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Electrodynamic formulation of special relativity from the first postulate

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Abstract. This work presents the form that the special theory of relativity takes when only the first postulate and the properties of homogeneity and isotropy of space and time are considered valid. The transformations of Lorentz coordinates are obtained in terms of a universal constant parameter k , developing from these the relativistic kinematics, dynamics and electrodynamics and their respective invariances before these transformations.

1. Introduction

Over time, relativity work has been developed from two postulates of special relativity enunciated by Einstein [1]. In our case the electrodynamic formulation is developed only taking into account the first postulate of special relativity [2], where the basic equations of electrodynamics are found.

Some researchers on the subject, such as N. David Mermin, in an article from 1983 [3], find the theorem of velocity addition, only counting on the principle of relativity and the properties of homogeneity and isotropy of space-time, arriving at a result that suggests a more general way of describing special relativity. In 1994 J. P. Hsu and L. Hsu [4], used the first postulate of special relativity to develop a theory called Taiji relativity, which is physically different from special relativity where it presents a simpler contextualization. Subsequently U. Molina and collaborators [2], who find the law of adding velocities from the first postulate and properties of homogeneity and isotropy, remaining in terms of a universal constant that depends on the velocity of the inertial frame S' respecting to S . Similarly in 2006 Ingrid Steffanel [5], as a continuation of the 2005 work by Molina, focuses on studying relativistic dynamics without the second postulate of relativity where are found expressions for energy, force and relativistic momentum.

In 2009 H.O. Di Rocco [6], was able to deduce equations for time dilation, length contraction, Doppler effect among other important aspects of the Special Theory of Relativity, TRE, in which it was not necessary to use Lorentz transformations or space-time diagrams. In 2008 M. J. Feigenbaum [7], obtains the relativistic kinematics and dynamics by making an extension of the Galilean theory without using the second postulate. Similarly in 2012 Peng Cheng Zou and colleagues [8], show that the invariance and constancy of the speed of light were originated from the principle of special relativity, but not from the arbitrary implementation of the second postulate. Also in 2009, A. Sfarti, [9], makes a special simplified theory with the first postulate.

On the other hand, in 2015 Alón Drory [10] speaks of the need for the second postulate in relativistic physics and that the principle of relativity together with the homogeneity and isotropy of



space restrict the form of transformations from one inertial system to another, thus maintaining the discussion whether or not the second postulate is necessary.

In this work it is proposed that the equations that make up the theory of special relativity depend on a constant parameter k , so it is enough to work on special relativity with only one postulate. A development is made on relativistic electrodynamics and the form taken by the basic equations of electromagnetism is analyzed.

2. Special relativity

This section deals with the fundamental concepts of the special theory of relativity.

2.1. Coordinated transformations of Lorentz

Given a S' system moving at constant speed v respecting to S , in the direction of the X e X' axes, see Figure 1. The Lorentz coordinate transformations in the axes the X and X' , when only the properties of homogeneity and isotropy in space and time are taken into account are [11,12],

$$\left. \begin{aligned} x' &= \gamma(x - vt) \\ x &= \gamma(x' + vt) \end{aligned} \right\} \quad (1)$$

The Equations (1) represent the transformations of the coordinates x and x' in the direction in which system S' moves respecting to S , when the second postulate has not been used yet.

When the second postulate of Einstein's Special Relativity is used, the Lorentz transformations are obtained, Equations (2), as they are known in the literature [12],

$$\left. \begin{aligned} t' &= \gamma \left(t - \frac{vx}{c^2} \right) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned} \right\} S \rightarrow S' \quad (2)$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, is the relativistic factor of Lorentz and c is the speed of light according to the second postulate.

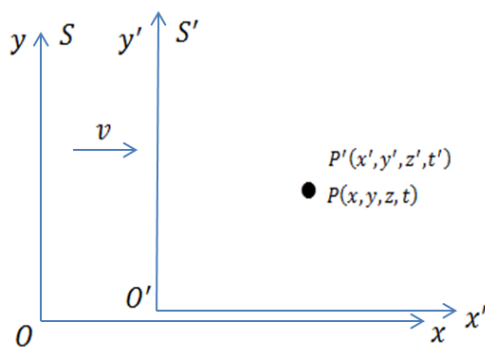


Figure 1. Movement of the system S' with constant speed v with regard to the system S , in the direction of the axes X e X'

Under these transformations of coordinates a whole theory is developed on the fundamental concepts of physics in general.

2.2. Transformations Lorentz velocities

According to the literature [12], the equations of the transformations of velocity, Equation (3), are expressed in the following form:

$$\left. \begin{aligned} u'_x &= \frac{u_x - v}{t - \frac{vu_x}{c^2}} \\ u'_y &= \frac{u_y}{\gamma \left(t - \frac{vu_x}{c^2} \right)} \\ u'_z &= \frac{u_z}{\gamma \left(t - \frac{vu_x}{c^2} \right)} \end{aligned} \right\} S \rightarrow S' \quad (3)$$

Which are found using the transformations of Lorentz (Equation (2)) and elemental algebra.

2.3. Relativistic dynamics

The relativistic linear momentum is defined for a particle of resting mass m_0 and velocity \vec{u} as:

$$\vec{p} = m\vec{u} = \gamma(u)m_0\vec{u} \quad (4)$$

In Equation (4), $m = \gamma(u)m_0$ is represented the moving mass of the particle. While the relativistic factor $\gamma(u)$, see Equation (5), specifies the form in which the velocity of the event is transformed according to the observer in the S system,

$$\gamma(u) = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \quad (5)$$

On the other hand, considering the definition of mechanical work, and using the theorem of work and kinetic energy, we arrive at Equation (6) on relativistic energy,

$$E = E_0 + E_c \quad (6)$$

where: $E_0 = m_0c^2$ is the energy at rest, $E = mc^2 = \gamma(u)m_0c^2$ is the total energy of the particle. While the relation momentum-energy, is expressed in Equation (7), in the form,

$$E^2 = E_0^2 + p^2c^2 \quad (7)$$

2.4. Four-position

A simple way of approaching physical concepts is by means of Four-vector notation. Given the S' system that moves at constant speed v respecting to the S reference system. The four-position for an observer located in the S system is defined by Equation (8), as,

$$X^\alpha = (ct, \vec{r}) = (x^0, x^1, x^2, x^3) \quad (8)$$

2.5. Four-speed

For an event with \vec{u} speed, an observer in the S system, the four-velocity is defined in Equation (9), as that derived from the four-position respecting to one's own time, that is,

$$U^\alpha = \frac{dX^\alpha}{d\tau} = \gamma(u)(c, \vec{u}) \quad (9)$$

2.6. Four-acceleration

It is defined, according to Equation (10), as the derivative of the four-velocity respecting to the own time, that is,

$$A^\alpha = \frac{dU^\alpha}{d\tau} = \gamma^2(u) \left[\frac{\vec{u} \cdot \vec{a}}{c} \gamma(u), \vec{u} \frac{\vec{u} \cdot \vec{a}}{c^2} \gamma(u) + \vec{a} \right] \quad (10)$$

2.7. Linear four-moment

The linear four-moment, as observed in Equation (11), is defined by the form,

$$P^\alpha = m_0 U^\alpha = m_0 \gamma(u)(c, \vec{u}) = \left(\frac{E}{c}, \vec{p} \right) \quad (11)$$

2.8. Electromagnetic field tensor

Given the $\vec{E} = (E_x, E_y, E_z)$ electric fields and $\vec{B} = (B_x, B_y, B_z)$ magnetic fields, the electromagnetic field tensor is defined, according to Equation (12), as,

$$(F^{\alpha\beta}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & cB_z & -cB_y \\ -E_y & -cB_z & 0 & cB_x \\ -E_z & cB_y & -cB_x & 0 \end{pmatrix} \quad (12)$$

2.9. Four-vector electrical current density

For an observer in the S system, a particle of charge density ρ and electric current \vec{J} , the four-density of current, see Equation (13), is defined,

$$J^\alpha = (c\rho, \vec{J}) \quad (13)$$

The transformation equations of these quantities maintain their invariance respecting to the Lorentz transformations, Equations (2). Any four-vector A^α for an observer in S is transformed for an observer in S' , according to Equation (14) in the form,

$$M'^\alpha = \Lambda_\beta^\alpha M^\beta, \alpha, \beta = 0,1,2,3 \quad (14)$$

where Λ_β^α are the elements of the Lorentz transformation matrix, which are defined according to Equation (15) as;

$$\Lambda_0^0 = \Lambda_1^1 = \gamma, \Lambda_1^0 = \Lambda_0^1 = -\gamma\beta, \Lambda_2^2 = \Lambda_3^3 = 1, \Lambda_\beta^\alpha = 0, \text{ en caso} \quad (15)$$

In special relativity independently of the referential inertial observer, the equations maintain their invariance respecting to the Lorentz transformations.

3. Special relativity of the first postulate

This section will deal with the fundamental concepts of special relativity, using only the first postulate, this arrive at the quantities of electromagnetism.

3.1. Lorentz coordinate transformations

Taking into account the two Equation (1), when the second postulate has not been used yet, introducing x' of the first equation into the second and clearing t' , and then using it to find the velocity according to an observer in S' . Developing, is found a constant amount with inverse units to the square of velocity, which from now on we will call k . When the postulate of the constancy of the speed of light is still not taken into account, the Lorentz transformations are replaced by Equation (16),

$$\left. \begin{aligned} t' &= \gamma(k, v)(t - kvx) \\ x' &= \gamma(k, v)(x - vt) \\ y' &= y \\ z' &= z \end{aligned} \right\} S \rightarrow S' \quad (16)$$

Now the relativistic factor of Lorentz is defined in Equation (17), as,

$$\gamma(k, v) = \frac{1}{\sqrt{1-kv^2}} \quad (17)$$

From the factor represented in Equation (17), it can be inferred that when $k = 0$, from which it follows that $\gamma = 1$, which is reduced to the transformations of Galileo, and when $k = \frac{1}{c^2}$, then the Lorentz factor are obtained for the special relativity of Einstein, treated in the previous section.

The Lorentz transformation matrix, Equation (18), only using the first postulate of special relativity, is as follows,

$$(\Lambda_{\beta}^{\alpha}) = \begin{pmatrix} \gamma & -\sqrt{k}v\gamma(k, v) & 0 & 0 \\ -\sqrt{k}v\gamma(k, v) & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (18)$$

3.2. Four-position

The four-position is defined by Equation (19), for an observer it is the S system, such as,

$$X^{\alpha} = \left(\frac{1}{\sqrt{k}}t, \vec{r} \right) = (x^0, x^1, x^2, x^3) \quad (19)$$

3.3. Four-speed

For a particle moving at \vec{u} speed, according to an observer in S, it is defined by Equation (20), the four-velocity as,

$$U^{\alpha} = \gamma(k, u) \left(\frac{1}{\sqrt{k}}, \vec{u} \right) \quad (20)$$

Being in this case and henceforth the relativistic factor for the particle, represented in Equation (21),

$$\gamma(k, u) = \frac{1}{\sqrt{1-ku^2}} \quad (21)$$

3.4. Four-acceleration

Given the \vec{a} acceleration of a particle in the S system, the four-acceleration is defined according to Equation (22),

$$A^{\alpha} = \frac{dU^{\alpha}}{d\tau} = \gamma^2(k, u) [\sqrt{k}(\vec{u} \cdot \vec{a})\gamma(k, u), \vec{u}(\sqrt{k} \vec{u} \cdot \vec{u})\gamma(k, u) + \vec{a}] \quad (22)$$

3.5. Relativistic four-moment

For a relativistic moment particle \vec{p} , the linear four-moment is defined by Equation (23),

$$P^{\alpha} = m_0\gamma(k, u) \left(\frac{1}{\sqrt{k}}, \vec{u} \right) = (\sqrt{k}E, \vec{p}) \quad (23)$$

Being $E = \frac{m}{k}$, the total energy of the particle and $\vec{p} = m_0\gamma(k, u)\vec{u}$, the relativistic linear momentum.

3.6. Relativistic momentum-energy relationship.

In this case the energy relations, Equation (24), are as follows,

$$E = E_c + E_0 \quad (24)$$

Being: E_c kinetic energy, $E = \frac{m}{k} = \frac{m_0\gamma(k,u)}{k}$ total energy and, $E_0 = \frac{m_0}{k}$ resting energy. While the momentum-energy relation is expressed by Equation (25), in the form,

$$E^2 = \frac{p^2}{k} + E_0^2 \quad (25)$$

4. Relativistic electrodynamics

Now are found the equations of the electromagnetic field, which maintain the same form as those known in the literature.

4.1. Electromagnetic field tensor

Following the Lorentz transformations, Equations (18) and in analogous form to Equation (12), the electromagnetic field tensor can be defined as shown in Equation (26),

$$(F^{\alpha\beta}) = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & \frac{1}{\sqrt{k}}B_z & -\frac{1}{\sqrt{k}}B_y \\ -E_y & -\frac{1}{\sqrt{k}}B_z & 0 & \frac{1}{\sqrt{k}}B_x \\ -E_z & \frac{1}{\sqrt{k}}B_y & -\frac{1}{\sqrt{k}}B_x & 0 \end{pmatrix} \quad (26)$$

According to the Lorentz transformation matrix found only with the first postulate of the theory of relativity, Equations (17), the transformations of the electromagnetic field $F'^{\alpha\beta} = \Lambda_{\mu}^{\alpha}\Lambda_{\nu}^{\beta}F^{\mu\nu}$, are invariant respecting to them.

4.2. Current four-vector

In this case the four-density of current would be expressed by Equation (27), in the form,

$$J^{\alpha} = \left(\frac{\rho}{\sqrt{k}}, \vec{j} \right) \quad (27)$$

Being ρ : the charge density and current density by Equation (28) as,

$$\vec{j} = \rho_0\gamma(k,u)\vec{u} = \rho\vec{u} \quad (28)$$

4.3. Four-gradient

The four-gradient is defined by Equation (29) as,

$$\nabla^{\mu} = \frac{\partial}{\partial x^{\mu}} = \left(\sqrt{k} \frac{\partial}{\partial t}, \nabla \right) \quad (29)$$

The four-gradient and the four-vector current density are chords to preserve or maintain the condition of continuity.

4.4. Four-divergence

Given any four-vector $\varepsilon^{\alpha} = (\varepsilon_0, \vec{\varepsilon})$ when applying the four-gradient of the Equation (29), it is obtained according to the Equation (30), the four-divergence, that is,

$$\nabla^{\mu}\varepsilon^{\mu} = \frac{\partial\varepsilon^{\mu}}{\partial x^{\mu}} = \sqrt{k} \frac{\partial\varepsilon^0}{\partial t} - \nabla\vec{\varepsilon} \quad (30)$$

4.5. Four-vector electric potential

Let φ be the electrical potential, \vec{A} the magnetic vector potential, defined in Equations (31) as:

$$\left. \begin{aligned} \vec{B} &= \nabla \times \vec{A} \\ \vec{E} &= -\nabla \varphi - \frac{\partial \vec{A}}{\partial t} \end{aligned} \right\} \quad (31)$$

The potential electromagnetic four-vector is defined by Equation (32) as,

$$A^\alpha = \left(\varphi, \frac{\vec{A}}{\sqrt{k}} \right) \quad (32)$$

With which they are transformed with the matrix elements (18) and would maintain invariant the transformations of the electromagnetic field.

5. Conclusions

In this paper is studied Einstein's Special Theory of Relativity taking into account his two postulates and the properties of homogeneity and isotropy of space and time. An analysis is made of some works like those of N. D. Mermin, U. Molina and collaborators where they carry out a comparison of the theorem of addition of velocities obtained from the first postulate and the Lorentz transformations finding that these are expressed in terms of a universal constant that depends on the velocity of the inertial frame S' respecting to S .

In the proposed work it is shown that the equations that conform the theory of special relativity depend on a free parameter k , which value is only a theoretical result, as has been corroborated by other authors such as U. Molina, Peng Cheng Zou, H.O. Di Rocco, Ingrid Steffanel, J.P. Hsu and Leonard Hsu and A. Sfarti, who propose that it is enough to work on special relativity with a single postulate. Subsequently the relativistic dynamics is developed and basically expressions for work, energy, four-velocity among others.

Finally, Relativistic Electrodynamics is studied, defining some concepts such as current density four-vector, electromagnetic field tensor, four-gradient, four-divergence and relativistic potential four-vector, considering the first postulate and the introduction of the constant k , which has velocity units squared, and which is adopted as a universal constant.

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