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Kalman filter-based stochastic subspace identification under mixed stochastic and periodic excitation

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Kalman filter-based stochastic subspace identification under mixed stochastic and periodic excitation

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CONTEXT

- Subspace-based system identification from output-only vibration measurements collected from structures in-operation
  - Modal analysis of civil, mechanical or aeronautical structures
  - Vibration modes are identified from eigenstructure of LTI system
- Intrinsic nature of the excitation may pose difficulties, e.g. presence of periodic inputs originating from rotating components of the structure, in addition to stochastic inputs
  - Identified eigenstructure contains both system and periodic modes
  - Consistency of the covariance-based subspace identification for measurements with oscillatory components showed in [1]
  - In practice periodic modes often disturb the estimation of close structural modes

AIMS

- Reconstruction of output signal where the periodic part is removed as preprocessing for engineering applications
- Identification of the eigenstructure of the stochastic system part only

MODELING

- System states are physical quantities of mechanical system i.e. displacements and velocities
- Periodic states represent the periodic excitation $u(t)$

Mechanical model

$$\mathcal{M}\ddot{z}(t) + \mathcal{C}\dot{z}(t) + \mathcal{K}z(t) = f(t) + \tilde{\mathbf{b}}u(t)$$
$$y(t) = L\ddot{z}(t) + v(t)$$
$$u(t) = \sum_{i=1}^{h} e_i \sin(\omega_i t + g_i)$$

$\Rightarrow$

Continuous-time combined state-space model

$$\dot{x}^{\text{sys}}(t) = A_c^{\text{sys}} x^{\text{sys}}(t) + \mathbf{b}u(t) + w(t)$$
$$y(t) = C^{\text{sys}} x^{\text{sys}}(t) + \mathbf{d}u(t) + v(t)$$

$\Downarrow$

Discrete-time combined state-space model

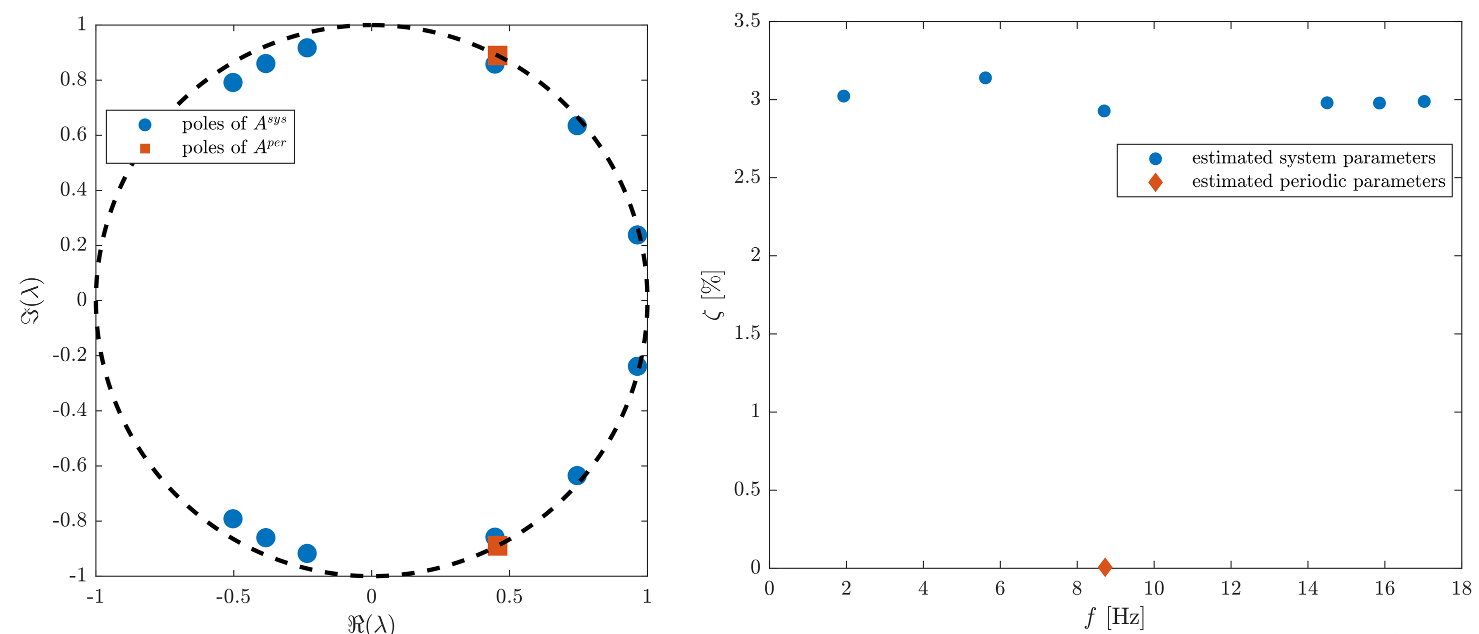
$$\begin{bmatrix} x_{k+1}^{\text{sys}} \\ x_{k+1}^{\text{per}} \end{bmatrix} = \underbrace{\begin{bmatrix} A^{\text{sys}} & A^{\mathbf{b}} \\ 0 & A^{\text{per}} \end{bmatrix}}_{=A} \begin{bmatrix} x_k^{\text{sys}} \\ x_k^{\text{per}} \end{bmatrix} + \begin{bmatrix} w_k \\ 0 \end{bmatrix}$$

$$y_k = \underbrace{\begin{bmatrix} C^{\text{sys}} & C^{\text{per}} \end{bmatrix}}_{=C} \begin{bmatrix} x_k^{\text{sys}} \\ x_k^{\text{per}} \end{bmatrix} + v_k$$

$\Leftarrow$

$$A^{\text{sys}}: \lambda_i^{\text{sys}}, \varphi_i^{\text{sys}} \rightarrow f_i^{\text{sys}}, \zeta_i^{\text{sys}}, \varphi_i^{\text{sys}}$$
$$A^{\text{per}}: \lambda_i^{\text{per}}, \varphi_i^{\text{per}} \rightarrow f_i^{\text{per}}, \zeta_i^{\text{per}}, \varphi_i^{\text{per}}$$

- Proposed modeling is equivalent to [1] up to similarity transform
- Identification of both system and periodic modes with the UPC algorithm [2]:



Left: Discrete-time poles of a 6 DOF chain system, Right: Estimated frequencies f_i^{sys}, f_i^{per} and damping ratios ζ_i^{sys}, ζ_i^{per} of 6 DOF chain system under combined random and periodic excitation.

REMOVAL OF THE PERIODIC SUBSIGNAL BY ORTHOGONAL PROJECTION AND SYSTEM IDENTIFICATION

- The goal is to reconstruct responses where the periodic signal is discarded and then to identify the observability matrix of the structural system: Γ_{sys}

$$\Gamma^{\text{sys}} = \left[(C^{\text{sys}})^T \; (C^{\text{sys}} A^{\text{sys}})^T \; \ldots \; (C^{\text{sys}} A^{\text{sys} p})^T \right].$$

Algorithm:

- Prediction of the periodic subsignal with the non steady-state Kalman filter

Similarity transform of the innovation state-space model into modal basis

$$\hat{x}_{k+1}^V = A^V \hat{x}_k^V + K_k^V e_k ,$$
$$y_k = C^V \hat{x}_k^V + e_k ,$$

with $V = [\Re(\Psi) \; \Im(\Psi)]$, where $\Psi = [\varphi_1^{\text{sys}} \ldots \varphi_m^{\text{sys}} \; \varphi_1^{\text{per}} \ldots \varphi_h^{\text{per}}]$, and $\hat{x}_k^V = V^{-1}\hat{x}_k$, $A^V = V^{-1}AV$, $C^V = CV$, $K_k^V = V^{-1}K_k$. This allows to select states corresponding to the periodic modes with a selection matrix S and subsequently approximate the periodic subsignal

$$\hat{y}_k^{\text{per}} = C^V S \hat{x}_k^V .$$

- Reconstruction of the output system response by projection of the raw output measurements onto the orthogonal complement of the periodic subsignal estimate

The orthogonal projection of the raw data Hankel matrix $\mathcal{Y}_{\text{raw}}$ onto the data Hankel matrix of the predicted periodic subsignal $\mathcal{Y}_{\text{per}}$ yields the decomposition

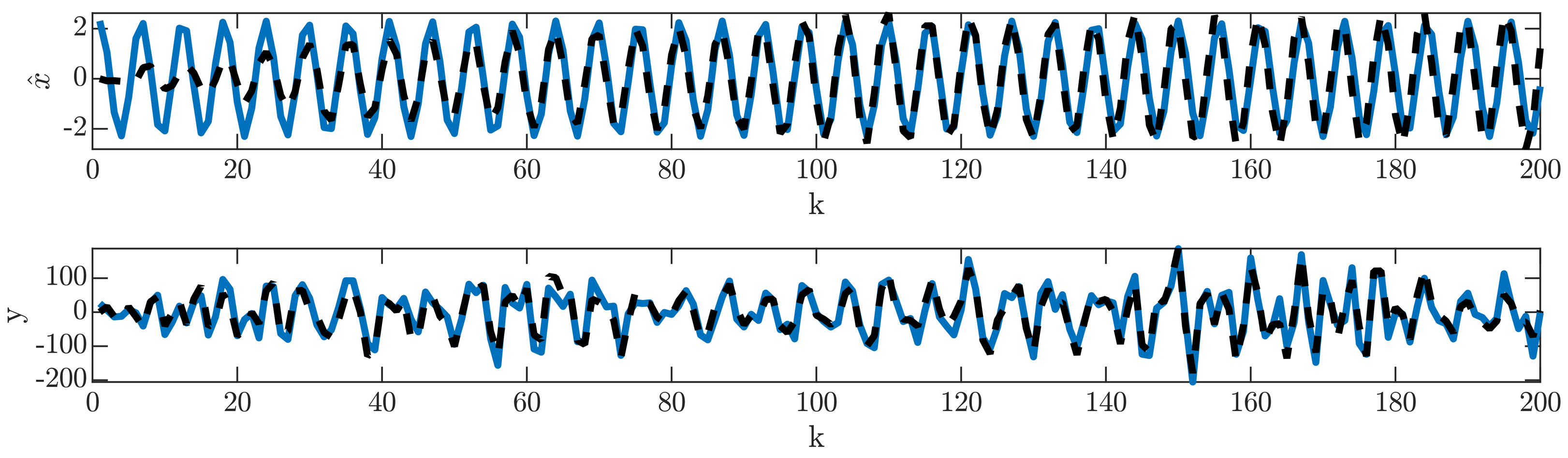
$$\mathcal{Y}_{\text{pro}}^- = \mathcal{Y}_{\text{raw}}^- / \mathcal{Y}_{\text{per}}^{-\perp} = \Gamma_{\text{sys}} \mathcal{Z}^- + \mathcal{K}\mathcal{E}^- + \mathcal{E}_{\mathcal{K}}^- ,$$
$$\mathcal{Y}_{\text{pro}}^+ = \mathcal{Y}_{\text{raw}}^+ / \mathcal{Y}_{\text{per}}^{+\perp} = \Gamma_{\text{sys}} \mathcal{Z}^+ + \mathcal{K}\mathcal{E}^+ + \mathcal{E}_{\mathcal{K}}^+ .$$

Γ_{sys} can be factorized from the projection similar to UPC

$$\mathcal{Y}_{\text{pro}}^+ / \mathcal{Y}_{\text{pro}}^- = \Gamma_{\text{sys}} \breve{\mathcal{Z}} .$$

NUMERICAL VALIDATION

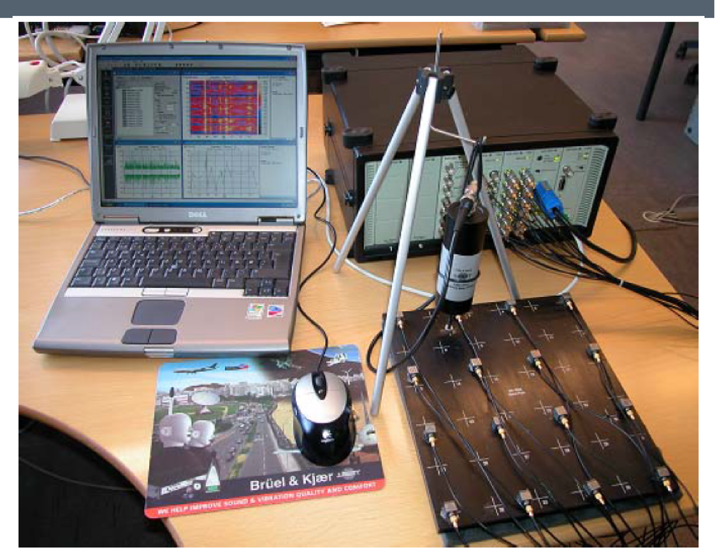
- Comparison of the exact and estimated states and system response of 6 DOF chain system under mixed periodic and random excitation:



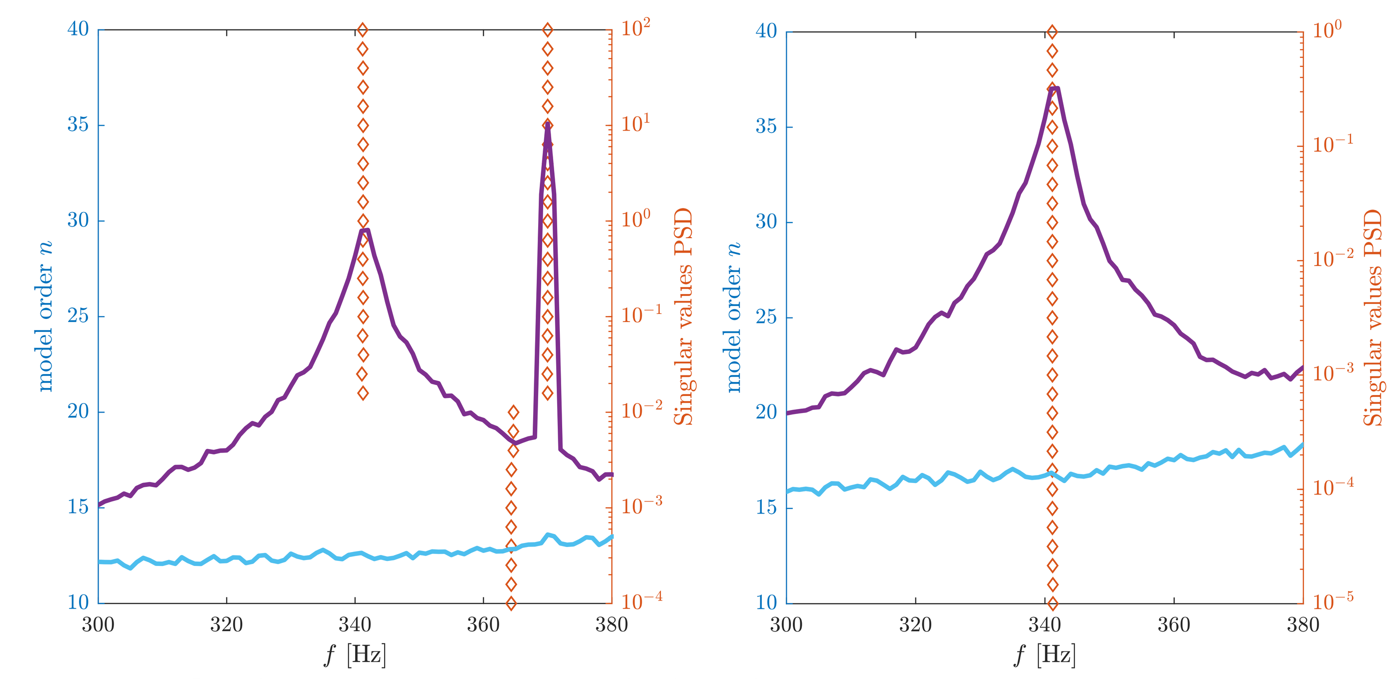
Top: Periodic states, Bottom: System response.

APPLICATION: MODAL ANALYSIS OF A PLATE

- Rectangular aluminum plate excited with a shaker
- Vibration measurements with 16 sensors at 4096 Hz with 491,520 samples
- Two experiments:
  - Random vibration
  - Random vibration with added sinusoidal signal of 370 Hz induced by the shaker
- Stochastic subspace identification and orthogonal projection of predicted periodic subsignal

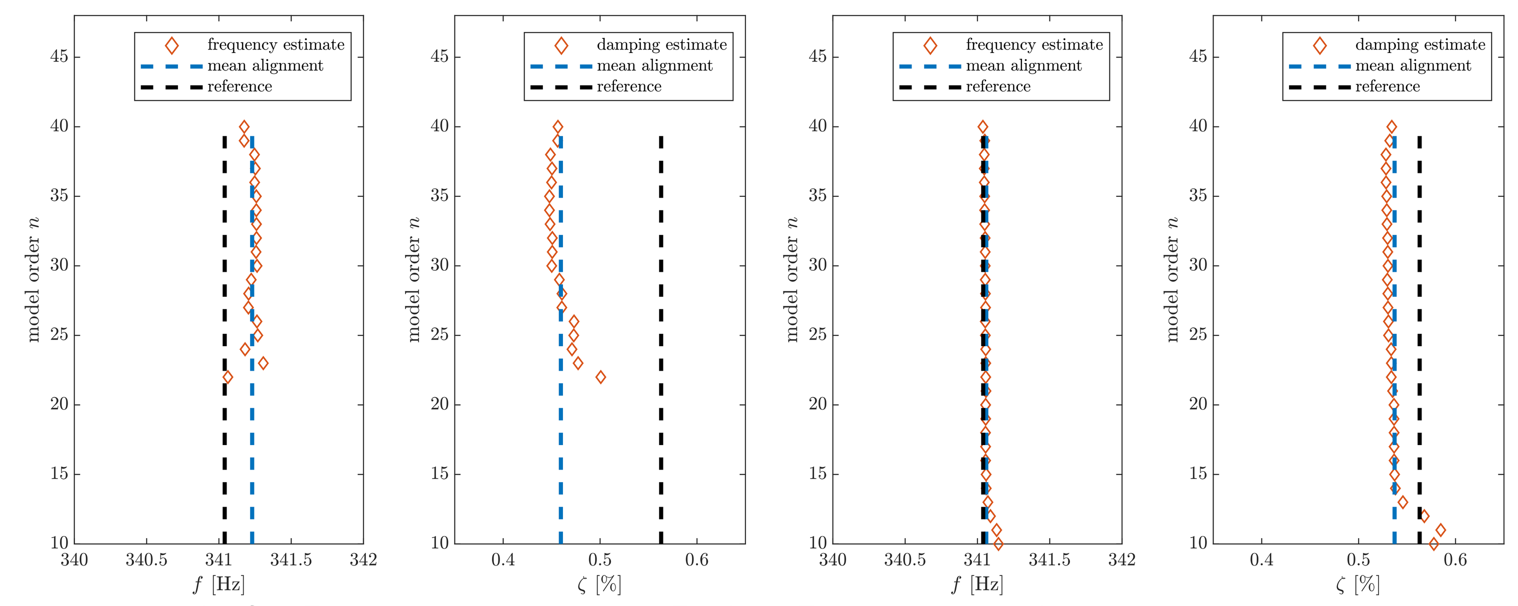


- Stabilization diagrams of modal frequencies with output measurements spectra:



Left: Raw measurements, Right: Reconstructed measurements.

- Frequencies and damping ratios estimated for model orders 10-40 and compared to estimates from random vibration experiment:



Left: Raw measurements, Right: Reconstructed measurements.

CONCLUSIONS

- Extension of the output-only stochastic and periodic state-space modeling to the UPC algorithm
- Consistent algorithm to reconstruct the stochastic part of the system response to mixed random and periodic excitation
- In practical application parameters estimated from the reconstructed measurements are closer to estimates from a comparable random vibration experiment

REFERENCES

[1] M. Favaro and G. Picci. Consistency of subspace methods for signals with almost-periodic components. *Automatica*, 48(3):514 – 520, 2012.

[2] P. van Overschee and B. de Moor. *Subspace Identification for Linear Systems.* Springer, 1st edition, 1996.

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