omit it from the discussion at this time.

$$
\text { (1.4) } \quad Y_{i r t s}={\underset{A}{i r t}}_{\alpha}^{K_{i r t s}^{\beta}} L_{i r t s}
$$

$i$ refers to 19 industries of which two (the total for the social sector and the total for industry and mining) are obtained as aggregates of the others, so there are 17 independent industries.
${ }^{13}$ The fact that Yugoslav social sector includes agriculture does not fmportantly distort these findings since the socialized part of agriculture is comparatively small and the capital/labor ratio in that branch has a movement similar to the aggregate social sector: 100 in 1551 ; . 92 in 1959; and finally, 1.13 in 1966.

| $\mathbf{r}$ | refers to 5 regions of which two (Yugos lavia and the South) are obtained as aggregates, so there are 3 independent regions: North, Serbia Proper and South less Serbia Proper. |
| :---: | :---: |
| $t$ | refers to the 13 years 1952 to 1964. |
| and $s$ | refers to the 12 size of firm categories (defined either by employment, capital stock or output). |

In addition to specifying a Cobb-Douglas function, (1.4) indicates that returns to scale, measured as the sum of $\alpha$ plus $\beta$, is a variable to be estimated from the data, and that both the capital and labor coefficients are allowed to vary by industry and by region. Different capital/labor coefficients for different industries is a specification that can hardly be questioned. Differing coefficients by regions, however, is a specification that may be unnecessary and one that we can and do test for.

All estimates are based upon the assumption that technical progress is neutral and disembodied. Consequently, there are no time subscripts to either alpha or beta. Besides being neutral and disembodied, we often will find it useful to assume that technological progress, as indicated by equation' (1.5), is smooth and exponential in its occurrence.

$$
\text { (1.5) } \quad A_{i r t}=\operatorname{Exp}\left(\lambda_{i r t}\right)
$$

Before beginning a discussion of the stochastic specifications of the regressions, it is necessary to briefly consider the broader sets of simulteneous equations from which we have lifted the production relationship (1.4).

The identification question was introduced earlier with the quotations fxom Professors Malinvaud and Brown. It was argued that in a study such as ours with limited objectives, it is possible to partially dodge the issue by accepting certain types of bias as being desirable. Management bias is an example of this. Beyond these effects there are many other sources of
possible bias, however, which we hope to eliminate by the selection of an appropriate model and estimator. Ideally, we need a theory of behavior for Yugos lav: enterprises, a theory which will tell how available resources, the decentralized market system, workers management, and centrally influenced investment allocation determine the capital and labor inputs. Unfortunately, in our opinion, no such theory is currently available, nor does any seem possible without extensive investigations of empirical behavior. While we will make some conjectures, these are too tentative to serve as the basis for deriving a set of simultaneous equations that can serve econometric needs. Consequently, we instead concentrate upon single equation methods that are the least subject to errors of model specification.

Ma゙:
Six single-equation estimators are tried. Some of these are completely bais-free if one grants their assumption. Generally, however, it is quite difficult to tell whether these assumptions are satisfied or not. For example, the use of lagged values of the independent variables as instrumental variables produces bias-free estimates if the lagged values are not correlated with the contemporary error term. It would seem that many of the transitory factors, such as weather which affect production in one year and produce a correlation between the error term and one of the input variables might not exist in subsequent years. On the other hand, one can also think of effects such as we have described for management and intra-industry investment allocation which would continue for long periods. While a variety of assumptions of this type underlie the different estimators, there is one assumption used by some of the estimators and not by others, that appears by us to be strongly justified by the realities of the Yugos lav economy. This is that the capital stock, save for the intra-industry investment allocation effect described above, is free of correlation with the error term.

This assumption of a zero correlation is based on two facts: first, investment is determined by the development plan and the intra-industry investment allocation mechanism, and not by the rate of interest. ${ }^{14}$ Second, there is a substantial lag between the initiation of new investment products and the time when their output first comes on stream, This lag is usually estimated tobe from three to four years in duration on the average. Consequently, changes in the capital stock this year are consequently decisions made sone years ago, decisions that are not apt to be influenced by the size of the cur. rent exror termo: Mundlak supports this point of viev even for capitalist economy by arguing that in a model using annual data, capital may be treated as a fixed factor. 15

Equation (1.6) gives the essential stochastic specifications:

$$
\text { (1.6) } \quad E_{i r t s}=H_{i r t} U_{i r t s}
$$

The error term $E$ is composed of two statistically independent components: the first term, $H$, measures those perturbations which are common to firms of all sizes, but which vary from year to year; and the second term, $U$, measures those perturbations which differ both from year to year, and from firm to firm. If the two variables $H$ and $U$ are uncorrelated with the inputs $K$ and $L$, then estimates of alpha and beta are unbiased estimates of the theoretical concepts which ve seek to measure. However, correlations between either of the two stochastic components and the inputs cause a biased parameter estimate. We shall call correlation between the inputs and $H$ "temporal bias," and correlation

[^2]between the inputs and $U$ "simultaneous equation bias." We next give a brief description of the theory underlying the various estimators used.

Change notation so that upper case letters denote natural logarithms, temporarily suppress the industry and region subscripts, and consider the relationship (1.4) and (1.6). We then have the following equations corresponding to (1.4) and (1.6):

$$
\begin{aligned}
& \text { (1.4a) } Y_{t s}=a_{t s}+\alpha K_{t s}+\beta K_{t s} \\
& \text { (1.6a) } E_{t s}=H_{t}+U_{t s}
\end{aligned}
$$

Temporal bias, the $H_{t}$ effect, may be eliminated by using "covariance estimates." ${ }^{16}$ A straightforward application of the covariance technique involves defining dummy time variables and estimating their coefficients which are unbiased estimates of $\bar{H}_{t}$. If one is not interested in knowing the values of $H_{t}$, but only in obtaining unbiased estimates of $\alpha$ and $B$, the same result may be obtained by defining the six variables of (1.4a) and (1.6a) as deviations from their annual means. Denoting annual deviates by lower case letters, we have, for example,

$$
y_{t s}=Y_{t s}-Y_{t},
$$

where $X_{t}$ is a simple average taken over the 12 size categories. If we use the annual deviates

$$
k_{t s}, \quad e_{t s} \text { and } y_{t s}
$$

in (1.4a), then $h_{t}$ is eliminated from (1.6a) and $e_{t s}$ equals $u_{t s}{ }^{17}$
This transformation, however, still does not remove the simultaneous equation bias which may be present if there is correlation between either $k_{t s}$

[^3]or $\ell_{t s}$ and $u_{t s .}$. Given our inability to specify a simultaneous equation model, we instead use the single equation techniques of grouping and instrumental variables to ameliorate this effect. The consequences of grouping firms in the cross section data according to the size of employments or fixed assets is discussed later in Section II. The technique of instrumental variables and its derivatives is discussed next.

The instrumental variables used are the lagged values of the independent variables $k_{t-1, s}$ and $l_{t-1, s}$. The standard technique is treated in any of the textbooks on econometrics and needs no description here. In addition to the standard estimator, however, we also use a hybrid proposed by Mund lak ${ }^{18}$ which requires some explanation. The Mundlak estimator is a combination of three estimators: the ordinary least squares estimator obtained from (1.4a) and (1.6a), denoted by ( $\bar{\alpha}, \bar{\beta}$ ); the covariance estimator denoted by ( $\hat{\alpha}, \hat{\beta}$ ) and the instrumental variable estimator obtained by using $K_{t-1, s}$ and $L_{t-1, s}$ as instruments for $K_{t s}$ and $L_{t s}$, and denoted by ( $\tilde{\alpha}, \tilde{B}$ ). ${ }^{19}$

Defining the covariance matrix of the independent variables for the estimators by $\bar{A}, \hat{A}$ and $\tilde{A}$, we have:

$$
\begin{aligned}
& \bar{A}=\left[\begin{array}{l}
k \\
L
\end{array}\right](K, L), \\
& \hat{\mathbf{A}}=\left[\begin{array}{l}
k \\
X^{\prime}
\end{array} \quad(k, l)\right. \\
& \tilde{A}=\left[\begin{array}{l}
K^{\prime}-1 \\
L^{\prime} \\
-1
\end{array}\right](K, L) .
\end{aligned}
$$

${ }^{18}$ Ibid., PP. $160-163$.
${ }^{19}$ If one is willing to concede our argument that no correlation exists between capital and the error term, then only labor need be used as an instrument. Estimators using only one instrumental variable, labor, are called Type 1; estimators using two are called Type 2.

between the inputs and $U$ "simultaneous equation bias." We next give a brief description of the theory underlying the various estimators used.

Change notation so that upper case letters denote natural logarithms, temporarily suppress the industry and region subscripts, and consider the relationship (1.4) and (1.6). We then have the following equations corresponding to (1.4) and (1.6):

$$
\begin{aligned}
& \text { (1.4a) } Y_{t s}=a_{t s}+\alpha K_{t s}+B K_{t s} \\
& \text { (1.6a) } E_{t s}=H_{t}+U_{t s}
\end{aligned}
$$

Temporal bias, the $H_{t}$ effect, may be eliminated by using "covariance estimates. "16 A straightforward application of the covariance technique involves defining dumny time variables and estimating their coefficients which are unbiased estimates of $\vec{H}_{t}$. If one is not interested in knowing the values of $H_{t}$, but only in obtaining unviased estimates, of $\alpha$ and $\beta$, the same result may be obtained by defining the six variables of (1.4a) and (1.6a) as deviations from their annual means. Denoting annual deviates by lower case letters, we have, for example,

$$
\dot{Y}_{t s}=Y_{t s}-Y_{t},
$$

where $Y_{t}$ is a simple average taken over the 12 size categories. If we use the annual deviates

$$
k_{t s}, \quad \ell_{t s} \text { and } y_{t s}
$$

in (1.4a), then $h_{t}$ is eliminated from (1.6a) and $e_{t s}$ equals $u_{t s}{ }^{17}$
This transformation, however, still does not remove the simultaneous equation bias which may be present if there is correlation between either kts

[^4]-25-
where $\mathrm{Y}, \mathrm{K}, \mathrm{L}, \mathrm{k}, \ell, \mathrm{K}_{-1}, \mathrm{~L}_{-1}$, are $\mathrm{N} \times 1$ vectors of observation. The ores: podding least squares parameter estimates are then
\[

$$
\begin{aligned}
& {\left[\begin{array}{l}
\bar{\alpha} \\
\bar{\beta}
\end{array}\right]=\bar{A}^{-1}\left[\begin{array}{l}
K^{\prime} \\
\bar{L}^{\prime}
\end{array}\right] Y, \underline{\text { Staple Least Squares Estimator }}} \\
& {\left[\begin{array}{l}
\hat{\alpha} \\
\hat{\beta}
\end{array}\right]=\hat{A}^{-1}\left[\begin{array}{l}
k^{\prime} \\
\ell^{\prime}
\end{array}\right] \quad y, \quad \text { Covariance Estimator }} \\
& {\left[\begin{array}{c}
\tilde{\alpha} \\
\tilde{\beta}
\end{array}\right]=\tilde{A}^{-1}\left[\begin{array}{c}
K_{-}^{\prime} \\
L^{\prime} \\
-1
\end{array}\right] \quad \text { Y. } \frac{\text { Instrumental Variables Estimator }}{\frac{\text { type } 2}{} \text { (doth capital and labor used }}}
\end{aligned}
$$
\]

The Mundlak estimator $(\alpha, \beta)$ is defined by

Where

$$
\left[\begin{array}{l}
\hat{\alpha} \\
\hat{\hat{b}}
\end{array}\right]=\hat{A^{-1}}\left[\begin{array}{l}
K^{\prime}-K^{\prime}-K^{\prime}-1 \\
L^{\prime}-\ell^{\prime}-L^{\prime}-1
\end{array}\right] \text { Ye Mundlak Estimator, Type 2 }
$$

$$
\hat{\hat{A}}=\left[\begin{array}{c}
K^{\prime}-k^{\prime}-K^{\prime}-1 \\
L^{\prime}-\ell^{\prime-}-L^{\prime}-1
\end{array}\right] \quad\left(K-k-K_{-1}, L-\ell-L_{-1}\right)
$$

That is, the variables from (1.4a) and (1.6a) are corrected to remove both temporal and simultaneous equation bias, but they still utilize the full range of the original data, which is present in the simple least squares estimator.

Although not unbiased, the Mundlak estimators are consistent under the assumption of profit maximization if two conditions are satisfied: one is that temporal changes in the prices of capital or labor and output are not corelated with the time effects, il $_{i}$; and other is chat changes in $H_{t}$ over time are Independent of the level of $H_{t}$. Even if we grant profit maximization, can we really expect these two subsidiary conditions to hold? From sheer ignorance, agnosticism concerning the latter condition might be granted; however, the former conditions, particularly the presumed independence of the wage rate
and temporal effects, is not apt to be so easily obtained. One important contributor to $H_{t}$ for the cross-section data is change in price of outputs (nondeflated output data is used). It is difficult to be confident that in either an Illyrian or Capitalistic Economy changes in wages are independent of changes in the price of outputs. These uncertainties must raise doubts about the Mundlak Estimator, both Type 1 and Type 2. These estimators are nevertheless included because they promise to be more efficient than other estimators with comparable bias. A less biased, less efficient estimator is discussed next.

One method of eliminating temporal and simultaneous equation bias is to use the combined estimator ( $\alpha, \beta$ ) which we call a covariance/instrumental estimator and which is given by
where

While this estimator is unbiased, it loses efficiency because all the lower case variables, being mean deviates, have a smaller range of values than does the original data. The Mundlak estimator improves efficiency by utilizing the full range of the original data. With the exception of what we will call a Reverse Covariance Estimator (described below on page 33 ), we have now introduced all the candidates.

How does the econometrician choose? The basic choice is between blas and efficiency, but even that choice is complicated by the existence of alternative model specifications; most importantly, should capital be assumed independent of the error term. Our very crude procedure is first, in advance of

## 29

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computing the estimates, to rank the estimators according to their expected freedom from bias; second, define error measures that can be applied to the estimates to judge how well they meet other a priori conditions we impose; and third, search among the estimates to find one that has an acceptable combination of freedom from bias and error. It is to be expected that freedom from bias and freedom from error will be inversely related.

Prior to attempting a ranking of the estimators, according to freedom from bias both the simple least squares and instrumental variable estimators may be completely eliminated as unacceptable. These estimators do not eliminate the temporal bias, $H_{t}$. Since the cross section data is not price deflated, $H_{t}$ will introduce significant bias unless some form of covariance estimator is used. We suggest the following ranking of the remaining estimators as a rough indicator of their freedom from bias; if we assume capital and the error terms are not correlated,

A1. Reverse Covariance
A2. Covariance/Instrumental, Type 1
A3. Mundlak, Type 1
A4. Covariance;
and if we assume capital and the error term are correlated,
B1. Covariance/Instrumental, Type 2
B2. Mundlak, Type 2
B3. Covariance
B4. Reverse Covariance.
No extended defense of these lists is planned or possible. Note, however that i.t would be unadmissably inefficient to use Type 2 estimators under the A classification, and it would introduce inadmissable bias to use Type 1 estimators under the B classification. For reasons already explained covariance/

## $3^{\circ}$ <br> $-28$

instrumental is superior to Mundlak, and with some trepidation, we place covariance after Mundlak. The reason why reverse covariance dominates the $A$ clagelfication is explained later.

Having obtained a ranking on the criterion of minimum bias, we must :
next define measures that indicate the extent to which an estimator violates
the a priori side conditions we wish to impose. Violation of these side condifions may be taken as evidence that 10 w efficiency and resulting high sthadard errors are at fault, or simply that an unacceptable degree of bias If present. The weakest such condition is that parameter values be positive, sifghtly stronger is the condition that they be both positive and statistically sidnfficant. A simple count of both these conditions over the $2 \times 19$ paramefer estimates computed for each estimator provides the best measure. If one feyilling to assume profit maximization and perfect competition, it is also mepningful to compute a coefficient of variation for the marginal products of epch fnput for each estimator. High values of the coefficient of variation whid' be indicative of low efficiency in the estimator. We do compute co-品.
efficients of variation for two estimators, but more from curiosity than confiftion. In summary, we seek the estimator that promises minimum bias, and Whinh does not generate an unacceptable number of non-positive parameter eq自mates.

## PART II

CROSS-SECTION ESTIMATES OF LABOR AND CAPITAL<br>OUTPUT ELASTICITIES

## INTRODUCTION

Our first task is to use the 1963 and 1964 cross-section data to estimate output elasticities for capital and labor. The objective is to obtain from this data unbiased, or at least consistent, estimates of output elasticities which will later be used as extraneous estimators in the time series analysis. A general discussion of the statistical model has been given. However, peculiaxities of the grouped, cross-section data require modification of the estimators presented on pages 22 to 24 in order to increase efficiency. Toward that end consider equation (2.1):

$$
\text { (2.1) } \quad Y_{i t s}=a_{i t s}+\alpha_{i} K_{i t s}+\beta_{i} L_{i t s}+H_{i t}+U_{i t s}
$$

where $i=1 \ldots 19 ; t=1962,1963,1964 ; s=1 \ldots 12$. All of the variables axe described earlier, but note that no attempt is made to estimate technological progress in this model. The shift parameter ${ }_{i t s}$ includes the effects not only of technological change, but also of annual changes in the prices of output, and in the prices of increments to the capital stock. It is an assumption of the analysis that equal output prices prevail for all firms in an industry. Actually, a somewhat less strict condition is sufficient: the average output price for all firms in each size group is the same. A similar condition is assumed for the price of increments to the capital stock. Although there was an extensive re-valorization of fixed assets in 1962, the 1963 and 1964 investments are in current prices. We must, therefore, presume that changes in the price of investment goods between 1962 and 1964 do not importantly disturb the distribution of the capital stock which is correctly
measured for 1962. Also concerning the capital stock, it is presumed that the different size categories all have the same ratio for equipment to structures so that the average length of life of capital goods for the different categories is the same.

To give the reader a better feel for the data, Table 3 presents for the year 1964 a sample of the data which we have available for each of the 19 industry aggregates defined in Table 1. The paricicular industry used in Table 3 is the most aggregate one available--that for the total productive part of the social sector. The most notable feature of this data is that the same set of firms is available by two different groupings: one grouping according to the number of employees, and the other according to the value of fixed assets. (The Statistics are also available, grouped according to gross value added and net value added; however, as will shortly be demonstrated, this information is superfluous since we only need data grouped according to each of the independent variables of the analysis.) Another feature is that the data in the tables is a sumation over all the firms in each size category; therefore, in order to convert these observations into the per firm measures of equation (2.1), it is necessary to divide each column of variables by the number of firms in that category. Since the number of firms varies from category to category, efficient least squares estimation requires, regardless of which estimator we use, that the estimates should be based upon a weighted regression with the weights being the square root of the number of firms. ${ }^{20}$ Throughout the analysis of the cross-section data, the square root of the number of firms is used as a weight enless ocherwise opecified.
${ }^{20}$ Edmund Malinvaud, Statisti=A1 Methods of Econometrics, (Chicago: Rand McNally \& Co., 1966), p.p. 242-246.

The existence of four sets of data according to four different grouping variables for the same industry and year presents at first glance a difficult decision--which grouping should be used. Fortunately, this question has been extensivil 1 by Yoel Haitousky 21 Haitovsky shows that when separate groupings are available by each of the independent variables it is more efficient to compute an estimate using all of the tables than to rely upon any one of them. This conbined regression can be described in the following way: compute mis-specified, separate regressions of the dependent variable on each one of the independent variables separately, using only the table of data grouped according to that independent variable; then combine these mis-specified regressions with correction terms that remove the bias caused by the mis-specifications. Although it is not our intention to reproduce all of Haitovsky's derivation, it is necessary to out line his methods since we extend his work to include instrumental variables, Mundlak reverse, and covariance estimators.

Consider the simplified version of our regression problem given by equation (2.2). Lower case letters indicate that all variables are annual mean deviates so that there is no intercept term, we also assume that $\varepsilon$ is independent of both of the inputs. Instead of first selecting one set of grouped data for fitting equation (2.2), we fit the two separate mis-specified regressions given by (2.3). The first equation of (2.3) is fitted to the data from the capital grouping only; henceforth we refer to this as grouping 1 ; and the second equation is fitted to the data from the employment grouping only;

-     - $\quad{ }^{1} 1_{\text {Yoel }}$ Haitovsky, "Unbiased Multiple Regression Coefficient; Estimated from One Way Classification Tables When the Cross Classifications are Unknown," The Journal of the American Statistical Association, Sept. 1966, Vol. 61, No. 315, pp. 720-728. This article is a revised version of Chapter 1 of the author's Ph.D. thesis presented to the Department of Economics, Harvard University.
henceforth grouping 2. Denoting the mis-specified estimates by bars, their least squares formula is given by (2.4). 22

Taking the expectations of ( $\overline{\alpha \beta}$ ), we discover that they equal the unbiased estimates of the correctly specified covariance model (2.2), which we denote by $(\hat{\alpha} \hat{\beta}$, plus an error bias term. This is expressed in (2.5). We may now substitute (2.4) into (2.5) and solve for the vector of unbiased estimates, thereby obtaining (2,6). Haitovsky obtains the variances of ( $\hat{\alpha}, \hat{\beta}$ ) in a similar manner.

A simple extension of this procedure obtains instrumental variable estimators. In the case under consideration we use lagged values of capital and labor as instruments. If we denote the unbiased instrumental variable estimates corresponding to equation (2.1) by ( $\tilde{\alpha}, \tilde{\beta}$ ) we have (2.7). The Mund lak estimator is obtained in a similar way, denoted by $(\hat{\hat{\alpha}}, \hat{\hat{\beta}}$ ) and presented in : equation (2.8).

The reverse covariance estimator must still be defined. We do so by simply changing the table subscripts in equation (2.6). This means, in terms of (2.4), that we estimate the sapital coefficient from the labor table, and the labor coefficient from the capital table. The reverse covariance estimator is obviously less efficient than the ordinary covariance estimator, but might it be less biased?

To answer this let ( $\alpha \%$, $\beta *$ ) denote the reverse covariance estimator. Our earlier ranking of estimators implied that reverse covariance is most biasfree if it is assumed that capital and the error term are not correlated, while labor and the error term are correlated. To prove this assertion,

22 In these formulas, the 1 or the 2 after the summation sign $\varepsilon$ indicates the Table, or equivalently, grouping basis, that is to be used in the summation. Thus we see that $\bar{a}$ is estimated solely from the data according to the first grouping, the capital basis, while $\bar{\beta}$ is estimated solely from the data according to the labor grouping.
calculate the expected value of the mis-specified regressions for both the ordinary and reverse covariance estimators. This is done in equation (2.9) where ( $\bar{\alpha}, \bar{\beta}$ ) is the mis-specified ordinary covariance estimator, and ( $\bar{\alpha} \bar{*}, \bar{\beta} \bar{x}$ ) is its reverse covariance counterpart.
(2.2) $y_{s}=\alpha k_{s}+\beta \ell_{s}+E_{s}$
(2.3) $\quad y_{s}=\bar{\alpha} k_{s}+E_{1 s}$
$\mathbf{y}_{s}=\bar{\beta}_{s}{ }_{s}+E_{2 s}$
(2.4) $\bar{\alpha}=\frac{\Sigma_{1} v k}{\Sigma_{1} k^{2}}$
$\bar{B}=\frac{\Sigma_{2} y \ell}{\Sigma_{2} \ell^{2}}$
(2.5) $\bar{\alpha}=\hat{\alpha}+\hat{\beta} \frac{\Sigma_{1} k \ell}{\Sigma_{1} k^{2}}$
$\bar{B}=\hat{\alpha} \frac{\Sigma_{2} k \ell}{\Sigma_{2} \ell^{2}}+\hat{B}$

(2.8) $\left[\begin{array}{c}\hat{\hat{a}} \\ \hat{\hat{B}} \\ \hat{\theta}\end{array}\right]\left[\begin{array}{c}\Sigma_{1}\left(K^{2}-\mathrm{k}^{2}-\mathrm{KK}-1\right) \\ \Sigma_{7}\left(K L-k \ell-K L_{-1}\right)\end{array}\right]\left[\begin{array}{l}\Sigma_{1}(L K-\ell l-L K-1) \\ \Sigma_{2}\left(L^{2}-\ell^{2}-L L_{-1}\right)\end{array}\right]\left[\begin{array}{c}\Sigma_{1}(Y K-y k-Y K-1) \\ \Sigma_{2}\left(Y L-v \ell-Y L_{-1}\right.\end{array}\right] \frac{\frac{\text { Mundlak }}{\text { EStirat }}}{\frac{\text { Type } 2}{2}}$

$$
\begin{align*}
& \begin{array}{r}
36 \\
-35-
\end{array} \\
& \begin{array}{c}
-36 \\
E(\bar{\alpha})=\alpha+\beta \frac{-25-}{\Sigma_{1} k \ell} \\
\Sigma_{1} k^{2} \\
E\left(\frac{\Sigma_{1} k F}{\Sigma_{1} k^{2}}\right)
\end{array}  \tag{2.9}\\
& E(\beta)=\alpha \frac{\Sigma_{2} \text { kl }}{\Sigma_{2} \ell^{2}}+\beta+E\left(\frac{\Sigma_{2} \ell E}{\Sigma_{2} \ell^{2}}\right) \\
& E\left(\alpha^{*}\right)=\alpha+\beta \frac{\Sigma_{2} k \ell}{\Sigma_{2} k^{2}} E\left(\frac{\Sigma_{2} k E}{\Sigma_{2} k^{2}}\right) \\
& E\left(\beta^{*}\right)=\alpha \frac{\Sigma_{1} k \ell}{\Sigma_{1} \ell^{2}}+\beta+\sigma^{\Sigma_{1} \ell E}\left(\frac{\Sigma_{1} \ell^{2}}{{ }^{2}}\right)
\end{align*}
$$

If ne assume that capital and the error term are not correlated but that labor and the error term are correlated, this gives
and

$$
\begin{aligned}
\quad E\left(\Sigma_{1} k \varepsilon\right) & =E\left(\Sigma_{2} k \varepsilon\right)=0, \\
\text { and } \quad E\left(\Sigma_{2} l \varepsilon\right) & \neq 0 .
\end{aligned}
$$

But what about $\mathrm{F}\left(\Sigma_{2} \ell \varepsilon\right)$ ? While it might seem that the presumed correlation between $l$ and $\varepsilon$ would make $E\left(\Sigma_{1} \ell \varepsilon\right) \neq 0$, this is not correct. Then using grouped data, if the grouping variable is itself independent of the error term, it may serve as an instrument to purge any other variables in that table of correlacion with $\varepsilon .{ }^{23}$ Immediately we see that all variables in the cantal table, Table 1, axe free of such correlation, and particularly $E\left(\Sigma_{I} \ell \varepsilon\right)=0$. This means that under the assumptions

$$
\begin{aligned}
& \mathrm{E}(\mathrm{l} \varepsilon)=0 \\
& \mathrm{E}(\ell \varepsilon) \neq 0,
\end{aligned}
$$

the covariance estimator (2. 6 ) is subject to simultaneous equation bias,
${ }^{23}$ See the discussion by Malfnvaud, op. cit., pD. 242-246.
but the corresponding reverse covariance estimator obtained by reversing the table subscripts is free of bias. This is why the reverse covariance estimator heads the A ranking of estimators. Of course, the reverse covariance estimator is less efficient. ${ }^{24}$

## COMPERIS ON OF THE CROSS-SECTION ESTIMATES

We begin our inspection in Table 4. by looking at estimates computed for only two sectors of the economy: the total social sector, and industry and mining. These sectors are the largest in the economy and both are aggregates of other branches whose parameters are estimated. Restricting attention to these two sectors enables us to focus on the sensitivity of the estimates to several sources of variation, specifically: variations in the regression weights; variation in the years for which the regression is run; and variation in the number of cells in the different size groupings.

While certain elements of Table 4 are not available because of lack of data, other elements are purposely omitted because, at an early state it became apparent that some variants were so ill-behaved that they would not be contenders for ultimate selections. Consequently, limited resources forced their exclusion. For example, Part B of the Table which uses the number of firms as weights in the regressions has a number of empty cells because the arguments in favor of square root of the number of firms as weights made it clear that the latter would finally be selected. Our inclusion here of the number of firms as weights is done to test the sensitivity of the results to

[^5]


## -40-

Consider first not the two input coefficients, but their sum, the scale coefficient. As would be expected, the scale coefficient shows greater stability than either of its components, $\alpha$ or $\beta$. Generally, all of the results from the 12 -cell data show returns to scale very close to unity. Excluding Section $C$, the range of the scale coefficient fo: both industry and mining and the total social sector is from .99 to 1.04 with a median value around 1.01 or 1.02. These values are not statistically significantly different from unity to allow rejection of the hypothesis of constant returns to scale. In none of the results, however, is the scale ccefficient forced to be unity; the presence of high multi-collinearity can cause this specification to explosively affect the estimates of the capital and labor coefficients. It is interesting that when square root weights are used, the 9 -celi data consistently gives lower estimates of the scale coefficients. The difference in each case is exactly 3 percentage points. A much greater difference in the scale cosfficients is found in the 9 -cell, 1963-67 regional data using square root weights (Part C). Comparing this data with the 9 -cell estimates from Section A, there is again a consistent difference, this time of 4 percentage points. We do not know why the 1963-67 data shows an important indication of decreasing returns to scale with a value of 95 but we would speculate that since tinis time period straddles the 1965 price reform it is possible that the rather dramatic changes in prices which occurred during that reform affected the large firms, which were under closer government surveillance, more negatively than it affected the small firms. If this is actually the cose, it would explain the dramatic shift to decreasing returns to scale which is brought about by including the post-reform years. In any event, the signifjcant alteration of the scale coefficient which occurs when we add these years validates cur restricting attention to
only the pre-reform years, thus assuring a more homogeneous sample with respect to prices, institutions, and behavior.

The labor coefficient estimates are in the high . 80's for all of the 12-cell data for either the total social sector of industry and mining. For the 9 -cell data, however, it is substantially less, somewhere in the low. $80^{\prime} \mathrm{s}$. Correspondingly, the capital coefficient, $\alpha$, tends to lie in the low teens for the 12-cell data, and in the high teens for.the 9-cell data. In Section $C$, the two capital coefficients according to the Mundlak estimators are slightly negative. The magnitude of these negative values suggests violation of the Mundlak assumptions in the longer time period rather, than a distortion due to sampling. We now turn to a consideration of parameter sensitivity from the point of view of the estimators rather than the data sample.

Except for the Mundlak estimators whose variance is not known and for which two coefficients are negative, the other estimators all generate coefficients that are statistically significant and positive. In order to establish the importance or unimportance of the correlation between capital and the error term, we contrast the Type 1 and Type 2 estimates for the covariancel instrumental and Mund lak estimators. For these two estimators, the use of both capital and labor as instruments reduces the capital coefficient and raises the labor coefficient by from 1 to 4 points. This is a very consistent result. However, it should not be interpreted to mean that the.introduction of capital as an instrumental variable has removed any significant bias, rather it is more likely that the consistent change of the parameters by a few points is due simply to the less-than-perfect correlation which exists between lagged capital and current capital. This causes labor to have a relatively more improved correlation with output than does capital. In any event, the differences are not large so that by selecting the Type 1 estimators we risk little.

At this point, along with the Type 2 estimators, we also discard the Mundlak estimators. The presence of the two negative capital ccefficients indicates that the assumptions of that estinator are not mat. If we compare the covariance/instrumental Type 1 estimators with either the ordinary covariance or reverse covariance estimators, we find that the former sesms to yield a higher capital coefficient estimate and a lower labor estimate. Here again, this result can be explained by the jess-than-perfect correlation which exists between lagged labor and current labor. This wculd cause the laboi coefficient for the covariance/instrumental, Type 1 estimator to be smaller than that for either of the covariance estimators.

The most intezestirg comparison is between the covariance and the reverse covariance estimators. Under our prefarred assumption that capital and the error term are not correlated, the reverse covariance estimator offers the best available means of removing bias caused by a correlation between labor and the error term. The reverse covariance estimator is superior in this respect to instrumental variable estimatcrs bacause the latier cannt remove such correla-, tions if the errors affecting the variables are associated :hrough time. inerefore, a comparison of the covariance and the reverse covariance estimators provides our best method for juagirg the importance of the bias generated by a possible correlation between labor and the ersor term. The result is surprising.

There are four blosks of data for which the two estimators may be compared. For these four blocks, none of the paranter estinates differs by more than one percentage point, sigrifying fiăt virtually icentical results are achieved whether we usc revers? covariance or covariance estimators. The conclusion must be that simultańcus equation bias resulting from a correlation between labor and tie frro term does not exist, at least not aner the assumptions of the moder. This also means that there is no reason fef further
considering the instrumental/covariance Type 1 estimators. The final comparison must be between reverse covariance, which has minimum bias, and ordinary covariance, which gives the same estimates for aggregate sectors but is more efficient. To select between these two we compare results for all nineteen sectors and five regions. First, however, a one-paragraph summary is given of the findings to this point.

The greatest economic import of Table 3 attaches to the consistency with which we find returns to scale of approzimately unity. Typical values of the capital and labor coefficients are . 15 and .85. This coatrasts significantly with the .25 and .75 values that are typically asserted for western economies. Of course, this has little real meaning until we examine the marginal products and income share in Yugoslavia. The greatest statistical import of Table 3 is that the estimates are quite stable for the six estimators we try, and also for the various data samples used. The largest change in estimates occurs when we go from the 12-cell data to the 9 -cell data which implies that consolidation of the extremes of the data may be dangerous. The similar results given by all the estimators, but particularly the nearly identical results for the ordinary and reverse covariance estimators is evidence that simultaneous equation bias is not important.

So far we have estailished thst the reverse covariance estimator is apt to be most bias-free, but that in practice, for the large aggregate sectors, there is almost no difference in the cstimates for reverse covariance and ordinary covariance. Since the ordjnary covariance estimators are more efficient they would seem to be supcrior. Estimates for the nineteen sectors confirm this judgment, Table 5 presents the capital, labor and seale coefficients for three estimators; ordinary covariance; reverse covariance; and covariance/ instrumental, Type 1 . In those cases where an estimators is not significantly

TABLE 5

positive at a .95 confidence level, the standard error of that coefficient is presented in parentheses. For the ordinary covariance estimator there is no coefficient in this table that is either negative or not significantly positive. In contrast, the reverse covariance estimator exhibits two negative values and four insignificantly positive values, while the covariance/instrumental, Type 1 estimator shows one negative value and one insignificantly positive value. One explanation of this is found in the standard errors of the coefficients. ${ }^{25}$ Typically, the standard errors for ordinary covariance are two-thirds to onehalf those for reverse covariance or instrumental/covariance.

In other regards, the conclusions of Table 4 hold for the disaggregate sectors of Table 5. Returns to scale are not importantly different from unity, although a number of the sub-branches of industry do show increasing returns to scale, particularly food, drink and tobacco (113), and metal making and using (120 and 121). The capital coefficient is açain in the teens, although the high teens rather than the low teens seem to be more characteristic. And the labor coefficient is generally in the high 30 's. Two industries show significant decreasing returns to scale: construction (003) and the miscellaneous sub-branch of industry (122). In both these cases, there are special circumstances at work and better estimates, described later, are presented in bold type.

The same data for industry and mining, but covering the five regions and presented in Table 6, shows similar results in all respects, except there are no negative or insignificantly positive values for either ordinary covariance or covariance/instrumental estimates. There is one negative and insignificantly
${ }^{25}$ Tables for standard errors are not presented because the paper is already overburdened with statistical measures.

## REGIONAL ELASTICITY ESTIMATES

FOR INDUSTRY AND MINING*

| Region | Covariance/Instrumental, |  |  | 0 | Cova |  | Reverse Covariance |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\alpha$. | ${ }_{\beta}^{\text {ype }} 1$ | $\alpha+3$ | $\alpha$ | B | $\alpha \times \beta$ | $\alpha$ | B | $\alpha \neq \beta$ |
|  | Year from 1963 to 1964. |  |  |  |  |  |  |  |  |
| Yugosiavia | . 19 | . 80 | . 9 90 | . 15 | . 84 | . 99 | . 16 | . 83 | . 99 |
| North | 17 | . 86 | 1.03 | . 10 | . 93 | 1.03 | . 08 | .96 | 1.04 |
|  | . 17 | . 86 |  |  |  |  |  |  |  |
| South | . 14 | . 80 | . 94 | . 14 | . 81 | . 95 | . 31 | . 62 | . 93 |
| Serbia Eroper <br> South less Serbia Proper | . 28 | . 65 | . 93 | . 28 : | . 64 | . 92 | . 27 | . 66 | . 93 |
|  | . 11 | .92 | 1.03 | . 03 | 1.01 | 1.04 | . 05 | . 99 | 1.04 |
|  |  |  | Ye | m 19 | 1967 |  |  | ; |  |
| Yugoslavia | . 14 | . 81 | . 95 | . 10 | . 85 | . 95 | . 11 | . 84 | . 95 |
| North | . 07 | . 92 | . 99 | . 03 | . 96 | . 99 | $\begin{aligned} & -.03 \\ & <.036 \end{aligned}$ | 1.05 | 1.02 |
| South | . 15 | . 78 | . 93 | . 11 | . 83 | . 94 | . 16 | . 76 | . 92 |
| Serbia Proper | . 18 | . 73 | . 91 | . 15 | . 76 | . 91 | . 15 | . 76 | . 31 |
| South less Serbia Proper | . 19 | . 79 | . 98 | . 14 | . 84 |  | . 15 | .82- | . 97 |

*All coefficients are significantly positive at a confidence level of .95. The covariance estimates for South less Serbia Froper for 1963-64, and the North for 1963-67, fail at the .975 level, however.
positive value for the reverse covariance estimator. The scale, capital, and labor coefficients, all satisfy reasonably well the standardized description given above. A surprising feature of Table 6 is that for 1963-64, Serbia proper has a very low measure for the labor coefficient and for returns to scale. The statistics for Serbia proper do not look so anamolous in. the longer 1963-1967 period both because the scale coefficient for all the other republics except Serbia Proper falls by 5 percentage points, and the Serbia proper capital coefficient loses 13 points while the labor coefficient gains 12 points. The outcome is that for the longer time period Serbia Proper is not so distinctly different from the other regions as it is for the $1963-64$ period. The reason for this is not nown.

In a pareto optimal economy the marginal products of labor and capital over sectors of the economy and regions are equal. A serious empirical application of this criterion involves many qualifications and modifications; nevertheless, a straightforward, naive comparison is not without merit. At the very least it can be an important indicator of unreasonable results. Table 7 presents the marginal products of capital and labor for the ordinary covariance estimator, and by way of contrast for the covariance/instrumental estimator. Contrasting the two aggregates, the total social sector and industry and mining, we find a good deal more difference can be attributed to the sectoral classification than to the estimator used. For both estimators, the marginal product of capital is significantly greater for the total social sector than it is for industry and mining, while just the reverse is true of the marginal product of labor. Since the control of investments is the strongest instrument in the hands of central policy-makers, this result is consistent with the idea that industry and mining is a priority sector whose growth is made possible by the

T:B:シ 7
SECTOPAL MARGINAL PRODUCT ESTIMATES
FOR 1963-64:

f:Marginal Products are computed at the weighted geometric mean. The weights are the square root of the number of firns per cell.
tComputed from the 17 sectors 002 to 122 by the formula $\dot{V}=100 S / \bar{X}$ where $S$ is the sample standard deviation and $\bar{X}$ is the sample mean.
infusion of large amounts of capital, so much capital that the rate of return is driven below what is available in other cectors. Later, in the section dealing with aggregation problems, the marginal product of apital for both of these two sectors is shown to ie biased downard by the process of linear aggregation.

While significance statements aise not available for the marginal products, a coefficient of variation can be used to macure che variability of the two estimators for the 17 disaggregate aectors. With a valce of 34.2 the coefficient of variation for the marginal pacduci of labor is icentical for ordinary covariance and covarianceifinstrmantal, but the coefficient of variation for the marginal products of capital is smaller for ondinary covariance, 61.4, than for covariance/instrumental, 79n6.

- Similar data is given in Table 0 for regional marginal products. Again, the regional classification is a much mace important de product than is the escimator. Another conclusion is that the marginal product of capital is lower in the North thon in the South, while the converse is true for the marginal product of labor. For the marginal product of labor this is to be expected due to the immobility of labor. Zor the marginal product of capital, however, expectations aze not so clonz sut, or the one hand, greater efficiency in the North causes average output per unit of capical to be high, which raises marginal pzoductivity; on the ocher hand, capital deepening has progressed further in the North-the capital/labor ratio is one-third larger than in the South-and this lowers marinal produstivity, The fact that the measured product is lower for the North suggests that capital deepening has been carried beyond what is optimal. ${ }^{26}$ This conslusion is reversed in the
${ }^{26}$ This conclusion conflices with that of D:. James Plummer who finds that capital is used mose effiriently in the ! !orth than in the South. Our study agrees with his in concinding that some reallocation of labor from South to North would be desirable. James Plummin, "Intcrifirm Production Function Analysis of Yugos lav Industrial nesource Alloaation," mineograph, Dac. 1969, p. 7.

Thale


REGIOMAL HAEGLHAL PRODUCT ESTIMATES FOR INDUSTRY AND MINING

| $\because$ | Covariance/Instr Type | menta | Covariance |  |
| :---: | :---: | :---: | :---: | :---: |
| Region | MPK |  | MPK | MPL |
| Ever - . - | Years from 1963 to 1964 |  |  |  |
| Yugoslavia | :14 | 1.21 | . 11 | 1.27 |
| - North | . 13 | 1.37 | . 08 | 1.47 |
| South | . 14 | 1.13 | .13 | 1.14 |
| - Serbia Proper | . 24 | . .94 | . 25 | . 94 |
| South less Serbia Proper | . 07 | 1.24 | . 02 | 1.35 |
|  | Years from 1963 to 1967 |  |  | -. |
| Yugoslavia | . 12 | 1.61 | . 08 | 1.69 |
| North | . 06 | 1.90 | . 02 | 1.99 |
| South | .13 | 1.44 | . 10 | . 1.53 |
| Serbia Proper | . 17 | 1.43 | . 15 | 1.49 |
| - South less | - . |  |  |  |
| $\therefore$ Serbia Proper |  | 1.34 | . 10 | 1.43 |

1963-67 data, but this appears related to the price reforms of 1965.
The really anomolous aspect of Table 9 is the large marginal product of capital for Serbia Proper generated by the 1963-64 data. More than the elasticity measures, the marginal products indicate that this is due to unknown aberrations in the $1963-64$ data. The longer 1963-67 period shows values for Sexbia Proper that are more in line with our expectations. If the regressions were run only on the $1965-67$ sub-sample, the results for Serbia Proper would be substantially closer to those for Yugos lavia as a whole. This leads to the conclusion that the marginal product of capital is low in the North and high in the South, while the converse is true of the marginal product of labor; and that the marginal product of capital and labor are about the same in Serbia Proper and the far South. Again, differences between the 1963-64 and 1963-67 results, weaken such conclusions.

## PROBLEMS OF AGGREGATION

The use of several estimators and different data samples increases confidence in the stability of the findings. Similarly, disaggregation by economic sectors and regions can be viewed as a replication of the experiment, a replication that also increases confidence in the stability of the estimates and confirms the existence of a relatively small capital coefficient and returns to scale near unity. This replication by disaggregation, however, burdens us with two issues not yet considered. First, in the time series analysis that follows, great simplification could be achieved if the capital and labor coefficients for any industry were the same for all regions. This hypothesis is easily confirmed or rejected by a "t-test" on the regional differences of the estimates for industry and mining. Second, for industry and mining and
for the total social sector there are estimates for both the aggregates and their sub-aggregate components. This raises the question of whether or not the aggregate coefficients for capital or labor are unbiased functions of the subaggregate coefficients. If they are not, the difference is called "aggregation bias." ${ }^{27}$ We begin with the simpler issue mentioned first, the hypothesis of regional equality.

For industry and mining the nine-cell, regionally disaggregate data may be used to test the hypothesis of regional equality. This is an important and convenient hypothesis, and one that is at times forced upon us. From Table 6, the maximum difference (covariance estimator, 1963-64 data) for the capital coefficient is .25 obtained as the difference between $\hat{\alpha}_{4}=.28$ and $\hat{\alpha}_{5}=.03$. For the labor coefficient, the maximum difference is obtained for the same category and is .37. Assuming the statistical independence of parameters estimated for different regions, the standard errors are:

and
$\mathbf{S}_{\boldsymbol{\beta}_{4}}+\hat{\beta}_{5}=.031$
The respective "t-statistics" for capital and labor are 15.7 and 11.8. These values are so large we may be assured that a significant difference exists regardless of the problems of multiple comparisons and of serial correlations of the errors which overstate these "t-statistics". (The assumed independence of parameters may understate it.) Even the smaller differences that exist when we compare the North with the South, still generate "t-statistics" of 2.5

27 Our discussion of aggregation bias follows R.G.D. Allen, Mathematical
Economics (New York: St. Martin's Press, 1957), pp. 694-724.
for capital and 3.7 for labor. With 30 degrees of freedom ${ }^{28}$ the critical Iimits are 2.36 for a significance level of . 025 , and 2.75 for a significance level of . 01. Thus, even the minimum differences tend to be significant. The hypothesis of a regional constancy in the coefficients must be rejected. We next test for aggregation bias.

Table 9 provides a comparison of two estimates of the output elasticities for the total social sector, and industry and mining: the first $(\hat{\alpha}, \hat{\beta})$, is the covariance estimate from Table 3 ; the second $(\bar{\alpha}, \bar{\beta})$, is obtained as a weighted ${ }^{29}$ sum of the sub-aggregates components of the two above sectors, also according to the covariance estimator. Since we reject the hypothesis of regional equality, we may also meaningfully compute the same statistics according to the threeregion disaggregation (only for industry and mining, of course). What do these differences show? For the sectoral aggregation, the capital coefficients are importantly smaller by about twenty-five per cent for the "Direct Regression" in comparison to the "Weighted Sum"; and the labor coefficients are only slightly larger for the total social sector by about five percent. The same comparison for the regional aggregate shows the capital cofficient slightly larger for the "direct regression" than for the "weighted sum," and the labor coefficient slightly smaller. What economic interpretation may be given to these differences?

To give an economic interpretation to the difference between the linear estimates $(\hat{\alpha}, \hat{\beta})$ and the geometric estimates $(\bar{\alpha}, \bar{\beta})$, we make the simplifying

28 The degrees of freedom are computed on the basis of 18 observations per table ( 9 cells for 2 years) and six parameters for both tables (capital and
labor, and four annual "shift" parameters, two per table). This gives $36-6=30$ labor, and four annu however, since the total numiver of firms is the same in both tables one cell is redundant so that the final outcome is $35-6=29$ degrees of freedom.
${ }^{29}$ The weights are the square roots of the average number of firms in the industry in any year. That is:

$$
\left(\begin{array}{ll}
T & N_{i t / T}
\end{array}\right) 1 / 2
$$



| $\begin{aligned} & \because \\ & \\ & \hline \end{aligned}$ | Direct <br> Regression |  |  | Weityted Sun of Sub-Aggregates |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 人 | $\hat{\beta}$ | $\hat{\alpha}+\hat{\beta}$ | $\bar{\square}$ | $\stackrel{\sim}{B}$ | $\bar{\alpha}+\bar{\beta}$ |
| Sectoral Aggregation (12-cell): |  |  |  |  |  |  |
| Tctal Social Sector (17 sub-aggregates) | . 13 | . 89 | 1.02 | . 17 | . 83 | 1.00 |
| Indus try and Mining <br> (12 sub-aggregates) | . 13 | . 89 | 1.02 | . 18 | . 87 | 1.05 |
| Regional Aggregation (9-cell) : |  |  |  |  |  |  |
| Indus try and Mining <br> . (3 sub-aggregates) | . 15 | . 84 | . 99 | . 13 | . 87 | 1.00 |

assumption of constant returns to scale $(\hat{\alpha}+\hat{\beta}=\bar{\alpha}+\bar{\beta}=1) .30$ On the basis of this assumption the production function may be expressed as,

$$
\begin{aligned}
& \text { (2.1) } \left.\quad Y_{i s}^{*}=\alpha_{i} k *_{i s}\right) \\
& \text { and } \quad \beta_{i}=1-\alpha_{i}, \\
& \text { where } Y_{i s}^{*}=Y_{i s}-l_{i s}, \\
& \text { and } \quad k_{i s}=k_{i s}-l_{i s}
\end{aligned}
$$

Consider the auxiliary regression.

$$
\begin{aligned}
& \text { (2.2) } k \%_{i s}=\delta_{i s} k *_{s}+\lambda_{i s} \\
& \text { where } k \%_{s}=\log \left(\Sigma_{i} K_{i s}\right)-\log \left(\Sigma_{i} L_{i s}\right), \lambda_{i s} \text { is a stochastic term, } \\
& \text { and } \alpha_{i s} \text { is a parameters. }
\end{aligned}
$$

Equation (2.2) expresses how the sub-aggregate capital/labor ratios are related to the aggregate capital/labor ratio for any size category.

The question we ask is, suppose (2.1) expresses the true micro-production function, what relationship will then exist between the $\alpha_{i}$ of that equation and an aggregate $\alpha$ obtained by first summing each variable over all sectoral sub-aggregates? That is, an $\alpha$ obtained from
(2.3) $y \psi_{s}=\sum_{i=1}^{I} y:_{i s}=\underset{i=1}{I} k *_{i s}+\varepsilon_{s}=\alpha k *_{s}+\varepsilon_{s}$

Substituting (2.2) into (2.1) and aggregating, we have


But (2.4) is of the same form as (2.3) so that a covariance estimator obtained from the former variables

$$
(2.5) \quad \hat{\alpha}=\sum_{i=1}^{I} \hat{\alpha}_{i} \hat{\delta}_{i s}
$$

${ }^{30}$ Since the statistical estimates of the scale coefficient for the total social sector and industry and mining differ from unity by only two percentage points, this specification is not arbitrary or misleading.

Furthermore, defining the "sum of sub-aggregates" estimate by

$$
\begin{aligned}
\bar{\alpha} & =\sum_{i=1}^{I} \quad \hat{\alpha}_{i} / I,
\end{aligned}
$$

we finally obtain

$$
(2.6) \quad \hat{\alpha}=\bar{\alpha} \quad \because I \operatorname{Cov}\left(\hat{c}_{i}, \hat{\delta}_{i s}\right)
$$

Equation (2.6) answers our original question. Where the "direct regression" estimate, $\hat{\alpha}$, is smaller than the "sum of sub-aggregates" estimate, ${ }^{31} \bar{\alpha}$, it implies that $\operatorname{Cov}\left(\hat{\alpha}_{i}, \hat{\delta}_{i s}\right)$ is negative. Cr, in more familiar terminology, it implies that industries with large capital coefficients have small capital/ labor ratios; and also the obverse, industries with large labor coefficients have large capital/labor coefficients. For the regional estimates, there is a tendency for the opposite results but the magnitude is too small to be important. These results have little meaning, however since it is differences in marginal products that govern the flow of resources.
\%.- As revealed in Table 10, the marginal products of labor (MPL) shows no important bias for either sectozal or regional aggregation, and the marginal product of capital (MPK) shows none for regional aggregation. There is, nevertheless, one important case of aggregation bias. For both the total social sector and industry and mining, the "direct regression" yields a MPK that is significantly lower than that produced by the "weighted sum." Application of the aggregation theory in the paragraphs above provides an explanation with economic import. The fact that $\hat{\alpha}$ is smaller than $\bar{\alpha}$ implies that there is a positive correlation between the marginal products and the capital/labor ratios of different industries--industries with high MPK's
${ }^{31}$ We use a weighted suman Table 8 to adjust for the fact that weighted $=$ regressions are used to obtain $\alpha$ and $\alpha_{1}$.


|  | Direct Regression MPK | MPL | Weig Sub- MPK | $\begin{gathered} \text { an of } \\ \text { ates } \\ \text { MPL } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Sectoral Aggregation (12-cell): |  |  |  |  |
| Total Social Sector (17 sub-aggregates) | . 15 | 1.15 | . 23 | 1.13 |
| Indus try and Mining <br> : (12 sub-aggregates) | . 10 | 1.34 | . 17 | 1.33 |
| Regional Agmegation (9-cell): |  |  |  |  |
| Industry and Mining . (3 subaggregates) | . 11 | 1.27 | . 11 | 1.29 |

: Marginal products are computed at the geometric mean of the crasisection data for 1963-64.
tend to have high capital/labor ratios. This is generally consistent with the view that profitability is an important criterion determining investment allocation in the Yugos lav economy.

## SECTION III

TIME SERIES ESTINATES OF NEUTRAL TECHNICAL PROGRESS:
1952 to 1964

The publicly available time series data is described in Section $I$. Before this information can be used for production function analysis, considerable effort must be expended in aggregation, deflation and so forth. So that we may come directly to the results, the description of the steps taken and methods used is relegated to an Appendix. The Appendix also contains a complete publication of the resultant statistical series for value added, employment, total fixed capital and equipment. These serics are presented for five regions and nineteen sectors for the years 1952 to 1966.

- The time series counterpart. of (2.4) is:
(3.1) $\mathrm{Y}_{\mathrm{irt}} \fallingdotseq a_{i r}+\alpha_{i r} K_{i r t}+\beta_{i r} L_{i r t}+\lambda_{i r t}+E_{i r t}$

$$
\text { where } \quad i=1 \ldots 19 \text { industries }
$$

$$
f=1 \ldots 5 \text { regions, and }
$$

$$
\therefore \quad \mathbf{T} \text { or } \mathrm{t}=1 \ldots 13 \text { years from } 1952 \text { to } 1964
$$

The variables $Y$, $K$ and $L$ are in logarithms, and $T$ is in natural integer units. To satisfactorily estimate the neutral technical progress coefficient $\lambda_{i r}$, it is necessary to make the assumption

$$
\begin{aligned}
& \text { (3.2) } \alpha_{i 1}=\alpha_{i 2} \ldots=\alpha_{i 5}=\bar{\alpha}_{i} \\
& \text { and } \quad \beta_{i 1}=\beta_{i 2} \ldots=\beta_{i 5}=\bar{\beta}_{i} \quad \text { for all } i,
\end{aligned}
$$

where $\bar{\alpha}_{i}$ and $\bar{\beta}_{i}$ are the ordinary covariance estimates obtained from Table 5.

To estimate $\lambda_{i r}$ we proceed in two steps: first, initial least squares estimates are computed for the coefficients of equation (3.1) without the benefit of the extraneous estimators utilized in assumption (3.2), and second, the capital and labor coefficients are restricted to the values prescribed by (3.2) and new estimates are computed for $a_{i r}$ and $\lambda_{i r}{ }^{0}{ }^{32}$.

The values of $\lambda(1)$ obtained in step 1 , and $\lambda(2)$ obtained in step 2, are found in Table 11. Results are presented only for Yugoslavia as a whole. These results strongly favor the $\lambda(2)$ coefficients which is based on the extraneous estimators and restricted regression. The large dispersion of $\lambda(1)$, even including negative values, occurs because the corresponding unrestricted estimates of $\alpha$ and $\beta$ are highly unstable (values that are negative or greater than 1.5 are common). The high multi-collinearity of the data together with varying amounts of underutilized capacity ${ }^{33}$ in both the capital and labor measures makes it impossible to estimate all three coefficients with only time series. The estimates for $\lambda$ (2) are much better. There are no negative values and the range, running 0.9 to 5.9 is not excessive.

Another test of the extraneous estimators is to compute how destructive assumption (3.2) is to the coefficient of multiple determination ( $\mathrm{R}^{2}$ ). A comparison of columns three and four of Table 11 reveals that only for agriculture (002) is there a large drop when the extraneous estimators are used:
${ }^{32}$ The same result is achieved by directly computing the single regression, $Y_{i r t}-\bar{\alpha}_{i} K_{i r t}-\bar{p}_{i} L_{i r t}=a_{i r}+\lambda_{i r} E+E_{i r t}$. This, however, would not permit a test of assumption (3.2). The technique of "restricted least squares" is described in Goldberger, op. cit., pp. 256-258.
${ }^{33}$ At this level of disaggregation there is little chance of calculating capacity utilization coefficients for capital, let alone labor. To our knowledge, no satisfactory data exists for making such computations, particularly in the early years.

$$
-60-
$$

T: $11^{\circ}$
SECTORAL ESTIMATES OF TECHIICAL PRCGRESS

|  |  | $\lambda(1)$ | $\lambda(2)$ | $\mathrm{R}^{2}(1)$ | $\mathrm{k}^{2}(2)$ | '7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total Social Sector | (000) | 2.1 | 3.8 | . 995 | . 991 | 3.93 |
|  |  |  |  |  |  |  |
|  | (001) | 8,9 | 4.5 | . 999 | . 997 | 5.10 |
| Industry \& Mining |  |  |  | . 979 | . 882 | 20.21 |
| Agriculture \& FishingConstruction | (002) | -8.4 | 4.3 | . 979 |  |  |
|  | (003) | 26.5 | 3.3 | . 884 | . 852 | 1.25 |
|  |  |  |  |  |  |  |
| :Transportation \& Communication | (004) | 7.5 | 5.0 | . 993 | . 984 | 4.85 |
|  |  |  | 2.1 | . 998 | . 981 | 31.00 |
| Handicraft $\therefore$ - | (005) | 9.3 | 2.1 |  |  |  |
|  | (006) | -1.2 | 1.6 | . 995 | . 989 | 6.03 |
|  |  |  | 5.2 | . 990 | 989 | 0.35 |
| Electricity | (111) | 7.2 | 5.2 | . |  |  |
| Coal $\&$ coal Mining | (112) | 5.6 | 4.2 | $=-.983$ | . 982 | 0.33 |
|  | (112) | 5.6 |  |  |  |  |
| Food, Drink \& Tobacoo | (113) | 12.1 | 0.9 | . 988 | . 942 | 16.49 |
| Textiles $\varepsilon$ clothing | (114) | -1.7 | 1.5 | -0.997 | . 995 | 2.18 |
|  |  |  | 4.2 | . 998 | . 987 | 26.50 |
| Timber \& Furniture | (115) | 2.0 |  |  |  |  |
| Paper, Printing \& Publishing. | (i16) | 10.6 | 3.8 | $=-.995$ | . 992 | . 71 |
| Leather, Rubber \& Footwear | (117) | 4.9 | 2.8 | . .994 | . 9.93 | 0.86 |
| Stone, Clay \& Glass | (118) | 10.2 | 4.2 | $\therefore .995$ | . 971 | 23.59 |
|  |  | 13.5 | $5: 8$ | -. 999 | :998 | 6.43 |
| Chemicals \& Petroleum | (119) |  |  | - 9 | 994 | 1.67 |
| Metal Using | (120) | 4.6 | 3.9 | . 996 | -994 |  |
| Metal Making | (121) | 6.8 | 5.9 | . 999 | . 993 | 27.94 |
|  | (122) | 12.6 | 2.1 | -. 985 | . 4.833 | 45.61 |

from . 970 to . 882. An $F$ test of (3.2) is made for each industry. ${ }^{34}$ A value of $\mathcal{F}$ greater than the critical limit $F .025=5.71$ causes a rejection at a . 025 significance level, of the hypothesis that (3.2) is a correct specification. For seven of the nineteen sectors wi.th $\mathcal{F}$ values over ten, the hypothesis expressed by (3.2) is strongly jected. For three others with values between five and six, acceptance or rejection is not clear cut. While a forceful acceptance of (3.2) is found for only one-half of the sectors, this is not a surprising or destructive outcome for the use of extraneous estimators. To the contrary, it is a rather strong outcome. As mentioned earlier, the unrestricted estimates contain many negative and ctherwise unacceptable coefficients. When comparison is made between the extraneous estimators and any set of "reasonable" output elasticities, the difference in the squared error is small. ${ }^{35}$ For this reason, we argce chat acceptance of (3.2) for one-half the sectors is a strong shouing.

The ultimate test of the extranecus estimator hypothesis, however, must be the reas onableness of the techaical progress coefficients they generate. Further evidence on this, in the form of ragional estimates, is found in Table 12. For Yugos 1avia and the North, all of the coefficients are positive but less than eight per ecnt. For the South, Scrbia Hroper and the South less Serbia Proper, four sectcrs show ai least one negative coefficient and three have at least one value greater than eight percont. With ninety-five
${ }^{34}$ The test statisinc is $\mathcal{F}=\frac{\eta-r}{q} \quad \frac{\operatorname{SSE}(2)-\operatorname{SSE}(1)}{\operatorname{SSE}(1)}$
where $\operatorname{SSE}(2)$ and $\operatorname{SSE}(1)$ are the sum of the squered errors computed with and without the specification (3.2), $h$ is the number of cbservations (13); $r$ is the number of parameters estimated (4); and $q$ is the number of extraneous restrictions imposed (2). Severai cuitical limits are F. $025=5.71$, F. $05=4.26$ and $\mathrm{F} .10=3.01$.
${ }^{35}$ This is concluded on the uesis of trial regressions using the parameter conffguration (. 50, .50) and (. 25, .75).

TABLE 12


| Total Social Sector | (000) | 3.8 | 3.7 | 3.3 | 3.7 | 2.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Industry \& Mining | (001) | 4.5 | 3.9 | 4.9 | 5.0 | 4.6 |
| Agriculture \& Fishing | (002) | 4.3 | 7.1 | -1.4 | -1.6 | 0.4 |
| Construction | (003) | 3.3 | 3.2 | 2.3 | 4.7 | -0.6 |
| Transportation' $\varepsilon$ Communication | (004) | 5.0 | 5.4 | 5.5 | 6.1 | 4.7 |
| Handicrafts | (005) | 2.1 | 0.7 | 2.0 | 2.1 | 1.8 |
| Retail Trade and other | (006) | 1.6 | 1.7 | 1.5 | 1.7 | 0.8 |
| Electricity | (111) | 5.2 | 1.8 | 12.4 | 12.6 | 12.5 |
| Coal \& Coal Mining | (112) | 4.2 | 4.7 | 4.6 | 4.7 | 4.3 |
| Food, Drink \& Tobacco | (113) | 0.9 | 2.3 | -3.6 | -3.2 | -4.0 |
| Textiles \& Clothing | (114) | 1.5 | 0.8 | 3.9 | 2.2 | 9.2 |
| Timber \& Eurniture | (115) | 4.2 | 2.9 | 0.7 | 2.1 | 4.8 |
| Paper, Printing \& Publishing | (116) | 3.8 | 2.8 | 5.4 | 2.7 | 13.2 |
| Leather, Rubber \& Footwear | (117) | 2.8 | 3.1 | 1.9 | 1.2 | 5.9 |
| Stone, Clay E Glass | (118) | 4.2 | 2.9 | 5.2 | 5.3 | 4.9 |
| Chemicals \& Petroleum | (119) | 5.8 | 6.0 | 4.8 | 6.7 | 1.8 |
| Netal Using | (120) | 3.9 | 3.3 | 5.3 | 5.6 | 5.4 |
| Metal laking | (121) | 5.9 | 4.1 | 7.8 | 8.1 | 8.2 |
| Miscellaneous | (122) | 2.1 | 0.4 | -2.4 | -.0.5 | -2.7 |

coefficients in all, these out liers are to be expected.
The footnote on page 52 relegates economic analysis to the companion papers which follow. Nevertheless, four observations and a generalization concerning technical progress are made. First, for the total social sector, the rate of neutral technical progress is between 2.7 and 3.7 for all of the regions of Yugos lavia. The North and Serbia Proper are both at the high end of this range and the South less Serbia Proper is at the low end. Second, for industry and mining, the pace of technical progress is quicker, but again it has a comparatively small range of 3.9 to 5.0 , and this time the North is at the bottom of the range while Serbia Proper and the South less Serbia Proper are at the top. Third, for agriculture the range is much larger, 7.1 to -1.6 , and this time the North is at the top while two southern regions are at the bottom. A scrutiny of the other large, one-digit sectors reveals only comparatively small regional variation. Four, for the branches of industry and mining, the southern regions do comparatively better versus the North in such non-agricultural, resource-oriented sectors as electricity (111), metal making (120) and metal using (121). The North, on the other hand, is superior In the consumer-oriented industries, food, drink and tobacco (113) and leather, rubber and footw.ear (117), on the high technology areas such as chemicals and petroleum (119).

The generalization is that the comparatively modest aggregate advantage of the North in dynamic efficiency is primarily due to its more marketoriented agriculture and food processing industries rather than advantages in the area of heavy industry. In contrast, the southern regions show significant superiority in the resource-oriented sectors (other than agriculture) and in the processing industrics associated with those resources.

The principal goal of this paper is to obtain disaggregate estimates of production function $c o c f f i c i e n t ~ s u i t a b l e ~ f o r ~ a n a l y z i n g ~ t h e ~ g r o w t h ~ o f ~ o u t-~$ put in Yugos lavia. This goal is met. Having gone this far, however, we take one more step and measure, for the Yugoslay social sector as a whole, the contribution of resounce mobilization, aconomies of scale, and neutral technical progress to output growth. Table 13 gives the rates of growth for output, inputs and the value of the scale coefficient. 36
Q.- The impressive growth rates of social sector enterprises is revealed here--value added in the social sector grows by nearly ien percent per year. This output growth, however, is matched by an equally impressive job of resource mobilization-capital and labor grom at over six percent per year.

The resultant residual for techaicel progress approaches four percent. . Roughly, we conclude that forty percent of output growth is due to technical progress and sixty percent to factor inputs. Since returns to scale are close co unity, its contribution is minimal. Similarly, since the rates of growth of capital and labor are nearly equal, the contribution of "capital deepening" is also slight.

There is a good deal of variability in these findings, but the explanation of growth in terms of "extensive cavelopment" with high rates of balanced resource mobilization and substantial tcehnical progress is not contradicted. If we could forget the large, comparatively stagnant private sector, output growth could even be described as balanced. A discussion of sectoral growth and development policies, however, is beyond the scope of this paper.
${ }^{36}$ The rate of technical progress is from a least squares regression and is a continuous race of growth; whreas, the rates of growth of capital, labor and output are annual compound rates of $g=0$ th. For this reason, the elasticity weighted ratc of resouzce gatow plas the rate of technical progress is not necessarily equal to the zate of output growth. This i.s to be revised.

T:SLE 13


|  |  |  | Capital" |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sector | (000) | 6.0 | 6.5 | 6.2 | 3.8 | 9.8 | 102 |
| tal Social Sector |  |  |  |  |  |  |  |
| Industry \& Mining | (001) | 6.8 | 7.4 | 7.0 | 4.5 | 11.3 | 102 |
| Agriculture \& Fishing | (002) | 7.2 | 8.0 | 7.1 | 4.3 | 8.7 | 98 |
| Construction | (003) | 4.0 | 9.3 | 5.0 | 3.3 | 7.0 | 93 |
| Transportation $\varepsilon$ Communication | (004) | 4.5 | 1.8 | 3.8 | 5.0 | 8.9 | 95 |
| Transportation \& Communication |  |  |  |  | 2.1 | 10.4 | 100 |
| Handicraft | (005) | 8.8 | 10.6 | 9.1 | 2.1 |  |  |
| Rettill Trade \& Other | (006) | 5.0 | 12.3 | 6.4 | 1.6 | 7.6 | 98 |
| Electricity | (111) | 7.0 | 9.4 | 7.8 | 5.2 | 13.7 | 101 |
| Coal $\varepsilon$ Coal Mining | (112) | 0.8 | 3.6 | 1.7 | 4.2 | 6.0 | 105 |
| Food, Drink \& Tobacco | (113) | 8.0 | 7.8 | 9.1 | 0.9 | 9.4 | 114 |
| Textiles \& Clothing | (114) | 7.8 | 6.8 | 8.1 | 1.5 | 9.6 | 106 |
| Timber \& Furniture | (115). | 6.4 | 3.0 | 5.5 | 4.2 | 8.4 | 99 |
| Paper, Printing \& Publishing | (116) | 9.6 | 12.8 | 9.8 | 3.8 | 13.4 | 97 |
| Leather, Rubber \& Footwear | (117) | 7.9 | 7.2 | 8.6 | 2.8 | 11.4 | 110 |
|  | (118) | 5.8 | 5.7 | 6.3 | 4.2 | 9.8 | 109 |
| Stone, Clay \& Gla |  |  |  |  |  |  |  |
| Chemicals \& Petroleum | (119) | 9.5 | 10.2 | 10.3 | 5.8 | 15.4 | 106 |
| Hetal Using | (120) | 9.2 | 7.2 | 10.1 | 3.9 | 14.8 | 112 |
|  | (121) | 3.7 | 5.7 | 4.4 | 5.9 | 9.9 | 115 |
| Metal Making |  |  |  |  |  |  |  |
| Miscellaneous | (122) | 22.7 | 9.0 | 2.1 | 8.5 | 105 |  |

* Annual Compound rate of growth from 1952 to 1964.
$\div$ The weights are the ordinary covariance: estimates from
:Continuous compound rate of growth from least square resression.


## DATA APPENDIX

Value added, employment and capital stock statistics are described in this appendix. Complete statistics for the years $1552-1966$, for five regions, and 21 industries are presented at the end. For the reader who is already familiar with Yugos lav statistical sources or who is only interested in the broad outlines, a fev sentences will suffice.

Value added, in constant 1966 dollars, is considered to be equivelent to the Yugoslav measure of "social product." Since orficial constant price series are not available for the branches of industry and mining, these missing series are estimated by the method of bi-proportional matrices. Employmant is measured on an average annual basis and is taken directly from the publications of the Federal Statistical Bureau. Capital stock statistics are more complex. In addition to our standard sectoral and geographic disageregation, we present a breakdown of fixed assets according to structures and equipment. The perpetual inventory method is used, and the base period is related to Ivo Vinsl:y's estimates after conversion to 1066 prices. A unique feature of the estimates is the use of durability weights for aggregating structures and equipment into total fixed assets.

The remaining pages are written for those who find this brief description insufficient.

## SECTION A

VALUE ADDED

The Yugoslav concept of "social product" principally differs from "gross value added" in Western terminology because aggregate measures eaclude value added originating in the service industries. Since, in this appendix, we only deal with productive (non-service) sectors of the economy, no problem is created by this discrepancy. The statistical yearbooks for 1264 through 1960 present social product in constant 1960 dinars by republics for the seven major economic sectors. For Yugos lavia as a whole, but not by republic, a further disaggregation into 22 sub-branches of industry is also available. Two transformations of this data are necessary: first, all series must be transformed from 1960 prices to 1966 prices; and second, constant price series must be estimated for our 12 branch disaggregation of industry and mining. The conversion to 1966 prices is easily performed by multiplying each sector by the percentage increase in prices between those two years. While this procedure does not allow for intra-sectoral price changes, these can be expected to be relatively unimportant in comparison with the inter-sectoral chances. In particular, by shifting to the 1566 price base we benefit from the major rationalization of prices which occurred in the 1065 reform. This reform caused significant upward revision of agricultural and raw materials prices in comparison with producer goods.

The problem of estimating a constant $1: 66$ price, regional series of social product for each of the 12 branches of industry and inining is resolved by applying the method of bi-proportional matrices. This method is available to us because the required data are available in current prices for each year, and the marginal totals for industry and mining and for the five regions are
available in both current and fixed 1966 prices. Thus, for each year we have a two-dimensional array of current price statistics (the rows being the 12 branches of industry and the columns being the five regions), whereas marginal totals in both current and fixed prices are available. What we wish to do is convert the elements of the two-dimensional table from current to 1966 price base.

In mathematically similar situations the method of bi-proportional matrices has been used in demographic analysis by Dering and Steffan and in up-dating input-output matrices by Bacharach. ${ }^{2}$ If we assume an independence of row and column effects, then the method of bi-proportional matrices has the characteristic that the derived cell estimates minimize the sum of the squared deviations of their final fixed price values from their original current price values. ${ }^{3}$

In practice, rather than first aggreçatinf republics into regions and aggregating the 22 Yugos lav sub-branches of industry into our 12, subbranches, we perform the bi-proportional estimation for the more disacgregate data and performed the ageregation afterwords. Since the amourt of price inflation in industrial branches was comparatively slight between 1952 and 1566, it is felt that with one exception no serious error was introduced by this procedure. For tobacco, where the product is definitely not homogeneous by regions and where different price trends exist for the various
$1_{11}$ On a Least Squares Adjustment of a Sampled Frequency Table When the Expected Marcinal Totals are Know," Annals of Mathematical Statistics, Vol. XI (1:4:0), pp. 427-444.
$\mathbf{2 " E s t i m a t i n g}^{\text {"En-negative Matrices fron Marginal Data," International }}$ Bconomic Review, Vol. 6, No. 3 (Sept. 1265), pp. 2Sl:-3lC.
${ }^{3}$ D. Friedlander, "A Technique for Estimating a Contingency Table, Given the Marginal Totals and Some Supplementary Data," Journal of the Royal Statistical Society, CXXIV, Series A, Part 3 (1261), pp. 412-420.
products, an important error may be present. Tobacco, however, is the only one of the 22 branches for which this effect vas pronounced.

## SECTION B

EMPLOYNENT
Employment in the social sector by industries and republics from 1552 to 1963 is given in Statistical Bulletin 3lc. Similar data for suissequent years is contained in the Statistical Yearbooks. From 1952 to $1 \geqslant 55$, the data in SB310 are obtained from monthly surveys of all firms in the social sector, and after 1955 from semi-annual surveys. Exclusions include apprentices, part time employed, overseas employed, etc. Since 1961 an alternate series obtained from the complex annual reports (KGI) is available. Except for agriculture, the difference between these two serfes is that the KGI series is based on a 12 -period average while the SB310 series is based on a 2-period average. Also, SB310 gives more complete coverage to seasonal employment in agriculture.

In general, the data on employment in the social sector appears quite reliable. Coverage with respect to the number of firms is virtually exhaustive. The principal problem would seem to be the omission of "moonlighters" (included only once as their principal occupation), temporary agricultural workers, and "dead brigades." The latter term refers to fictitious or parttime workers who appear as full-time employees on payroll lists, principally in order to reduce the enterprise's taxes: ${ }^{4}$ The "bricades" presumably are included in the employment staisistics but there are no published estimates of

[^6]their magnitude. This study assumes their numbers are negligible and no adjustments are made in the employment data which are taken directly from SB31C and since 1963 from the Statistical Yearbooks.
$\because:$
$\square$

PART I. ESTIMATION OF THE CAPITAL STCCK

## Introduction

All firms in the social sector of the Yugos lav economy are required to report, in detail, the nature of their capital account transactions with the bank on whom credits are drawn. This provides the bank with a complete set of investment data distinguishing investments in inventory, equipuent, and structures from other transactions of the enterprises. This data is published in highly disaggregate form, by three digit branches of the economy, republics and autonomous regions, private and social sectors (the private sector investments are obtained by much cruder estimates), and by technical types of investment (total, structures, equipment, and other), and provides an unusually sound statistical base for estimating capital stock according to the perpetual inventory method. The recent publication of this data by the Institute for Economic Investments in five volumes entitled Investments 18:6-1966, and totaling over one thousand pages, makes a critical contribution to the underlying data block by converting all investments into 1966 prices. These statistics serve as the basis for our capital stock estimates.

Perhaps the most serious possible flaw in these statistics is that, by accident or design, the enterprises may understate reported investments
by using bank credits granted for inventory financins to purchase fixed assets. During the years preceeding the 1965 . Reform, there are numerous allegations of this practice in the newspapers. Insofar as this erroneous reporting exists, it can be expected to dampen reported investments during periods of high demand accompanied by tight bank credits.

Our capital stock estimates are by no means the first for Yugos lavia. The investment data has been available for some years and has been imaginatively and painstakingly exploited by Dr. Ivo Vinski in a long series of publications analyzing the growth of Yugoslav capital stock. Vinski's
work is based on the investment series described above. His estimates of the base period capital stock axe derived from a detailed inventory of
structures and equipment in the social sector made by the government in 1953. ${ }^{1}$
More recently, in 1062 and 1966, the government revalued the capital stock of enterprises. Among other things, this revalorization is designed to increase the value of capital assets upon which the firm must pay rent.
${ }^{1}$ A partial list of the most important of Dr. Vinski's works on the Yugos lav capital stoc: may je helpful. The results of the 1953 census of fixed assets are presented in English in "National Wealth of Yugos lavia at the end of 1953," Income and Wealth, Series VII (London: Bowes and Bowes, 1559), pages 160-192. These estimates for 1953 are extended to the Repuiblics of Yugoslavia in the publication Prociena Nacionalnon Doratstva po nodruciina Jugoslaviavi ie (Zagrei: Ekonomski Institut, 105シ). Using the perpetual inventory method the regional estimates are then used to prepare capital stock estimates for the entire post war period in 185s prices with the result being presented in Procjena Rasta Fiksnih Fondova Do Jugoslavenskin Renublikana od 1946 do 1960 (Zagreb: Ekonomski Institut, 1565). ilore recently, a six sector breakdown for Yugoslavia as a whole is given in 1962 prices for the years 1944 to 1964 in the article "Rat Fiksnih Fondova Jugos lavije od 1844 do 1964," Ekonomist, Broj for 1965, pp. 667-679. Estimates for the prevar period are also available in "National Product and Fised Assets in the Territory of Jugoslavia: 1:09-1955," Income and Wealth, Series IX (London: Bowes and Bowes, 1961), pp. 2C6-233.
${ }^{2}$ The 1962 revalorization of fixed assets serves as a basis for the capital stock series presented by Gojko Grdjic, "

These two sources of initial capital stock, the 1953 Survey which under lies Vinski's work, and the 1962 and 1966 revalorization, are both used by us to obtain our base year capital stock figures.

## CONRRIBUTION OF THE NEU ESTIMATES

We believe that our estimates make two sisnificant contributions to the existing capital stock figures, as well as a number of minor improvements. The two important contributions are: first, the use of durability veights when aggregating over equipment and structures; and second, the presentation of a disaggregate series of capital stock for the sub-branches of industry by regions and investment type. The need to weight equipment and structures by their respective durabilities arises because, even under idealized circumstances, the dollar cost of an investment good is not a satisfactory measure of that item's contribution to output. For example, assume there are two identical machines, $A$ and $B$ which produce one unit of output except that $A$ has an average length of life of 10 years while machine $B$ has an average length of life of one year. In a perfectly competitive econony which equalizes the discounted value of expected future receipts, the price of machine $A$ will be ten times that of machine $B$. Wile dollar expenditure on each of the machines is a satisfactory measure of the cost of the investment goods, it is an inadequate measure of their contribution to current production. Specifically, a dollar of investment in machine $B$ produces ten times the current output that a dollar investment in machine A does. To properly agcregate machines with different life expectancies we must first weight the capital goods by their respective durabilities.

The proper procedure for doing this and the required assumptions are detailed by Haavelmo. ${ }^{7}$

For practical reasons we distinguish only betveen two types of investments, structures and equipments. Each of these aggregates is assumed to have its own average length of life. Let $K^{*}$ denote the unweighted sum of the dollar value of structures, $S$, and equipment, $E$. This is the marnitude of fixed assets which the enterprise reports for accounting parposes and is the definition given in (1). In contrast, our measure of fixed assets, which utilizes the durability weights $C_{i}^{s}$ and $C_{i}^{e}$, is given by the variable $K$ in equation (2). These weights depend upon the rate of interest, $P$;
(1) $\quad \mathrm{K}_{\mathrm{i}}=\mathrm{S}_{\mathrm{i}}+\mathrm{E}_{\mathrm{i}}$
(2) $K_{i}=S_{i} C_{i}^{s}+E_{i} C_{i}^{e}$

$$
\begin{aligned}
& c_{i}^{s}=\frac{2-e^{-p_{M}}}{1-e^{-p M_{i}^{e}}} \\
& c_{i}^{e}=\frac{1-e^{-p_{M}}}{1-e^{-p M_{i}^{e}}}
\end{aligned}
$$

the average length of life of equipment $M_{i}^{e}$; the average length of life of structures $M_{i}^{s}$; and an arbitrary normalization coefficient M. Given

7 Trygve Havelmo, A Study in the Theory of Investment (Chicaro: University of Chicago Press, 1260), ppc 97-102. See also the discussion of this topic in the context of investment functions by Svi Griliches, "Capital Stock in Investment Functions" in Measurement in Economics, Ed. Carl Christ and Others (Stanford: Staniord University Press, 1863), pp. 115-137.

The necessary assumptions concerning marl:et equilibrium used by Haavelmo are: (1) that the rate of interest, $P$, is expected to remain constant over the life of investment goods; (2) that the annual deflated income from owning capital goods is expected to remain constant over their life; and (3) that the purchase valuc of capital goods is equal to their discounted future income stream. These are heady requirements, particularly for a Socialist economy, but in some ways they appear to we better satisfied for the unique blend of socialistic planning and enterprise decentralization that constitutes the Yugos lav economy than they would be for the typical capitalist econony. For example, at least in theory, the central planning of investments should eliminate many of the uncertainties that are associated with uncoordinated, independent investment decisions. These uncertainties cause investments in particular areas to have high riek premiums thet raise the rate of intcrest which is to be used in discountine future receipe streams. Indeed, our estimation problems for the variable $P$ are quite simple since: for the great majority of firms, an unchanging charge of $6 \%$ per annum was the lending rate of the Xugos lav governnent.
estimates of these four coefficients we may construct a capital stock series for the variables $K$ whose usefulness in production function analysis is markedly superior to the variable $K \%$. The magnitude of the differences in the coefficients $C^{s}$ and $C^{e}$, and the significant differential in the rate of growth of $S$ and $E$ in the Yugoslav economy suggests that Havelmo's conjecture that ..."It is my fuess that such a procedure (conversion to an equalidurability basis), even if it is very rough and approximate, would be a definite improvement over the customary, but unfounded, method of measuring K simply as $\mathrm{S}+\mathrm{E} .{ }^{\circ}{ }^{\text {© }}$

The second important contribution of our capital stock serics is a disaggregation of industry into its sub-branches. Until this time, there has been no capital stock series available for these branches either for Yugoslavia as a whole or by regions. Our estinates, available by five regions, are presented for 12 branches of industry. These twelve branches represent an aggregation of the 22 branches available in the Yugoslav three digits classifications. The aggregation used is presented in Table 1. The regional disaggregation of capital stock into our five categories is particularly difficult to make since it requires a division of the Repuslic of Serbia into its components, the Uza Podruce, the Vojovdina, and the Kosmet. For time periods prior to 1952 there is very little data availaile for these autonomous regions. The abovementioned publication of the IEI presents, for the first time publicly, investment data for these areas. Among the minor improvements we would include the conversion of all of our series to 1966 prices. Vinski's regionally disacgregate data is
$3^{3}$ Ibid., p. 101.
only available in 1966 prices and his most recent national data is in 1962 prices. Our use of the post-1965 reform prices embodies the rationalizations of the price system which is an important goal of that reform. Another distinctive feature, if not an unmixed improvement, is the use of exponential decay in estimating retirements. Vinski's capital stoc! estimates deduct a retirement component apparently based upon the assumption of a "one horse shay." That is, an item of capital with an expected average length of life $M$ produces for exactly li years and then becomes totally obsolete and is replaced. In contrast, exponential decay assumes that, in each year a fraction $\frac{1}{M}$ of the still-existing capital stock is subject to replacement. While there is scant empirical evidence for choosing betveen these two assumptions, retirement according to exponential decay is considerably simpler for computational purposes and is more pleasing to our a priori intuition. ${ }^{9}$ Computational simplicity is achieved because recirements in any given period are a function only of the existing unretired capital stock and do not depend upon the time stream of past investments. We turn now from our discussion of what is new about our capital stoc!: series to a more detailed discussion of the method used, and particularly of the major problems encountered.

## PROBLEMS OF ESTIMATION

Estimation of capital stock according to the perpetual inventory method demands the availaility of two sets of data: One for investments and the other for a base period measure of capita1. In aduition to these
${ }^{9}$ A discussion of this is available in havelino, Ibid., p. 127, and in Griliches, op. cit., p. 119. An empirical study of the importance of this assumption is Given by Helen StoneTice, "Depreciation, Obsolescence, and the Nieasurement of the Ageregate Capital Stocl: of the United States, 1900-1062." The Reviev of Income and Wealth, Series 13, No. 2, June 1567, pp. 115-154.
two requirements and their attendant problems, our use of durability weights when aggregating structures and equipment means that we must somehow oitain estimates of the averace lengths of life for these two types of investment. Since the IEI investment data described above is made to order for our purpose, no further discussion of this most critical item is required. Therefore, ve concentrate our discussion on the estimates of base period capital stock, and the average length of life of equipinent and structures. As a preliminary to these discussions equations (3) throuch (7) present the formulas used in computation. Equations (3) and (4) define the stock of structures and the retirement of structures as:
(3) $s_{i j t}=s_{i j t-1}+C_{i}^{s} 1_{i j t}^{s}-R_{i j t}^{s}$, and 50.
(4) $\quad R_{i j t}^{s}=\frac{S_{i j t-1}}{M_{i}^{s}}$

Equations (5) and (6) define the stock of equipment and the retirement of equipment as:
(5) $E_{i j t}=E_{i j t-1}+C_{i}^{e} I_{i j t}^{e}-R_{i j t}^{e}$, and
(6) $\quad r_{i j t}^{e}=\frac{E_{i j t-1}}{M_{i}^{e}}$

Total capital stock is then obtained as the direct sum.

$$
\text { (7) } \quad K_{i j t}=E_{i j t}+S_{i j t}
$$

In the above, $l_{i j t}^{s}$ and $l_{i j t}^{e}$ refer to investment in structures and equinment, where $i$ refers to industry, $j$ to region, and $t$ to time, and $C_{i}^{e}$ and $C_{i}^{s}$ are as defined in (1). A value of $P$ of .06 and $\widetilde{M}$ of 21.1 is selected. The latter Is the average length of life we estimate for the total capital stoc: in the productive part of the social sector of the economy. :

## the probleif of average lengty of life

Consider first the problem of estimatinc the average length of life of equipment and structures, $M^{e}$ and $M^{s}$. Lackiñ bocin a table describinc the expected length of life of physical items of capital stock, as well as an enumeration of the various types of physical capital, we must instead use financial data on depreciation changes and the value of fixed assets to infer these lengths of life or for each of the industry groups and for structures, equipment, and total capital. Eowever, even usinc this indirect procedure, lack of data prohibits us from deriving recional estimates of each of these magnitudes. Actually, this may be an advantage since regional differences in depreciation rates nay reflect differences in depreciation policy rather than differences in the durability of capital goods. (A leading Yugoslavic economist suggests that during this period the southern republics are more inclined to undereatinate depreciation in order to increase distributable earnings than are the northern republics who are more confident that contributions to the depreciation fund will ultimately become available to the enterprise itself so that such contributions are both a tax offset to current income and a source of future investment fund.) In any event, our application of national coefficients to the various republics presumes that the durability of capital zoode does not vary regionally, at least not within the 10 sectors for which we na:e estimates. Our length of life estimates are based upon the fact that Yugoslav enterprises computc depreciation according to the straight line basis. 10

[^7]According to this procedure depreciation in any year where an enterprise is computed as a simple fraction $1 / M$ of the book value of all undepreciated assets. Given data on the book value of equipment and structures, and data on the annual flow of depreciation charges which are attributable to equipment and to structures, it is a simple matter to estimate $M$ as the ratio of the book value of capital to the depreciation flow. In practice our data is an average for the years 1963, 1964 and 1965. The choice of these periods is predicated on the fact that the revalorization of capital at the end of 1962 provides a good initial point, that the second revalorizam tion of capital in 1966 makes the incorporation of this and later years misleading, and that an average value over three years reduces noise. The sources of our data are given in a foot note to Table 2. The cited Statistical Bulletins are unusual in that they present the accumulated depreciation fund separately for equipment and structures, thus making it possible to estimate depreciation over the three year period as the difference between the end period depreciation fund in 1965 and the initial depreciation fund in 1962. A valid objective to this procedure is that it neglects that totally depreciated assets are constantly being removed from both the book value of fized assets account and the depreciation fund account. While it would be possible to estimate the magnitude of these removals by (our first) using round estimates of $M$ and then going back and obtaining a second round set of $M$ corrected for this phenomena, it is not felt that this would alter the estimates sufficiently to justify the additional labors. The complete set of average length of life estimates used in our durability aggregation are presented in Table 2. For the Total Productive Sector, an average length of life for both structures and equipment of 21.1 years
( 15.9 years for equipment and 33.5 years for structures) appears to be a reasonable magnitude. For individual sectors, the high values for Transportation and Communication, and Handicraft appear proper, as does the 10 w value for Construction, and Industry and Mining. The rather 1ow, 16.4 estimate for Agriculture appears somewhat surprising to this author but it is not unreasonable. Our estimates for the sub-branches of industry present some difficulties since, in a few cases, removal of items from the depreciation fund does casse unduly small values for depreciation that result in unusually long lengths of life, in one case infinite. To correct for this we impose the restriction that $i^{\mathbf{s}}$ be no greater than 50 years, and $M^{e}$ be no greater than 25 years. In the cases where these restrictions are imposed, the unconstrained values are given in parenthesis.

## THE PROBLEM OF THE INITIAL CAPITAL STCCK

The most difficult problem is to obtain base year estimates of the capital stock. For the six major sectors of the economy there is no serious problem since we have Dr. Vinski's estimates for 1946 available by republics in 1956 prices. For these sectors only three adjustments are necessary: (1) use the implicit lEl investment price deflators to adjust to the 1966 price base; (2) separate the Uze Podruce and Vojvodina from the aggregate for Serbia in order to compute our North-South aggregates; and (3) remove estimates for the private sector from Vinski's totals which are for both the private and social sectors. The solution to the first problem is already stated, the sulution to the second problem is identical to the method we used to estimate the branch data described
below, and the solution to the third problem, the separation of social and private sector capital stock, uses estimates for agricultural and handicraft also developed by Vinski but which are not widely known. ${ }^{11}$ Using Vinski's data it is possible to estimate an initial capital stock for any year since 1946. From one point of view the most satisfactory year would be 1953 since that is the date of the capital census from which Vinski oitains his estimates. Thus for 1953, his use of the one-horse-shay replacement assumption has no bearing on the estimates made for that single year. This is not true of other years. Nevertheless, this is not the base year which we choose for making our estimates. The reason for this we now explain. The estimation of a base year capital stock value for the six major sectors may not be a problem, but the estimations of this variable for the twelve sub-branches of industry is. Consequently, our selection of a base year is designed to facilitate our estimation for the sub-branches. With respect to this problem there is no really satisfactory solution. However, there is one important factor which suggests that even substantial estimation errors for the base year 1946 may be unimportant to the value of the capital stock for the years after 1952--the years which are our principal concern. This factor is simply that, particularly in the branches of industry, investment growth is so great that by 1952 it swamps any errors which are made In the initial capital stock values for 1946. Cur tactic then is to make very crude estimates for 1946 and rely on the rapid growth of investment 'until 1952 to make our errors unimportant. For this reason we elect to use 1946 as our base year for estimating the capital stock. The growth of investment after that date also tends to make the replacement error induced by using Vinski's estimates relatively unimportant.

11 Ivo Vinski, Procjera Rasta Fiksnih Fondova Jusos lavije od 1046 do

Estimation of capital stock for the branches of industry in $18: 6$ is done by projecting backwards the average capital-output ratio for the years 1963, 1964 and 1965 to 1046 , and multiplying this figure by estimates of output measured in 1966 prices for that year. This is an extremely crude procedure both because the capital-output ratio is not constant over the 20 year period and because adequate regional data on real output is not available for 1946, particularly not for the autonomous provinces. A partial solution to the problem of changing capital output ratios is obtained by forcing our total for industry in 1546 to be equal to Vinski's. This is equivalent to assuming that the decrease for allbranches is the same as that for industry as a whole. The absence of satisfactory output statistics for the period before 1952 causes us to use indexes of real physical product as proxies for a true index of social product. Some measure of the crudeness of these two procedures may be obtained by comparing our unconstrained original estimates with the Vinski total for Yugoslav industry in 1946 (after adjustment to 1966 prices). Our original estimates are $62 \%$ of the Vinski estimates for 1946. The fact that our estimates are below Vinski's is consistent of the observation that over the entire 20 year period the Yugoslav capital output ratio has fallen. Therefore, it is appropriate to look upon our correction of this figure to the Vinski total as a correction for the decrease in the capital-output ratio. Although we present our initial capital stock estimates for 1946 to the critical view of scholars, in order to cmphasize the crudity of the early period estimates, we do not present capital stock estimates for the period 1947-1951. After 1952 it is judged that the errors of this estimation procedure become unimportant.


$\therefore$

YUGOSLAV PRODUCTION STATISTICS， 1952 TO 1966

| 1．${ }^{\text {a }}$ | 1952 | 1953 | 1954 | ：1955 ${ }^{\prime}$ | 1956 | ＇1957 | 1958 | 1959 | 1960 | 1961 | 1962 | 1963 | 1984 | ， 1965 | 2966 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| th！30\％${ }^{\text {a }}$ | ：CO： 171 | 59：979 | 670635 | 748094 | 788151 | 855915 | 928761 | 991351 | 1071960 | 1127675 | 1165043 | 1221960 | 1318742 | 1377504 | 1355000 |
| cripltal | \％：773 | 50013 | 37314 ． | 40594 | 42815 | 44890 | 45918 | 49416 | 53164 | 57570 | 62416 | 67757 | 73949 | 78314 | 83.273 |
| Fnurbaint | 50074 | 16，544 | 20587 | 22574 | 23980 | 25267 | 26688 | 28430 | 31062 | 34108 | 37254 | 41213 | 45786 | 48057 | 522？5 |
| VイLUE ADOED | 1587 | －311 | 9505 | 10847 | 11918 | 13983 | 15649 | 17481 | 19867 | 21223 | 22783 | 26336 | 30575 | 33118 | 35236 |
| is：3！ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 ：i， | 1952 | 1953 | 1954 | 1955 | － 1956 | 1957 | 11958 | 1959 | －1950 | 1961 | 1962 | 1963 | 1964 | 1665 | 1966 |
| i A．joik | 31：398 | $3: 12044$ | 377799 | 415095 | 434951 | 468334. | 503199 | 534746 | 574074 | 599404 | 615211 | 644629 | 690750 | 715321 | 090200 |
| ESidTAL | ： 9515 | 1：3\％ 0 | 16 ¢54 | 17754 | 18420 | 19290 | 20190 | 21340 | 23107 | 25167 | 27372 | 29763 | 32251 | 33605 | 34986 |
| Endirbicat a | 8113 | ii， 24 | 9427 | 9936 | 10358 | 10922 | 11581 | 12390 | 13650 | 15052 | 16571 | 18249 | 20022 | 20990 | 21932 |
| VR：－ut ADDED | 4503 | $\because 57$ | 5678 | 6409 | 6809 | 7894 | 8484 | 9562 | 10864 | 11702 | 12512 | 14315 | 16262 | 17483 | 18797 |
| StuTh |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ！ | 1952 | 1433 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1950 | 1961 | 1952 | 1963 | 1964 | 1985 | 1 150́ |
| 1．nem | 24：073 | $\therefore \therefore \div 735$ | 292836 | 332999 | 353200 | 387581 | 425562 | 456605 | 497886 | 528271 | 549837. | 577331 | 627992 | $0622.33^{\circ}$ | 659800 |
| CRJITAL | ； $3: 25$ | 1：447 | 20460 | 22239 | 24396 | 25600 | 26728 | 29076 | 30057 | 32503 | 35044 | 37994 | 41698 | 44710 | 48207 |
| ［CUIPGENT | 791 | 9720 | －11160 | 12638 | 13622 | 14346 | 15107 | 16040 | 17406 | 19057 | 20683 | 22964 | 25763 | 27857 | 30363 |
| VALUE ACDED | ． 3 324 | $\therefore 254$ | 3820 | 4438 | 5109 | 6058 | 7165 | 7918 | 9002 | 9520 | 10271 | 12021 | 14313 | 15635 | 16439 |
| SGUTH LESS S | EMTIA Pr | UPI： |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1552 | 1953 | 1954 | 1955 | 1956 | 1957 | 19.58 | 1959 | 1960 | 1961 | 1962 | 1963 | 1964 | 1955 | 1966 |
| LALOBE： | 128304 | 14.698 | 146507 | 169098 | 180340 | 197513 | 214736 | 227398 | 247436 | 253516 | 269945 | 287949 | 310118 | 331688 | 332400 |
| Cnlital． | 5230 | 7124 | 7906 | 9090 | 9853 | 10395 | 11021 | 11733 | 12627 | 13009 | 14459 | 15420 | 16060. | 17960 | 15537 |
| EかJすpricit | $\bigcirc 127$ | ¢ぃ35 | 4104 | 4921 | 5452 | 5758 | 6196 | 6693 | 7359 | 8072 | 8709 | 9419 | 10290 | 11121 | 12203 |
| VALU：ADDED | 1454 | 1515 | 1215 | 2107 | 2408 | 2693 | 3220 | 3402 | 3953 | 4314 | 4664 | 5540 | 6589 | 7356 | 7743 |
| SEitiga Profer |  |  |  |  |  |  |  |  |  |  | －： |  |  |  |  |
| ＇ | ！ 6 | 10：53 | 1954 | 1955 | 1056 | 1957 | 1958 | 1959 | 1950 | 1961 | 1962 | 1953 | 1964 | 1965 | 1066 |
| Lasoir | 121909 | 12：137 | 146229 | 163901 | 172860 | 189963 | 210826 | 229207 | 250450 | 269653 | 279892 | 20.9382 | 3：7874 | 330575 | 327400 |
| CABPTAL | 8：9132 | 10：3．10 | 12513 | 13710 | 14505 | 15170 | 15674 | 16307 | 17400 | 18960 | 20543 | 22545 | 25014 | 26750 | 20679 |
| coubinatit ： | ：7759 | －1：2 | 7015 | 7078 | 8132 | 8552 | 8388 | 9311 | 10019 | 10957 | 11947 | 13520 | 15：479 | 16723 | 16150 |
| VALUE AOUED | 1570 | 1\％39 | 2012 | 2330 | 2701 | 3390 | 4045 | 4436 | 5050 | 5207 | 5606 | 6431 | 7723 | 82：\％ | $86: 5$ |



|  | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1960 | 1961 | 1962 | 1963 | 1964 | 1965 | 1066 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LASOR | 139501 | 135575 | 144734 | 155329 | 152086 | 160261 | 173039 | 185183 | 195886 | 203161 | 223715 | 231032 | 246388 | 250826 | 246000 |
| CAiPITAL | 20166 | 29983 | 29813 | 29857 | 30149 | 30671 | 31426 | 32483 | 33694 | 34845 | 35813 | 36633 | 38197 | 39029 | 40062 |
| Eoulpment | 12373 | 11993 | 11736 | 11639 | 11818 | 12207 | 12631 | 13166 | 13683 | 14164 | 14652 | 15072 | 15949 | 16490 | 17219 |
| Vilue adoed | 2229 | 2432 | 2598 | 3077 | 3163 | 3630 | 4005 | 4500 | 5352 | 5582 | 5860 | 6295 | 6765 | 7205 | 7636 |
| H |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1952 | 1553 | 1954 | 1955 | i956 | 1957 | 1958 | 1959 | 1960 | 1961 | 1962 | 1963 | 1964 | 1965 | 1956 |
| Lainot | 7+10s | 71725 | 76105 | 85567 | 81181 | 85135 | 93298 | 99614 | 104588 | 111342 | 121631 | 129344 | 136227 | 138443 | 133300 |
| Cabital | 10080 | 1ちり58 | 15878 | 15891 | 16021 | 16169 | 16609 | 17064 | 17478. | 17856. | 18344 | 18965 | 19059 | 20020 | 205.36 |
| FGOIPPIEPT | -6070 | 6700 | 6595 | 6598 | 6698 | 6816 | 7078 | 7354 | 7597 | 7828 | 8090 | $\therefore 8361$ | 8835 | 9151 | 9609 |
| VRLUE RDDED soum | 1145 | 1231 | 1425 | 1641 | 1755 | 2022. | 2212 | 2395 | 2779 | 2878 | 3098 | 3406 | 3696 | 3853 | 3940 |
|  | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1960 | 1961 | -1962 | 1963 | 1964 | 1965 | 2966 |
| Lreor | 615393 | 630150 | 66529 | 69752 | 70905 | 75126 | 79741 | 85569 | 91298 | 96819 | 102084 | 101688 | 110161 | 112383 | 112700 |
| CAMITML | 1:405 | 14030 | 13935 | 13955 | 14129 | . 14502 | 14816 | 15419 | 16216 | 16989 | 17469 | 17767 | 18538 | 19009 | 19527 |
| ECiITPENT | ¢ذu3 | 5:193 | 5141 | 5041 | 5121 | 5391 | 5553 | 5802 | 6086 | . 6336 | 6563 | 6711 | 7114 | 7339 | 7610 |
| VALUE NODEO | 1083 | 1200 | 1173 | 1437 | 1408 | 1608 | 2793 | 2104 | 2573 | 2704 | 2752 | 2889 | 3069 | 3352 | 3695 |
| SOUTI LESS SEİBIA PROPEES |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1902 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1950 | 1961 | 1962 | 1963 | 1964 | 196,5 | 1966 |
| Laisois | 23632 | 29033 | 32126 | 33324 | 33842 | 354.65 | 37511 | 40075 | 44144 | 47249 | 50631 | 51959 | 55130 | 56690 | 56000 |
| Cnidjol | b339 | 5314 | 5235 | 5348 | 5549 | 5888 | 60371 | 6411 | 6918 | 7379 | 7656 | 7854 | 8041 | A108 | 8277 |
| coulbatit | 2159 | 2090 | 2033 | 2014 | 2109 | 2336 | $2399^{\text {i }}$ | 2564 | 2757 | 2911 | 3056 | 3183 | 3328 | 3406 | 3531 |
| Vatue Nubeo | 415 | 501 | 469 | 577 | 603 | 658 | 721 | 848 | 1003 | 1085 | 1125 | 1188 | 1308 | 1373 | 1505 |
| SEREIA PROPER |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1052 | 1053 | 1954 | 1955 | 1956 | 1957 | $\therefore 1959$ | -1959 | 1950 | -1961 | 1962 | 1963 | 1964 | 1965 | 1966 |
| LATSOR | 30501 | 34:317 | 36503 | 36433 | 37063 | 39661 | 42230 | , 45494 | 47154 | 49570 | 51453 | 49729 | 550.31 | 55693 | 56700 |
| CAPITAL | 15747 | 1/16 | 8550 | 8618 | 8579 | 8614 | $8780^{\circ}$ | '9009 | 9299 | 9610 | 9813 | 9913 | 10498 | 10902 | 11250 |
| E.OUI ${ }^{\text {PIMEPIT }}$ | $\therefore 343$ | 3293 | 3108 | 3026 | 3012 | 3055 | 3155 | 3239 | 3329 | 3425 | 3506 | 3528 | 3786 | 3934 | 4079 |
| value adoed | 068 | 730 | 704 | 806 | 805 | 950 | 1072 | 1256 | 1570 | 1619 | 1637 | 1701 | 1762 | 1979 | 2190 |




## YUGOSLAVAPRODUCTION STATISTICS, 1952 TO 1966

COAL AND COAL MINING





YUGOSLAV PRODUCTION STATISTICS， 1952 TO 1966
IIMBER AND FURNITURE

|  | $\therefore 53$ | ： $0: 3$ | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1960 | 1961 | 1962 | ． 1963 | 1964 | 1955 | 19.55 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LA， | ：\％$\%$ | ¢．$\because: 3$ | 3887 | 43630 | 40257 | 5470 | 85564 | 96503 | 108124 | 113259 | 123211 | 129174 | 134343 | 135466 | 130900 |
| CABIアAL | $\because 97$ | $\because 2$ | 484 | 1283 | 1237 | 555 | 2459 | 2528 | 2659 | 2834 | 2995 | 3174 | 3365 | $\times 479$ | 3574 |
|  | i！ 6 | 1：30 | 234 | 540 | 543 | 276 | 1129 | 1175 | 1265 | 1375 | 1481 | 1610 | 1739 | 1029 | 1902 |
| Vicut．inciou | 713 | i；7 | 49 | 443 | 465 | 60 | 1963 | 1185 | 1333 | 1421 | 1590 | 1762 | 2030 | 2143 | 2202 |
| －サいろくTil |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1.52 | 1.163 | 1954 | 1955 | 1956 | 2957 | 1958 | 1959 | 1960 | 1961 | 1962 | .1953 | 19.64 | 1985 | 1900 |
| 1 $\because$ ： | $\therefore \therefore 29$ | 8129 | 27503 | 72513 | 76649 | 49084 | 52123 | 57141 | 62861 | 64410 | 67979 | 66982 | 68071 | 234．5 | 65800 |
|  | ：$: 143$ | 1．1．31 | 1074 | 2352 | 2368 | 1329 | 133＇t | 1367 | 1416 | 1470 | 1525 | 1.590 | 1500 | 1709 | 1754 |
|  | 4 | $\bigcirc 36$ | $522^{\circ}$ | 1064 | 1006 | 568 | 570 | 599 | 635 | 671 | 709 | 756 | 804 1072 | 245 | 5 |
| Yiai m MnOLD | 4， 23 | $\therefore 90$ | 380 | 821 | 825 | 540. | 521 | 649 | 692 | 700 | 891 | 954 | 1072 | 140 | 5 |
| Scaish |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1：52 | 1：53 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1060 | 1961 | 1962 | 1963 | 1954 | 1065 | 1050 |
| LA：Bu： | 5． 2.509 |  | 22242 | 28483 | 30303 | 32174 | $33 \div 41$ | 39302 | 45263 | 48849 | 55232 | 62292 | 65472 | 65971 | 65： 00 |
| Cが号が | 1.130 | $\because 24$ | 513 | 1079 | 1031 | 1100 | 1124 | 1161 | 1243 | 1363 | 1469 | 1524 | 1765 | 1771 | 1020 |
| ERUX：3－： | －1， | $\therefore 36$ | 228 | 524 | 526 | 540 | 554 | 576 | 630 | 705 | 772 | 854 | 935 | $\begin{array}{r}984 \\ \hline 008\end{array}$ | 1010 |
| VRLu天 A0UEO | 200 | $\therefore 08$ | 337 | 378 | 360 | 375 | 447 | 536 | 641 | 661 | 9 | 808 | 965 | 1008 | 2067 |
| SOUTH LESS SETETA PRUFL！ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1.952 | ：$\because 33$ | 1934 | 1955 | 1956 | 1957 | 1958 | 1959 | 1950 | 1961 | 1952 | 1963 | 1964 | 1963. | 1080 |
| L．A．tid |  | ： 0 934 | 44875 | 4070 | 24912 | 25963 | 25522 | 30015 | 34946 | 36741 | 33653 | 46374 | 47548 | 49630 | 49400 |
| crios ral | 2.35 | ¢ $6: 51$ | 1274 | 538 | 489 | 481 | 472 | 467 | 472 | 487 | 508 | 540 | 569 | 591 | 612 |
| Eculbielsur | $\because 1$ | 1．14 | $54 \hat{3}$ | 265 | 213 | 208 | 201 | 199 | 200 | 217 | 234 | 260 | 282 | 301 | 318 |
| Vraile Mujiod | － | $\therefore 57$ | 305 | 50 | 300 | 304 | 359 | 429 | 503 | 515 | 515 | 591. | 730 | 773 | 610 |
| SER：SIA PRJPER |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| － | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1953 | 1959 | 1960 | 1901 | 1962 | 1963 | 1964 | 1965 | 1060 |
| LASU： | 3：23 | 7：\％53 | 5346 | $2 \div 213$ | 81253 | 6206 | 6919 | 8747 | 10317 | 12103 | 16574 | ：5913 | 17824 | 1734： | 15700 |
| Chisfat | － | 2343 | 520 | 502 | $2 \div 29$ | 584 | 619 | 663 | $7 \div 0$ | 340 | 935 | 1019 | 119 | 1557 | 1180 |
| Fowiragnt | ：224 | 1062 | 253 | 220 | 1108 | 297 | 319 | 345 | 394 | 459 | 510 | 506 | 6238 | 661 | 276 |
| VALUE A！nEO | 39 | －：95 | 43 | 328 | 915 | $7 i$ | 88 | 107 | 138 | 145 | 104 | 213 | 235 | 2：5 | 2 J |










[^0]:    ${ }^{6}$ A study in the Theory of Investment (Chicago: University of Chicago Press, 1960).

[^1]:    - (1) Yugos lavia; (2) North (Slovenia, Croatia and Vojvodina); (3) South (Bosnia and Hercegovina, Montenegro, Macedonia, Serbia proper, the Kosmet); (4) Serbia proper; (5) South less Serbia proper.

[^2]:    ${ }^{14}$ Given the substantial inflation of the past two decades, the State levy of less than six per cent on fixed assets, and the interest charge on borrowed funds are not sufficiently great to serve to ration investment funds.

    $$
    { }^{15} \text { Mund lak, op. cite., p. } 146 .
    $$

[^3]:    ${ }^{16}$ For a discussion of the general theory of covariance estimators, see Henry Scheffe, The Analysis of Variance (New York: John Wiley \& Son, 1959), pp. 192-220.
    ${ }^{17}$ We are free to paramaterize our model so that $\Sigma_{h_{t}}=h_{0}=0$.

[^4]:    ${ }^{16}$ For a discussion of the general theory of covariance estimators, see Henry Scheffe, The Analysis of Variance (New York: John Wiley \& Son, 1959), pp. 192-220,
    ${ }^{17}$ We are free to paramaterize our model so that $\Sigma_{h_{t}}=h_{0}=0$.

[^5]:    ${ }^{24}$ A related bias-free estimator could be obtained by using ordinary covarlance applied only to one table, the capital table. However, experiment. 6 not reported here revealed this estimator to be less attractive than the two table reverse covariance estimators described above.

[^6]:    ${ }^{4}$ Benjamin Ward, "The Firm in Illyria: Marlet Syndicalism", American Economic Revicu, Vol. 40 , p. 504.

[^7]:    ${ }^{10}$ Dragomin Vojnic, Investicijc na Podruciu iucos 1avije 1947-1 5 , (Zagreb: Ekonomski Institut, 1960), p. 1כC.

