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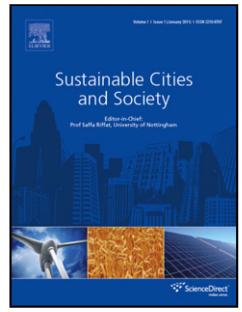


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# Optimization of Household Energy Consumption towards Day-ahead Retail Electricity Price in Home Energy Management Systems

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Highlights

(1) Investigation of interaction between market trading strategies, including wholesale and retail markets, and the behavior of household appliances.

(2) Discussion about the role of schedulable household appliances in secure operation of the distribution network in near real-time conditions.

(3) Determination of retail strategies in the electricity market based on household behavior considering electricity price uncertainties.

**Abstract:** In this paper, a novel approach is proposed to optimize the behavior of household appliances towards retail electricity price. At the supply-side, a distributed generation-owning retailer participates in the wholesale electricity market, i.e. day-ahead and intraday trading floors. Considering the uncertainties associated with electricity price and wind/solar power, the retailer determines the retail price using stochastic programming. At the demand-side, smart household prosumers take the advantage of two kinds of storage capacities: (1) thermal storage capacity of thermostatic devices and (2) electrical storage capacity of batteries integrated with roof-top photovoltaic panels. The Home Energy Management System (HEMS) determines the operational strategies of appliances, including thermostatically controlled, uninterruptible and curtailable appliances, in response to the retail price. The HEMS uses a heuristic Forward-Backward Algorithm (F-BA) to minimize the energy cost of the thermal appliances satisfying the residents' comfort. To prevent from creating peak demands in the power system, a Peak Flattening Scheme (PFS) is suggested. Investigating the interaction between household consumption and network security, the household demands are located at different buses of a distribution network relieving congestion in weak lines. Finally, the proposed approach is implemented as a case for Danish sector of Nordic Electricity Market.

**Keywords:** Retailer, HEMS, heuristic approach, electricity price, household appliances

## 1. Introduction

### 1.1. Motivation and Problem Description

In restructured electricity markets, retailers are profit-based entities which purchase electricity from the wholesale market with volatile prices and sell it to the consumers with a specified tariff [1]. Electricity price and demand level are the most important uncertainties for retailers. At the demand-side, the consumption pattern of household consumers depends on the behavior of appliances which have different characteristics. Therefore, an optimization of the appliances' behavior can modify the consumption pattern of households considerably. Improving the consumption pattern of households, residential retailers can determine market strategies in an optimized manner to make more profit. On the other side, consumers can experience a reasonable reduction in electricity bills.

Adjusting the consumption of households, the security of the distribution network may be jeopardized if the operational constraints of the network are neglected. In addition, a peak demand may be created due to the shift of consumption to the low-cost hours. In such a situation, congestion occurrence in some weak lines of distribution network may cause serious problems to deliver the electrical energy. In this way, the financial loss is imposed on the Distribution System Operator (DSO). In order to overcome the problem, smart Home Energy Management System (HEMS) can optimize the behavior of household appliances to increase the profit of retailers and consumers and to reduce the system stress.

As a result, adjusting the consumption pattern of households without considering the network constraints not only does not optimize the operation strategies but also may put the security of the distribution network at risk. To sum up, optimization of household consumption needs to be done with considering the effect of the electricity market on one side and the impacts on the distribution network on the other side. These kinds of studies need general knowledge about electricity market, thermal-electrical characteristics of household appliances, and power distribution network. The aforementioned aspects have strong correlations with the household comfort that make them hard to strike the right balance between the concepts without disturbing the consumer convenience. For this reason, a comprehensive study is needed to address all the issues satisfying the household comfort.

### 1.2. Literature Review

Smart HEMS is defined as the optimal system providing energy management services in order to efficiently manage and monitor electricity consumption, generation, and storage in the smart houses [2]. The overall architecture of programming for a smart HEMS includes (1) household appliances (2) objective function (3) demand response programs and (4) optimization approach.

Regarding household appliances, most of the studies discuss the HEMS for the devices with thermal storage capability, including Heating, Ventilation, and Air Condition (HVAC) [3], Electric Water Heater (EWH) [4] and refrigerator [5]. Electrical batteries [6] and Plug-in Hybrid Electric Vehicles (PHEV) [7] are flexible schedulable devices which attract many attentions in recent years. Wet appliances, including clothes dryer, washing machine, and dishwashers, are time-shiftable loads whose consumption can be scheduled in low-cost hours [8]. Due to the increased penetration of Renewable Distributed Generation (RDG) in the distribution network, wind turbines and solar panels take a more active role in HEMS, especially in recent years [9].

The main objective of the HEMS studies is to minimize the cost of energy consumption [10]. However, addressing a single objective, the HEMS may fail to find the optimized strategy. For this reason, many studies consider multi-objective to schedule household consumption. Inconvenience is the most important secondary objective of the HEMS. Some studies consider penalties for any inconvenience imposed on the residents. In this way, the aim can be maximizing the comfort level [11] or minimizing the penalties [12]. The load profile is the third objectives of the HEMS. Most of the papers schedule the household consumption to reduce Peak to Average Ratio (PAR) of load profile [13]. Emission is another objective of HEMS and is related to the greenhouse gases produced and emitted to the environment because of electricity production in the residential sector [14]. Paper [15] surveyed the environmental impacts of HEMS and promoted technologies of decarburization to achieve a low-carbon human life. Therefore, minimization of energy cost, maximization of consumer satisfaction, minimization of Peak-to-Average Ratio (PAR) and minimization of carbon emission are multiple objectives in the literature.

In order to set a comprehensive Demand Response Program (DRP) in the HEMS, detailed knowledge about the characteristics of the thermal/electrical behavior of household appliances is needed. To model the household appliances, they are classified into different categories according to their ability to respond to requests for load reduction. Table 1 describes a categorization of response classes for household appliances [16]- [17].

**Table 1**. Classification of household responses
"TCA: Thermostatically Controlled Appliances, RPV: Roof-Top Photovoltaic"

| Main Class | Subclass | Main Feature | Appliances | Reference |
|---|---|---|---|---|
| **C-TCA** | - | Thermal storage capacity | EWH, HPS, HVAC, Refrigerator | [16] |
| **Non-TCA** | Curtailable | Load curtailment without shifting excess consumption to later times | Lighting System | [18] |
| | Uninterruptible | Shiftable loads with the same consumption at later times | Wet Appliances, e.g. Dishwasher, Washing Machine, Clothes Dryer | [19] |
| | Interruptible | Interruptible demand with the ability to resume the remaining consumption at a later time | Electric Vehicle | [20] |
| **Uncontrollable** | Energy Consumer | Nonprogrammable/Essential devices, high priority to convenience | Audio/vision Devices, Emergency Lighting, Electric Oven | [21] |
| | Energy Generator | Intermittent/weather-dependent generation | RPV, Wind Micro Turbine | [22] |
| **Storage** | - | Storing energy in low-cost times to discharge in high-cost times | PHEV, Electrical Battery | [17] |

Based on Table 1, the household appliances can be classified into four groups: (1) Controllable Thermostatically Controlled Appliances (C-TCA) (2) Controllable Non-thermostatically Controlled Appliances (Non-TCA) (3) Uncontrollable Appliances (UA) and (4) Energy Storage System (ESS). Detailed information about the features and associated appliances are described in the Table.

As mentioned above, most of HEMS studies use multi-objective methods to optimize the household energy consumption. In accordance with [23], weighted sum, bounded objective, physical programming, and Pareto front are multi-objective methods with higher prominence in the literature. In order to optimize the household consumption, three approaches are used in the literature as follows: (1) mathematical programming (2) heuristic approaches and (3) meta-heuristic approaches. From viewpoint of linearity or nonlinearity of the objective function, the mathematical programming can be solved using Linear Programming (LP) [24], Mixed Integer Linear Programming (MILP) [25] and Mixed Integer Non-Linear Programming (MINLP) [26]. Heuristic approaches are mental shortcuts which employ practical approaches to problem-solving not guaranteed to be optimal or perfect. Teacher Learning Based Optimization (TLBO) [27], List Processing [28] and Markov Decision Process (MDP) [29] are some heuristic algorithms proposed to optimize the household energy consumption. Regarding the meta-heuristic approaches, Harmony Search Algorithm [30], Genetic Algorithm [31], Ant Colony Optimization (ACO) [32] and Particle Swarm Optimization [33] are discussed in some studies.

As reviewed above, most of the studies aim to minimize the electricity cost of consumers without considering the interaction of household consumption and market strategies. Moreover, the network constraints are not addressed in the literature. Regarding the retail pricing in the literature, the electricity price is determined based on the agreed tariff independent of households behavior.

### 1.3. Paper Contributions

In conclusion, in the literature, many studies exist that address home energy management systems. Therefore, what are missing in the literature can be stated as follows:

(4) Investigation of interaction between market trading strategies, including wholesale and retail markets, and the behavior of household appliances.
(5) Discussion about the role of schedulable household appliances in secure operation of the distribution network in near real-time conditions.
(6) Determination of retail strategies in the electricity market based on household behavior considering electricity price uncertainties.

This study focuses on these gaps in the literature. The paper considers a retailer who participates in the wholesale market with uncertain electricity price. The retailer faces four classes of household consumers with different consumption patterns. Regarding the uncertainties associated with electricity price and wind/solar power, the retailer offers an electricity price to the residential consumers based on the hourly settlement. Optimizing the household consumption, insecure operation conditions of the distribution network, including power congestion and voltage reduction, are eliminated through rescheduling of the controllable appliances. Therefore, the contributions of this paper can be stated as follows:

(1) Offering retail electricity price to the residential consumers considering the uncertainties associated with the wholesale market prices.
(2) Comparing the profile of retail electricity price for traditional and smart household consumers.
(3) Rescheduling of the home energy consumption to relieve power congestion in the distribution network.
(4) Proposing a heuristic approach to optimize the operation of a wide variety of household appliances reducing the time and computational burden of the problem.

### 1.4. Paper Organization

The paper is organized as follows: section 2 describes the general framework of the proposed approach. The mathematical formulations of the problem, including wholesale market, retail strategies and HEMS model, are presented in section 3. Simulations and analysis of results are provided in section 4. Finally, the conclusions and recommendations for future studies are illustrated in section 5.

## 2. Problem Statement

In this approach, the market trading strategies have been proposed for an electricity retailer who supplies the energy needs of different classes of smart household consumers.

In the supply side, the retailer participates in two trading floors of the wholesale electricity market, i.e. day-ahead and intraday markets, to procure the required energy. In addition, the retailer can procure some parts of the energy from wind self-generation facilities. The aim of the retailer is to determine the trading strategies, especially retail price, to bring it an acceptable predefined profit.

In the retail side, the residential consumers purchase their energy from the retailer based on the hourly settlement. The households are equipped with smart home energy management systems (S-HEMS) receiving/sending data from/to the DSO through a two-way communication system. In this study, the DSO is responsible for delivering energy to the consumers. Interrupting the energy delivery, the DSO must pay penalty cost to the residential consumers according to the terms and conditions.

The price-based demand response program is considered to describe the response of households to the offered retail price. In this way, the HEMS plays the role of interface between household appliances and the retailer. The HEMS with two-way communication system receives the offered retail price and optimizes the operation of household appliances to minimize the energy cost satisfying the household convenience.

In the demand side, the HEMS aims to determine the operational strategies of household appliances minimizing the electricity bills and satisfying the comfort constraints of the residents. The HEMS determines the operational strategies of three groups of household appliances according to their characteristics: (1) controllable thermostatically controlled appliances (C-TCA) (2) Controllable non-thermostatically controlled appliances (Non-TCA) and (3) Roof-top photovoltaic panels integrated with the electrical storage system (RPV-ESS). In the first category, the household appliances with thermal storage capacity, e.g. electric water heater (EWH), refrigerator and heat pump system (HPS), are considered. The second category is classified into two subcategories as (1) uninterruptible appliances and (2) curtailable appliances. The uninterruptible appliances are devices that must run through a complete set of operations before completing their task. This kind of appliances is generally modeled such that they consume a fixed quantity of power for a specific quantity of consecutive time steps [34]. In this study, dishwashers, washing machines, and clothes dryers are considered as the uninterruptible appliances. Curtailable demands are the appliances whose energy consumption can be curtailed in response to the electricity price or system request. In this approach, the lighting system is considered as the curtailable demand whose energy consumption can be changed in response to the offered retail price. Note that the HEMS users can specify an emergency section for the lighting system that will not be dimmed by the HEMS. In addition to the controllable appliances, there are some uncontrollable appliances, e.g. audio/vision and cooking devices, whose consumptions are not controlled by the HEMS. The electricity consumption of such appliances is modeled by an hourly load profile. In the third category, the operation of roof-top photovoltaic panels integrated with electrical batteries is optimized to store and dispense energy maximizing households' payoff.

This paper determines strategies to optimize the operation of household appliances considering the interaction between household consumers, distribution network and electricity market. In the other word, this paper aims not only to optimize residential consumptions but also to optimize trading strategies of the retailer with considering the security constraints of the distribution network. In this way, the impact of HEMS optimization on the determination of retail electricity price is investigated. To achieve the aim, first of all, the retail electricity price is determined for traditional household consumers without HEMS. It means that the retailer determines the electricity price for household consumers who do not have access to the HEMS to know the electricity price. Afterward, it is supposed that the household consumers are equipped with the HEMS in a smart grid structure. In this structure, the HEMS optimizes the operation of the household appliances based on the offered retail price. The HEMS improves the consumption pattern to minimize the cost of energy consumption. Optimizing the household consumption, the retailer recalculates the electricity price for the optimized consumption pattern. Finally, the profiles of the offered retail price for two structures, i.e. traditional and smart grids, are compared.

Moreover, in order to prevent power interruption, the DSO predicts the probability of congestion occurrence in weak lines of the distribution network and reschedule the operation of TCAs using a flexible comfort band to relieve the congestion. The presented method makes it possible to deliver power to the consumers without any interruption preventing from imposing penalty cost to the retailer. In addition, by using the integrated power of RPVs, the needs for grid expansion and conventional backup capacity can be reduced or postponed.

Considering the abovementioned facts, the problem is proposed by a two layers model. In the first layer, the retailer participates in the wholesale electricity market to determine the retail price. This layer uses stochastic programming and Auto Regressive Integrated Moving Average (ARIMA) as the modeling and forecasting approaches, respectively. In the second layer, the operation of household appliances is optimized by the HEMS. This layer is optimized through heuristic optimization approaches and mathematical programming. The problem in the second layer is modeled as a master problem and two sub-problems. The master problem includes the optimal scheduling of household appliances by the HEMS. Flattening of load profile and mitigating power congestion are objective functions of the two sub-problems that are imported to the constraints of the master problem as new constraints. Figure 1 shows a schematic diagram for the general structure of the problem.

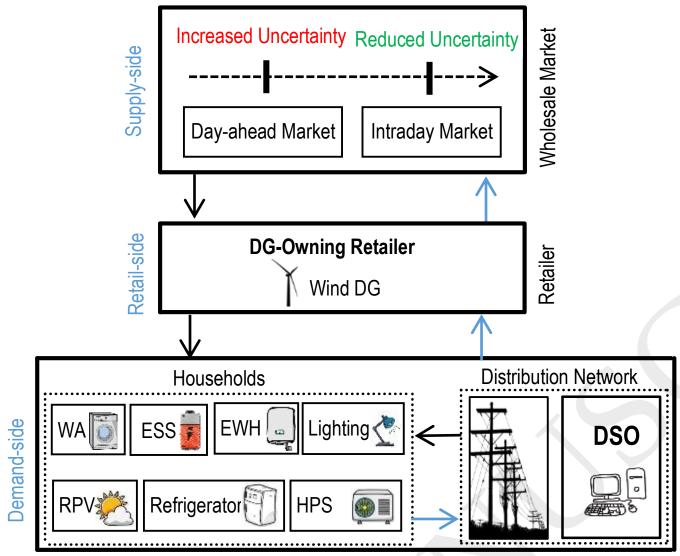



**Figure 1.** The general structure of the proposed approach

## 3. Mathematical Model of the Problem

The aim of the retailer is to participate in the electricity market to obtain an acceptable profit. The HEMS aims to minimize the operation cost of household appliances. In addition, the HEMS tends to minimize the penalty cost of energy interruption in cooperation with DSO. In this way, the needs for grid reinforcement can be postponed and the DSO takes the advantage of using available grid capacity without needing to install new infrastructures. In the following sub-sections, the mathematical structure of the proposed approach is presented.

### 3.1. Wholesale Electricity Market

The objective function of the retailer is to maximize the profit of participation in two trading floors of the wholesale electricity market emphasizing increased use of green energy portfolio. The objective function of the retailer in the wholesale market is described as follows:

$$\underset{\left(P_t^{DA}(\omega),P_t^{ID}(\omega)\right)}{\text{Maximize}}\left[\text{Profit}^{\text{retailer}}\right] \\ =\left[\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_T}\pi(\omega)\times\left[\lambda_t^{D}\times P_t^{D}(\omega)\right]\right]+\left[\sum_{\omega=1}^{N_\omega}\sum_{t=1}^{N_T}\pi(\omega)\times\left[\lambda_t^{DA}(\omega)\times P_t^{DA}(\omega)+\lambda_t^{ID}(\omega)\times P_t^{ID}(\omega)\right]\right] \quad (1)$$

Subject to:

$$P_t^{DA}(\omega)=P_{t,S}^{DA}(\omega)-P_{t,P}^{DA}(\omega) \quad (2)$$

$$P_t^{ID}(\omega)=P_{t,S}^{ID}(\omega)-P_{t,P}^{ID}(\omega) \quad (3)$$

$$-P_t^{D}(\omega)\leq P_t^{DA}(\omega)\leq P_t^{D}(\omega)+\sum_{i=1}^{N_{DG}}P_i^{W,max} \quad (4)$$

$$-P_t^{D}(\omega) \le P_t^{ID}(\omega) \le \sum_{i=1}^{N_{DG}} P_i^{W,max} \tag{5}$$

$$P_i^{W,min} \le P_{t,i}^{W}(\omega) \le P_i^{W,max} \tag{6}$$

$$P_t^{D}(\omega) + P_t^{DA}(\omega) + P_t^{ID}(\omega) - P_{t,i}^{W}(\omega) = 0 \tag{7}$$

Where t, ω and i are the indices of time, scenarios (associated with the uncertain variable in scenario tree) and self-generation facilities of the retailer, respectively. In this way, $N_T$, $N_\omega$, and $N_{DG}$ refer to the number of time hours, scenarios and wind resources, respectively. Regarding the electricity price, variables, $\lambda_t^{DA}$ and $\lambda_t^{ID}$ present the electricity price of day-ahead and intraday markets, respectively. Moreover, $\lambda_t^{D}$ indicates the price offered to the consumers. Considering the power variables, $P_{t,S}^{DA}$ ($P_{t,P}^{DA}$), $P_{t,S}^{ID}$ ($P_{t,P}^{ID}$) are sold (purchased) power in (from) day-ahead and intraday markets, respectively. Note that the subscripts S and P show the sold and purchased power in the electricity market, respectively. In addition, $P_t^{D}$ and $P_{t,i}^{W}$ are the load level and wind power, respectively. Note that $P_t^{DA}$ and $P_t^{ID}$ are the net power traded in day-ahead and intraday markets, respectively. In Eq. (1), $\lambda_t^{DA}$ and $\lambda_t^{ID}$ are input stochastic variables which are specified by the probabilistic scenarios with occurrence probability π(ω). Solving the stochastic programming, $P_t^{DA}$ and $P_t^{ID}$ are determined as the output variables.
The objective function, Eq.(1), comprises two terms: (1) the expected profit from selling energy to the consumers (2) the expected profit/cost from trading energy in two market floors, i.e. day-ahead and intraday markets.

Constraints (2) and (3) describe the net amount of power traded in day-ahead and intraday markets, respectively. Constraint (4) limits the amount of power traded in the day-ahead market to the summation of clients' demand and total installed capacity of wind turbines. Constraint (5) limits the amount of power traded in the intraday market to the total installed capacity of wind turbines. The main reason for limiting the power traded in day-ahead and intraday markets is to prevent from speculating in the retail market. The wind power of intermittent self-generation facilities is bounded through (6). Constraint (7) describes the global balance of power for all retail strategies.

### 3.2. Retail Electricity Price

In the literature on the retail electricity market, there are two kinds of structures: (1) Single-retailer structure with monopolistic competition (2) Multi-retailer structure with competitive competition. In the monopolistic competition, only one retailer is considered for the problem. The logical reason for this structure is to avoid increasing the complexity of the problem. On the contrary, instead of monopolistic competition, there is pure competition (perfect market) in some research studies. In these studies, two or more retailers are considered to enhance the retailers' competitiveness. In such structure, each retailer is pressured to attract more consumers to the retail electricity market. It is evident that if a retailer cannot propose a competitive price to the clients, it may lose some customers [1]. Many studies about the multi-retailer structures use Game Theory approaches to model the competition between rival retailers [35]. The main reason for using the single-retailer structure is to avoid the complexity of the Game Theory approaches.
As a result, in a multi-retailer structure, the consumers can decline the electricity price of one retailer and purchase electricity from the other retailers. In this paper, in order to concentrate on the HEMS and avoid the complexity of Game Theory approaches, a single retailer is proposed. In this approach, the retailer determines the hourly electricity price for the consumers. Therefore, because of the single-retailer structure, the consumers cannot decline the offered electricity price. For this reason, in order to prevent from exercising market power by the retailer, the electricity price is determined considering a predefined profit percentage for the retailer.

In fact, the electricity price is determined to bring the retailer an acceptable profit percentage and minimize the expected procurement cost. For this reason, considering a predefined profit percentage η, the retailer's income equals η × Cost [36]. In accordance with the income and cost functions of the retailer in Eq. (1), the retail electricity price offered to the consumers is determined as follows:

$$\mathrm{Profit}^{\mathrm{retailer}}(\omega)=\mathrm{Income}^{\mathrm{retailer}}(\omega)-\mathrm{Cost}^{\mathrm{retailer}}(\omega) \tag{8}$$

$$\lambda_t^D=\frac{\eta\times\left[\sum_{\omega=1}^{N_\omega}\mathrm{Cost}^{\mathrm{retailer}}(\omega)\right]-\left[\sum_{\omega=1}^{N_\omega}\mathrm{Income}^{\mathrm{market}}(\omega)\right]}{\sum_{\omega=1}^{N_\omega}\pi(\omega)\times P_t^D(\omega)} \tag{9}$$

$$\lambda_t^D=\frac{\begin{pmatrix}\eta\times\min\left(\sum_{\omega=1}^{N_\omega}\pi(\omega)\times\left[\lambda_t^{DA}(\omega)\times P_{t,P}^{DA}(\omega)+\lambda_t^{ID}(\omega)\times P_{t,P}^{ID}(\omega)\right]\right)\\ -\max\left(\sum_{\omega=1}^{N_\omega}\pi(\omega)\times\left[\lambda_t^{DA}(\omega)\times P_{t,S}^{DA}(\omega)+\lambda_t^{ID}(\omega)\times P_{t,S}^{ID}(\omega)\right]\right)\end{pmatrix}}{\sum_{\omega=1}^{N_\omega}\pi(\omega)\times P_t^D(\omega)} \tag{10}$$

The Eq. (8)-(10) determine the retail price based on the forecasted values of day-ahead and intraday electricity prices. The ARIMA approach forecasts the scenarios of electricity price 24 hours prior to the energy delivery time. In a more complicated model, the problem can be discussed considering adjustment for the forecasted values to reduce the forecast error. Using multi-stage stochastic programming or online scheduling, the applicability of the problem can be increased. In this way, a trade-off between tractability and complexity may be needed.

Note that the income factor η is defined according to the Risk Bearing Capacity (RBC) of the retailer to attract new customers or preserve current ones.

### 3.3. HEMS Operation Scheduling

In this study, HEMS is a smart control system which optimizes the operation strategies of all controllable appliances. This unit has two-way communication with the system operator to receive hourly retail electricity prices or requests of load reduction when the system security is jeopardized. The main aim of the HEMS is to minimize the cost of energy consumption satisfying the comfort constraints of occupants. Comfort bands of the household appliances are determined and set into the HEMS by the occupants.
In order to optimize the operation of appliances, the HEMS classifies the household devices into three main categories according to the operational characteristics: (1) TCA (2) Non-TCA (3) RPV-ESS. For each group of appliances, a heuristic or mathematical programming approach is considered to optimize the consumption behavior.

First of all, the appliances with thermal storage capacity, e.g. EWH, HPS, and refrigerator, are optimized by a heuristic Forward-Backward Algorithm (F-BA) to meet the temperature requirements of the households. Secondly, for non-TCAs, there are two different kinds of appliances with distinctive characteristics, including uninterruptible and curtailable appliances. In this way, the wet appliances, e.g. washing machine, dishwasher, and clothes dryer, are uninterruptible demands which can be shifted to low energy cost periods satisfying the consecutive time constraints. On the other hand, the curtailable demands can be curtailed in response to the electricity price without any temporal consequences. Curtailable demands are often a function of electricity price. In this study, the lighting system is considered as the curtailable demand. For example, if household consumers dim the lighting demand in response to the high electricity price, they are not expected to increase the lighting consumption at later periods to make up for the reduced consumption. In order to make a mathematical model fitting the characteristics of the curtailable demands, a combined demand function, including linear, exponential, potential and exponential responses, is proposed [37]. Finally, the HEMS optimizes charging/discharging strategies of the integrated RPV-ESS based on maximizing (minimizing) the profit (energy cost) of households. In this way, a linear programming approach is used to determine the optimized strategies of the RPV-ESS. Considering the aforementioned facts, the objective function of the HEMS is to minimize the overall energy cost as well as maximize the ESS-RPV's payoff as following multi-objective function:

$$\mathrm{OF}=\left(\alpha\times \mathrm{OF}_1\right)+\left(\beta\times \mathrm{OF}_2\right) \tag{11}$$

s.t.

$$\alpha=\frac{\overline{P}^{D}}{\overline{P}^{battery}+\overline{P}^{D}},\ \beta=\frac{\overline{P}^{battery}}{\overline{P}^{battery}+\overline{P}^{D}} \tag{12}$$

In the multi-objective function (OF), the objectives $OF_1$ and $OF_2$ are linked through the weighting factors $\alpha$ and $\beta$. The objectives $OF_1$ and $OF_2$ are stated as follows:

$$\mathrm{OF}_1=\min\left[\sum_{t=1}^{N_T}(P_t^{TCA}+P_t^{Wet}+P_t^{LDT})\times\lambda_t^{D}\right] \tag{13}$$

$$\mathrm{OF}_2=\max\left[\sum_{t=1}^{N_T}\lambda_t^{D2G}\times P_t^{Dch}\right] \tag{14}$$

$$P_t^{D}=P_t^{TCA}+P_t^{Wet}+P_t^{L}+P_t^{NC} \tag{15}$$

$$P_t^{TCA}=P_t^{EWH}+P_t^{HPS}+P_t^{R} \tag{16}$$

$$P_t^{Wet}=P_t^{wm}+P_t^{dw}+P_t^{cd} \tag{17}$$

Where $OF_1$ and $OF_2$ are the minimization of energy cost and maximization of household's payoff, respectively. $P^{TCA}$, $P^{Wet}$, $P^{L}$, $P^{LDT,}$ and $P^{NC}$ are the demand for TCAs, wet appliances, total lighting, controllable lighting, and non-controllable appliances, respectively. $P^{EWH}$, $P^{HPS,}$ and $P^{R}$ are the demand of electric water heater, heat pump, and refrigerator, respectively. $P^{wm}$, $P^{dw}$ and $P^{cd}$ are the demand for washing machine, dishwasher and clothes dryer, respectively. $P^{Dch}$ and $\lambda^{D2G}$ are power and price of electrical energy injected from ESS into the grid. In addition, $\overline{P}^{battery}$ and $\overline{P}^{D}$ are the upper capacity of ESS and household demand, respectively.

Optimizing Eq. (11), all the consumption strategies are imported into a final step to make the collective decision about the operation of appliances. This is a primary decision made in the master problem without considering the requirements of the distribution network and load profile. The final decision is made after importing the constraints imposed on the master problem by means of the two sub-problems.

The detailed descriptions of the HEMS control algorithms for different appliances are illustrated in the following subsections.

### 3.4. Thermostatically Controlled Appliances

Thermostatically controlled appliances have thermal storage capability which is an important feature for price-based demand response programs. The feature makes it possible to store energy during low-price hours to supply the consumption during the high-price hours. Note that the scheduling of the TCAs needs general knowledge about thermal dynamics of the appliances. The thermal dynamics are often modeled by Newton's Laws of Cooling and are controlled in a state space environment. In the following subsections, the thermal behavior of the TCAs is described.

#### 3.4.1. Electric Water Heater

The thermal dynamic behavior of an electric water heater is modeled considering the heat exchange with the environment and with cold water inflows. The temperature of hot water in the electric water heater is described as follows [38]:

$$\theta_{hw}^{t}=\theta_{a}^{t}+\left(R_w\times P^{EWH}\right)-\left(\frac{M-m^t}{M}\right)\left(\theta_a^t-\theta_{hw}^{t-1}\right)\exp\left(\frac{-\tau}{R_w\times C_w}\right) \tag{18}$$

$$P^{EWH}=\frac{Q^{EWH}}{3600}\times X^{EWH}_{HEMS} \quad (19)$$

$$Q^{EWH}=m^{t}\times\rho^{water}\times\Delta\theta_{hw} \quad (20)$$

$$\left(\underline{\theta}_{hw}\pm\theta^{tol}_{hw}\right)\leq\theta^{t}_{hw}\leq\left(\bar{\theta}_{hw}\pm\theta^{tol}_{hw}\right) \quad (21)$$

Where $\theta_{hw}$ and $\theta_a$ are temperatures of hot water and environment, respectively. $R_w$ and $C_w$ are thermal resistance and capacitance of the EWH, respectively. M is capacity of tank and $m^t$ is hot water usage of households in time slot t in Kg. $P^{EWH}$ and $Q^{EWH}$ are energy needed to increase the temperature of cold water inflows in kWh and kJ, respectively. $X^{EWH}_{HEMS}$ is a decision binary variable made by the HEMS to turn the EWH on ($X^{EWH}_{HEMS}=1$) or turn it off ($X^{EWH}_{HEMS}=0$). Note that a tolerance interval $[-\theta^{tol}\ \theta^{tol}]$ is considered to prevent from high-frequency switching when HEMS maintains the temperature near the lower/upper comfort band. Eq. (18) describes the temperature of hot water as a function of hot water usage and electrical energy consumption. Eq. (19) and (20) denote the energy consumption of the EWH in kWh and kJ, respectively. The inequality (21) enforces the comfort band defined by the households.

### 3.4.2. Refrigerator

In order to describe thermal behavior of the refrigerator, the thermal model, in accordance with Grey Box model, is characterized by a thermal mass and thermal resistance through a single state model as follows [39]:

$$\frac{d\theta_c}{dt}=\frac{1}{C_{r,c}\times R_c}(\theta_a-\theta_c)-\frac{1}{C_{r,c}}(COP\times P^R)+\vartheta \quad (22)$$

$$P^{R}=\frac{Q^{R}}{3600}\times X^{R}_{HEMS} \quad (23)$$

$$Q^{R}=U_{cs}(\theta_a-\theta_c)-\left(m_{cs}\times\rho_{cs}\times\Delta\theta_c\right) \quad (24)$$

$$\left(\underline{\theta}_{c}\pm\theta^{tol}_{c}\right)\leq\theta^{t}_{c}\leq\left(\bar{\theta}_{c}\pm\theta^{tol}_{c}\right) \quad (25)$$

Where $\theta_c$ is the temperature of the refrigeration chamber. $R_c$ and $C_{r,c}$ are thermal resistance and thermal mass of the refrigeration chamber, respectively. COP is the overall coefficient of performance defined as the ratio between the thermal power extracted at evaporator side and the refrigerator electrical consumption. $P^R$ and $Q^R$ are energy needed to decrease the temperature of the refrigeration chamber in kWh and kJ, respectively. $U_{cs}$ is the overall transmittance coefficient from the refrigeration chamber to the ambient, $m_{cs}$ and $\rho_{cs}$ are the cold storage mass and its specific heat capacity, respectively. $X^{R}_{HEMS}$ is a decision binary variable made by the HEMS to turn the refrigerator on ($X^{R}_{HEMS}=1$) or turn it off ($X^{R}_{HEMS}=0$). Note that a tolerance interval $[-\theta^{tol}\ \theta^{tol}]$ is considered to prevent from high frequency switching when HEMS maintains the temperature near the lower/upper comfort band. $\vartheta$ is a random function indicates the stochastic pattern of door opening for the household refrigerator.
The Eq. (22) describes the temperature of the refrigeration chamber. Eq. (23) and (24) denote the energy consumption of the refrigerator in kWh and kJ, respectively. The inequality (25) enforces the comfort band defined by the households.

### 3.4.3. Heat Pump System

A heat pump is a device that transfers heat from a low-temperature zone to a higher temperature zone using mechanical work. Although a heat pump can provide both heating or cooling, in cooler climates heating is, of course, more common [40]. In this study, a third order linear model to describe the thermal behavior of the HPS is used as follows [41]:

$$\frac{d\theta_r}{dt}=\frac{U_{fr}}{C_{p,r}}(\theta_f-\theta_r)-\frac{U_{ra}}{C_{p,r}}(\theta_r-\theta_a)+\frac{1-\rho}{C_{p,r}}(P^S) \quad (26)$$

$$\frac{d\theta_f}{dt}=\frac{U_{wf}}{C_{p,f}}(\theta_w-\theta_f)-\frac{U_{fr}}{C_{p,f}}(\theta_f-\theta_r)+\frac{\rho}{C_{p,r}}(P^S) \quad (27)$$

$$\frac{d\theta_w}{dt}=\frac{\eta\times P^{HPS}}{C_{p,w}}-\frac{U_{wf}}{C_{p,w}}(\theta_w-\theta_f) \quad (28)$$

$$P^{HPS}=\frac{Q^{HPS}}{3600}\times X^{HPS}_{HEMS} \quad (29)$$

$$Q^{HPS}=(m_w^{tank}\times\rho^{water}\times\Delta\theta_w)-Q^{wf} \quad (30)$$

$$\left(\underline{\theta}_r\pm\theta_r^{tol}\right)\le\theta_r^t\le\left(\bar{\theta}_r\pm\theta_r^{tol}\right) \quad (31)$$

Where $\theta_r$, $\theta_f$ and $\theta_w$ are the room air temperature, floor temperature and water temperature in the floor heating pipes, respectively. Moreover, $C_{p,r}$, $C_{p,f}$ and $C_{p,w}$ denote the heat capacity of the room air, of the floor and of the water in the floor heating pipes, respectively. $U_{fr}$, $U_{ra}$ and $U_{wf}$ describe the heat transfer coefficients between floor and room, room air and ambient, water and floor, respectively. $P^{HPS}$ and $P^S$ are energy extracted from the electrical supply and from the solar irradiation, respectively. $\eta$ is the coefficient of performance of heat pump and $\rho$ is the fraction of solar irradiation emitted on the floor. $m^{tank}$ is the mass of water in the tank and $Q^{wf}$ is the heat transferred from water to floor.

The differential equations (26)-(28) describe the thermal behavior of the HPS for the temperature of the room, of the floor and of the water in the pipes, respectively. Equations (29)-(30) denote the work of compressor in kWh and kJ, respectively. The inequality (31) enforces the comfort band of the room temperature.

### 3.4.4. Forward-Backward Algorithm for TCA

Due to the performance of successive market floors, the proposed approach should respond to the DR programs with appropriate response time, especially for short notice programs. The thermal dynamics of the TCAs have non-linear behavior because of exponential time-dependent terms in Newton's Laws of Cooling. This feature increases the computational time burden of the problem. Therefore, the optimization may be failed especially when the response time is crucial. To reduce the computational burden of the problem, a heuristic Forward-Backward Algorithm (F-BA) is suggested in this paper. The F-BA minimizes the energy cost of the TCAs satisfying the residents' convenience. Moreover, due to fast convergence of the algorithm, it can be used to respond to the mid/short notice DR programs. The proposed algorithm uses the thermal characteristics of the TCAs to obtain the lowest energy consumption strategy in response to the offered electricity price. This algorithm is described as follows:

**Step 1:** Sort the offered electricity price in ascending order: $\Lambda=\left\{\forall i=1,...,N_T:\hat{\lambda}_t^D(i)\le\hat{\lambda}_t^D(i+1)\right\}$

**Step 2:** Based on the predicted consumption pattern of TCAs, calculate the electrical energy needed to meet the demand.

**Step 3:** Turn on the TCA for the hour associated with the lowest electricity price: $\left(t_{i=1},\hat{\lambda}_t^D(1)\right)$

**Step 4:** Check the temperature of all time spots on the horizon. If all the temperature values are within the comfort band, stop the problem, otherwise, go to the next step.

**Step 5:** Turn on the TCA for the next hour ($i\rightarrow i+1$) associated with the lowest electricity price: $\left(t_{i+1},\hat{\lambda}_t^D(i+1)\right)$

**Step 6:** Check the temperature of all time spots on the horizon. There are three states:

**State 1:** If all the temperature values are within the comfort band, stop the problem.

**State 2:** If there is at least one temperature between $t_i$ and $t_{i+1}$ exceeding the comfort band, use the backward operator in step 7.

**State 3:** If all the temperature values of interval $[t_i \ t_{i+1}]$ are within the comfort band and there is at least one temperature between $t_{i+1}$ and $t_{i=NT}$ exceeding the comfort band, use the forward operator in step 8.

**Step7:** Remove $t_{i+1}$ (turn TCA off at $t_{i+1}$) and go back to step 5 to resolve the problem for the interval [$t_i$ $t_{i+1}$].
**Step8:** Preserve $t_{i+1}$ (keep TCA on at $t_{i+1}$) and go back to step 5 to solve the problem for the interval [$t_{i+1}$ $N_T$].
The heuristic approach optimizes the value of $P^{TCA}$ in the objective function Eq. (12). Figure 2 depicts a schematic diagram of the forward-backward performance in the proposed F-BA.

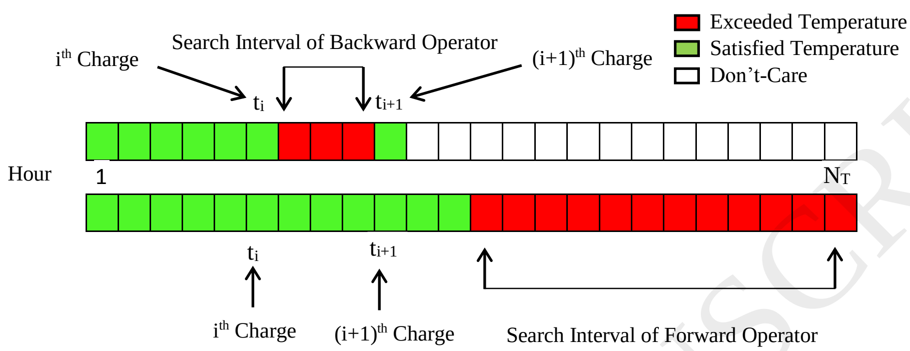



**Figure 2**. Performance of forward and backward operators in the F-BA

### 3.5. Uninterruptible Demands

The Household wet appliances, i.e. washing machine, clothes dryer, and dishwasher, are uninterruptible demands whose consumption can be shifted to durations associated with low electricity price (off-peak hours). Generally, the uninterruptible loads must run through a complete set of operations before completing their task. Uninterruptible loads are generally modeled such that they consume a fixed quantity of power for a specific quantity of consecutive time steps. In the literature on household demand response, the uninterruptible loads are also called deferrable, shiftable or time-shiftable loads [42].
It is worth mentioning that the operation of washing machine and clothes dryer must be done in consecutive times. Otherwise, it may damage the clothes disturbing the convenience of the households. Regarding the dishwashers, scheduling logic for the table-top dishwasher is different from large-size dishwashers. The table dishwashers may be needed to operate two or three times a day for a family of two or three. However, the large-size dishwashers can be operated once a day shifted to the hour with the lowest electricity price. Considering the mentioned facts, the scheduling logic for the wet appliances can be stated as following rule-based approach:

- Washing Machine

There are two different states for the operation of the washing machine as follows:

1. If $(N_w+N_d)>1$: choose the $(N_w+N_d)$ consecutive hours with lowest electricity price while total energy consumption of the washing machine plus clothes dryer is minimized.
2. If $(N_w+N_d)\leq 1$: choose the hour associated with the lowest electricity price to operate the washing machine and dryer.

Where $N_w$ and $N_d$ are the duration of operation time for washing machine and clothes dryer, respectively.

- Dishwasher

The operation states of the dishwasher can be presented as follows:

1. If $(D_{dw}>1)$: choose $D_{dw}$ consecutive hours with the lowest electricity price satisfying the constraint enforces that the operation time of the device is after the time of putting dishes in the machine.
2. If $(D_{dw}=1)$: choose the hour associated with the lowest electricity price to operate the machine.

Where $D_{dw}$ describes the usages number of the dishwasher in a day. For table-top dishwashers, we consider $D_{dw}>1$ which means the use of the machine is more than one time a day. For large-size dishwashers, it is considered as $D_{dw}=1$.

The above rule-based approach optimizes the value of $P^{Wet}$ in the objective function Eq. (17).

### 3.6. Curtailable Demands

Curtailable appliances are the demands whose consumption can be curtailed in response to the increased electricity price or request from system operator when there is a power shortage in the power system or the system security is jeopardized. In this study, the lighting system of households is considered as the curtailable demand whose energy consumption can be increased/decreased in response to the decreased/increased electricity price. In order to describe the behavior of households to the offered electricity price, a combined demand function, including linear, potential, logarithmic and exponential model is proposed.

To set the proposed model into the HEMS, the households import their expected electricity price during high demand periods of the lighting system. If the offered electricity price is lower than the expected price, the HEMS increases the demand for the lighting system. Adversely, if the offered retail price is higher than the expected price, the HEMS begins to dim the light. Note that the emergency lighting consumption is not allowed to be dimmed during the HEMS optimization. For this reason, the HEMS users can specify an emergency section for the lighting system that will not be dimmed by the HEMS. Considering the mentioned facts, the model is formulated as follows:

$$DR_t^L = P_{t-1}^{LDT,lin} \times [1+\varepsilon_{lin}\frac{\lambda_t^D - \lambda_t^D}{\lambda_t^D}] + P_{t-1}^{LDT,pot} \times [\frac{\lambda_t^D}{\lambda_t^D}]^{\varepsilon_{pot}} + P_{t-1}^{LDT,exp} \times \exp[\varepsilon_{exp}\frac{\lambda_t^D - \lambda_t^D}{\lambda_t^D}] + P_{t-1}^{LDT,log} \times [1+\varepsilon_{log} \times \ln\frac{\lambda_t^D}{\lambda_t^D}] \quad (32)$$

$$P_t^L = P_t^{LDI} + P_t^{LDT} \quad (33)$$

$$P_t^L = (1-DR_t^L) \times P_{t-1}^L + P_t^{LDT} \quad (34)$$

$$P_{t-1}^L = (DR_t^L \times P_{t-1}^L) + P_t^L - P_t^{LDT} \quad (35)$$

$$DR_t^L \leq \overline{DR}^L \quad (36)$$

Where $P^L$, $P^{LDI}$ and $P^{LDT}$ describe the total power of the lighting system, the power of delay intolerant and power of delay tolerant demands of the lighting system, respectively. $P^{LDT,X}$ describes the delay tolerant consumption of the lighting system for demand function X. $\tilde{\lambda}^D$ is the reference retail price expected by the households. $DR^L$ and $\overline{DR}^L$ are the amount of demand reduction and the maximum allowable value of demand reduction in the lighting system. Note that the curtailable part of the lighting system is defined as $P^{LDT}$ and the uncurtailable part (emergency demand) is defined as $P^{LDI}$. The maximum allowable reduction of the lighting demand is enforced by inequality (36).

The above flexible demand function optimizes the value of $P^{LDT}$ in the objective function Eq. (17).

### 3.7. Solar-Storage System

The households are equipped with roof-top solar photovoltaic panels integrated with an electrical storage system. Generally, there are two kinds of electrical batteries for household applications. The first is the batteries which are designed to be charged from the grid during low-cost hours and to be discharged into the power grid when the electricity price is high. The second category is the batteries which are used as a part of photovoltaic panels to store the electrical energy extracted from the panels. It is evident that the storage capacity and the electrical characteristics of the two kinds of batteries are different. In this paper, the latter is used in the RPV-ESS. In fact, the RPV-ESS is charged from the photovoltaic panels and cannot be charged from the grid. On the other hand, the energy is injected into the distribution grid during high electricity price. However, the power may be injected into the grid during normal electricity price due to the definite capacity of batteries. The HEMS receives the weather data about solar irradiation, temperature and wind speed to forecast the solar power in the next 24 hours. The HEMS determines the discharging strategies to maximize the households' payoff. The HEMS uses linear programming to optimize the operational strategies of the RPV-ESS as follows:

$$OF_2 = \max\left[\sum_{t=1}^{N_T} \lambda_t^{D2G} \times P_t^{Dch}\right] \tag{37}$$

$$SOC_t = SOC_{t-1} + (\delta^{ch} \times P_t^{Ch}) - (\frac{P_t^{Dch}}{\delta^{Dch}}) \tag{38}$$

$$0 \le P_t^{Ch} \le \overline{P}^{RPV} \tag{39}$$

$$0 \le P_t^{Dch} \le \overline{P}^{battery} \tag{40}$$

$$\gamma_t^{Ch} = \frac{SOC_t - SOC_{t-1}}{\delta^{ch}} \le \overline{\gamma_t^{Ch}} \tag{41}$$

$$\gamma_t^{Dch} = (SOC_{t-1} - SOC_t) \times \delta^{Dch} \le \overline{\gamma_t^{Dch}} \tag{42}$$

$$\underline{SOC_t} \le SOC_t \le \overline{SOC_t} \tag{43}$$

Where $P^{Ch}$ and $P^{Dch}$ are charging and discharging power, respectively. $\delta^{Ch}$ and $\delta^{Dch}$ are charging and discharging efficiency, respectively. $\gamma^{Ch}$ and $\gamma^{Dch}$ are the rate of charging and discharging which are bounded to the maximum allowable value as $\overline{\gamma}^{Ch}$ and $\overline{\gamma}^{Dch}$, respectively.

Eq. (37) describes the objective function of HEMS for RPV-ESS operation. Eq. (38) denotes a state-transition equation to describe the battery's state of charge (SOC) at time t. The inequalities (39) and (40) bound the charging and discharging power to the capacity of RPV and battery, respectively. Charging and discharging rates of the batteries are presented by (41) and (42). The SOC of the batteries is bounded by (43).

### 3.8. Peak Flattening Scheme

In the price-based demand response programs, if great values of demand are shifted to the periods with the lowest price, a new peak demand may be created in the daily load profile. One of the objectives of the DRPs is to use the appliances' capability to flatten the load profile. Therefore, if the HEMS shifts a considerable part of the demand to the hour associated with lowest electricity price, the only work the HEMS has done is to shift the peak demand of households from high-price periods to the low-price periods. Absolutely, by increasing the penetration of HEMS in the distribution network, the created peak may become a permanent peak. In this situation, the profile of electricity price is affected noticeably. In order to prevent from creating the mentioned problem, a Peak Flattening Scheme (PFS) is proposed in this paper. The PFS integrated with the HEMS not only is able to optimize the energy consumption but also reduces the Peak to Average Ratio (PAR) of the load profile.

The PFS proposes two rules to reduce the PAR. The logical programming of PFS is illustrated in Algorithm 1. First of all, the rule in line 2 describes that total demand ($P^{TCA} + P^{Wet} + P^{LDP}$) shifted to k hours with lowest electricity piece must be lower than a predefined energy value ($E^{max}$). Secondly, lines 3-5 distribute the shiftable energy to an interval of the lowest electricity price ($\Omega$=[t-$\alpha$, t+$\alpha$]) instead of one hour associated with the lowest electricity price. The reason is that during night hours, the electricity price is usually low and there is barely any noticeable difference between the electricity prices for two consecutive hours. In this situation, for a strictly inflexible HEMS, a great value of energy is shifted to the hour with lowest electricity price, in contrast; the fairly flexible HEMS distributes the energy to an interval ($\Omega$) in which the variation of electricity price is lower than a predefined value ($\zeta$). This approach reduces the PAR of the load profile instead of creating a new peak in the night hours.

**Algorithm 1. Peak Flattening Scheme**

Line 1: **for** t=1:k **do**:
Line 2: $\sum_{t=1}^{t=k}(P^{TCA} + P^{Wet} + P^{LDP}) \le E^{max}$
Line 3: **end for**
Line 4: **for** t $\in [t-\varphi, t+\varphi]$ **do**:

Line 5: **if** $|\lambda_t - \lambda_{t\pm\varphi}| \le \zeta$;

Line 6: $\mathrm{P}^{\mathrm{DT}} \rightarrow \frac{\sum_{\mathrm{t-\varphi}}^{\mathrm{t+\varphi}} \mathrm{P}^{\mathrm{DT}}}{2\alpha+1}$

Line 7: **end if**

Line 8: **end for**

### 3.9. Congestion Management

When the energy consumption in the residential area of distribution network increases, power congestion may occur in some weak lines. If an interruption occurs in the power system, the network operator must pay penalty cost to the consumers. In order to minimize the penalty cost and increase the security of the distribution network, DSO can decrease the probability of congestion occurrence through managing the comfort band of TCAs for households who participate in emergency DRPs voluntarily. In this situation, if the comfort band of TCAs changes one or two Celsius degrees, the power consumption of the households decreases without disturbing their convenience. Therefore, the HEMS can help the DSO to mitigate the congestion to prevent power interruption. Incorporating Flexible Comfort Band (FCB) into the optimization approach, the HEMS can prevent interruptions during peak demand hours.

If the rate of network congestion is represented by the following probability calculation, the congestion rate will appear in the distribution network when σ has a positive value:

$$\sigma = \mathrm{Prob}\left\{ \mathrm{P}_{zz'} \ge \overline{\mathrm{P}_{zz'}} \right\} \tag{44}$$

Where $\mathrm{P}_{zz'}$ shows the power flow in the distribution line from bus z to bus z'. In power system studies, a predefined critical value is defined for σ such as $\sigma_{critical}$. It means if σ is bigger than the critical value, the network congestion will occur in some lines, hence the congestion management is essential. The network operator aims to minimize the outage cost which is paid to the consumers due to power interruption. The DSO determines the probability of congestion occurrence in near real-time condition when the power flow of distribution lines approaches the safety bound. Therefore, the objective function of the congestion management can be stated as follows:

$$\mathrm{Minimize}\left(\mathrm{Cost}^{\mathrm{congestion}}\right) = \sum_{t=1}^{N_T} \sum_{z=1}^{N_D} \sum_{k=1}^{N_L} \varpi_z \times \left[ \mathrm{P}_{\mathrm{t,k}}^{\mathrm{Con}}(\mathrm{t}) - \mathrm{P}_{\mathrm{t,k}}^{\mathrm{Cap}}(\mathrm{t}) \right] \tag{45}$$

s.t.

$$\mathrm{F}(|\mathrm{V}_z|, \delta_{zz'}, \mathrm{P}_{zz'}, \mathrm{Q}_{zz'}) = 0 \tag{46}$$

Where z and k are the indices of buses and lines in the distribution network, $\mathrm{P}^{\mathrm{Con}}_{\mathrm{t,k}}$ and $\mathrm{P}^{\mathrm{Cap}}_{\mathrm{t,k}}$ are the power flow of line k at time t during congestion occurrence and after congestion relief, respectively. $\varpi_z$ denotes the penalty cost of outage occurrence in bus z ($/MWh). Note that the Eq. (46) refers to the general constraints of the load flow problem in the distribution network.

To minimize Eq. (45), firstly, we should determine 'which buses should be selected for load shedding'. For this reason, shift factor A is calculated to determine load shedding plans. This factor shows the approximate change in the line flow due to a change in the bus load and is derived from the DC load flow. Shift factor A is a linear sensitivity factor which indicates the contribution of load level to the distribution lines capacity as follows [43]:

$$A_{k,z} = \frac{\Delta P_k}{\Delta P_z^D} \tag{47}$$

where $A_{k,z}$ is the power shift factor for line k because of change in the load of bus z; $\Delta P_k$ is the change in the power flow of line k due to change in the load of bus z, $\Delta P_z^D$.

Now, in order to relieve the network congestion, the priority for load shedding is provided considering the following objects:
1) Select the load with the most impact on the congested lines, which means a load with the highest shift factor ($A_{max}$).
2) Select the load with the minimum outage cost ($\varpi_{z,min}$).

Finally, the division of $A_{max}$ and $\varpi_{z,min}$ determines the load which has the minimum load shedding cost and the most impact on the congestion relief. Therefore, the division is formulated as follows:

$$\mu_{k,z} = \frac{A_{k,z}^{\max}}{\varpi_z^{\min}} \tag{48}$$

where $\mu_{k,z}$ is the priority factor of load shedding, $A_{k.z}^{max}$ is the maximum shift factor, and $\varpi_z^{min}$ is the minimum outage cost. Therefore, the load with minimum outage cost and maximum impact on congestion relief is selected for load shedding. Note that the priority list for load shedding is arranged according to the descending order of priority factor $\mu_{k,z}$.

The Eq. (47)-(48) determine the candidate buses for congestion management. Afterward, in order to mitigate the congestion, the DSO determines the demand value which must be shed (or decreased) using load shedding (or FCB) approach. The shed (reduced) demand value is calculated as follows:

$$\Delta P_{t,Congestion}^{D} = \sum_{k=1}^{N_L} \left[ P_{t,k}^{Con}(t) - P_{t,k}^{Cap}(t) \right] \tag{49}$$

To relieve the congestion, in the traditional distribution system, the DSO has to shed $\Delta P^D_{t,Congestion}$ demand (load shedding). In contrast, in the smart structure, the DSO sends a request to the HEMS to turn off (or turn down) the TCAs until $\Delta P^D_{t,Congestion}$ demand reduction is obtained (FCB).

### 3.10. Electricity Market Performance

Figure 3 depicts a schematic diagram to show how the residential retailer participates in two trading floors of the electricity market. In order to take part in the DR program, first of all, the retailer participates in the day-ahead market based on the forecasted price of day-ahead and intraday markets (first stage). In the first stage, the electricity prices of the two markets are unknown and are considered as uncertain variables. Although the retailer participates in the day-ahead market, it forecast its operation in the intraday market based on the generated scenarios of electricity price. In this way, all the household appliances are incorporated into the decision-making procedure to optimize the energy cost of the consumers. In fact, the day-ahead market behaves as an energy market to schedule the operation strategies of the household appliances, including TCAs and non-TCAs.

Approaching the energy delivery time, the uncertainty level associated with the price of electricity market decreases noticeably. Therefore, in order to incorporate the certainty gained on the electricity price into the consumption schedule, the retailer participates in the intraday market to purchase/sell the deficit/surplus of its energy (second stage). In the second stage, the day-ahead market was cleared before; as a result, the day-ahead price is realized. The electricity price of the intraday market is considered as an uncertain variable. Due to approaching the market clearance, the electricity price of the intraday market can be updated with lower uncertainty. In this way, on short notice, the TCAs can be switched off/on to provide a spinning reserve to the power system 10 to 60 minutes prior to the energy delivery time. To sum up, in the first stage, the energy consumption of all household appliances is scheduled in the day-ahead market. Afterward, in the second stage, only the TCAs are scheduled in the intraday market on short notice.

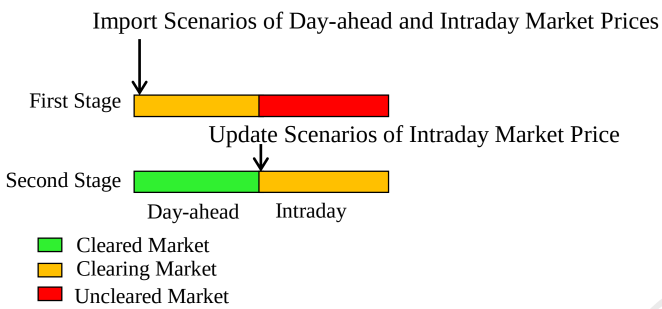



**Figure 3.** Market clearance procedure in the two-stage stochastic programming

### 3.11. Electricity Market Participants

In order to describe the main duties of the market participants, i.e. retailer, DSO and end-users, the whole procedure of the problem is broken down into different tasks in Figure 4. In addition, the associated software for coding and optimizing the problem is illustrated in each stage. In the electricity market, the retailer aims to determine the electricity price for the consumers considering a predefined profit percentage. To achieve the aim, the retailer takes the following steps:

**Step 1)** The retailer generates price scenarios for day-ahead and intraday markets using ARIMA.
**Step 2)** The retailer participates in the day-ahead market based on the forecasted price of day-ahead and intraday markets (first stage of the stochastic programming).
**Step 3)** The retailer participates in the intraday market to purchase/sell the deficit/surplus of its energy (second stage of the stochastic programming).
**Step 4)** The retailer determines the electricity price for the consumers considering the optimized operation in the two trading floors of electricity markets, i.e. day-ahead and intraday markets.

The HEMS receives the retail electricity price through the two-way communication. The HEMS optimizes the operation of the household appliances using the F-BA algorithm. Optimizing the problem, the HEMS sends the profile of the energy consumption to the retailer/DSO for the next 24 hours. This profile is used for two main objectives as follows:

(1) The retailer uses the profile to optimize the strategies of energy procurement in the electricity market floors.

(2) The DSO uses the profile of energy consumption to analyze the possible congestion occurrence in weak lines of the distribution network.

In order to relieve congestion, the DSO performs the congestion management according to the algorithm presented in section 3.9. The DSO determines the load points with the most impact on the congestion occurrence. The request of load reduction is sent to the candidate demands to relieve the congestion in the peak hours of the day.

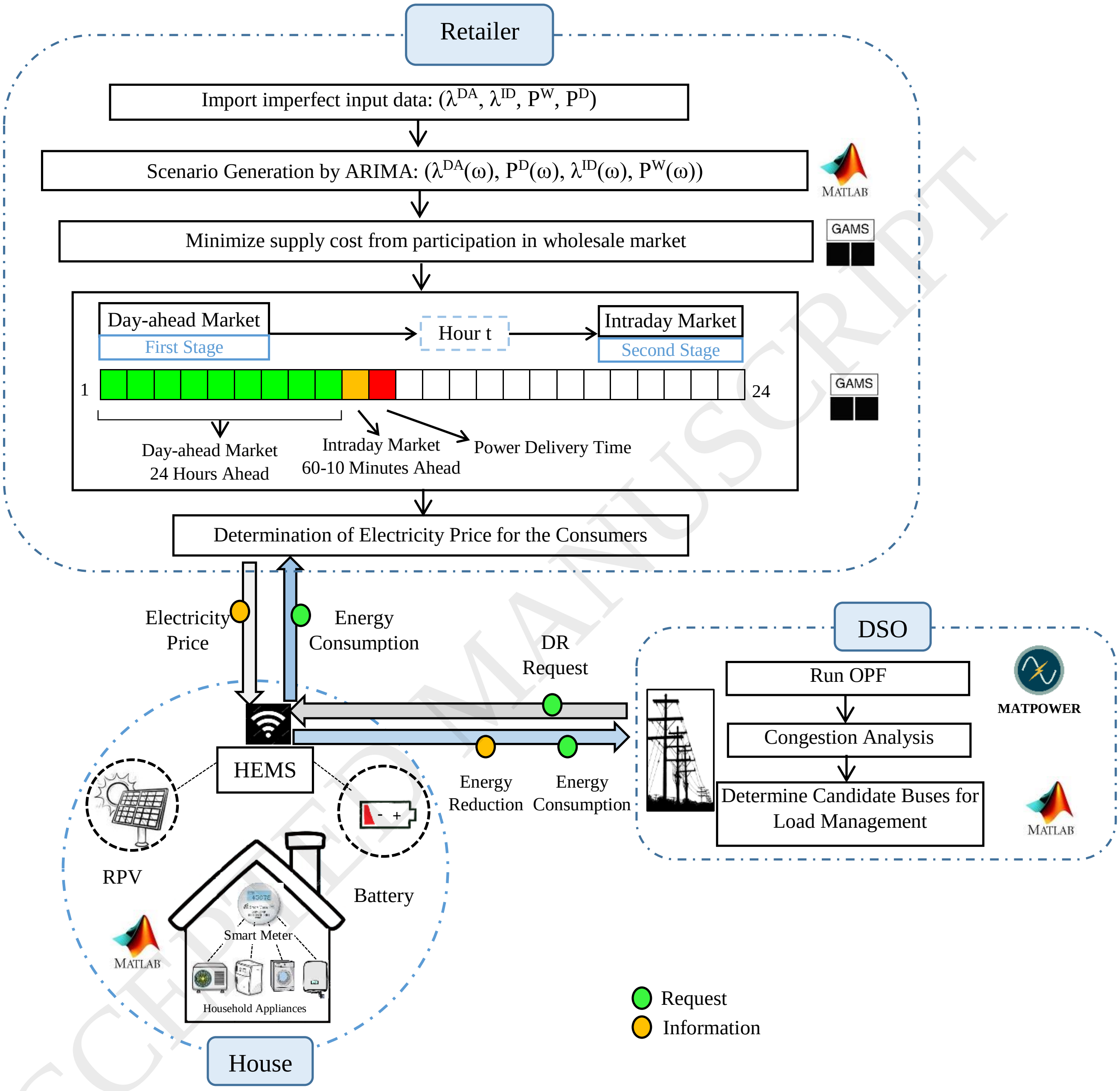



**Figure 4**. Problem breakdown structure with associated tasks and simulation/optimization software

## 4. Numerical Studies

In this section, the proposed approach is implemented on a case study from the Danish sector of the Nordic Electricity Market and a residential part of European LV distribution network.

### 4.1. Input Data

In this paper, a single retailer is considered to supply the energy consumption of a residential area during 24 hours of a day. The retailer procures its required energy from two trading floors of the Nordic Electricity Market [44], including day-ahead and intraday markets. In addition, the retailer has wind power self-generation facilities. The generation capacity of wind power is 10 MW. To hedge against the price uncertainty of the Nordic Electricity Market, a time series-based Seasonal Auto-Regressive Integrated Moving Average approach (S-ARIMA) is used. Moreover, to forecast the wind power, a Non-seasonal ARIMA model is considered. The historical data used for fitting the process of electricity price and wind power correspond to the days between September 2017 and February 2018. The number of scenarios generated by the ARIMA model is 50, 50 and 50 for the day-ahead market, intraday market, and wind power, respectively. Therefore, a total number of scenarios equals 125000 which is tractable. Figure 5 depicts the scenarios generated for electricity prices of Nordic Electricity Market.

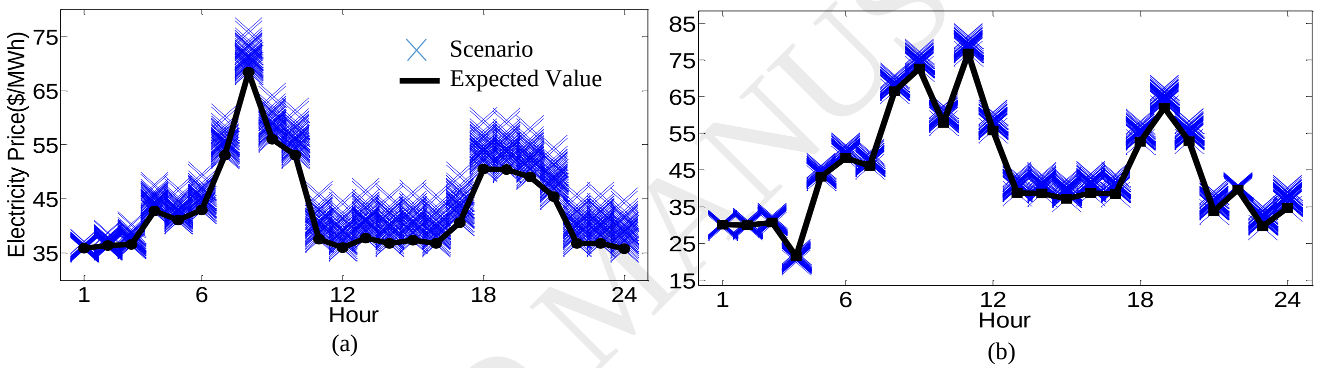



**Figure 5**. Electricity price scenarios for (a) day-ahead and (b) intraday markets , ARIMA Model $(p,d,q)\times(P,D,Q)_s=(1,0,1)\times(1,1,1)_{24}$

Regarding the consumption pattern, four classes of households are considered with different occupancy patterns. Figure 6 depicts the consumption pattern of household appliances for different classes [45]. To clarify the concept of thermal storage capability, the consumption profiles for TCAs and non-TCAs are described separately. It is worth mentioning that the comfort band of the TCAs in the traditional system is considered as follows:

(1) Electric Water Heater: The comfort band is the temperature of water between 60-70 $^0$C
(2) Heat Pump System: The comfort band is the temperature of room between 20-22 $^0$C
(3) Refrigerator: The comfort band is the temperature of chamber between 3-5 $^0$C

Table 2 illustrates the characteristics of the occupancy patterns.

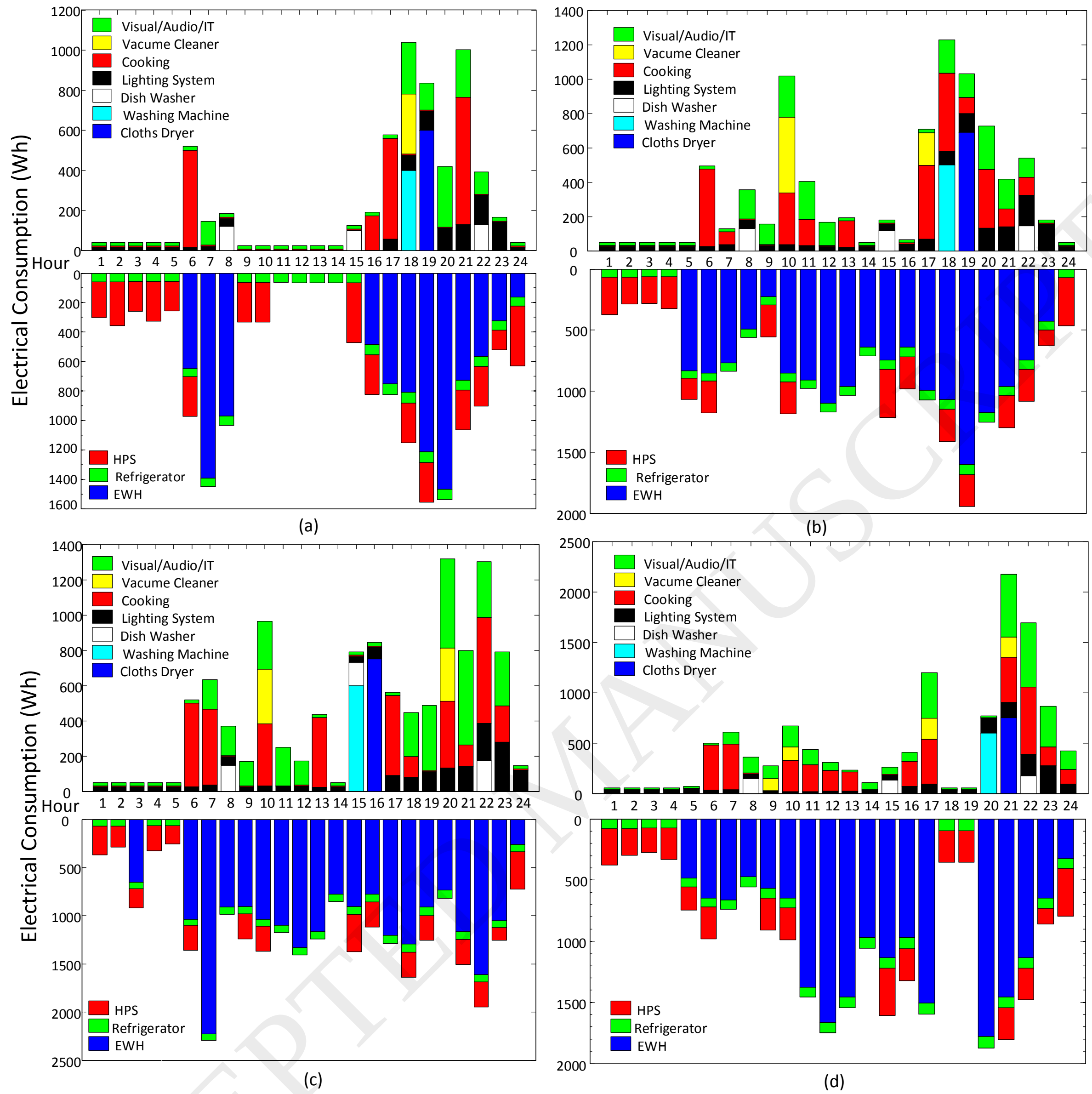



**Figure 6.** Traditional consumption of the household appliances for (a) class A (b) class B (c) class C and (d) class D

**Table 2.** Occupancy characteristics of households

| Class | Number of Occupants | Type of Occupants |
|---|---|---|
| **A** | 1 | One full-time employee |
| **B** | 2 | Two pensioners |
| **C** | 3 | Two pensioners with one full-time employee |
| **D** | 3 | One full-time employee with a housewife and student child |

In order to investigate the impact of HEMS on the operation of the distribution network, the residential subnetwork of the European LV Distribution Network Benchmark [46] is modeled in Figure 7. Distribution of household classes is described in the single line diagram. For example, bus 11 includes 17, 15, 10, 4 households of class A, class B, class C, and class D, respectively.

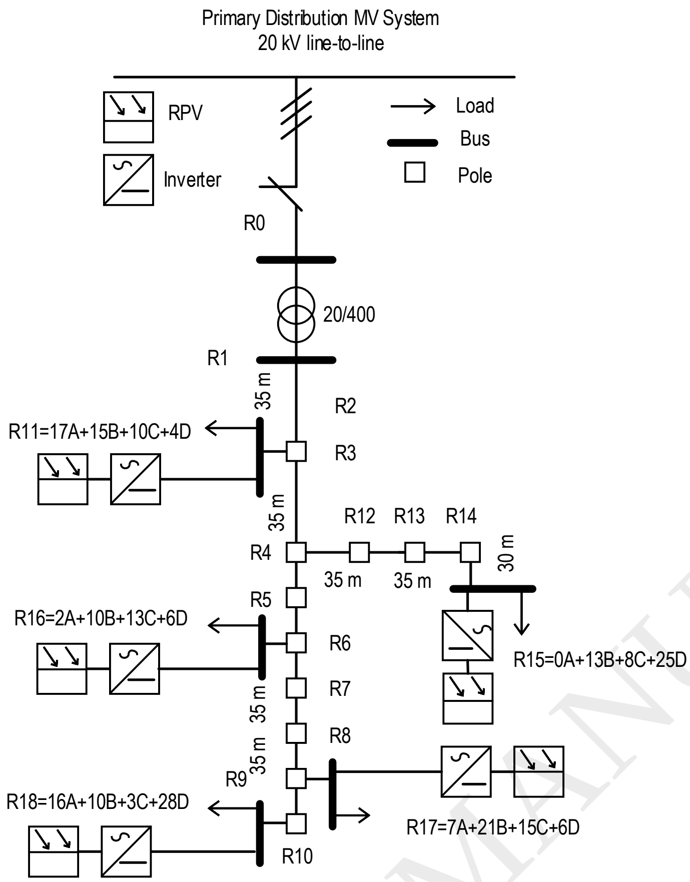



**Figure 7**. The topology of European LV distribution network, residential subnetwork *[46]*

Each household of classes A, B, C, and D is equipped with RPV-ESS with capacity of 1, 1.5, 2 and 2 kW, respectively. Performance data and technical characteristics of the RPV-ESS are described in Table 3. Estimated parameters of TCAs, i.e. EWH, HPS, and refrigerator, are described in the Appendix.

**Table 3**. RPV-ESS characteristics and simulation data

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| Solar Capacity (kW) | [1-2] | $SOC_0$ (%) | 30 |
| Battery Capacity (kWh) | [1-2] | $\delta^{Ch}$ (%) | 90 |
| Location | Copenhagen, Denmark | $\delta^{Dch}$ (%) | 90 |
| $SOC^{max}$ (%) | 100 | $\Upsilon^{Ch}$ (%) | 80 |
| $SOC^{min}$ (%) | 20 | $\Upsilon^{Dch}$ (%) | 80 |

## 4.2. Results and Discussions

This section presents the simulation results of the suggested approach. The problem is coded in three software, including GAMS 24.1.2, MATLAB 2014R and MATPOWER 4.0. The stochastic programming approach is coded in GAMS and solved using the CPLEX solver. The results of the electricity market are imported to the MATLAB to optimize the operational strategies of the household appliances. The MATPOWER is used to run optimal power flow on the distribution network. The abovementioned software is linked through GDX (GAMS Data eXchange) interface files [47].

The retailer participates in two trading floors of the wholesale electricity market. The retailer offers electricity price to the consumers through the HEMS for the next 24 hours considering the uncertainties associated with wholesale electricity price and wind power. The HEMS optimizes the operation of the household appliances in response to the electricity price. Note that the retail price is offered in an hourly-based dynamic pricing scheme. It is worth mentioning that the retailer is considered as a price-taker agent in the wholesale market. It means that the retailer has no market power capability in either of the aforementioned market floors to change the market clearing price.

The optimized strategies of the HEMS to minimize the cost of energy consumption are depicted in Figure 8. As the bar graphs reveal, the HEMS schedules the operation of TCAs, including EWH, HPS, and refrigerator, mainly in the low price hours, i.e. 1-3, 11-16 and 22-24. Regarding the HPS, the energy consumption is generally scheduled in the hours with the lowest electricity price, i.e. 1-3 and 11-13. For the EWH, the energy consumption is shifted to the low price hours next to the time of hot water usage. The reason is that the thermal model of EWH minimizes heat loss of the storage tank when it is not in use. Refrigerators are appliances whose thermal storage capacity is lower than the EWH and HPS. In addition, the energy consumption of refrigerators depends mainly on the door-opening pattern. For this reason, the refrigerator is operated in some high price hours, e.g. 7 and 19, to satisfy the comfort band of the households. There is a similar pattern for the EWH when the hot water demand is high. For example, due to the high demand for hot water during 20-22 p.m., the HEMS has to turn the device on despite the energy stored during low price hours, i.e. 14-17. In these hours, the HEMS uses low energy as much as possible to maintain the temperature of the water near the lower level of the comfort band. It is evident that the energy consumption of EWH in high price durations, i.e. 19-21 is lower than the low price durations. i.e. 14-17 p.m.

Based on the graphs, the energy consumption of EWH not only is shifted to the low price durations by the F-BA but also it is widely distributed within the different hours of low price durations through PFS. In fact, if the PFS is omitted from the HEMS, the main consumption of TCAs is scheduled in one or two low price hours. In contrast, when the PFS is incorporated into the HEMS, the energy consumption of TCAs are distributed within an interval of low price hours instead of one or two low price hours. In this situation, the HEMS prevent from creating a new peak consumption during low price hours.

Figure 9 makes a comparison between the general demand of retailer for both smart and traditional consumption patterns. As the graph reveals, shifting a great value of consumption to low price durations, not only a peak consumption is not created, but also a reduction of 8% in the value of PAR has been obtained. As a result, flat consumption is observed during low price hours, i.e. 1-3 and 11-13.

Figure 10 describes the temperature and consumption behavior of the EWH in response to the retail price for household class A. Based on the graph, the HEMS takes advantage of storage capacity to heat the water in low price hours meeting the demand in high price hours. In contrast, in the traditional consumption pattern, the EWH is heated during hot water usage, regardless of electricity price. In this situation, the EWH operates in the hours 7-8 and 19-21 when the electricity price is high. The comfort band of thermostat setting may be different for different occupants. In this paper, the comfort band of EWH is considered between 60 $^0$C to 70 $^0$C. Regarding the temperature behavior, the F-BA iterates until the temperature of all time spots satisfy the comfort band. The converged iteration shows the optimized consumption strategy.

Figure 11 illustrates the temperature and consumption behavior of HPS. Thermal comfort for the indoor temperature of the buildings is defined between 20 $^0$C and 22 $^0$C according to the ASHRAE standard [48]. As the graph reveals, the HEMS turns on the HPS in two low-cost durations, i.e. 1-3 and 11-12, to maintain the indoor temperature of the building within the comfort band. It is evident that the thermal dynamics of the building is much slower than the dynamics of HPS. Therefore, heating the HPS in hours 11-12, the peak of indoor temperature takes place in hours 16-17. Based on the graph, the F-BA iterates to obtain the lowest cost strategy satisfying the preferable temperature band of residents.

Thermal and consumption behaviors of the refrigerator for household class A is depicted in Figure 12. The comfort band for the chamber temperature is considered between 3 $^{0}$C and 5 $^{0}$C. Comparing the response of the HEMS to the offered electricity price with the traditional consumption pattern, it is shown that the refrigerator can be operated during low-cost hours to meet the temperature band during high-cost hours. It is worth mentioning that the operation of a refrigerator depends heavily on the door-opening pattern. For this reason, the operation of the refrigerator is mainly scheduled for the low-cost hours next to the high frequency of the door opening.

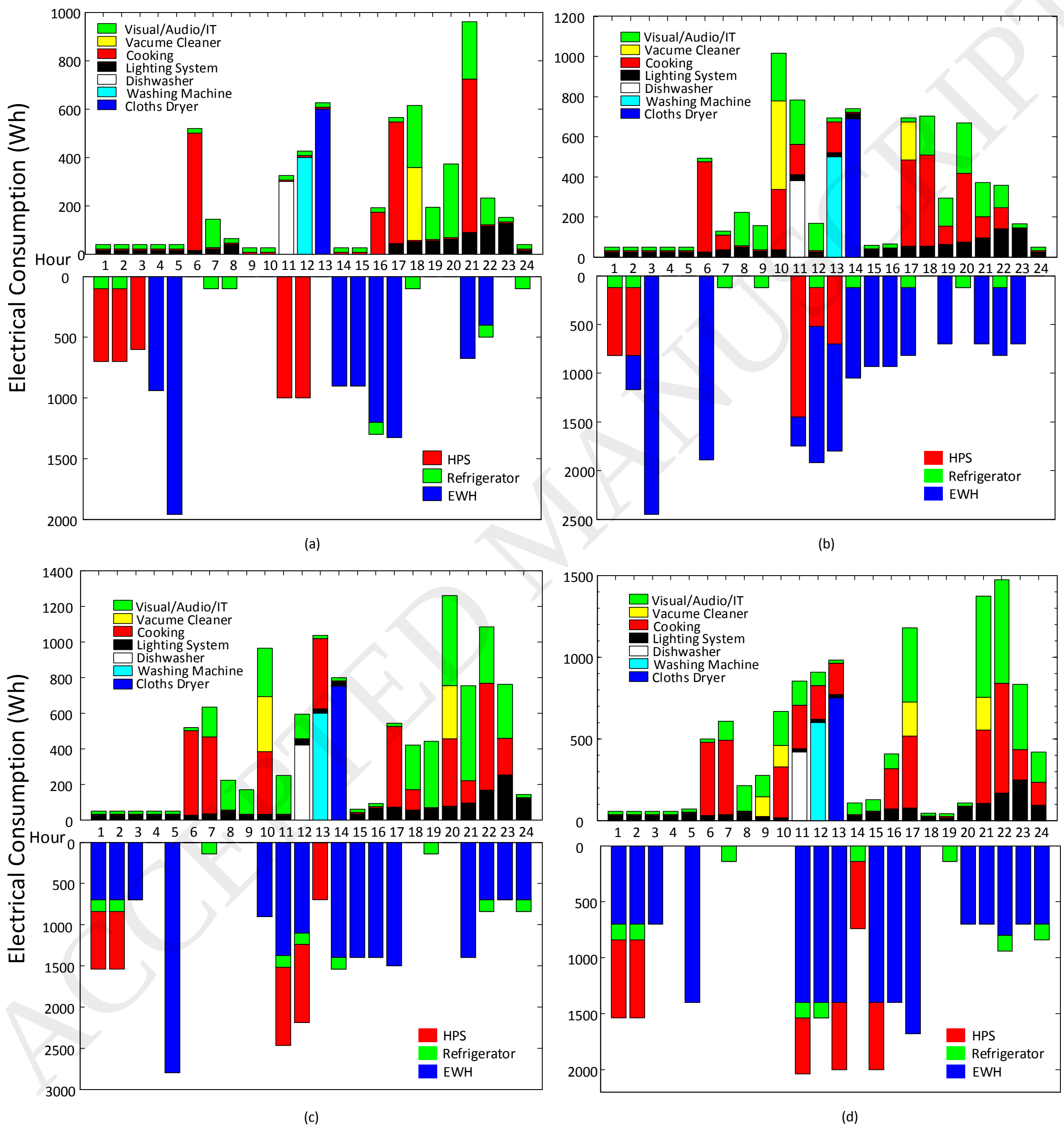



**Figure 8.** Smart consumption scheduled by HEMS for (a) class A (b) class B (c) class C and (d) class D

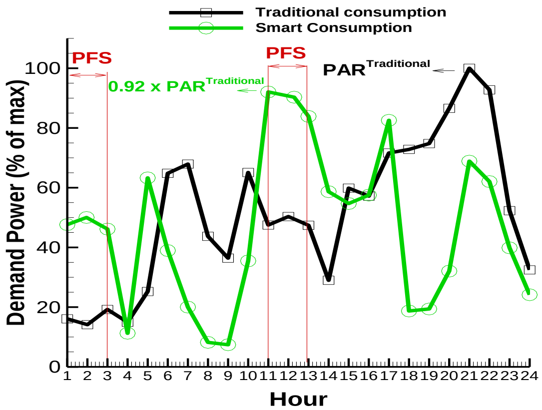
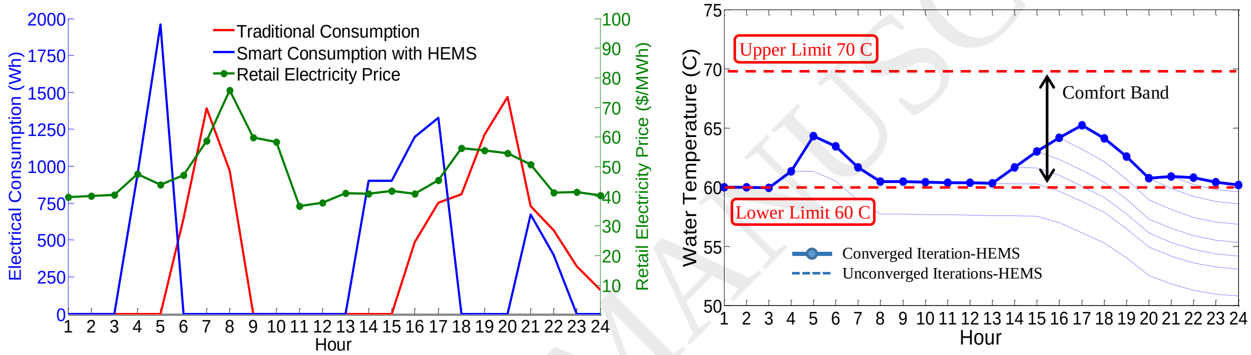
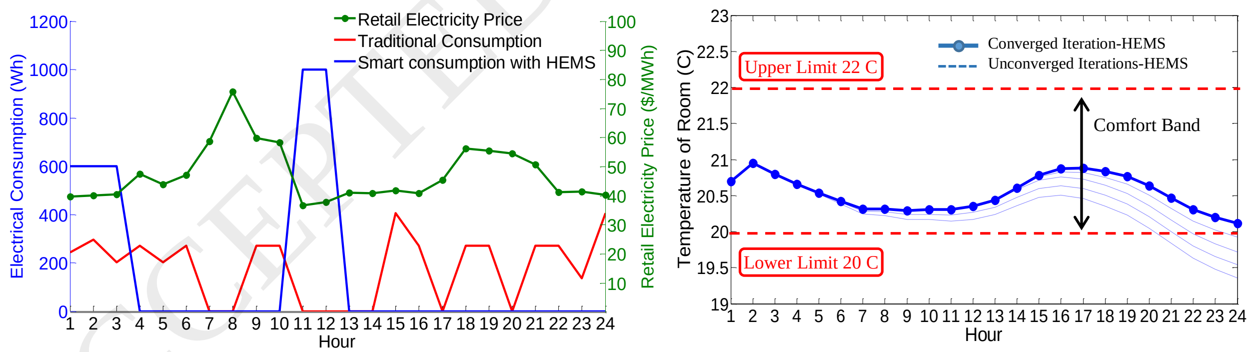



**Figure 9.** Total demand of retailer for smart and traditional consumption patterns




(a) (b)

**Figure 10**. The response of EWH to the HEMS for (a) Consumption behavior and (b) Temperature behavior




(a) (b)

**Figure 11.** The response of HPS to the HEMS for (a) Consumption behavior and (b) Temperature behavior

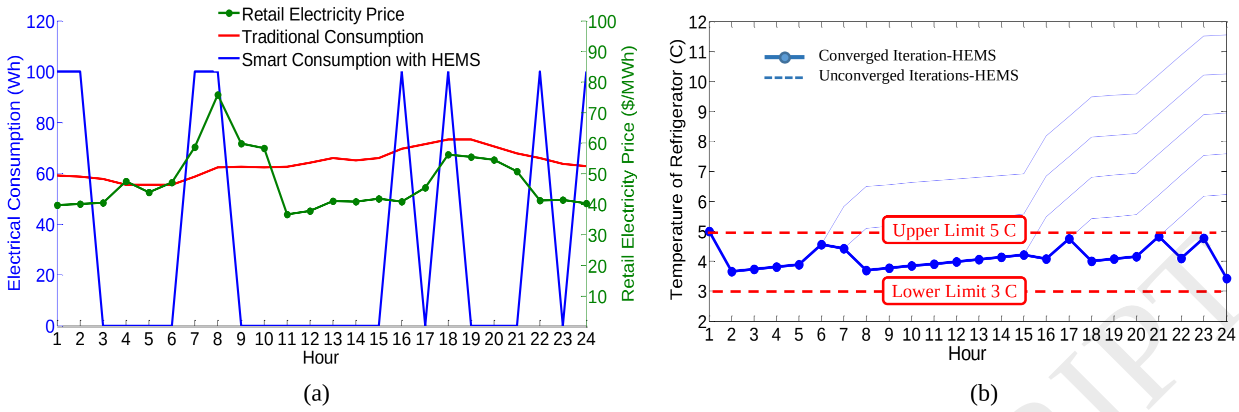



**Figure 12.** The response of refrigerator to the HEMS for (a) Consumption behavior and (b) Temperature behavior

Figure 13 illustrates the optimized operation of the lighting system in response to the electricity price when the retail price is more than the expected price. Based on the line graph, the HEMS dims the lighting demand during high price hours. It is evident that the delay-intolerant demand (emergency demand) is not dimmed during night hours, i.e. 17-21, by the HEMS.

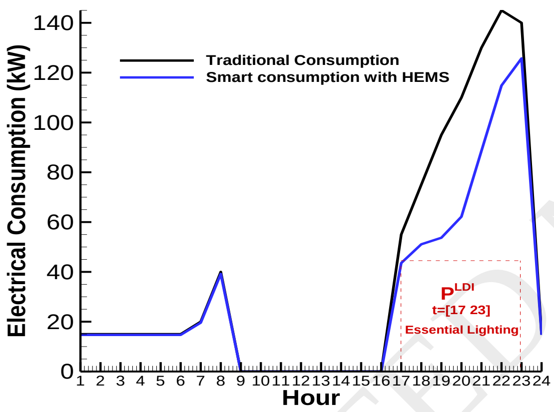



**Figure 13.** Optimized operation of the lighting system

The operation strategies of RPV-ESS is depicted in Figure 14. The ESS is charged from the RPV panels and is discharged into the grid to maximize the household's payoff. As the bar graph reveals, the most energy is injected into the grid at hour 18 when the electricity price is relatively high. Note that the power injection at hour 13 is due to the upper capacity limit of the storage system. In addition, because of low irradiation at 8 a.m., the ESS cannot inject the high value of power to the grid.

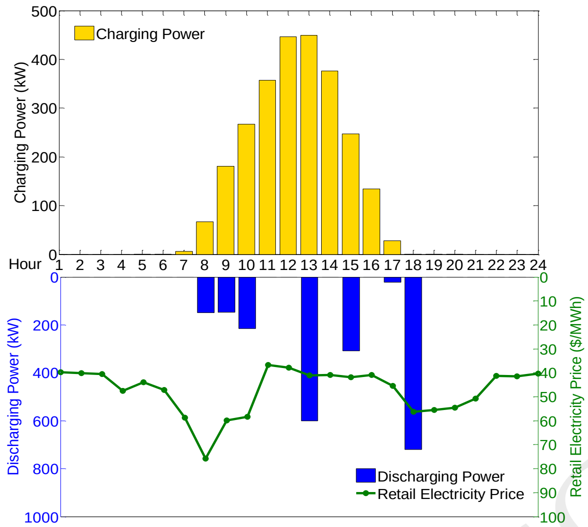



**Figure 14.** Optimized operation of ESS-RPV for 1 kW solar panel located in Copenhagen $(55.72,12,38)_{x,y}$ *[49]*

Figure 15 describes the voltage magnitude of buses 17 and 18 for the smart consumption during the day. The simulation results show that a congestion occurrence in the distribution network causes a voltage reduction in two buses of the radial network, i.e. buses 17 and 18, at hour 11. In order to mitigate the congestion, the DSO has two different options: (1) congestion management through load shedding (traditional approach) (2) congestion management through the participation of the HEMS in demand reduction (smart approach).

Taking traditional or smart option, outage cost and inconvenience may be imposed on the DSO and households, respectively. Table 4 illustrates the simulation results of the congestion management for two options. Regarding the traditional decision, the DSO has to shed 25 % and 37 % of the demand at buses 17 and 18 to relieve congestion, respectively. In this situation, an outage cost and a great inconvenience are imposed on the DSO and households, respectively. In this way, only consumers of buses 17 and 18 participate in the congestion management. In contrast, in the smart approach, a DR request is sent to the households through the HEMS to increase/decrease the upper/lower comfort band of refrigerator/EWH. The request is sent to all consumers at the residential subnetwork. Figure 16 shows the consumption behavior of the EWH and refrigerator during congestion management. As the figure reveals, the EWH and refrigerator are switched off at hour 11 and the operation temperature changes lower than 1 $^{0}$C. The results show that by using the FCB for all consumers, the DSO can mitigate the congestion in weak lines preventing from load shedding. In this situation, no outage cost is imposed on the network operator.

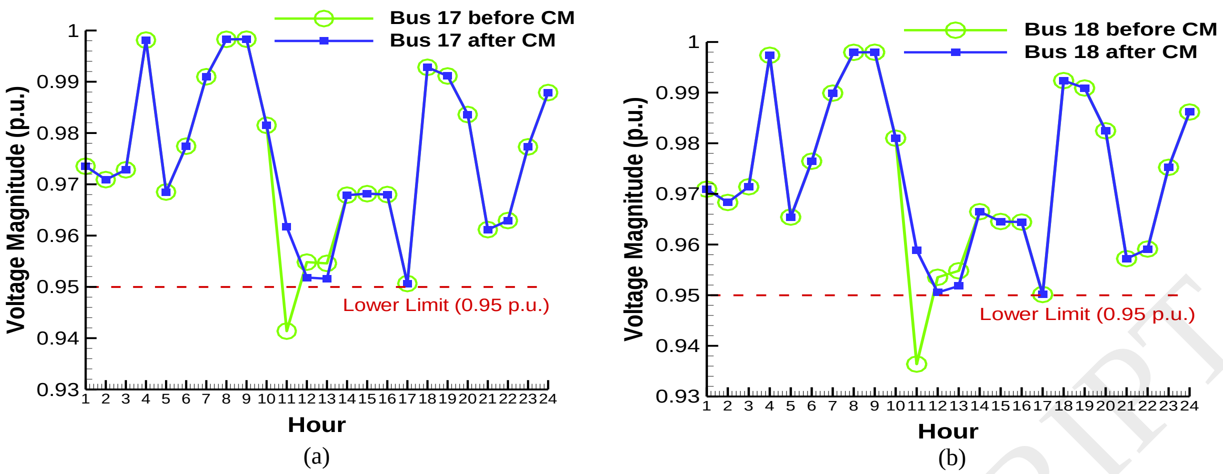



**Figure 15.** Voltage magnitude of buses 17 and 18 before and after congestion management (CM)

**Table 4**. Congestion management, associated cost, and inconvenience

| **Bus number** | **Congestion Occurrence** | **Mitigate Congestion** | | | | |
|---|---|---|---|---|---|---|
| | | | **Load Shedding** | | **HEMS Cooperation** | |
| | **Demand Level (kW)** | **Demand Level** | **Cost imposed on system** | **Inconvenience** | **Cost imposed on system** | **Inconvenience** |
| R11 | 145 | 145 | 0 | No | No | Yes, FCB |
| R15 | 126 | 126 | 0 | No | No | Yes, FCB |
| R16 | 80 | 80 | 0 | No | No | Yes, FCB |
| R17 | 120 | 90 | Fine Cost | 25% interruption | No | Yes, FCB |
| R18 | 135 | 85 | Fine Cost | 37 % interruption | No | Yes FCB |

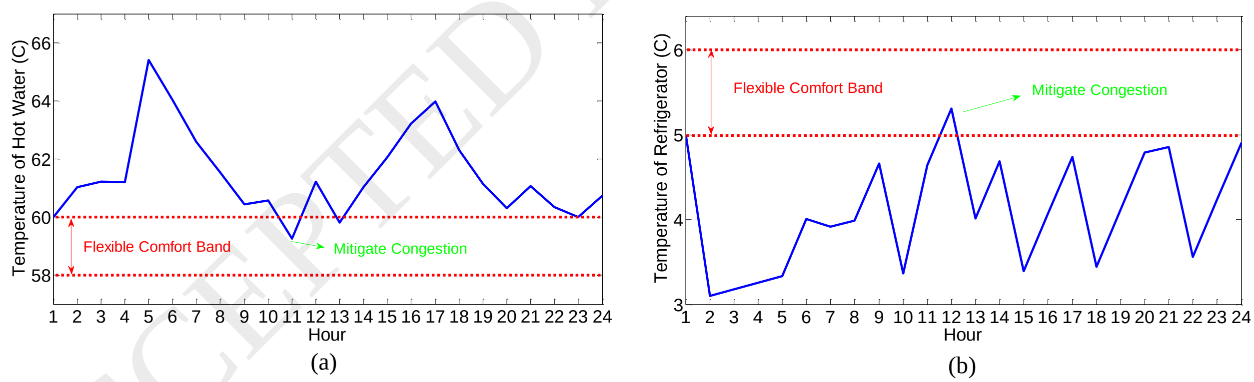
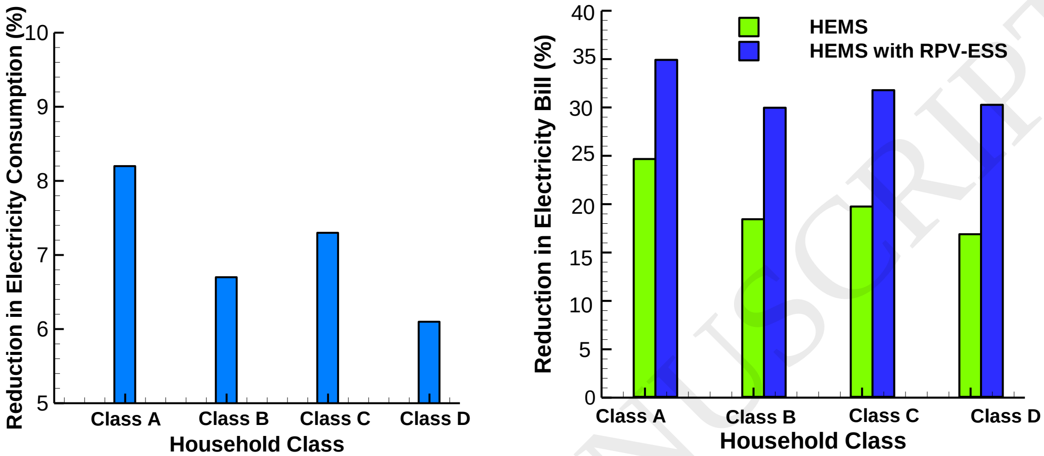



**Figure 16.** Flexible Comfort Band of TCAs during congestion management for (a) EWH and (b) refrigerator

Figure 17 describes the reduction in the electricity consumption of the TCAs and the total electricity bills due to HEMS optimization. First of all, regarding Figure 17(a), it is shown that HEMS control reduces the electricity consumption of the TCAs from approximately 6.1 to 8.3 % for different household classes in comparison with the traditional system. The reason is that the HEMS schedules the operation of TCAs near the lower/upper comfort band when the electricity price is high/low. Adversely, in the traditional system, the operation of the TCAs are scheduled regardless of the electricity price. Therefore, the TCAs have experienced a moderate reduction in electricity consumption during HEMS optimization. Secondly, the reduction in electricity bills is depicted in Figure 17(b). According to the bar graph, the HEMS causes 18 % to 25 % reduction in electricity bills for different household classes. Integrating the smart RPV-ESS with the HEMS, the electricity bills reduce 10 % more than the stand-alone HEMS. It is worth mentioning that the reduction of electricity bills is the result of two cases as follows:

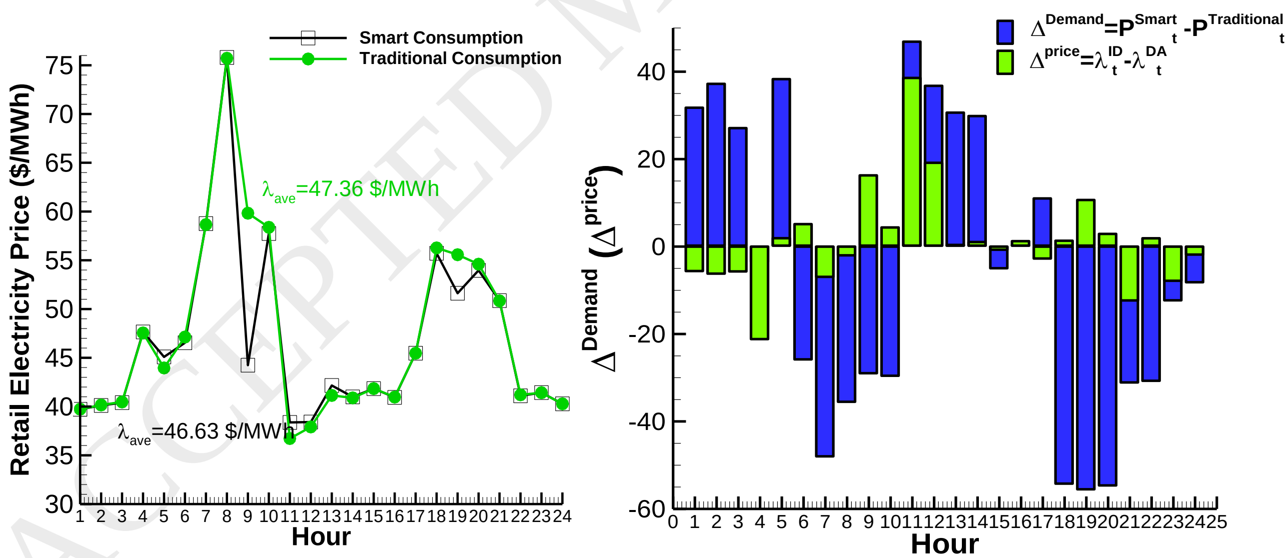
(1) Decreasing the electrical energy consumption
(2) Shifting the electrical energy consumption from high-cost hours to low-cost hours

Moreover, as the graph reveals, the most reduction in electricity bills occurs for household class A. It shows that the flexibility of the family of 1 is more than the families of 2 and 3 in response to the offered electricity price.




(a) (b)

**Figure 17**. Reduction in (a) Electricity Consumption (b) Electricity bills for different household classes




(a) (b)

**Figure 18.** The interaction between retail price and household consumption (a) comparative profile of retail price (b) difference between price and demand variables

In order to investigate the interaction between retail electricity price and HEMS optimization strategies, the daily profile of retail price for two consumption patterns, i.e. traditional and smart consumption, is described in Figure 18. The comparative profile of the retail price is illustrated in subfigure 18(a). To obtain the comparative profile, first of all, the retailer determines the electricity price for traditional consumers who do not have access to the HEMS. In this way, the households do not know the dynamic prices and cannot optimize their consumption pattern. Afterward, it is assumed that the consumers are equipped with the HEMS. The households have access to the dynamic retail price and optimize their consumption in response to the offered price. Changing the demand profile of consumers, the retailer determines the retail price for the new hourly electricity consumption. The comparative results of offered price for two states are described in subfigure 18(a). In this figure, there is barely any noticeable difference between the price of two consumption patterns except hours 9 and 19. In these two hours, the electricity price of smart consumption has experienced a considerable reduction. To find the reason of this problem, the difference between the values of two variables are depicted in subfigure 18(b) as follows:

(1) Difference between electricity price of day-ahead and intraday markets ($\Delta^{Price=}\lambda^{ID}$-$\lambda^{DA}$)

(2) Difference between electricity demand of households for traditional and smart consumption patterns ($\Delta^{Demand}$=$P^{Smart}$-$P^{Traditional}$)

Based on the bar graph, the hours 9 and 19 have two common features as follows: (which are not observed in the other hours)

(1) $\Delta^{Demand}$ is a negative large enough value

(2) $\Delta^{Price}$ is a positive large enough value

It means that for the hours when the difference between wholesale price of day-ahead and intraday markets is high, if the households decrease the electricity consumption considerably, the retailer can offer lower retail price maintaining the same level for its profit. Therefore, the retailer has decreased the retail price for hour 9 from 59 $/MWh to 44 $/MWh (a reduction of 25 %) and for hour 19 from 55 $/MWh to 51 $/MWh (a reduction of 7 %). In addition, the operation of the HEMS has resulted in a 1.5 % reduction in the average retail price during 24 hours. As a result, by using the HEMS, the retailers can offer lower electricity price to the households while their profit remains unchanged.

The economic interpretation is that if two abovementioned conditions are satisfied, the dependency of the retailer on the intraday market with high electricity price reduces; therefore, the probability of procurement from the day-ahead market with lower electricity prices increases. As a result, considering a predefined profit percentage, the retailer can offer lower electricity price to the consumers.

Regarding the computation and time burden of the optimization approach, the problem is run in a modern laptop with Intel Pentium CPU at 2.5 GHz and 4 GB of RAM. The computation time for three groups of appliances, i.e. TCAs, interruptible and curtailable appliances is about 5 seconds for one household. Simulating four classes of households, the computation time increases to 12 seconds which is reasonably well for a 24-hour ahead scheduling. In addition, when a congestion occurs in the distribution network, the MATPOWER requires 3 seconds to perform load flow in the distribution network.

## 5. Conclusion

In this paper, a comprehensive approach to Home Energy Management Systems (HEMS) is proposed studying the short-term effects from the electricity market to the local distribution network. Regarding the uncertainties associated with electricity price, the retailer uses stochastic programming to offer retail price to the consumers. The HEMS receives the hourly retail price and uses optimization approaches, i.e. Forward-Backward heuristic algorithm and linear programming, to minimize the operation cost of appliances. Reducing Peak to Average (PAR) of load profile and mitigating Congestion Rate (CR) are the objectives of two sub-problems which are added to the constraints of the master problem.

The results show that the HEMS can use a fast heuristic approach to minimize the energy consumption cost of thermostatically controllable appliances (TCAs). The proposed approach has a functional flexibility to incorporate the PAR reduction and CR mitigation into the problem. Using the suggested load function, the lighting demand of households can respond to the offered electricity price according to the expected flexibility of the residents. It is shown that by using the proposed HEMS, the households can experience a noticeable reduction in the electricity bills while the profit percentage of the retailer remains unchanged. In addition, the HEMS has strategic flexibility to relive CR of the distribution network by using the Flexible Comfort Band (FCB). Furthermore, the proposed Peak Flattening Scheme (PFS) distributes the shiftable demands to the interval of low price hours to reduce the PAR. Reducing the PAR, sub-optimality may occur in the problem; therefore, a trade-off between PAR reduction and cost minimization is inevitable. Studying the interaction between the retailer and HEMS, the retailer can offer lower electricity price to the households, especially in the hours when the dependency on the intraday market is low.

## Appendix

The following table describes the estimated parameters of TCAs.

**Table A.1.** Estimated parameters for TCAs, EWH *[7]*, refrigerator *[39]* and HPS *[40]*

| Appliance | Parameter | Unit | Value |
|---|---|---|---|
| HPS | $U_{fr}$ | kJ/$^0$Ch | 624 |
| | $U_{ra}$ | kJ/$^0$Ch | 28 |
| | $U_{wf}$ | kJ/$^0$Ch | 28 |
| | $C_{p,r}$ | kJ/$^0$C | 810 |
| | $C_{p,f}$ | kJ/$^0$C | 3315 |
| | $C_{p,w}$ | kJ/$^0$C | 836 |
| | $\eta$ | Scalar | 3 |
| | $\rho$ | Scalar | 0.1 |
| | $\rho^{water}$ | kJ/$^0$C kg | 4.18 |
| | $m_W^{tank}$ | kg | 200 |
| Refrigerator | $C_{r,c}$ | kWh/$^0$C | 8937 |
| | $R_c$ | $^0$C/kW | 1.4749 |
| | COP | Scalar | 0.58 |
| EWH | $R_W$ | $^0$C/kW | 1.52 |
| | $C_W$ | kWh/$^0$C | 863.4 |
| | M | kg | 200 |

## Declaration of interests: none.

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