University of Nevada,
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A Geostatistical Study of the Manhattan Gold Deposit

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mining Engineering

by

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ABSTRACT

Geostatistical reserve estimation techniques are divided between those which assume an underlying distribution function for the data and those that do not. This thesis applies a parametric technique, Lognormal Kriging, and a non-parametric technique, Probability Kriging, to the structurally controlled Manhattan gold deposit located in Manhattan, Nevada. The average grade of the deposit is suitable for bulk mining and heap leaching, however, substantial portions of the deposit are high grade mill ore. These high grade pods are small in size relative to the dimensions of the exploration drilling grid. Therefore, interpretation of the exploration data by other than a probabilistic method will give no indication of the presence of a high grade pod other than the average grade of a block. Probability kriging estimates a distribution function for each estimated block. The estimated distribution may be used to determine average grades and tonnages of blocks above specified cutoff grades.

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LIST OF SYMBOLS

z(x)	realization of a random variable
$D^2(V/G)$	dispersion of grades of blocks with support V within the domain g
$D^2(v/G)$	dispersion of grades of coves with support v within the domain g
m	mean
Zc	cutoff grade
f(x)	probability distribution of z(x)
F(x)	distribution function of z(x)
$\int x$	without limits indicates integrating over the space defined by x
smu	selective mining unit
OK	ordinary kriging
LNK	lognormal kriging
IK	indicator kriging
PK	probability kriging
IDW	inverse distance weighting
MG	multigaussian kriging
A	a stationary field
E(z(x))	expected value of the random variable z(x)
μ_2	the second moment about the mean, the variance
$\mu_{1,1}$	the covariance
h	a vector distance
$\gamma(h)$	experimental variogram
E(z(x)I z)	conditional expectation
i(x;z)	indicator function
i(x;z)	expected value of indicator function

ϕ (A;zc)	realized proportion of the random function
E{*I(x;z)*}	conditional distribution function of the random
u(x)	uniform transform of the realization z(x)
$\sum_i$	summation over i
ϕ^*(A;zc)	the estimate of the spatial distribution for a panel A, given the cutoff grade zc
$i(x_j$; zc)	the indictor transform of the data value at location j, given the cutoff grade zc
$u(x_j)$	the uniform transform of the data value at location j
λ_j	the weight applied to the indictor transform of the data value at location j
v_j	the weight applied to the uniform transform of the data value at location j
N	the number of data in the kriging neighborhood
μ_1 and μ_2	lagrangian multipliers
σ^2_{PK}	probability kriging estimation variance
T(A;zc)	the tonnage recovered above a cutoff grade zc, within A
$Q(zc_k)$	the quantity of metal above the cutoff grade zc
nc	the number of cutoff grades
m^*(A;zc)	estimated mean grade above a cutoff grade zc
[A]	the data covariance matrix
[B]	the matrix of covariances between the data and the point or block to be estimated
[X]	the matrix of kriging weights
T_{+zc}/T	the proportion of tonnage in a block above a cutoff grade
u_b	the mean grade of a block
Q_{+zc}/Q	the proportion of the total quantity of metal above a cutoff grade

Chapter 1 Introduction

To properly evaluate a mineral deposit with exploration drilling data both the spatial distribution and the concentration of mineralization must be identified. Over estimation of recoverable reserves may result in the loss of investment capital, while underestimation of reserves may result in a lost opportunity for investment. The correct application of geostatistical theory will result in improved reserve estimates, and reduce the risk of error in investment decisions.

Development drilling projects have as their objective to establish a mineral inventory and a deposit block model. The mineral inventory may be best represented graphically in the form of a grade-tonnage curve. Such a diagram facilitates the selection of cutoff grades for design purposes and economic evaluation. The block model is the basis for production scheduling and the final mine design. The integration of reserve estimation and cost estimation algorithms results in blocks being assigned net dollar values. For the purpose of long-term mine planning the exact location of each selective mining unit (smu) size block with a positive net value that lies within a large panel is unimportant. It is adequate at the long-term planning stage to have an estimate of the distribution of grades within the large panel. From such a distribution the tonnage, mean grade, and quantity of metal above a given cutoff grade may be calculated.

1.1 The scope of the problem

The gold-silver deposits currently being exploited in the Western U.S. may be generally categorized as follows: sedimentary rock-hosted; volcanic rock-hosted or associated with hot springs; and deposits associated with intrusive rocks. The specific attributes of a given deposit are unique unto itself and the reserve estimation problem will be similarly unique.

For an estimation technique to reliably estimate the reserves of a deposit the technique must not be limited in its ability to quantify the spatial variability within that deposit. Traditional estimation techniques such as inverse distance weighing and estimation by polygons are not intended to quantify 3-D spatial variability. Ordinary kriging, through the use of the variogram can quantify spatial variability in 3-D; however, ordinary kriging is limited by the assumption that the distribution of values is multivariate normal. Deviations from normality by the distribution of grades of an exploration data set may result in ordinary kriging producing a less than optimal result.

A problem faced in estimating recoverable reserves based on samples taken from either a core drill or reverse circulation rotary drill is that the support size of the sample is much smaller than the minimum size block that can be distinguished as ore or waste during the mining process. Figure 1.1 indicates the theoretical relationship between

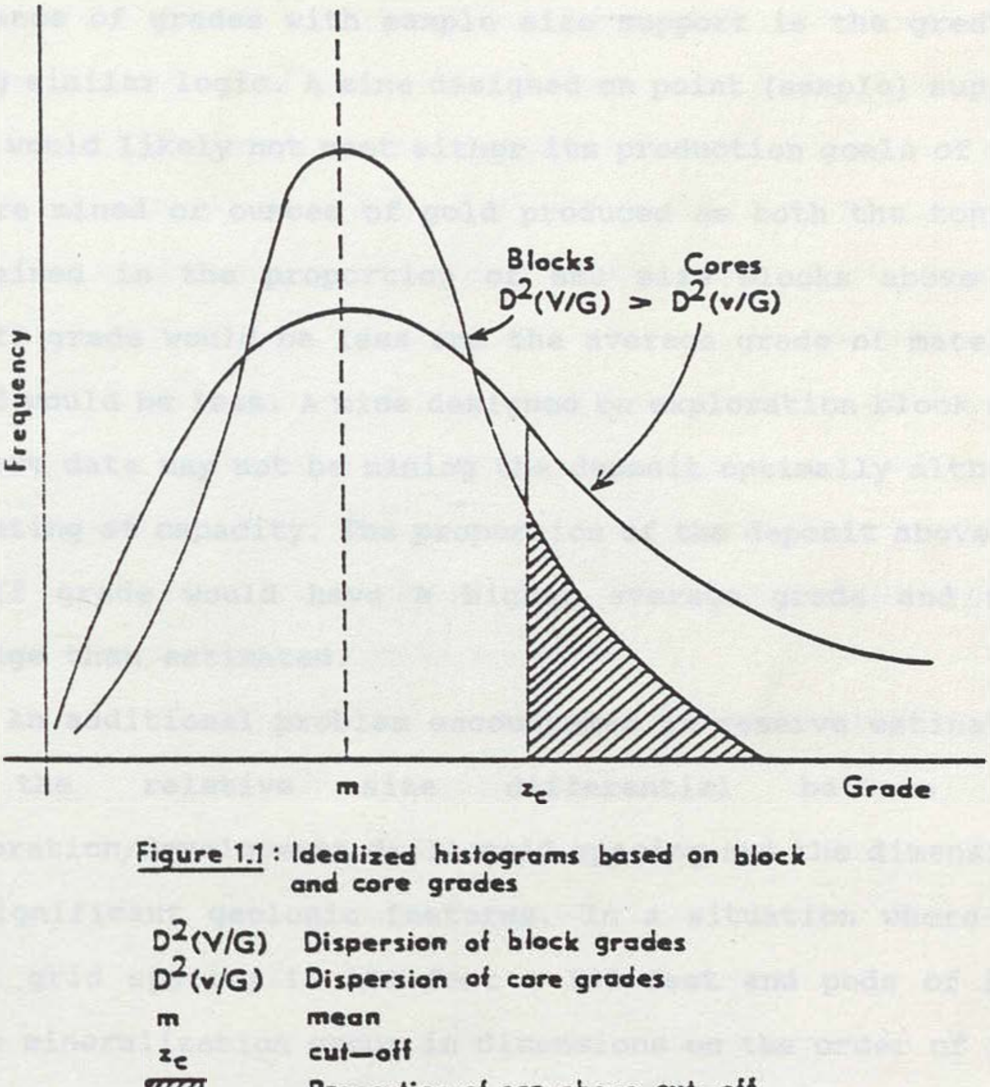


Figure 1.1 : Idealized histograms based on block and core grades

$D^2(V/G)$ Dispersion of block grades

$D^2(v/G)$ Dispersion of core grades

m mean

z_c cut-off

[hatched box] Proportion of ore above cut-off based on blocks exploration size blocks, smu size blocks, and samples. Following the premise of the permanence of shape hypothesis, all support sizes maintain the same mean grade. The variance of the exploration size block grades is the least as would be expected from the volume-variance relationship, and the variance of grades with sample size support is the greatest using similar logic. A mine designed on point (sample) support data would likely not meet either its production goals of tons of ore mined or ounces of gold produced as both the tonnage contained in the proportion of smu size blocks above the cutoff grade would be less and the average grade of material mined would be less. A mine designed on exploration block size support data may not be mining the deposit optimally although operating at capacity. The proportion of the deposit above the cutoff grade would have a higher average grade and more tonnage than estimated.

An additional problem encountered in reserve estimation is the relative size differential between the exploration/development drill grid spacing and the dimensions of significant geologic features. In a situation where the drill grid spacing is 100 feet x 100 feet and pods of high grade mineralization occur in dimensions on the order of 10 - 20 feet in length, and 5 - 20 feet in width, interpretation of the exploration data by other than a probabilistic method will give no indication other than the average grade as to the presence of a high grade pod or internal waste within a given area.

Also, gold deposits typically have a positively skewed distribution of grades, with a small percentage of high grade data residing in the tail of the distribution. Although a small percentage of data, frequently these ultrahigh grades represent a significant portion of the total contained metal in the deposit. The presence of these data often results in erratic and essentially useless variograms of the untransformed values. If the spatial variability of these data deviates substantially from that of the mean of the deposit, then use of either traditional estimation techniques or ordinary kriging may result in the overestimation of the true range of influence of the high grades, and consequently, the overestimation of the reserves.

To summarize the problem, the mining industry requires that a reserve estimation technique:

1) be capable of using all representative data without trimming;
2) provide information as to the spatial distribution of intermingled ore and waste zones;
3) accurately estimate the grade distribution of smu size blocks within an exploration size panel.

1.2 The theory of random variables

A mineral deposit may be thought of as a sample space with an infinite number of potential sample points. When the mineral deposit is sampled a value is assigned to a particular location. Associating values over a set of points within a sample space is, in effect, defining a function over the points of a sample space. By definition, this real-valued function, defined over the sample space, is a random variable. In the case of a mineral deposit, where the sample space is infinite, the random variable is a continuous random variable. If establishing the quantity of gold contained per unit mass of rock is the desired goal, then at any location 'x' an assayed grade may be represented by a random variable Z(x). The total set of samples taken for all locations x in the deposit defines the distribution function for the random variable, Z(x). Because the distribution function takes into consideration the entire mineral deposit, it must account for both the random nature of the deposit from point to point, as well as the overall spatial structure.

If Z(x) is a continuous random variable, the function expressed:

$$F(x) = P(Z(x) <= x) = \int_{-\infty}^{x} f(t)\,dt \qquad (1.1)$$

where f(t) is the value of the probability density function of Z(x) at t, is called the distribution function, or the cumulative distribution function of Z(x).

Certain properties hold for the distribution function in the continuous case; that is F(- inf.) = 0, F(inf.) = 1, and

$F(a) <= F(b)$ where $a < b$.

It follows then that if $f(x)$ and $F(x)$ are, respectively, the values of the probability distribution and the distribution function of $Z(x)$ at x, then:

$$P(\ a <= Z(x) <= b\ ) = F(b) - F(a) \tag{1.2}$$

for any real constants a and b with $a <= b$, and

$$f(x) = dF(x)/dx \tag{1.3}$$

where the derivative exists.

In the case of evaluating a mineral deposit we are interested in what the value of the random variable $Z(x)$ will be at a sampling location. If $Z(x)$ is a continuous random variable and $f(x)$ is the value of its probability density at x, the expected value of this random variable is

$$E(Z(x)) = \int_{-\infty}^{\infty} x * f(x)\, dx \tag{1.4}$$

where the integral exists.

Among the mathematical expectations that are of importance to statistics and have relevance when establishing the distribution of grades in a mineral deposit are the mean of the distribution and the variance. The mean is defined as the first moment about zero of the distribution of $Z(x)$. Moments of the distribution of a random variable serve to describe the shape of the distribution of the random variable, ie. the shape of the graph of its probability distribution or probability density. The spread or dispersion of the distribution of a random variable is calculated by the second moment about the mean. This quantity is known as the variance of Z(x). It was shown in Figure 1.1 that the variance is instrumental in determining both the tonnage above a given cutoff grade and the average grade above cutoff. The variance is denoted symbolically,

$$u_2=E[(Z(x)-u)^2]=\int_{-\infty}^{\infty}(x-u)^2*f(x)\,dx \qquad (1.5)$$

when Z(x) is a continuous random variable. Similarly, the dispersion variance ($D^2(v/V)$) may be denoted:

$$D^2(v/V)=1/V\int_V E[(Z_v(x)-u)^2dx \qquad (1.6)$$

where V is the total volume of the deposit, and $Z_v(x)$ is the estimate of the grade at the location x.

The random function in describing the complete set of random variables must account for the random element of variance from point to point, as well as the overall spatial variability. In the case of a mineral deposit the grades are distributed partly in a random fashion, and partially structured. A relative degree of continuity exists dependent upon the mineralizing process which took place. Without this continuity estimation of ore reserves would not be possible as spatial variability would be purely random.

Given that some of the points within a mineral deposit are not independent of one another, then a quantification of the measure of the relationship between two points would be of value. The relationship between two random variables $Z(x_1)$ and $Z(x_2)$ is called the covariance. Where $Z(x_1)$ and $Z(x_2)$ are continuous random variables the covariance may be represented

$$u_{1,1}=E[(Z(x_1-u_1(Z(x_2)-u_2)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1-u_1)(x_2-u_2)f(x_1,x_2)\,dx_1dx_2 \quad (1.7)$$

Note that if there is a high probability that high values of $Z(x_1)$ go with high values of $Z(x_2)$, or if low values of $Z(x_1)$ go with low values of $Z(x_2)$, the covariance will be positive; if there is a high probability that high values of $Z(x_1)$ go with low values of $Z(x_2)$, or vice versa, then the covariance will be negative.

The variability of two values $z(x_1)$ and $z(x_2)$ at two separate locations x_1 and x_2 may be evaluated on the basis of the vector h separating the two points. In geostatistics, the variogram is defined as the expectation of the difference between the random variables $[Z(x_1) - Z(x_2)]^2$, and may be expressed:

$$2\,\gamma\,(x_1,x_2) = E\,[(Z(x_1) - Z(x_2))^2]. \quad (1.8)$$

For statistical inference to be possible, several realizations must be available. If the variogram function is only dependent upon the vector distance h between x_1 and x_2, then statistical inference is possible. This assumption is known as the assumption of stationarity, which formally has two parts:

1) the mathematical expectation, $E(Z(x)) = u$, exists, and is independent of the support size;
2) for each pair of random variables a covariance exists and is dependent upon the

separation distance:

$$C(h) = E[Z(x+h) * Z(x)] - u^2 \quad (1.9)$$

where h is a vector.

The act of sampling, be it by hand, with an exploration drill hole, or a production blast hole, fixes the value of the random function at a discrete set of points. To estimate any of the remaining unsampled locations given the sample data involves estimating conditional probability distributions at the unsampled locations. The best possible estimator of the realization at an unknown point is the conditional expectation,

$$E_n Z_o = E(Z(x_o) | Z_1, Z_2, Z_3, \ldots Z_n) \quad (1.10)$$

where Z_a, a = 1 to n, are the n available data. In a mining situation, this estimator is the conditional estimator of the true block grades given the values of the exploration drill hole composites in the estimation neighborhood. How each of the estimation techniques used in this thesis apply conditional expectation to estimate the grade and tonnage above a particular cutoff is discussed in Chapter 2.

1.3 Previous work

Since the inception of non-parametric estimation techniques (Journal, 1984) (Sullivan, 1985) several case studies have been made to compare the performance of the non-parametric estimators with the parametric methods of ordinary kriging (OK), lognormal kriging (LNK), and multigaussian kriging (MG).

Verly and Sullivan (1985) estimated local recoveries with point support at the Jerritt Canyon mine using MG and probability kriging (PK). They concluded that the estimators performed equally well.

Kim et. al (1987a), examined indicator kriging (IK) and OK on a Carlin-type deposit with a positively skewed grade distribution. IK was used to estimate grade distributions for several panels, while OK estimates were limited to average grade. The OK blocks were accepted into or rejected from the mineral inventory on the basis of the estimated average grade. Kim concluded that IK estimates were unbiased, where as OK tended to underestimation.

Lindsey (1987) estimated recoverable reserves at the Golden Sunlight gold deposit in Montana in a case study comparing PK and OK. Exploration data distributions for this deposit were also positively skewed. Estimation of global recoverable reserves using PK were significantly more accurate than when using OK, particularly at higher cutoff grades. Lindsey concluded that PK out performed OK on an overall basis.

Van Brunt and Taylor (1989) compared PK, OK, and inverse distance weighting (IDW) using exploration data to the blast hole data refined exploration block model currently in use at the Manhattan Mine, which is calculated using a discriminator kriging approach. They concluded that PK performed admirably, while severe overestimation was encountered with OK and IDW.

Kim et. al (1987b), compared the distributions of grades estimated for individual panels on a bench of a Carlin-type gold mine using IK, LNK, and PK. The PK and IK estimators calculate these distributions directly, whereas for LNK to estimate a distribution an assumption that the distribution of grades in each panel is lognormally distributed must be made. The study was confined to comparison of results at only one cutoff grade. The distribution of blasthole data within each panel was used as the 'actual' database for comparison. Kim concluded that the additional variography required to perform PK versus IK was warranted given the superior performance. He also concluded that if the underlying assumption of lognormality is valid then LNK is a viable alternative for reserve estimation.

The calculation of local recoverable reserves requires the estimation of the proportion of values above a given cutoff grade, ie. the realization of the distribution of the random variable Z(x) at a series of cutoff grades. This proportion may be thought of as a conditional probability.

Parker and Switzer (1975) investigated the use of conditional probability distribution functions in ore reserve estimation. Given n data in a neighborhood, the number of ore and waste smu's in a large exploration size panel were estimated. Further applications included the determination of exploration drillhole locations, valuation of mineable blocks in pit design, and estimation of ultimate pit limits.

Journal (1980) discussed the generation of lognormal conditional distribution functions as applied to local reserve estimation, and the use of cross validation to verify the results. In the case of the Imouraren uranium deposit the exploration data values were lognormally distributed, and the use of a lognormal transformation on the data was therefore suitable.

Journal (1983) developed the indicator function, which is binomially distributed, with expected value

$$E\{I(x;z)\} = 1 * \text{Prob}\ \{Z(x) <= z\} + 0 * \text{Prob}\ \{Z(x) > z\}$$
$$= \text{Prob}\ \{Z(x) <= z\} = F(z), \qquad (1.11)$$

for all x in a stationary field. If the stationary field is designated 'A', then the proportion of the distribution below a particular cutoff grade may be defined:

$$\varnothing(A;z) = 1/A \int_A i(x;z)\,dx \qquad (1.12)$$

Where $\varnothing(A;z)$ is the realized proportion of the random function. Kriging over a series of cutoffs would then define a step-wise cumulative distribution of grades. The indicator transform forms the basis for the development of the PK estimator, which also estimates conditional probability distributions.

1.4 Scope and organization

The objective of this thesis is two-fold:

1) To compare the distribution of grades within an exploration size panel, as calculated by PK and LNK, to the distribution of blasthole grades

within each panel.

2) To evaluate the usefulness of mapping conditional probabilities, as calculated from exploration data using PK, in identifying the spatial distribution of mineralization during the exploration/development phase of a mining project.

The development of the non-linear estimators used to calculate the distributions of grades within exploration size blocks is discussed in Chapter 2. A brief history of the district and summary of the geology as it is presently understood follows in Chapter 3. Chapter 4 covers the statistical analysis of the exploration data and the selection of cutoff grades. In Chapter 5, covariance as related to structural analysis and the estimation of variograms using transformed data will be discussed. Variogram models will be tested by cross validation. Chapter 6 summarizes the kriging results by comparing the distribution

s calculated from exploration data with those derived from the blastholes at several cutoff grades. In Chapter 7, mapping conditional probabilities derived from exploration data using PK will be compared to contoured blasthole maps. The application of this technique to the evaluation of mineral deposits is discussed.

Chapter 2 Probability Kriging and Lognormal Kriging

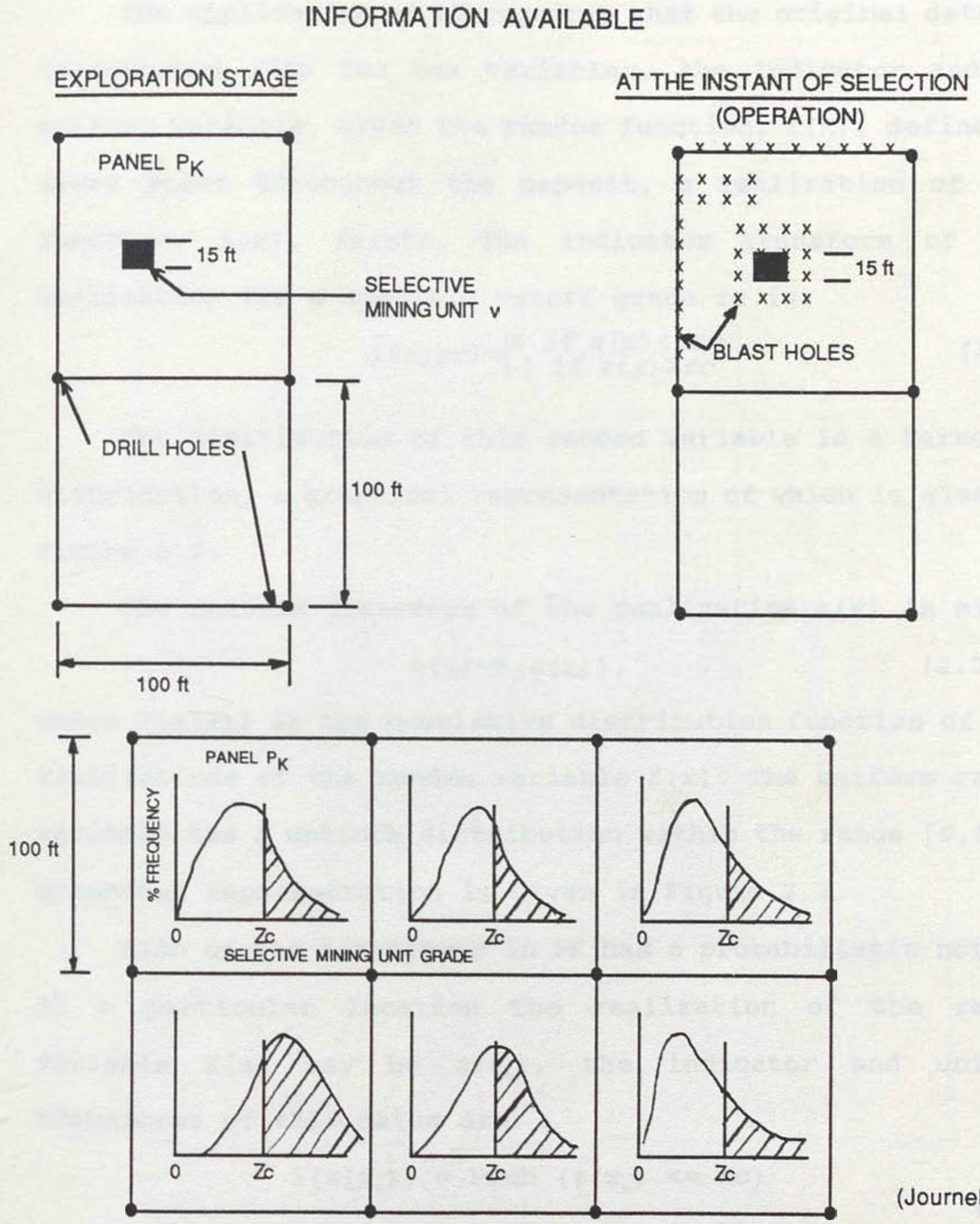
The principles of random variables have been discussed in Chapter 1. The introduction of PK, a non-parametric estimator, and LNK, a parametric estimator, will be covered in this chapter. The end result of using each of these estimators is an estimate of the spatial distribution of point support grades within a block in the deposit (Figure 2.1). PK estimates this distribution directly after simple transformations of the original data. LNK, based on the hypothesis of multivariate lognormality of the data, estimates the spatial distribution of an individual block by using the kriged mean grade of the block, and the logarithmic variance of the samples in the kriging neighborhood plus the kriging variance to determine the exact shape of the lognornal distribution of grades.

2.1 Probability kriging

PK was developed to fill the need for a simple estimator of local recoverable reserves in the mining industry (Sullivan, 1985). This estimator utilizes the estimate of the conditional probability distribution function to determine the average grade and proportion of a block or panel above a given cutoff grade.

The conditional distribution function of the random variable Z(x), at location x, given 'n' neighborhood data, may be denoted:

$$F_x\ (Z|n) = \text{Prob}\ \{Z(x) <= Z|n\}. \qquad (2.1)$$



(Journel,1980)

FIGURE 2.1: Graphical representation of the spatial distribution of point support grades within a block.

This relationship forms the basis for the PK estimator.

2.1.1 Indicator and uniform transforms

The application of PK requires that the original data be transformed into two new variables, the indicator and the uniform variable. Given the random function, Z(x), defined at every point throughout the deposit, a realization of this function, z(x), exists. The indicator transform of this realization for a specific cutoff grade zc is:

$$i(x;zc)=\begin{cases}0 \text{ if } z(x)<=zc\\1 \text{ if } z(x)>zc\end{cases} \qquad (2.2)$$

The distribution of this random variable is a Bernoulli distribution, a graphical representation of which is given in Figure 2.2.

The uniform transform of the realization z(x) is simply

$$u(x)=F(z(x)), \qquad (2.3)$$

where F(z(x)) is the cumulative distribution function of the realizations of the random variable Z(x). The uniform random variable has a uniform distribution within the range [0,1]. A graphical representation is given in Figure 2.3.

Each of the transforms in PK has a probabilistic notion. At a particular location the realization of the random variable Z(x) may be $z(x_a)$, the indicator and uniform transforms of this datum are:

$$i(z(x_a)) = \text{Prob } (z(x_a) <= xc)$$

and, $$u(x_a) = \text{Prob } (Z(x) <= z(x_a)). \qquad (2.4)$$

Furthermore, each transform varies between 0 and 1, thereby reducing the effects of high grade data.

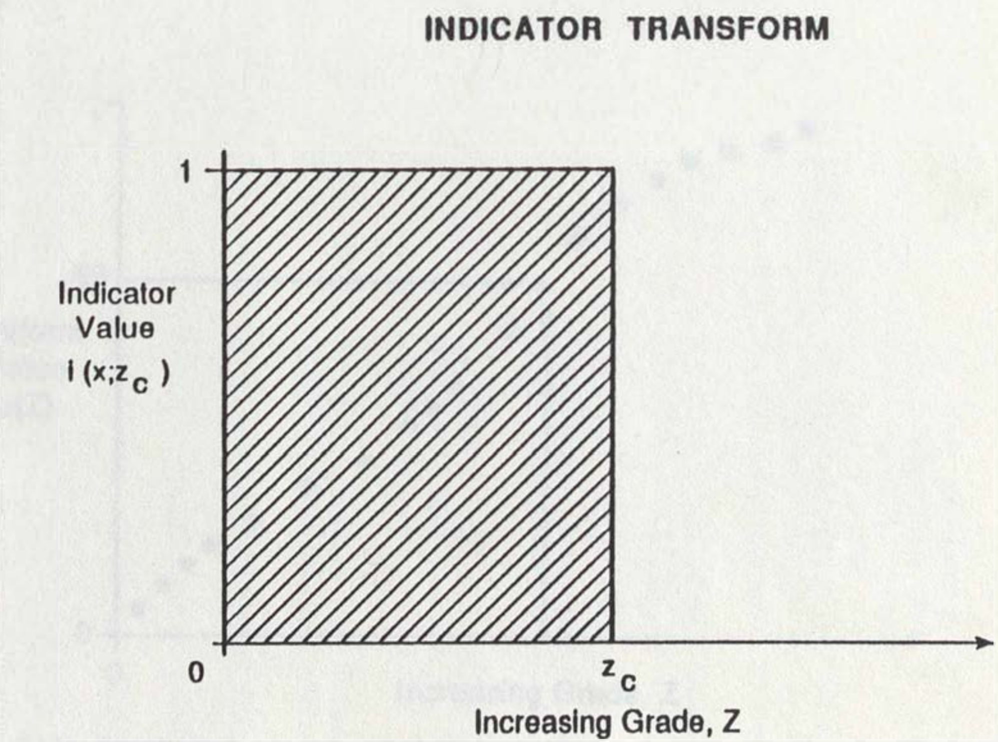


FIGURE 2.2: Graphical representation of the indicator transform.

(Lindsey, 1987)

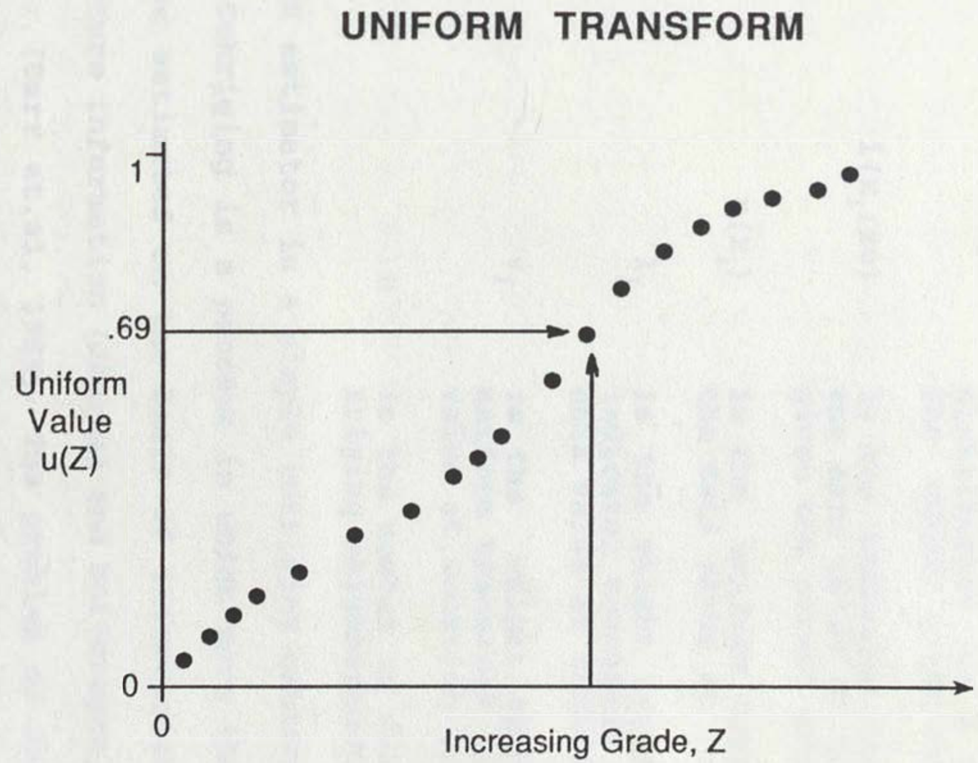


FIGURE 2.3: Graphical representation of the uniform transform.

(Lindsey, 1987)

2.1.2 The PK estimator and cokriging

The PK estimator, a linear combination of the indicator and uniform random variables, estimates the conditional distribution function of grades with point support within a panel. The equation has the form:

$$\phi^{*}(A;ZC)=\sum_{j=1}^{n}\lambda_j i(x_j;xc)+\sum_{j=1}^{n}\nu_j u(x_j) \qquad (2.5)$$

where; $\phi^{*}(A;zc)$ is the estimate of the spatial distribution for a panel A, given the cutoff grade zc,

$i(x_j;zc)$ is the indicator transform of the data value at location j, given the cutoff grade zc,

$u(x_j)$ is the uniform transform of the data value at location j,

λ_j is the weight applied to the indicator transform of the data value at location j,

ν_j is the weight applied to the uniform transform of the data value at location j,

N is the number of data in the kriging neighborhood.

The PK estimator is a simple cokriging estimator.

Cokriging is a process in which more than one variable can be estimated on the basis of intervariable and spatial structure information (Journal and Huijbregts, 1978); (Myers, 1982); (Carr et.al, 1985). The problem of undersampling one variable with respect to another led to the development of cokriging (Matheron, 1971). It is theorized that increased precision in reserve estimation will occur if the spatial correlation of both variables, in a two variable process, is considered. In PK, the two variables are transforms of the original random variable, and both variables exist at all locations. So, for PK to improve on the estimate of OK, the coregionalization of the indicator and uniform variables must not be intrinsic, that is, the direct and cross variograms are not proportional to a single basic model. This point will be addressed in Chapter 5.

2.1.3 Non-bias conditions

For the PK function to be unbiased, two constraints must be met:

$$\sum_{j=1}^{N} \lambda_j = 1 \qquad \sum_{j=1}^{N} \nu_j = 0 \tag{2.6}$$

The estimation variance of the PK estimator is minimized subject to these constraints. The minimized PK estimation variance may be expressed:

$$\sigma^2_{PK}(A;zc) = \overline{C_i}(A,A,zc) - u_1(zc) - \sum_{a=1}^{N} \lambda_a(zc)\overline{C_i}(A,x_a,zc) - \sum_{a=1}^{N} \nu_a(zc)\overline{C_{ui}}(A,x_a,zc) \tag{2.7}$$

where u_1 and u_2 are Lagrangian multipliers. Because the sum of the weights applied to the uniform transform equals zero, u_2 does not appear in the equation. In this study, the conditions (2.6) were used when 8 or more data were located in the kriging neighborhood. When less than 8 data were located in the neighborhood, a less stringent constraint was applied.

$$\sum_{j=1}^{N} (\lambda_j(z)+\nu_j(z))=1 \qquad (2.8)$$

It is likely that this constraint will apply greater weight on the uniform variable. Isaacs (1984) determined that this procedure will help to increase the resolution of the PK estimator when only a few data are present.

2.1.4 Estimation of grade and tonnage above cutoff

The estimated proportion of the unknown spatial distribution of the random variable Z(x) below a particular cutoff grade has been defined above. The tonnage recovered above a cutoff grade zc, based on a point support smu is:

$$T(A;zc)=1-\emptyset^*(A;zc) \qquad (2.9)$$

Similarly the estimation of the grade above a cutoff grade also involves the proportion $\emptyset^*(A;zc)$ (Figure 2.4). Sullivan (1985) has defined two methods of estimating the quantity of metal above a cutoff grade, the direct method, and the indirect method.

The direct method is expressed:

$$Q(zc_k)=[\emptyset^*(A;zc_{a+1})-\emptyset^*(A;zc_a)]\,c_{a+1} \qquad (2.10)$$

where, $Q(zc_k)$ is the quantity of metal above the cutoff grade zc,

nc is the number of cutoff grades,

c_{a+1} is a measure of the central

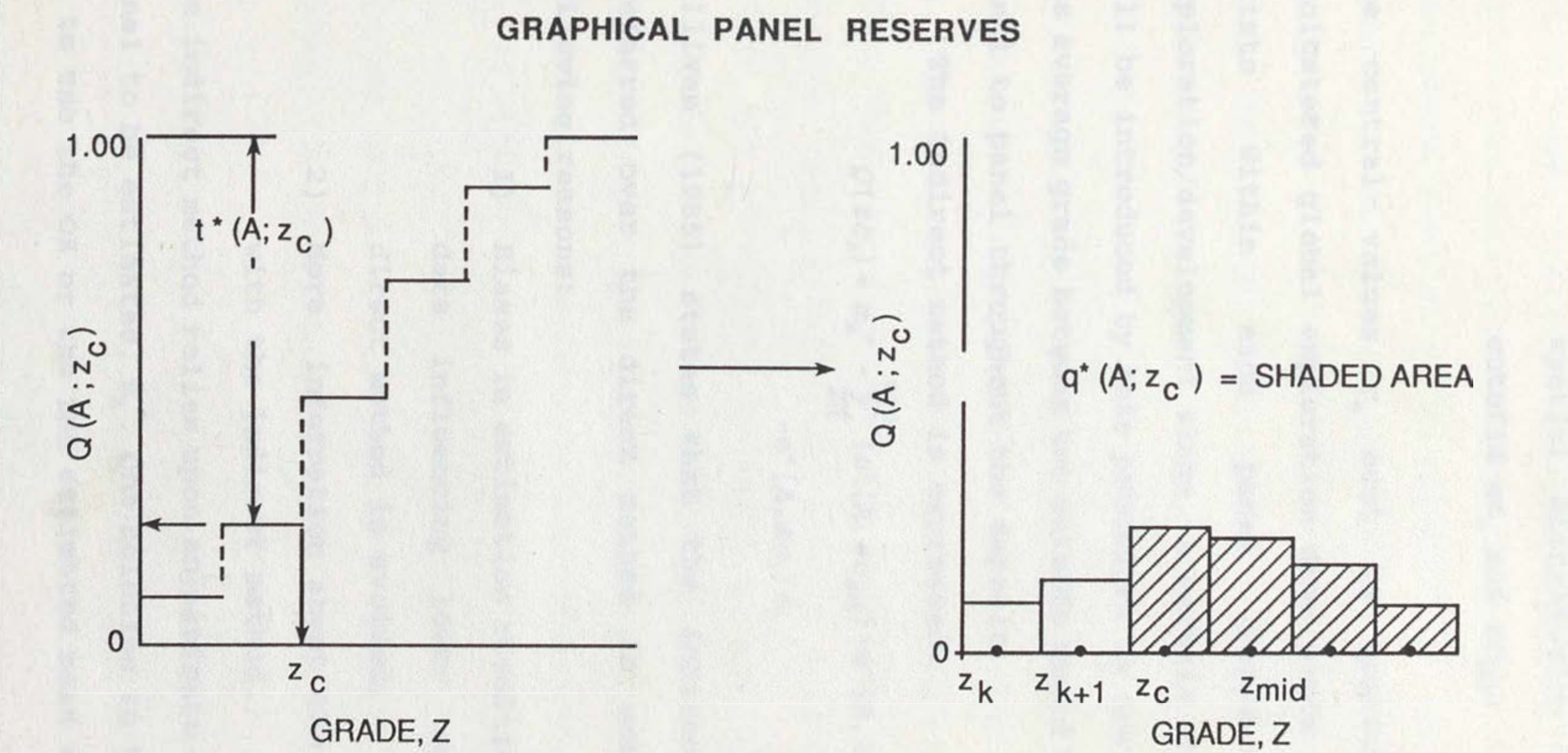


FIGURE 2.4: Graphical representation of a reserve function. (Lindsey, 1987)

tendency of the grade of the spatial distribution between cutoffs zc_a and zc_{a+1}.

The central values c_a must be approximated from the declustered global exploration data since insufficient data exists within each panel neighborhood at the exploration/development stage. A certain amount of smoothing will be introduced by this procedure as one would expect that the average grade between two cutoffs would vary somewhat from panel to panel throughout the deposit.

The indirect method is expressed:

$$Q(zc_k) = m_a^* - \sum_{a=1}^{nc-1} (\phi^*(A, zc_{a+1}) - \phi^*(A, zc_a)\; c_{a+1} \\ -\phi^*(A, zc_1)\, c_1 \qquad (2.11)$$

Sullivan (1985) states that the indirect method may be preferred over the direct method in some cases for the following reasons:

1) Biases in estimation resulting from high grade data influencing lower cutoffs using the direct method is avoided.
2) More information about the deposit is used with the indirect method.

The indirect method relies upon an estimate of the mean of the panel to be estimated, m_A^*. One solution to this problem might be to use the OK or the LNK estimated mean for the panel, but since a non-parametric solution is desired this approach will not be taken. Instead, as with the direct method, the global exploration data will be called upon for an estimate for the mean grade. Now, given the quantity of metal above cutoff by either method, the mean grade above cutoff may be estimated as:

$$m^{*}(A,zc)=Q(zc_{k})/1-\phi(A;zc). \qquad (2.12)$$

2.2 Lognormal Kriging

2.2.1 The logarithmic transformation and kriging system

In comparison to PK, the mathematics of LNK is quite simple. It has been determined that the distribution of grades in many gold deposits is lognormal (Krige, 1966); (Rendu, 1978). When such a distribution is present it is expected that superior reserve estimation occurs when the original data values are normalized by using a logarithmic transform, a variogram of the transformed data is calculated, the corresponding kriging weights and solution is arrived at, and the kriged estimate is then back transformed. The transform may be expressed:

$$Y=\ln(z(x)). \qquad (2.13)$$

The LNK system of equations is identical to OK. The system is a linear arrangement of matrices:

$$[A][X]=[B] \qquad (2.14)$$

where, [A] is the data covariance matrix,

[X] is the matrix of kriging weights to be solved for,

[B] is the matrix of covariances between the data and the point or block to be estimated.

In this thesis it will be assumed that the true mean grade of the deposit is unknown, and therefore OK is used rather than simple kriging (SK).

2.2.2 Unbiasedness

To be an unbiased estimator, the kriging weights of the LNK estimator must sum to unity. In the point support case, the OK estimator of the transformed variable Y(x) may be expressed:

$$Y^{*}_{OK}(x) = \sum_{a=1}^{N} \lambda_a \, Y(x_a) \qquad (2.15)$$

The unbiased condition will be met if

$$\sum_{a=1}^{N} \lambda_a = 1 \qquad (2.16)$$

for each kriging neighborhood.

Subject to the above unbiased condition, the estimation variance of the LNK estimator is minimized. The estimation variance is:

$$E[\,|Y(z(x)) - Y^{*}_{OK}(z(x))|^2] = \sigma^2_{OK}(z(x))$$
$$= \sigma^2 - \sum_{a=1}^{N} \sigma(z(x_a - z(x)) + u \qquad (2.17)$$

where, u is the Lagrangian multiplier,

N is the number of data in the kriging neighborhood,

$Y(z(x))$ is the transformed value of the actual but unknown grade,

$\sigma(x_1 - x_0)$ is the covariance associated with the points x_1 and x_0.

Additionally, a correction factor must be included to account for the bias associated with the back transform:

$$Z^*(x_0) = Exp\ (Y(Z^*(x_0))). \quad (2.18)$$

The unbiased back transform is:

$$Z^*(x_0) = B\{Exp\ [Y(z^*(x_0)]\} \quad (2.19)$$

where B, the correction factor, is determined through cross validation.

2.2.3 Estimation of tonnage and mean grade above cutoff

Once it has been determined that the sample grades are distributed lognormally, an assumption that the true but unknown distribution of actual grades is multivariate lognormal can be made. Matheron (1971) and Krige (1951) have postulated the permanency of the lognormal pattern, which implies that once drill hole samples have been proven to be distributed lognormally, the distribution of grades in blocks of any size can also be assumed to be distributed lognormally. Based upon this assumption, and utilizing the kriged mean grade for each panel and the logarithmic variance, the proportion of the distribution of grades within a block exceeding a cutoff can be determined. With this value, the tonnage and mean grade above a cutoff follow directly.

Following the procedure outlined by Rendu (1981), the proportion of tonnage in a panel above a cutoff grade zc is:

$$T_{+zc}/T = 1 - \varnothing\{1/(\sigma_{eW})\,Ln(zc/u_B) + \sigma_{eW}/2\} \quad (2.20)$$

where, σ_{eW} is the sum of the kriging variance and the logarithmic sample variance of the data in the kriging neighborhood,

u_B is the mean grade of the block,

$\varnothing(zc)$ is the proportion of the area under the cumulative normal distribution less than the cutoff grade zc.

The proportion of the total quantity of metal in a block above a cutoff grade zc is expressed:

$$Q_{+zc}/Q = 1 - \{1/(\sigma_{eW})\,Ln\,(zc/u) - \sigma_{eW}/2\} \quad (2.21)$$

The ratio of the average grade above a cutoff grade zc to the average grade of a block is:

$$u_{+zc}/u = (Q_{+zc}/Q)/(T_{+zc}/T) \quad (2.22)$$

where u_{+zc} is the mean grade above the cutoff grade zc.

Chapter 3 The Manhattan Gold Deposit

Exploration and production data from the Manhattan Mine used in this study were generously made available by Echo Bay Minerals Co. The Manhattan Mine is located 50 miles north of Tonopah, NV, in the Big Smoky Valley, on the western flank of the Toquima Range (Figure 3.1).

3.1 History of the Manhattan district

The gold discovery which led to the development of the Manhattan Mine occurred in April, 1905, by John C. Humphrey, when his party had paused to rest while traveling from Belmont, to the Seyler Ranch near Peavine. By March, 1906, 3000 people lived in and around Manhattan. Development of the district was temporarily halted by the fire in San Francisco later in 1906. Placer mining began in earnest in 1908 after a brief economic depression had stopped capital investment in lode mines (Ferguson, 1924). District production in both lode and placer mining peaked for the first time during the years 1913-1915, and then declined as many of the rich surface deposits were depleted.

In 1917, the discovery of rich ore in the lower levels of the White Caps Mine signalled the beginning of a second albeit brief boom period. Ferguson (1924) reports that only one of the larger mines was operating and little exploration was in progress at the time of his visit in 1924.

The Manhattan Gold Dredge Co. operated a large dredge in Manhattan Gulch from 1938-1947, recovering approximately

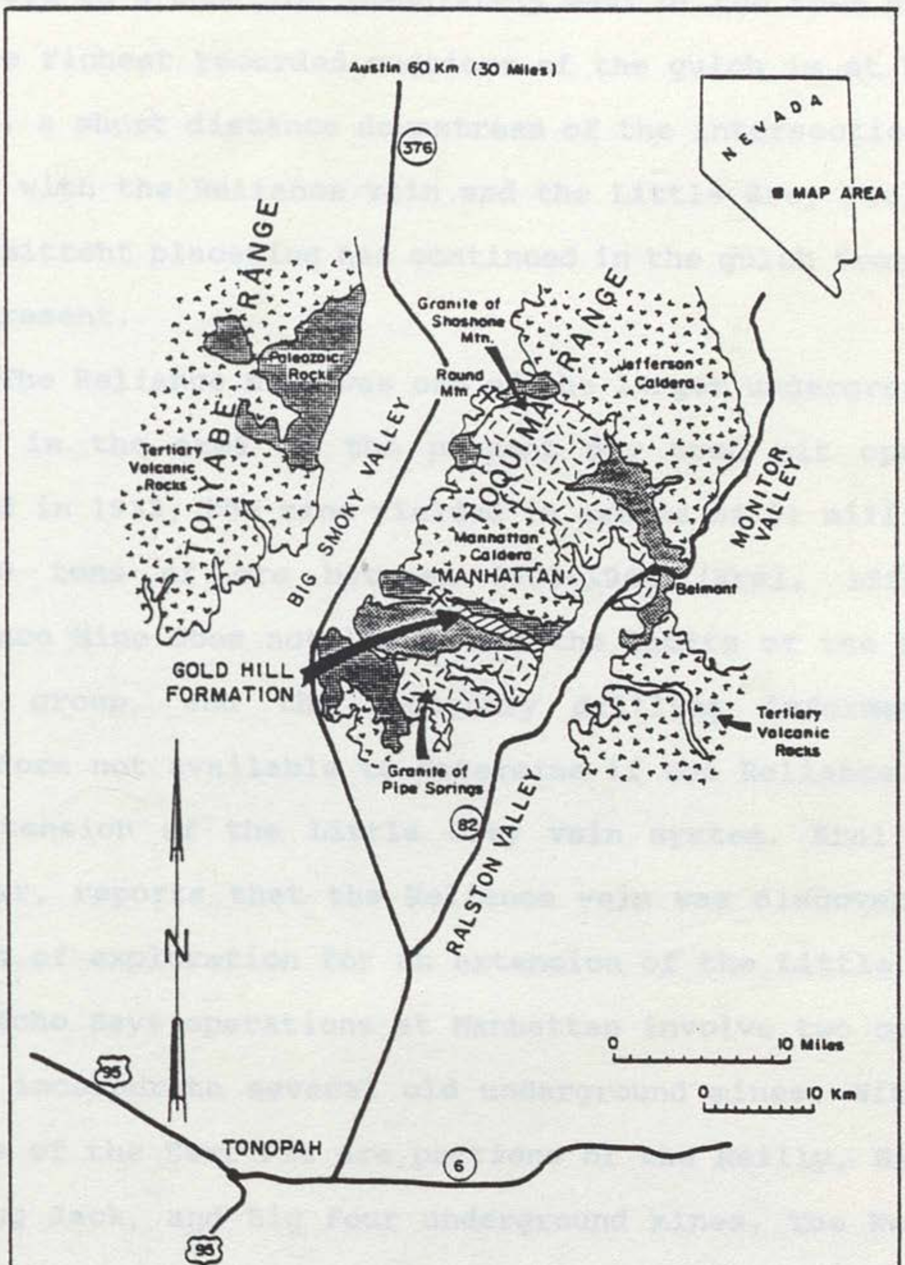


FIGURE 3.1 Location map.
(Maddry et.al. 1987)

300,000 troy ounces of gold. The operation began at the west end of the gulch in the Big Smoky Valley, and proceeded eastward to a position immediately west of the town site. One of the richest recorded sections of the gulch is at Wolftone Point, a short distance downstream of the intersection of the gulch with the Reliance vein and the Little Grey Fault zone. Intermittent placering has continued in the gulch from 1947 to the present.

The Reliance Mine was one of the larger underground lode mines in the area of the present day open pit operation. Opened in 1932, the mine yielded in excess of $1 million from 60,000 tons of ore between 1935-1941 (Kral, 1951). The Reliance Mine does not lie within the limits of the Echo Bay claim group, and the necessary drilling information is therefore not available to determine if the Reliance vein is an extension of the Little Grey vein system. Kral (1951), however, reports that the Reliance vein was discovered as a result of exploration for an extension of the Little Grey.

Echo Bays operations at Manhattan involve two open pits which incorporate several old underground mines. Within the limits of the East Pit are portions of the Reilly, Big Pine, Jumping Jack, and Big Four underground mines. The West Pit, the subject of this study, incorporates the Little Grey mine.

3.2 District geology

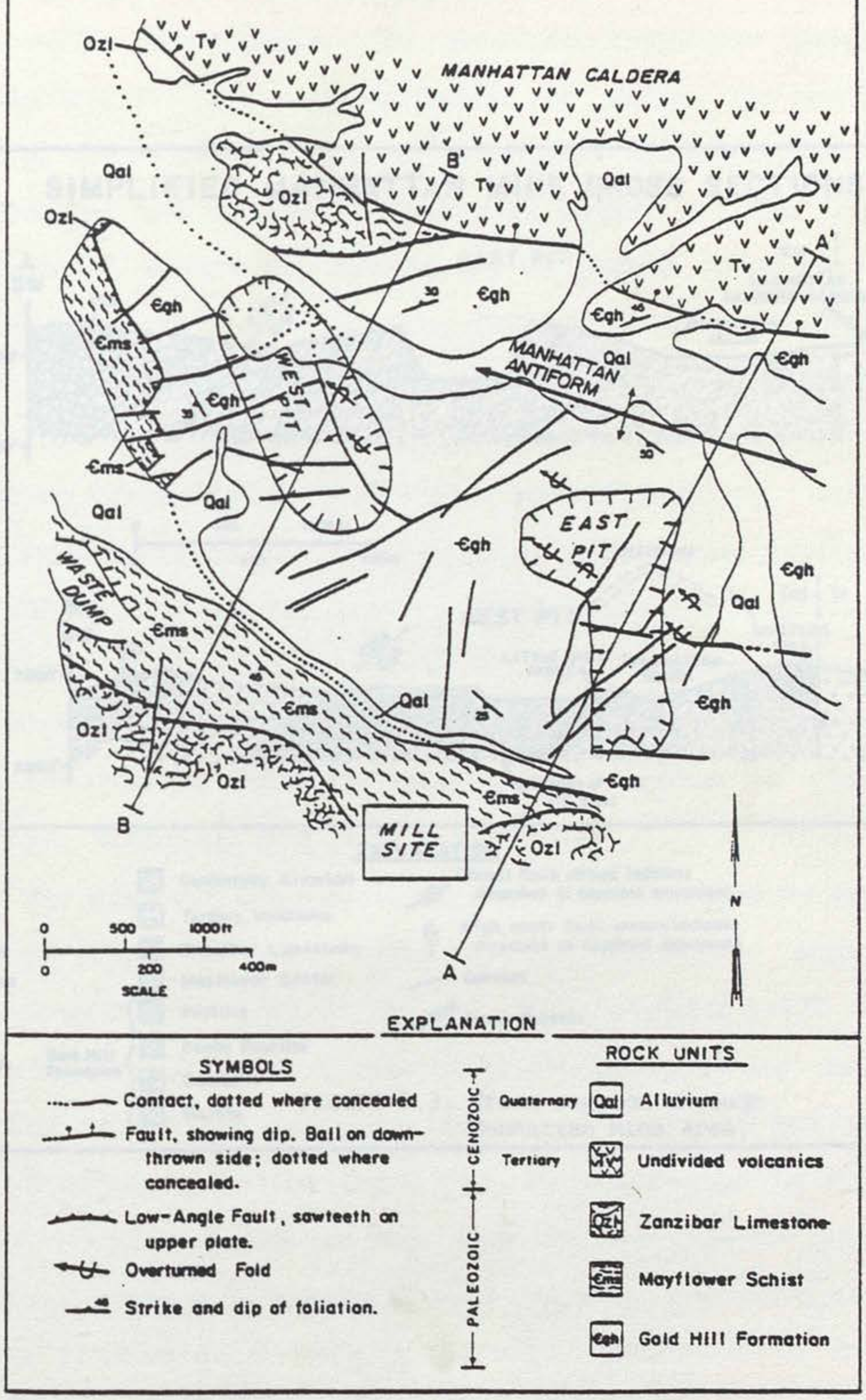
Exposed rocks in the Manhattan district include Cambrian and Ordovician siliciclastics and carbonates, Cretaceous granite, and Tertiary volcanic rocks from the Manhattan caldera (Maddry et al, 1987). The Paleozoic rocks are present in thin thrust sheets that are locally intensely folded. The thrust sheets strike east-west or northwest in general, and dip to the south. The Manhattan caldera lies approximately 0.4 miles north of the Manhattan gold deposit (Figure 3.2). Paleozoic rocks are truncated, overlain, or included in the caldera as floating fragments (Figure 3.3). The Tertiary volcanic and volcanoclastic rocks are described in detail by Shawe (1984). The age of the volcanics varies from 23 to 26 Ma.

Gold mineralization in the vicinity of the Manhattan mine is found in the Gold Hill Formation (Maddry et al., 1987). The Gold Hill Formation. consists of phyllite, sandy phyllite, quartzite, and marble. Mineralization is preferentially found in the sandy phyllite and the quartzite due to their brittle characteristics during deformation. Replacement of marble has resulted in some economic mineralization, but it is minor for the most part.

3.3 West Pit orebody

The West Pit orebody currently being mined, and the focus of this study, was originally mined as the Little Grey mine, an underground, open stope operation. The dimensions of the ore shoot were reported by Ferguson (1924) to be 100 feet x 300 feet x 5-20 feet (drift length, height, and stope width, respectively). The orebody today is considered to be 1200' in

Fig.3.2 GEOLOGIC MAP OF THE MANHATTAN MINE AREA



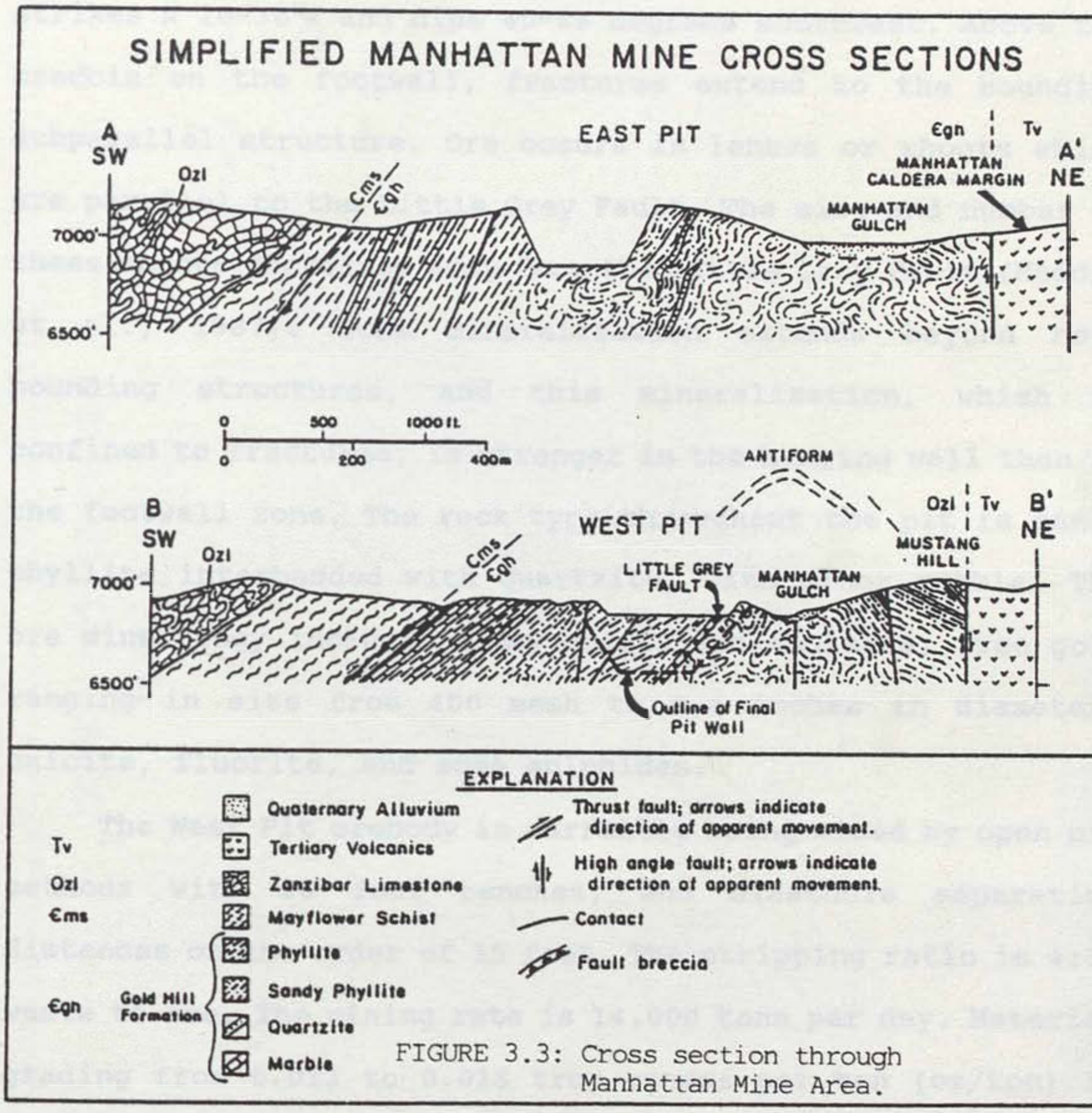


FIGURE 3.3: Cross section through Manhattan Mine Area.

strike length, 300' in dip length, and of variable width. The Little Grey Fault zone (Figure 3.4), which is bounded by two subparallel structures, the footwall structure being the Little Grey Fault, is approximately 200' thick. The fault zone strikes N 10-30°W and dips 45-65 degrees southwest. Above the breccia on the footwall, fractures extend to the bounding subparallel structure. Ore occurs in lenses or shoots which are parallel to the Little Grey Fault. The size and number of these shoots decreases away from the Little Grey Fault (Maddry et al., 1987). Some mineralization extends beyond both bounding structures, and this mineralization, which is confined to fractures, is stronger in the hanging wall than in the footwall zone. The rock type throughout the pit is sandy phyllite interbedded with quartzite, with minor marble. The ore mineralogy includes drusy quartz and adularia, free gold ranging in size from 400 mesh to 0.4 inches in diameter, calcite, fluorite, and some sulphides.

The West Pit orebody is currently being mined by open pit methods with 20 foot benches, and blasthole separation distances on the order of 15 feet. The stripping ratio is 4:1, waste to ore. The mining rate is 14,000 tons per day. Material grading from 0.013 to 0.018 troy ounces per ton (oz/ton) is hauled directly to the leach pad. Material grading 0.019 to 0.059 oz/ton is crushed and sent to the leach pad, and material grading in excess of 0.060 oz/ton is crushed and sent to the floatation circuit in the mill. Selection of ore and

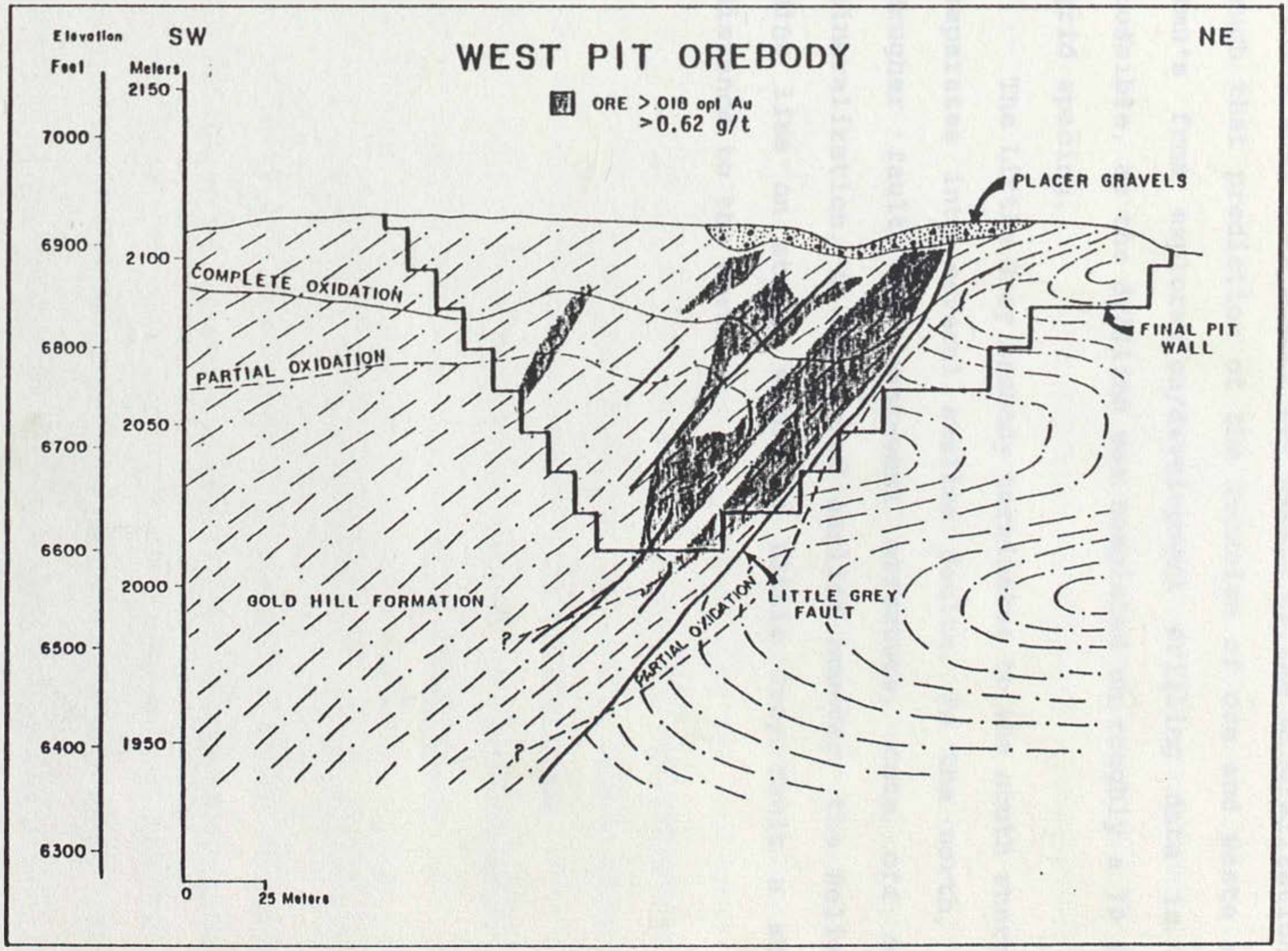


FIGURE 3.4 West pit orebody. (Maddry et.al., 1987)

waste is based on OK of blasthole assays to calculate the average grade of a 15 foot x 15 foot x 20 foot selective mining unit size block. The nature of the mineralization is such that prediction of the location of ore and waste size smu's from exploration/development drilling data is not possible, as the drilling was completed on roughly a 70 foot grid spacing.

The Little Grey orebody terminates to the south where it separates into several smaller faults. To the north, the Brugher fault, an east-west structure, cuts off most mineralization. As mentioned earlier however, the Reliance Mine lies on strike with the Little Grey fault a short distance to the north.

Chapter 4 Statistical Analysis of the Data

Prior to the application of any geostatistical techniques a univariate statistical analysis of the data will be performed. The purpose of this analysis is four- fold:

1) to select indicator cut off grades with which the PK algorithm will estimate conditional probability distribution functions;
2) to confirm the presence of a lognormal distribution of exploration data that will justify the use of LNK;
3) to examine the population for the presence of bimodality and outliers;
4) to calculate frequency distributions of point support grades within large exploration size blocks using production data to be compared with results utilizing exploration data.

A total of 1,386 exploration composites exist in the West Pit database. Of these, 431 are located in the hanging wall, 790 are within the Little Grey Fault zone, and 165 are located in the footwall. Exploration drilling in the West Pit is oriented along the strike of the Little Grey fault. Drill grid spacing is roughly 70 feet, with some angle holes included to intercept the fault zone in its true thickness. Samples are taken in 5 foot intervals, and assays are then composited into 20 foot benches. The effects of spatial clustering of the data were negated by declustering according to the cell method (Journal, 1983) using a 100 x 100 x 20 foot block system.

Subdivision of the database on the basis of structural position and the presence of drusy quartz and adularia mineralization on fractures incorporates into the estimation process the natural physical boundaries of the deposit. Also, the assumptions of second order stationarity will be more closely adhered to. Statistics of the exhaustive data set and the individual zones are given in Table 1.

4.1 Selection of indicator cutoff grades

The PK algorithm as described in section 2.1 approximates the conditional probability distribution function given the 'n' neighborhood data for a block or point location by estimating the realization of the random function at a series of cutoff grades. If cutoff grades are chosen so that the data are divided into subgroups containing equal portions of the data, then the unknown distribution will be approximated most efficiently. Plotting the data into histograms (Figures 4.1, 4.2, 4.3, and 4.4) allows for visual inspection of the distribution of composite grades, and the output from such a program generally gives some measure of the relative frequency of data within each class.

TABLE 1. STATISTICS OF EXPLORATION DRILLING DATA

	EXHAUSTIVE EXPLORATION DATA	HANGING WALL	LITTLE GREY FAULT	FOOTWALL
Number of Data	1386	431	790	165
Mean (oz/ton)	0.018	0.006	0.026	0.012
Variance	0.003	0.000	0.006	0.002
Coefficient of Variation	3.24	2.06	2.85	3.54

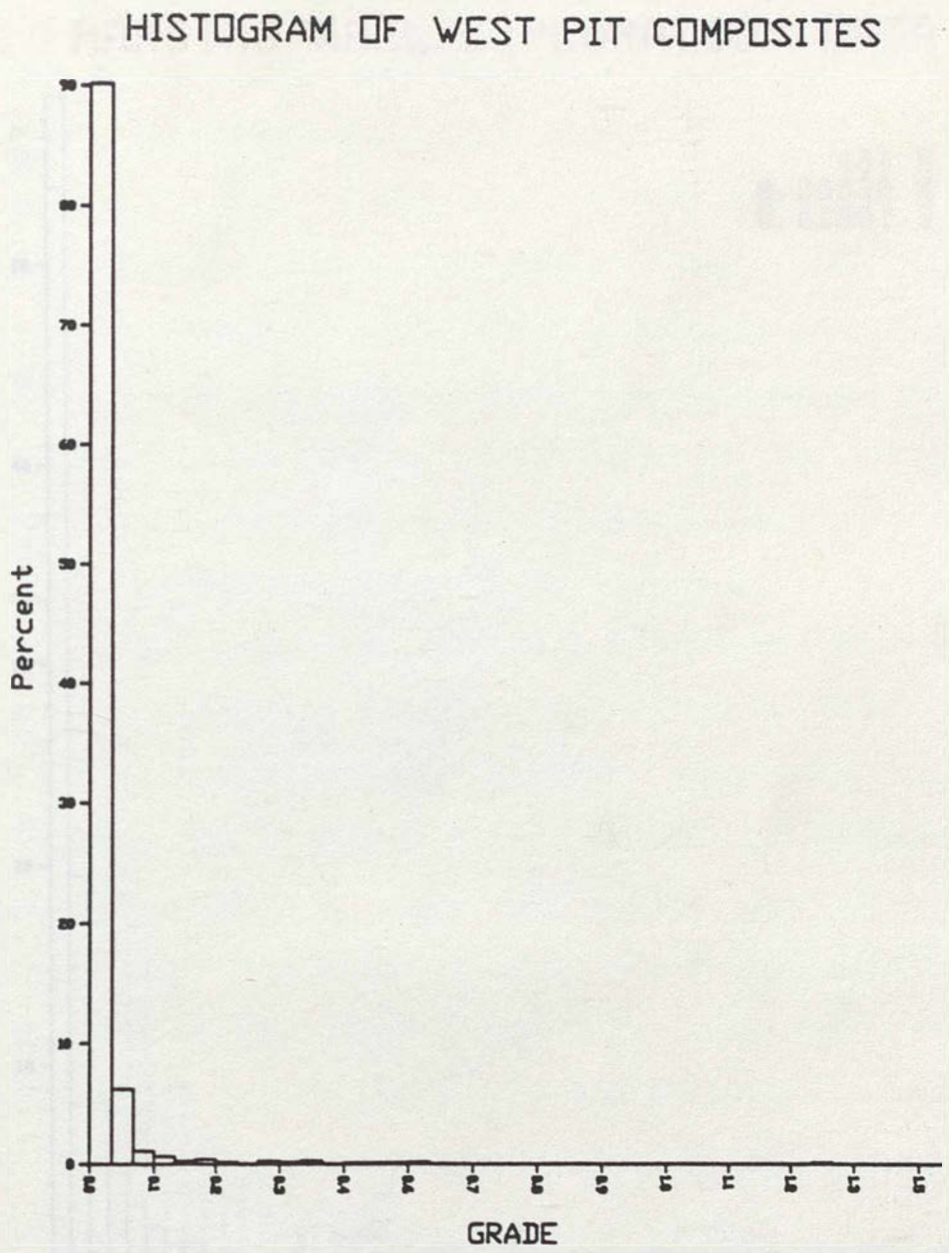


Figure 4.1

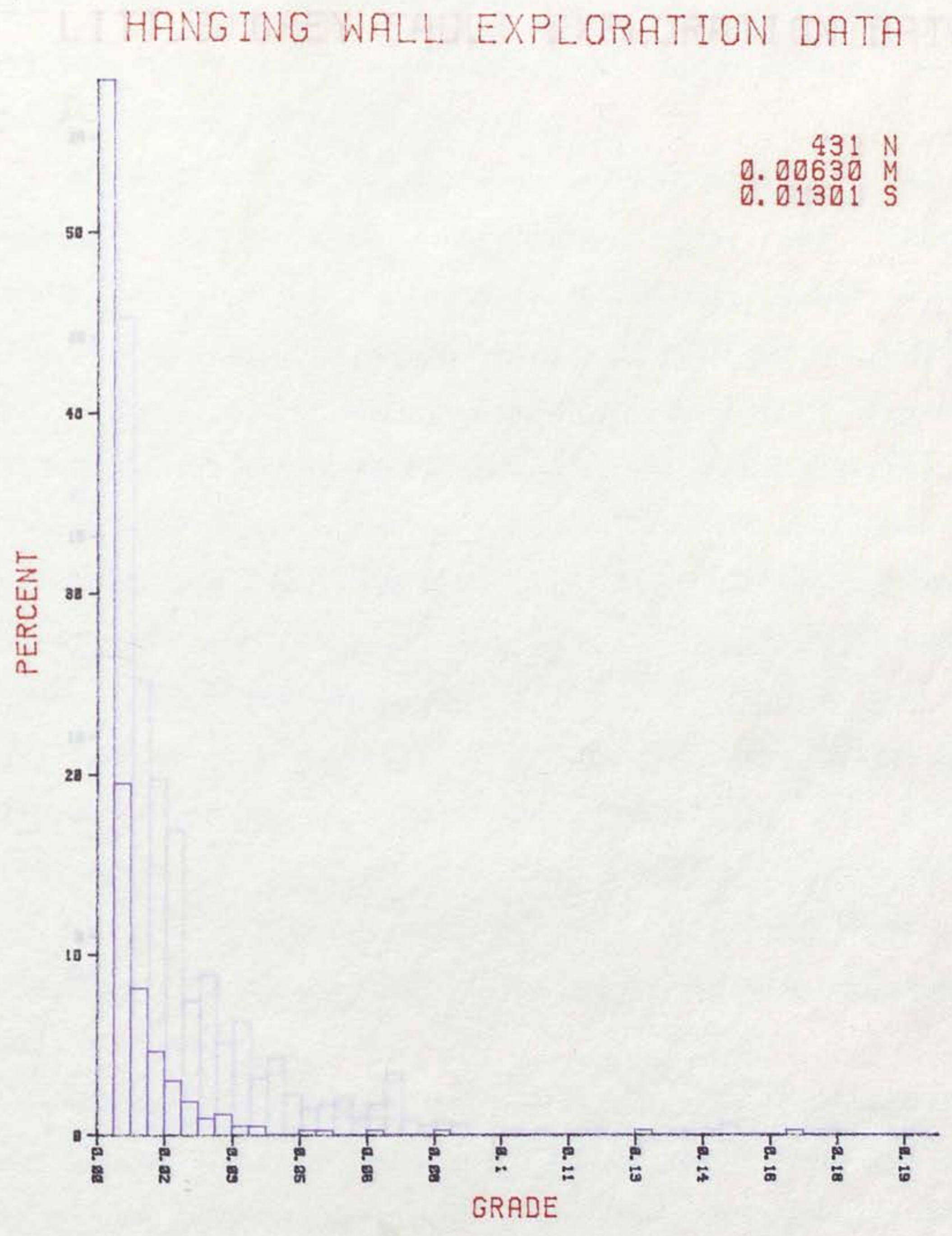

HANGING WALL EXPLORATION DATA
431 N
0.00630 M
0.01301 S
PERCENT
50
40
30
20
10
0
0.00
0.02
0.03
0.05
0.06
0.08
0.1
0.11
0.13
0.14
0.16
0.18
0.19
GRADE

Figure 4.2

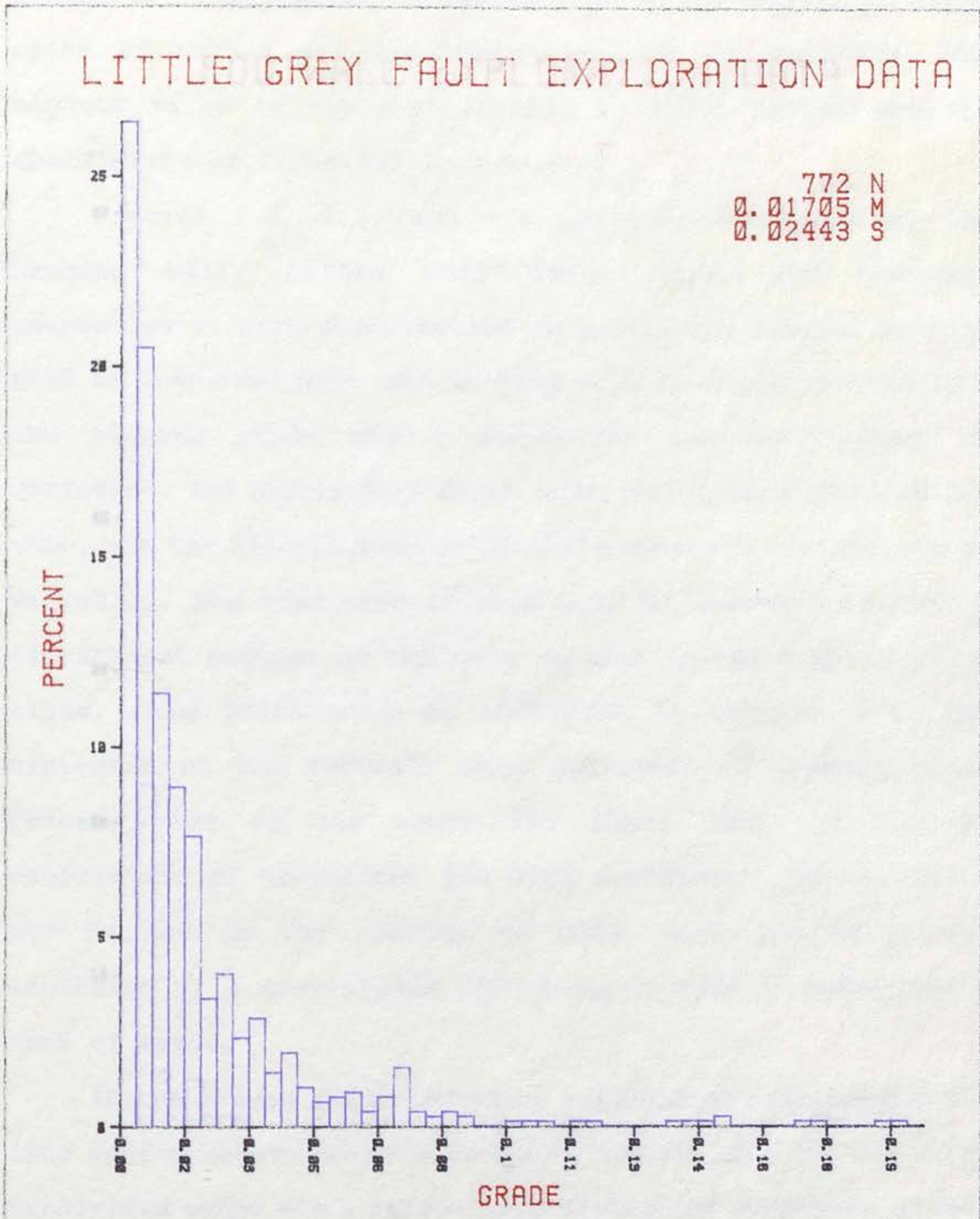

LITTLE GREY FAULT EXPLORATION DATA
772 N
Ø.Ø17Ø5 M
Ø.Ø2443 S
PERCENT
GRADE

Figure 4.3

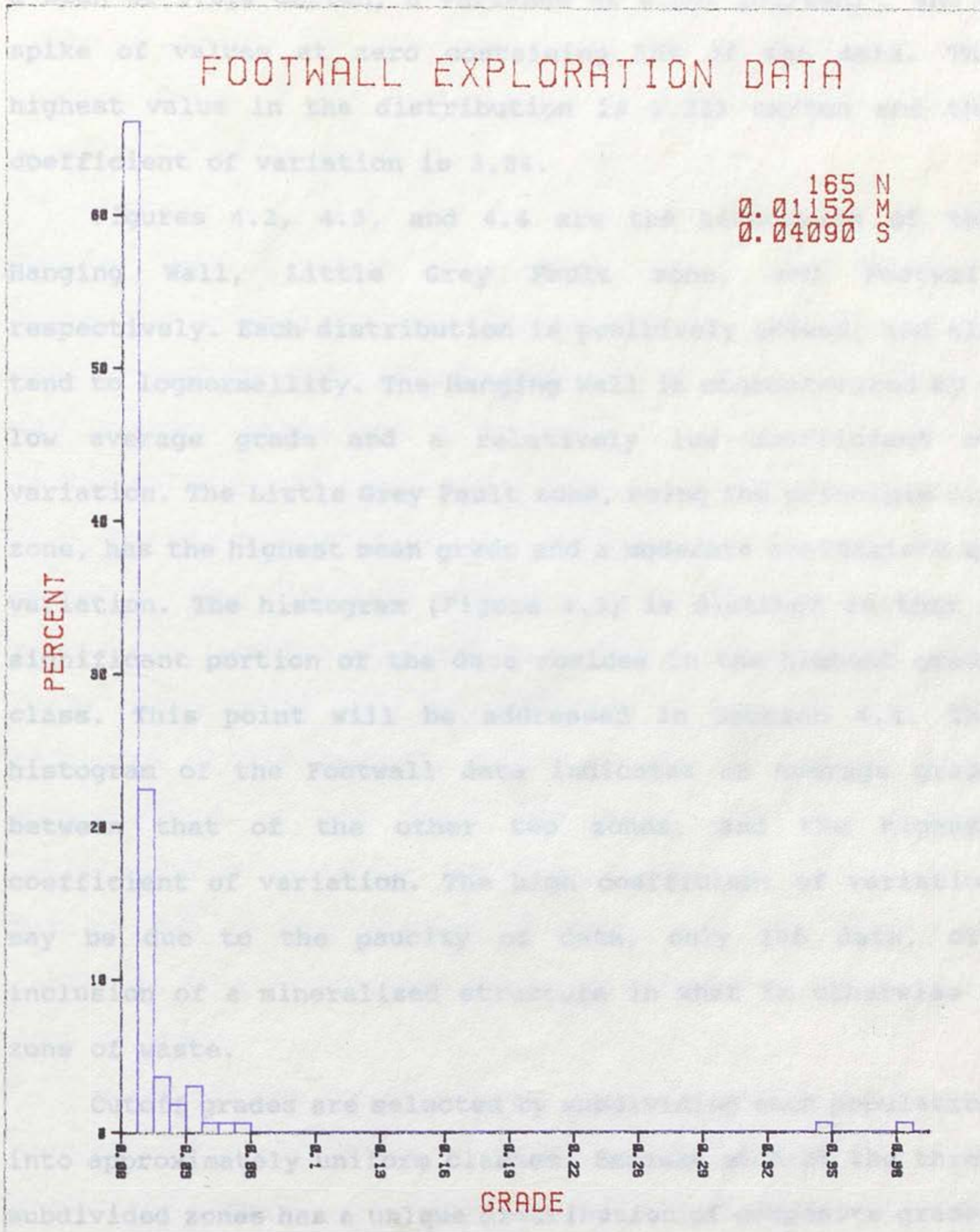

FOOTWALL EXPLORATION DATA
165 N
0.01152 M
0.04090 S
PERCENT
GRADE
0
10
20
30
40
50
60

Figure 4.4

Figure 4.1 is a histogram of the exhaustive West Pit composite database. The distribution is positively skewed with a mean of 0.018 oz/ton, a variance of 0.004 $(oz/ton)^2$, and a spike of values at zero containing 35% of the data. The highest value in the distribution is 1.313 oz/ton and the coefficient of variation is 3.24.

Figures 4.2, 4.3, and 4.4 are the histograms of the Hanging Wall, Little Grey Fault zone, and Footwall respectively. Each distribution is positively skewed, and all tend to lognormallity. The Hanging Wall is characterized by a low average grade and a relatively low coefficient of variation. The Little Grey Fault zone, being the principle ore zone, has the highest mean grade and a moderate coefficient of variation. The histogram (Figure 4.3) is distinct in that a significant portion of the data resides in the highest grade class. This point will be addressed in Section 4.4. The histogram of the Footwall data indicates an average grade between that of the other two zones, and the highest coefficient of variation. The high coefficient of variation may be due to the paucity of data, only 165 data, or, inclusion of a mineralized structure in what is otherwise a zone of waste.

Cutoff grades are selected by subdividing each population into approximately uniform classes. Because each of the three subdivided zones has a unique distribution of composite grades the cutoff grades selected for each will be different. This

Figure 4.1 is a histogram of the exhaustive West Pit composite database. The distribution is positively skewed with a mean of 0.018 oz/ton, a variance of 0.004 $(oz/ton)^2$, and a spike of values at zero containing 35% of the data. The highest value in the distribution is 1.313 oz/ton and the coefficient of variation is 3.24.

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Cutoff grades are selected by subdividing each population into approximately uniform classes. Because each of the three subdivided zones has a unique distribution of composite grades the cutoff grades selected for each will be different. This

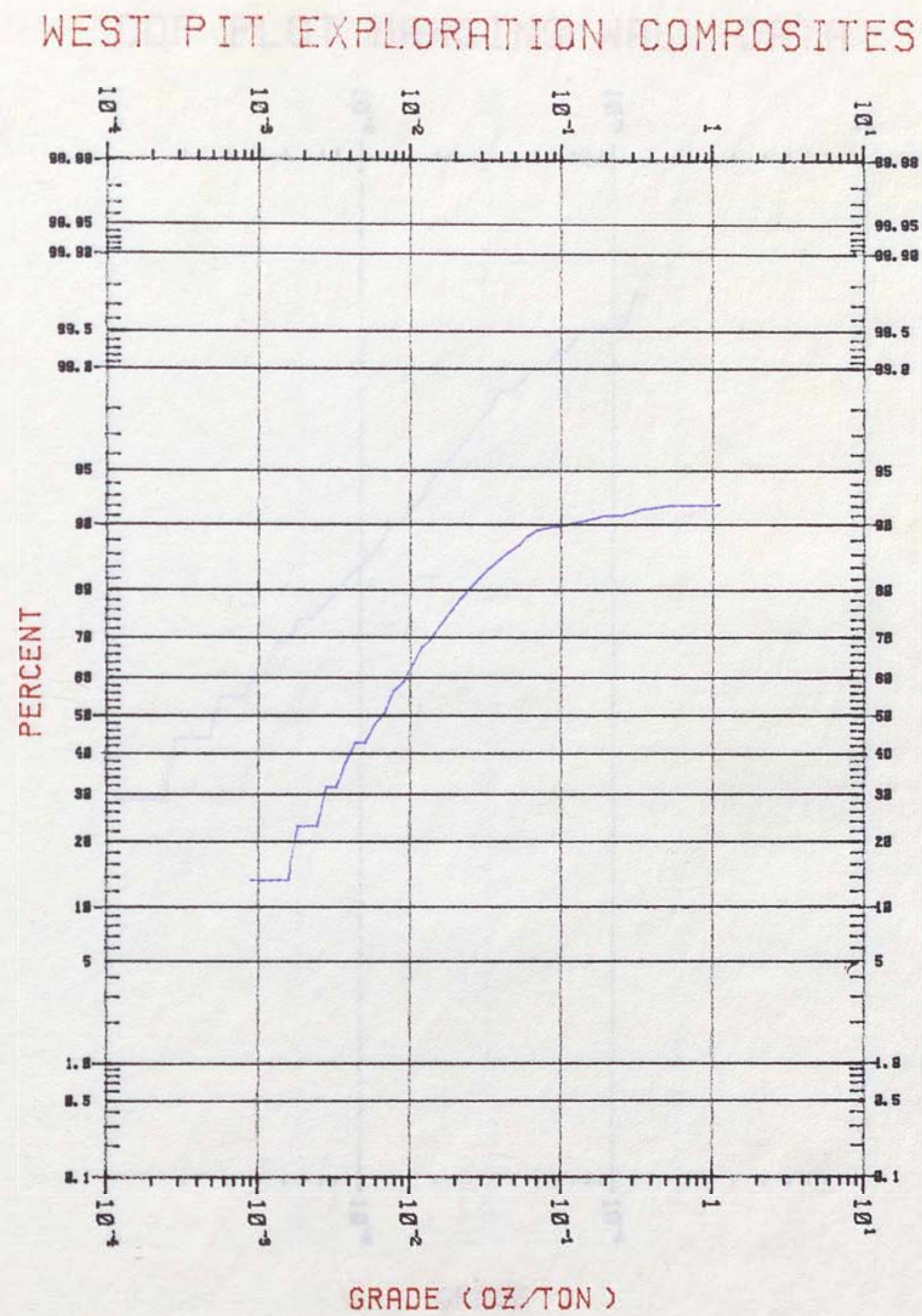
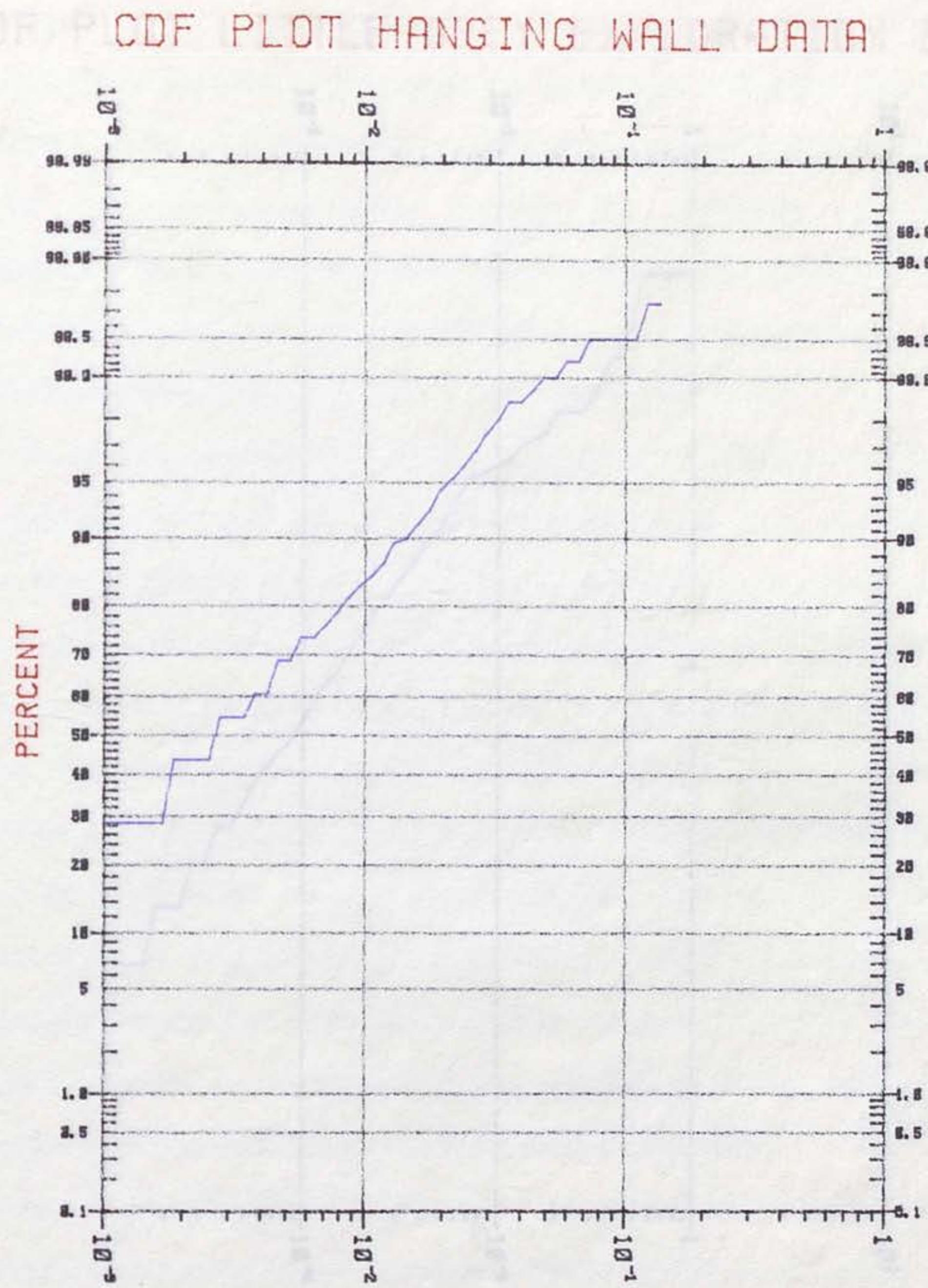

WEST PIT EXPLORATION COMPOSITES
PERCENT
GRADE (OZ/TON)
99.99
99.95
99.90
99.5
99.0
95
90
80
70
60
50
40
30
20
10
5
1.0
0.5
0.1
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^{1}

Figure 4.5



CDF PLOT HANGING WALL DATA
PERCENT
GRADE
99.99
99.95
99.90
99.5
99.0
95
90
80
70
60
50
40
30
20
10
5
1.0
0.5
0.1
10^{-3}
10^{-2}
10^{-1}
1

Figure 4.6

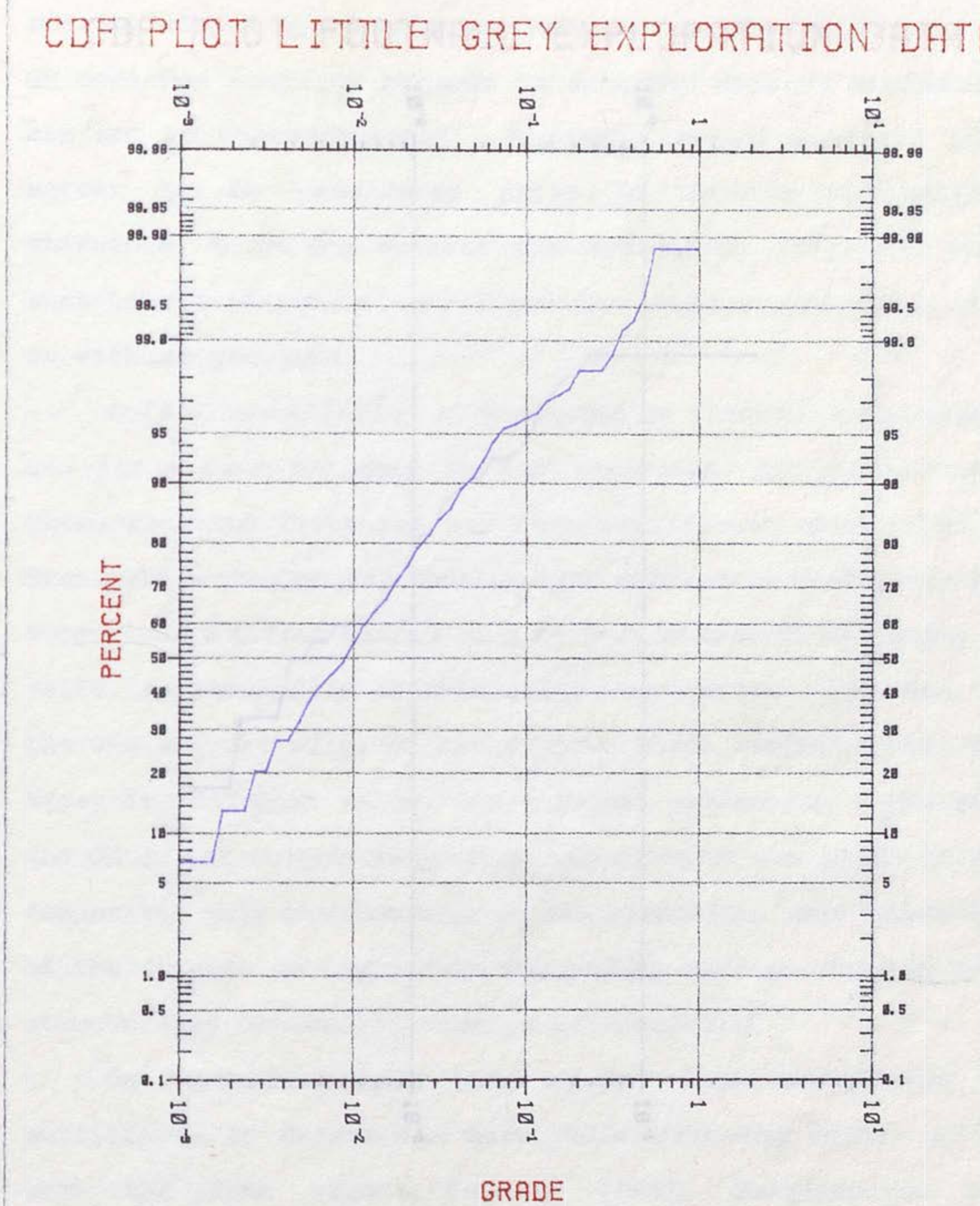

CDF PLOT LITTLE GREY EXPLORATION DATA
PERCENT
GRADE
99.99
99.95
99.90
99.5
99.0
95
90
80
70
60
50
40
30
20
10
5
1.0
0.5
0.1
1

Figure 4.7

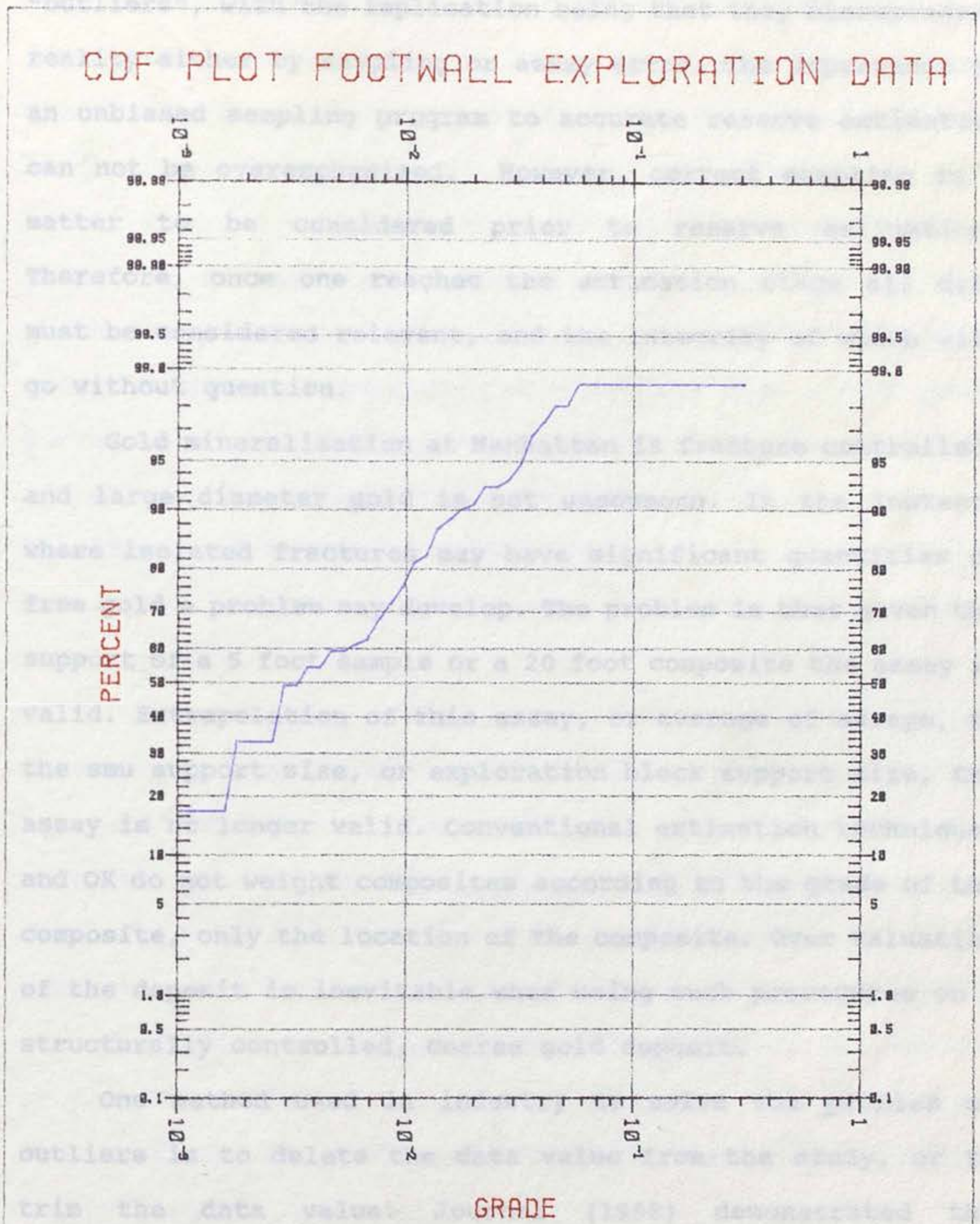

CDF PLOT FOOTWALL EXPLORATION DATA
PERCENT
GRADE
10^{-3}
10^{-2}
10^{-1}
1
99.99
99.95
99.90
99.5
99.0
95
90
80
70
60
50
40
30
20
10
5
1.0
0.5
0.1

Figure 4.8

have a small percentage of the data residing at the end of the tail of the distribution. These data have refered to as "outliers", with the implication being that they misrepresent reality either by sampling or assay error. The importance of an unbiased sampling program to accurate reserve estimation can not be overemphasized. However, correct sampling is a matter to be considered prior to reserve estimation. Therefore, once one reaches the estimation stage all data must be considered relevent, and the integrity of which will go without question.

Gold mineralization at Manhattan is fracture controlled, and large diameter gold is not uncommomn. In the instance where isolated fractures may have significant quantities of free gold a problem may develop. The problem is that given the support of a 5 foot sample or a 20 foot composite the assay is valid. Extrapolation of this assay, or average of assays, to the smu support size, or exploration block support size, the assay is no longer valid. Conventional estimation techniques and OK do not weight composites according to the grade of the composite, only the location of the composite. Over valuation of the deposit is inevitable when using such procedures on a structurally controlled, coarse gold deposit.

One method used in industry to solve the problem of outliers is to delete the data value from the study, or to trim the data value. Journal (1988) demonstrated the consequences of deleting or trimming extreme data by calculating the quantity of metal contained in the extreme class of data, and then calculating the percentage of underestimation associated with deleting or trimming various percentages of the data. It was shown that, for a deposit where the economic cutoff grade is at or near the mean grade of the deposit, deleting as little as 1% of the data could result in under estimation by as much as the expected rate of return of the deposit. Journal concluded that in the case where two mineralization types are intermixed, ie. a high grade fracture controlled mineralization and a low grade pervasive mineralization, consideration should be given to a non-parametric, probabilistic approach, which calculates the likelihood of a type of mineralization to be present at a particular location, and evaluates each mineralization type independently.

This procedure is highly regarded by the author. It presupposes, however,that enough of the right kind of data has been collected during the exploration program to make such calculations, and for this study, none of the data have been deleted or cut back to a lower grade. The success of the PK estimator will depend upon its ability to model the spatial characteristics of the high grade mineralization. The success of the LNK estimator will depend upon how much the actual distributions of grades within each block deviate from lognormallity.

4.4 Determination of the frequency distributions of blasthole grades

In this study, the distribution of point support grades within exploration size blocks calculated from exploration data will be compared to the distribution of the grades of production blastholes within each block. Two benches in the West Pit, the 6840 and 6820 benches, have been subdivided into 139 blocks with dimensions 100 x 100 x 20 $feet^3$ (Figures 4.9 and 4.10). The production blastholes which lay within each block are sorted, and the distribution of the grades is examined. The actual tonnage in a block above a cutoff grade is:

$$T_{+zc} = (n/N)\ 18{,}000 \text{ tons} \quad (4.1)$$

where, n is the number of blastholes whose assayed grade is greater than zc, the cutoff grade;

N is the total number of blastholes in the block;

18,000 tons is the tonnage assumed to be contained in a 100 x 100 x 20 $foot^3$ block.

The actual grade of the proportion of a block above a cutoff grade zc, is:

$$m^*_{zc} = [\sum_{j=1}^{N} x_j]/n \quad (4.2)$$

where x_j is the assay of a blasthole greater than the cutoff grade zc.

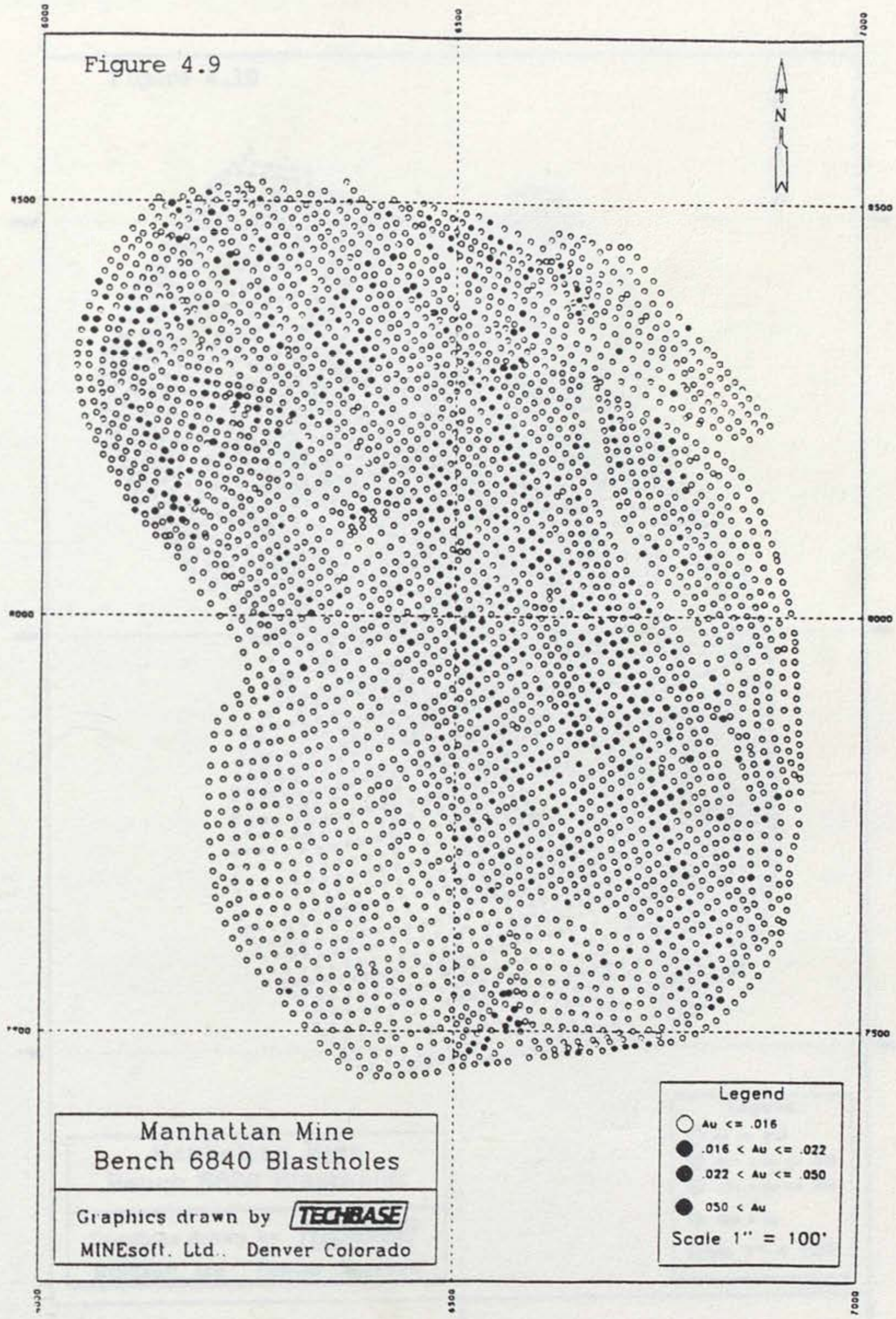

N
8500
8000
7500
Manhattan Mine
Bench 6840 Blastholes
Graphics drawn by TECHBASE
MINEsoft, Ltd., Denver Colorado
Legend
Au <= .016
.016 < Au <= .022
.022 < Au <= .050
.050 < Au
Scale 1" = 100'

Figure 4.9

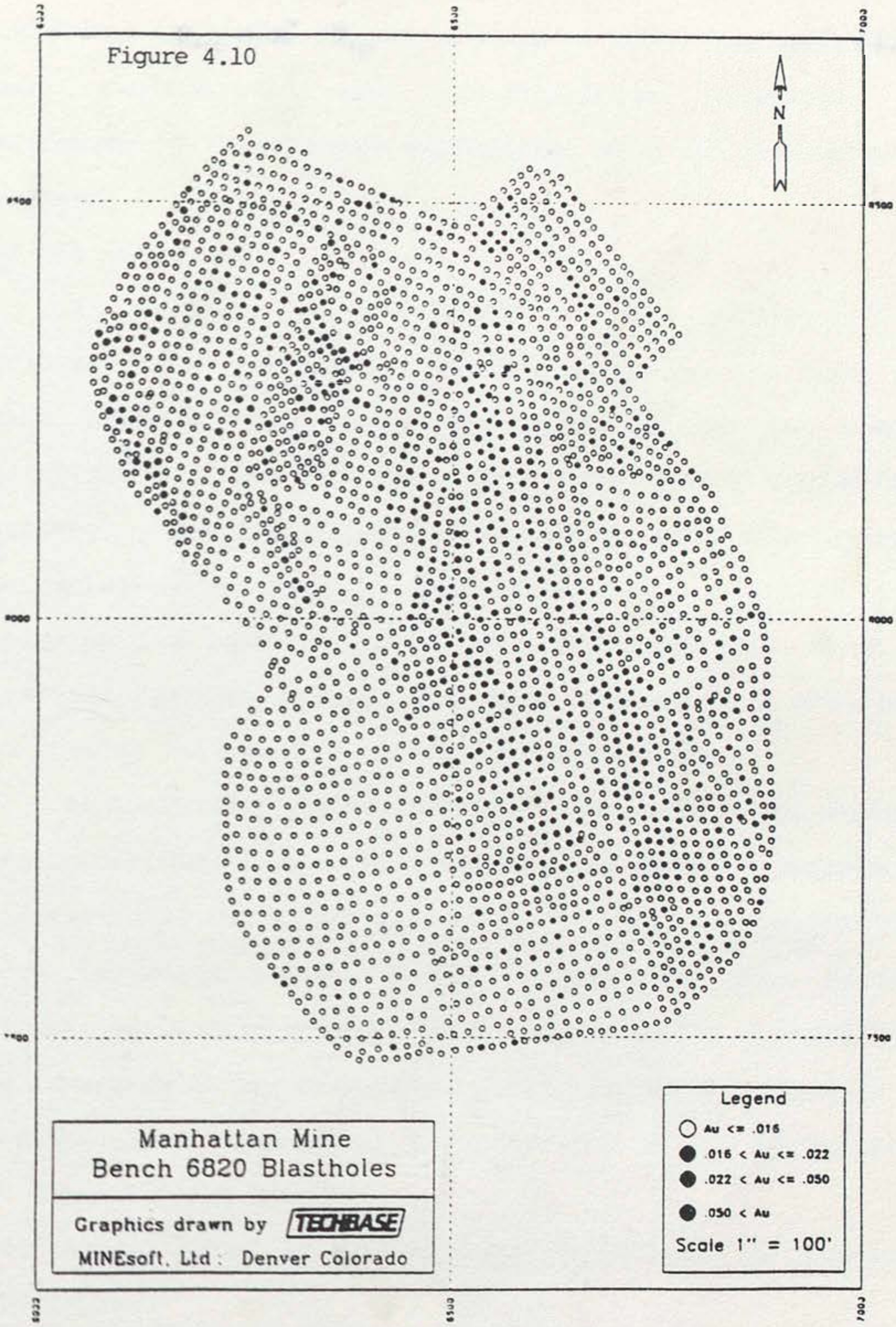


Figure 4.10
N
Manhattan Mine
Bench 6820 Blastholes
Graphics drawn by TECHBASE
MINEsoft, Ltd : Denver Colorado
Legend
Au <= .016
.016 < Au <= .022
.022 < Au <= .050
.050 < Au
Scale 1" = 100'

The actual quantity of metal contained in a block above a cutoff grade zc, is:

$$Q_{+zc} = m^{*} \; T_{+zc} \qquad (4.3)$$

Chapter 5 Variography

This chapter discusses covariance as related to structural analysis, and the process of using the variogram to model regionalized and coregionalized variables. The variograms of transformed exploration data are presented and analyzed.

5.1 The variogram

In Section 1.2 covariance and the concept of the variogram were introduced. The variogram is the tool with which geostatistics models the structure of the spatial variability of the distribution of the random variable of interest. Given the assumption of second order stationarity, the variogram is a function only of the vector distance h that separates two points, x, and x+h. The manner in which the variogram function changes for small changes in h determines the spatial variability of the random variable.

At Manhattan, the deposit has strong structural control. The mineralizing fluids, being channelled by these structures, preferentially deposited gold on the surface of the fractures where favorable physio-chemical conditions were present. Fracture controlled mineralization shows better continuity in the direction of the fractures. Continuity in the plane of the fractures is a function of the direction of the transport of the mineralizing fluids.
A structural model of the spatial variability is therefore anisotropic.

The expression of the variogram function:

$$2\gamma(x_1,x_2)=E[(Z(x_1)-Z(x_2))^2]$$

is a measure of the degree of correlation of the two random variables $Z(x_1)$ and $Z(x_2)$. As the distance separating the two points increases, the correlation decreases, and the mean quadratic deviation between the two random variables increases. Thus, the variogram function increases with increasing h, the vector describing the separation distance.

In practice, the variogram function increases to a particular value h, and then stabilizes. The separation distance at which the function stabilizes is called the range of the function, and represents the distance at which no correlation exists between the two random variables $Z(x_1)$, and $Z(x_2)$. The variables are then independent. The value of the variogram function at this point is called the sill. The sill value is the a priori variance of the random function.

Of particular interest in the study of regionalized variables is the behavior of the variogram function near the origin. Theoretically, when h = 0, the variogram function $\gamma(h) = 0$. However, in a structurally controlled gold deposit with coarse gold, this is seldom the case. Rather, a discontinuity exists at the origin, and as h approaches zero, $\gamma(h)$ does not equal zero. This local variability is a manifestation of the random component of a mineral deposit, and is termed the nugget effect. In addition to the small scale variability of the mineralization, sampling and assay errors contribute to the nugget effect.

5.2 Lognormal transform variograms

5.2.1 LN transform variography

Having determined that the composited exploration grades are distributed approximately lognormally, the spatial variability of each mineralized zone is studied through variograms of the lognormally transformed data values. Variograms are calculated along the general trend of fracturing (x-direction), down dip of the trend of fracturing (y-direction), and perpendicular to the trend of fracturing (z-direction)(Figures 5.1, 5.2, and 5.3). Experimental variograms are calculated using lag distances of 35 feet. The parameters of the variograms calculated in each zone are summarized in Table 2.

5.2.2 Interpretation of the LN transform variograms

Figure 5.1 presents the variograms calculated in the Hanging Wall zone. Figure 5.2 presents the variograms calculated in the Little Grey Fault zone, and Figure 5.3 presents the variograms calculated in the Footwall zone.
All of the variograms are modelled with a simple spherical model, with the exception of the variogram calculated in the z-direction within the Little Grey Fault zone, which exhibits no structure.

The variograms are anisotropic with the principle axis being parallel to the strike of the Little Grey fault, and the second major axis being parallel to the dip direction of the

Figure 5.1

Experimental Variogram for WP1EXLNGRADE

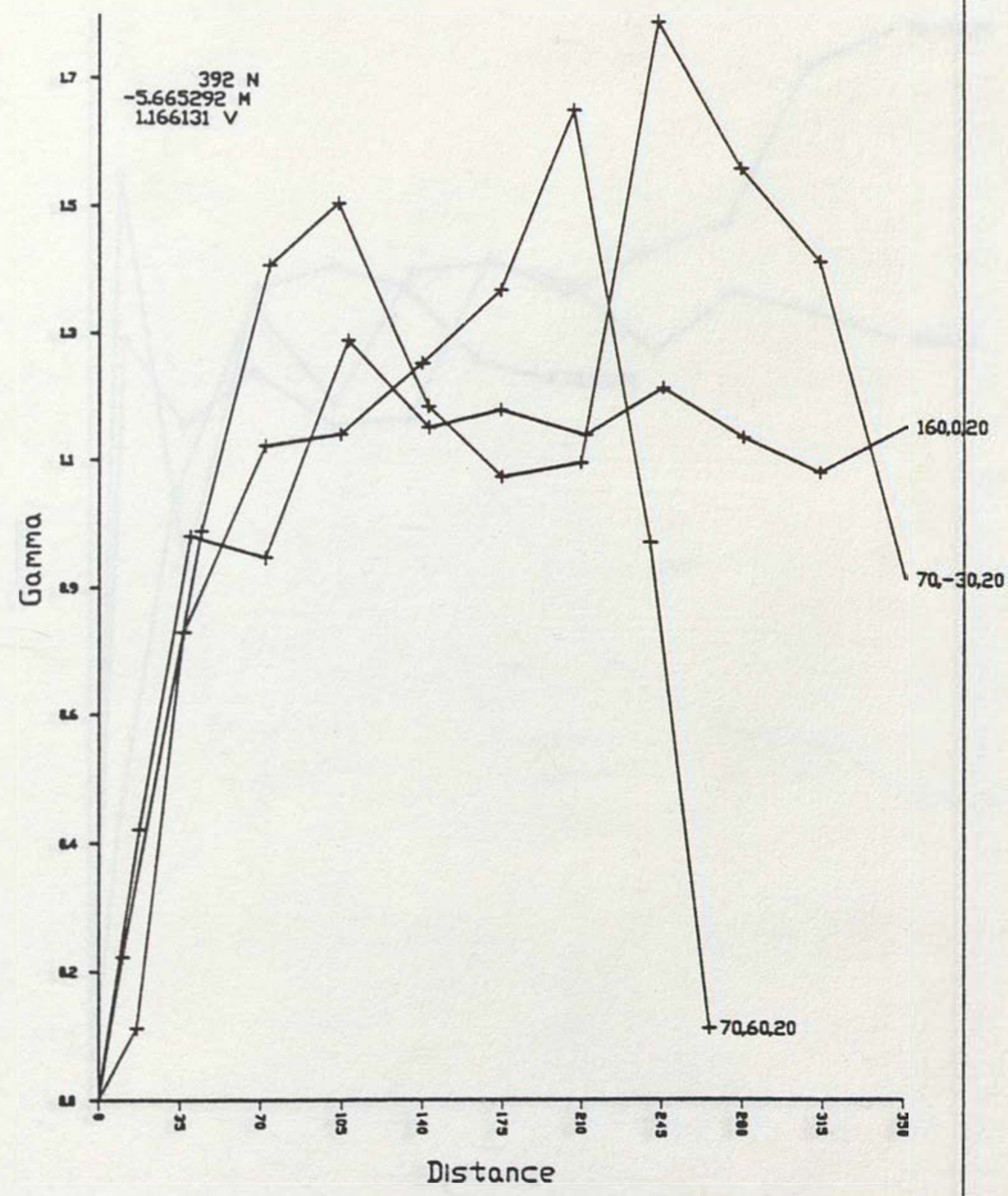


Figure 5.2

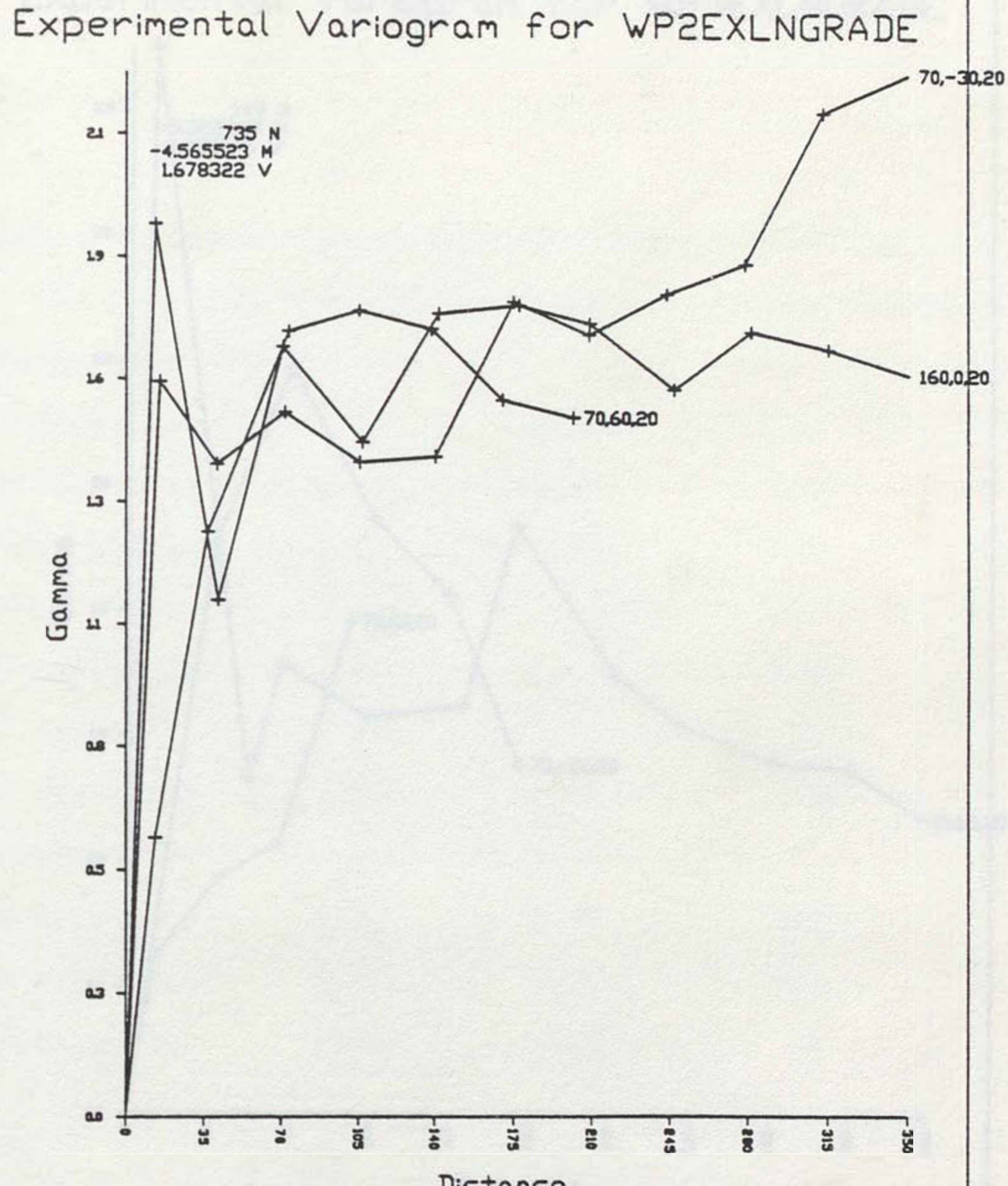

Experimental Variogram for WP2EXLNGRADE
735 N
-4.565523 M
1.678322 V
70,-30,20
160,0,20
70,60,20
Gamma
Distance

Figure 5.3

Experimental Variogram for WP3EXLNGRADE

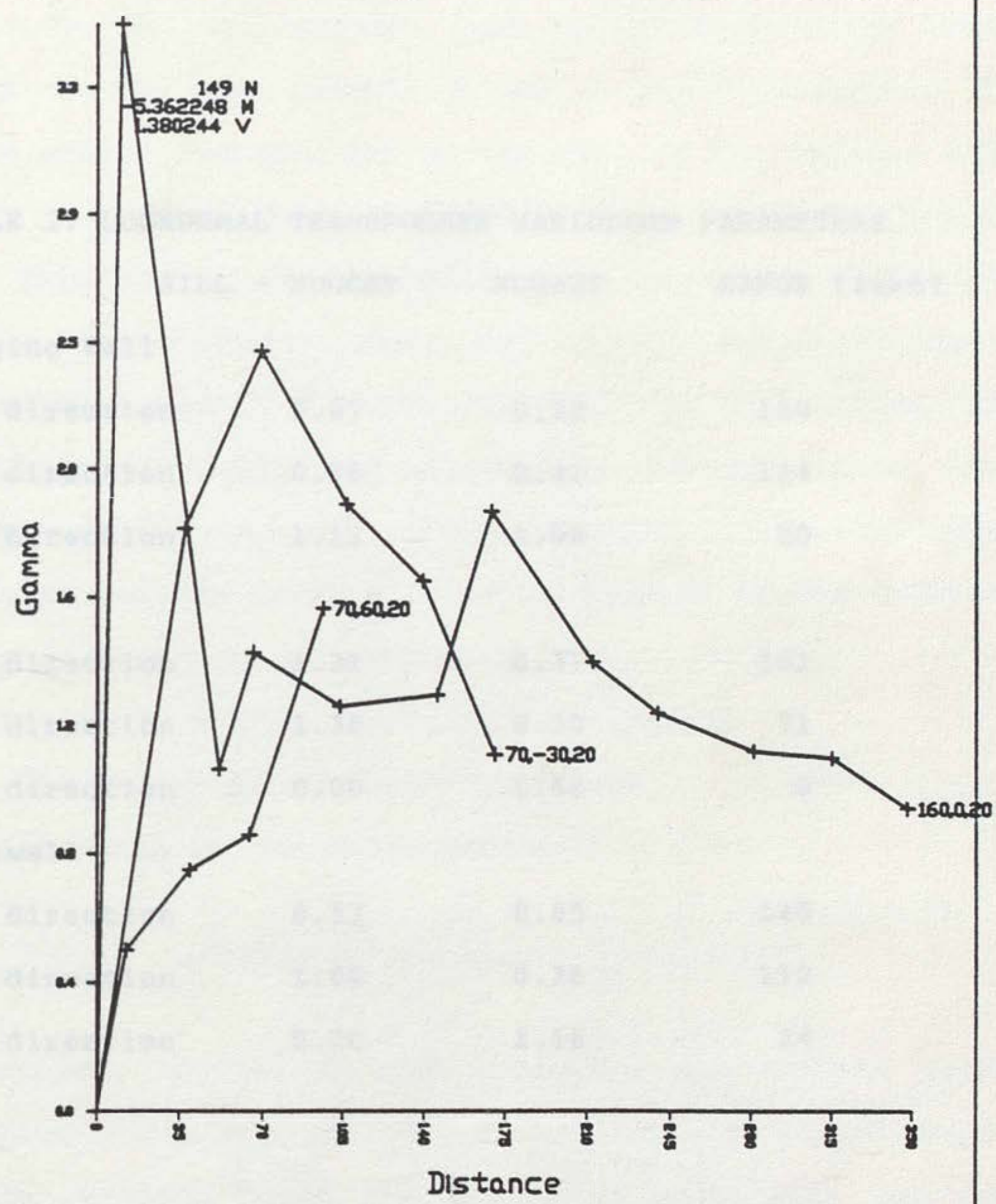


TABLE 2. LOGNORMAL TRANSFORMED VARIOGRAM PARAMETERS

	SILL - NUGGET	NUGGET	RANGE [feet]
Hanging Wall			
x - direction	0.87	0.30	150
y - direction	0.76	0.41	114
z - direction	1.17	0.00	80
LGF			
x - direction	1.31	0.37	101
y - direction	1.38	0.30	71
z - direction	0.00	1.68	0
Footwall			
x - direction	0.53	0.85	148
y - direction	1.00	0.38	112
z - direction	0.20	1.18	24

structure. The z-direction in the Hanging Wall and the Footwall shows a slight hole effect which is to be expected since these variograms are calculated across the fabric of the fracture sets. The z-direction variograms in the Little Grey Fault zone and the Footwall zone have a substantial nugget effect for the same reason. In the x- and y-directions, the nugget effect averages 30% of the sill in the Hanging Wall, 20% in the Little Grey Fault zone, and 45% in the Footwall zone. This implies that the mineralization in the Little Grey Fault zone is most continuous, and that the mineralization in the Footwall zone is the most random. In the Little Grey Fault zone, kriging weights will be higher for nearby samples, while in the Footwall zone, the increased nugget effect will result in higher weights being applied to samples further from the points or blocks to be estimated.

5.3 Indicator, uniform, and cross variograms

5.3.1 The PK system of linear equations

Returning to the PK algorithm:

$$\emptyset^*(A;zc)=\sum_{j=1}^{N}\lambda_j i(x_j;zc)+\sum_{j=1}^{N}v_j u(x_j)$$

The weights applied to the indicator and uniform random variables in the PK algorithm are determined by minimizing the kriging variance using the Lagrangian multiplier technique (Sullivan, 1985).

The PK system of equations follows:

$$\sum_{j=1}^{N} \lambda_j \gamma_I (x_i - x_j; zc) + \sum_{j=1}^{N} \nu_i \gamma_{Iu} (x_i - x_j; zc) + \mu_1 = \gamma_I (x_i - x; zc)$$

and,

$$\sum_{j=1}^{N} \lambda_i \gamma_{Iu} (x_i - x_j; zc) + \sum_{j=1}^{N} \nu_j \gamma_u (x_i - x_j) + \mu_2 = \gamma_{Iu} (x_i - x; zc)$$

if,

$$\sum_{j=1}^{N} \lambda_j = 1$$

and,

$$\sum_{j=1}^{N} \nu_j = 0$$

Where, zc is the cutoff grade;

N is the number of samples in the neighborhood;

is the weight applied to the indicator variable;

is the weight applied to the uniform variable;

μ_1 and μ_2 are Lagrangian multipliers;

$\gamma_I(x_i - x_j; zc)$ is the indicator variogram value for the distance vector $x_i - x_j$ at cutoff grade zc;

$\gamma_{uI}(x_i - x_j; zc)$ is the cross uniform - indicator variogram value for vector $x_i - x_j$ at cutoff grade zc;

$\gamma_u(x_i - x_j)$ is the uniform variogram value for vector $x_i - x_j$.

To solve this system of equations, direct variograms of the indicator and uniform random variables must be calculated, as well as a cross variogram to model the coregionalization of the two random variables.

The variography requirement for PK is extensive. In a 3-D study such as this one, indicator, uniform, and cross variograms must be calculated in each of the x, y, and z directions specified in Section 5.2. Also, the indicator and cross variograms are calculated for each cutoff grade specified in Section 4.2. A summary of the equations used to model these variograms is given in Tables 3, 4, and 5, and the complete set of variograms is in Appendix A.

5.3.2 Positive - definiteness

To eliminate the possibility of obtaining a negative estimation variance, the matrix of covariances used to establish the kriging weights must be positive-definite (Sullivan, 1985). This requirement is satisfied when the following inequalities are met:

$$C^i_j > 0, \text{ and } C^u_j > 0,$$

$$C^u_j * C^i_j - (C^{ui}_j)^2 > 0, \text{ for all j.}$$

TABLE 3. HANGING WALL VARIOGRAM PARAMETERS

	Cutoff Grade oz/ton	Nugget	Sill - Nugget	Range
Indicator				
	0.001	0.135	0.085	150
	0.002	0.125	0.145	170
	0.003	0.120	0.125	110
	0.004	0.125	0.117	130
	0.005	0.110	0.090	40
	0.006	0.145	0.060	175
	0.010	0.055	0.085	100
	0.016	0.065	0.025	110
	0.020	0.051		0
Uniform		0.035	0.055	160
Cross				
	0.001	-0.040	-0.065	120
	0.002	-0.060	-0.070	160
	0.003	-0.060	-0.060	130
	0.004	-0.060	-0.055	140
	0.005	-0.055	-0.050	140
	0.006	-0.070	-0.025	180
	0.010	0.000	-0.068	120
	0.016	0.000	-0.035	80
	0.020	-0.030		0

TABLE 4. LITTLE GREY FAULT ZONE VARIOGRAM PARAMETERS

	Cutoff Grade oz/ton	Nugget	Sill - Nugget	Range [feet]
Indicator				
	0.002	0.055	0.100	125
	0.004	0.075	0.125	125
	0.006	0.140	0.115	211
	0.010	0.175	0.060	120
	0.012	0.165	0.085	200
	0.016	0.081	0.135	75
	0.022	0.060	0.120	60
	0.030	0.035	0.115	75
	0.050	0.000	0.095	70
Uniform		0.029	0.053	130
Cross				
	0.002	0.000	-0.072	120
	0.004	-0.045	-0.060	200
	0.006	-0.045	-0.072	150
	0.010	-0.060	-0.065	110
	0.012	-0.055	-0.065	145
	0.016	-0.040	-0.075	210
	0.022	-0.030	-0.065	160
	0.030	0.000	-0.075	100
	0.050	0.000	-0.047	100

TABLE 5. FOOTWALL VARIOGRAM PARAMETERS

	Cutoff Grade oz/ton	Nugget	Sill - Nugget	Range [feet]
Indicator				
	0.002	0.090	0.150	80
	0.004	0.100	0.140	100
	0.010	0.117	0.042	120
	0.014	0.066	0.033	140
Uniform		0.050	0.033	160
Cross				
	0.002	-0.060	-0.060	80
	0.004	-0.070	-0.068	150
	0.010	-0.044	-0.036	120
	0.014	-0.022	-0.021	120

This requirement has implications upon the models of the variograms such that:

1) Any structure observed in the cross variogram must also be present in the uniform and indicator variograms.

2) A structure appearing in either the indicator or the uniform variogram will not necessarily appear in the cross variogram.

All variograms used in the PK portion of this study have covariance matrices which are positive-definite.

5.3.3 Interpretation of PK variography

In general, the variograms calculated in the PK portion of the study were quite well behaved. The indicator variograms all displayed two characteristics:

1) decreasing range with increasing cutoff grade;

2) decreasing nugget value as a percentage of the sill value with increasing cutoff grade.

The interpretation of decreasing range with increasing cutoff grade is that low grade material occurs throughout the deposit, and that high grade mineralization is limited to isolated pockets. In fact, ore and waste is mixed together throughout the mine.

The interpretation of decreasing nugget value as a percentage of the sill is that much of the error involved in assaying limited quantities of gold is eliminated at higher cutoff grades (Carr, et.al, 1985).

The indicator variograms display the same anisotropic characteristics as the LN transformed variograms. Also, the indicator variograms in the z-direction within the Little Grey Fault Zone show a lack of structure.

Cross variograms between the indicator and uniform variables were calculated at each cutoff grade. The most striking feature of the cross variograms is that the gamma values are negative. This is a result of the negative corellation between indicator and uniform data.

Uniform variograms were also calculated in each of the three zones. The smoothness of the variograms is evidence of the robustness of the uniform transform to outlier data.

5.4 Cross validation

Cross validation was developed to provide a quantitative guide to the accuracy of variogram models. The procedure first calls for calculation of a variogram from the data. Then, each data is removed one at a time, and the experimental variogram is used to krig a value at the point of the removed datum using the remaining samples. The results may be reported in terms of the mean squared error, or by examining the plot of the actual data values versus the estimated. The underlying premise of cross validation is that a variogram which results in unbiased estimation at known data points will produce unbiased results at unknown locations.

5.4.1 Cross validation with LNK

Cross validation of LN transformed variograms serves two purposes. First, the accuracy in terms of unbiased estimation is evaluated following the procedure outlined above. Secondly, cross validation is used to calculate a correction factor to be used in the back transform procedure after kriging. The correction factor, B, introduced in Equation 2.19, is the ratio of the original mean of the data to the estimated mean of the data when initially a correction factor of 1.0 (no correction) is used in the cross validation process.

5.4.2 Cross validation with PK

Cross validation with the PK estimator is slightly more involved than the original cross validation technique. Rather than estimate a grade at each original datum location, the PK estimator is used to calculate a conditional probability distribution function at the missing data location. As with conventional cross validation, this procedure is repeated at all data locations. In this study, the cross validation of PK variography is accomplished by estimating the expected value of the conditional probability distribution function calculated at each datum location and comparing it to the original datum value.

5.4.3 Cross validation results

The results of the cross validation portion of the study are presented in a series of scatter plots of actual versus cross validated data (Figures 5.4 a,b, 5.5 a,b, and 5.6 a,b) for each zone, and in Table 6. A better correlation of cross validated to actual data was achieved with the indicator,

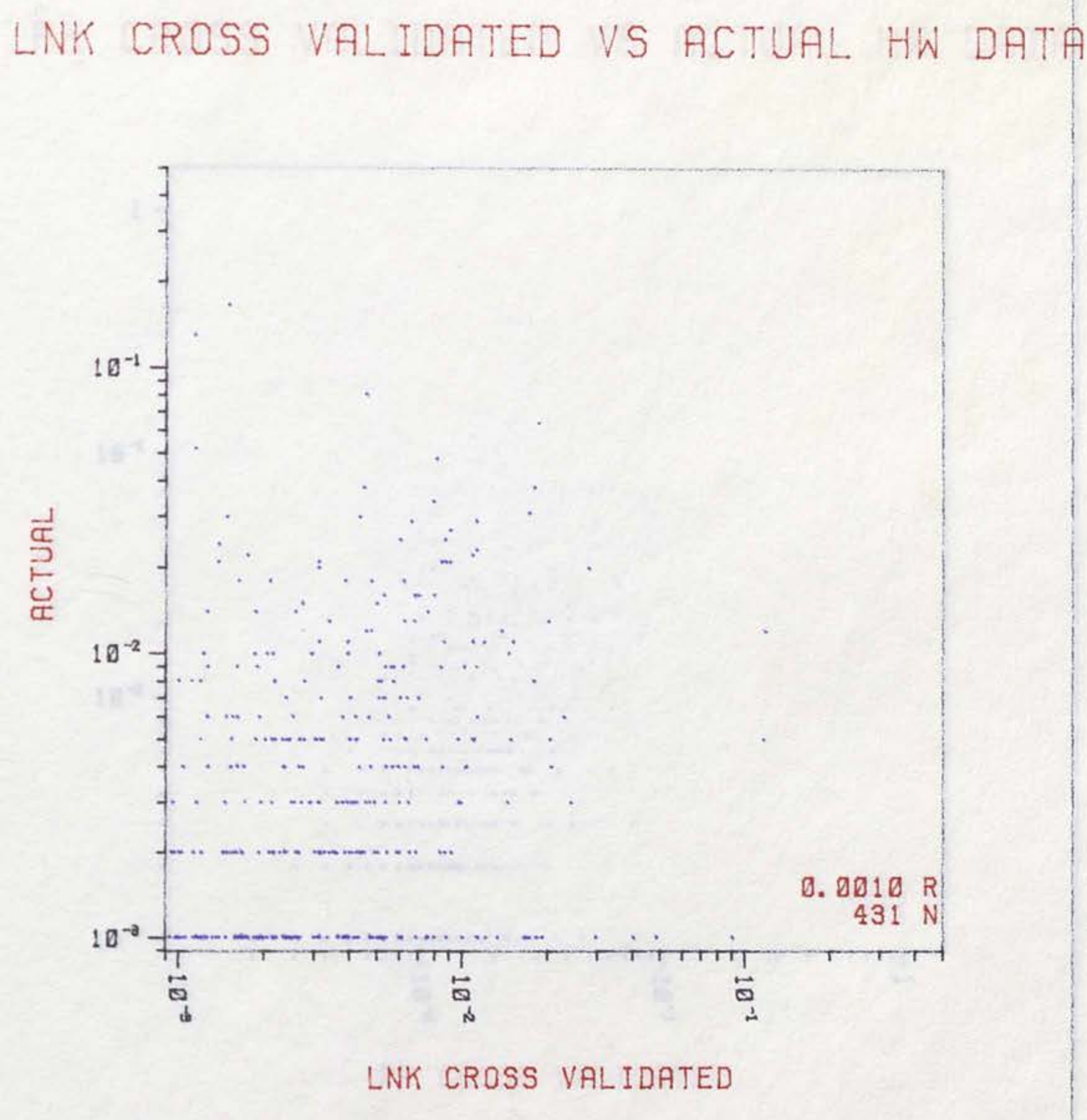

LNK CROSS VALIDATED VS ACTUAL HW DATA
ACTUAL
10^{-1}
10^{-2}
10^{-3}
0.0010 R
431 N
10^{-3}
10^{-2}
10^{-1}
LNK CROSS VALIDATED

Figure 5.4 a

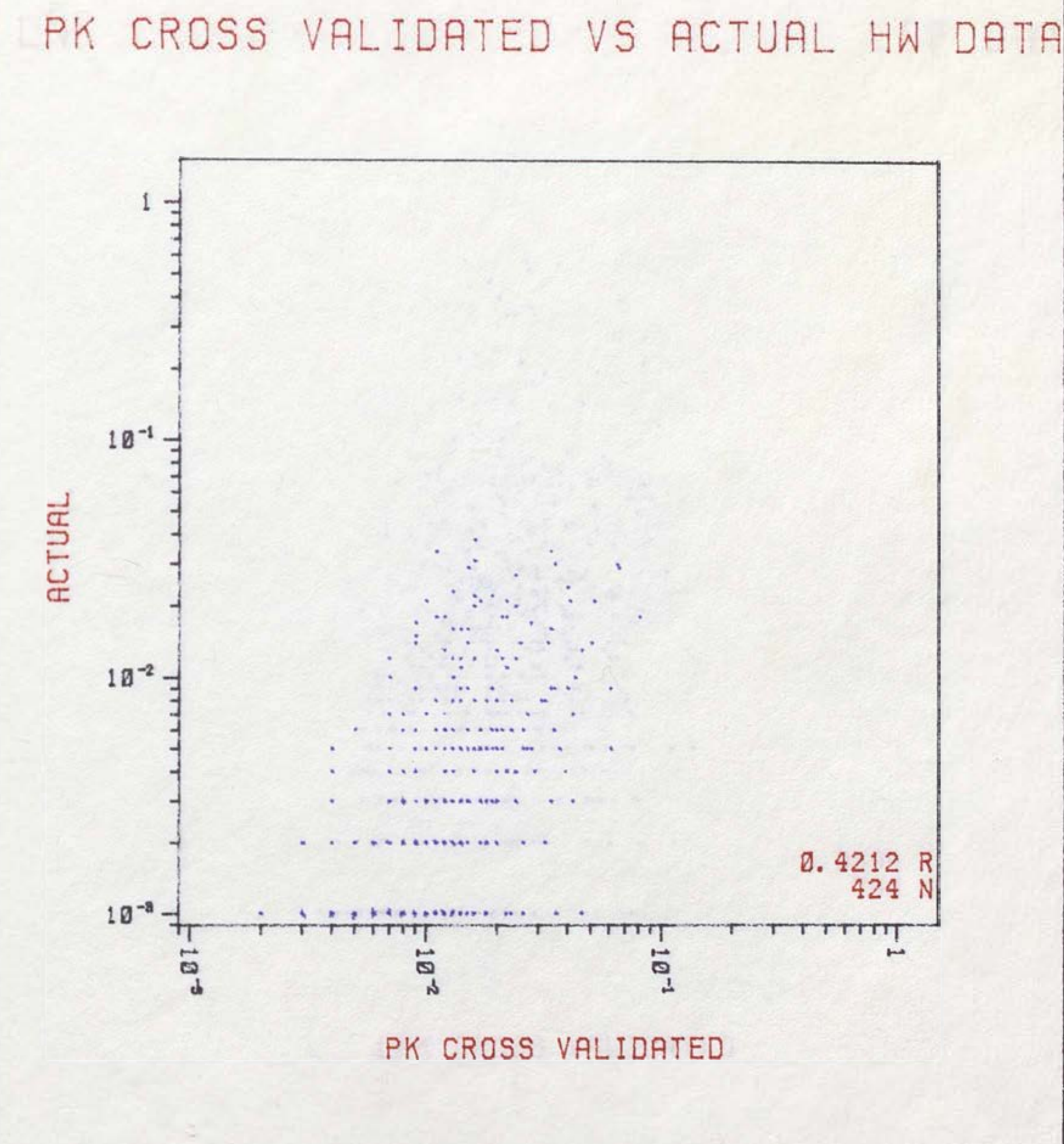

PK CROSS VALIDATED VS ACTUAL HW DATA
ACTUAL
1
10⁻¹
10⁻²
10⁻³
10⁻³
10⁻²
10⁻¹
1
0.4212 R
424 N
PK CROSS VALIDATED

Figure 5.4 b

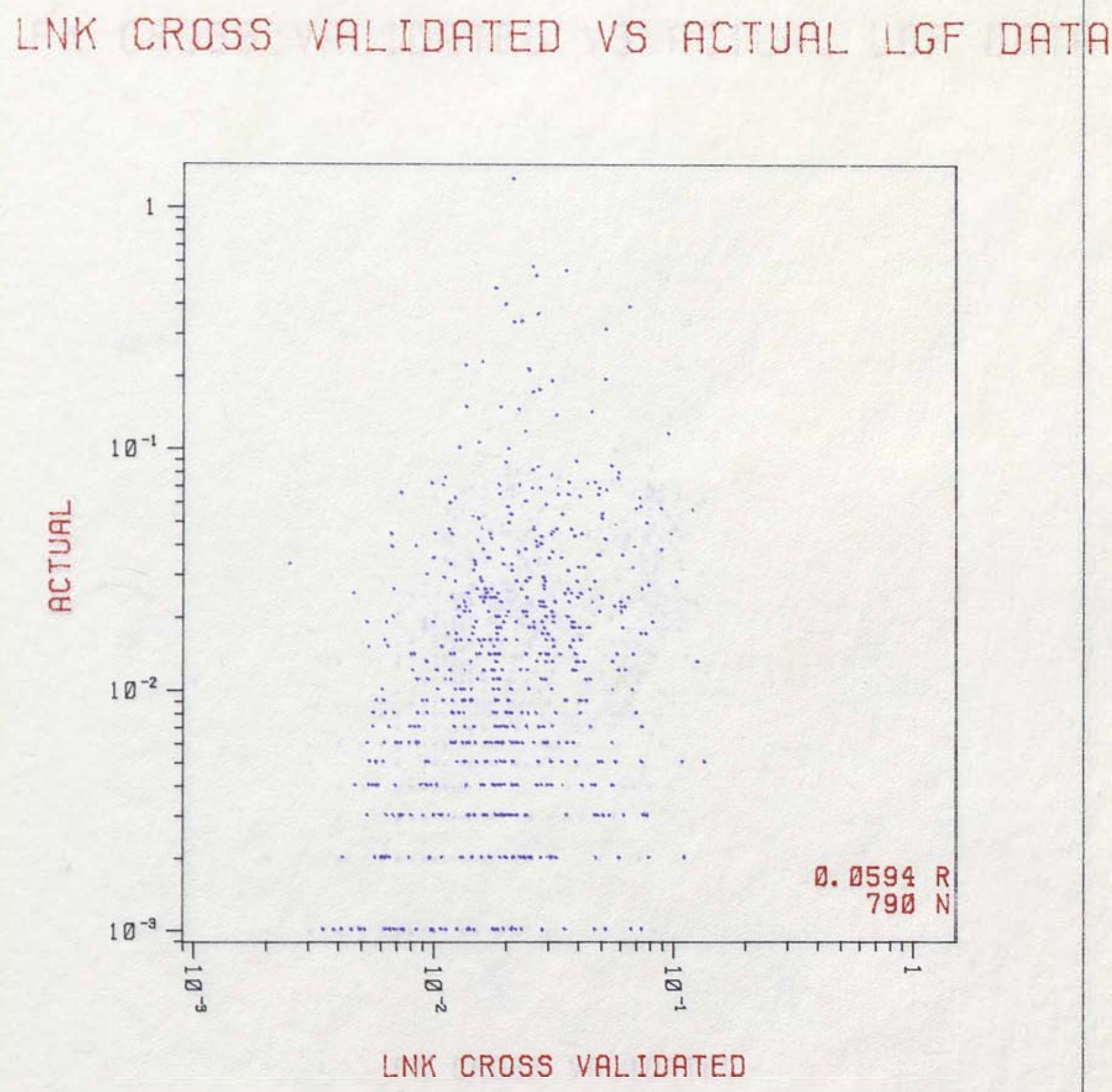

LNK CROSS VALIDATED VS ACTUAL LGF DATA
ACTUAL
1
10⁻¹
10⁻²
10⁻³
10⁻³
10⁻²
10⁻¹
1
LNK CROSS VALIDATED
0.0594 R
790 N

Figure 5.5 a

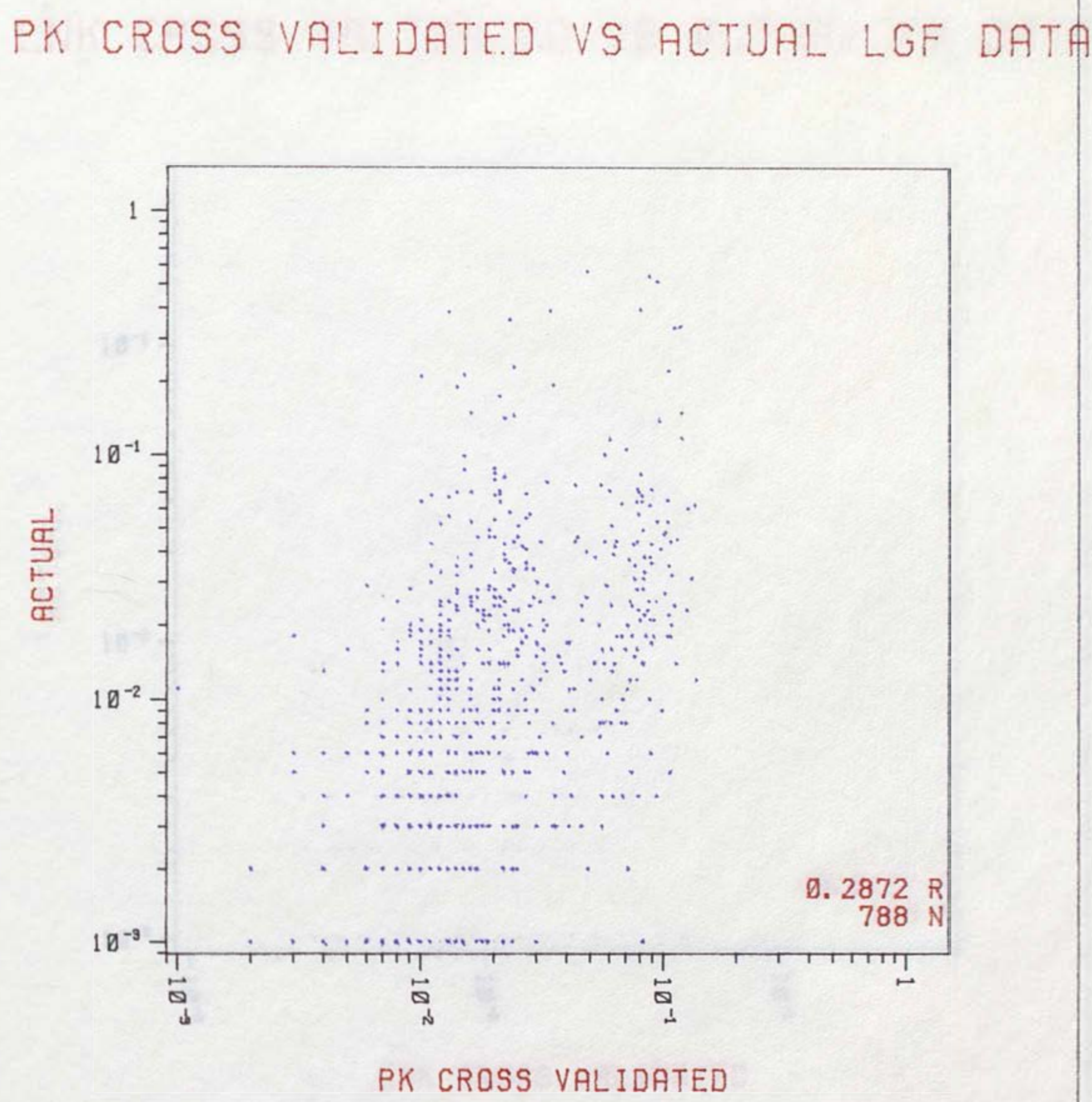

PK CROSS VALIDATED VS ACTUAL LGF DATA
ACTUAL
1
10⁻¹
10⁻²
10⁻³
0.2872 R
788 N
10⁻³
10⁻²
10⁻¹
1
PK CROSS VALIDATED

Figure 5.5 b

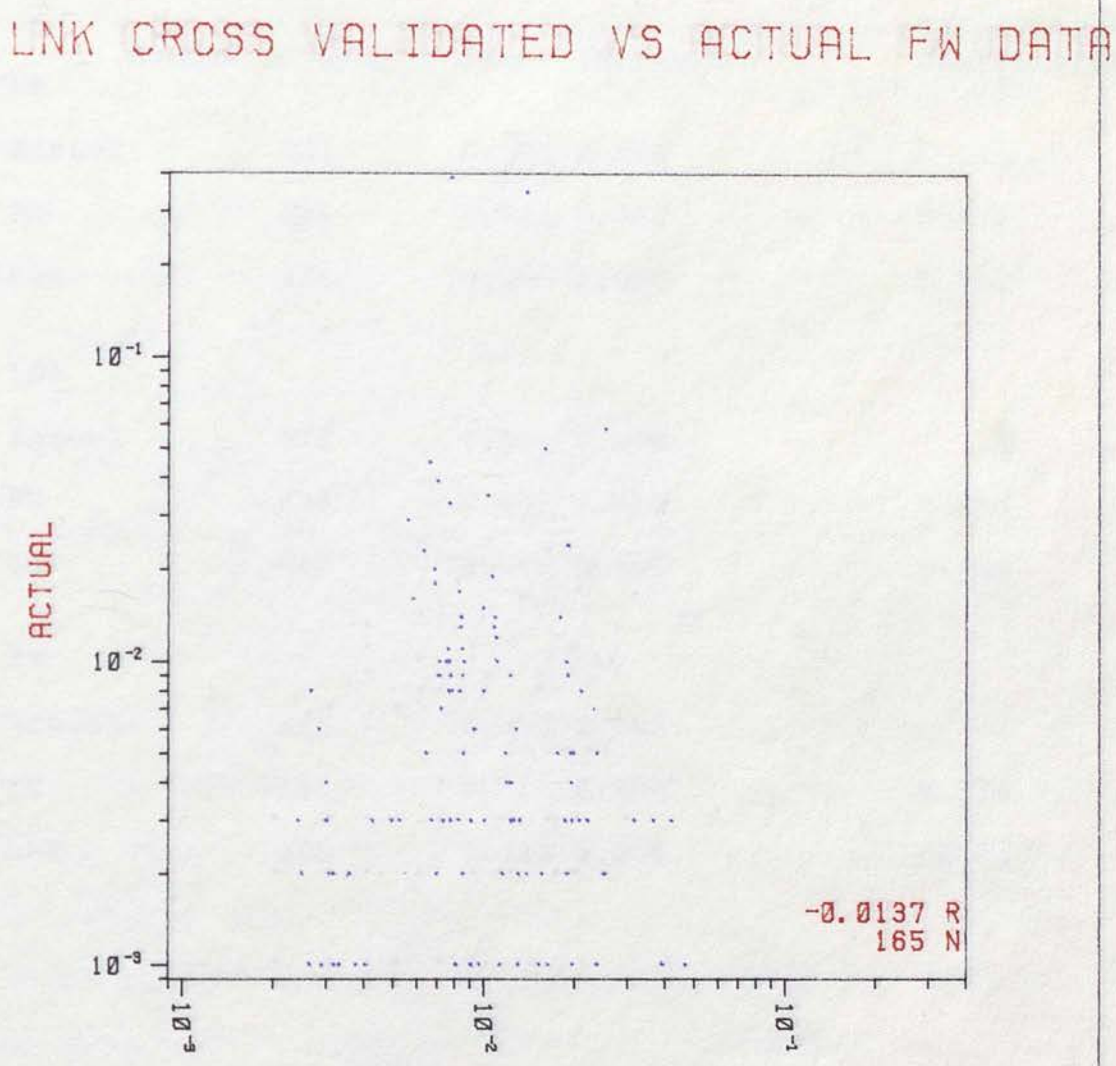

LNK CROSS VALIDATED VS ACTUAL FW DATA
ACTUAL
10⁻¹
10⁻²
10⁻³
-0.0137 R
165 N
10⁻³
10⁻²
10⁻¹
LNK CROSS VALIDATED

Figure 5.6 a

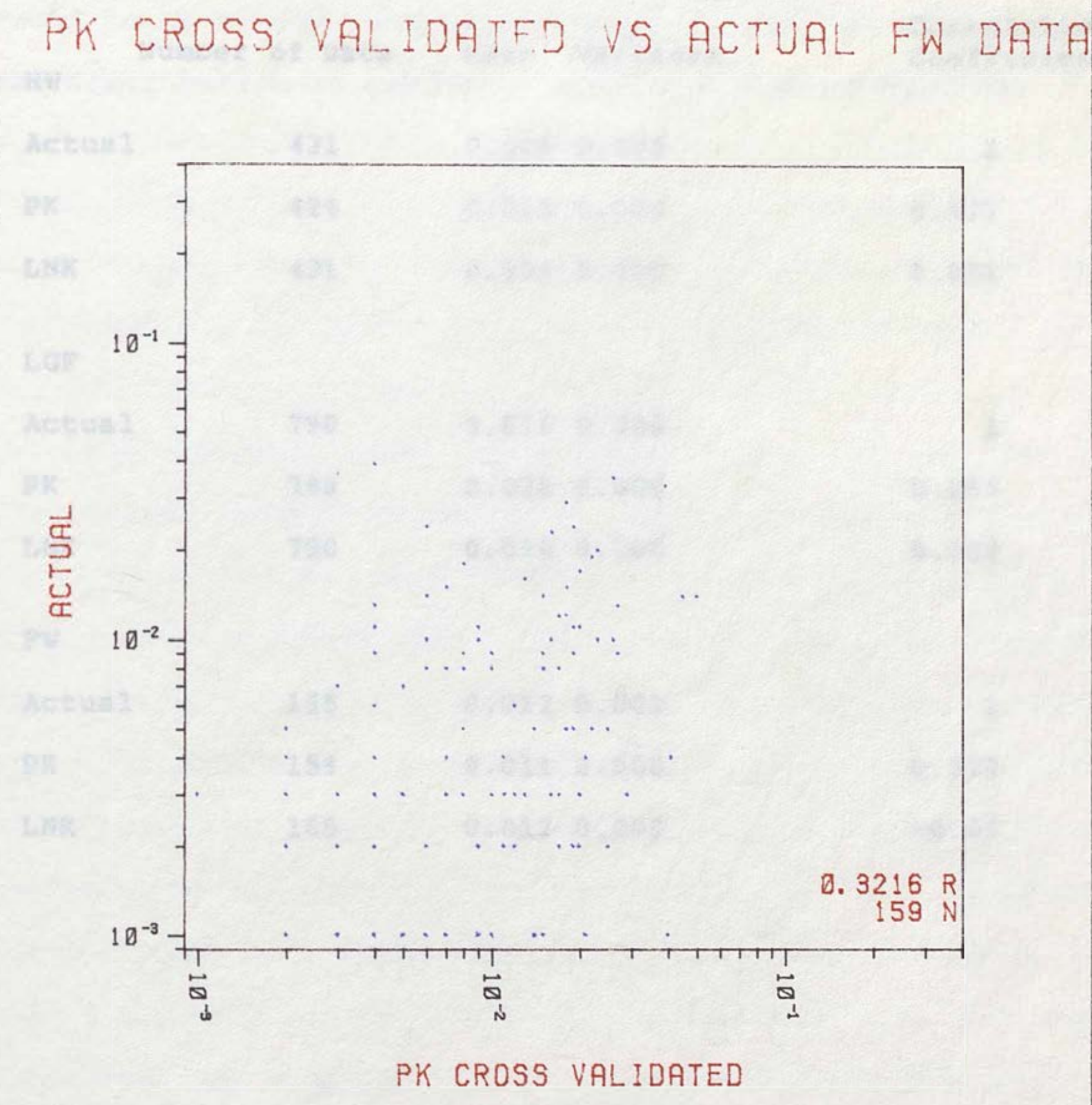

PK CROSS VALIDATED VS ACTUAL FW DATA
ACTUAL
PK CROSS VALIDATED
0.3216 R
159 N

Figure 5.6 b

TABLE 6. CROSS VALIDATION STATISTICS

	Number of Data	Mean	Variance	Correlation Coefficient
HW				
Actual	431	0.006	0.000	1
PK	424	0.015	0.000	0.421
LNK	431	0.006	0.000	0.001
LGF				
Actual	790	0.026	0.006	1
PK	788	0.028	0.000	0.287
LNK	790	0.026	0.000	0.059
FW				
Actual	165	0.012	0.002	1
PK	159	0.011	0.000	0.322
LNK	165	0.012	0.000	-0.01

uniform, and cross variography required for PK than with the LN transformed variograms used for LNK. In general, both techniques over estimated low grades, and under estimated high grades, however, the mean grade of each zone was accurately estimated by both techniques. Therefore, neither method is found to be biased, although it appears that LNK will smooth the distribution of grades in each zone more so than PK.

Chapter 6 Probability and Lognormal Kriging Results

This chapter presents the results of kriging exploration data using PK and LNK to determine the distribution of point support grades within exploration size blocks. The total tonnage and quantity of metal above a series of cutoff grades are the values which are examined in the comparison. The basis for the evaluation of these estimation techniques is the distribution of blasthole grades within each exploration size block. Although this distribution does not represent the true distribution of point support grades within a block, it does represent the ultimate sampling grid for this deposit.

6.1 Application of lognormal kriging

LNK in this paper is accomplished by OK of the LN transformed random variable Z(x). Panels 100 feet x 100 feet in size are block kriged in 2-dimensions, utilizing a 4 x 4 discretization matrix. OK is applied to each panel to estimate the logarithmic mean grade and kriging variance for each panel. In actual reserve estimation, the logarithmic variance of the smu size block in the deposit is added to the kriging variance to estimate the total variance of the lognormal distribution in a block. The objective of this study is to evaluate the point distribution of grades within a block, therefore, the logarithmic sample variance of the data used to estimate

the mean grade of each panel is added to the kriging variance to obtain the total variance of the lognormal distribution assumed for each panel (Rendu, 1981); (Kim, 1987). The proportion of a panel above a given cutoff grade is then used to estimate tonnage and quantity of metal above the cutoff, as developed in Section 2.2.3.

6.2 Application of probability kriging

PK in this paper is accomplished by OK of a linear combination of the uniform and indicator transforms of the original random variable. Blocks 100 feet x 100 feet x 20 feet are kriged in 3-dimensions, utilizing a 5 x 5 x 2 discretization matrix. PK estimates directly the proportion of point support grades within a block above a given cutoff grade (Sullivan, 1985), therefore, no affine correction is necessary. These proportions correlate directly to the tonnage of a block above a cutoff grade. The quantity of metal above a cutoff grade is easily derived from these proportions as described in Section 2.1.4.

6.3 Kriging results

The results of this section of the study are presented in a series of scatter plots, and two tables. The respective correlation coefficients and the mean of each of the groups of data are given in Tables 7 and 8. The correlation coefficient, r, relates the percent variation in the dependent variable that is attributible to the
independent variable.

6.3.1 Tonnage comparison

Tonnage comparisons are completed by plotting the proportion of a block estimated to exceed the cutoff grade in question, versus the proportion of blastholes within the block with grades exceeding the cutoff grade. The results plotted at each cutoff grade are displayed in Figures 6.1 - 6.5.

The correlation coefficients presented in Table 7 indicate that a strong linear relationship exists between the estimated tonnages and the actual data at the lower cutoff grades. This relationship does not exist at the higher cutoff grades, indicating that bias exists either in both estimation techniques, and/or in the simplistic method in which the actual tonnage was estimated.

6.3.2 Quantity of metal comparison

The quantity of metal comparison in this study is based on the number of blastholes within a panel exceeding a specified cutoff grade, and their average grade. The equations defining quantity of metal for both PK and LNK are presented in Sections 2.1 and 2.2, respectively. The results of the quantity of metal comparison show a poor correlation of estimated to actual values at all cutoff grades (Figures 6.6 - 6.10); (Table 8). At each cutoff grade, PK significantly underestimates both the actual value and the estimated LNK quantity of metal. The reason for this underestimation cannot be accounted for soley by error involved in the PK estimation technique. To the contrary, it

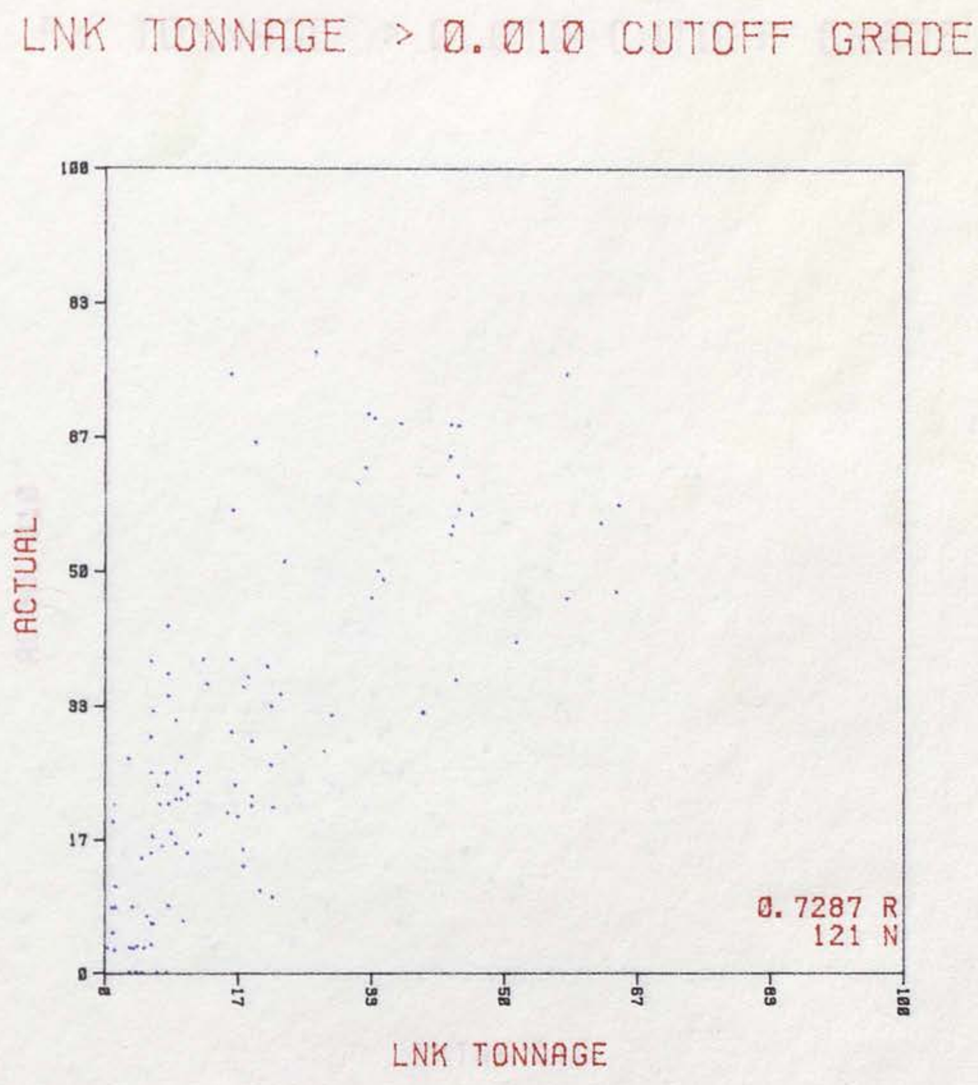

LNK TONNAGE > 0.010 CUTOFF GRADE
ACTUAL
LNK TONNAGE
0.7287 R
121 N
100
83
67
50
33
17
0
0
17
33
50
67
83
100

Figure 6.1 a

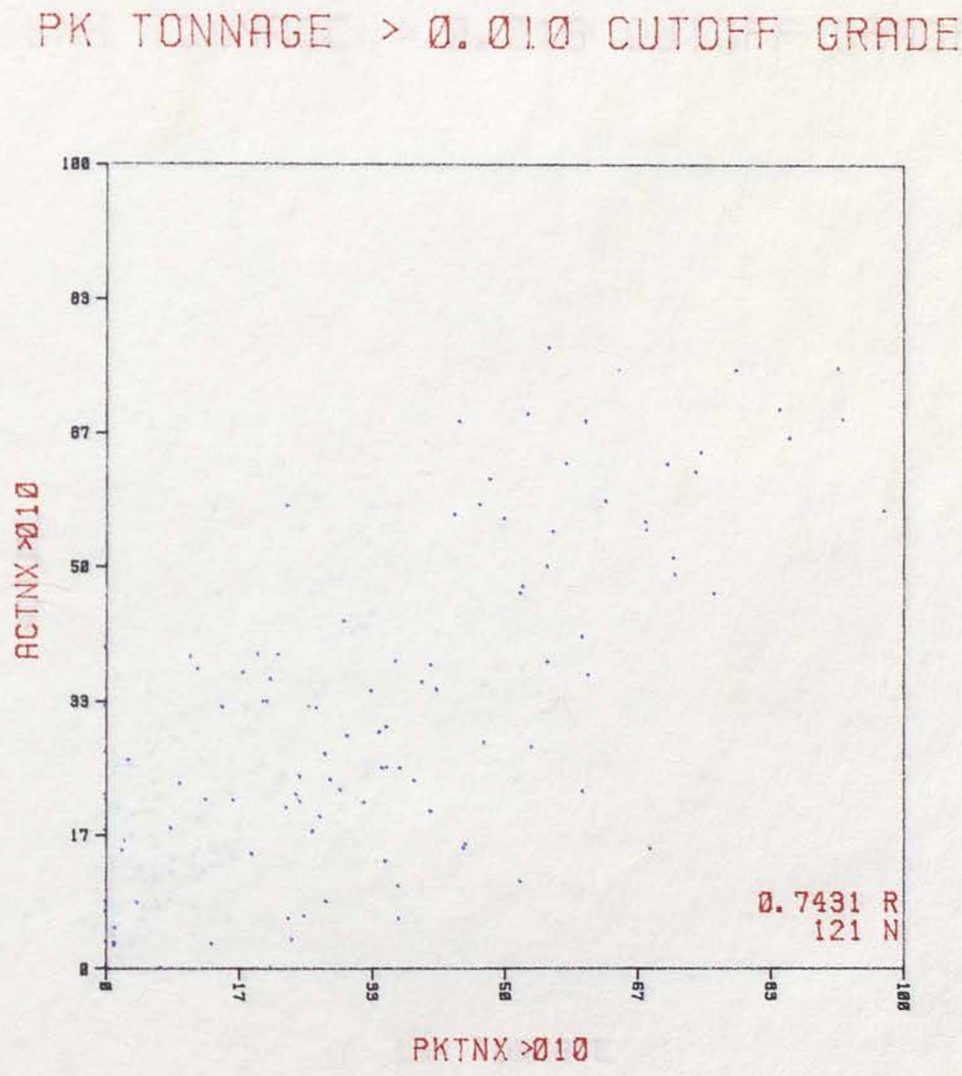

PK TONNAGE > Ø.Ø1Ø CUTOFF GRADE
ACTNX >Ø1Ø
PKTNX >Ø1Ø
0.7431 R
121 N

Figure 6.1 b

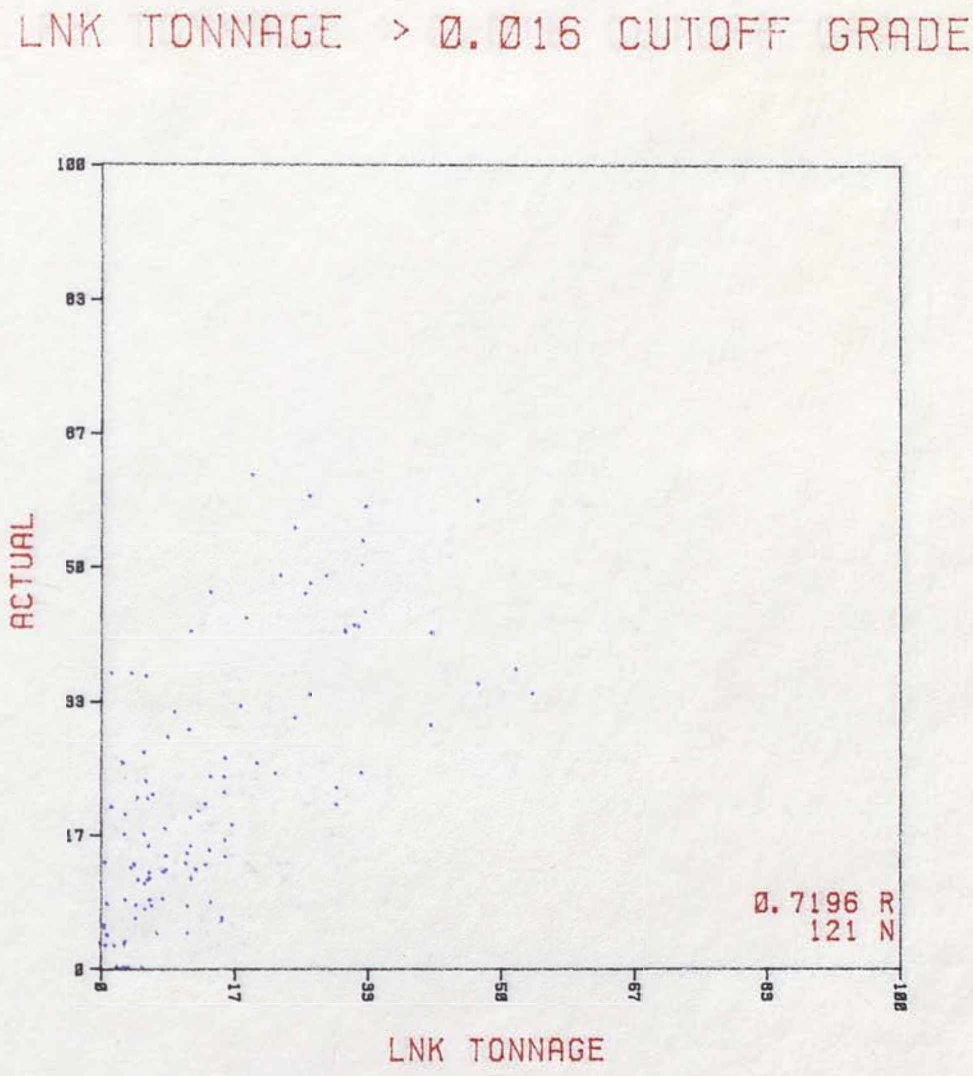

LNK TONNAGE > 0.016 CUTOFF GRADE
ACTUAL
LNK TONNAGE
0.7196 R
121 N

Figure 6.2 a

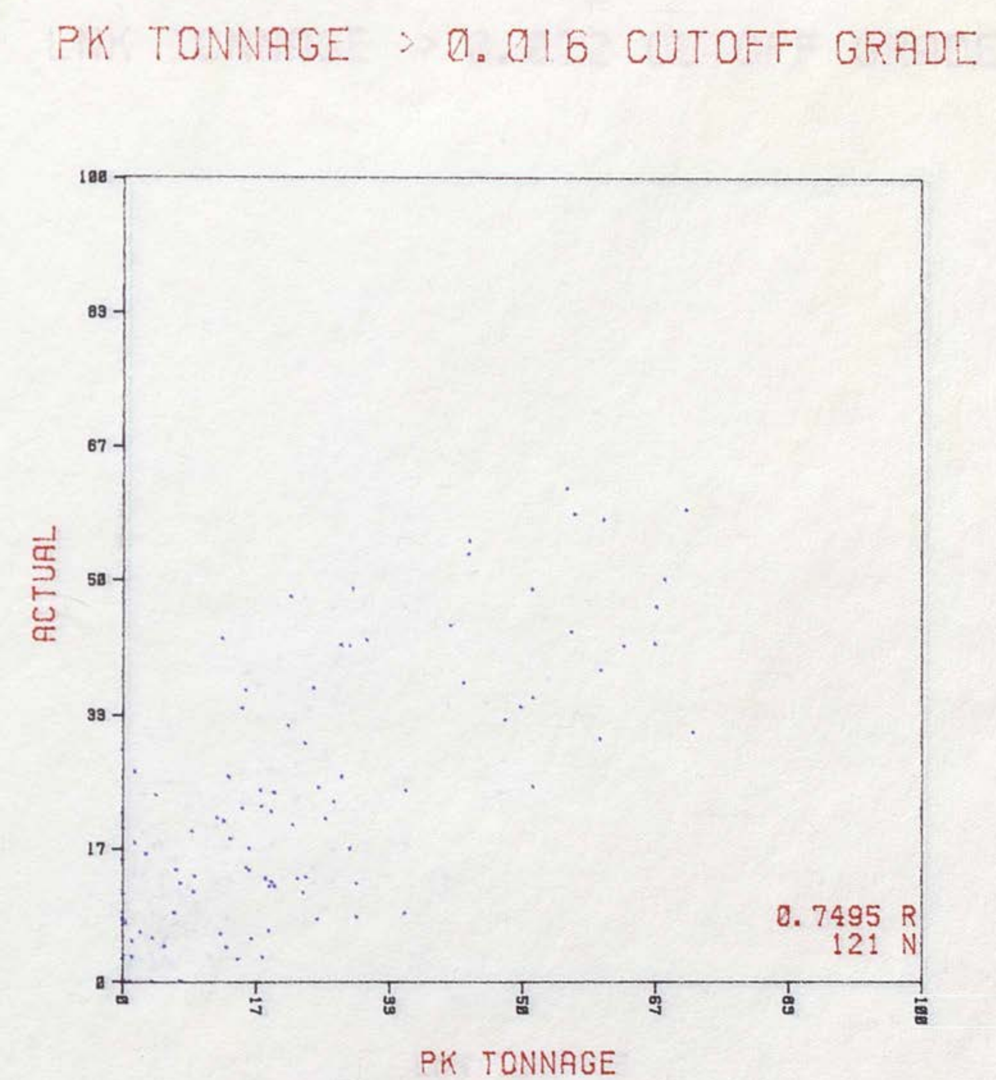


Figure 6.2 b

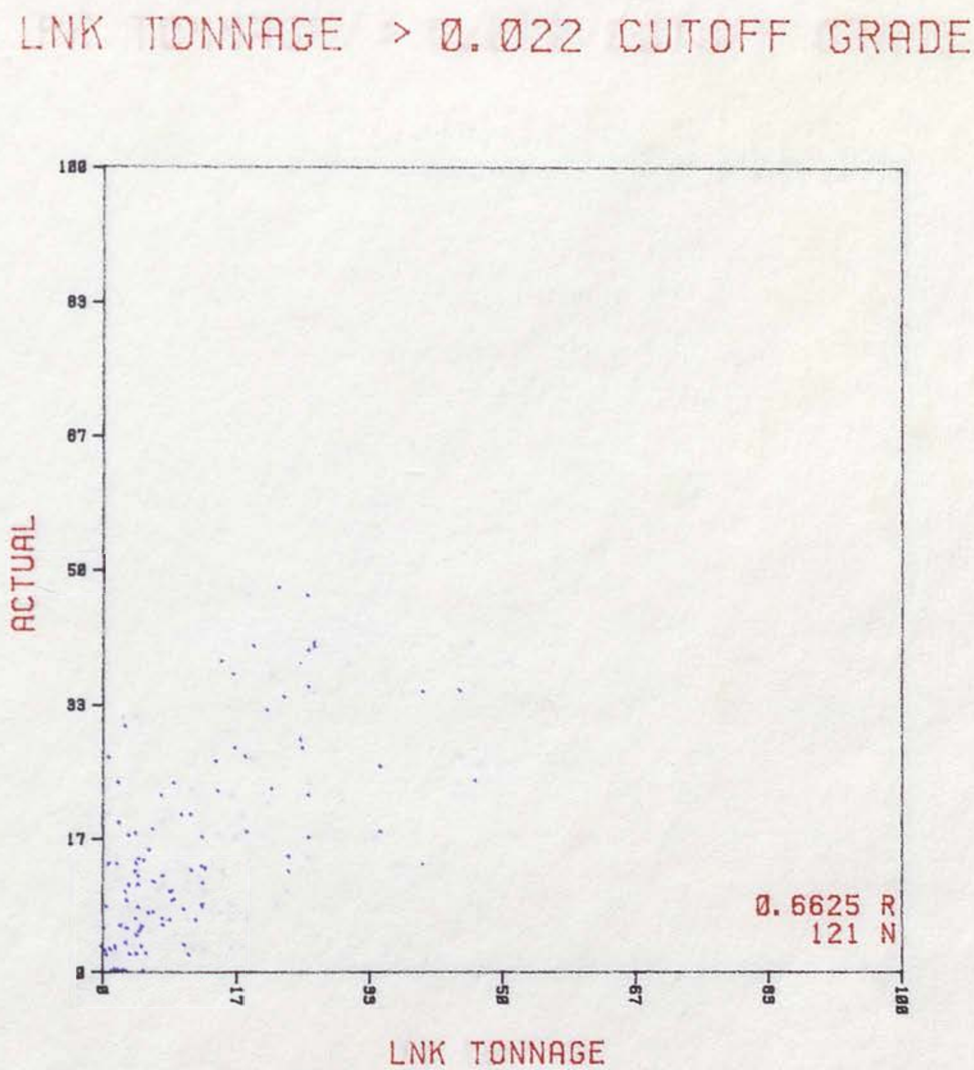

LNK TONNAGE > Ø.Ø22 CUTOFF GRADE
ACTUAL
100
83
67
50
33
17
0
Ø.6625 R
121 N
0
17
33
50
67
83
100
LNK TONNAGE

Figure 6.3 a

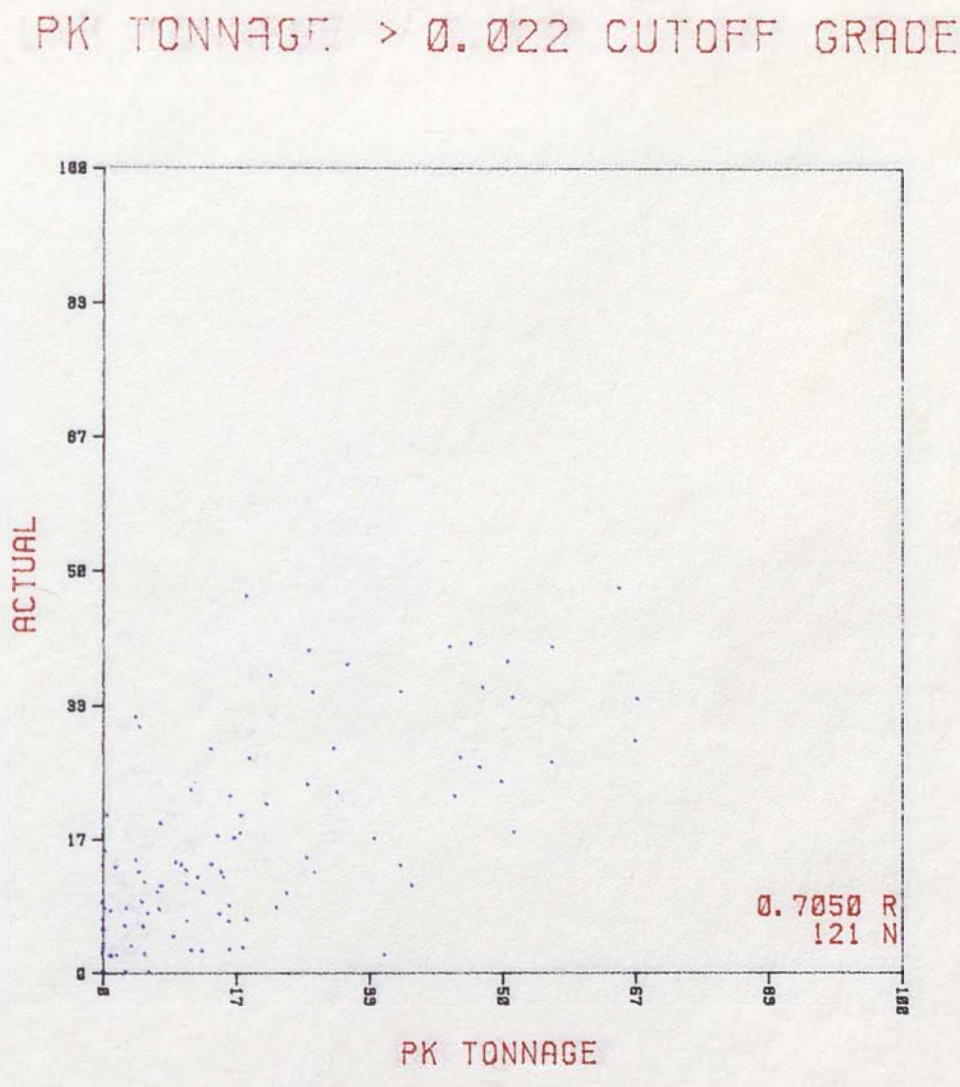


Figure 6.3 b

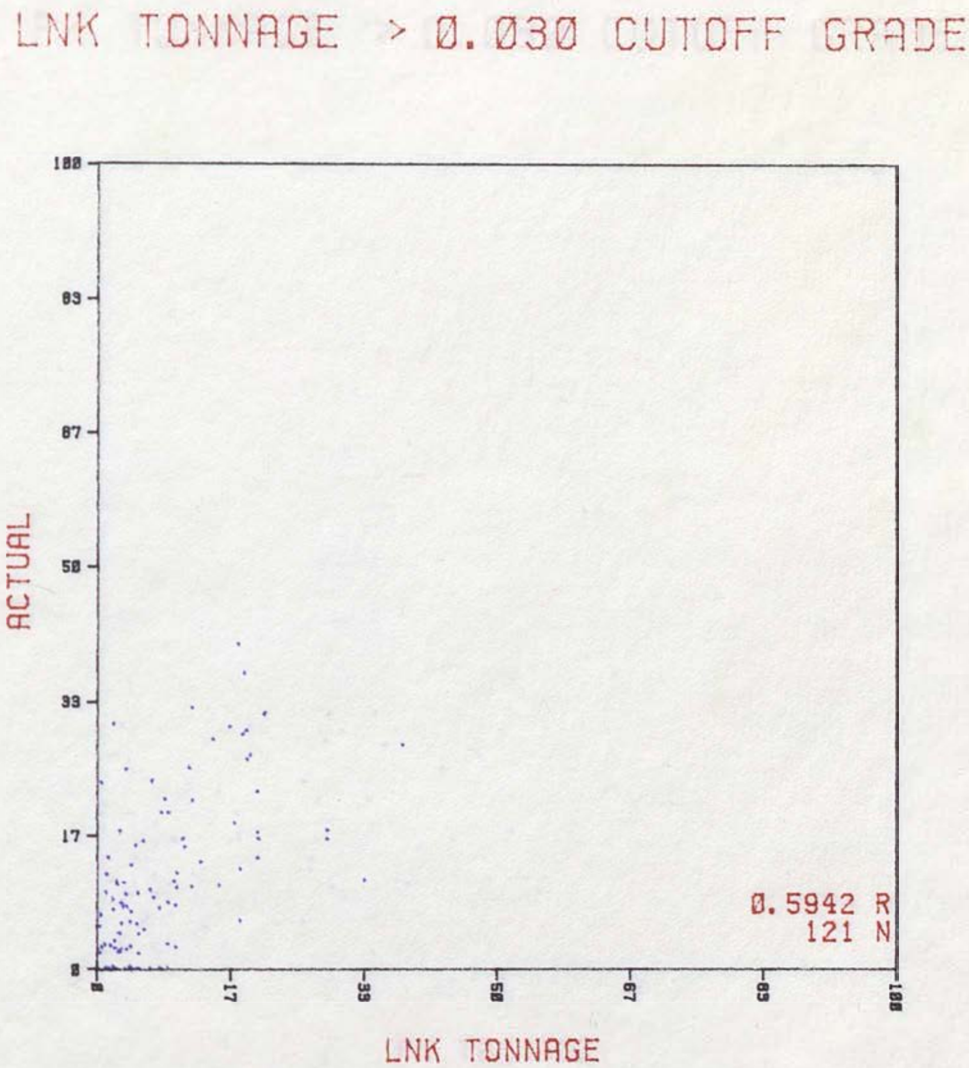

LNK TONNAGE > Ø.Ø3Ø CUTOFF GRADE
100
83
67
50
33
17
0
ACTUAL
Ø.5942 R
121 N
0
17
33
50
67
83
100
LNK TONNAGE

Figure 6.4 a

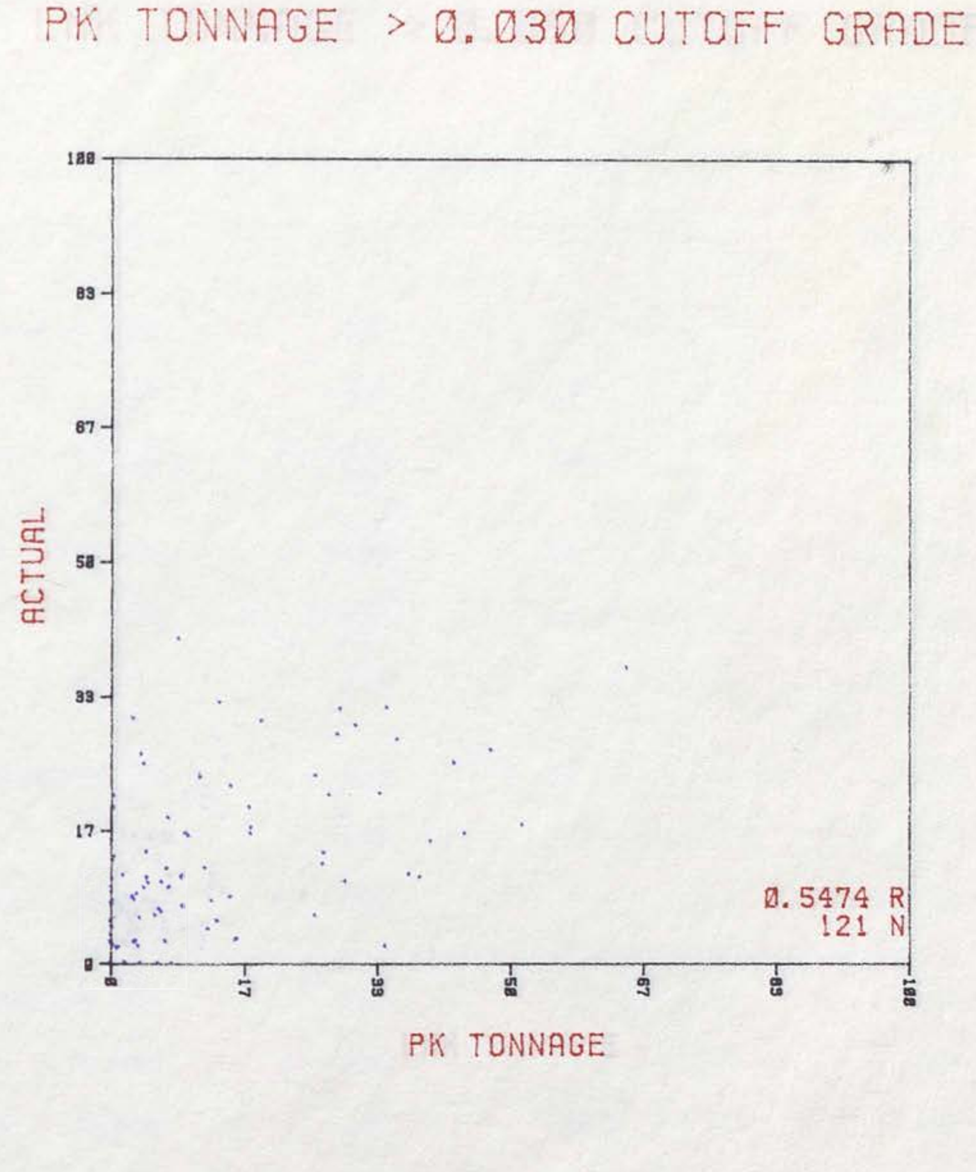


Figure 6.4 b

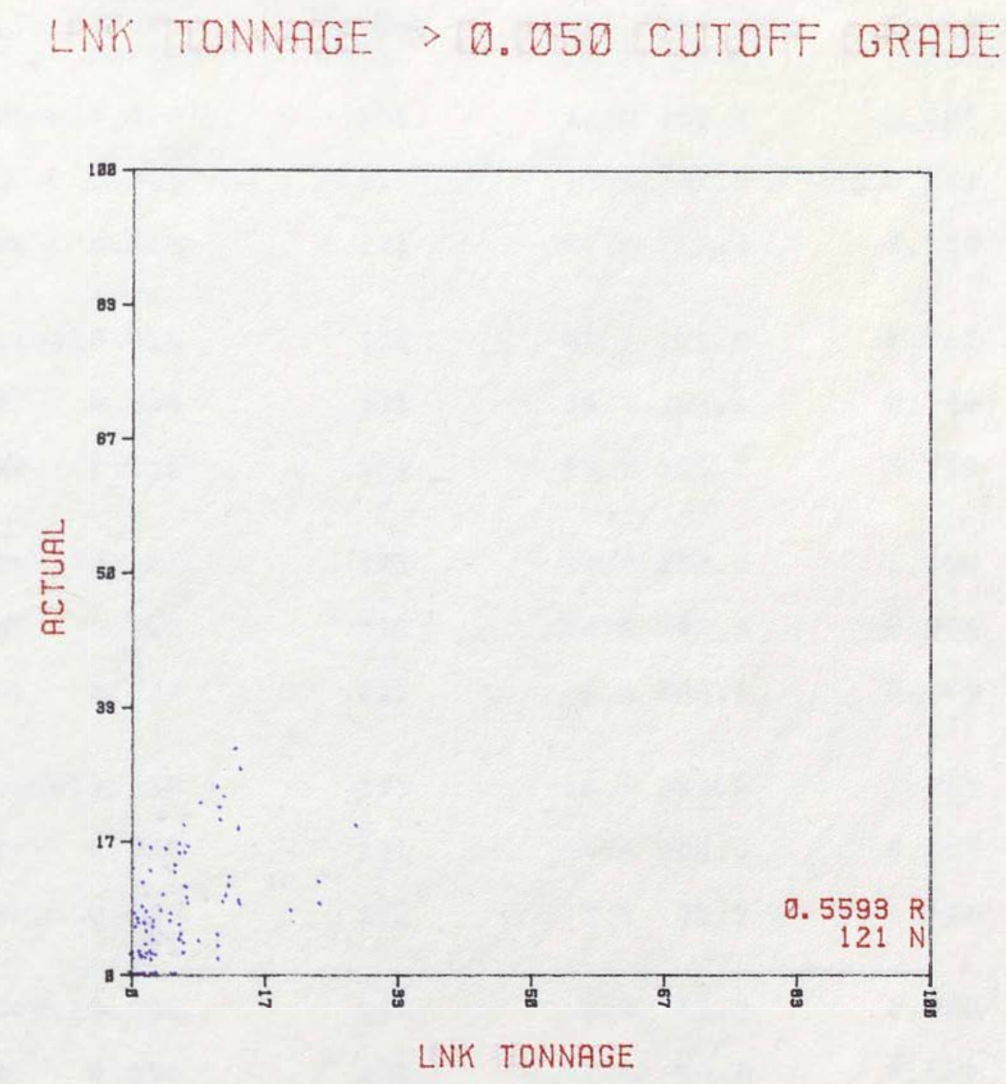

LNK TONNAGE > 0.050 CUTOFF GRADE
ACTUAL
LNK TONNAGE
0.5593 R
121 N

Figure 6.5 a

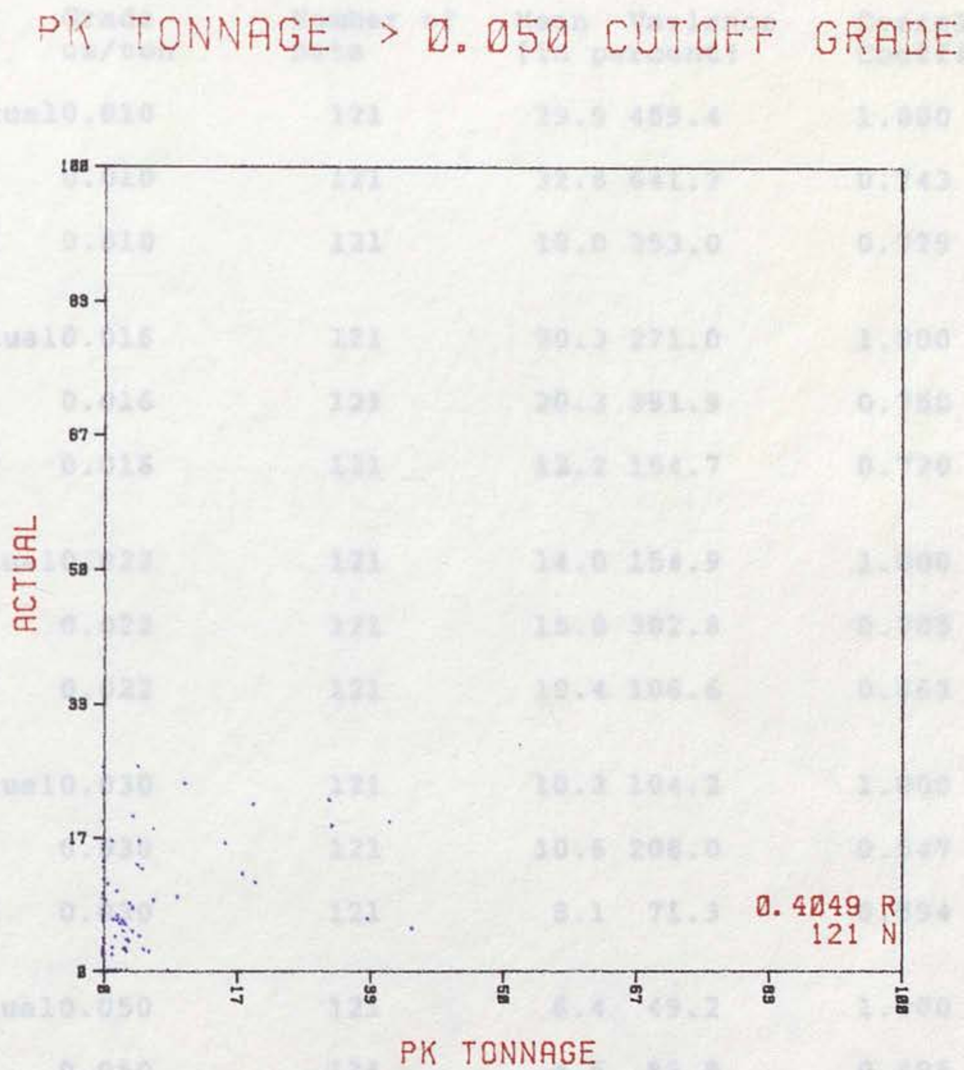

PK TONNAGE > 0.050 CUTOFF GRADE
100
83
67
50
33
17
0
ACTUAL
0
17
33
50
67
83
100
PK TONNAGE
0.4049 R
121 N

Figure 6.5 b

TABLE 7. TONNAGE COMPARISON STATISTICS

	Cutoff Grade oz/ton	Number of Data	Mean	Variance	Correlation Coefficient
			[in percent]		
Actual	0.010	121	29.9	459.4	1.000
PK	0.010	121	32.8	641.7	0.743
LNK	0.010	121	18.0	253.0	0.729
Actual	0.016	121	20.3	271.0	1.000
PK	0.016	121	20.3	391.9	0.750
LNK	0.016	121	13.2	154.7	0.720
Actual	0.022	121	14.0	154.9	1.000
PK	0.022	121	15.0	302.8	0.705
LNK	0.022	121	10.4	106.6	0.663
Actual	0.030	121	10.3	104.2	1.000
PK	0.030	121	10.6	208.0	0.547
LNK	0.030	121	8.1	71.3	0.594
Actual	0.050	121	6.4	49.2	1.000
PK	0.050	121	4.6	95.8	0.405
LNK	0.050	121	5.2	31.4	0.559

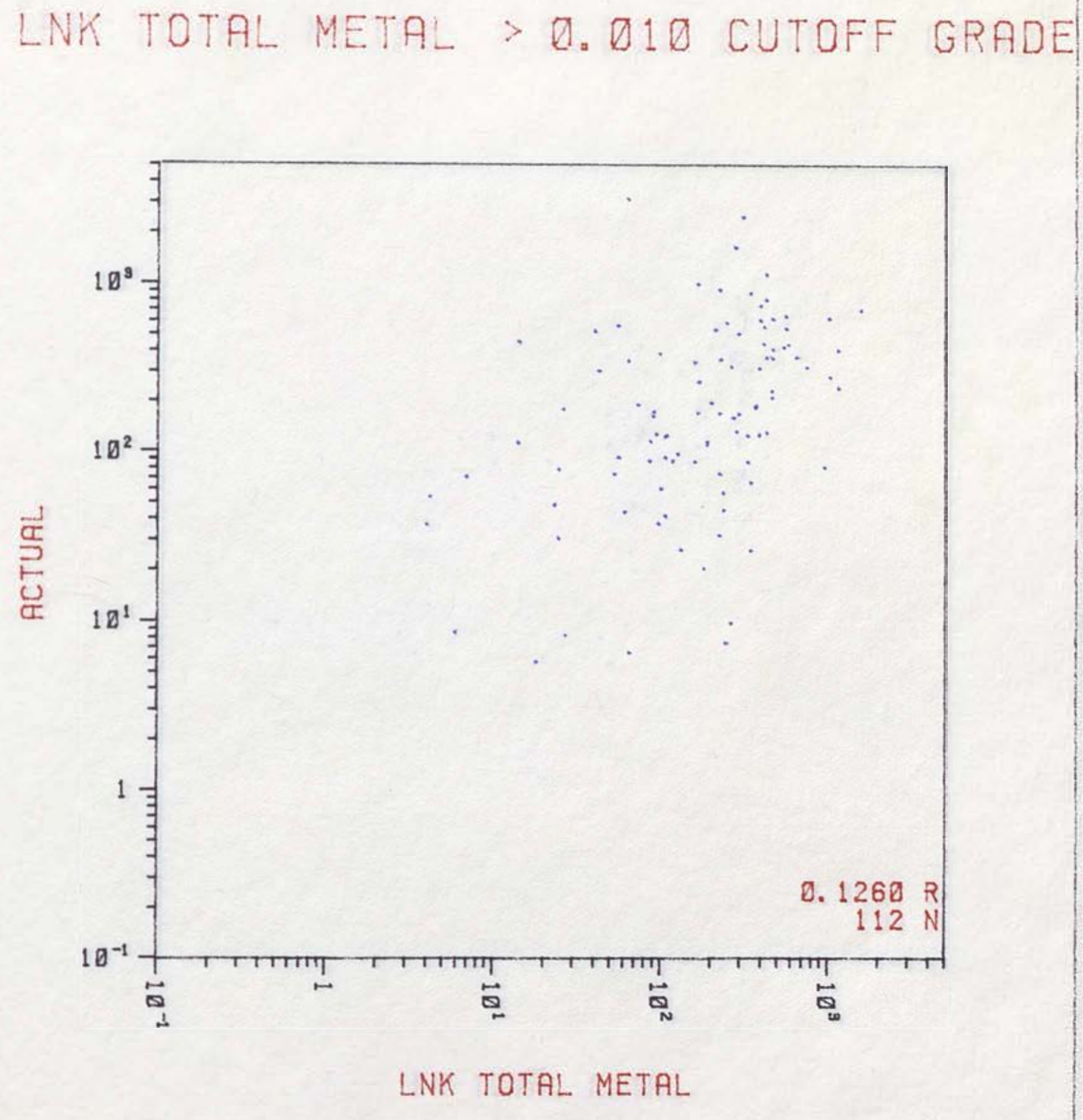

LNK TOTAL METAL > 0.010 CUTOFF GRADE
ACTUAL
LNK TOTAL METAL
0.1260 R
112 N
10^{-1}
1
10^{1}
10^{2}
10^{3}

Figure 6.6 a

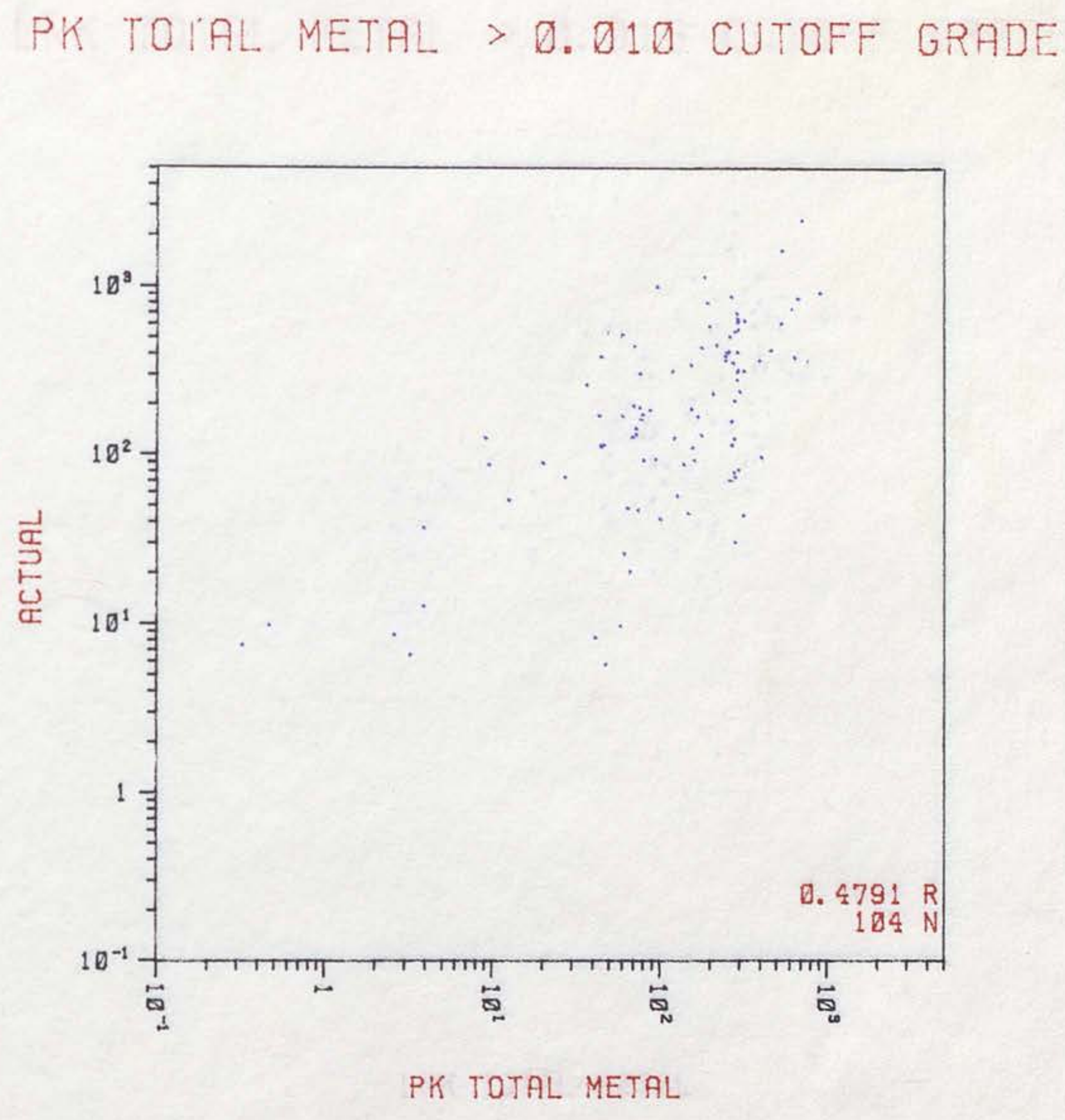

PK TOTAL METAL > 0.010 CUTOFF GRADE
ACTUAL
10³
10²
10¹
1
10⁻¹
0.4791 R
104 N
10⁻¹
1
10¹
10²
10³
PK TOTAL METAL

Figure 6.6 b

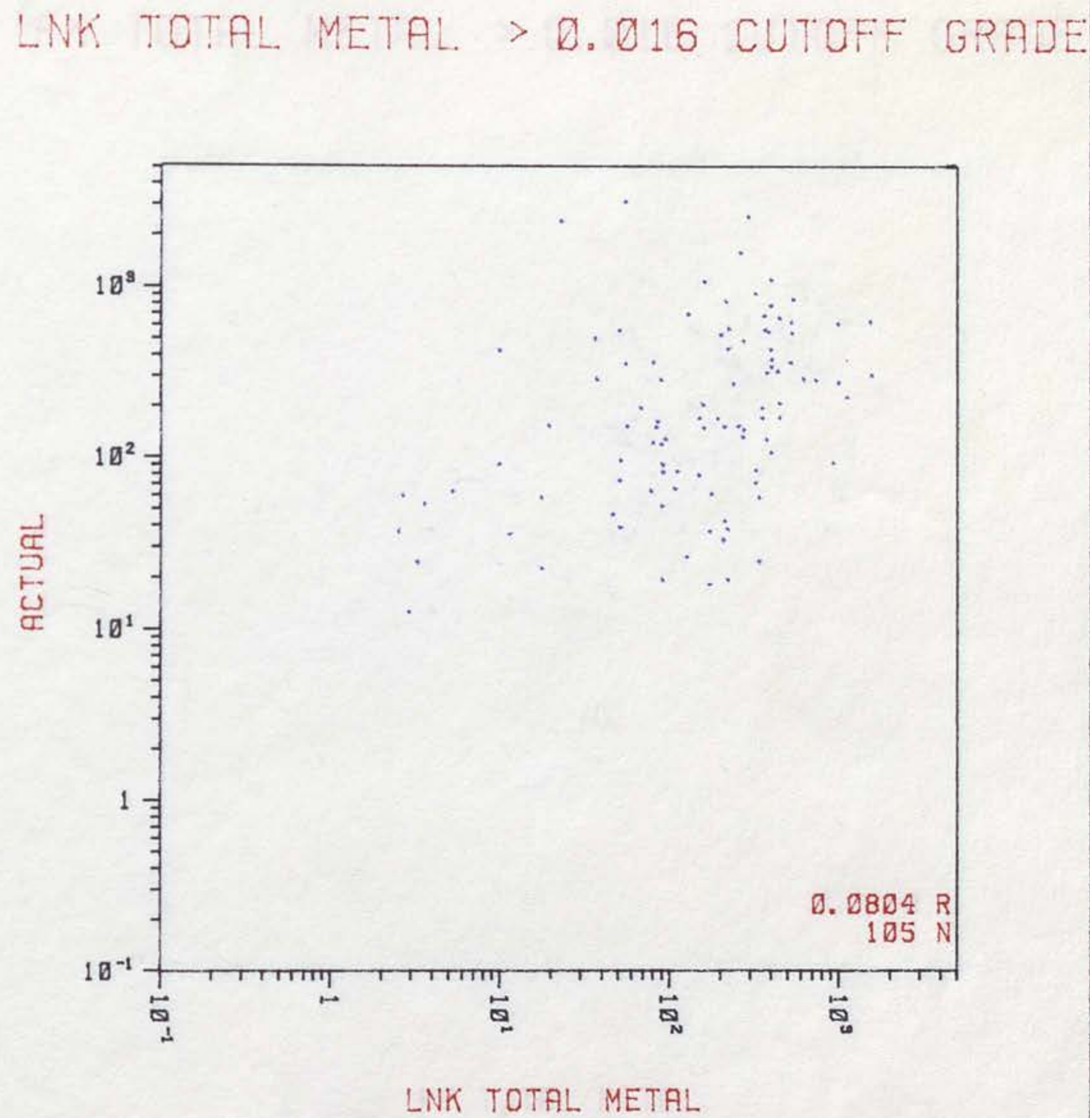

LNK TOTAL METAL > 0.016 CUTOFF GRADE
ACTUAL
LNK TOTAL METAL
0.0804 R
105 N

Figure 6.7 a

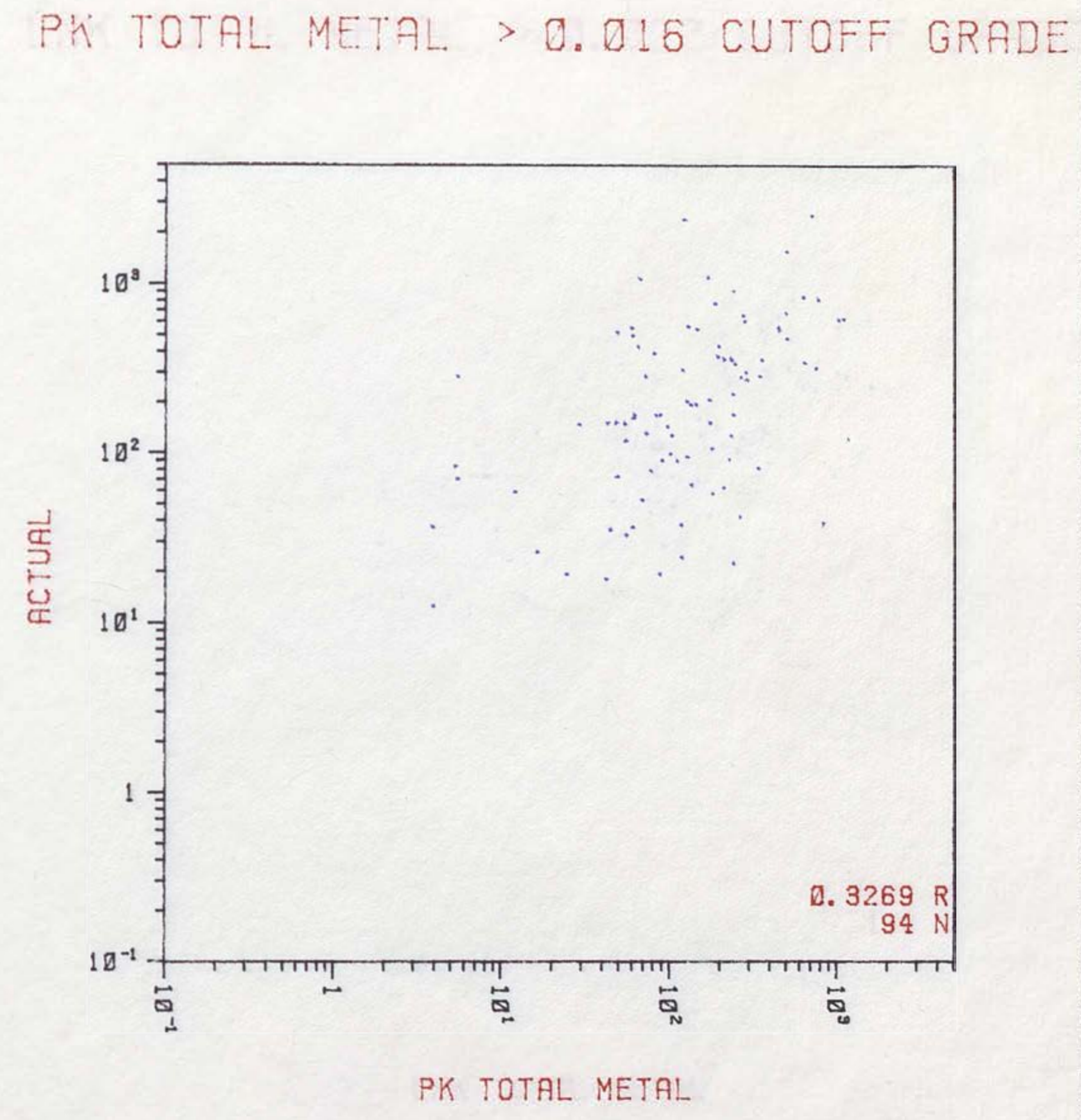

PK TOTAL METAL > 0.016 CUTOFF GRADE
ACTUAL
PK TOTAL METAL
0.3269 R
94 N

Figure 6.7 b

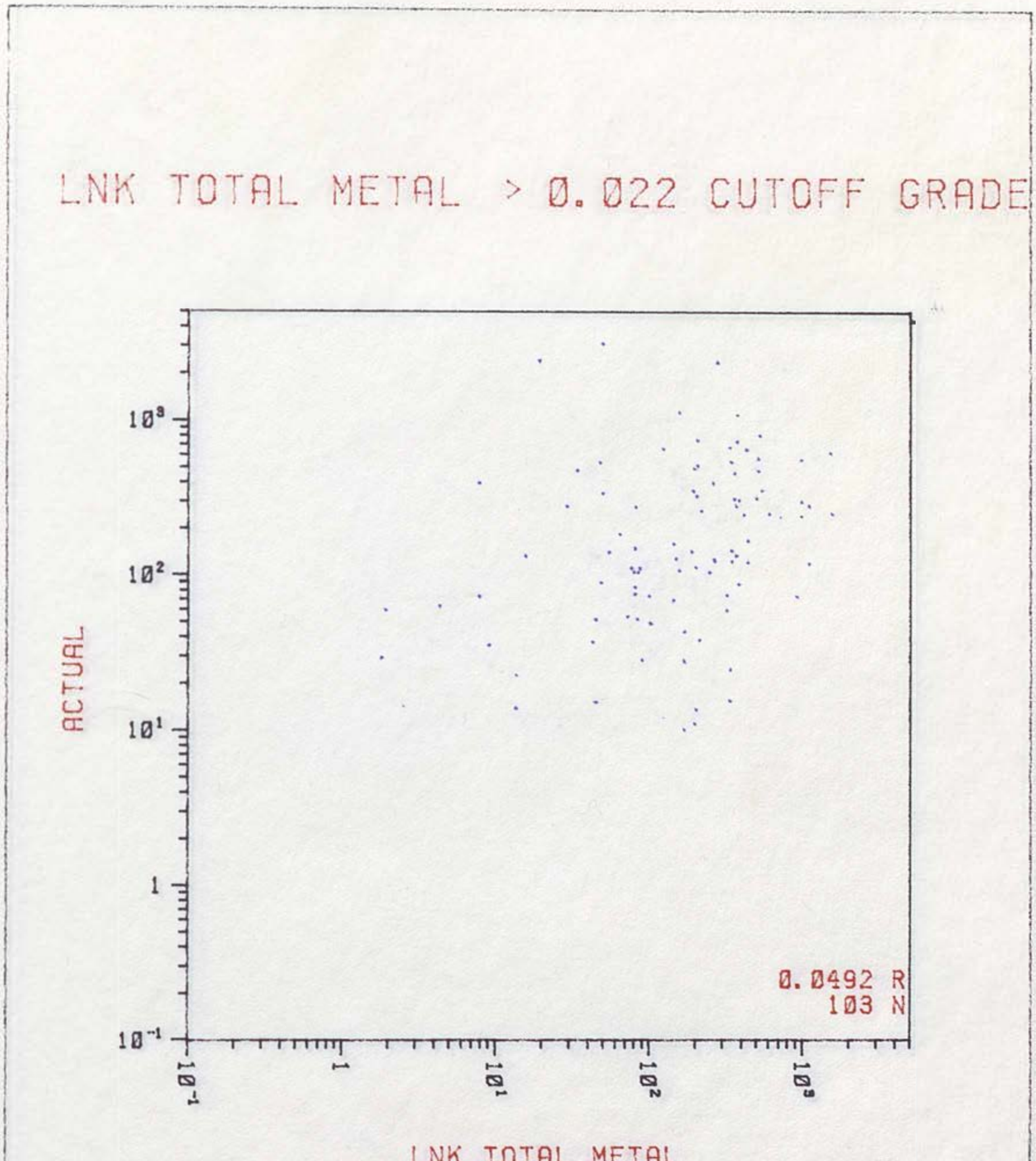

LNK TOTAL METAL > 0.022 CUTOFF GRADE
ACTUAL
10³
10²
10¹
1
10⁻¹
10⁻¹
1
10¹
10²
10³
LNK TOTAL METAL
0.0492 R
103 N

Figure 6.8 a

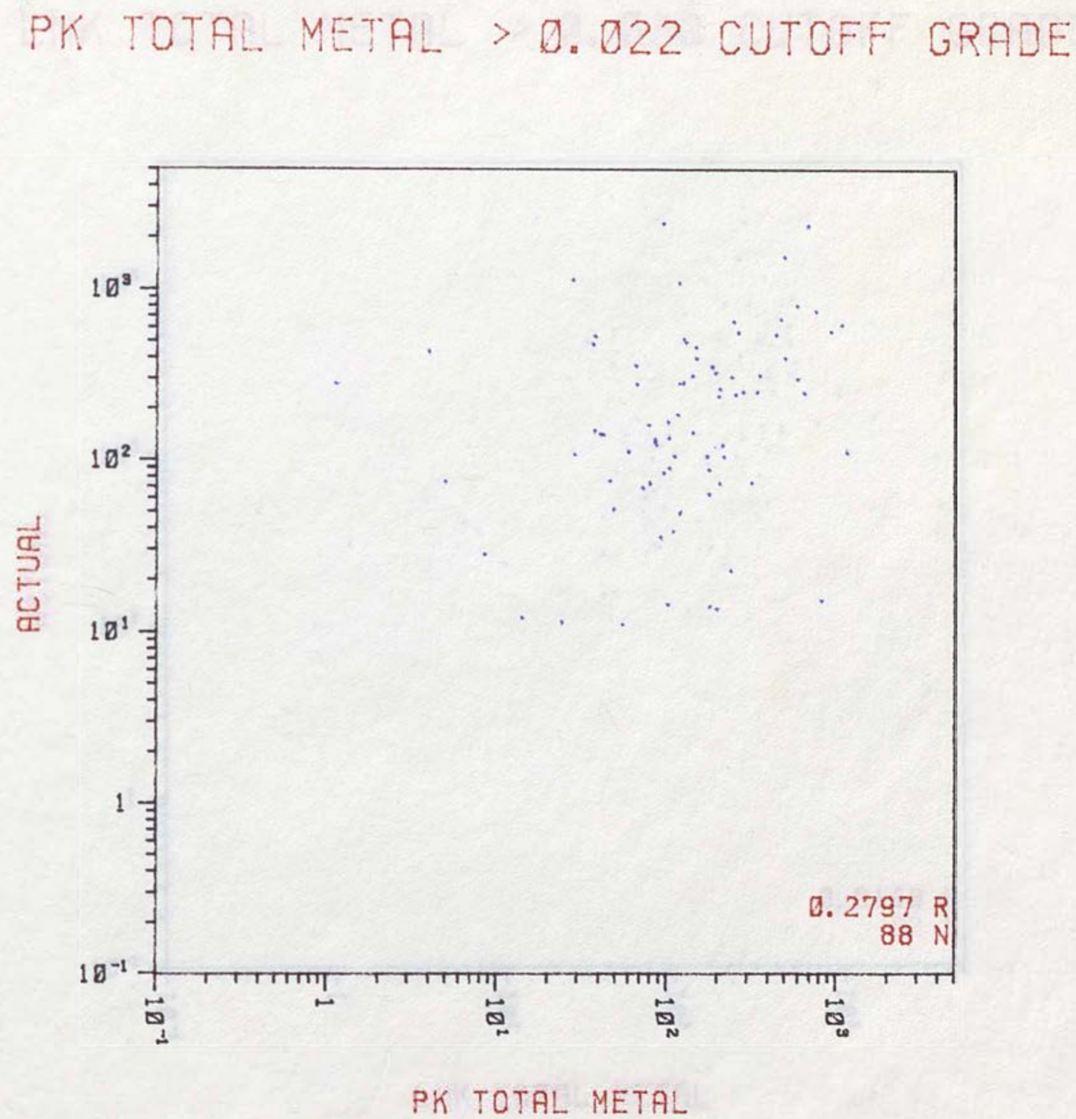
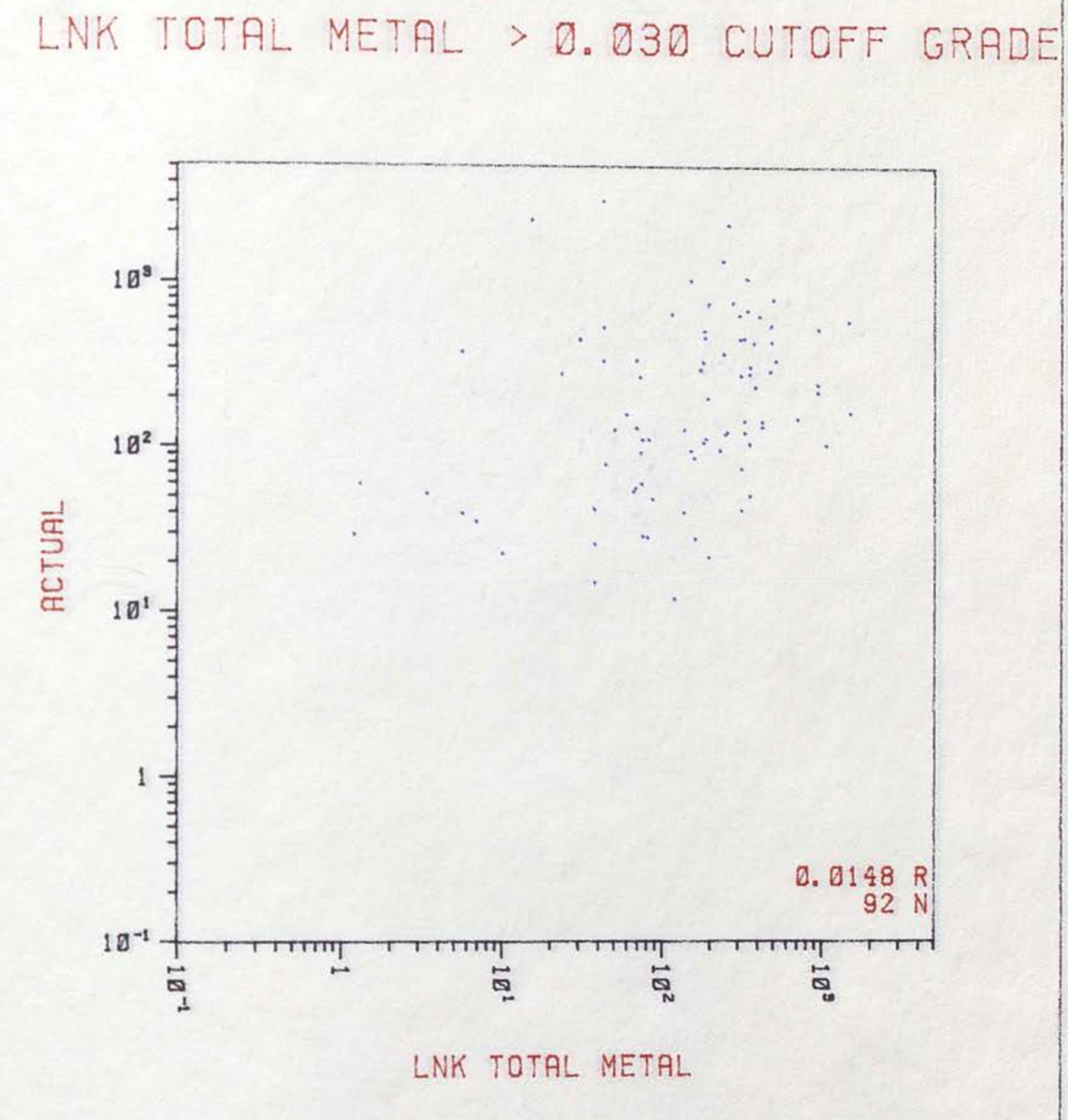

PK TOTAL METAL > 0.022 CUTOFF GRADE
ACTUAL
10³
10²
10¹
1
10⁻¹
0.2797 R
88 N
10⁻¹
1
10¹
10²
10³
PK TOTAL METAL

Figure 6.8 b



LNK TOTAL METAL > 0.030 CUTOFF GRADE
ACTUAL
10^3
10^2
10^1
1
10^{-1}
0.0148 R
92 N
10^{-1}
1
10^1
10^2
10^3
LNK TOTAL METAL

Figure 6.9 a

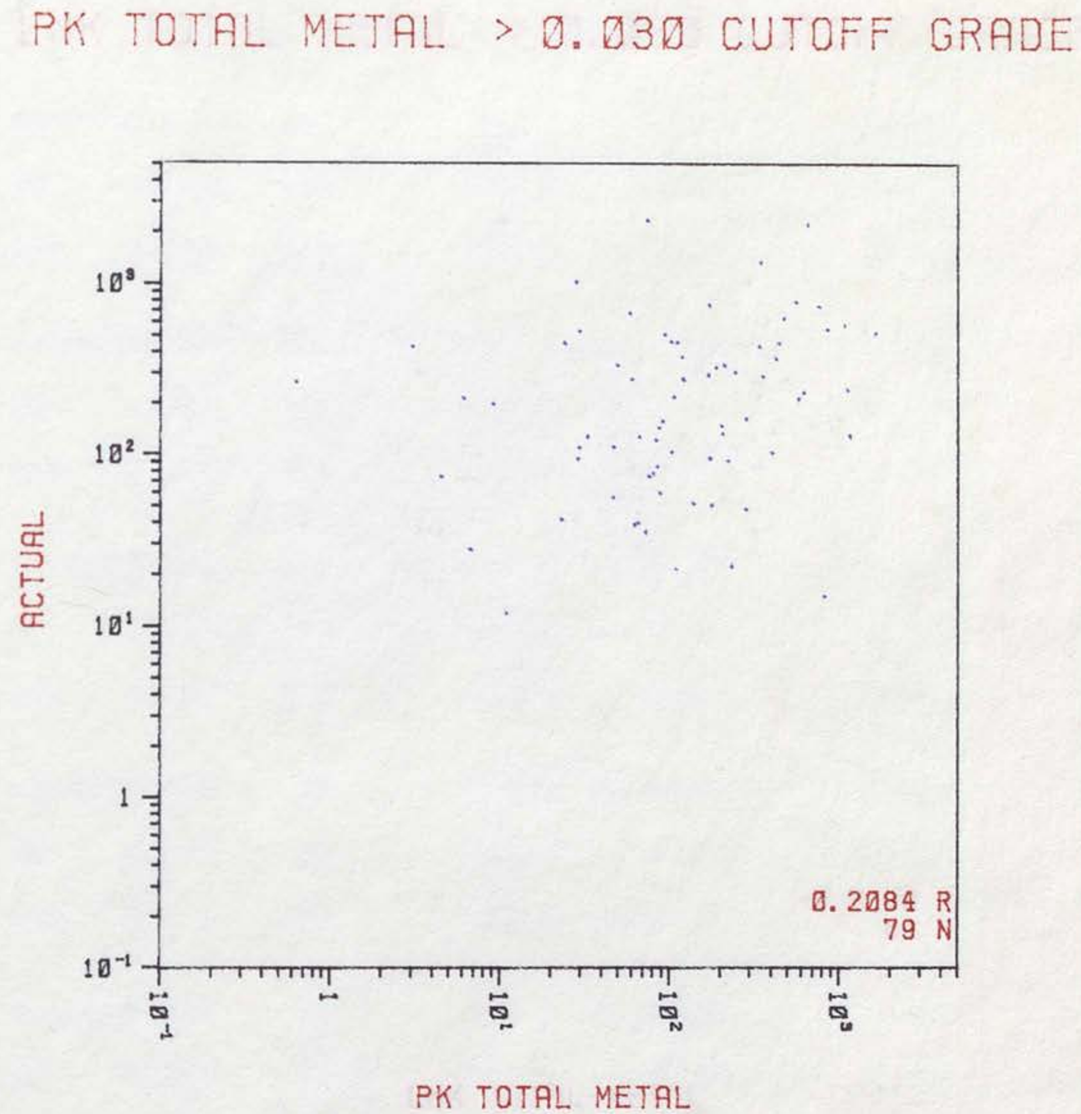
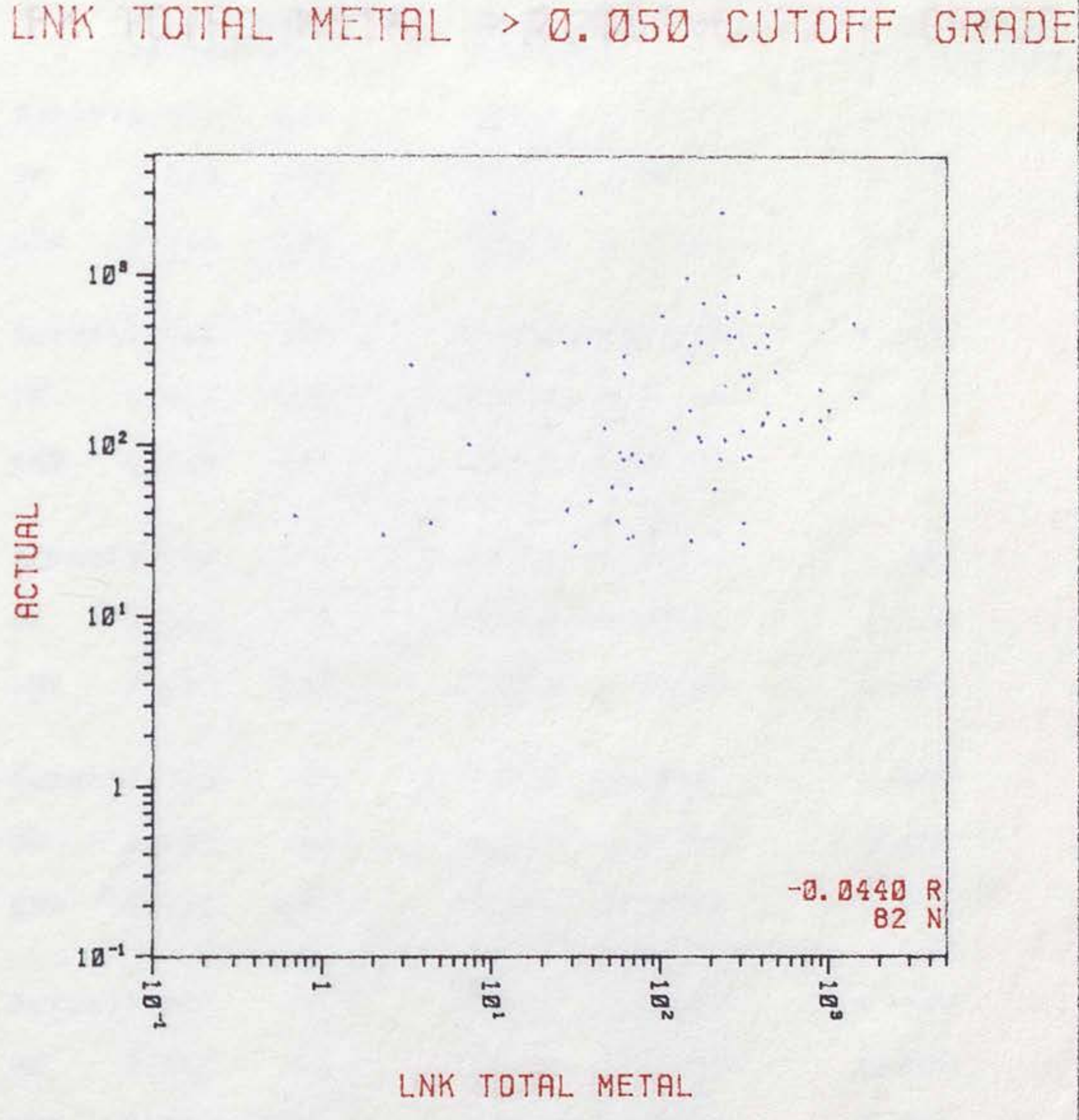

PK TOTAL METAL > 0.030 CUTOFF GRADE
ACTUAL
10³
10²
10¹
1
10⁻¹
10⁻¹
1
10¹
10²
10³
0.2084 R
79 N
PK TOTAL METAL

Figure 6.9 b



LNK TOTAL METAL > 0.050 CUTOFF GRADE
ACTUAL
LNK TOTAL METAL
-0.0440 R
82 N

Figure 6.10 a

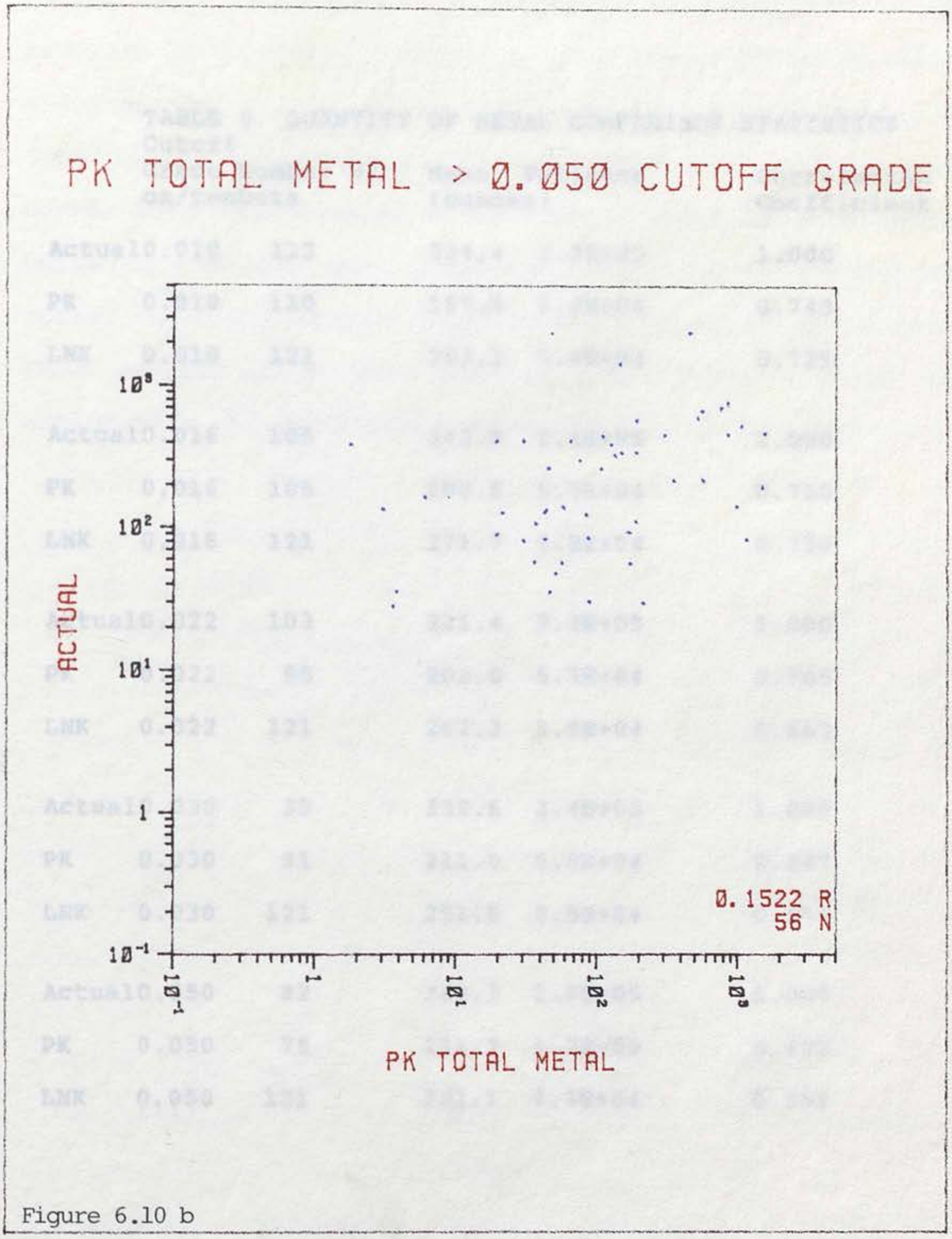

PK TOTAL METAL > 0.050 CUTOFF GRADE
ACTUAL
PK TOTAL METAL
0.1522 R
56 N

Figure 6.10 b

TABLE 8. QUANTITY OF METAL COMPARISON STATISTICS

	Cutoff Grade oz/ton	Number of Data	Mean [ounces]	Variance	Correlation Coefficient
Actual	0.010	112	334.4	2.2E+05	1.000
PK	0.010	110	187.6	3.2E+04	0.743
LNK	0.010	121	283.1	9.4E+04	0.729
Actual	0.016	105	343.8	2.4E+05	1.000
PK	0.016	105	200.8	5.7E+04	0.750
LNK	0.016	121	271.7	9.2E+04	0.720
Actual	0.022	103	321.4	2.4E+05	1.000
PK	0.022	95	203.0	5.7E+04	0.705
LNK	0.022	121	262.3	8.9E+04	0.663
Actual	0.030	92	332.6	2.4E+05	1.000
PK	0.030	91	211.8	9.0E+04	0.547
LNK	0.030	121	251.6	8.5E+04	0.594
Actual	0.050	82	340.3	2.6E+05	1.000
PK	0.050	75	221.7	1.2E+05	0.405
LNK	0.050	121	231.1	7.7E+04	0.559

is evident at the Manhattan Mine that the quantity of gold indicated to be present by blasthole assays is not reconciled at the mill (Veek, 1989). This problem has resulted in the practice of trimming of high blasthole grades before kriging, and expansion of the smu size to twice that of a 15 x 15 x 20 $foot^3$ block. In addition, it is of interest to note that the number of blocks considered to contain metal above a cutoff grade is constant when using LNK, compared to PK and the actual where the number of blocks declines at higher grades. This aspect could have serious implications during the exploration and development of any mineral prospect.

Chapter 7 Mapping Conditional Probabilities

PK, a cokriging estimator, produces as an output a conditional probability distribution function at each estimation location. The probabilistic nature of these distributions provides information not only at the datum location but also at locations where no data exists. Mapping conditional probabilities during the exploration or development drilling phase may therefore provide useful information as to the spatial distribution of mineralization.

In this study, PK, with a 50 foot x 50 foot x 20 foot block size, is used to produce conditional probability distributions on a grid dense enough to capture the gross spatial features of the mineralization. For comparison, a contoured blasthole map is used (Figure 7.1). A contoured conditional probability map for grades in excess of the 0.016 oz/ton cutoff grade are plotted in Figure 7.2. The probability contours on this map accurately depict the outline of low grade gold mineralization on the production blasthole map within the Little Grey Fault zone, and identifies a large zone of internal waste. Plotting contours of conditional probabilities of grades above the 0.050 oz/ton cutoff grade provides different information. High probability contours clearly indicate the location of breccia controlled mineralization along the Little Grey Fault and the bounding hanging wall structure. The contour
map also indicates that substatial internal waste is present

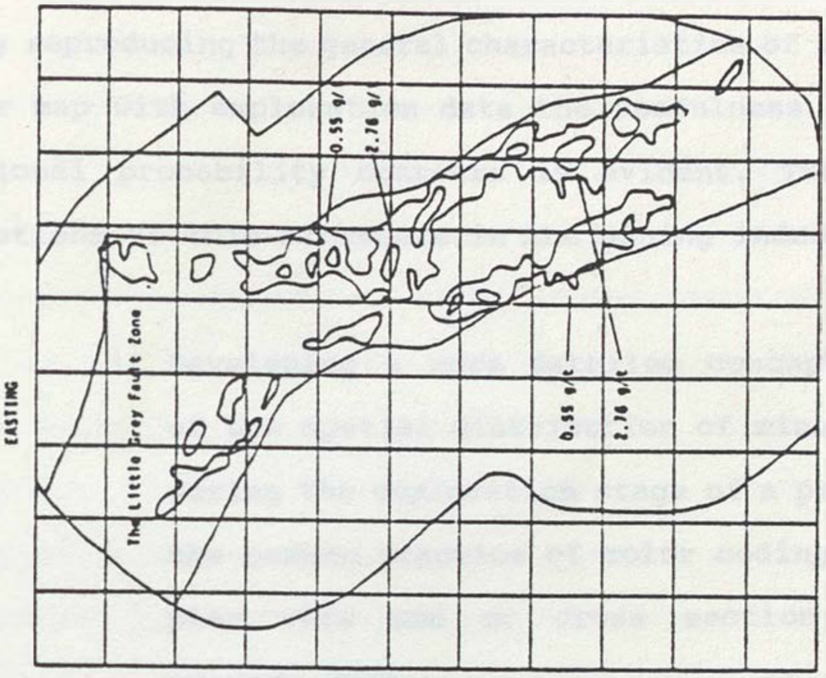


Figure 7.1

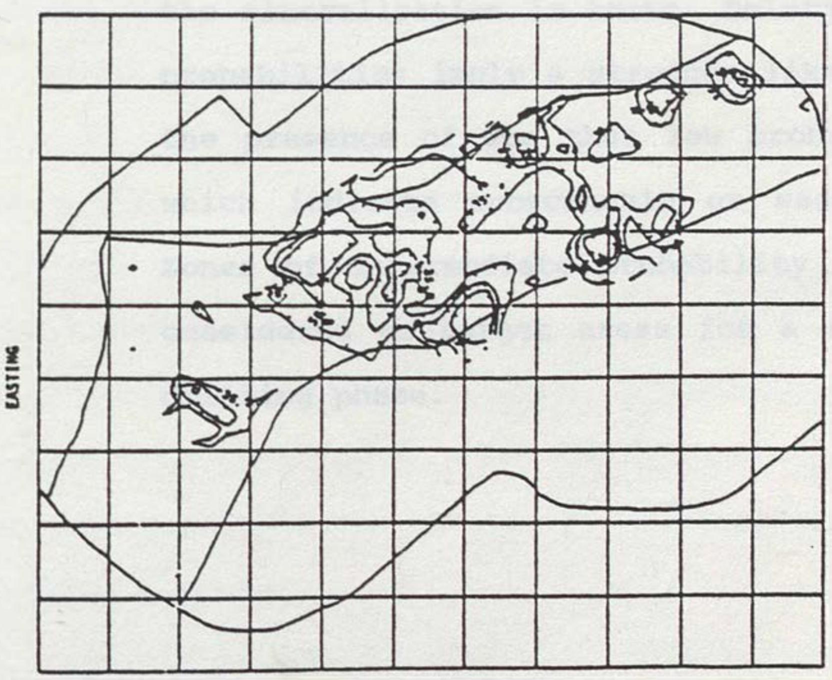


Figure 7.2

in the fault zone.

By reproducing the general characteristics of a blasthole contour map with exploration data the usefulness of mapping conditional probability contours is evident. Two possible applications of this procedure in the mining industry are;

1) Developing a more detailed conceptual model of the spatial distribution of mineralization during the exploration stage of a project than the common practice of color coding blocks in plan view and or cross section based on average grade;

2) The contours provide a quantitative measure of the certainty with which the location of the mineralization is known. Relatively high probabilities imply a stronger likelihood of the presence of ore than low probabilities, which indicate subeconomic or waste zones. Zones of intermediate probability should be considered as target areas for a subsequent drilling phase.

Chapter 8 Conclusions and Suggestions For Further Work

In this study, the application of PK and LNK, to a structurally controlled, coarse gold deposit, is developed. The results of the study suggest that PK is a better estimator of the distribution of point support grades in an exploration size panel than either the actual blastholes located in the block, or LNK, for this deposit. The study also exposes a problem with the LNK approach, that being overestimation of high grade tonnage. A likely explanation for this over estimation is the characteristic long tail of the lognormal distribution which contains some very high grade values and the fact that the true distribution of point support grades may not be lognormal in all panels. In Table 8, as the cutoff grade is increased, both PK and the actual algorithms find fewer blocks with mineralization beyond the cutoff grade. LNK, however, finds mineralization in excess of all cutoff grades, in all blocks.

The approach of using the distribution of blasthole grades within an exploration size panel as actual data was less than successful in this case study. It is evident that the number of samples within the panel, approximately 45, was insufficient to accurately identify the true distribution of point support grades within the panel. Further work on this subject might include:

1) Implementation of sampling stations at various locations prior to milling to better define what

the actual data is at Manhattan;

2) Testing the use of 10 foot benches as a possible remedy for grade control;

3) Variography indicated that mineralization was quite continuous down dip in the Little Grey Fault zone; rotation of the coordinate system so that blocks are aligned with the plane of mineralization may yield improved results;

4) The benefits of mapping conditional probabilities in ore reserve estimation should be tested further, a variety of deposit types should be examined, including simulated deposits.

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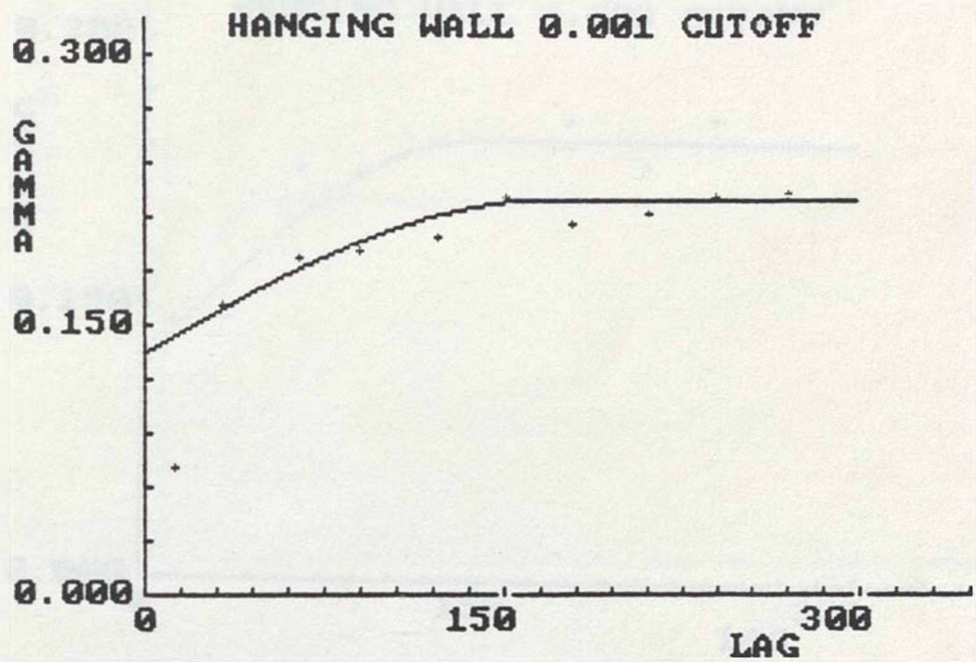
APPENDIX

WP COMPOSITE VARIOGRAMS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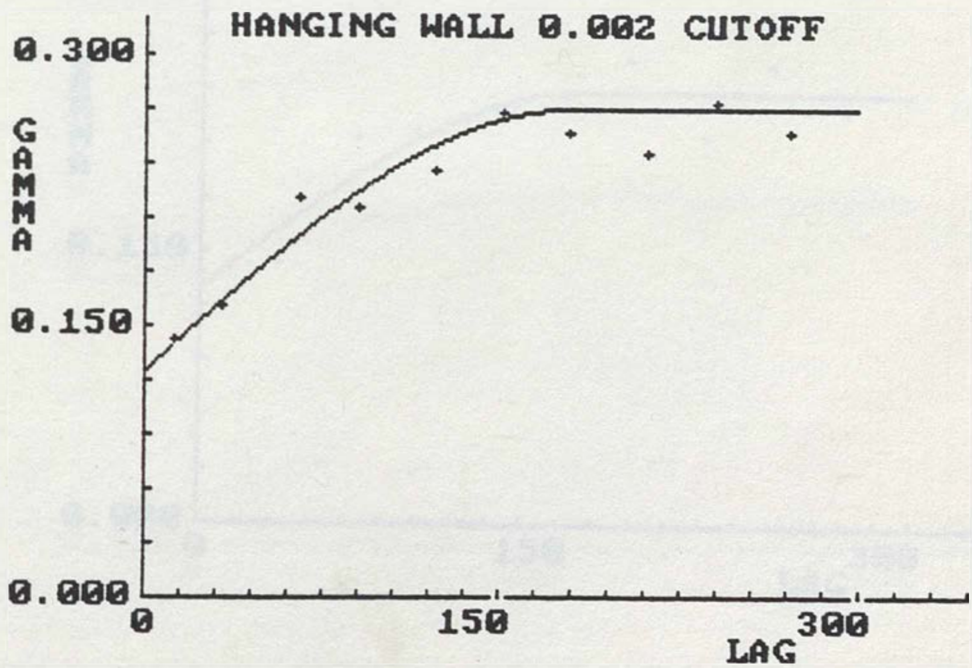
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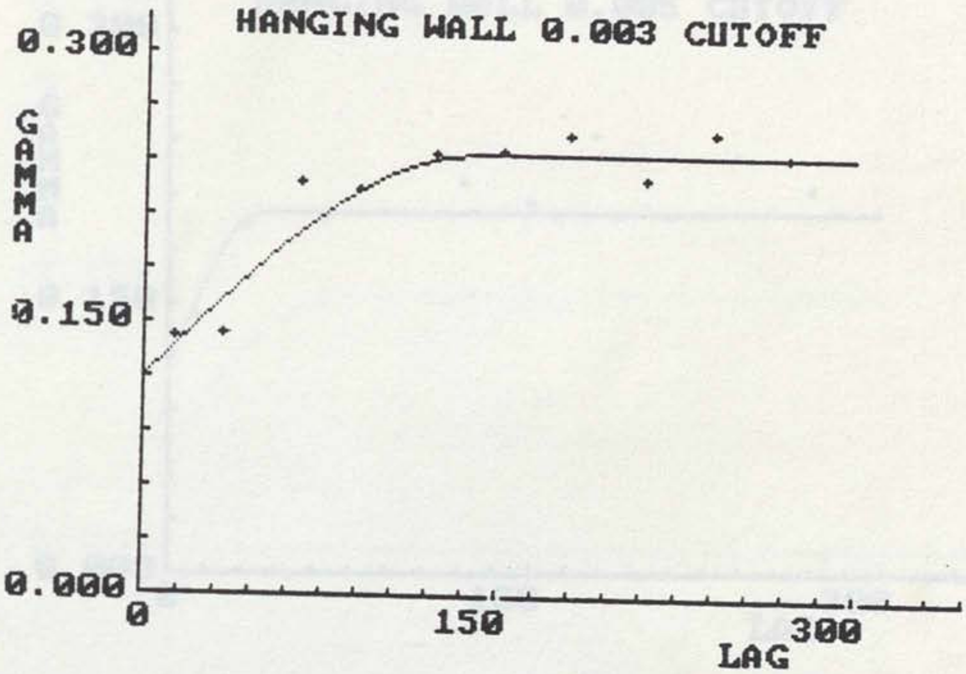
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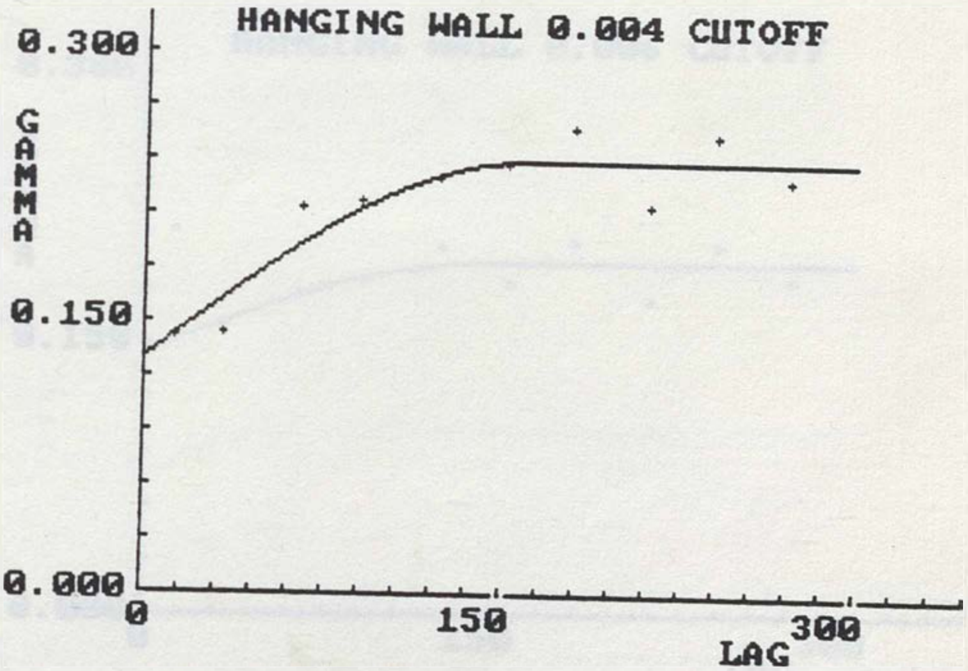
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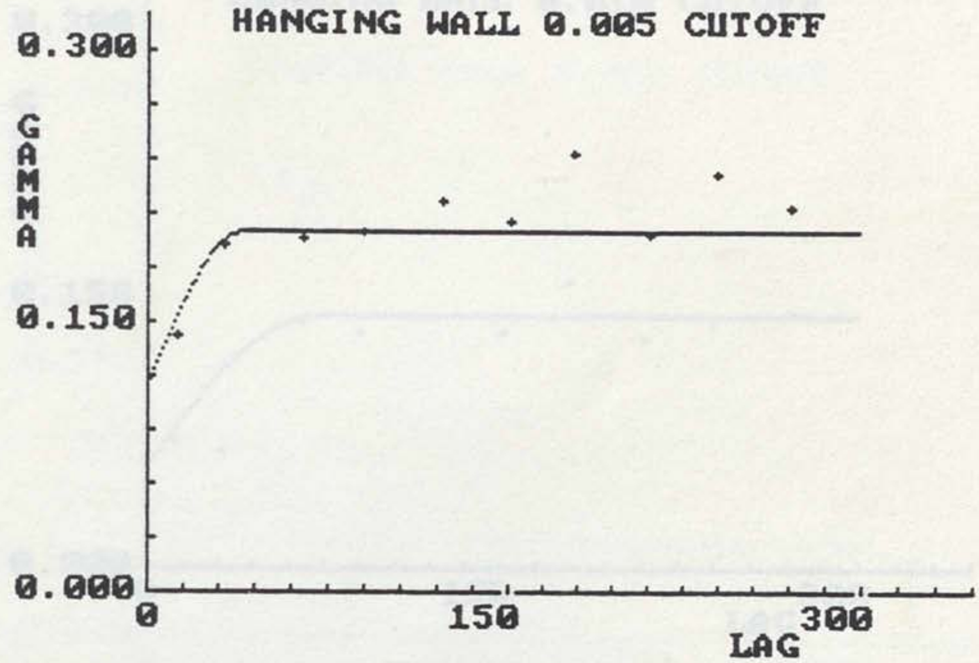

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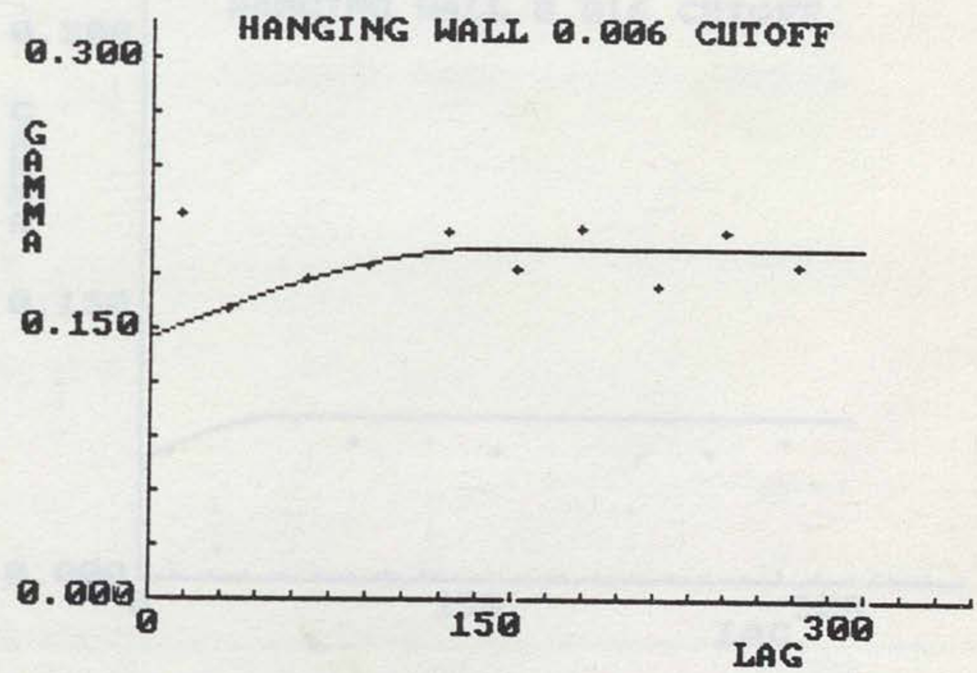

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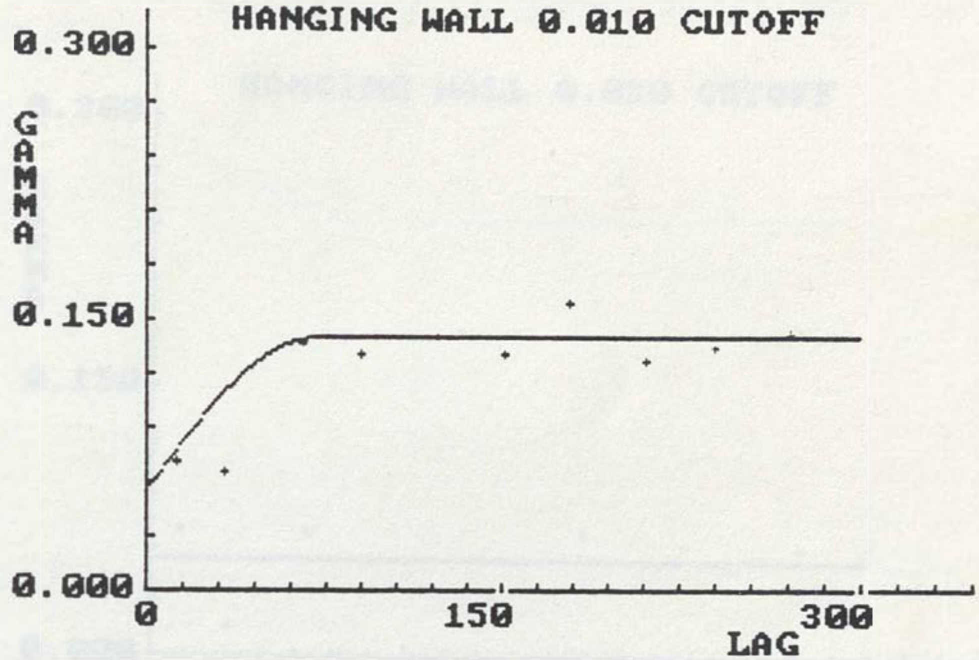
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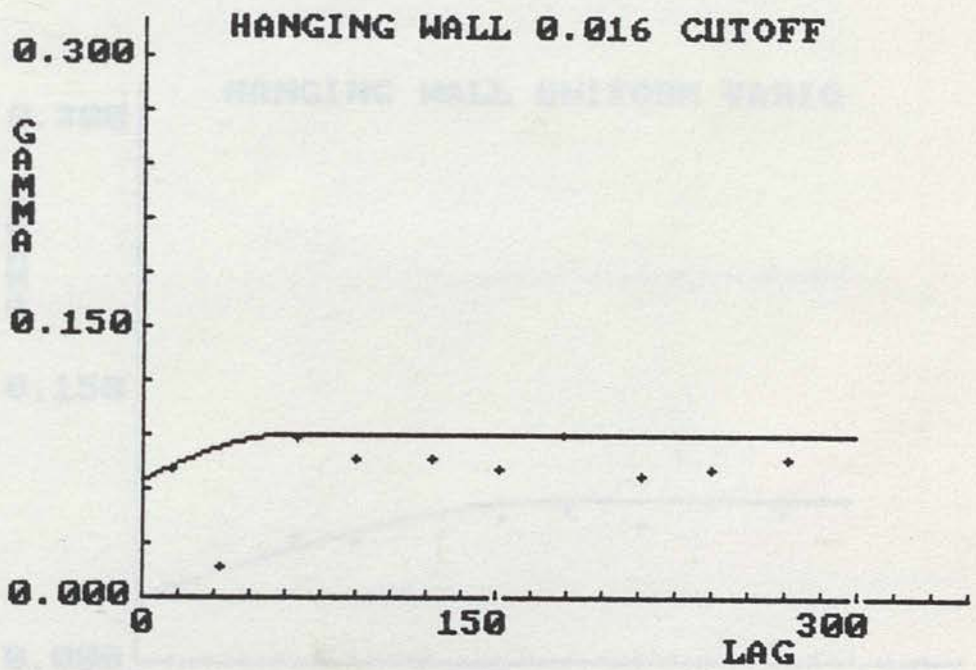
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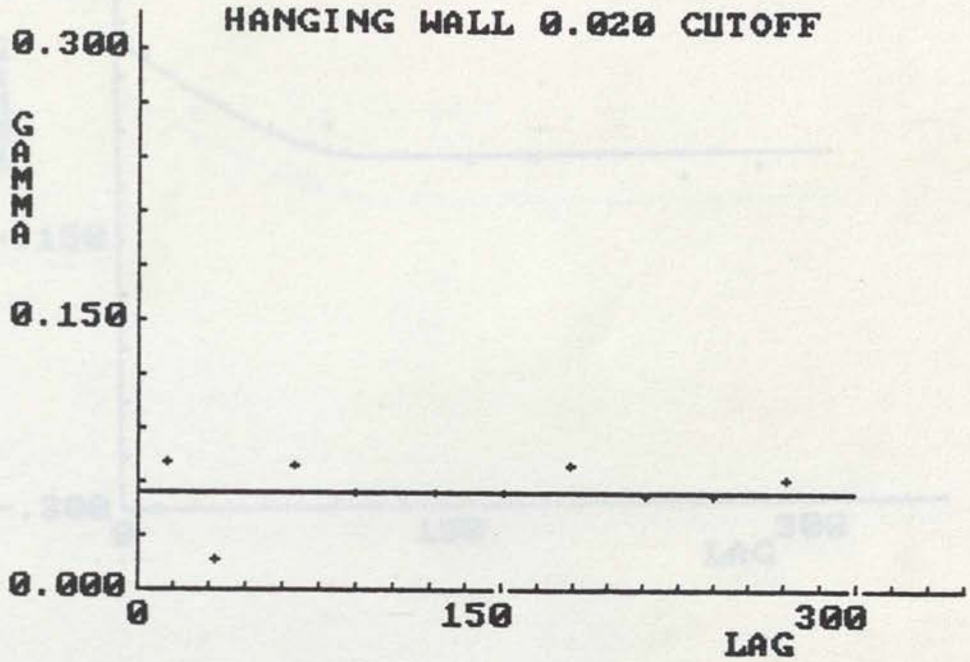
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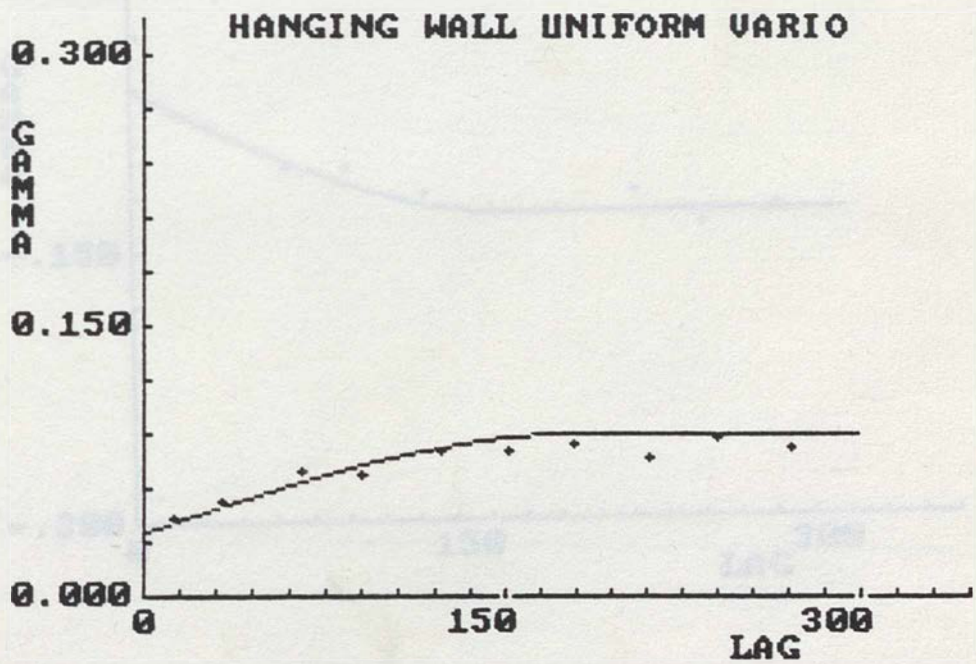
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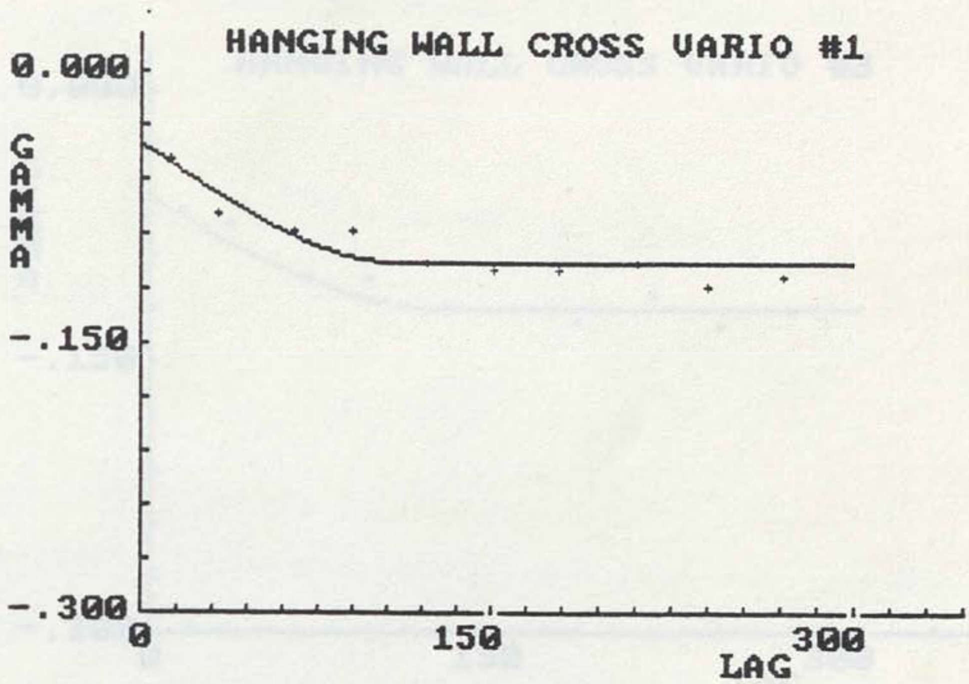
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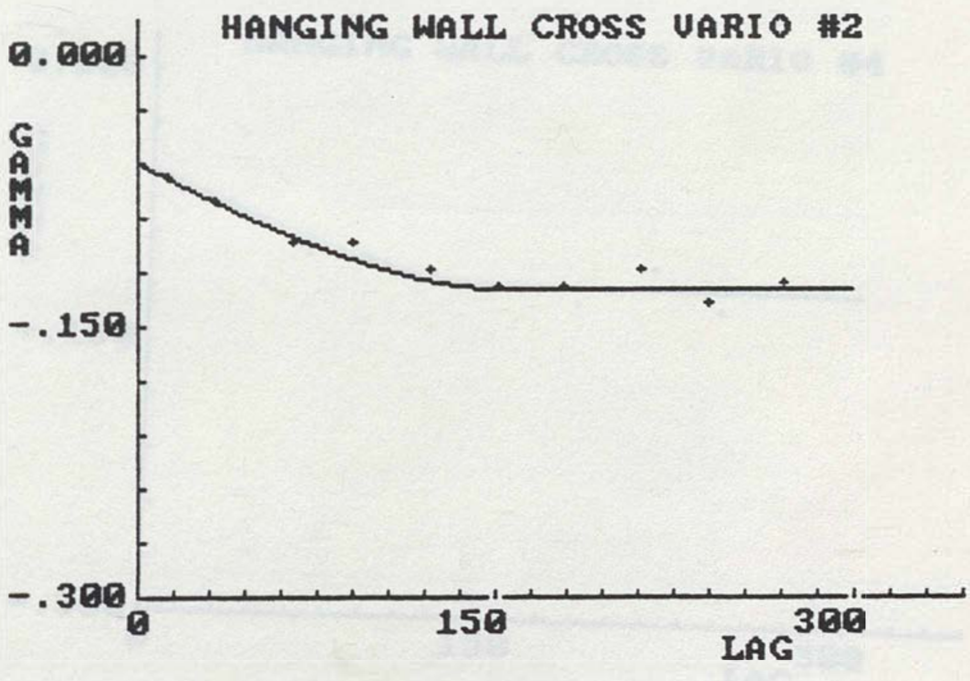
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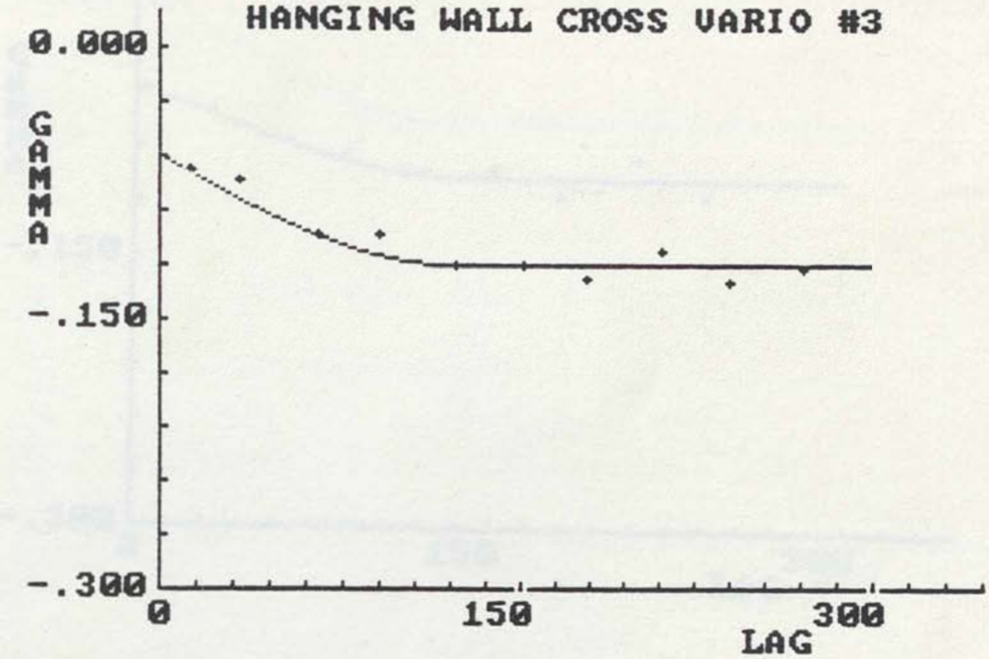
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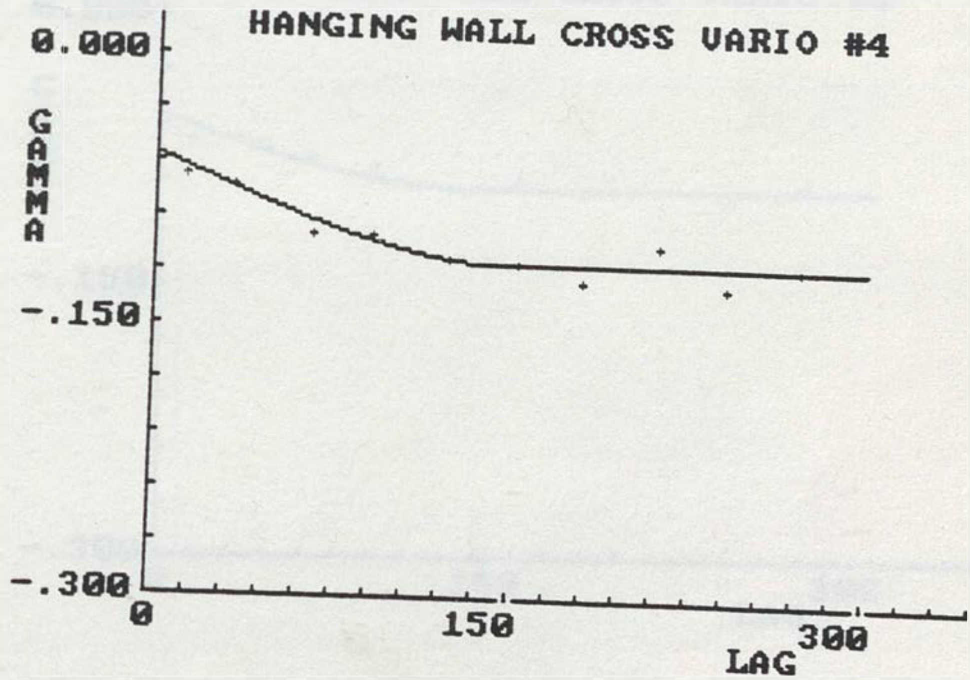
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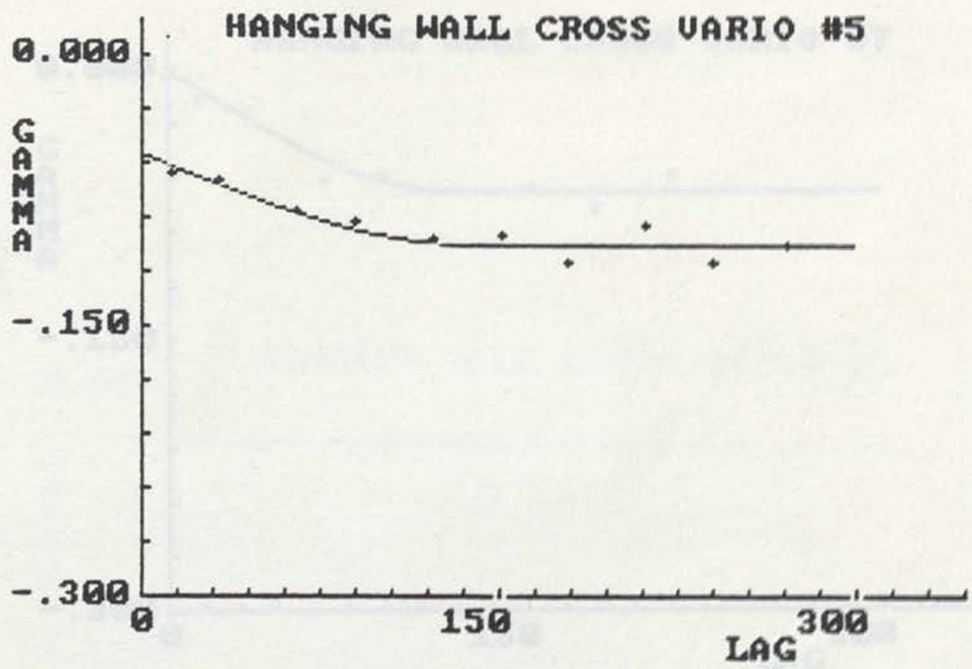
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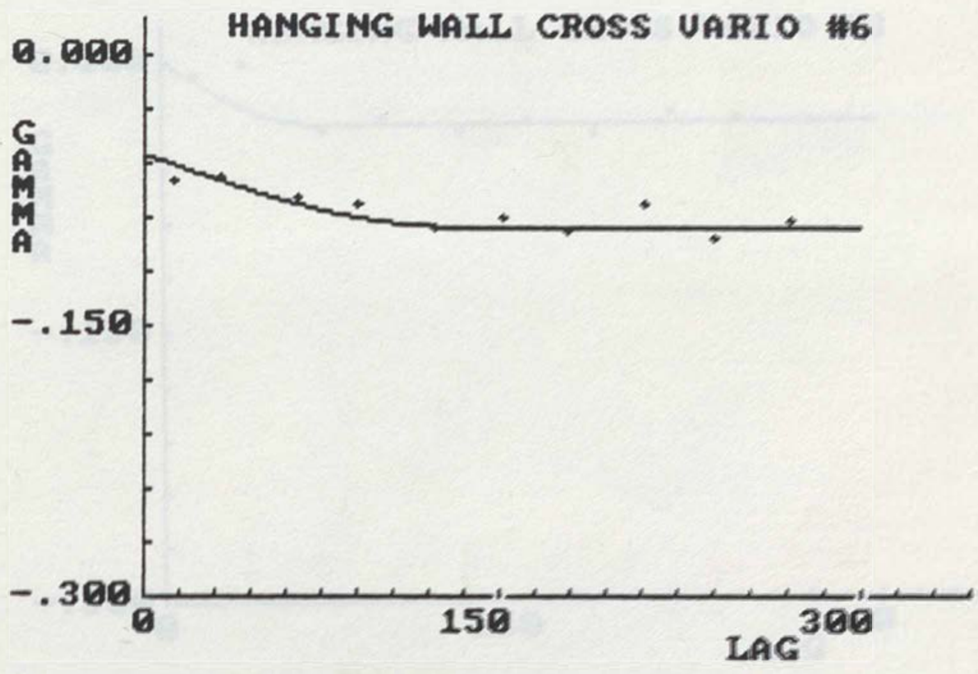
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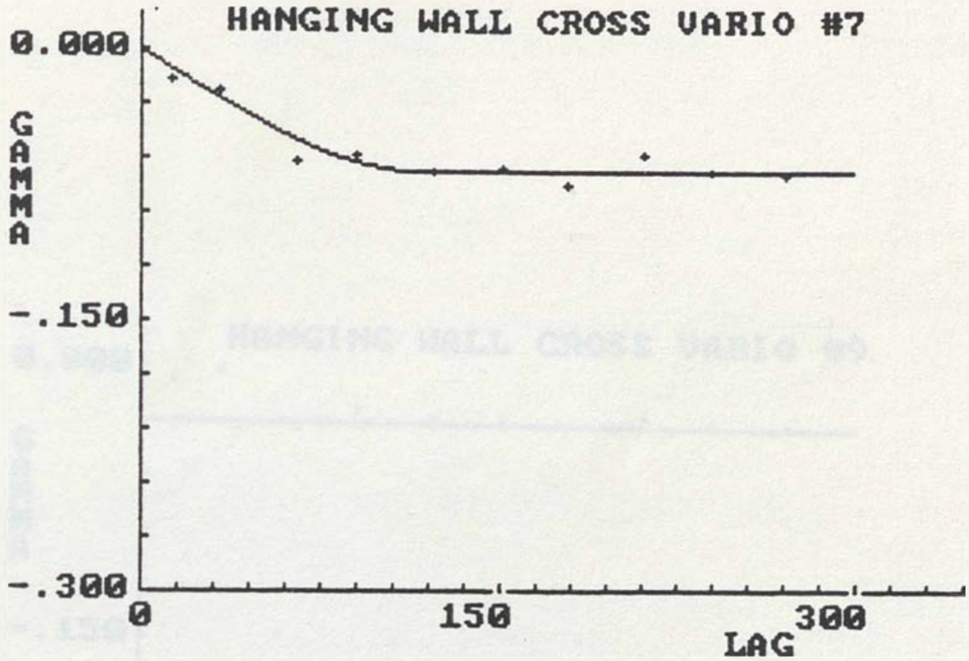
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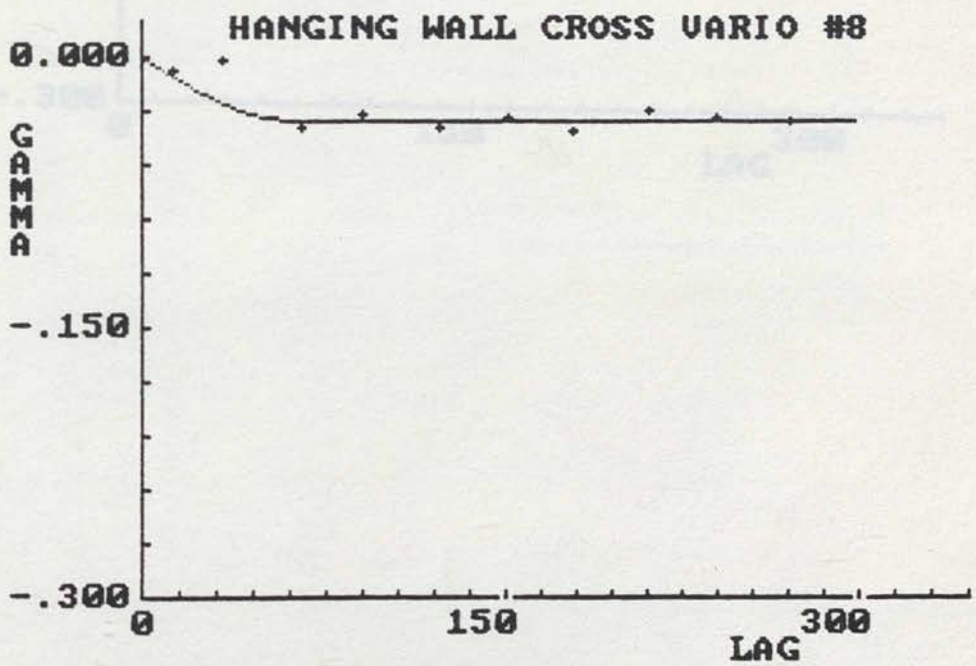
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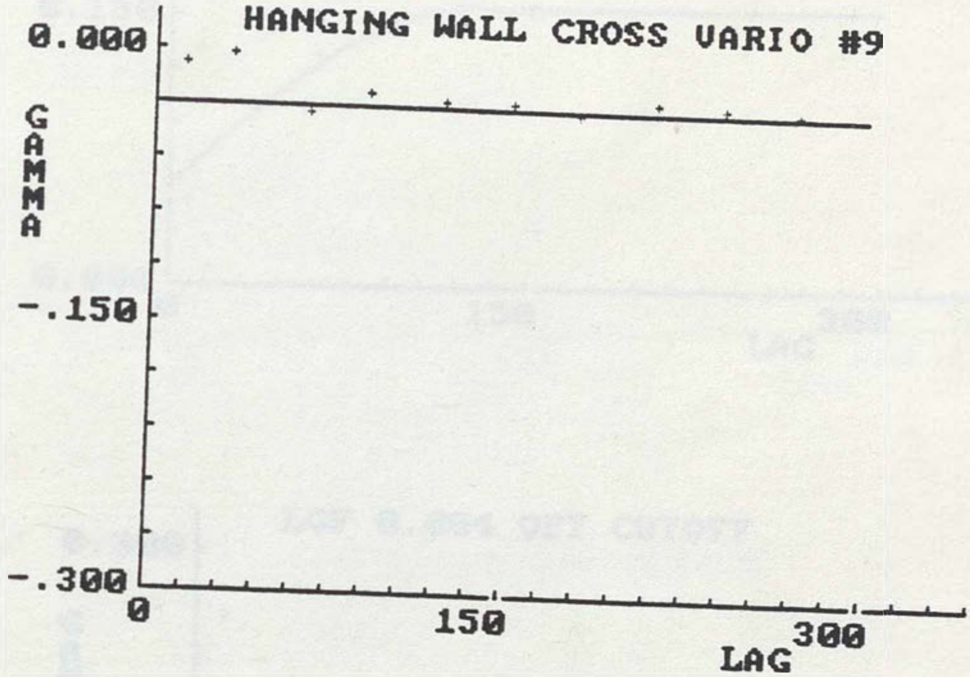
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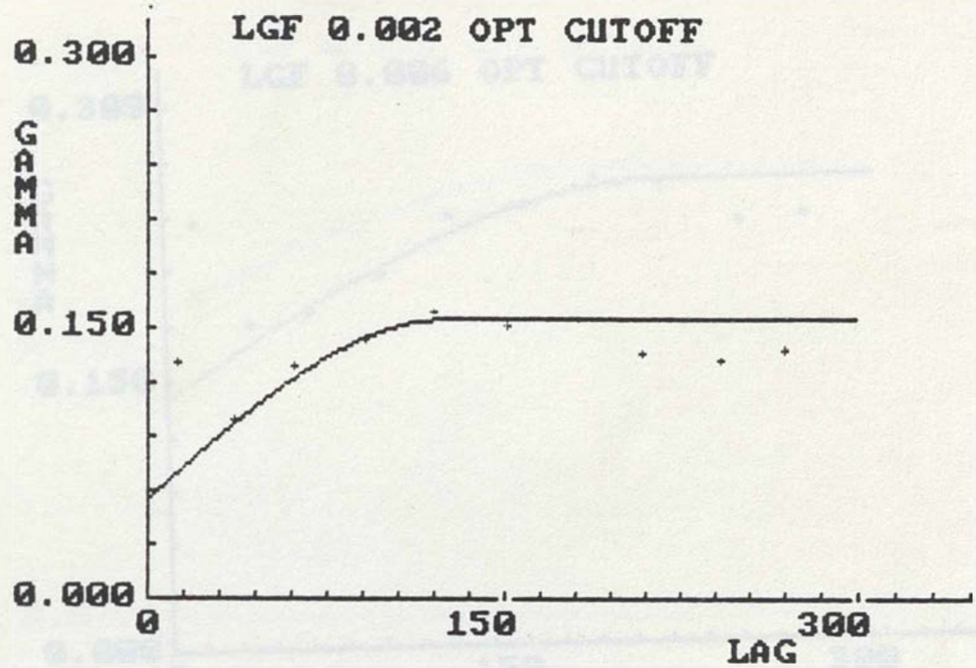
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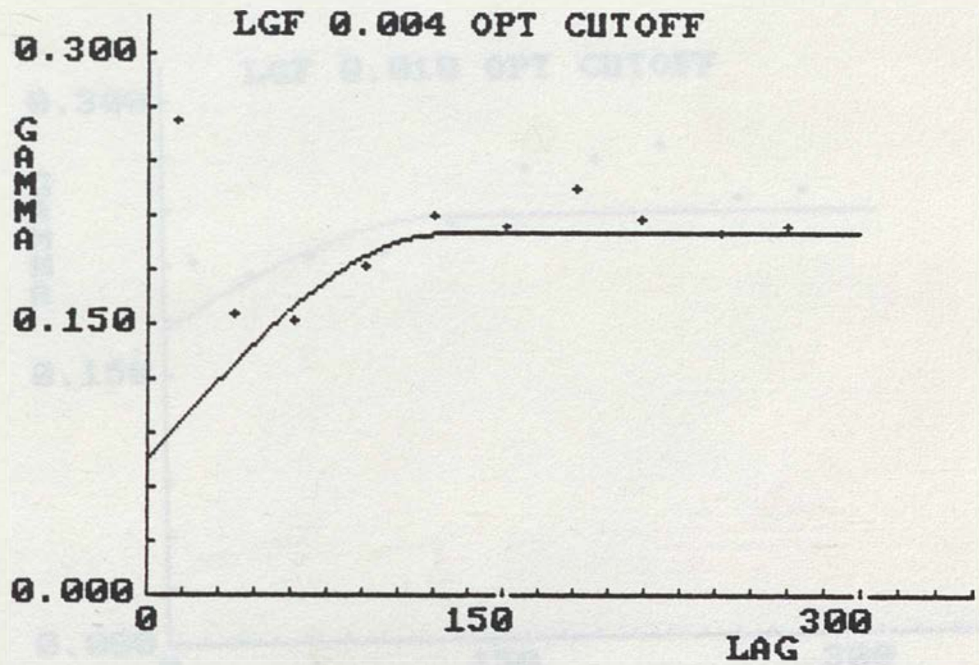
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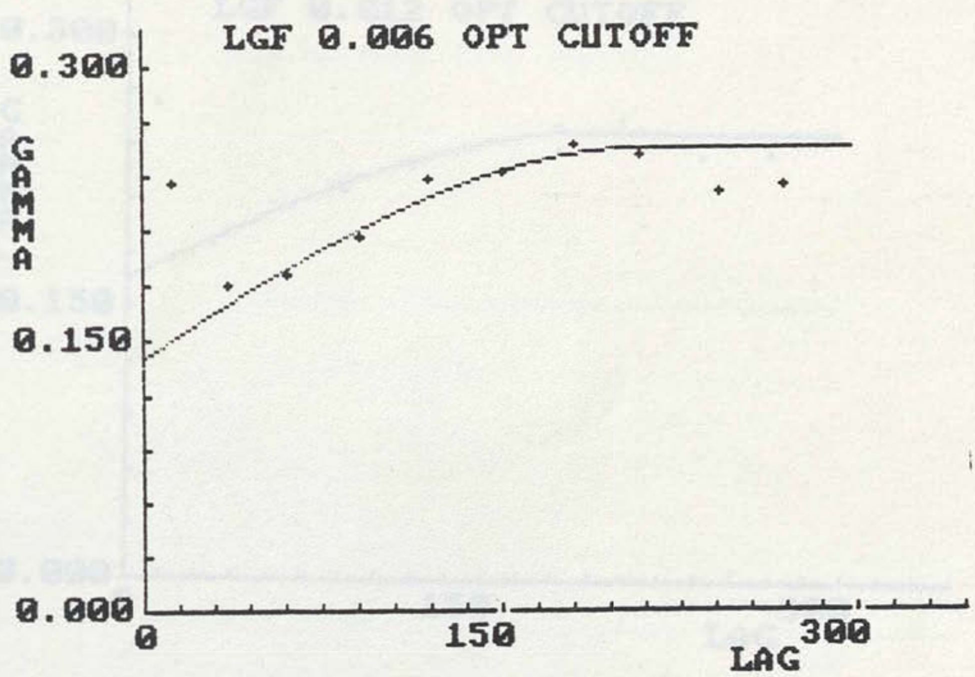
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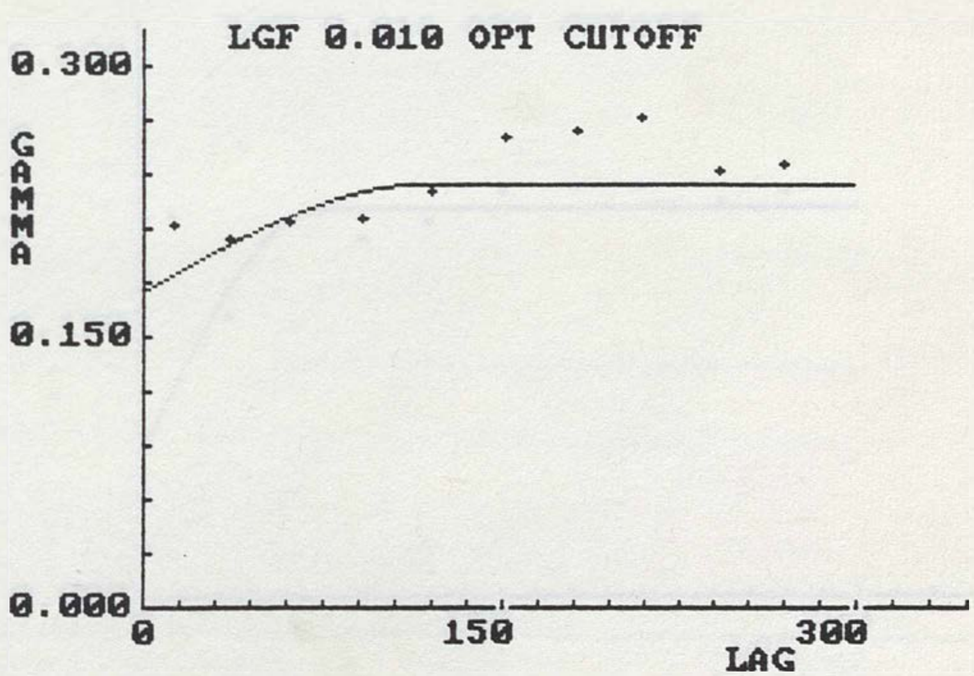
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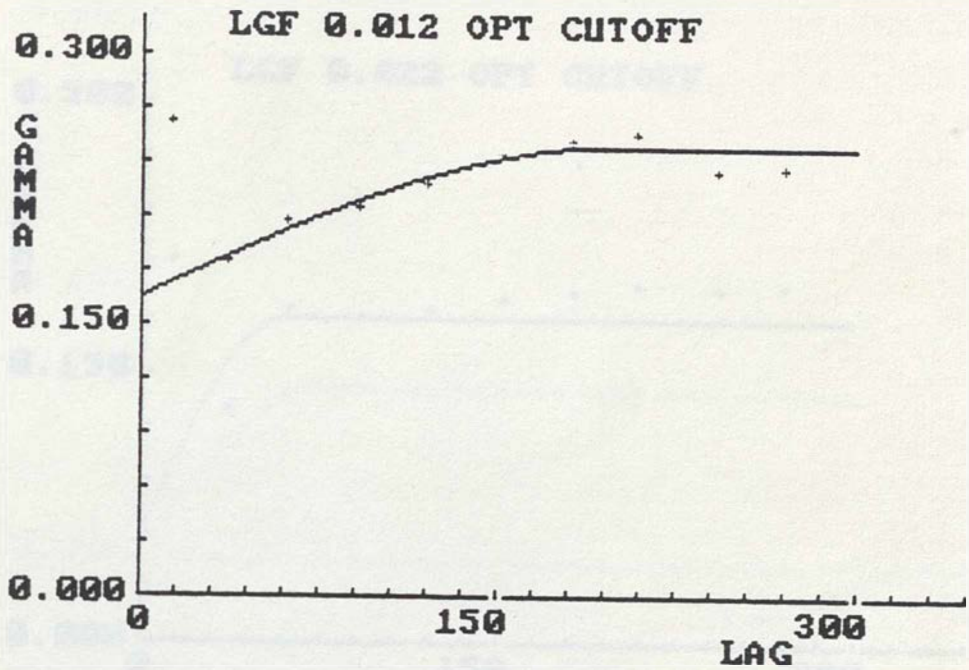
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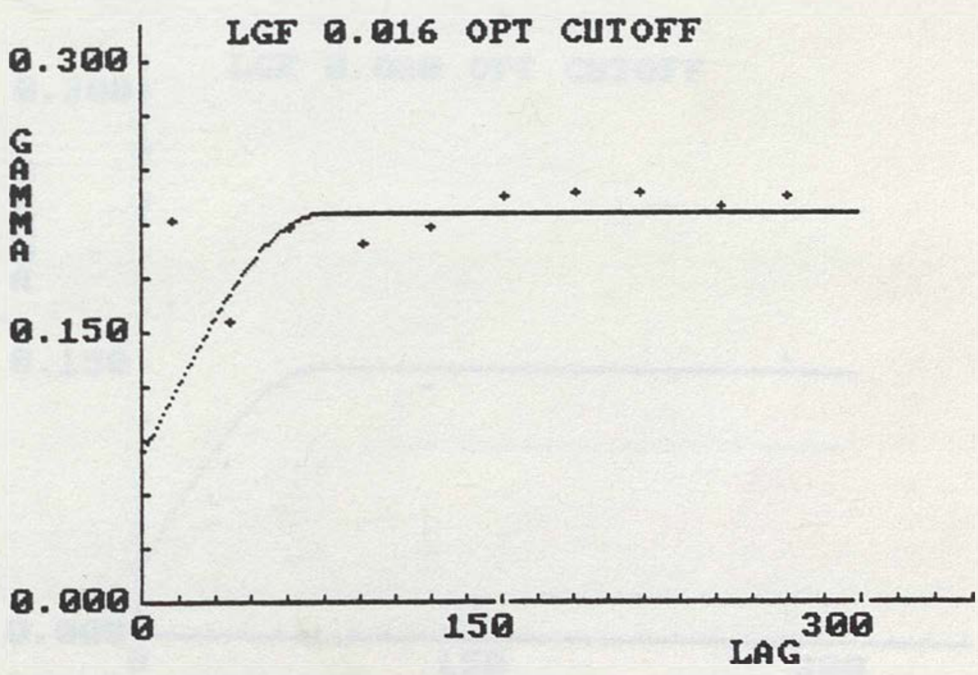
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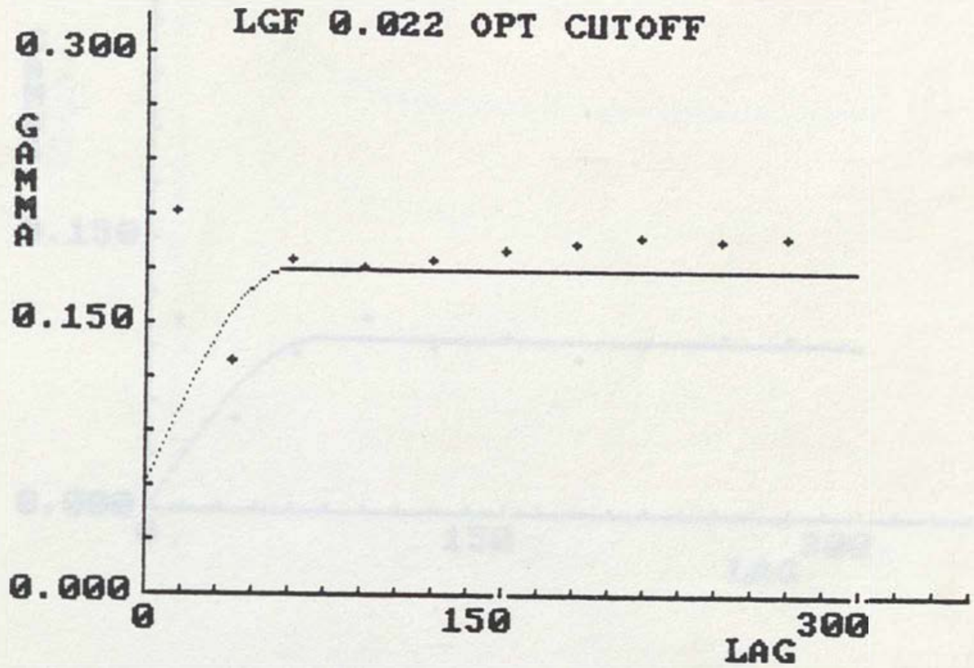
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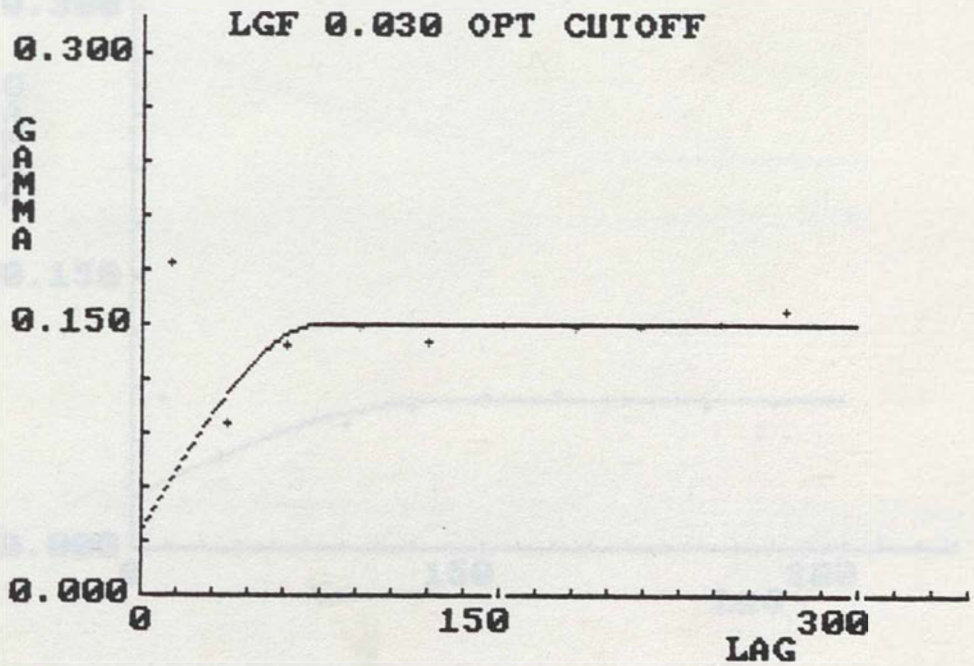
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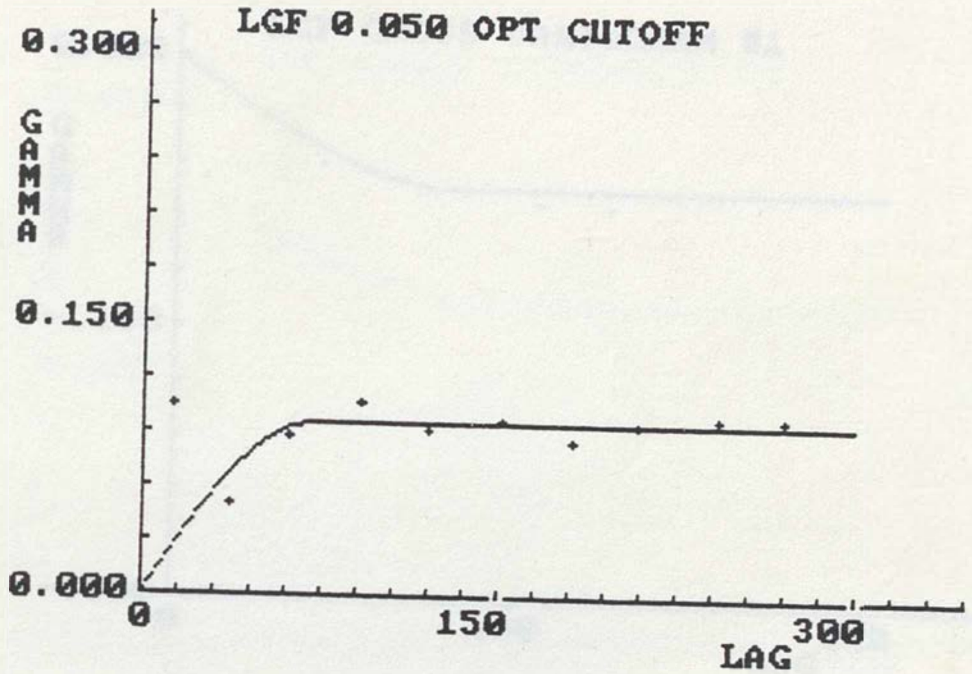
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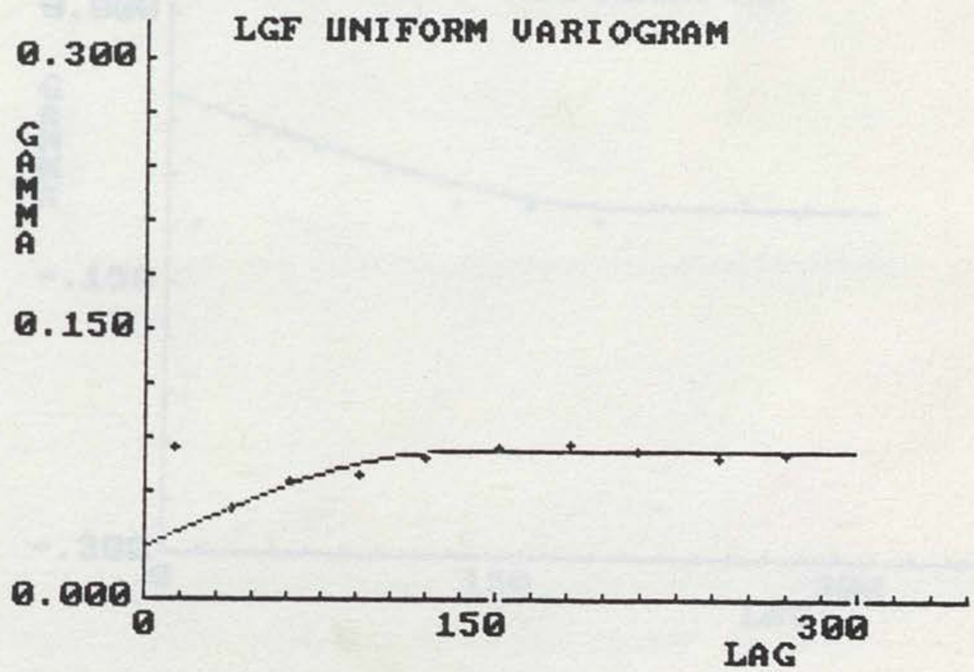
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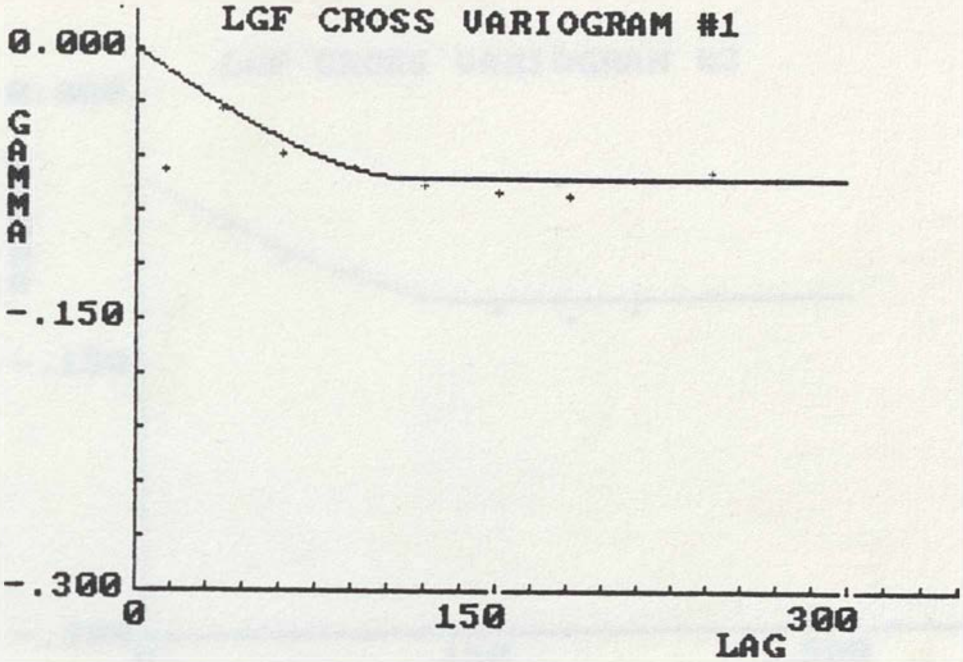
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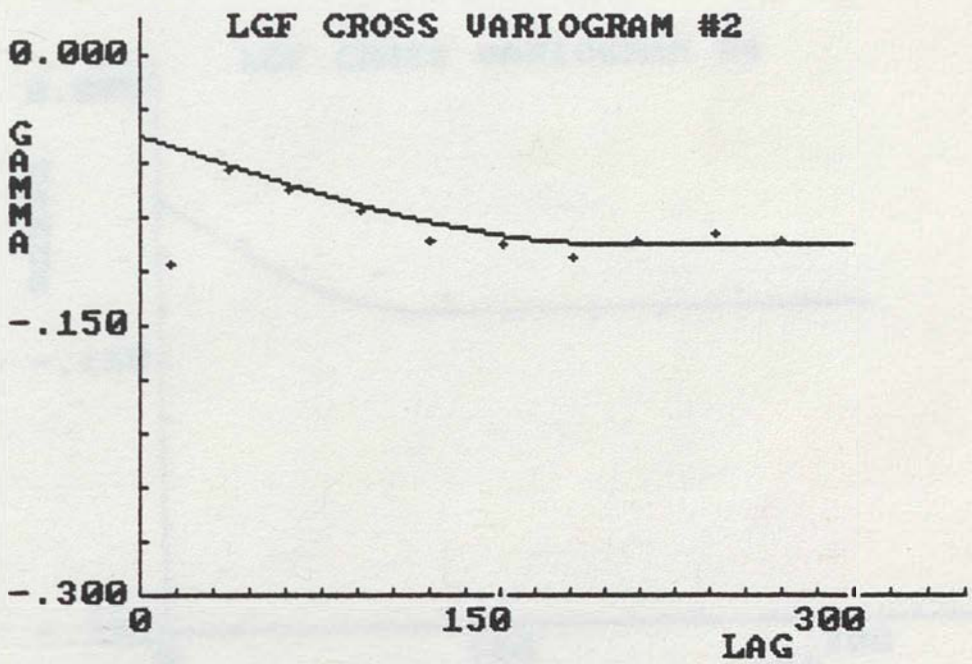
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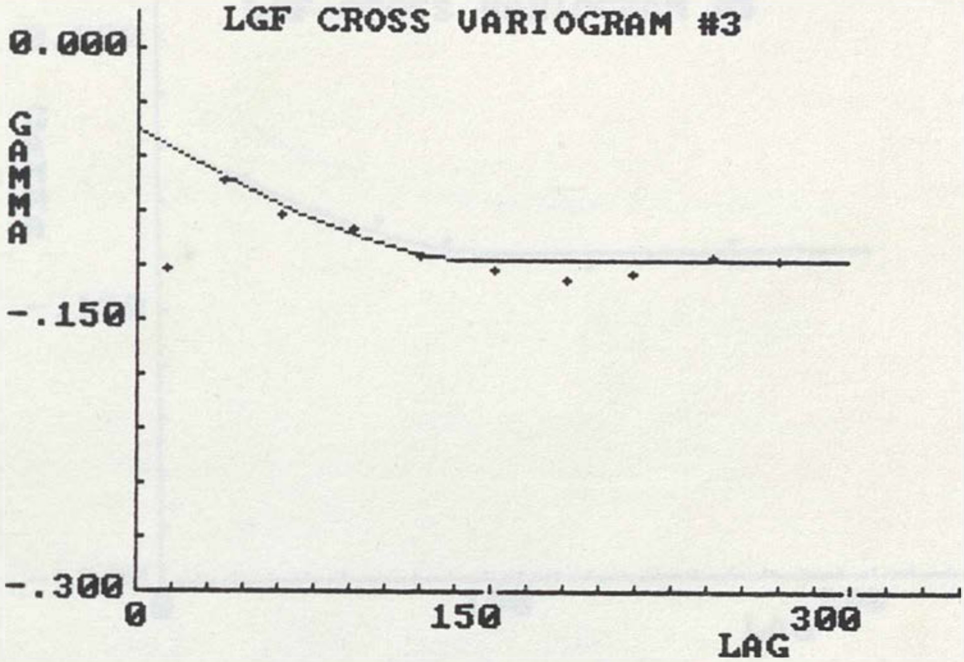
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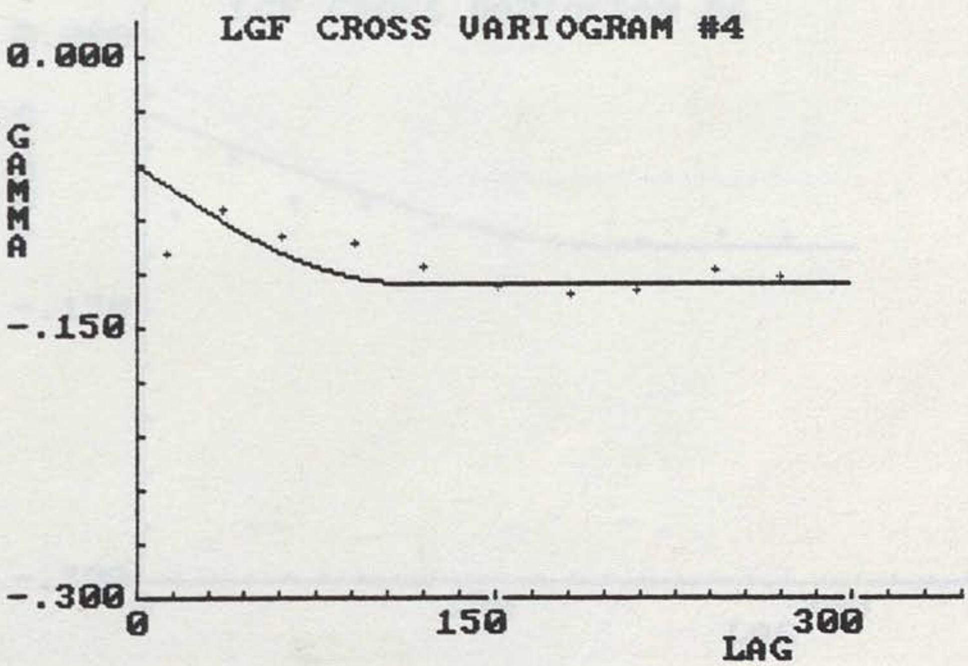
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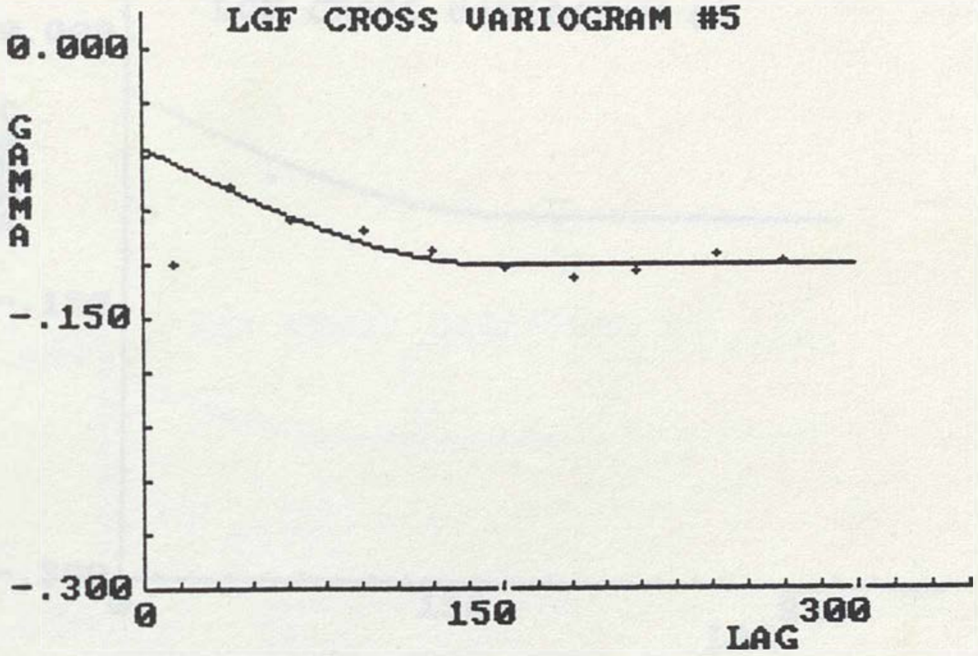
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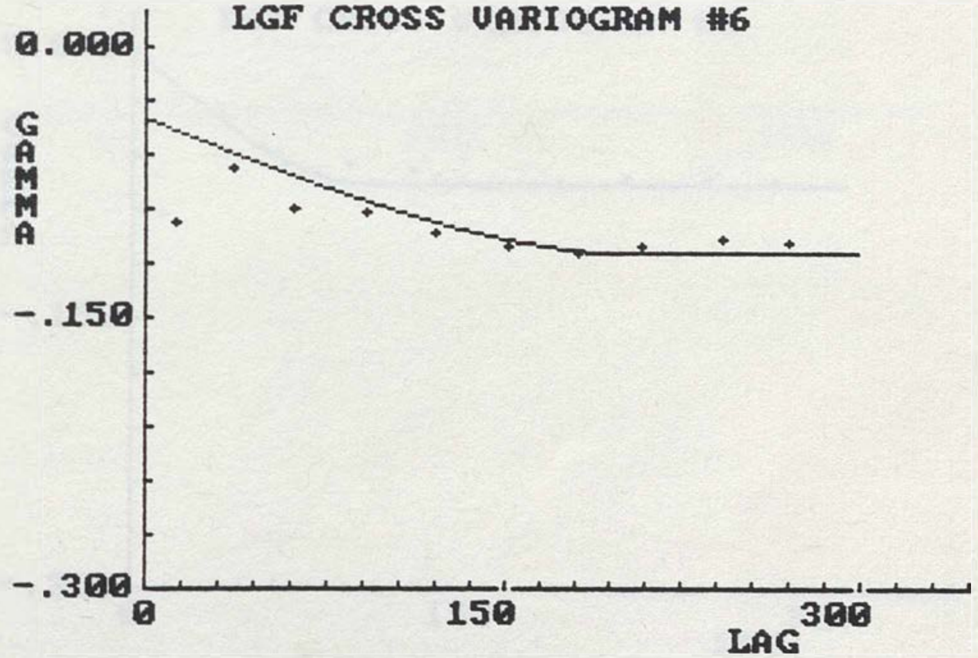
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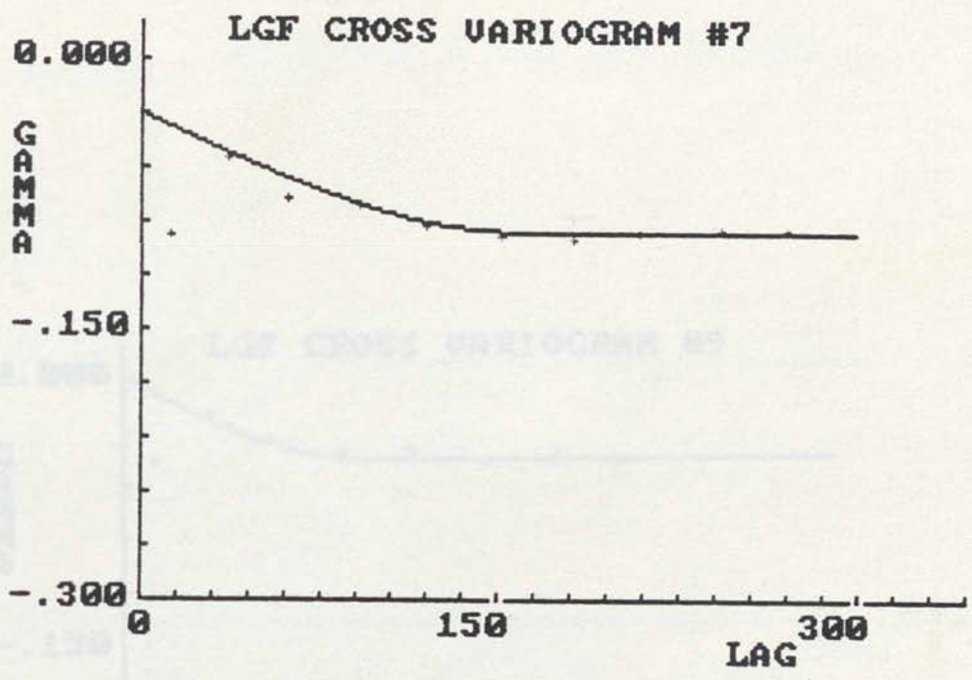
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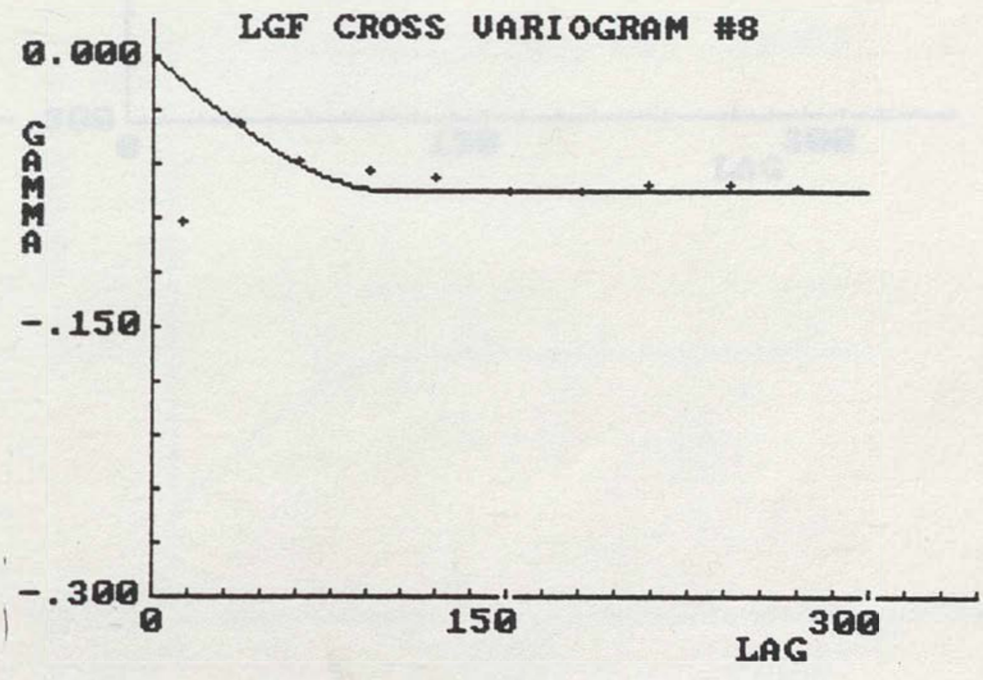
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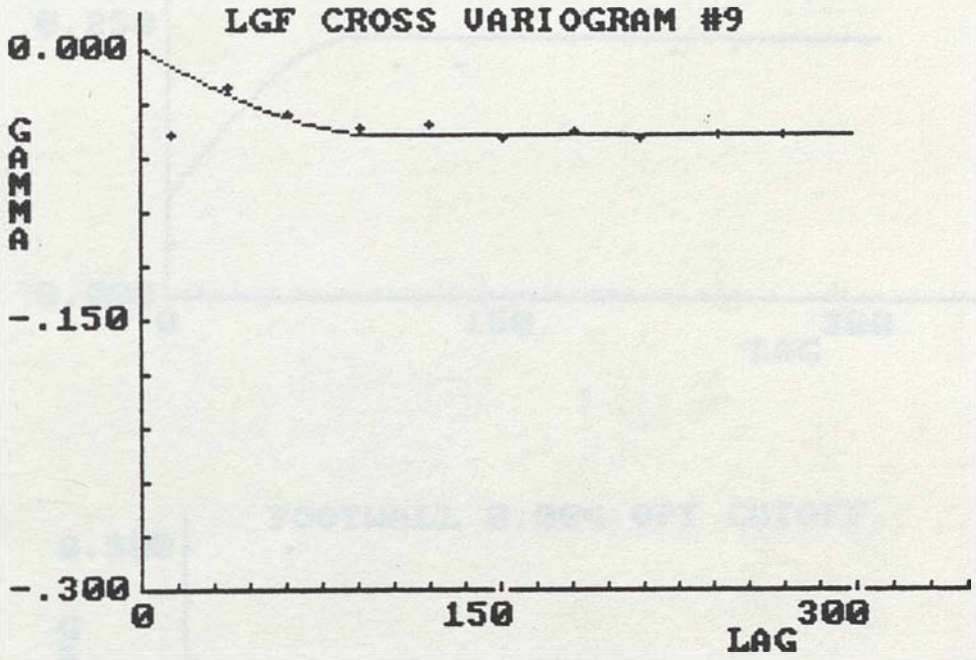
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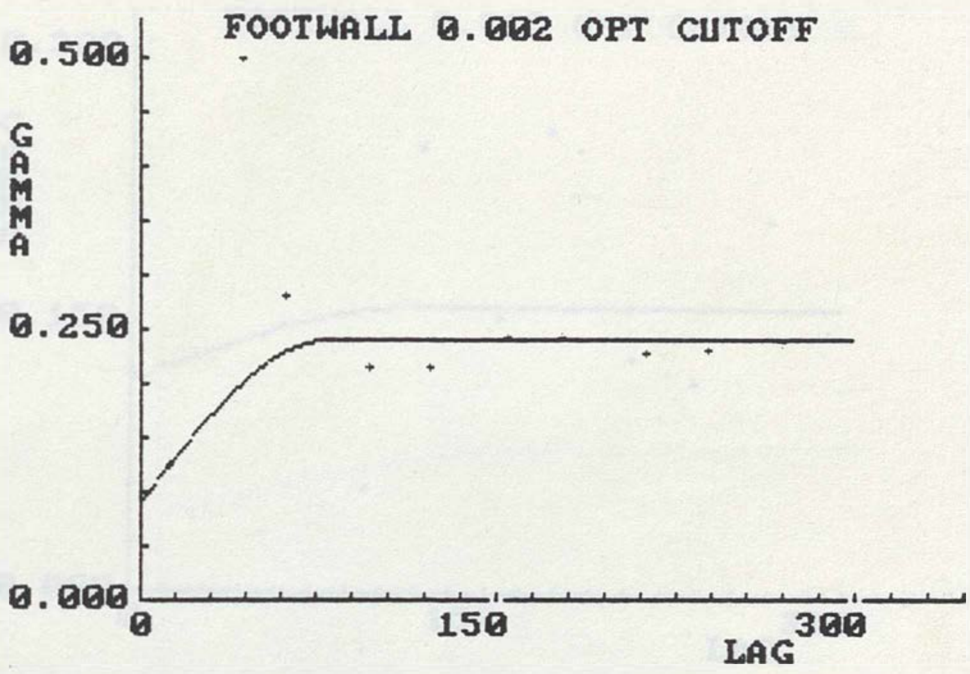
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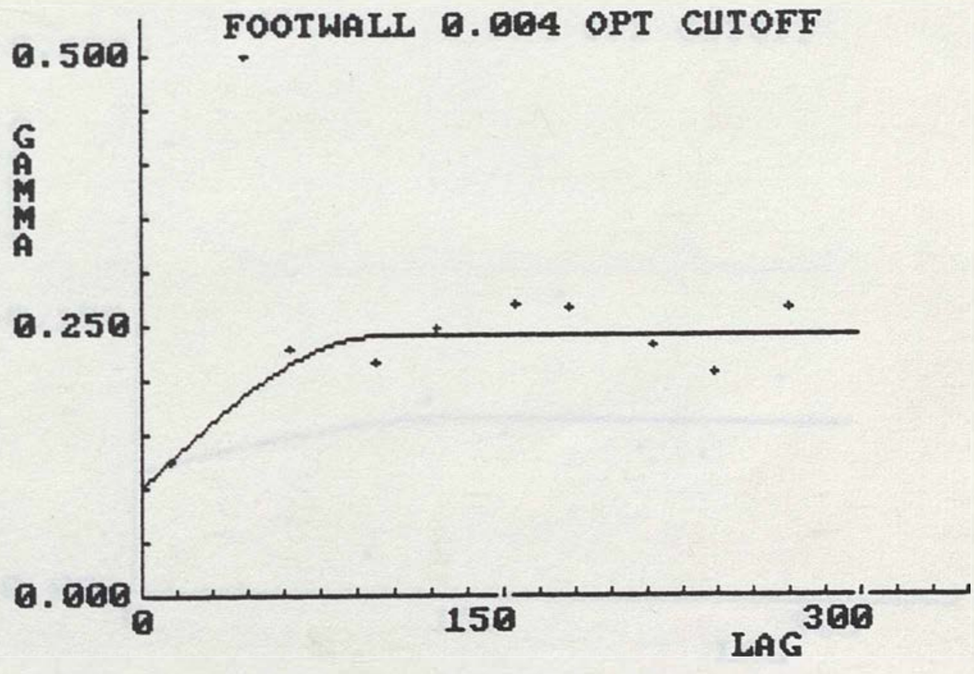
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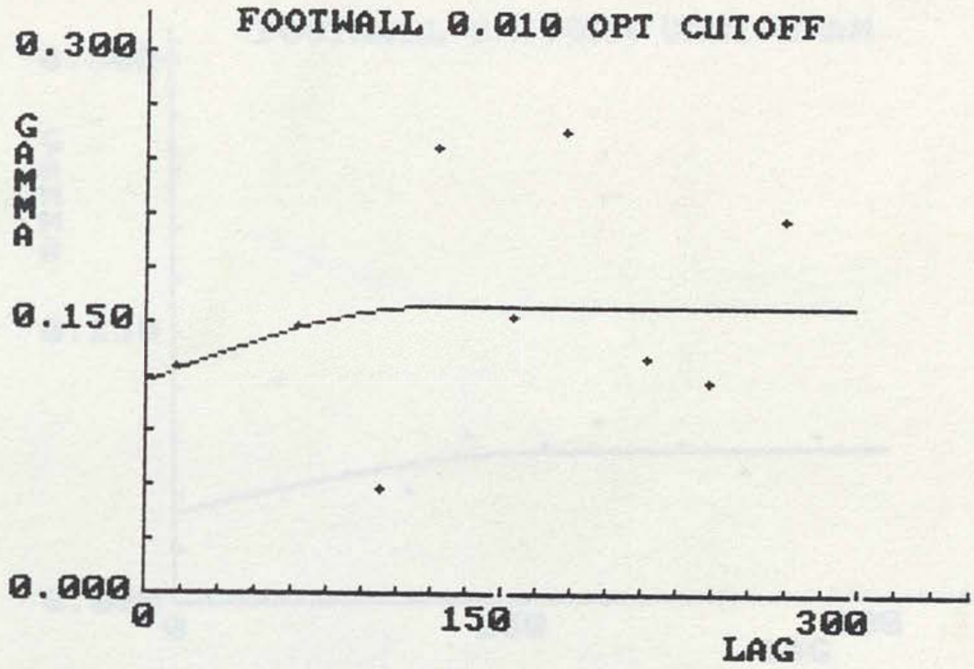
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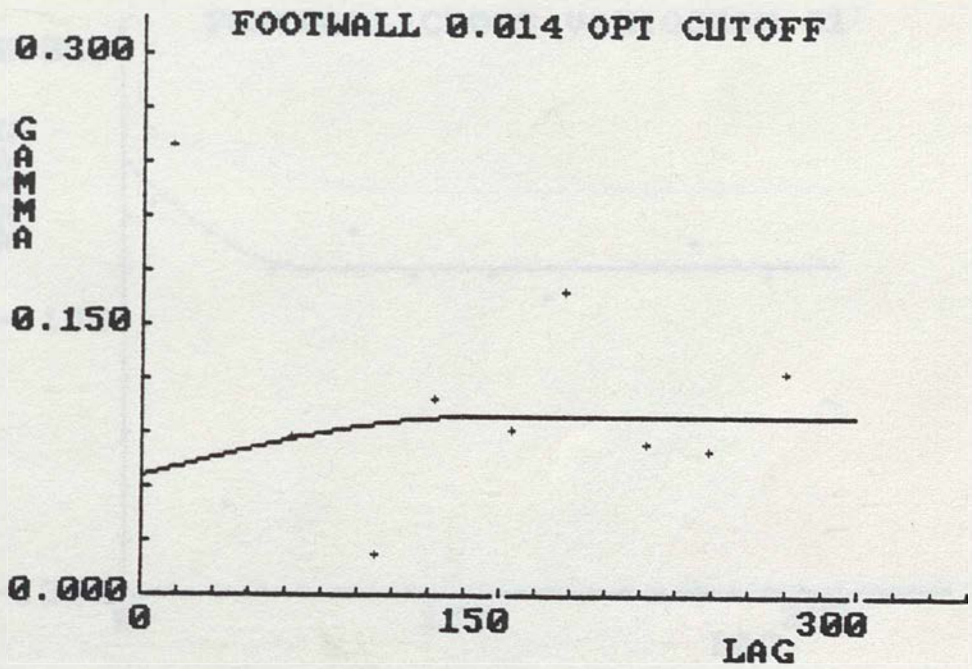
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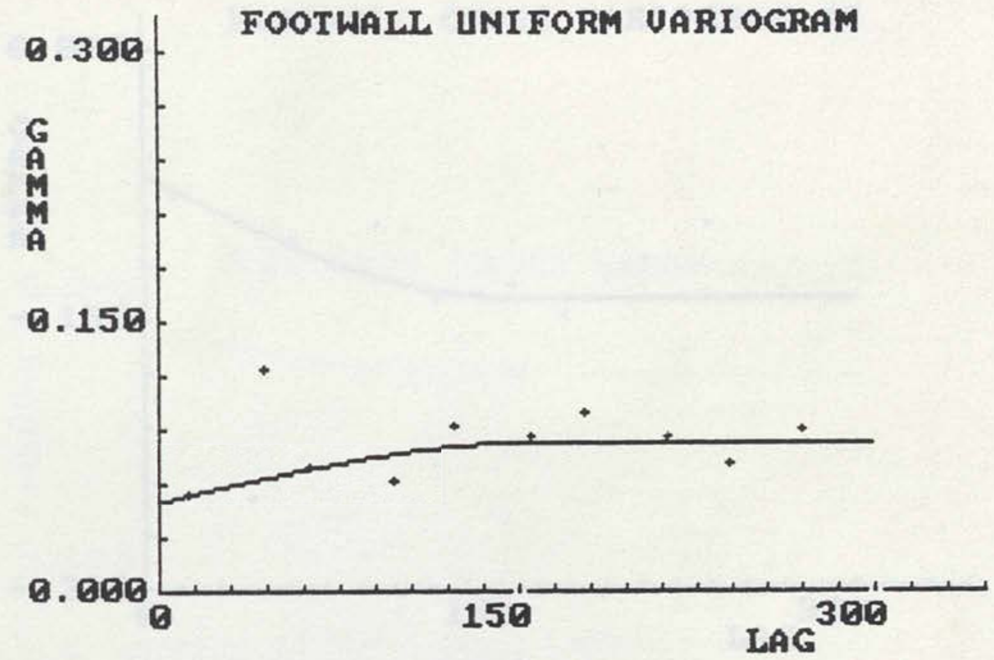
LAG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150
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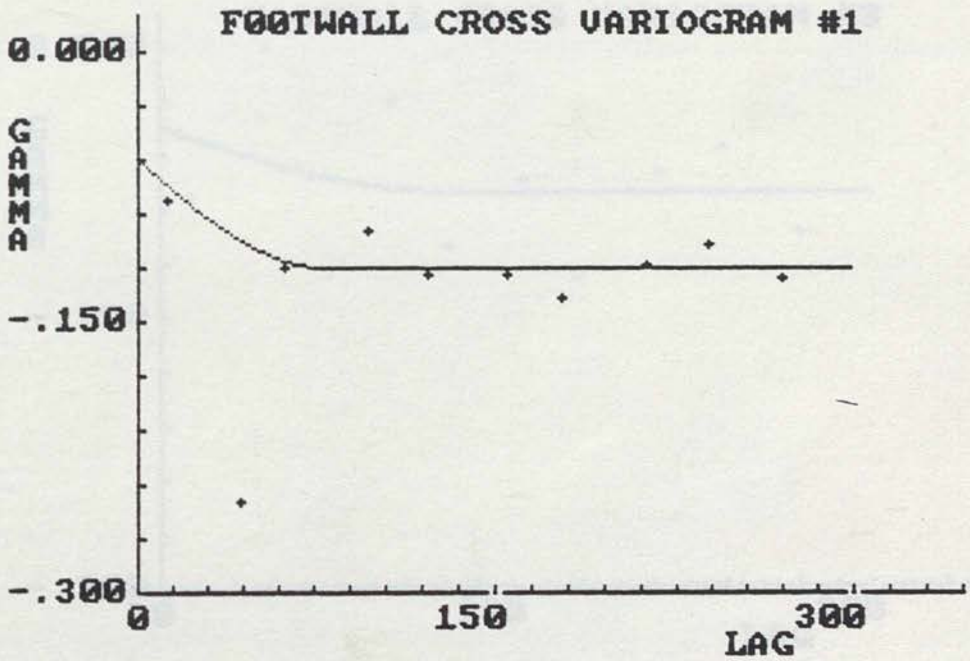
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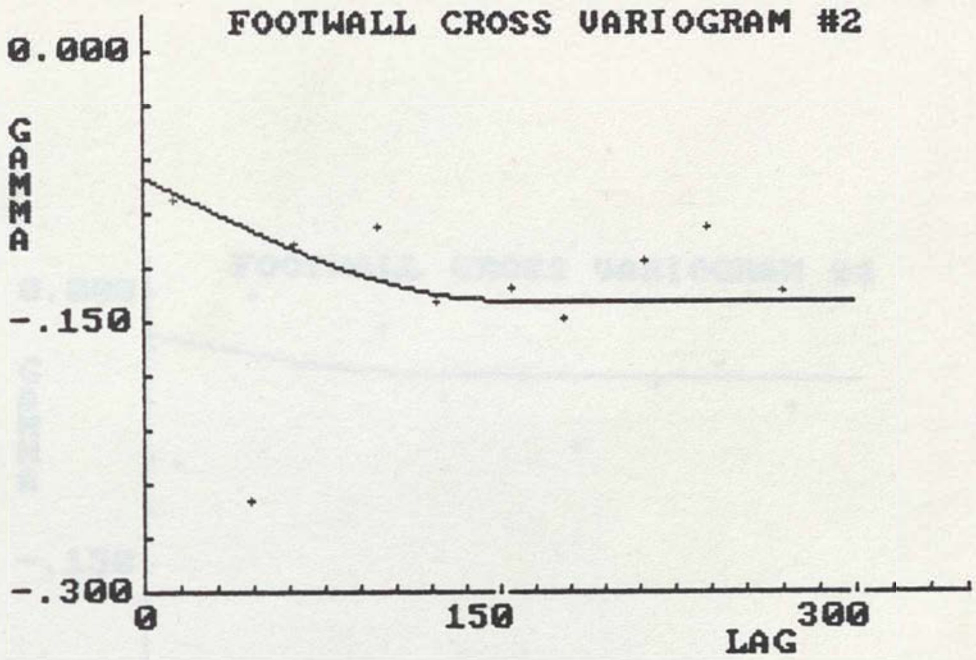
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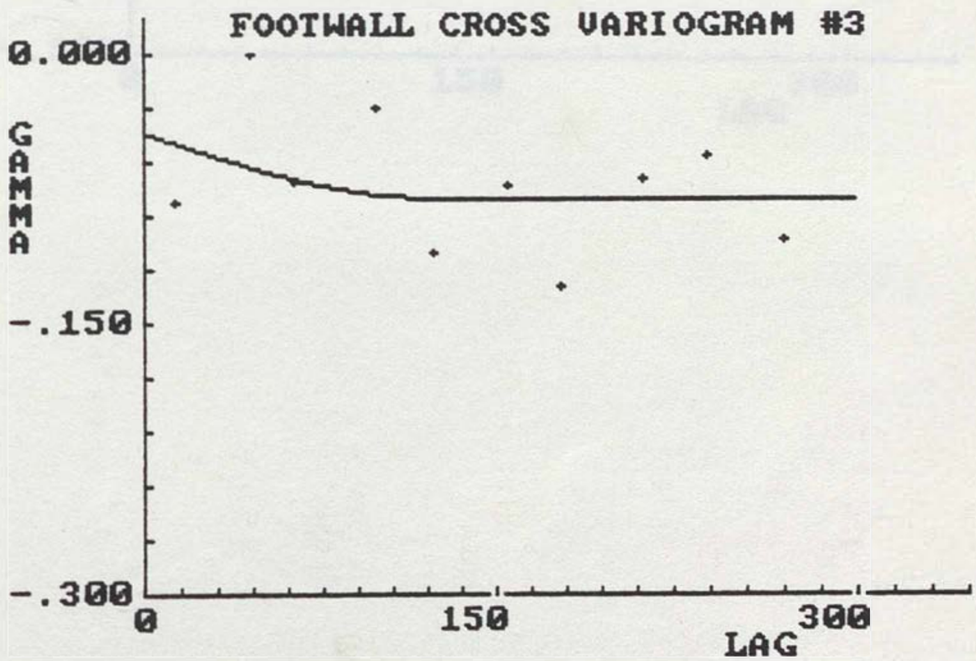
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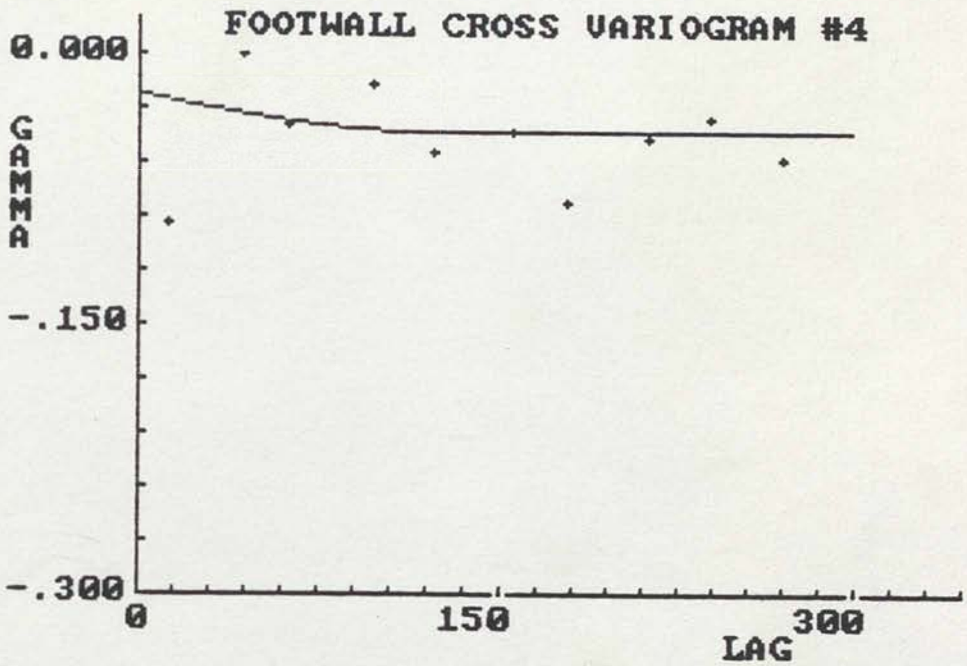
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300
LAG