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Joint Statistical Modeling of Received Power, Mean Delay, and Delay Spread for Indoor Wideband Radio Channels

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***Abstract*—We propose a joint statistical model for the received power, mean delay, and rms delay spread, which are derived from the temporal moments of the radio channel responses. Indoor wideband measurements from two different data sets show that the temporal moments are strongly correlated random variables with skewed marginals. Based on the observations, we propose a multivariate log-normal model for the temporal moments, and validate it using the experimental data sets. The proposed model is found to be flexible, as it fits different data sets well. The model can be used to jointly simulate the received power, mean delay, and rms delay spread. We conclude that independent fitting and simulation of these statistical properties is insufficient in capturing the dependencies we observe in the data.**

***Index Terms*—temporal moments, mean delay, rms delay spread, multivariate log-normal, wideband radio channels, indoor propagation.**

I. Introduction

Characterization of radio channel properties such as the received power, mean delay, and rms delay spread are imperative for the design of communication systems. These statistics are computed from the moments of the instantaneous power of the received signal, known as *temporal moments*. Used since the 1970s [1], the temporal moments are ubiquitous in wireless communications literature, and are also used in simulations of communication systems. More recently, temporal moments have been used as summary statistics for parameter estimators for stochastic channel models [2]–[4]. In applications such as these, where more than one of the temporal moments are used simultaneously, knowledge of their statistical properties, including their dependencies, is beneficial.

Empirical averages and cdfs of received power, mean delay, and rms delay spread are reported frequently in the literature. In [5], Awad et al. survey the empirical data on the delay properties of the indoor radio channel, including the mean delay and rms delay spread, and fit marginal models to a large number of available data sets. They obtained normal, Weibull, or log-normal distributions as the best fit models, with Rayleigh, Rician, and Poisson being the other considered distributions. Although clearly handling a model selection problem, the selection was done by evaluating the best fit without adjusting for model complexity. Similar results were obtained in [6] where the rms delay spread was empirically modeled as a normally distributed random variable. The expected values of temporal moments are connected to parameters describing the model for the impulse responses for in-room scenarios [7]. This observation was further deepened in [8], [9] which show how the arrival rate of a process changes the variance of the mean delay and rms delay spread. It is wide-spread practice to report only empirical marginal distributions of rms delay spread and to disregard the dependencies between moments. Moreover, independent modeling of rms delay spread is prevalent in the literature, while its dependency with received power and mean delay is not.

An exception is Greenstein et al. [10] who modeled the path gain[1] and delay spread jointly. They proposed a joint log-normal distribution for the path gain and rms delay spread based on intuitive arguments, and later validated it empirically using a wide range of outdoor measurements available from literature. However, Greenstein et al. did not consider mean delay. Moreover, they proposed a fixed correlation coefficient of –0.75 between path gain and rms delay spread, which might not be able to account for the variability observed in measurements.

In this contribution, we extend the Greenstein model to jointly characterize the mean delay, along with received power and rms delay spread. We propose a multivariate log-normal model for the temporal moments, from which we can obtain mean delay and rms delay spread using a simple transformation. We find that this easy-to-use model is flexible enough to capture the variability observed in data. We also provide a method to estimate parameters of the model so that it can be easily fitted to new measurements, something that the Greenstein model lacked. Finally, the model is validated using indoor channel impulse response measurements from two different campaigns.

II. Temporal Moments

Consider the case where measurements of the channel transfer function are recorded using a vector network analyzer (VNA) in a single-input, single-output (SISO) set-up. The transfer function is sampled in the measurement bandwidth B at N_s frequency points with separation $\Delta f = B/(N_s - 1)$.

[1]Greenstien et al. defined path gain as the ratio of received power to transmitted power.

The model for the measured transfer function, Y_n, at frequency sample n reads

$$Y_n = H_n + W_n, \quad n = 0, 1, \ldots, (N_s - 1), \tag{1}$$

where H_n is the transfer function and W_n is measurement noise modeled as independent and identically distributed (iid) circularly symmetric Gaussian random variables. The time domain signal, $y(t)$, is obtained by the discrete-frequency, continuous-time inverse Fourier transform as

$$y(t) = \frac{1}{N_s} \sum_{n=0}^{N_s-1} Y_n \exp\left(j2\pi n \Delta f t\right), \tag{2}$$

where $y(t)$ has period $1/\Delta f$.

A particular realization of $y(t)$ can be summarized in terms of its temporal moments defined as

$$m_k = \int_0^{\frac{1}{\Delta f}} t^k |y(t)|^2 \mathrm{d}t, \quad k = 0, 1, \ldots, (K - 1). \tag{3}$$

In total K temporal moments are computed "instantaneously" per realization of the received signal, i.e. without "averaging" over multiple realizations. Thus, having N_{real} realizations of the channel results in the $N_{\text{real}} \times K$ dimensional matrix, $\mathbf{M} = \left[\mathbf{m}^{(1)}, \ldots, \mathbf{m}^{(N_{\text{real}})}\right]^T$, where $\mathbf{m}^{(i)} = \left[m_0^{(i)}, m_1^{(i)}, \ldots, m_{K-1}^{(i)}\right]$. The SI unit for the k^{th} temporal moment is s^k.

The instantaneous received power, P_0, equals m_0, while the instantaneous mean delay, $\bar{\tau}$, or the instantaneous rms delay spread, τ_{rms}, are obtained as transformations of the temporal moments as

$$\bar{\tau} = \frac{m_1}{m_0}, \quad \text{and} \quad \tau_{\text{rms}} = \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2}. \tag{4}$$

Note that the unit of $\bar{\tau}$ and τ_{rms} is in seconds. For the purpose of our discussion, we will focus on the first three temporal moments, i.e. (m_0, m_1, m_2), as they suffice for the received power, mean delay, and rms delay spread. We refer to (m_0, m_1, m_2) as *temporal moments* and $(P_0, \bar{\tau}, \tau_{\text{rms}})$ as *standardized moments* of $|y(t)|^2$. Note that in (4), we make no attempt to compensate or remove the effect of a finite measurement bandwidth, i.e. the standardized moments are computed of the received signal, $y(t)$, and not of the channel impulse response.

III. Measurement Data and Observations

A. Dataset from Lund University

In [11], mm-wave measurements of the channel transfer function are recorded at 60 GHz using a VNA in a SISO set-up. The measurement is conducted in a small room of dimensions $3 \times 4 \times 3$ m^3 using a 25×25 virtual planar array, giving $N_{\text{real}} = 625$ realizations of the channel. Frequency bandwidth used is 4 GHz, with $N_s = 801$ frequency sample points. This gives a signal observation time of $1/\Delta f = 200$ ns in the time domain. Temporal moments are computed for this dataset for the non-line-of-sight (NLOS) case, and the density estimates and scatter plots are shown in Fig. 1a.

TABLE I
Sample Pearson correlation coefficients between standardized and temporal moments of data.

	$\hat{\rho}_{P_0,\bar{\tau}}$	$\hat{\rho}_{P_0,\tau_{\text{rms}}}$	$\hat{\rho}_{\bar{\tau},\tau_{\text{rms}}}$
Lund	-0.28	-0.35	0.53
Lille	-0.55	-0.21	0.85
	$\hat{\rho}_{m_0,m_1}$	$\hat{\rho}_{m_0,m_2}$	$\hat{\rho}_{m_1,m_2}$
Lund	0.94	0.34	0.54
Lille	0.93	0.39	0.69

B. Dataset from Lille

The 60 GHz channel sounder developed in [12] measures the frequency transfer function using a VNA in bandwidth $B = 2$ GHz at $N_s = 1601$ sample points by steps of $\Delta f = 1.25$ MHz. This results in a signal observation time of $1/\Delta f = 800$ ns. Measurements were taken in a computer laboratory of floor area 7.15×5.2 m^2 at 26 sites, covering the whole room. At each site, 250 measurements were carried out. We use a subset of this data, specifically, line-of-sight (LOS) measurements with $N_{\text{real}} = 500$ realizations obtained from the first two sites having the same distance between transmitter and receiver. Density estimates and scatter plots of temporal moments for this dataset is shown in Fig. 1b.

C. Observations

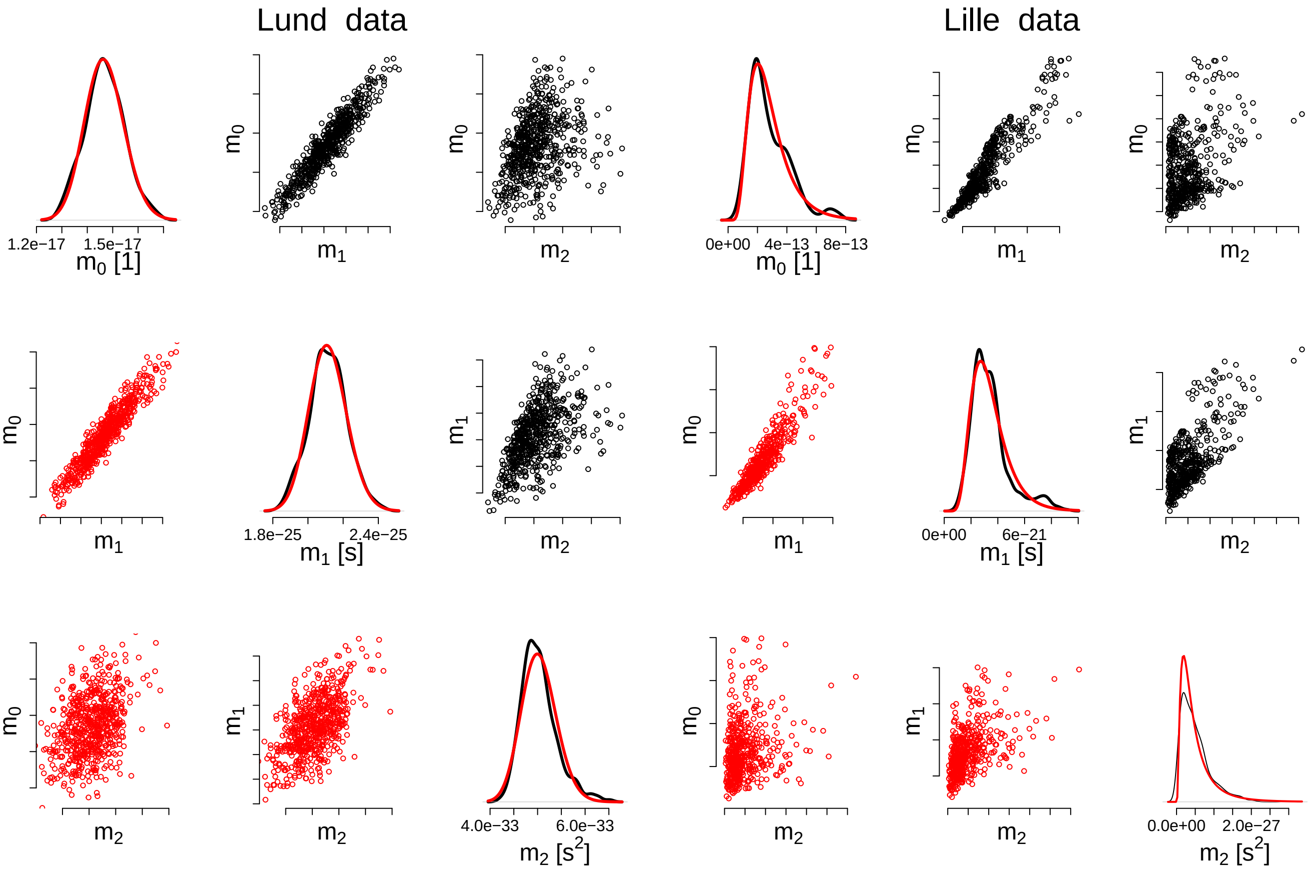
The marginal distributions of the temporal moments appear to be skewed, more so for the Lille data. The scatter plots fan out towards the top-right of each plot, giving rise to the skewed marginals. It is evident from the scatter plots that the temporal moments are correlated random variables, suggesting that the standardized moments could be correlated as well. This conjecture is indeed found to be true from the correlation coefficients between the temporal and standardized moments for the two data sets reported in Table I. The Pearson correlation coefficient between random variables A and B is

$$\rho_{A,B} = \frac{\operatorname{cov}(A, B)}{\sigma_A \sigma_B}, \tag{5}$$

where $\operatorname{cov}(\cdot, \cdot)$ is the covariance operator and σ is the standard deviation. The sample Pearson correlation coefficients for both temporal and standardized moments are reported in Table I. We observe that the correlation between received power and rms delay spread is less than the value proposed by Greenstein et al. [10]. Moreover, the correlation of standardized moments across the two data sets vary significantly, while the correlation of temporal moments seems more stable.

IV. Proposed model

The temporal moments are non-negative, correlated random variables with skewed marginals. Therefore, we propose to use a multivariate log-normal distribution to model the temporal moments. In principle, one could use a multivariate Gaussian distribution or copulas [13] to model the dependency structure. However, given the support for log-normality of standardized



(a) (b)

Fig. 1. Density estimates and scatter plots of temporal moments obtained from (a) Lund data and (b) Lille data (shown in black) vs. those simulated from the fitted proposed model (shown in red). The scales of the corresponding scatter plots are the same. Number of points simulated is same as in the measurements, i.e. $N_{\text{real}} = 625$ for Lund data and $N_{\text{real}} = 500$ for Lille data. Correlation coefficients of temporal moments are reported in Table I and the parameter estimates are in Table II.

moments in the literature, coupled with the aim to have a general yet simple-to-use model, we propose a multivariate log-normal distribution to model the vector, $\mathbf{m} = (m_0, m_1, m_2)$, of first three temporal moments, i.e. $K = 3$. The K-variate log-normal distribution, which is the exponential transform of a multivariate Gaussian, has the pdf

$$f(\mathbf{m};\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{\prod_{k=0}^{K-1}(m_k)^{-1}}{\sqrt{(2\pi)^K \det \boldsymbol{\Sigma}}} \times \exp\left(-\frac{1}{2}(\ln(\mathbf{m})-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\ln(\mathbf{m})-\boldsymbol{\mu})\right), \quad (6)$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the mean vector and covariance matrix of the associated multivariate Gaussian pdf. The entries of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are defined as $\mu_k = \mathbb{E}[\ln m_k]$ and $\Sigma_{kk'} = \operatorname{cov}(\ln m_k, \ln m_{k'})$, for $k, k' = 0, 1, 2$, respectively. In contrast, the means and covariances of m_0, m_1, and m_2 are functions of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ as

$$\mathbb{E}[m_k] = \exp\left(\mu_k + \frac{1}{2}\Sigma_{kk}\right), \quad \text{and} \quad (7)$$

$$\operatorname{cov}(m_k, m_{k'}) = e^{\left(\mu_k+\mu_{k'}+\frac{1}{2}(\Sigma_{kk}+\Sigma_{k'k'})\right)}\left(e^{\Sigma_{kk'}}-1\right). \quad (8)$$

The multivariate log-normal is a positive distribution which models the skewed marginals better than a Gaussian.

In principle, the received power, mean delay and rms delay spread could be the quantities modeled using the multivariate log-normal distribution. In practice, we do not observe any qualitative difference between one or the other. However, here we chose to model the temporal moments as their means and covariances are easier to compute analytically, given a channel model, as compared to the standardized moments due to the non-linear transformation. Using the model of the temporal moments, the standardized moments can be simulated via the one-to-one transformation given in (4).

A. *Estimation of parameters*

Fitting the matrix of temporal moments $\mathbf{M}$, obtained from N_{real} independent realizations of the channel impulse responses, to the proposed model requires the estimation of the mean vector, $\boldsymbol{\mu}$, and the covariance matrix, $\boldsymbol{\Sigma}$. This can be achieved by maximizing the likelihood of the data as

$$(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}) = \underset{\boldsymbol{\mu},\boldsymbol{\Sigma}}{\operatorname{argmax}} \prod_{i=1}^{N_{\text{real}}} f\left(\mathbf{m}^{(i)};\boldsymbol{\mu},\boldsymbol{\Sigma}\right). \quad (9)$$

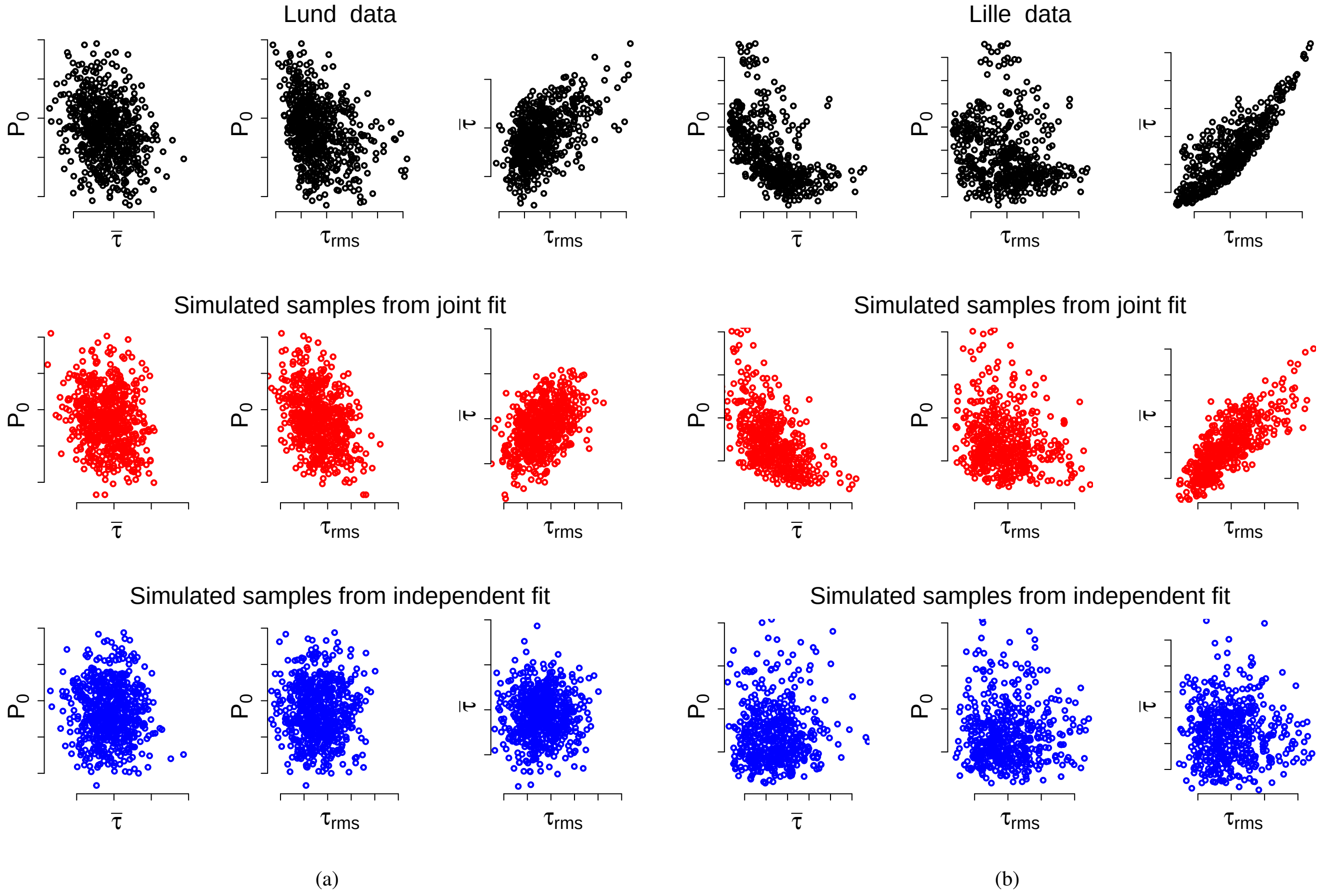


Fig. 2. Scatter plots of received power, mean delay, and rms delay spread obtained from (a) Lund data and (b) Lille data (in black) vs. those simulated from the proposed model (in red). The samples simulated by independently fitting P_0, $\bar{\tau}$, and $\tau_{\rm rms}$ to the data are shown in blue. The scales of the corresponding scatter plots are the same. Number of points simulated is same as in the measurements, i.e. $N_{\rm real} = 625$ for Lund data and $N_{\rm real} = 500$ for Lille data. Correlation coefficients of standardized moments are reported in Table I and the parameter estimates are in Table II.

Since $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the parameters of the associated Gaussian, the maximum likelihood estimates $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$ are

$$\hat{\boldsymbol{\mu}} = \frac{1}{N_{\rm real}} \sum_{i=1}^{N_{\rm real}} \ln \mathbf{m}^{(i)}, \quad \text{and} \tag{10}$$

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{N_{\rm real}} \sum_{i=1}^{N_{\rm real}} \left(\ln \mathbf{m}^{(i)} - \hat{\boldsymbol{\mu}}\right)\left(\ln \mathbf{m}^{(i)} - \hat{\boldsymbol{\mu}}\right)^T . \tag{11}$$

The estimates obtained after fitting the model to the two data sets are reported in Table II.

B. Simulation from the model

Simulation from the proposed model is straightforward. To generate one realization, three steps should be performed:

1) Draw $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
2) $\mathbf{m} = \exp(\mathbf{x})$ (entry-wise exponential)
3) Compute $\bar{\tau}$ and $\tau_{\rm rms}$ from (4) (optional)

V. Model Validation

We check the validity of the model by fitting it to the two experimental datasets available, and then qualitatively investigating the simulated data against the measurements.

TABLE II
Parameter estimates obtained after fitting.

	Lund data			Lille data		
$\hat{\mu}^T$	-38.8	-56.8	-74.4	-29.0	-47.2	-63.2
$\hat{\Sigma}$	2.8×10^{-3}	2.5×10^{-3}	1.4×10^{-3}	0.22	0.17	0.12
	2.5×10^{-3}	2.6×10^{-3}	2.1×10^{-3}	0.17	0.15	0.19
	1.4×10^{-3}	2.1×10^{-3}	5.3×10^{-3}	0.12	0.19	0.70

A. Simulation of temporal moments

We estimate the parameters of the proposed model for the two datasets, and then simulate temporal moments using the methodology described in the previous section. The results are shown in Fig. 1 for both Lund and Lille data, and the parameter estimates are in Table II. The model appears to fit the marginals well, even for the very skewed case of Lille data. The high correlation between the temporal moments, especially between m_0 and m_1, is well captured by the model. For both Lund and Lille data, the fanning out of the scatter plots is well represented by the model.

B. Simulation of P_0, $\bar{\tau}$, and $\tau_{\rm rms}$

We now compare the scatter plots of received power, mean delay, and rms delay spread obtained from the proposed model with those from the measurements in Fig. 2. Additionally, we also show the samples obtained by independently fitting log-normal pdfs to the standardized moments from the measurements. We observe a strong positive correlation between the mean delay and rms delay spread obtained from measurements. The received power, however, is negatively correlated with mean delay and rms delay spread. This dependency structure between the standardized moments is captured well by the proposed joint model. In contrast, any information on the correlation between the variables is lost when simulating them independently.

VI. Conclusions

Observing that temporal moments of channel impulse responses, and hence their standardized moments, are dependent random variables, we propose to model them as jointly log-normal random variables. The proposed model is simple, easy to use, and analyze. We find that the model is flexible enough to fit measurements exhibiting contrasting behaviors. This model can be used to jointly simulate received power, mean delay, and rms delay spread. We validated the joint model using experimental data obtained from indoor environments.

Independent fitting and simulation of received power, mean delay, and rms delay spread leads to loss of correlation observed in the measurements, and these should be simulated jointly. Therefore, reporting only their marginal distributions, e.g. in the form of plots of their empirical cdfs, is insufficient. Instead, their means and covariances should be reported.

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