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Analysis of the Non-Smooth Dynamical Effects and Suitable Operation and Control Strategies for Digital Displacement Units

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ABSTRACT

Newly emerging digital hydraulic pump-motor units enable high efficient operation and thereby span widely with respect to potentially application use. The digital displacement machine (DDM) provides four-quadrant operation at lower losses compared to conventional fluid power machines, due to the possibility of activating/deactivating individual pressure chambers. However, its non-smooth digital effects introduces pressure oscillations and complicates the control system development. This possibly explains why most state-of-the-art control strategies neglects the transient machine behavior when synthesizing controllers for the actuated plant. Since the dynamical behavior and control performance is greatly dependent on the chosen operation strategy and rotational speed of the machine, this paper identifies how the suitable operation and control strategy changes as function of the rotational speed. The paper shows that the DDM dynamics should only be neglected for a machine with a relatively high number of cylinders, relatively high displacement operation and if the rotational speed of the DDM is significantly faster than the plant dynamics.

KEYWORDS: Fluid Power, Digital Displacement, Hybrid system, Control model, Event-driven

1 INTRODUCTION

Energy efficient fluid power systems have received increasing research interest in the past decade both in academia and industry. Application utilization spans from cylinder drives [1, 2], to transmissions and power take-off systems [3, 4, 5, 6, 7, 8, 9]. In this paper, the digital displacement technology is considered which has great potential for use in all of the above mentioned applications. The machine uses the radial piston type design, where cylinders are radially located on a eccentric shaft, resulting in a reciprocating piston motion. The use of digital control valves located between each pressure chamber and high and low pressure manifold enables each chamber to be controlled individually. Thereby, the efficiency may be improved significantly, especially at part load operation. Additionally, the technology provides great redundancy and scalability due to its modular design. However, several challenges remains for the technology to be deployed on the market, which explains the large work focusing on design and performance optimization of the machine [6, 10, 11, 12, 13, 14, 15, 16]. For the technology to be competitive to existing solutions, proper displacement control is considered of major importance, but is complicated by the non-smooth digital effects. Most state-of-the-art control strategies for the digital displacement machine neglects the transient machine behavior, and determines the cylinder chamber activation sequence based on either offline optimization or estimation techniques in a look-ahead angle [17, 18, 19, 20, 21, 22]. However, since the activation of the pressure chamber is conducted as function of the shaft angle and not time, the response time of the DDM is proportional to the rotational shaft speed. This paper investigates the dynamics of the digital displacement machine (DDM) as function of machine speed and operation strategy and shows that the dynamics should not always be neglected if proper operation and stability are to be guaranteed. Furthermore, the paper shows that the suitable operation and control strategy changes as function of machine speed and proposals on suitable strategies are provided accordingly. To simplify the dynamical analysis, the investigation is based on a relatively simple mathematical model, which only captures the fundamental dynamics of the DDM.

2 MATHEMATICAL MODEL

The relatively simple mathematical model is established based on the illustration of the DDM and a single pressure chamber shown in Fig. 1.

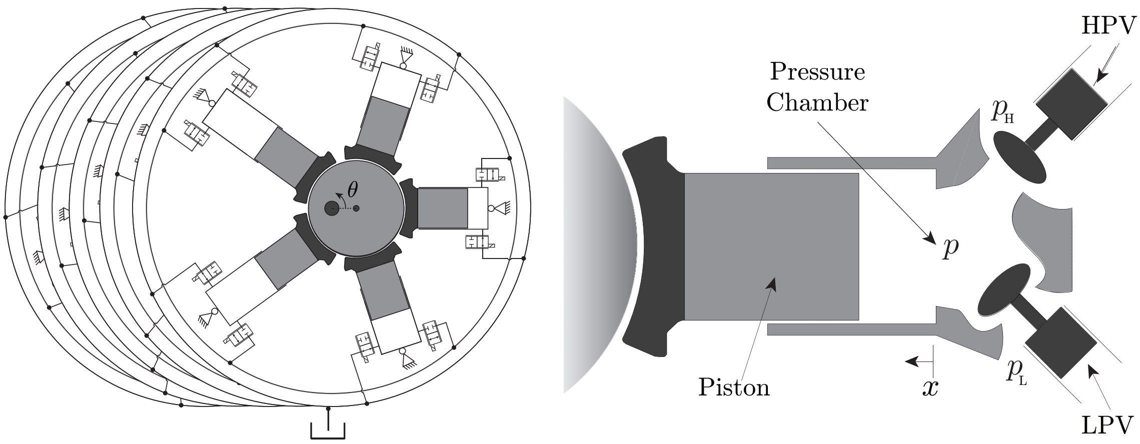


Figure 1: Illustration of the radial piston type digital displacement machine (25 cylinder machine) and definition of variables used for the mathematical model [23, 24].

The piston displacement for the i'th cylinder, x_i, is described as function of the shaft angle by

$$x_i = r_\mathrm{e}\,(1-\cos(\theta_i)) \qquad \theta_\mathrm{i} = \theta + \frac{2\,\pi}{N_\mathrm{c}}\,(i-1) \qquad i \in \{1,\ldots,N_\mathrm{c}\} \tag{1}$$

where r_e is the eccentric shaft radius and N_c is the number of cylinders. The displacement volume is thus given as $V_\mathrm{d} = 2\,r_\mathrm{e}\,A_\mathrm{p}$, where A_p is the piston area. The chamber volume for the i'th cylinder, V_i, and its time derivative are then given by

$$V_i = \frac{V_\mathrm{d}}{2}\,(1-\cos(\theta_i)) + V_0 \qquad \dot{V}_i = \frac{V_\mathrm{d}}{2}\,\sin(\theta_i)\,\dot{\theta} \tag{2}$$

where A_p is the piston area and V_0 is the minimum chamber volume. The continuity equation is used to describe the pressure build-up for the i'th cylinder given by

$$\dot{p}_i = \frac{\beta_\mathrm{e}(p_\mathrm{i})}{V_\mathrm{i}} \left(Q_{\mathrm{H},i} - Q_{\mathrm{L},i} - \dot{V}_i\right) \tag{3}$$

where β_e is the pressure dependent effective bulk modulus. Q_L and Q_H are the flows through the low and high pressure valve respectively. The orifice equation is used to describe the flows through the valves and are given to be

$$Q_{\mathrm{L},i} = \frac{x_{\mathrm{L,i}}}{k_\mathrm{f}} \sqrt{|p_i - p_\mathrm{L}|}\,\mathrm{sign}\,(p_i - p_\mathrm{L}) \qquad Q_{\mathrm{H},i} = \frac{x_{\mathrm{H,i}}}{k_\mathrm{f}} \sqrt{|p_\mathrm{H} - p_i|}\,\mathrm{sign}\,(p_\mathrm{H} - p_i) \tag{4}$$

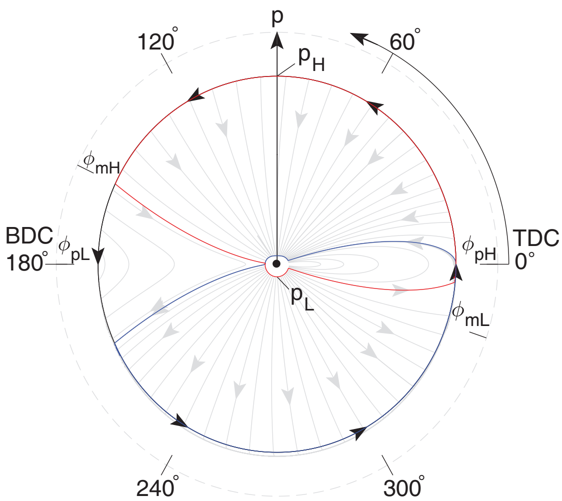
where $x_\mathrm{L} \in [0,1]$ and $x_\mathrm{H} \in [0,1]$ are normalized valve plunger positions, while k_f is the valve flow coefficient. When considering the fundamental machine dynamics it is deemed sufficient to model the valves as a simple first order system given as

$$\dot{x}_{\mathrm{H,i}} = \frac{1}{\tau_\mathrm{v}} \left(u_{\mathrm{H,i}} - x_{\mathrm{H,i}}\right) \qquad \dot{x}_{\mathrm{L,i}} = \frac{1}{\tau_\mathrm{v}} \left(u_{\mathrm{L,i}} - x_{\mathrm{L,i}}\right) \tag{5}$$

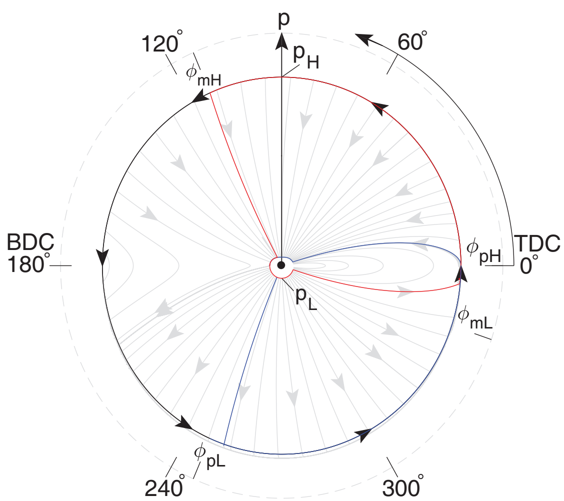
where u_H and u_L are the inputs to the high and low pressure valve respectively, while t_v is the valve time constant. The torque contribution from the i'th pressure chamber is derived to be given by

$$\tau_i = \frac{d\,V_i\,(\theta_i)}{d\theta}\, p_i = \frac{V_\mathrm{d}}{2} \sin(\theta_i)\, p_i \tag{6}$$

The dynamical behavior of the pressure build-up in a pressure chamber is investigated by simulation the pressure trajectories in a polar plot as presented in [23]. The pressure trajectories for a pumping and motoring stroke is shown in Fig. 2a for a full-stroke response and in Fig. 2b for a partial-stroke response.



(a) Full stroke operation phase plot.



(b) Partial stroke operation phase plot.

Figure 2: Polar phase plot of the pressure trajectories of a single pressure chamber. Red trajectory indicates a motoring stroke and blue trajectory indicates a pumping stroke [23, 24].

An active stroke is initiated by closing the low pressure valve (LPV) at ϕ_{mL} and ϕ_{pL} for motoring and pumping respectively. As the pressure exceeds the high pressure due to compression, the high pressure valve (HPV) is passively opened. The pressure level remains high until the active stroke is ended by closing the HPV at ϕ_{mH} and ϕ_{pH} for motoring and pumping respectively. It is seen that the last part of the motoring stroke and the first part of the pumping stroke in full-stroke operation is not utilized. This is done such that passive valve opening of the LPV due to the generated suction force may be utilized to reduce the energy losses.

It is seen that the pressure dynamics with respect to the angle is relatively fast, why it is considered valid to neglect the transient pressure build-up. As a result, the flow and torque throughput may either be active or inactive depending on the binary input, $\bar{u} \in \{0,1\}$, given by

$$\mathcal{D}_i = \frac{V_{\mathrm{d}}}{2} \sin(\theta_i)\, \bar{u}_i \qquad Q_{\mathrm{H},i} = \mathcal{D}_i\, \dot{\theta}_i \qquad \tau_i = \mathcal{D}_i\, p_i \tag{7}$$

where $\mathcal{D}_{\mathrm{i}}$ is the displacement throughput of the i'th pressure chamber. An illustration of the piston position, chamber pressure, valve plunger positions and displacement throughput is shown in Fig. 3 for full stroke operation.

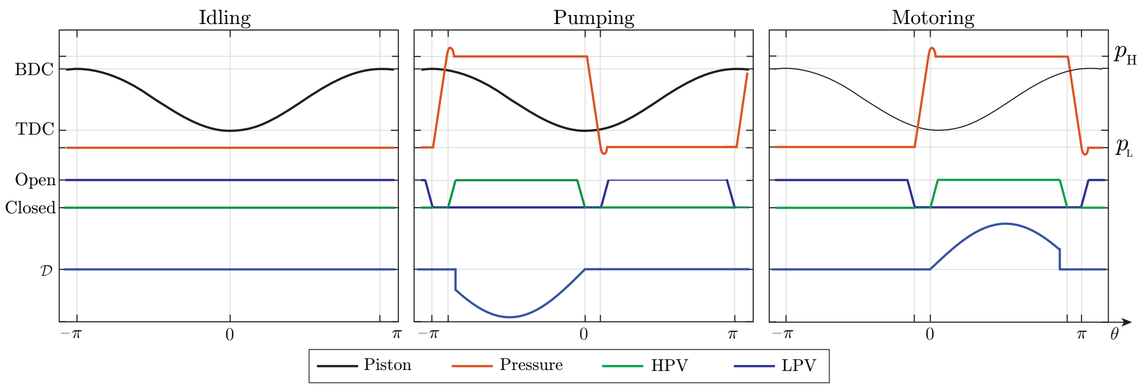


Figure 3: Illustration of the machine response in full stroke operation.

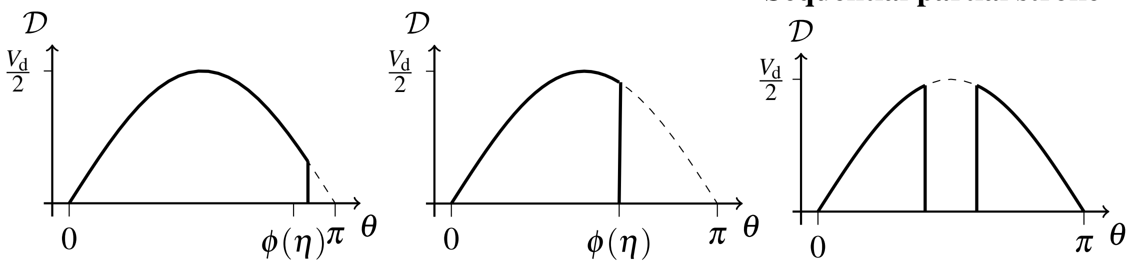
It is seen that the displacement response may be simplified to be a fraction of a sine wave, when neglecting the pressure dynamics. In the remainder of this paper, this simplification is used to describe the input-output relation of the DDM, whereby the complexity of dynamical analysis is significantly reduced.

The different operation strategies considered in this study are shown in Fig. 4 together with a comparison of the important characteristics. Since every operation strategy has its associated advantages and disadvantages, they should be applied accordingly to obtain the best possible performance. The dynamical behavior of the operation strategies are also vastly different, such that the suitable control strategy for each operation strategy is different and should be applied accordingly. This paper investigates the non-smooth dynamical effect of the different operation strategies and provides proposals on suitable control strategies for each operation strategy.

3 THE NON-SMOOTH DYNAMICAL EFFECT OF THE DDM

Most state-of-the-art control strategies for digital displacement units neglects the dynamical behavior of the DDM when designing the control system for the actuated plant. This paper investigates under which operation conditions it is sufficient to neglect the DDM dynamics and how the operation conditions affects the dynamical behavior. This is done

Figure 4: Operation strategies for the digital displacement machine [24].



Full stroke	**Partial stroke**	**Sequential partial stroke**
• Most simple and energy efficient strategy, due to switching at low flow and pressure levels for only a fraction of cylinders	• Medium energy efficient strategy, due to switching at higher flow rates, but low pressure difference for every cylinder	• Least energy efficient strategy, due to occasionally opening against high pressure difference multiple times every stroke
• Small and fast valves incapable of opening against high pressure	• Small and fast valves incapable of opening against high pressure	• Larger and slow valves capable of opening against high pressure
• Control update rate is proportional to the machine speed.	• Control update rate is proportional to the machine speed	• Control update rate is speed independent and only constrained by valve switching time
• Displacement is quantized by the number of cylinders per revolution and not changeable instantaneously	• Displacement is continuously controlled per revolution and not changeable instantaneously	• Displacement is instantaneously quantized by the number of cylinders and continuous per revolution

by comparing the response of a continuous first order plant actuated by the DDM to the response of an ideal infinitely fast actuator (corresponding to neglecting the DDM dynamics). The comparison is illustrated in Fig. 5 where a full-stroke delta-sigma modulation strategy is considered [25, 26]. η is the constant displacement fraction input, $\bar{\eta}$ is the binary input sequence determine whether the current cylinder chamber should be active or inactive. y_c is the total output displacement of all pressure chambers and y_p is the plant response. The plant is scaled with the constant $\pi/(\eta_m N_c)$ to obtain unity dc-gain and enable comparison to the ideal actuated plant. The constant $\eta_m \approx 0.96$ is the maximum full stroke displacement fraction. The plant eigen-frequency, ω_n, represents the plant inertness and is expected to have a large influence on the severity of pulsations in the plant response due to its filtering property. The difference between the plant response with and without the DDM, Δy, gives a measure of the digital effect introduced by the DDM. Simulation results for investigation of the DDM dynamical and non-smooth effect is shown in Fig. 6 for both a full-stroke pulse density modulated (PDM) and a partial-stroke pulse width modulated (PWM) digital displacement machine. A 5 cylinder DDM with a displacement fraction input of $\eta = 0.5$ is considered for simplicity of illustration.

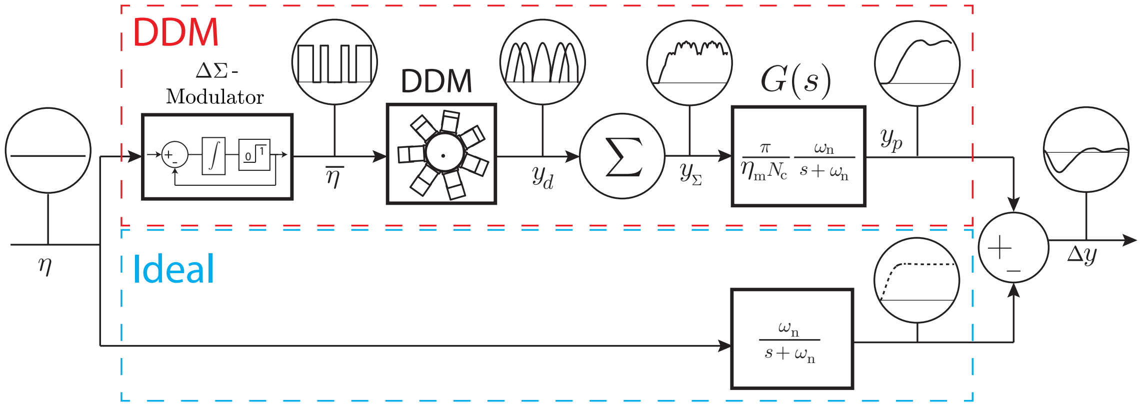


Figure 5: Strategy for investigation of the digital effect of the DDM. A full-stroke delta-sigma modulation strategy is considered [24].

It is seen that every second cylinder is activated for the full stroke period with the PDM strategy and every cylinder is activated for half of the stroke period with the PWM strategy. The red line indicates the angle delay, θ_d, in the response from the actuation decision is made until the first committed output. The dashed line shown for the plant response, y_p, is the ideal actuated response and thus has no delay. The important dynamical characteristics of the DDM may be divided into three parts; time-delay, response time and steady state energy of the oscillations. The time-delay is known to introduce phase-lag to a closed loop control system and is therefore of high importance with respect to ensuring stability. The DDM response angle, ϕ, from zero to maximum displacement is equal to the angle where the active stroke part is ended and is determined by the displacement fraction as

$$\eta = \frac{V(\phi)}{V(\pi)} = \frac{\frac{V_d}{2}\,(1-\cos(\phi))}{\frac{V_d}{2}\,(1-\cos(\pi))} = \frac{(1-\cos(\phi))}{2} \qquad (8)$$
$$\phi(\eta) = \mathrm{acos}(1-2\,\eta) \qquad\qquad \eta \in [0,\, \eta_m]$$

Therefore, the DDM response time is inversely proportional to the machine speed, ω given by

$$T_r = \frac{\phi(\eta)}{\omega} \qquad (9)$$

where $\eta = \eta_m$ for a full stroke operated machine. The energy in the steady state part of the difference signal, Δy, is a measure of the severity of the oscillation caused by the DDM. This measure is calculated by the squared average energy in the steady-state response of the difference signal and is evaluated for 10 full revolutions indicated by the blue line in Fig. 6 and is given by

$$E = \frac{1}{20\,\pi} \int_{\theta_0}^{\theta_0+20\,\pi} \Delta_y(\theta)^2 d\theta \qquad (10)$$

where θ_0 is the initial angle during steady state operation. Since the steady state energy is dependent on four variables; displacement fraction, number of cylinders, machine speed

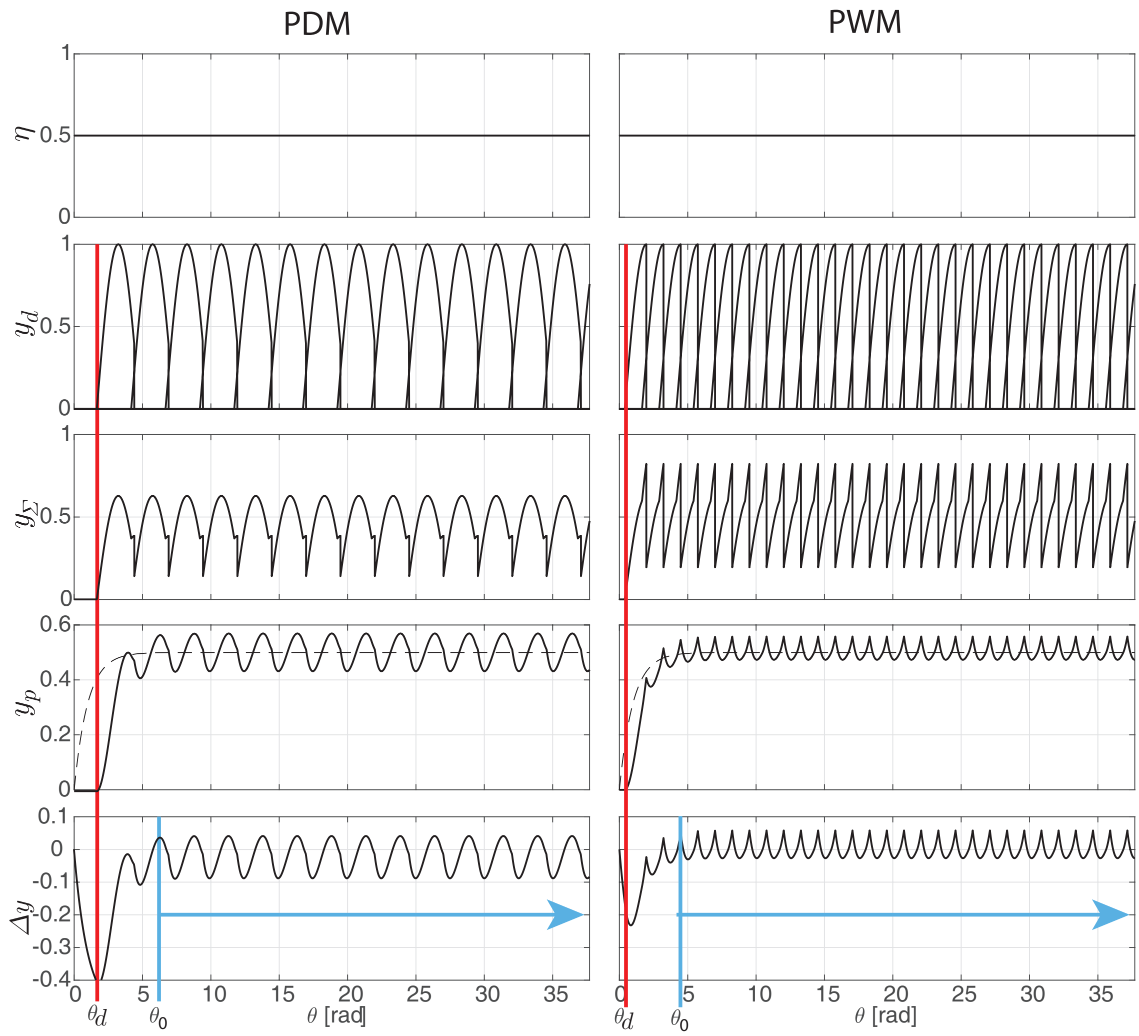
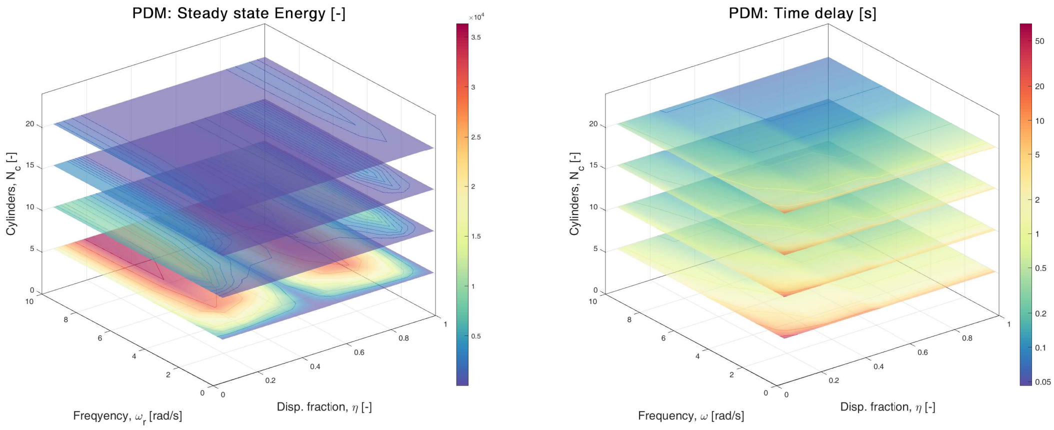


Figure 6: Simulation results for analysis of the digital effect. Red line indicates the DDM response delay and the blue line indicates the initial steady state angle [24].

and plant eigen-frequency, a relative eigen-frequency, ω_r, is introduced and given by

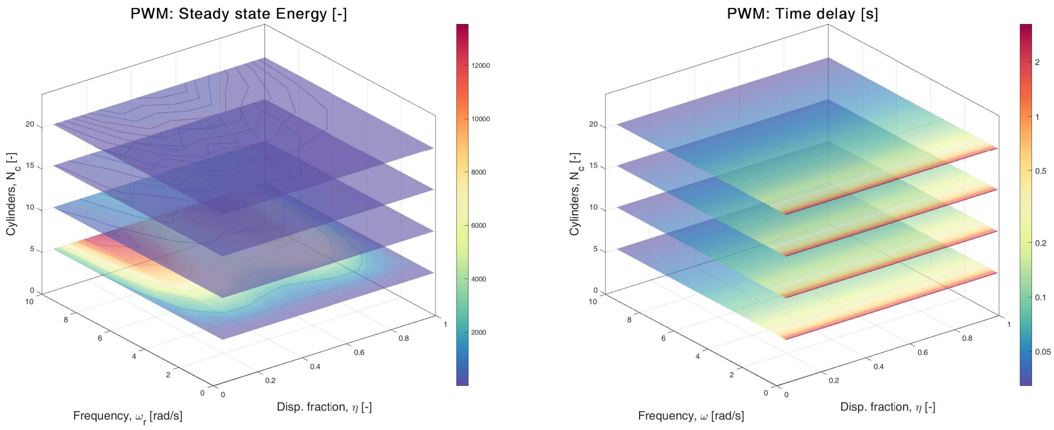
$$\omega_\mathrm{r} = \frac{\omega_\mathrm{n}}{\omega} \tag{11}$$

The relative frequency hence reduces the number of variables by one and exploits that the response in the angle domain is identical for a constant ω_r. Results for the time-delay and steady state energy is shown in Fig. 7 for a PDM operated DDM and in Fig. 8 for a PWM operated DDM. The response time is not shown, since it may be directly calculated from (9) and is only dependent on the machine speed for a PDM strategy.
It is seen that the energy in the steady state response is decreasing for a higher number of cylinders and a lower relative frequency. Additionally, the energy is approximately reflected around $\eta = 0.5$, since actuating a single cylinder yields the same oscillation amplitude as deactivating a single cylinder in full-stroke. However, the relative oscillation energy with respect to the input, E/η, is much larger for low displacements, since the energy in the input signal is significantly lower. For PDM operation, the time delay is seen to be decreasing for a higher number of cylinders and a higher machine speed. For PWM operation the time-delay is always the minimum delay since all cylinders are activated, while the minimum time-delay for PDM operation occurs when the current cylinder is



(a) Steady state energy. (b) Time delay.

Figure 7: Pulse Density Modulation.



(a) Steady state energy [24]. (b) Time delay.

Figure 8: Pulse Width Modulation [24].

activated.

It is clear that the dynamics of the DDM can only be neglected in the control design process if the number of cylinders is relatively high, the relative frequency, ω_r, is relatively low and the displacement fraction is relatively high. To avoid unmodeled phase when omitting the DDM dynamics it is required that $\omega > 10\,\omega_n$. However, providing specific numbers about when it is considered sufficient to neglect the DDM dynamics is not done in this paper, since it is difficult to give a conclusive answer. A further investigation of the introduced phase-lag as function of displacement for a PDM operated machine is made. The discrete $\Delta\Sigma$-modulator has the form given by

$$\bar{\eta}(k) = \begin{cases} 1 & \text{for} \quad \sum_{k=0}^{n} \eta(k) - \bar{\eta}(k) \geq \frac{1}{2} \\ 0 & \text{for} \quad \sum_{k=0}^{n} \eta(k) - \bar{\eta}(k) < \frac{1}{2} \end{cases} \tag{12}$$

The discrete integrated displacement fraction error, which is the input to the quantizer always has a value of $q(k) \in \left[-\frac{1}{2}, \frac{3}{2}\right[$. The maximum delay in the output occurs if the integrator is on its boundary values, where the number of samples needed before the next cylinder is activated is given by $n_d = \text{floor}(1/\eta)$. The time-delay in the output is then calculated to be given by

$$T_\mathrm{d} = \frac{\theta_\mathrm{d}}{\omega} \qquad \theta_\mathrm{d} = \frac{2\,\pi}{N_\mathrm{c}}\,n_\mathrm{d} \qquad \rightarrow \qquad T_\mathrm{d} = \frac{2\,\pi}{\omega\,N_\mathrm{c}}\,n_\mathrm{d} \tag{13}$$

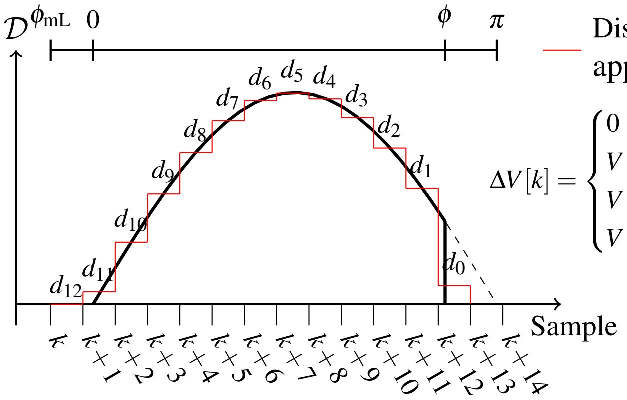
Since the activation of the pressure chambers with continuous dynamics is done at discrete angles, the system is essentially a hybrid dynamical system. To simplify the analysis a frequency response of the DDM, where the delay-time is included is conducted based on a discrete approximation. The discrete approximation of the machine dynamics is done by utilization of the method presented in [27].

3.1 Frequency response based on DLTI model

Considering a motoring stroke, the decision to motor or idle is taken at ϕ_mL, where the LPV is either closed to activate the chamber or held open to idle the chamber. The discrete displacement fraction between samples is then calculated by

$$\begin{aligned} \Delta\mathcal{D}[k] &= \frac{\Delta V[k]}{\theta_\mathrm{s}} = \frac{\left(V\left(\theta[k+1]\right) - V\left(\theta[k]\right)\right)\,N_\mathrm{c}}{2\,\pi} \\ \theta[k] &= \phi_\mathrm{mL} + \theta_\mathrm{s}\,(k-1) \qquad\qquad k \in \{1,\ldots,N_\mathrm{c}\} \end{aligned} \tag{14}$$

where $\theta_\mathrm{s} = 2\,\pi/N_\mathrm{c}$ is the constant sampling angle. The discrete approximation of the displacement throughput is done by use of (15) and results in the impulse response shown in Fig. 9 for a 25 cylinders machine.



$$\Delta V[k] = \begin{cases} 0 & \theta[k], \theta[k+1] \notin [0\,;\phi] \\ V(\theta[k+1]) - V(\theta[k]) & \theta[k], \theta[k+1] \in [0\,;\phi] \\ V(\theta[k+1]) - V(0) & \theta[k] < 0 < \theta[k+1] \\ V(\phi_\mathrm{mH}) - V(\theta[k]) & \theta[k] < \phi < \theta[k+1] \end{cases} \tag{15}$$

Figure 9: Discrete approximation of displacement throughput of the digital displacement machine.

It is seen that $p = 12$ samples is needed to describe the full stroke. The shown displacement fractions are evaluated accordingly to $d_\mathrm{k} = \Delta\mathcal{D}[p-k]$ and results in the discrete transfer function given by

$$G(z) = \frac{\mathcal{D}(z)}{u(z)} = \frac{d_\mathrm{p}\,z^p + d_\text{p-1}\,z^{p-1} + \cdots + d_1\,z + d_0}{z^p} \tag{16}$$

A frequency response plot is made of the discrete transfer function, where the machine operates at three different rotational speeds $\omega \in \{1,\ 10,\ 100\}$ rad/s. The frequency response is shown in Fig. 10, where the time-delay as function of displacement fraction

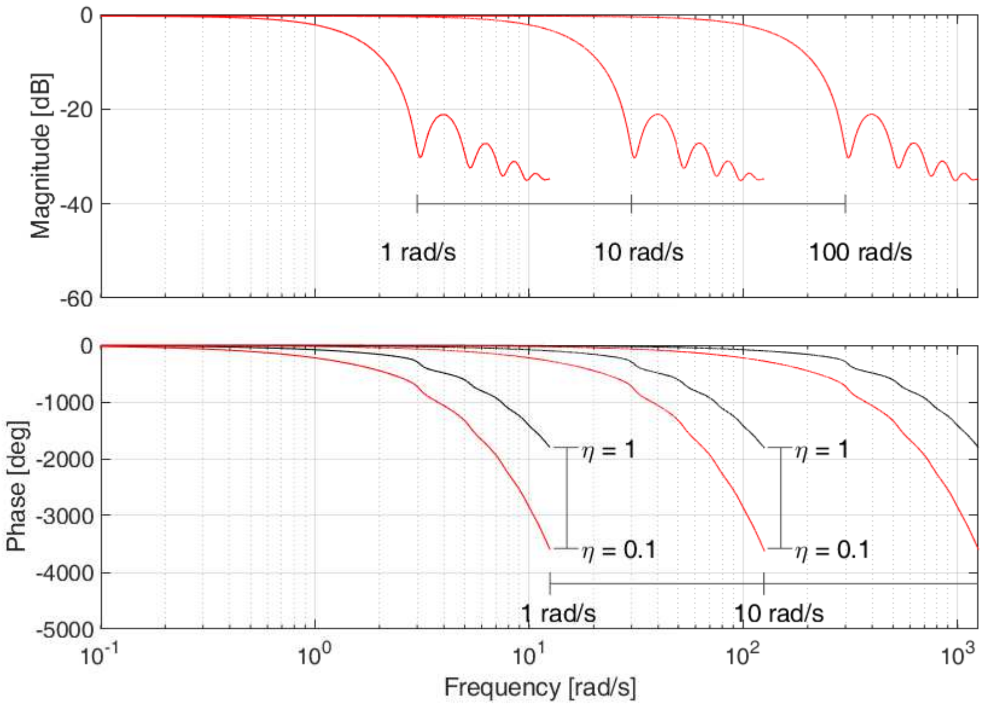


Figure 10: Frequency response of the DDM for varying rotational speed and displacement fractions in full-stroke operation.

input is shown. It is seen that the bandwidth of the DDM is directly proportional to the rotational speed and that the time-delay due to full stroke operation may cause stability problems for the closed-loop system if it is neglected during control design. As the displacement fraction goes towards zero, the time-delay goes towards infinity. However, if the desired displacement fraction is very low, it follows that the system is close to being at an equilibrium point and thus the very large time-delay might not be of major concern. Therefore, the included time-delay in the control model should not be a worst-case scenario, but should neither be completely omitted with respect to stability analysis.

By varying the closing angle by the displacement fraction, $\phi(\eta)$, the frequency response of a partial stroke operated DDM is generated and is shown in Fig. 11. It should be noticed that the frequency response has been generated in the angle domain, so the frequency should be interpreted as radians per radians of shaft angle. It is seen that the frequency response varies significantly with displacement fraction input. Since the number of samples that the displacement is committed over varies with the displacement fraction, the model order varies for partial stroke operation. With respect to control development a varying model structure depending on the input may be considered. Alternatively, it is seen that the highest phase-lag is introduced for a full stroke, such that a conservative approximation would be to linearize for $\eta = \eta_{\mathrm{m}}$ [28].

4 OPTIMAL OPERATION AND CONTROL STRATEGY

Since the displacement of the DDM is committed as function of the shaft angle, the rotational speed has a large influence on the performance of the different operation strategies. The following section shows that the suitable choice of operation strategy and control

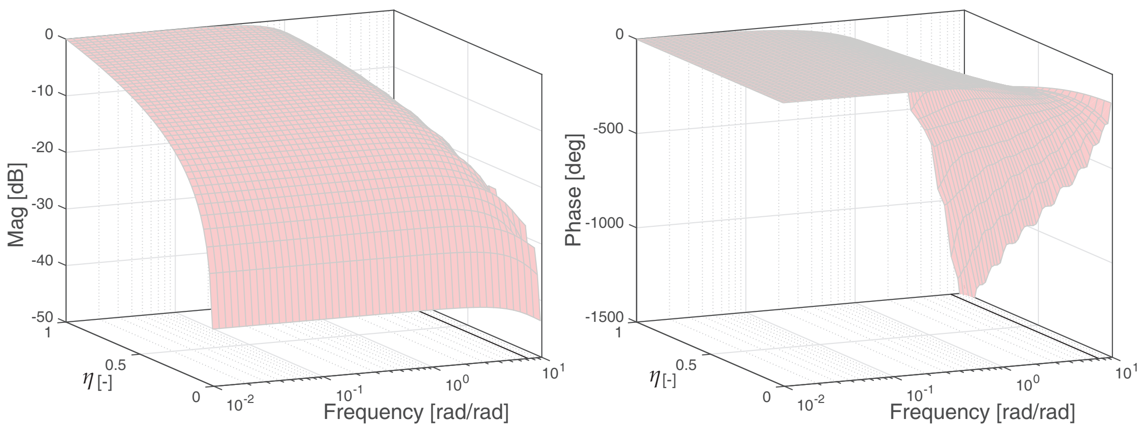


Figure 11: Frequency response of the DDM for varying displacement fractions in partial-stroke operation [28].

strategy varies with the speed of the machine. The control performance of the three operation strategies (full, partial and sequential partial) stroke is compared through simulation of the simplified model with sinusoidal displacement outputs. To provide a more clear view of the results, the number of cylinders has been reduced to 10. In this simulation study only reference tracking performance of a displacement fraction input is considered. For the full-stroke mode, the delta-sigma PDM strategy is used and for partial stroke the PWM strategy is used. To simplify the problem of determine the actuation sequence for the sequential partial-stroke operation, the full stroke is divided into 5 sections such that a cylinder activation/deactivation is made simultaneously for all cylinders. This is a rough simplification, since the cylinders in reality could be switched at any angle for an improved tracking performance. An illustration of the division of the full stroke into sequences, where the cylinders may be activated/deactivated, is shown in Fig. 12. The resulting displacement fraction output as function of combination of active/inactive pressure chambers is shown in Fig. 13.

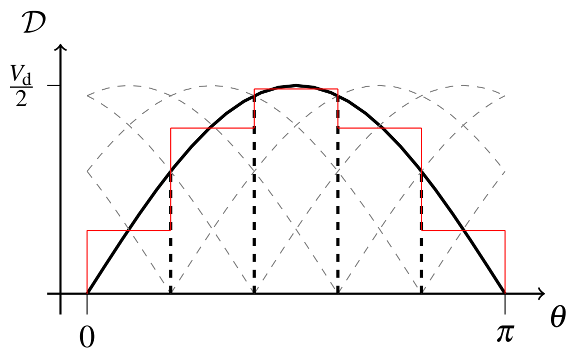


Figure 12: Operation strategies for the digital displacement machine.

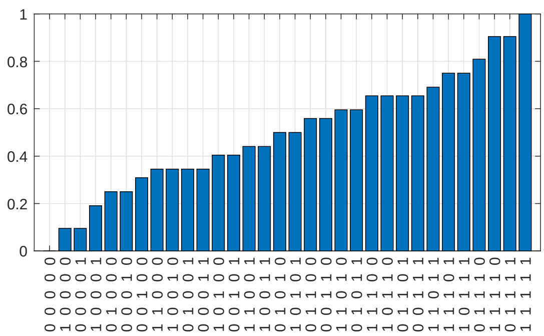


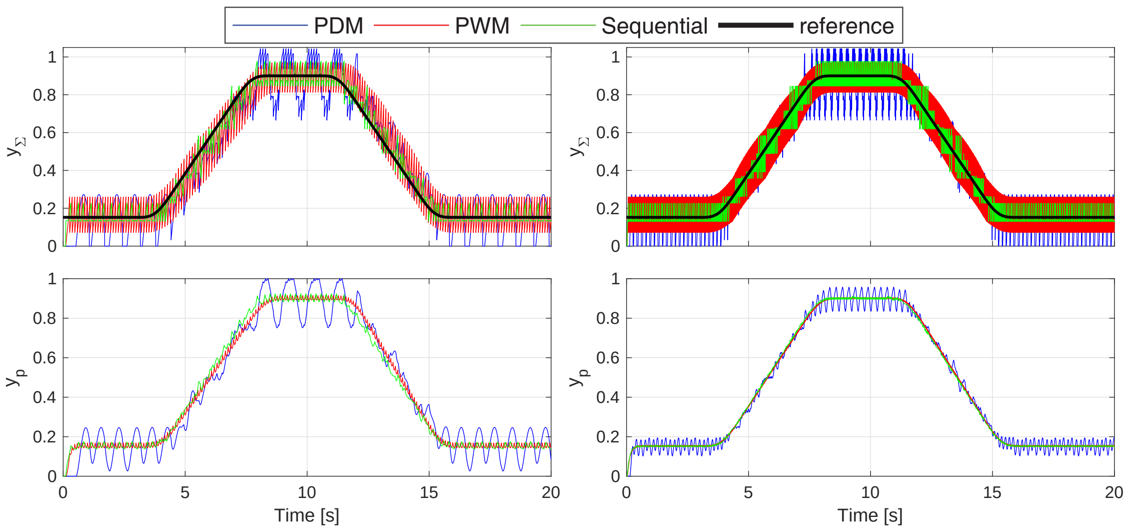
Figure 13: Displacement fractions outputs as function of the combinations

The red line on the left figure shows the discrete displacement between samples, which combinations are used to generate the displacement fraction combinations shown in the right figure. A displacement fraction of 1 is thus obtained by activating all 5 cylinders in the motoring stroke part. The combination that yields the lowest displacement error is thus used at the current activation decision. An integrator is furthermore introduced to account for the displacement error over time, similar to the discrete integrator in the PDM strategy. No outer control loop is considered, since the objective is solely to compare the different operation strategies.

Since sequential partial stroke requires larger valves capable of generating a higher force to open against high pressure differences, the switching time is slower than for the fast valves which may be used for full and partial stroke operation. Switching time of valves for the full and partial stroke operation has been reported to be below $T_s <$ 5 ms [29, 30], while the switching time of commercially available valves applicable for sequential partial stroke is in the range of $T_s \in [20-25]$ ms [31]. If it is considered that the switching time for opening and closing must at maximum occupy 10 % of an activation and it should be possible to at least divide the full stroke into two sections, the maximum rotational speed of the DDM becomes

$$\omega \leq \frac{2\pi}{2 \cdot T_s \cdot 10} \approx 21\,\text{rad/s} \tag{17}$$

It is evident that the switching time of the valves sets a direct requirement to the maximum possible speed when utilizing sequential partial stroke. Since this simulation study considers infinite fast switchings, the problem with respect to maximum speed of the DDM in partial stroke has not been taken into account. The simulated displacement outputs of the three different operation and corresponding control strategies are shown in Fig. 14. The plant responses of the first order system actuated by the DDM are shown in Fig. 5.



(a) $\omega = 5\,\text{rad/s}$ (b) $\omega = 20\,\text{rad/s}$

Figure 14: Response of digital displacement machine and first order plant for a PDM, PWM and sequential partial stroke strategy, $\omega_n = 10\,\text{rad/s}$.

It is seen that the full-stroke PDM strategy performs poorly at low speed, which is due the control update rate being proportional to the machine speed and the many time-delays introduced due to only fully activating a fraction of cylinders. In general the sequential partial stroke yields the best tracking performance, but one has to bear in mind that instantaneous switching is considered. Improved performance may likely be obtained by allowing valve switching at any angle for partial and sequential partial stroke operation. For the rotational speed of 20 rad/s the machine dynamics is 10 times higher than the plant dynamics $\omega_n = 10\,\text{rad/s}$, in which case the dynamics of the machine may be neglected when considering a frequency response and neglecting the time-delays due to the binary input. However, it is seen that the plant response has noticeable fluctuations and simply neglecting the DDM dynamics may cause poorer tracking performance and might even stability problems.

One newly proposed and mention-worth control strategy is the co-called "creep mode" operation [31], which is significantly different compared to the conventional operation strategies. The strategy relies on closing all valves (both the HPV and LVP), where after the system finds a force equilibrium due to the friction and load force. Then movement in the clockwise direction may be obtained by opening a single HPV that pressurizes a chamber that yields positive torque or by opening a single LPV that de-pressurizes a chamber that yields negative torque. Shaft movement thus occurs until a new equilibrium is reached, where after the valve is closed to lock the shaft in the new position. The strategy may afterwards be continued to obtain small shaft movements in the desired direction. The strategy yields the lowest possible step movement in the shaft position and is thus very good for precise shaft position control. However, due to the many valve actuations at high pressure difference, the strategy is not very energy efficient.

Based on the presented investigation and results regarding the different operation strategies, a summarization is made and is used to suggest suitable control strategies.

- **Full stroke:** Is the favorable choice for high-speed machine operation, where energy efficiency is important. Furthermore, it is considered the simplest strategy since valve switchings are done at fixed angles. Due to the high speed, the control update rate is fast and a static feedback control law with a pulse-density modulator is desired. In case of a relative slow dynamical plant, relatively high number of cylinders and operation at relatively high displacement the DDM dynamics may be omitted in the control design. Alternatively, continuous, discrete or hybrid dynamical approximation may be used to describe the DDM dynamics [32, 27, 33, 26, 34, 35].

- **Partial stroke:** Is the favorable choice for low/medium speed machine operation, where both tracking performance and energy efficiency are important. Since multiple objectives are important and the time between samples are significantly higher due to the lower machine speed, a model-predictive control strategy is considered favorable. Both discrete linear time invariant (DLTI) and time variant (DLTV), as well as hybrid mixed logical dynamical programming model predictive strategies has been suggested [28, 36]. Additional, objectives like frequency content of output is also possible to optimize with respect to if desired.

- **Sequential partial stroke:** Is the favorable choice for very low speed operation, where control tracking performance is important and energy efficiency is of less importance. In sequential partial stroke, there are an infinite number of switching combinations that yields similar output, but the energy losses may be very different. Since valve opening against high pressure is conducted, the energy losses may be very high if not considered. Therefore, a model predictive control strategy is also deemed the suitable strategy for this operation. However, only a simple strategy where the stroke is divided into fixed sections has been published, leaving potential for further improvements [37].

- **Creep mode:** Is the favorable choice for very accurate DDM shaft position control in the vicinity of standstill. So far only a single paper has been published on the subject [31], which activates cylinders sequentially or in parallel. For smoother operation, further research could include overlapping the actuation of cylinders and model predictive strategies taking into account the many activation combinations.

Since the suitable operation strategy and control strategy changes as function of the rotational frequency, switching of strategies occur if the machine is operated in a broad speed range. Therefore, it is proposed to investigate the use of supervisory control, which is also a class of hybrid dynamical systems due to the switching event behavior. A proposal on a supervisor for control of a DDM is illustrated in Fig. 15.

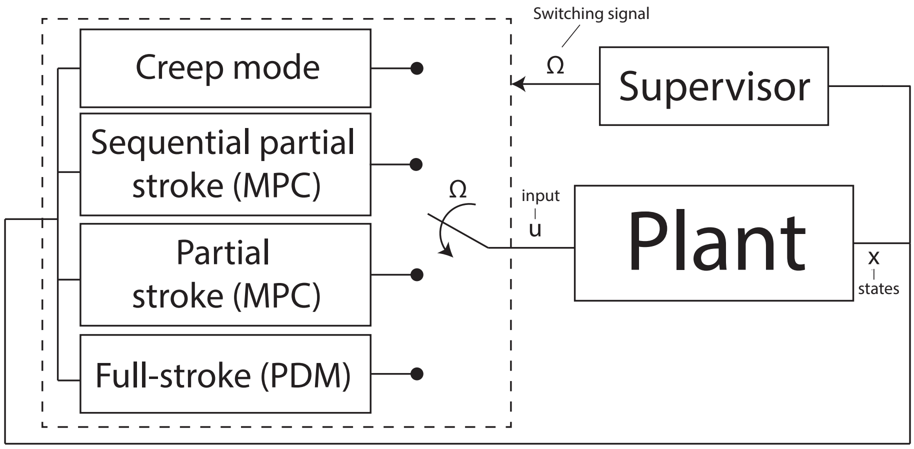


Figure 15: Supervisory control strategy for control of digital displacement machine at various rotation frequencies.

It is seen that the operation strategy and control law is switched by the switching signal, Ω, which in this case is the measured shaft speed. However, the influence of the switching event on stability for such complex problem is expected to be very complicated. However, supervisory control theory does exist and is still undergoing a rapid development [38]. It is evident, that many control related challenges remain for the DDM to be operated and controlled optimally, where this paper only discusses and proposes in which direction to pursue with respect to control development.

5 CONCLUSION

The paper shows that the dynamics of digital displacement machine (DDM) should not always be neglected to ensure proper control performance and stability. Since the response time of the DDM in full and partial stroke operation is proportional to the rotational shaft speed, the non-smooth dynamical effects are substantial for low speed operation. Especially, if the actuated plant dynamics is relatively fast compared to the DDM dynamics and that the number of cylinders and displacement fraction are relatively low. The four operation strategies; full stroke, partial stroke, sequential partial stroke and creep mode are compared. The energy efficiency is highest in the given order, while the energy efficiency is highest in the reverse given order. To avoid poor control performance at low speed, it is deemed favorable to alter the operation strategy in the reverse given order with respect to the rotational shaft speed. In partial and sequential partial stroke operation there is an infite number of combination that yields the same displacement output over a revolution, but the energy efficiency and induced pressure pulsations may be very different as function of the valve switching angle. Therefore, it is deemed favorable to apply a model-predictive control strategy for these operation modes, where the low control update rate ensures online solving capability. At higher speed in full-stroke operation, a static pulse-density-modulation based control law is assessed to be favorable. If the machine is to operate at both low and high speed in varying operation modes, an illustrative supervisory control strategy is proposed to ensure proper operation and stability in the vicinity of the switching event.

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