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CLASSROOM DIALOGUE AS A FRENCH BRAID:
A CASE STUDY FROM TRIGONOMETRY

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Studies have shown that dialogues in teaching that follow the model of IRE (initiation, response, evaluation) are widely used in teaching despite it having some deficiencies. Based on the theories of collective learning and the social nature of thought, knowledge creation and learning, we argue in this paper that a 'captivating' dialogue which at a first glance seemed to follow the model of IRE can be understood as part of a process that initiates the students into the practices of school mathematics including learning to speak mathematically. We illustrate this point with an example from an upper secondary Norwegian class in trigonometry. Here the teacher combined asking open-ended problems to the students with IRE questions during a joint review of problem-solving using the principles of Polya.

INTRODUCTION

It is not uncommon to hear experienced teachers value the classroom interaction that develops between the teacher and the class while the class is introduced to and guided through mathematical reasoning or problem-solving. On the other hand, the literature on mathematics teaching tells that classroom interaction between the teacher and the students in general is dominated by obstacles for learning and pseudo discussions in the form of IRE (Initiation-Response-Evaluation), the Topaz effect, and 'Guess what the teacher has in mind' for instance seen in a state-of-the-art report by the Norwegian National Center for Mathematics Education (Nosrati & Wæge, 2014, p. 7).

This mismatch of decades of research and advice from the literature and what teachers do in practice motivated us to analyse examples of such dialogues. As Kilpatrick (1988) stated in a similar discussion of a discrepancy between the advice of educational researchers and the teachers' actions: "What is it that teachers know that others do not?"

This paper therefore aims to characterise and discuss a 'captivating' dialogue observed in a Norwegian upper secondary mathematics classroom in trigonometry, which at a first glance appeared to follow the IRE model. The concept of captivating dialogue is meant to capture a particular element of what is often referred to in classroom teaching as a joint review of mathematical reasoning or problem-solving, different from a teacher's lecturing. The captivating dialogue is guided by the teacher, focuses on the collective learning, and the utterances of each student become pieces of the class's shared knowledge.

THEORETICAL FRAMEWORK

Initiation-Response-Evaluation, IRE

IRE has multiple times been identified as the most frequently type of interaction in classrooms. Often it consists of at least 90% of the teacher's questions (eg. Heritage & Heritage, 2013; Hogan et al., 2014). It is regarded as an effective way for a teacher to check factual knowledge or recall but it may be criticized for not promoting understanding or higher order thinking because it gives no room for the individual student to elaborate on their answers or articulate their opinions or doubts, neither does it support classroom discussions.

However, sometimes what at first appears to be an IRE interaction can in fact be a dynamic teaching and learning encounter (Forman, 1989). Burbules and Bruce (2001) also argue that IRE interactions are beneficial to learning as part of a review and rehearsal. For example, some students can feel motivated by being able to answer a direct question and be rewarded for it; which can lead to greater confidence and motivation. If used in a skillful manner in the right context, it may even become more than plain rehearsal. Tainio and Laine (2015) shows example of Finnish teachers' emotion work through IRE classroom interactions. Other studies point to the importance of IRE as forming a predictable classroom routine and that the IRE interactions serves the purpose of aiding the students in learning the knowledge that previous generations have built up (Roth & Gardener, 2012). Hogan et al. (2014) also refers to a number of studies arguing that IRE is neither good nor bad in itself, it depends on the purposes and the particular occasions.

It therefore appears that although IRE interactions clearly have shortcomings, many studies also point to benefits of IRE interactions. Understanding the benefits of IRE depends on the understanding of learning, which we will discuss below.

Discursive approach and collective learning

Cobb et al. (2011) write about the collective learning of the classroom community in terms of the evolution of classroom mathematical practices. Lerman (1994) also argues for a shift in focus from the individual's 'understanding' to the social nature of thought, knowledge-creation, and learning. Later he writes: "In the mathematics classroom, interactions should not be seen as windows on the mind but as discursive contributions that may pull others forward into their increasing participation in mathematical speaking/thinking, in their zones of proximal development" (Lerman, 2002, p. 89). Thus, in a cultural, discursive psychological view, the students' answers should not be interpreted in terms of their grasping or understanding certain concepts, explanations or relations, but rather that the answers are interpreted as acts of participation. This is in line with Sfard (1998) who argues for perceiving learning as a combination between acquisition and participation. Participating indicates learning as a process of becoming a member of a community, thus taking part and being part of the conversation. In the Discursive Approach to mathematics education by Sierpinska (2005), the teachers' role in classroom conversations is characterised by an obligation to lead the discussion

in the direction of relevant mathematical ideas, themes, and issues. Apparently, this leading and direction of the discussion may sometimes be observed and misinterpreted as cases of teachers shaping the 'Topaz effect' (Brousseau, 1997).

In Lerman (2002) the principles of a cultural, discursive psychology are outlined and operationalised as tools of research with illustrating examples. Research in cultural, discursive psychology in mathematics teaching and learning includes the following elements: i) Intersubjectivity and internalisation, ii) The zone of proximal development and semiotic mediation, iii) Positioning and voice in classroom mathematical practices, iv) Social relationships, v) Mathematical artefacts, and vi) Development as a process of thinking/speaking mathematics. For the purpose of this paper, vi) is of particular interest as the learning of school mathematics is seen as an initiation into the practices of school mathematics including learning to speak mathematically. The teacher therefore has a vital role in showing what is approved within the discourse. In this view, the captivating dialogue may be interpreted as the teacher's introduction and development of elements of acceptable constructions by means of collective inclusion of the class' students into the teacher's review of mathematical reasoning or problem-solving etc.

Research Question

How can a teacher initiate the students into the practices of school mathematics in a captivating dialogue, which at first appears to include series of IRE interactions?

METHODOLOGY

The data consisted of video recordings of teaching in eight Norwegian upper secondary classrooms as part of the EU research project KeyCoMath (http://keycomath.eu/) about students' strategies for creative problem-solving (Andresen, 2015, 2017). The aim of the project was to develop and study teaching that encourages students' activity, inquiry, and autonomy. The recordings (around 30 hours in all) were done during the autumn of 2013 by one of the authors and translated below by the other.

This paper focuses on one sequence (15 minutes) and discusses the interactions between the students and the teacher (Tom). The discussions are illustrated with excerpts from the transcription of the sequence. The micro-contexts of these interactions are analyzed using Conversation Analysis (Have, 2007) where the focus is the "one phenomenon, the in-situ organization of conduct, and especially talk in interaction" (p. 27). The choice of just one sequence is made to be able to have some level of detail.

DATA AND ANALYSIS

Tom teaches a science class with 13 students at a private upper secondary school in Norway. Polya's (1985) scheme for problem-solving was introduced in the previous lesson. This lesson was spent on introduction of the use of trigonometry for solving problems. The next lessons were planned to be group work on a larger problem-solving project: modelling a Ferris wheel. Due to constraints of the length of the paper, we will

only show two pieces of the classroom dialogue, one from the beginning and one towards the end of the sequence where Tom reviews the students' work with a problem from a task picked from the National written examination the previous year. In the problem discussed in the excerpts, the students were supposed to find the length x where the angle α would be the biggest (see Figure 1, left). As part of the problem, they were asked to use the formulas for sin (u-v) and cos (u-v) to verify a given expression of tan (u-v) (see Figure 1, right).

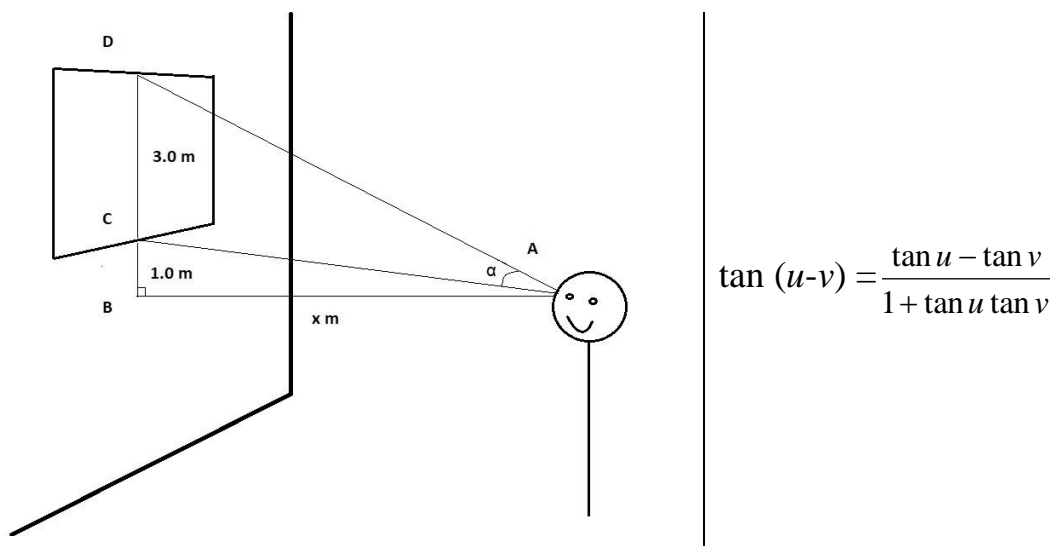$$\tan (u\text{-}v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Figure 1: Figure accompanying the problem (redrawn by the authors due to copyright) and the expression of tan (u-v).

During the sequence Tom writes on the blackboard, frequently turns around and moves forward when talking with the class and the turns back, writes etc. The atmosphere in the classroom is calm and friendly without tensions, the students concentrate on the teacher's review which aims to support their thinking more deeply about how to solve a problem in the actual case as well as in general.

Two minutes into the recording Tom talks while writing on the board and frequently turning around facing the students:

Tom: Did any of you have the need to clean up what you have been doing? … Polya's four steps. … Sometimes we do not have time to do this … to clean up the problem-solving and not just be happy with the answer and clear the thinking towards the end, it is a good thing to do. Think about, what did I actually use? The first thing I used was as Student 1 said an argument that *u*-*v* is actually two angles in play … The difference between two angles becomes a new angle. And then you can use that tan to the angle is equal to sin to cos to the angle. There were no cheap points in this problem, you see. [laughing] How was it with sin? [a little silence] sin is the difference…?

Student 2: Sin *u*, cos *v*, plus [Tom writes a formula on the board while two students discuss minus or plus for 10 seconds].

Tom: Minus or plus?

Student 3:   Minus.

Tom: Yes. … [Another student says Yes] What is most interesting here is not to remember this. It is not what mathematics is.

Here, the last three lines of the teacher–student exchange apparently follow the IRE model. However, seen in context this exchange serves to constructively conclude the longer discussion between two students about minus or plus. Tom redirects focus towards the principles of cleaning up problem-solving. Far from neglected, the two students' discussion become a minor part of the sequence's overall goal, which is emphasised in the lines after "Yes".

In the second excerpt towards the end of the sequence, Tom is aiming to understand the students' thinking:

Student 4:   Thinking that, the basic that tan is sin to cos. You can try to divide with cos and see how this goes. This is very well.

Tom: So actually a kind of trial and error?

Student 4:   Yes actually.

Student 5:   Take cos, pick out.

Tom: Aha, so you recognised that one of this - so it looks like this is minus and this is minus this is plus. And if it was the one we have to divide by the same as what we have. This was what you were thinking?

Student 5:   Yes.

Tom: Aha. Then here we have minus and minus and plus and plus. Here we have $\sin u$ divided by $\cos u$, and $\sin v$ divided by $\cos v$, how does this look, I think that this is actually what you discovered. But can you actually just take an expression and divide it by something? Is that allowed? [Some seconds silence] You don't know. Can you just divide by $\cos u$ or $\cos v$? Or how? [some hands showing].

Student 2:   [Mumbling] numerator and denominator, it is ok to divide by the same.

Tom: So actually, I multiply by one over cos u times cos v and then I multiply cos u times cos v. Is this what you are saying.

Student 2:   Hmmm.

Tom: Are we allowed to do this?

Student 6:   It is usual calculation with fractions.

Tom: Yes, it is usual calculation with fractions. Is it allowed?

     [Several silent seconds].

Students: [Mumbling] denominator.

Tom: Yes, the denominator. What about this?

Students:	[Mumbling; then several silent seconds].	
Tom:	Does the problem state any limitation?	
Students:	[Mumbling; then several silent seconds].	
Tom:	What is it that we cannot take tan to?	
Student 7:	90 degrees.	
Tom:	90 degrees. Or 270 degrees. Can u-v be 90 or 270? If u is 270 and v is 90 for instance, they could be like this, right? You can take tan to 180, look here, if this is 270 and this is 90. It is working.	

In this part several students participate in the interaction with Tom. First Tom meets the students in what they have done, then he challenges them. The students draw on their pre-understanding about fraction and contribute to the joint construction of the class' knowledge. The 'mumbling' seems to be part of the student–teacher interaction and might be interpreted as a sign of the students' work in progress. Like in the first excerpt, Tom interchanges between open and closed questions and deliberately uses brief and clear IRE-like exchanges to constructively conclude the review's single steps. The conclusions are added to the class' shared knowledgebase.

DISCUSSION

In the first excerpt, Tom used an IRE interaction to constructively conclude a discussion between two students about if a minus or a plus should be used in the formula. Alternatively, the students, who already had spent some time on the task before the sequence might have spent further time on settling this detail. This would result in getting side-tracked from the original purpose of the sequence. The individual student's understanding of all details is not important at that moment. Each student can later check it out or ask the ones that do know. By now, collective learning (Cobb et al., 2010) has taken place and *the class* knows that it is minus. Tom focuses on the class as a whole and makes sure that the class is on the path he has decided for this sequence. This is in line with Sierpinska's (2005) views of the role of the teacher as someone who has the responsibility of leading a class in a relevant direction.

One main characteristic of this captivating dialogue in the classroom is the students' joint focus on the review of mathematical reasoning. Focus is not on individual students or on single answers; the teacher directs the review and keeps it intentionally proceeding on path, in accordance with the planned learning trajectory. In line with this, the classroom dialogues will not necessarily reveal whether each single student has learnt what the teacher had set as the learning goal since the teacher–student exchanges do not serve as windows to the students' minds. In line with Lerman (2002), the students' contributions are not necessarily a window into the student's mind, but an act of participation. However, following Sfard (1998), though, the students who attended the review have learnt from it since learning may (also) be interpreted as progressive participation in existing and joint activity. It is worth mentioning that the

teaching sequences were aimed at developing problem-solving skills – it was not a drill-sequence.

The concept of 'captivating dialogue' is distinct from what is commonly understood as an IRE dialogue in important aspects: i) The questions do not serve as a means for checking the student's mind in the form of checking knowledge or ways of thinking ii) Since focus is on the review, not on the individual student, the teacher will not necessarily pick the respondent. iii) The response to a student's answer should be interpreted as inclusion of the student into the joint activity, rather than assessment of the student's performance. The term 'captivating' was vindicated by the students' engagement in attempts to answer the questions and by their participation, sometimes only through mumbling.

CONCLUSIONS

In this paper we wish to argue for seeing teacher–student interactions in the classroom in context with their aims and goals, content, and the roles they play for the development of new interactions. We have discussed a case of classroom interactions that at first appeared to have a lot in common with IRE interactions; they indeed showed IRE-pattern but when analysed within a framework of collective learning, we could interpret them in terms of an initiation into school mathematics practice and communication. One question the analysis raises is, if the interaction could be characterised as following the IRE model or not. From one perspective it was IRE, as we observed a teacher centred dialogue which many times where short questions that were then answered, evaluated by the teacher, then followed by a new question. A simple count of percentage of different types of question would however never suffice to catch the context of these questions, the function that each question and answer has in the class interaction and the development of them.

In the title of the paper we use the term 'French braid' which is a certain type of way to weave/braid hair into a tail where gradually more and more of the hair is braided into the rest. This is a metaphor which we find describes well how the teacher braids the input of the different students into the shared knowledge of the class in this collective learning process.

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