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Chen, Jiaqi and Lee, Sang-Ho

Chonnam National University, Chonnam National University

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Jiaqi Chen and Sang-Ho Lee

Graduate School of Economics, Chonnam National University, Republic of Korea

77 Yongbong-ro, Bukgu, Chonnam National University, Gwangju, 61186, Republic of Korea

Jiaqi Chen is a doctoral student of Chonnam National University (chenjiaqi@naver.com); Sang-Ho Lee is a professor of Chonnam National University and corresponding author (sangho@jnu.ac.kr) Tel: +82-62-530-1553, Fax: +82-62-530-1559

Abstract

This paper investigates strategic choices between duopolistic firms' R&D investments and government's output subsidies in an endogenous timing game with research spillovers. We show that a simultaneous-move game among three players appears at equilibrium if the spillovers are very low while government leadership with both firms' simultaneous-move game appears otherwise. We also show that government followership appears unless the spillovers are low or high, while both the government leadership and followership outcomes are socially desirable at equilibrium. However, a single firm's leadership equilibrium appears if the spillovers are high, but it causes a welfare loss.

Keywords: Endogenous timing game; Research spillovers, R&D investments; Output subsidies;

JEL Classifications: L13, L20, H20

1. Introduction

Recently the rapid development of new technologies in the world has forced firms and policy makers to foster R&D (research and development) investments which can provide new products in the market places or reduce their costs during production activities.[1] Since R&D exhibits externality features associated with research spillovers and its significant policy implications on innovation activities, works on R&D policies are currently gaining importance in oligopolistic industries and have become highly popular in public economics.[2]

Many researchers and policy makers have proposed various policies to encourage firms' R&D investments and spillovers directly and indirectly. Among the effective policy alternatives in the real world, governments are continuously increasing subsidization toward research institutions and organizations.[3] While the R&D subsidies have received much policy attentions, there have been some debates about their effectiveness.[4] As an effective option, governments might choose to subsidize output rather than R&D performances in cases where output enhancement is politically more popular and more straightforward.[5]

In the literature of output subsidies and R&D activities, a significant number of studies have concluded that the presence of R&D spillovers in which returns from R&D are inappropriate, is critical to assess the welfare effect of governmental intervention. For example, Lee (1998) found the

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[1] Regarding empirical works on the assessment of R&D activities in the literatures, See, for example, Mairesse and Mohnen (2004), Conte et al. (2009), Marinucci (2012), and Blanco et al. (2018).

[2] The research spillovers is usually defined as institutional and/or technological factors. A significant number of studies have confirmed that R&D spillovers exist and their implications on innovation and competition policy are presently significant in the oligopolies economies. See, for example, d'Aspremont and Jacquemin (1988), Kamien et al. (1992), Poyago-Theotoky (1995, 1999), Beath et al. (1998), Amir (2000), Gil Molto et al. (2011), Kesavayuth et al (2018), and Leal et al. (2020) among others.

[3] For instance, EU institutions have reaffirmed the commitment to R&D policies in which the budgets of the research Framework Programs have grown exponentially, from EUR 3.3 billion in 1984 to EUR 80 billion in Horizon 2020. In 2014, the Research, Innovation and Science Policy Experts (RISE) high-level group has also proposed double this budget, which would lead to a 7-year budget of more than EUR 120 billion in current prices for the next period.

[4] See Kauko (1996), Leahy and Neary (1997), Rebolledo and Sandonis (2012), and Lee and Tomaru (2017).

[5] Some examples of output subsidies include European agriculture subsidies, ethanol production subsidies in the United States, fertilizer subsidies, electric car production subsidies, and so on. See more discussions by World Trade Organization, "The Economics of Subsidies," for background on output subsidies.

significance of R&D spillovers in the design of output subsidy policy under asymmetric information. Leahy and Neary (1997) addressed the strategic relationship between optimal output/R&D subsidies and firms' R&D investments in a domestic market while Yoon and Choi (2018) considered these relations under international trade. Poyago-Theotoky (2002) analyzed environmental tax/subsidy policy toward firm's investment on abatement technology while Haruna and Goel (2017) emphasized the role of public institutes and compared output and R&D subsidies in mixed markets.[6]

In a formal set-up of regulatory framework between government and firms' R&D investments, it is assumed that firm's decision is sequentially chosen after the realization of subsidy policy. That is, the subsidy rate is exogenously fixed when the firms determine their R&D decisions in a committed policy setting where the government can credibly commit to its policy rule and firms maximize their objectives under the committed time-consistent policy framework. However, Leahy and Neary (1997) highlighted the firm's strategic decision in a different timing of investment and government policy where the firm chooses its R&D in advance before the government announces its policy. Then, the firm might induce the government to adjust its non-committed policy to be favorable to the firm in a time-inconsistency framework.[7]

This paper is the first to investigate an endogenous timing game on strategic choices between firm's R&D investment and government policy.[8] Specifically, in the presence of research spillovers, we examine an observable delay game with three-period and three-player model, extending the formulation of two-period and two-player model by Hamilton and Slutsky (1990) in a homogeneous duopoly market.

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[6] In the mixed markets where the public firm competes with private firms, the literature revealed the importance of subsidy policies in R&D competition. For example, Lee et al. (2017) examined output subsidy while Gil-Molto et al. (2011) considered R&D subsidy. Kesavayuth and Zikos (2013) compared output and R&D subsidies, while Lee and Tomau (2017) analyzed the optimal policy mix of output and R&D subsidies.

[7] For more analysis, see Petrakis and Xepapadeas (1999), Lutz et al. (2000), Requate and Unold (2001, 2003), Dijkstra (2002), and Poyago-Theotoky and Teerasuwannajak (2002). Recent works have also analyzed the opportunistic behaviors in different contexts where regulated firms are heterogeneous in their objectives. See, for example, Xu et al. (2017), Xu and Lee (2018), Lee et al. (2018), Leal et al. (2018), Garcia et al. (2018), Lian et al. (2018), and Escrihuela-Villar and Gutiérrez-Hita (2018).

[8] For discussions on the endogenous timing game, see Dowrick (1986), Robson (1990), Hamilton and Slutsky (1990), Amir and Stepanova (2006), and Amir (2006). A number of works applied this game into a mixed duopoly model where firms have different objectives. See, White (1996), Pal (1998), Matsumura (1999), Barcena-Ruiz (2007), Matsumura and Ogawa (2010), Naya (2015) and Lee and Xu (2018) among others.

In our extended model with an R&D-relevant game, therefore, firms determine their cost reducing R&D investments, either simultaneously or sequentially, while the government also chooses output subsidy policy, either simultaneously or sequentially. And then given endogenously chosen their decisions of timing, both firms play Cournot output competition in the last stage. This structural enhancement of the model allows us to anticipate when either the government or the firm is likely to play either a leader or a follower in making their decisions.

In the analysis, there are in total 27 subgames of choosing the timing of movement but our analysis provides the equilibrium results with 5 cases as follows. First, a simultaneous-move game among three players appears at equilibrium if the spillovers rate is very low while the government leadership (as a first-mover) equilibrium with a simultaneous-move game between the firms appears otherwise. Thus, the spillovers rate is crucial to determine the strategic timing of the game between the government and firms. In particular, if the spillovers rate is not so low, the commitment of output subsidy policy is attainable in an equilibrium in which both firms choose their R&D investments simultaneously after observing government policy. These findings are consistent to the previous results in the absence of governmental policy. For example, Leal et al. (2020) showed that both firms choose simultaneous game when the spillovers rate is low while sequential game when the spillovers rate is high.[9] Our analysis highlights the role of government policy in determining the strategic choices of R&D activities between the firms. In particular, our findings show that if the spillovers rate is not low, the government has strong incentive to be a leader in the equilibrium.

Second, the government followership (as a last-mover) equilibrium appears if the spillovers rate is intermediate. In special, the government followership equilibrium accompanying with a simultaneous-move game between the firms appears if the spillovers rate is relatively low while the government followership equilibrium accompanying with a sequential-move game between the firms appears if the spillovers rate is relatively high. This implies that the government might not commit to output subsidy rate and chooses its policy option opportunistically after observing firms' R&D investments if the spillovers rate is neither high nor low. This also shows a mixture of timing between the firms at

[9] In the absence of government policy, Leal et al. (2020) considered an endogenous R&D timing game and showed that the equilibrium is contrast to the endogenous output timing game in Hamilton and Slutsky (1990) who showed that only a sequential-move game appears at equilibrium.

equilibrium with government followership, either simultaneously or sequentially. Further, a sequential-move game between firms yields that a leading firm provides higher output and R&D investment but its profit is lower.

Third, if the spillovers rate is high, a single firm's leadership with a simultaneous-move game between the government and the other firm might appear at equilibrium. That is, a higher spillovers rate might induce a firm's leadership and thus a sequential-move game between firms. However, there are multiple equilibria where the government might choose to be a follower in a sequential-move game between the firms or a first-mover with government leadership in a simultaneous-move game between the firms.

Finally, we compare the welfare effects of the equilibrium outcomes in an endogenous timing game. We show that either the government leadership (as a first-mover) or the government followership (as a last-mover) with a simultaneous-move game between the firms can provide the highest welfare if the spillovers rate is not high. We also show that the government followership (as a last-mover) with a sequential-move game between the firms can provide the highest welfare when the spillovers rate is intermediate. However, there is a welfare loss in the equilibrium under either a simultaneous-move game among three players or a single firm's leadership with a simultaneous-move game between the government and the other firm.

The structure of this paper is as follows. In section 2, we introduce the basic model of a duopoly model with R&D investments and output subsidy. We then classified 8 subgames in an endogenous timing game in section 3 and analyze the fixed timing game in section 4. In section 5, we examine the equilibrium of an endogenous timing game and provide welfare comparisons. We conclude the paper in section 6.

2. The basic model

We consider a duopoly market in which two firms (1 and 2) produce homogeneous goods. The inverse demand function is denoted as $P = a - Q$ where P is the market price, $Q = q_1 + q_2$ is the market total output, and q_i is the output of firm *i=1,2*. The consumer surplus is denoted as $CS = \frac{Q^2}{2}$.

Following d'Aspremont et al. (1988) in a standard model of cost-reducing R&D investment with research spillovers, we assume that the cost functions in output production and R&D investment, respectively, are ex-ante identical between the firms and given as:[10]

$$C(q_i, x_i) = \left(c - x_i - \beta x_j\right)q_i + q_i^2 \quad \text{and} \quad \Gamma(x_i) = r x_i^2 \quad for \quad i = 1{,}2 \ \text{ and } \ i \neq j. \tag{1}$$

where x_i is the amount of R&D investment for firm i and $\beta \in [0,1]$ is the R&D spillovers rate. The initial cost c is reduced by each firm's R&D investment, x_i, and rival's R&D investment, βx_j, where $a > c > 0$. This means that depending on the research spillovers rate, R&D investment not only reduces a firm's own cost by x_i per unit of output, but also the rival firm's cost by βx_j per unit of output. Note that the firm has to spend x_i^2 to implement cost-reducing R&D, due to the R&D investment causes decreasing returns to scale.

We also assume that each firm receives an output subsidy, $s > 0$, which denotes the per-unit subsidy rate to output, financed by government. Then, the profit function of the firm is given as follows:

$$\pi_i = \left(a - q_i - q_j\right)q_i - \left(c - x_i - \beta x_j\right)q_i - q_i^2 - x_i^2 + s \ q_i, \qquad i = 1{,}2 \ \text{ and } \ i \neq j. \tag{2}$$

The social welfare is defined as the sum of consumer surplus and firms' profit minus total subsidy, which is given as follows:

$$W = CS + \pi_1 + \pi_2 - s(q_1 + q_2). \tag{3}$$

In this paper, we investigate the strategic choices between firm's R&D investment and government's output subsidy policy in the presence of research spillovers. We consider a multi-stage game where both government and firms first choose their output subsidy rate and cost reducing R&D investments, respectively, either simultaneously or sequentially given the rate of spillovers, and then later firms play Cournot competition in the last output stage. Note that we consider a simultaneous Cournot duopoly in output competition but allow for sequential Stackelberg games in their R&D investments and government's output subsidy policy. This structural enhancement of the model allows us to anticipate when either the government or firm is likely to play as either a leader or a follower in

[10] The model with linear demand and quadratic cost functions is a standard formulation and popularly used in the literature on the cost-reducing innovation. See, for example, Gil Molto et al. (2011), Kesavayuth and Zikos (2013), and Leal et al. (2020).

making their decisions regarding output subsidy policy and R&D investment, respectively.[11]

3. The subgames in an endogenous choice

In the analysis, we examine an endogenous timing between firm's R&D investment and government's output subsidy policy. We specifically extend an observable delay game in a two-period and two-player framework formulated by Hamilton and Slutsky (1990) into the three-period and three-player framework where the government (G) and both firms (firm F1 and firm F2) choose their timing to move among $T_k = 1,2,3$ where $k = G, F1, F2$ in determining their output subsidy and R&D choices, respectively. If all players choose the same period, it yields the equilibrium of a simultaneous-move game. If all players choose different period, it yields the equilibrium of a successive sequential-move game. Otherwise, the various equilibrium of both simultaneous-move and sequential-move games with leadership or followership emerges. Table 1 provides the payoff matrix of the game.

Table 1: Payoff Matrix of the Endogenous Choice Game

Government	$T_G = 1$			$T_G = 2$			$T_G = 3$		
Firm1 / Firm2	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$
$T_{F2} = 1$	(1,1,1)	(1,2,1)	(1,3,1)	(2,1,1)	(2,2,1)	(2,3,1)	(3,1,1)	(3,2,1)	(3,3,1)
$T_{F2} = 2$	(1,1,2)	(1,2,2)	(1,3,2)	(2,1,2)	(2,2,2)	(2,3,2)	(3,1,2)	(3,2,2)	(3,3,2)
$T_{F2} = 3$	(1,1,3)	(1,2,3)	(1,3,3)	(2,1,3)	(2,2,3)	(2,3,3)	(3,1,3)	(3,2,3)	(3,3,3)

In the matrix, the order of parentheses indicates the timing to move, i.e., $T_k = 1,2,3,$ of each player, $k = G, F1, F2$. There are 27 subgames of choosing the timing of movement (T_G, T_{F1}, T_{F2}) in total, but due to its symmetry, we can reduce to 8 cases as follows:

(1) Case I: simultaneous-move game {(1,1,1), (2,2,2) and (3,3,3)}

(2) Case II: simultaneous-move game between government and one firm and sequential-move game by

[11] Regarding the sequencing R&D decisions with Cournot competition in outputs, see Amir et al. (2000) in a private duopoly and Leal et al. (2020) in a mixed duopoly with corporate social responsibility.

the other firm's followership {(1,1,2), (1,1,3), (2,2,3); (1,2,1), (1,3,1), (2,3,2)}

(3) Case III: simultaneous-move game between the firms and sequential-move game by the government's leadership {(1,2,2), (1,3,3), (2,3,3)}

(4) Case IV: successive sequential-move game with government's leadership {(1,2,3), (1,3,2)}

(5) Case V: simultaneous-move game between the firms and sequential-move game by the government's followership {(2,1,1), (3,1,1), (3,2,2)}

(6) Case VI: simultaneous-move game between government and one firm and sequential-move game by the other firm's leadership {(2,1,2), (3,1,3), (3,2,3); (2,2,1), (3,3,1), (3,3,2)}

(7) Case VII: successive sequential-move game by the government's intermediation {(2,1,3), (2,3,1)}

(8) Case VIII: successive sequential-move game by the government's followership {(3,1,2), (3,2,1)}

In each case, in the last stage of output competition both firms decide their outputs simultaneously. We solve the subgame perfect Nash equilibrium of these games by backward induction.

4. Fixed timing game

In the last stage, given the output subsidy rate and R&D investments, the first order condition of the firm yields the following outputs:[12]

$$q_1 = \frac{1}{15}(3a - 3c + 3s + 4x_1 - x_2 - x_1\beta + 4x_2\beta)$$
$$q_2 = \frac{1}{15}(3a - 3c + 3s - x_1 + 4x_2 + 4x_1\beta - x_2\beta) \quad (4)$$

Now, we examine the 8 cases arranged in the above description, respectively, and then compare the equilibrium results.

Case I. *(T_G, T_{F1}, T_{F2})={(1,1,1), (2,2,2), (3,3,3)}*

This case considers a simultaneous-move game in which the government and both firms are all followers. Then, the government chooses output subsidy to maximize social welfare in Eq. (3) while each firm chooses its R&D investment to maximize the profits in Eq. (2) simultaneously. The first order

[12] It can be shown that the second-order conditions are satisfied.

conditions yield the following symmetric equilibrium outcomes between the firms:

$$s^I = \frac{15(a-c)}{2(26-3\beta+\beta^2)}$$

$$x_1^I = x_2^I = \frac{(a-c)(4-\beta)}{26-3\beta+\beta^2}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case I.

$$q_1^I = q_2^I = \frac{15(a-c)}{2(26-3\beta+\beta^2)}$$

$$\pi_1^I = \pi_2^I = \frac{(a-c)^2(193+16\beta-2\beta^2)}{2(26-3\beta+\beta^2)^2}$$

$$W^I = \frac{(a-c)^2(193+16\beta-2\beta^2)}{(26-3\beta+\beta^2)^2}$$

Case II. *(T_G, T_{F1}, T_{F2})={(1,1,2), (1,1,3), (2,2,3); (1,2,1), (1,3,1), (2,3,2)}*

This case considers a variant of sequential-move game that the government and firm *i* play a simultaneous-move game first and then firm *j* follows later, where $i \neq j$ *and* $i = 1,2$. Then, by backward induction, firm *j* chooses its R&D investment to maximize its profit. The resulting response function of firm *j*'s R&D is given by:

$$x_j = \frac{2(4-\beta)(3a-3c+3s-x_i+4x_i\beta)}{193+16\beta-2\beta^2}$$

Taking this reaction function into consideration, the government and firm *i* choose output subsidy and R&D investment, respectively and simultaneously. Then, the first order conditions yield the following asymmetric equilibrium outcomes between the firms:

$$s^{II} = \frac{(a-c)(286721+250218\beta-3486\beta^2-14224\beta^3+684\beta^4-168\beta^5+32\beta^6)}{1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8}$$

$$x_i^{II} = \frac{2(a-c)(78884+11219\beta+813\beta^2+1583\beta^3-1378\beta^4-48\beta^5+56\beta^6-4\beta^7)}{1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8}$$

$$x_j^{II} = \frac{2(a-c)(4-\beta)(19475+9673\beta-1553\beta^2+402\beta^3-16\beta^4-40\beta^5+4\beta^6)}{1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case II.

$$q_i^{II} = \frac{(a-c)(292781+139107\beta-6831\beta^2-6154\beta^3+204\beta^4+72\beta^5-4\beta^6)}{1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8}$$

$$q_j^{II} = \frac{15(a-c)(19475+9673\beta-1553\beta^2+402\beta^3-16\beta^4-40\beta^5+4\beta^6)}{1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8}$$

$$\pi_i^{II} = \frac{2(a-c)^2(1517+595\beta-69\beta^2-20\beta^3+2\beta^4)^2(31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6)}{(1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8)^2}$$

$$\pi_j^{II} = \frac{2(a-c)^2(193+16\beta-2\beta^2)(19475+9673\beta-1553\beta^2+402\beta^3-16\beta^4-40\beta^5+4\beta^6)^2}{(1018760+273067\beta-87193\beta^2+24242\beta^3-3674\beta^4+884\beta^5+188\beta^6-104\beta^7+8\beta^8)^2}$$

$W^{II} = (2(a-c)^2(148151879270 + 125894893231\beta + 7660552520\beta^2 - 4916046932\beta^3 + 2354395127\beta^4 - 504765428\beta^5 - 86966808\beta^6 + 39873768\beta^7 - 12992100\beta^8 - 162752\beta^9 + 783392\beta^{10} - 60704\beta^{11} - 12112\beta^{12} + 1792\beta^{13} - 64\beta^{14}))/(1018760 + 273067\beta - 87193\beta^2 + 24242\beta^3 - 3674\beta^4 + 884\beta^5 + 188\beta^6 - 104\beta^7 + 8\beta^8)^2$

Note that (i) $q_i^{II} \geq q_j^{II}$, $x_i^{II} \geq x_j^{II}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{II} \gtrless \pi_j^{II}$ if $\beta \lesseqgtr 0.25$. It implies that the leading firm provides higher output and R&D investment regardless of research spillovers rate while its profit is lower (higher) when the spillover rate is high (low).

Case III. *(T_G, T_{F1}, T_{F2})={(1,2,2), (1,3,3), (2,3,3))*

This case considers a variant of sequential-move game that the government is a leader while both firms follow and then choose R&D to maximize their profits by observing the output subsidy rate. The first order conditions of the firms yield the following response functions:

$$x_1 = x_2 = \frac{2(a-c+s)(4-\beta)}{67-6\beta+2\beta^2}$$

Using these reaction functions, the government chooses output subsidy to maximize social welfare. Then, the first order condition yields the following symmetric equilibrium outcomes between the firms:

$$s^{III} = \frac{(a-c)(217+154\beta-38\beta^2)}{788-244\beta+68\beta^2}$$

$$x_1^{III} = x_2^{III} = \frac{15(a-c)(4-\beta)}{394-122\beta+34\beta^2}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case III.

$$q_1^{III} = q_2^{III} = \frac{225(a-c)}{788 - 244\beta + 68\beta^2}$$
$$\pi_1^{III} = \pi_2^{III} = \frac{225(a-c)^2(193 + 16\beta - 2\beta^2)}{8(197 - 61\beta + 17\beta^2)^2}$$
$$W^{III} = \frac{225(a-c)^2}{788 - 244\beta + 68\beta^2}$$

Case IV. *(T_G, T_{F1}, T_{F2})={(1,2,3), (1,3,2)}*

This case considers a successive sequential-move game with the government's leadership. Then, by backward induction, firm *j* chooses R&D investment to maximize its profit. The first order condition provides the response function of firm *j*'s R&D:

$$x_j = \frac{2(4-\beta)(3a - 3c + 3s - x_i + 4x_i\beta)}{193 + 16\beta - 2\beta^2}$$

Taking this response function into consideration, firm *i* chooses R&D investment to maximize its profit. The first order condition provides the response function of firm *i*'s R&D:

$$x_i = \frac{2(a-c+s)(1924 + 39\beta + 62\beta^2 + 32\beta^3 - 36\beta^4 + 4\beta^5)}{31841 + 8880\beta - 2518\beta^2 + 768\beta^3 - 228\beta^4 + 64\beta^5 - 8\beta^6}$$

Using these two response functions of the firms, the government chooses output subsidy to maximize social welfare. The first order condition yields the following asymmetric equilibrium outcomes between the firms:

$$s^{IV} = \frac{(a-c)(48909433+71999516\beta+12402940\beta^2-5526068\beta^3+1733908\beta^4-537704\beta^5-116984\beta^6+72400\beta^7-25088\beta^8+5088\beta^9-352\beta^{10})}{12(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$
$$x_i^{IV} = \frac{(a-c)(7133+2596\beta-510\beta^2+124\beta^3-28\beta^4)(1924+39\beta+62\beta^2+32\beta^3-36\beta^4+4\beta^5)}{6(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$
$$x_j^{IV} = \frac{(a-c)(4-\beta)(3388175+2502774\beta-122546\beta^2-13828\beta^3+56640\beta^4-31520\beta^5+5864\beta^6-1056\beta^7+112\beta^8)}{6(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case IV.

$$q_i^{IV} = \frac{(a-c)(50936753+36527462\beta+765302\beta^2-1550452\beta^3+159320\beta^4-70912\beta^5-88\beta^6+1952\beta^7-112\beta^8)}{12(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$

$$q_j^{IV} = \frac{5(a-c)(3388175+2502774\beta-122546\beta^2-13828\beta^3+56640\beta^4-31520\beta^5+5864\beta^6-1056\beta^7+112\beta^8)}{4(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$

$$\pi_i^{IV} = \frac{(a-c)^2(31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6)(263921+167382\beta-7176\beta^2-5704\beta^3+1224\beta^4-528\beta^5+56\beta^6)^2}{72(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})^2}$$

$$\pi_j^{IV} = \frac{(a-c)^2(193+16\beta-2\beta^2)(3388175+2502774\beta-122546\beta^2-13828\beta^3+56640\beta^4-31520\beta^5+5864\beta^6-1056\beta^7+112\beta^8)^2}{72(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})^2}$$

$$W^{IV} = \frac{(a-c)^2(7133+2596\beta-510\beta^2+124\beta^3-28\beta^4)^2}{12(14851035+6166730\beta-1962522\beta^2+323914\beta^3+10604\beta^4-45852\beta^5+42340\beta^6-14632\beta^7+3624\beta^8-656\beta^9+48\beta^{10})}$$

Note that (i) $q_i^{IV} \geq q_j^{IV}$, $x_i^{IV} \geq x_j^{IV}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{IV} \gtrless \pi_j^{IV}$ if $\beta \lesseqgtr 0.25$. Thus, the same results with Case II occur under the asymmetric outcomes.

Case V. *(T_G, T_{F1}, T_{F2})={(2,1,1), (3,1,1), (3,2,2)}*

This case considers a variant of sequential-move game with the government's followership in which both firms choose R&D before government decides the output subsidy rate. Then, by backward induction, the government chooses the output subsidy to maximize social welfare. The first order condition provides the response function of the government:

$$s = \frac{1}{8}\left(2a - 2c + (x_1 + x_2)(1+\beta)\right)$$

Using this, both firms choose R&D to maximize their profits simultaneously. The first order condition yields the following symmetric equilibrium outcomes between the firms:

$$s^V = \frac{12(a-c)}{41-6\beta+\beta^2}$$

$$x_1^V = x_2^V = \frac{(a-c)(7-\beta)}{41-6\beta+\beta^2}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case V.

$$q_1^V = q_2^V = \frac{12(a-c)}{41-6\beta+\beta^2}$$

$$\pi_1^V = \pi_2^V = \frac{(a-c)^2(239+14\beta-\beta^2)}{(41-6\beta+\beta^2)^2}$$

$$W^{V} = \frac{2(a-c)^2(239+14\beta-\beta^2)}{(41-6\beta+\beta^2)^2}$$

Case VI. *(T_G, T_{F1}, T_{F2})={(2,1,2), (3,1,3), (3,2,3); (2,2,1), (3,3,1), (3,3,2)}*

This case considers a variant of sequential-move game where firm *i* moves first and then both the government and firm *j* follow simultaneously. The first order conditions of the government and firm *j* provide the following response functions:

$$s = \frac{2(a-c)(41+5\beta-\beta^2) + x_i(37+47\beta+8\beta^2-2\beta^3)}{2(152+11\beta-\beta^2)}$$
$$x_j = \frac{(4-\beta)(6a-6c-x_i(1-7\beta))}{152+11\beta-\beta^2}$$

Using these response functions, firm *i* chooses R&D to maximize its profits. The first order condition yields the following asymmetric equilibrium outcomes between the firms:

$$s^{VI} = \frac{(a-c)(11129+3922\beta-681\beta^2+268\beta^3-58\beta^4)}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$
$$x_i^{VI} = \frac{2(a-c)(3293+483\beta+304\beta^2+34\beta^3-72\beta^4+8\beta^5)}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$
$$x_j^{VI} = \frac{4(a-c)(4-\beta)(367+133\beta-30\beta^2+20\beta^3-4\beta^4)}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case VI.

$$q_i^{VI} = \frac{2(a-c)(5624+1927\beta-231\beta^2-32\beta^3+2\beta^4)}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$
$$q_j^{VI} = \frac{30(a-c)(367+133\beta-30\beta^2+20\beta^3-4\beta^4)}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$
$$\pi_i^{VI} = \frac{4(a-c)^2(37+10\beta-2\beta^2)^2}{38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6}$$
$$\pi_j^{VI} = \frac{8(a-c)^2(193+16\beta-2\beta^2)(367+133\beta-30\beta^2+20\beta^3-4\beta^4)^2}{(38287+8646\beta-3335\beta^2+1020\beta^3-342\beta^4+128\beta^5-16\beta^6)^2}$$

$W^{VI} = (4(a-c)^2(104404857 + 82160894\beta + 908031\beta^2 - 960888\beta^3 + 1753338\beta^4 - 754440\beta^5 + 196548\beta^6 - 32448\beta^7 - 7224\beta^8 + 2304\beta^9 - 128\beta^{10}))/(38287 + 8646\beta - 3335\beta^2 + 1020\beta^3 - 342\beta^4 + 128\beta^5 - 16\beta^6)^2$

Note that (i) $q_i^{VI} \geq q_j^{VI}$, $x_i^{VI} \geq x_j^{VI}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VI} \gtrless \pi_j^{VI}$ if $\beta \lesseqgtr 0.193$. Thus, the same results with Case II occur under the asymmetric outcomes, but the threshold of spillovers rate is lower.

Case *VII*. *(T_G, T_{F1}, T_{F2})={(2,1,3), (2,3,1)}*

This case considers a successive sequential-move game where firm *i* moves firstly, the government moves secondly and then firm *j* follows lastly. Then, firm *j* decides its R&D investment to maximize its profit. The first order condition provides the following response function:

$$x_j = \frac{2(4-\beta)(3a-3c+3s-x_i+4x_i\beta)}{193+16\beta-2\beta^2}$$

Using this response function, the government chooses subsidy to maximize welfare. The first order condition provides the following response function:

$$s = \frac{2(a-c)\left(1661+967\beta-51\beta^2-74\beta^3+8\beta^4\right)+x_i(1273+3789\beta-130\beta^2+206\beta^3-132\beta^4+16\beta^5)}{12(1042+109\beta-24\beta^2+8\beta^3-\beta^4)}$$

Using these response functions, firm *i* chooses R&D to maximize its profits. The first order condition yields the following asymmetric equilibrium outcomes between the firms:

$$s^{VII} = \frac{3(a-c)(-6139027-5415180\beta-82967\beta^2+281048\beta^3-46074\beta^4+10192\beta^5+3056\beta^6-1008\beta^7+64\beta^8)}{-65121239-16333554\beta+6705577\beta^2-1412924\beta^3+22684\beta^4-17840\beta^5-40772\beta^6+11440\beta^7+568\beta^8-288\beta^9+16\beta^{10}}$$

$$x_i^{VII} = \frac{2(a-c)(5480921+2154286\beta+583804\beta^2+129199\beta^3-134048\beta^4-20648\beta^5+7600\beta^6+280\beta^7-152\beta^8+8\beta^9)}{65121239+16333554\beta-6705577\beta^2+1412924\beta^3-22684\beta^4+17840\beta^5+40772\beta^6-11440\beta^7-568\beta^8+288\beta^9-16\beta^{10}}$$

$$x_j^{VII} = \frac{4(a-c)(4-\beta)(-620863-304167\beta+27340\beta^2-21277\beta^3-2223\beta^4+2734\beta^5-28\beta^6-60\beta^7+4\beta^8)}{-65121239-16333554\beta+6705577\beta^2-1412924\beta^3+22684\beta^4-17840\beta^5-40772\beta^6+11440\beta^7+568\beta^8-288\beta^9+16\beta^{10}}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case VII.

$$q_i^{VII} = \frac{12(a-c)(1580714+785343\beta-43451\beta^2-30505\beta^3+4803\beta^4-449\beta^5-139\beta^6+36\beta^7-2\beta^8)}{65121239+16333554\beta-6705577\beta^2+1412924\beta^3-22684\beta^4+17840\beta^5+40772\beta^6-11440\beta^7-568\beta^8+288\beta^9-16\beta^{10}}$$

$$q_j^{VII} = \frac{30(a-c)(620863+304167\beta-27340\beta^2+21277\beta^3+2223\beta^4-2734\beta^5+28\beta^6+60\beta^7-4\beta^8)}{65121239+16333554\beta-6705577\beta^2+1412924\beta^3-22684\beta^4+17840\beta^5+40772\beta^6-11440\beta^7-568\beta^8+288\beta^9-16\beta^{10}}$$

$$\pi_i^{VII} = \frac{4(a-c)^2(1517+595\beta-69\beta^2-20\beta^3+2\beta^4)^2}{65121239+16333554\beta-6705577\beta^2+1412924\beta^3-22684\beta^4+17840\beta^5+40772\beta^6-11440\beta^7-568\beta^8+288\beta^9-16\beta^{10}}$$

$$\pi_j^{VII} = \frac{8(a-c)^2(193+16\beta-2\beta^2)(620863+304167\beta-27340\beta^2+21277\beta^3+2223\beta^4-2734\beta^5+28\beta^6+60\beta^7-4\beta^8)^2}{(65121239+16333554\beta-6705577\beta^2+1412924\beta^3-22684\beta^4+17840\beta^5+40772\beta^6-11440\beta^7-568\beta^8+288\beta^9-16\beta^{10})^2}$$

$$W^{VII} = (2(a-c)^2(604454818565702 + 499033155530783\beta + 31059959705818\beta^2 - 8689146368490\beta^3 + 16905272721089\beta^4 - 1114914042353\beta^5 - 663813877578\beta^6 + 344122205936\beta^7 - 134890269479\beta^8 - 30428529928\beta^9 + 12467839048\beta^{10} + 1041752576\beta^{11} - 457612524\beta^{12} - 13852400\beta^{13} + 9563888\beta^{14} - 178752\beta^{15} - 108608\beta^{16} + 9728\beta^{17} - 256\beta^{18}))/(65121239 + 16333554\beta - 6705577\beta^2 + 1412924\beta^3 - 22684\beta^4 + 17840\beta^5 + 40772\beta^6 - 11440\beta^7 - 568\beta^8 + 288\beta^9 - 16\beta^{10})^2$$

Note that (i) $q_i^{VII} \geq q_j^{VII}$, $x_i^{VII} \geq x_j^{VII}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VII} \gtrless \pi_j^{VII}$ if $\beta \lessgtr 0.191$. Thus, the same results with Case II occur under the asymmetric outcomes, but the threshold of spillovers rate is lower.

Case VIII. *(T_G, T_{F1}, T_{F2})={(3,1,2), (3,2,1)}*

This case considers a successive sequential-move game where both firms move sequentially and the government follows lastly. The first order condition of the government provides the following response function:

$$s = \frac{1}{8}(2a - 2c + (x_i + x_j)(1+\beta))$$

Using this response function, firm *j* chooses R&D to maximize its profit. The first order condition of the firm provides the following response function:

$$x_j = \frac{(7-\beta)(6a - 6c - x_i(1-7\beta))}{239 + 14\beta - \beta^2}$$

Using these response functions, firm *i* chooses R&D to maximize its profits. The first order condition yields the following asymmetric equilibrium outcomes between the firms:

$$s^{VIII} = \frac{(a-c)(13855 + 4742\beta - 768\beta^2 + 362\beta^3 - 47\beta^4)}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$

$$x_i^{VIII} = \frac{8(a-c)(1015 + 135\beta + 128\beta^2 + 32\beta^3 - 15\beta^4 + \beta^5)}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$

$$x_j^{VIII} = \frac{2(a-c)(7-\beta)(577 + 202\beta - 51\beta^2 + 32\beta^3 - 4\beta^4)}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$

Then, putting these equilibrium results into (4) and then (2) and (3), we obtain the equilibrium outcomes in this case VIII.

$$q_i^{VIII} = \frac{2(a-c)(6931 + 2318\beta - 156\beta^2 - 22\beta^3 + \beta^4)}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$
$$q_j^{VIII} = \frac{24(a-c)(577 + 202\beta - 51\beta^2 + 32\beta^3 - 4\beta^4)}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$
$$\pi_i^{VIII} = \frac{8(a-c)^2(29 + 8\beta - \beta^2)^2}{47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6}$$
$$\pi_j^{VIII} = \frac{4(a-c)^2(239 + 14\beta - \beta^2)(577 + 202\beta - 51\beta^2 + 32\beta^3 - 4\beta^4)^2}{(47321 + 9492\beta - 4402\beta^2 + 1092\beta^3 - 471\beta^4 + 112\beta^5 - 8\beta^6)^2}$$
$$W^{VIII} = \frac{4(a-c)^2(159163953+120253250\beta+588897\beta^2-233052\beta^3+2814021\beta^4-1105614\beta^5+304095\beta^6-25080\beta^7-5814\beta^8+960\beta^9-32\beta^{10})}{(47321+9492\beta-4402\beta^2+1092\beta^3-471\beta^4+112\beta^5-8\beta^6)^2}$$

Note that (i) $q_i^{VIII} \geq q_j^{VIII}$, $x_i^{VIII} \geq x_j^{VIII}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VIII} \gtreqless \pi_j^{VIII}$ if $\beta \lesseqgtr 0.143$. Thus, the same results with Case II occur under the asymmetric outcomes, but the threshold of spillovers rate is lower.

5. Endogenous timing game

We now compare the equilibrium outcomes and find an equilibrium of an endogenous timing game between both firms and the government. We first compare the profits of the firms, given the choice of the government, and find the equilibrium choices of (T_G, T_{F1}, T_{F2}) in the subgames.[13]

Lemma 1.

(1) Suppose $T_G = 1$. Then, the equilibrium outcomes between the firms are (1,1,1) if $\beta \leq 0.054$; (1,2,2) if $0.054 < \beta \leq 0.464$; either (1,2,3) or (1,3,2) if $0.464 < \beta$.

(2) Suppose $T_G = 2$. Then, the equilibrium outcomes between the firms are (2,1,1) if $\beta \leq 0.062$; (2,3,3) if $0.062 < \beta$.

(3) Suppose $T_G = 3$. Then, the equilibrium outcomes between the firms are (3,1,1) if $\beta \leq 0.143$. either (3,1,2) or (3,2,1) if $0.143 < \beta \leq 0.61$. either (3,1,3) or (3,2,3) or (3,3,1) or (3,3,2) if $0.61 < \beta$..

Lemma 1 shows that there are several different equilibrium timings depending on the rate of spillovers and the movement of government. It states that (1) if the government decides to act in the first period, then case I, case III, case IV can an equilibrium; (2) if the government decides to act in the

[13] Note that the threshold of spillovers rate for each equilibrium outcome is represented by numbers with three decimal place for expositional convenience. All the proofs of lemmas and propositions are provided in Appendix.

second period, then case V and case III can be an equilibrium; (3) if government decides to act in the last period, then case V, case VIII, and case VI can be an equilibrium. Therefore, neither case II nor case VII can be an equilibrium.

We now compare the welfare ranks of $\mathrm{W}(\mathrm{T_G}, \mathrm{T_{F1}}, \mathrm{T_{F2}})$, given the firms' choices in their movements.

Lemma 2.

(1) Suppose that $T_{F1} = T_{F2} = 1$. *Then,* $W(1,1,1) \gtreqless W(2,1,1) = W(3,1,1)$ *if* $\beta \lesseqgtr 0.096$*;*

(2) Suppose that $T_{F1} = 1, T_{F2} = 2$. *Then,* $W(1,1,2) \gtreqless W(2,1,2)$ *if* $\beta \lesseqgtr 0.101$*;* $W(2,1,2) \gtreqless W(3,1,2)$ *if* $\beta \lesseqgtr 0.096$*;* $W(1,1,2) \gtreqless W(3,1,2)$ *if* $\beta \lesseqgtr 0.098$*;*

(3) Suppose that $T_{F1} = 1, T_{F2} = 3$. *Then,* $W(1,1,3) \gtreqless W(2,1,3)$ *if* $\beta \lesseqgtr 0.01$*;* $W(2,1,3) > W(3,1,3)$ *for all* $\beta \in [0,1]$.

(4) Suppose that $T_{F1} = 2, T_{F2} = 1$. *Then,* $W(1,2,1) \gtreqless W(2,2,1)$ *if* $\beta \lesseqgtr 0.101$*;* $W(1,2,1) \gtreqless W(3,2,1)$ *if* $\beta \lesseqgtr 0.098$*;* $W(2,2,1) \gtreqless W(3,2,1)$ *if* $\beta \lesseqgtr 0.096$*;*

(5) Suppose that $T_{F1} = T_{F2} = 2$. *Then,* $W(2,2,2) \gtreqless W(3,2,2)$ *if* $\beta \lesseqgtr 0.096$*;* $W(1,2,2) \gtreqless W(3,2,2)$ *if* $\beta \lesseqgtr 0.098$*;* $W(1,2,2) \geq W(2,2,2)$ *for all* $\beta \in [0,1]$*;*

(6) Suppose that $T_{F1} = 2, T_{F2} = 3$. *Then,* $W(2,2,3) \gtreqless W(3,2,3)$ *if* $\beta \lesseqgtr 0.101$*;* $W(1,2,3) \gtreqless W(3,2,3)$ *if* $\beta \lesseqgtr 0.098$*;* $W(1,2,3) \geq W(2,2,3)$ *for all* $\beta \in [0,1]$.

(7) Suppose that $T_{F1} = 3, T_{F2} = 1$. *Then,* $W(1,3,1) \gtreqless W(3,3,1)$ *if* $\beta \lesseqgtr 0.101$*;* $W(1,3,1) \gtreqless W(2,3,1)$ *if* $\beta \lesseqgtr 0.01$*;* $W(2,3,1) > W(3,3,1)$ *for all* $\beta \in [0,1]$*;*

(8) Suppose that $T_{F1} = 3, T_{F2} = 2$. *Then* $W(1,3,2) \gtreqless W(3,3,2)$ *if* $\beta \lesseqgtr 0.098$*;* $W(2,3,2) \gtreqless W(3,3,2)$ *if* $\beta \lesseqgtr 0.101$*;* $W(1,3,2) \geq W(2,3,2)$ *for all* $\beta \in [0,1]$*;*

(9) Suppose that $T_{F1} = T_{F2} = 3$. *Then,* $W(1,3,3), (2,3,3) \geq W(3,3,3)$ *for all* $\beta \in [0,1]$.

Using Lemma 1 and Lemma 2, we can obtain the following proposition.

Proposition 1. *The equilibrium of an endogenous timing game is as follows:*

(1) If $0 \leq \beta \leq 0.054$, $(T_G, T_{F1}, T_{F2}) = (1,1,1)$

(2) If $0.054 < \beta \leq 0.062$, $(T_G, T_{F1}, T_{F2}) = (1,2,2)$

(3) If $0.062 < \beta \leq 0.096$, $(T_G, T_{F1}, T_{F2}) = \{(1,2,2), (2,3,3)\}$

(4) If $0.096 < \beta \leq 0.098$, $(T_G, T_{F1}, T_{F2}) = \{(1,2,2), (2,3,3), (3,1,1)\}$

(5) If $0.098 < \beta \leq 0.143$, $(T_G, T_{F1}, T_{F2}) = \{(2,3,3), (3,1,1)\}$

(6) If $0.143 < \beta \leq 0.61$, $(T_G, T_{F1}, T_{F2}) = \{(2,3,3), (3,1,2), (3,2,1)\}$

(7) If $0.61 < \beta \leq 1$, $(T_G, T_{F1}, T_{F2}) = \{(2,3,3), (3,2,3), (3,3,2)\}$

Proposition 1 shows some interesting equilibrium outcomes in an endogenous timing game: First, a simultaneous-move game (case I) appears if the spillovers rate is very low ($0 \leq \beta \leq 0.054$) while the government leadership (as a first-mover) with a simultaneous-move game between the firms (case III) appears otherwise ($0.054 \leq \beta \leq 1$). Thus, the spillovers rate is crucial to determine the strategic timing of the game between the government and firms. In particular, if the spillovers rate is not so low, the commitment of output subsidy policy is attainable in an equilibrium while both firms choose their R&D investments simultaneously after observing government policy. These findings are consistent to the previous results in the absence of government in which both firms choose simultaneous game when the spillovers rate is low while sequential game when the spillovers rate is high.[14] Our analysis highlights the role of government policy in determining the strategic choices of R&D activities between the firms. It is noteworthy that our findings show unique equilibrium outcomes if the spillovers rate is low ($0 \leq \beta \leq 0.096$) in which the government has strong incentive to be a leader in the equilibrium.

Second, the government followership (as a last-mover) appears if the spillovers rate is intermediate ($0.096 \leq \beta \leq 0.61$). In special, the government followership accompanying with a simultaneous-move game between the firms (case V) appears if $0.096 \leq \beta \leq 0.143$ while the government followership accompanying with a sequential-move game between the firms (case VIII) appears if $0.143 \leq \beta \leq 0.61$. This implies that the government might not commit to output subsidy rate and chooses its policy

[14] For example, Hamilton and Slutsky (1990) and Amir and Stepanova (2006) examined an endogenous timing game and showed that only a sequential-move game between the firms appears in the equilibrium. While Amir (2000) and Leal et al. (2019) considered an endogenous R&D timing game and examined how R&D spillovers can affect the equilibrium of R&D choices game.

option opportunistically after observing firms' R&D investments if the spillovers rate is intermediate.[15] This in turn yields that a mixture timing between the firms either simultaneously or sequentially. Further, a sequential-move game between the firms produces the results that a leading firm provides higher output and R&D investment but its profit is lower.

Finally, a variant of sequential game where a sequential-move game by one firm's leadership and a simultaneous-move between government and the other firm (case VI) appears if $0.61 \leq \beta \leq 1$. That is, if the spillovers rate is high, a sequential-move game between the firms appears and at least one of the two firms prefers to be a leader. Note that there are also multiple equilibria in this range of spillovers wherein the government leadership (as a first-mover) with a simultaneous-move game between the firms (case III) also appears.

6. Welfare comparisons

We now compare the welfare consequences of the equilibrium outcomes of an endogenous timing game between both firms and the government. We first compare welfare ranks among 8 cases in choosing the timing of movement.

Proposition 2. *The welfare comparisons provide the followings:*

(1) Case III provides the highest welfare if $0 \leq \beta \leq 0.055$*;*

(2) Case IV provides the highest welfare if $0.055 < \beta \leq 0.099$*;*

(3) Case V provides the highest welfare if $0.099 < \beta \leq 0.143$*;*

(4) Case VIII provides the highest welfare if $0.143 < \beta \leq 1$.

Proposition 2 shows the importance of spillovers rate in the welfare ranks: First, the government leadership (as a first-mover) with a simultaneous-move game between the firms (case III) provides the highest welfare when the spillovers rate is very low ($0 \leq \beta \leq 0.055$). Second, the government leadership (as a first-mover) with a sequential-move game between the firms (case IV) provides the highest welfare when the spillovers rate is low ($0.055 \leq \beta \leq 0.099$). Third, the government

[15] Lee et al. (2019) compared irreversible R&D and flexible R&D in the presence of government policy, and highlighted the significant role of research spillovers, which yields contrasting results in equilibrium outcomes.

followership (as a last-mover) with a simultaneous-move game between the firms (case V) provides the highest welfare when the spillovers rate is not low ($0.099 \leq \beta \leq 0.143$). Finally, the government followership (as a last-mover) with a sequential-move game between the firms (case VIII) provides the highest welfare when the spillovers rate is high ($0.143 \leq \beta \leq 1$). Therefore, the mixture timing between the government and both firms in either simultaneous or sequential manner is not socially desirable.

Finally, proposition 1 and proposition 2 yield the following proposition:

Proposition 3. *The equilibrium outcome of an endogenous timing game can be socially desirable in the following cases that (i)* $(T_G, T_{F1}, T_{F2}) = (1,2,2)$ *if* $0.054 < \beta \leq 0.055$ *(ii)* $(T_G, T_{F1}, T_{F2}) = (3,1,1)$ *if* $0.099 < \beta \leq 0.143$*, and (iii)* $(T_G, T_{F1}, T_{F2}) = \{(3,1,2), (3,2,1)\}$ *if* $0.143 < \beta \leq 0.61$*. Otherwise, the equilibrium outcomes are not socially desirable.*

Proposition 3 provides the welfare consequences of the equilibrium outcomes of an endogenous timing game: In particular, it implies that (i) the equilibrium with the government leadership (as a first-mover) and a simultaneous-move game between the firms (case III) provides the highest welfare when $0.054 \leq \beta \leq 0.055$; (ii) the equilibrium with the government followership (as a last-mover) and a simultaneous-move game between the firms (case V) provides the highest welfare when $0.099 \leq \beta \leq 0.143$; (iii) the equilibrium with the government followership (as a last-mover) and a sequential-move game between the firms (case VIII) provides the highest welfare when $0.143 \leq \beta \leq 0.61$.

7. Conclusion

This paper is the first to consider the strategic relationship between R&D investments and output subsidies in the presence of research spillovers in an observable delay game with three-period and three-player model. We investigated an endogenous timing game among the government and firms under Cournot output competition, and found that the spillovers rate is crucial to determine the strategic timing of the game.

First, a simultaneous-move game appears at equilibrium if the spillovers rate is very low while the government leadership (as a first-mover) appears otherwise. That is, if the spillovers rate is not very low, the government has strong incentive to be a leader, which is socially desirable. Second, the government followership (as a last-mover) appears at equilibrium if the spillovers rate is intermediate, which can provide the highest welfare irrespective of whether the firms play a simultaneous-move game or a sequential-move game. That is, if the spillovers rate is intermediate, the government has strong incentive to be a follower, which is also socially desirable. Finally, a sequential-move game by a firm's leadership with a simultaneous-move game between government and the other firm appears at equilibrium if the spillovers rate is high. However, there is a welfare loss in this equilibrium.

Although there are limitations because of its model-specific assumptions in our model, we believe that our implications on R&D policy are useful in the endogenous timing game. However, it is required to examine the robustness of our findings under other market structures with differentiated products in oligopolistic competition. Finally, considering the increasing importance of the other governmental instruments, extending this model into mixed markets with/without R&D subsidy policy remains for the future research.

Appendix

A1. Proof of Lemma 1

(1) When $T_G = 1$, firm 1's profit ranks are as follows: (i) $\pi_1(1,1,1) \gtreqless \pi_1(1,2,1)$ if $\beta \lesseqgtr 0.054$; (ii) $\pi_1(1,1,2) \gtreqless \pi_1(1,2,2)$ if $\beta \lesseqgtr 0.054$; $\pi_1(1,1,2) \gtreqless \pi_1(1,3,2)$ if $\beta \lesseqgtr 0.06$; $\pi_1(1,2,2) \gtreqless \pi_1(1,3,2)$ if $\beta \lesseqgtr 0.353$; (iii) $\pi_1(1,1,2) \gtreqless \pi_1(1,2,3)$ if $\beta \lesseqgtr 0.058$; $\pi_1(1,2,2) \gtreqless \pi_1(1,2,3)$ if $\beta \lesseqgtr 0.464$.

Also, firm 2's profit ranks are as follows: (i) $\pi_2(1,1,1) \gtreqless \pi_2(1,1,2)$ if $\beta \lesseqgtr 0.054$; (ii) $\pi_2(1,2,1) \gtreqless \pi_2(1,2,2)$ if $\beta \lesseqgtr 0.054$; $\pi_2(1,2,1) \gtreqless \pi_2(1,2,3)$ if $\beta \lesseqgtr 0.06$; $\pi_2(1,2,2) \gtreqless \pi_2(1,2,3)$ if $\beta \lesseqgtr 0.353$; (iii) $\pi_2(1,2,1) \gtreqless \pi_2(1,2,2)$ if $\beta \lesseqgtr 0.054$; $\pi_2(1,2,1) \gtreqless \pi_2(1,3,2)$ if $\beta \lesseqgtr 0.058$; $\pi_2(1,2,2) \gtreqless \pi_2(1,3,2)$ if $\beta \lesseqgtr 0.464$.

Hence, comparing the profits ranks between the firms provides an equilibrium choices of timing between the firms when $T_G = 1$.

(2) When $T_G = 2$, firm 1's profit ranks are as follows: (i) $\pi_1(2,1,1) \gtreqless \pi_1(2,2,1)$ if $\beta \lesseqgtr 0.595$; $\pi_1(2,1,1) \gtreqless \pi_1(2,3,1)$ if $\beta \lesseqgtr 0.062$; $\pi_1(2,2,1) \gtreqless \pi_1(2,3,1)$ if $\beta \lesseqgtr 0.052$; (ii) $\pi_1(2,1,2) > \pi_1(2,2,2)$ for all $\beta \in [0,1]$; $\pi_1(2,3,2) \gtreqless \pi_1(2,1,2)$ if $\beta \gtreqless 0.063$; $\pi_1(2,2,2) \gtreqless \pi_1(2,3,2)$ if

$\beta \lesseqgtr 0.054$; (iii) $\pi_1(2,1,3) > \pi_1(2,2,3)$ for all β $\pi_1(2,3,3) \gtreqless \pi_1(2,1,3)$ if $\beta \gtreqless 0.062$; $\pi_1(2,2,3) \gtreqless \pi_1(2,3,3)$ if $\beta \lesseqgtr 0.054$.

Also, firm 2's profit ranks are as follows: (i) $\pi_2(2,1,1) \gtreqless \pi_2(2,1,2)$ if $\beta \lesseqgtr 0.595$; $\pi_2(2,1,1) \gtreqless \pi_2(2,1,3)$ if $\beta \lesseqgtr 0.062$; $\pi_2(2,1,2) \gtreqless \pi_2(2,1,3)$ if $\beta \lesseqgtr 0.052$; (ii) $\pi_2(2,2,1) > \pi_2(2,2,2)$ for all β; $\pi_2(2,2,3) \gtreqless \pi_2(2,2,1)$ if $\beta \gtreqless 0.063$; $\pi_2(2,2,2) \gtreqless \pi_2(2,2,3)$ if $\beta \lesseqgtr 0.054$; (iii) $\pi_2(2,3,1) > \pi_2(2,3,2)$ for all β ; $\pi_2(2,3,3) \gtreqless \pi_2(2,3,1)$ if $\beta \gtreqless 0.062$; $\pi_2(2,3,2) \gtreqless \pi_2(2,3,3)$ if $\beta \lesseqgtr 0.054$.

Hence, comparing the profits ranks between the firms provides an equilibrium choices of timing between the firms when $T_G = 2$.

(3) When $T_G = 3$, firm 1's profit ranks are as follows: (i) $\pi_1(3,1,1) \gtreqless \pi_1(3,2,1)$ if $\beta \lesseqgtr 0.143$; $\pi_1(3,1,1) \gtreqless \pi_1(3,3,1)$ if $\beta \lesseqgtr 0.595$; $\pi_1(3,2,1) > \pi_1(3,3,1)$ for all β; (ii) $\pi_1(3,1,2) > \pi_1(3,1,1)$ for all $\beta \in [0,1]$; $\pi_1(3,3,2) \gtreqless \pi_1(3,1,2)$ if $\beta \gtreqless 0.61$; $\pi_1(3,1,1) \gtreqless \pi_1(3,3,2)$ if $\beta \lesseqgtr 0.595$; (iii) $\pi_1(3,1,3), (3,2,3) > \pi_1(3,3,3)$ for all β.

Also, firm 2's profit ranks are as follows: (i) $\pi_2(3,1,1) \gtreqless \pi_2(3,1,2)$ if $\beta \lesseqgtr 0.143$; $\pi_2(3,1,1) \gtreqless \pi_2(3,1,3)$ if $\beta \lesseqgtr 0.595$; $\pi_2(3,1,2) > \pi_2(3,1,3)$ for all β; (ii) $\pi_2(3,2,1) > \pi_2(3,1,1)$ for all β ; $\pi_2(3,2,3) \gtreqless \pi_2(3,2,1)$ if $\beta \gtreqless 0.61$; $\pi_2(3,1,1) \gtreqless \pi_2(3,2,3)$ if $\beta \lesseqgtr 0.595$; (iii) $\pi_2(3,3,1), (3,3,2) > \pi_2(3,3,3)$ for all β.

Hence, comparing the profits ranks between the firms provides an equilibrium choices of timing between the firms when $T_G = 3$.

A2. Proof of Proposition 1

Using Lemma 1 and Lemma 2, we can obtain the following relations where the government and both firms do not want to deviate from their choices

(1) .$(T_G, T_{F1}, T_{F2}) = (1,1,1)$ *can be an equilibrium if* $0 \leq \beta \leq 0.054$.

(2) $(T_G, T_{F1}, T_{F2}) = (1,2,2)$ *can be an equilibrium if* $0.054 < \beta \leq 0.098$.

(3) $(T_G, T_{F1}, T_{F2}) = (2,3,3)$ *can be an equilibrium if* $0.062 < \beta \leq 1$.

(4) $(T_G, T_{F1}, T_{F2}) = (3,1,1)$ *can be an equilibrium if* $0.096 < \beta \leq 0.143$.

(5) $(T_G, T_{F1}, T_{F2}) = \{(3,1,2), (3,2,1)\}$ *can be an equilibrium if* $0.143 < \beta \leq 0.61$.

(6) $(T_G, T_{F1}, T_{F2}) = \{(3,2,3), (3,3,2)\}$ *can be an equilibrium if* $0.61 < \beta \leq 1$.

Arranging these results provides the equilibrium of endogenous timing game.

A3. Proof of Proposition 2

Due to the symmetry, we have the equivalence in welfare rankings in the same case. Hence, we need to compare the following 8 cases in the W(T_G, T_{F1}, T_{F2}). Then, we have the following welfare comparisons:

(1) Case I: $W(1,1,1) = W(2,2,2) = (3,3,3)$

- $W(1,1,1) \gtreqless W(1,1,2)$ if $\beta \lesseqgtr 0.055$;
- $W(1,1,1) \leq W(1,2,2)$ where equality holds only if $\beta = 0.053$.
- $W(1,1,1) \gtreqless W(1,3,2)$ if $\beta \lesseqgtr 0.055$.
- $W(1,1,1) \gtreqless W(2,1,1)$ if $\beta \lesseqgtr 0.096$.
- $W(1,1,1) \gtreqless W(2,1,2)$ if $\beta \lesseqgtr 0.096$.
- $W(1,1,1) \gtreqless W(2,1,3)$ if $\beta \lesseqgtr 0.095$.
- $W(1,1,1) \gtreqless W(3,1,2)$ if $\beta \lesseqgtr 0.096$.

(2) Case II: $W(1,1,2) = W(1,1,3) = W(2,2,3) = W(1,2,1) = W(1,3,1) = W(2,3,2)$

- $W(1,1,2) \geq W(1,2,2)$ if $0.055 \leq \beta \leq 0.122$ or $0.643 \leq \beta \leq 1$.
- $W(1,1,2) \leq W(1,3,2)$ where equality holds only if $\beta = 0.058$.
- $W(1,1,2) \gtreqless W(2,1,1)$ if $\beta \lesseqgtr 0.098$.
- $W(1,1,2) \gtreqless W(2,1,2)$ if $\beta \lesseqgtr 0.101$.
- $W(1,1,2) \gtreqless W(2,1,3)$ if $0 \leq \beta \leq 0.1$.
- $W(1,1,2) \gtreqless W(3,1,2)$ if $\beta \lesseqgtr 0.098$.

(3) Case III: $W(1,2,2) = W(1,3,3) = W(2,3,3)$:

- $W(1,2,2) \underset{<}{\overset{\geq}{=}} W(1,3,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.055$.
- $W(1,2,2) \underset{<}{\overset{\geq}{=}} W(2,1,1)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.098$.
- $W(1,2,2) \underset{<}{\overset{\geq}{=}} W(2,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.1$.
- $W(1,2,2) \underset{<}{\overset{\geq}{=}} W(2,1,3)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.099$.
- $W(1,2,2) \underset{<}{\overset{\geq}{=}} W(3,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.098$.

(4) Case IV: $W(1,3,2) = W(1,2,3)$:

- $W(1,3,2) \underset{>}{\overset{\leq}{=}} W(2,1,1)$ if $\beta \underset{<}{\overset{\geq}{=}} 0.099$.
- $W(1,3,2) \underset{>}{\overset{\leq}{=}} W(2,1,2)$ if $\beta \underset{<}{\overset{\geq}{=}} 0.102$.
- $W(1,3,2) \leq W(2,1,3)$ if $0.101 \leq \beta \leq 1$.
- $W(1,3,2) \underset{<}{\overset{\geq}{=}} W(3,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.099$.

(5) Case V: $W(2,1,1) = W(3,1,1) = W(3,2,2)$:

- $W(2,1,1) \underset{<}{\overset{\geq}{=}} W(2,1,2)$ if $\beta \underset{<}{\overset{\geq}{=}} 0.096$.
- $W(2,1,1) \geq W(2,1,3)$ if $0.097 \leq \beta \leq 0.756$.
- $W(2,1,1) \underset{<}{\overset{\geq}{=}} W(3,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.143$.

(6) Case VI: $W(2,1,2) = W(3,1,3) = W(3,2,3) = W(2,2,1) = W(3,3,1) = W(3,3,2)$:

- $W(2,1,2) < W(2,1,3)$ for all $\beta \in [0,1]$.
- $W(2,1,2) \underset{<}{\overset{\geq}{=}} W(3,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.096$.

(7) Case VII: $W(2,1,3) = W(2,3,1)$:

- $W(2,1,3) \underset{<}{\overset{\geq}{=}} W(3,1,2)$ if $\beta \underset{>}{\overset{\leq}{=}} 0.097$.

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