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INSTRUMENTAL MEDIATIONS AND STUDENTS’ IDENTITIES

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INSTRUMENTAL MEDIATIONS AND STUDENTS’ IDENTITIES

Abstract – This theoretical article considers the relation between the use of digital tools in the teaching and learning of mathematics and the development of students’ identities as mathematical writers, within their upper secondary school education. The theoretical outset is the instrumental approach in mathematics education, which studies how students appropriate digital artifacts as tools. We argue that the focus on epistemic and pragmatic mediations, which has long been prevalent within this approach, can benefit from being augmented with a focus on the students’ identity work. The interest for students’ identity work in relation to their use of digital tools in mathematics grew out of a longitudinal ethnographic case study of Danish upper secondary students in the subject of mathematics. Using data excerpts from this study we provide empirical examples of an identity perspective on the use of digital tools in mathematics education and discuss possible ways to incorporate “identity” and ”identity work” in the instrumental approach.

Key words: student identity, instrumental approach, technology, case study.

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INTRODUCTION

The construction of students' socially situated identities as writers is receiving growing attention by several scholars, mainly in first language education (Hyland, 2009). The increased use of technology in text production significantly expands the possible genres and modalities of students' productions of texts and thus also affects the way students – consciously or unconsciously – take on different social identities as writers in their written texts and in the social act of writing (Ivanič, 2006; Gee, 2003; Kress, 1997). In upper secondary mathematics programs – at least in Denmark – the use of technology, both as a tool for solving tasks and as the main medium for students' production of mathematical texts, has increased dramatically over the last decade. From year 2018 all national written examinations in mathematics at the upper secondary level are partly digital, which means that the students are obliged to produce their written exam papers using digital tools. At many of the examinations in the Danish upper secondary education, including exams in the subject of mathematics, students already use different types of aids, including ICT. In this article we address the question of what might be the interplay between students' *identity work* (Gee, 2003) and their use of technology for producing texts in the subject of mathematics. In our empirical investigation of this question, we attempt to sustain a perspective on identity as something we do, which is in line with the literacy tradition and its identity work (Ivanič, 2006; Gee, 2003), in combination with the mathematics education framework known as the *instrumental approach* (Artigue, 2002; Guin, Ruthven & Trouche, 2005).

The main theoretical thesis of the article is that a focus on epistemic and pragmatic mediations, a core aspect of the instrumental approach (see later section), which has long been prevalent in mathematics education research on technology, could benefit from being augmented with a focus on students' identity work. The background for this theoretical point is our reflections concerning certain empirical phenomena that we observed in relation to a recent longitudinal ethnographic case study of Danish students' development as mathematical writes (Iversen, 2014). The purpose of that particular case study was not primarily the investigation of students' use of mathematical digital tools, e.g. Computer Algebra Systems (CAS), rather the focus was on the students' mathematical writing process in general. But as empirical data from the study were collected it became clear that these tools played a central part of the students' development as mathematical writers as well as in the students' general mathematical development. At the same time it became clearer to us that the students' use of such tools in their production of mathematical texts was much more multifaceted than we originally had imagined, but also that the instrumental approach in combination with the notion of identity work might capture and better describe the students' development as mathematical writers. In support of our thesis, we therefore draw on the empirical data from the case study of Danish students' development as mathematical writers (Iversen, 2013; 2014), and we analyze the data presented by means of a "methodological frame" consisting of a combination of the instrumental approach to the use of tools in mathematics education and knowledge of writers' socially situated identities from first language education research. But before we present the frame for our analysis, we first present the theoretical constructs making up the basis for it, i.e. the constructs of the instrumental approach and students' identity work. The data, which is of a Danish upper secondary student called Emil, is made up from three excerpts of data, each illustrating Emil's identity work with digital tools in relation to his production of texts in mathematics. Each excerpt is followed by a sub-analysis. The sub-analyses are wrapped up in the final concluding discussion.

INSTRUMENTAL MEDIATIONS

The instrumental approach (Guin, Ruthven & Trouche, 2005) addresses students' use of technology when learning mathematics from the perspective of appropriating digital tools for solving mathematical tasks. It builds on two different frameworks; one from cognitive ergonomics (mainly Verillon & Rabardel, 1995) and one from mathematics education, conceptualizing how learning and development occurs through the evolution of schemes (Vergnaud 1996; 2009). Trouche (in Guin et al., 2005, p. 148) describes instrumental genesis as a bidirectional process of instrumentation directed toward the subject and instrumentalization directed toward the artifact, and suggests that "this process goes on through the emergence and evolution of schemes [...] while performing tasks of a given type." This approach views computational artefacts as mediating between user and goal (Rabardel & Bourmaud, 2003). An important aspect of this conceptualisation is that humans have goals on various levels, and that goals of smaller actions can feed into larger plans (Nardi, 1996). Furthermore, the approach presupposes a continuation and dialectics between design and use, in the sense that a pupil's goal directed activity is shaped by his use of a tool (the process often referred to as *instrumentation*) and simultaneously, the goal directed activity of the pupil reshapes the tool (the process often referred to as *instrumentalization*) (Rabardel & Bourmaud, 2003).

When describing students' work with technology, the instrumental approach discriminates between epistemic mediations and pragmatic mediations (Artigue, 2002; Guin et al., 2005;

Rabardel & Bourmaud, 2003). In the words of Rabardel and Bourmaud (2003, p. 668), epistemic mediations to the object are "mediations aiming mainly at getting to know the object (its properties, its evolutions in line with the subject's actions, etc.)" while pragmatic mediations to the object are "mediations concerning action on the object (transformation, regulation management, etc.)" In relation to mathematical work, this distinction operationalizes the difference between learning with technology and merely using technology to solve tasks. Epistemic mediations relate to goals that are internal to the user, i.e. affecting his or hers conception of, overview of, or knowledge about something. Pragmatic mediations on the contrary relate to goals outside of the user, i.e. making a change in the world.

Rabardel and Bourmaud use a microscope and a hammer as examples of these two types of mediations, whereas Lagrange (2005) refers to experimental uses of computers (as epistemic) and to the mathematical technique of "pushing buttons"as pragmatic.In the words of Rabardel and Bourmaud:

> mediations aiming mainly at getting to know the object (its properties, its evolutions in line with the subject's actions, etc.) that we call epistemic mediations to the object. The microscope is a good example of an artifact organized around this type of relation. [...]
>
> mediations concerning action on the object (transformation, regulation management, etc.) that we call pragmatic mediation to the object. The hammer is an example of an artifact primarily organized around this type of component. (Rabardel and Bourmaud, 2003, p. 668-669)

Finally, Rabardel and Bourmaud (2003, p. 669) introduce sensitivity to a broader conception of the *orientation* of the mediation. Instrumented mediations may be directed towards (a combination of) the object of an activity (the solution of a task), other subjects (classmates, the teacher), and oneself (as a reflective or heuristic process), this direction of the mediation becomes important for the identity construction that may be affiliated with the instrumental approach. When looking at identity, it makes sense to look at reflective mediations defined as mediations directed towards the self. Mediations may be directed towards objects, towards other people, and toward the self. Mediations towards others are described as *interpersonal mediations*, although the "the subject is also in relation with him/herself. He/she knows, manages and transforms him/herself." Rabardel and Baurmaud (2003 p. 669) describe this as "'reflexive mediations' through which the subject's relation to him/herself is mediated by the instrument". Reflective mediations may thus be considered as related to identity. Knowing, transforming, and managing yourself is related to your identity. Still, obtaining specific goals may also be considered as part of one's identity, and so may communication.

Hence, in the instrumental approach, identity is something that cuts across the different kinds of mediations and which relates to the entire mediating system. In the following section we will address the notion of “identity work” to clarify and articulate the way artifacts relate to the building of identity.

IDENTITY WORK

In her recent and comprehensive review, Darragh (2016) goes through the use of the concept of identity in 188 studies related to mathematics education. One of her main conclusions is that identity research within this area usually falls under one of two distinct paradigms. Either identity is regarded as an acquisition, thus fitting within a psychological frame; or it is regarded as an action which fits within a sociological frame. Darragh connects the first paradigm to Erikson, referring to this as the Eriksonian identity, meaning that it is something a person possesses and that it becomes coherent and consistent over time. The second paradigm is due to Mead, the Meadian identity, and is something a person does and which is multiple, at times contradictory and, not least, socially constituted. Darragh remarks: “Many of the key theorists drawn on by mathematics education researchers would fit their definitions within a Meadian view. Identity is generally agreed to be multiple or referred to in the plural. Furthermore, these influential theorists treat identity in terms of an action rather than an acquisition.” (p. 9). Based on her review and characterization of the use of the identity concept in mathematics education research, Darragh makes the following proposal:

> I suggest that by defining identity as something we do, be it identity-work or identity as performative, we form a sociological understanding and distinguish this concept from the others of a psychological paradigm. This sociological perspective of identity provides us with the opportunity to differently view peoples' experiences of mathematics learning and teaching; it provides something new. (Darragh 2016, p. 11).

Within a literacy perspective, Ivanič (1998, 2004, 2006) argues that students' learning is closely linked to processes of identification, meaning the extent to which students identify with the values, beliefs, goals, and activities that prototypical participants in the learning activities represent. Ivanič writes:

> The default meaning of ‘identification’ turns ‘identity’ from a noun to a verb: it treats identity not as a state but as a process […]. There is also a stronger, more active meaning of ‘identification’ which is found particularly in Wenger (1998) and Gee (2003, 2005): a desire to identify. Identification in this strong sense is essential to full participation, and is what makes identity work happen. When participating in an activity, it will make a massive

> difference whether a person does or doesn't identify with the sort of people who are its 'subjects', and whether they take to themselves its 'object(s)'. (Ivanič, 2006, p. 14)

Further, he suggests that identification is "the key factor in learning, in language learning and in the transformation of practices across contexts" (Ivanič, 2006, p. 1), and we share the view with Ivanič and others (e.g. Gee, 2001, 2003, 2005; Hall, 1996; Kress, 1997, Lave & Wenger, 1991; Wenger, 1998) that processes of identification is an important element of students' learning processes. In continuation, we see identification and identity as useful analytical concepts, when one wishes to address important aspects of teaching-learning processes in mathematics.

The concept of identity has in recent years become prominent, both in writing research (Hyland, 2009) and in mathematics education (e.g. Braathe, 2008; Darragh, 2016; Grootenboer & Zevenbergen, 2008; Heyd-Metzuyanim, Lutovac & Kaasila, 2016; Sfard & Prusak, 2005; Steentoft & Valero, 2009). Hence, Gee's suggestion to take the concept of identity as an analytical lens has found resonance in several areas of educational research (Gee, 2001). According to Steentoft and Valero (2009), the rapidly increasing use of the concept of identity in mathematics education is related especially to the fact that more and more studies take socio-cultural and post-structuralist theories as their starting point. We add that this development likely goes hand in hand with the growing number of studies in mathematics education that in one way or another focus on the concept of discourse (e.g. Ryve, 2011), since such a focus often means attention to the construction and negotiation of social identities in different contexts (De Fina, 2011).

As Gee (2001) emphasizes, the concept of identity is used in many different ways in educational research. In what follows, we adhere to the part of the research literature that describes identity as a social phenomenon (i.e. Median as opposed to Eriksonian), which is mutually co-constructed between different participants in a specific situation, and which is linked to the discourses that are available to participants in the given situation. In the words of Hyland (2009, p. 70), "identity is something we do; not something we have." All of us do identity all the time, and this doing has been coined as *identity work* by Gee (2003). In this way, identity can be understood as negotiated ways of participating in different social groups, cultures and institutions. And one of these ways of participating could be a certain way of using digital tools in the writing assignments in relation to the teaching and learning of mathematics.

Hence, in their mathematical writing, students are positioned and are positioning themselves in relation to the different socially situated identities that are available to them in a given school culture, for example, by affiliating or not affiliating themselves with positions that are available to them in given teaching-learning situations – caricatures might include the "dutiful school student", the "mathematician", or the "CAS-expert". Identities as such are not immutable positions that people can choose to take on or not take on. They are never conclusively determined and thus always – in principle – negotiable and therefore possible to change, even within educational institutions typically embedded in asymmetrical power structures (Ivanič, 2004). Hence, when we study the identity work of a student in the context of his written mathematical assignments, we do so in a context, that is influenced both directly by the teacher and his/her attitude to technology, by classmates, and by the institutional frames of the activities. Our focus, however, is on how the student presents himself related to the use of technology in mathematical activities to changing teachers.

INSTRUMENTAL MEDIATIONS OF IDENTITY WORK

As evident from the presentation of theoretical constructs above, students' pragmatic mediations (p) and students' epistemic mediations (e) in relation to use of technology have been well described and researched, as has the relationship between students' pragmatic and epistemic mediations (p-e). Students' identities in relation to use of technology in mathematical activities (i) have also to some extent been the topic of study (e.g. Iversen, 2014). However, as pointed to previously, it is not only the relationship between students' pragmatic and epistemic mediations which affect students' use of technology, the relationships between these two types of mediations and students' socially situated identities, respectively, are important as well – we shall refer to them as (i-p) and (i-e). In fact, also the relationship between all three 'entities' (p-i-e) may be in play at given times for given students in relation to a given mathematical activity involving use of technology. An illustration is provided in Figure 1.

The illustration in Figure 1 shall serve as the basis for our analysis of the empirical cases to be presented later. In a sense, we use the continuum along the two axes as outset for our analysis – that is the axis i-p (identity and pragmatic mediations) and the axis i-e (identity and epistemic mediations). We thus combine the constructs of identity and identity work, from Gee (2003) and Ivanič (2006), with the constructs of epistemic and pragmatic mediations ending in mathematical work as obtained from Artigue (2002) and Guin et al. (2005).

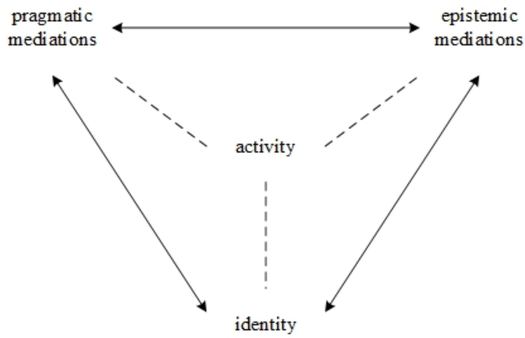


Figure 1. - A schematic illustration of the interplay between the three 'entities' of students' pragmatic mediations, their epistemic mediations, and their socially situated identities in relation to use of technology in mathematical activities.

To illustrate the importance of the identity dimension in students' mathematical work involving technology, we offer a case study of a Danish upper secondary school student, Emil. More precisely, we offer three sub-cases, two which may be seen as illustrating the i-p and the i-e axes of Figure 1, respectively, and one which illustrates a situation involving the interplay of all three axes (p-i-e). But before we get to an introduction of the case student and the presentation and analysis of the three sub-cases, the specific educational setting in which the empirical data were generated should be explained.

EDUCATIONAL SETTING AND CASE DATA

In Denmark students can choose between different types of upper secondary education programs, for example the Gymnasium (general upper secondary examination program), the higher technical examination program, the higher commercial examination program, etc. Danish upper secondary education is most often three years, and students may take mathematics at one of three levels (C, B or A), depending on the number of years, e.g. if a student has mathematics every year for all three years, the student will have A-level, which is the highest level (at the higher technical examination program, all students must have at least B-level). Within the past decade, CAS and other digital technologies have found their way into everyday use in the mathematics programs of all types of Danish upper secondary education – and at the final written exam, students are now assumed to have access to and be familiar with at least one CAS tool. As for the everyday use of CAS, the ministerial orders for mathematics emphasize that CAS should be used not only to solve mathematical problems but also to assist the learning of the subject of mathematics. The ministerial orders are, however, not very prescriptive in terms of exactly *how* CAS should be used, and CAS use in everyday educational practice differs greatly from teacher to teacher and from school to school (Jankvist, Misfeldt & Marcussen, 2016).

Our case student, Emil, was a technical upper secondary school student with mathematics at A-level. During his three years of mathematics at upper secondary school, Emil and his class experienced four different teachers (which is not usual), and each of these teachers had their own CAS policy (Jankvist, Misfeldt & Iversen, accepted), which the students were exposed to. Now, the first teacher was only there for a brief period of time in the beginning of the students' first year, so the effect to this teacher's CAS policy diminished to almost nothing over the three year period. This however was not the case of the second teacher, who insisted that every posed mathematics task should, to the extent possible, be solved without any use of CAS. The third teacher, on the other hand, insisted on the exact opposite, i.e. that CAS be used whenever possible. The fourth and final teacher left it up to the students to decide when to make use of and not make use of CAS. Worth noticing is that Emil and his fellow students would not only restrict their technology use to a single CAS program – which is otherwise often the case – but made use of several different graphical calculators and mathematics software programs (*TI-Nspire*, *Maple*, *Graph*, etc.) as part of the written dimension of their mathematics education.

METHODOLOGICAL APPROACH

The data excerpts presented and analyzed in the following sections are drawn from a longitudinal, ethnographic study of students' mathematical writing and their writing development in the subject of mathematics (Iversen, 2014). This field study took place over a two-year period (2011-2013) and consisted of eight case studies of individual students from different types of Danish upper secondary schools. In the present article we draw on a case study of the student Emil, and re-interpret selected empirical data from the point of view of the theoretical constructs and frame presented above.

The purpose of Iversen (2014) was to study students' mathematical writing in its "natural" environment. The eight students were followed for at least one school year each during which focus was on their written mathematical work and this in such a way that the investigation did not deliberately influence on the students' mathematical activities. During this time period, a significant amount of diverse data was generated through classroom observation, interviews and conversations with teachers and students as well as physical and web-based collections of a wide range of written texts related to mathematics education.

The purpose of using these relatively diverse data types has been to allow for “thick descriptions” (Geertz, 1973) and “thick participation” – that is a socialization of the field through which it becomes possible to understand and interpret the studied phenomena through an alternate use of insider and outsider perspectives. In the following analysis, focus is mainly on Emil’s written mathematical work, but this is done based on participation and thick descriptions as reported in (Iversen, 2014).

THE STUDENT EMIL AND HIS MATHEMATICAL FOREGROUND

For Emil, technology plays a major role in his perception of himself as a student of mathematics. As we shall see in the following interview excerpt, this perception does not only concern his current educational situation, but also his future situation – what Skovsmose (2005) refers to as a student’s *mathematical foreground*. The excerpt is taken from a context in which, Iversen discusses the use and relevance of the mathematics software program *Maple* with Emil:

> **Emil**: Actually yesterday, I decided what I want to study. I want to go to the Technical University of Denmark and study mathematics and technology.
> **Iversen**: And you have to use Maple there?
> **Emil**: Firstly, I know that it would be a good idea for me to learn it, to know a bit more than the basics, more than adding two numbers...
> **Iversen**: But you already know more than the basics. You master it quite well. Of course, as you mention, there is more to be learnt. But the fact that you’re good at this, how important is that? You say that you decided to study mathematics and technology, but you’ve probably had an idea of this that you wanted to go in that direction? Maybe this has had an effect on the way that you’ve been working and writing in your mathematics class?
> **Emil**: I’ve spent more time on mathematics than on any other subject, because I wanted it more. And from the perspective that I wanted to study it further, it has been an argument for trying to make more out of it, and try to get an even better foundation from the beginning. Also because I know that so many people say it just gets harder at university. So, I’ve tried to give myself a good start.
> (Interview with Emil, May 9, 2012, translated from Danish)

To sum up, Emil is a rather mathematically skilled and also ambitious student. As seen from above, he not only has a genuine interest in the subject of mathematics he is also interested in pursuing further studies of the subject, and possibly a future career involving mathematics (and technology). In this respect, he views digital tools, such a *Maple*, as an integrated part of his future studies and career. It appears that his motive for putting effort into his written work in mathematics also may have to do with a wish of getting a good start with any continued studies of the subject.

SUB-CASE 1: IDENTITY AND PRAGMATIC USE OF TECHNOLOGY

The first example of Emil's written mathematical work, which we have chosen to present, is an answer to a calculus task, where he has chosen to use ICT (e.g. *Maple*) extensively, and in a rather pragmatic fashion. When the teacher responds in writing to Emil's work, we see how this feedback is feeding into Emil's identity work – but this we shall return to in the sub-analysis. First, the task, Emil's answer, and the teacher's feedback.

Figure 2 gives the task ("Task 5"), a calculus task, which reads: "On the picture parts of the graphs for the functions $f(x)$ and $g(x)$ are shown. The graphs and the x-axis demarcates an area, which is colored in gray. A) Calculate x_T, the x-coordinate for the area's center of gravity."

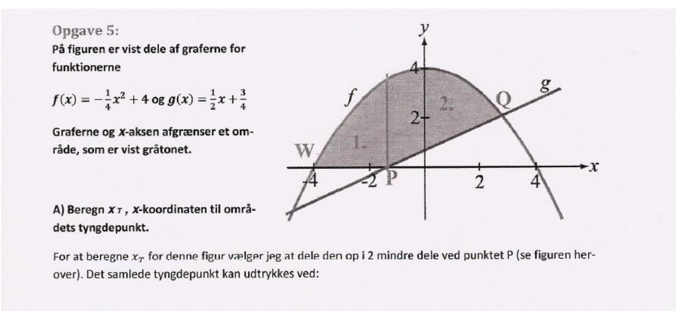
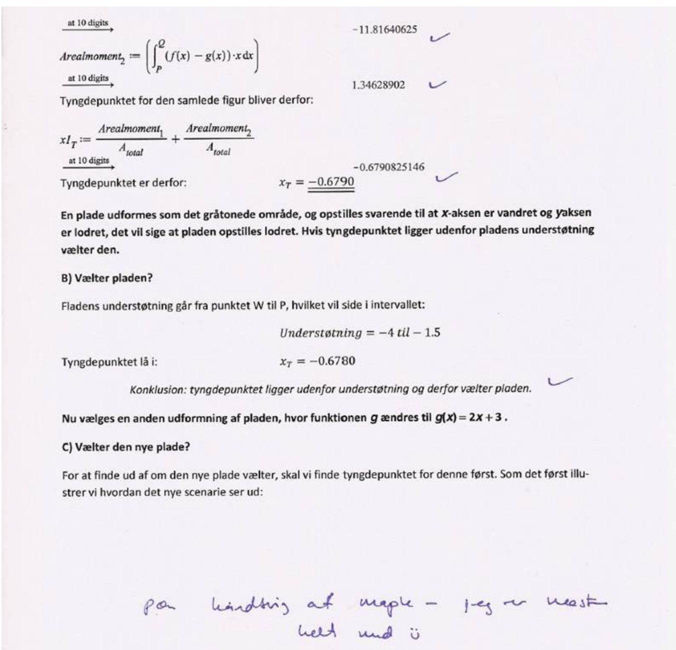


Figure 2. - The formulation of "Task 5" and first part of Emil's answer to the task. From December 2011, Emil's 3rd year of General Upper Secondary School.

At the bottom of Figure 2, Emil begins his answer to the task: "To calculate x_T for this figure, I first split it into 2 smaller parts at the point P (cf. figure above). The unified center of gravity may be expressed by:" The answer then continues on the next page where Emil uses the three points P, Q, and W. These are points that Emil has inserted on the figure in order for him to analytically express the center of gravity.

We will not go through Emil's solution step by step, but we provide a brief summary in order to give a general idea of how Emil uses ICT. The first thing that happens is that Emil writes up an expression for the center of gravity x_T. Emil then describes that he needs to know the coordinates of the points P, Q, and W. He notices that W can be read of the figure (or found analytically by solving $f(x) = 0$), and that Q and P can be found by solving $f(x) = g(x)$ and $g(x) = 0$, respectively. Using *Maple*, he defines the functions $f(x)$, $g(x)$, and a function $h(x) = 0$, and finds values for the x-coordinates for the points Q and P. Knowing the three points, he is able to calculate the two parts of the initial expression for the center of gravity, using *Maple*, the functions $f(x)$, $g(x)$, and $h(x)$, and the coordinates of *P, Q,* and *W* (see Figure 3). The teacher's final feedback is important to notice: "Nice handling of *Maple* – I can almost follow it :-)"



$\xrightarrow{\text{at 10 digits}}$ -11.81640625

$Arealmoment_2 := \left(\int_P^Q (f(x) - g(x)) \cdot x \, dx\right)$

$\xrightarrow{\text{at 10 digits}}$ 1.34628902

Tyngdepunktet for den samlede figur bliver derfor:

$xI_T := \frac{Arealmoment_1}{A_{total}} + \frac{Arealmoment_2}{A_{total}}$

$\xrightarrow{\text{at 10 digits}}$ -0.6790825146

Tyngdepunktet er derfor: $x_T = -0.6790$

En plade udformes som det gråtonede område, og opstilles svarende til at *x*-aksen er vandret og *y*aksen er lodret, det vil sige at pladen opstilles lodret. Hvis tyngdepunktet ligger udenfor pladens understøtning vælter den.

B) Vælter pladen?

Fladens understøtning går fra punktet W til P, hvilket vil side i intervallet:

$Understøtning = -4 \; til -1.5$

Tyngdepunktet lå i: $x_T = -0.6780$

Konklusion: tyngdepunktet ligger udenfor understøtning og derfor vælter pladen.

Nu vælges en anden udformning af pladen, hvor funktionen *g* ændres til $g(x) = 2x + 3$.

C) Vælter den nye plade?

For at finde ud af om den nye plade vælter, skal vi finde tyngdepunktet for denne først. Som det først illustrer vi hvordan det nye scenarie ser ud:

Figure 3. - Fourth and final part of Emil's answer to "Task 5", question A. Also, questions B and C of "Task 5" are shown here alongside Emil's answers and the teacher's comments. From December 2011, Emil's 3rd year of General Upper Secondary School.

SUB-ANALYSIS 1: IDENTITY AS THE “CAS-WIZARD”

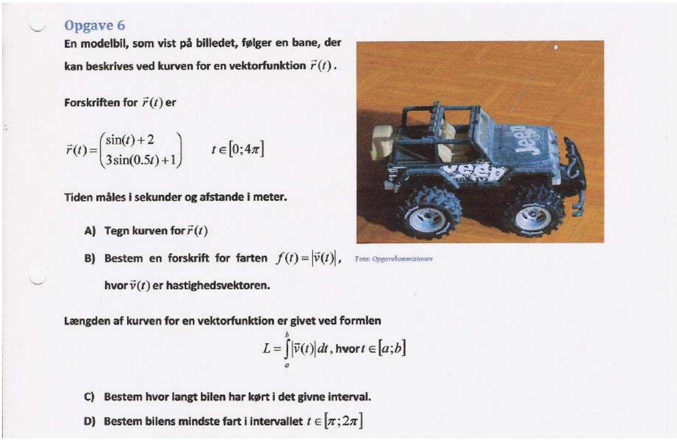
The above example shows the interplay between Emil’s pragmatic use of technology and the construction of his mathematical identity. We notice that the teacher assigns Emil a position as an expert user of CAS. Hence, Emil is acknowledged for his competency with mathematical tools, which is likely to affect his future identity work (Ivanič, 1998, 2006; Gee, 2003). In relation to the specific task and the teacher, Emil is positioned as an efficient “CAS wizard”, able to solve difficult mathematical problems with a few omnipotent lines of code. The teacher acknowledges the skills and describes that she can “almost” follow the solution strategy. The teacher is of course very present in this piece of data, since she is actively co-constructing Emil’s identity as a CAS-wizard.

Whether it in this situation is Emil’s intention to create an image of himself as a “CAS wizard” or not, it is clear that there is identity work going on between the teacher and the student. And it is very likely that such a designation of position affects Emil’s future work with technology in the subject of mathematics.

SUB-CASE 2: IDENTITY AND EPISTEMIC USE OF TECHNOLOGY

The second example shows how Emil is using technology both in order to improve his own understanding in a given mathematical situation and in order to meet the teacher’s implicit suggestions of what good mathematical practice is. In that sense, the second example shows how epistemic use of digital technology can also be identity work – because it involves how the teacher sees Emil; as a student and as a mathematician. We present the task, Emil’s answer to the task, and an excerpt from an interview in which Emil comments on his answer.

The task (“Task 6”) in Figure 4 reads: “A remote control car, as shown on the picture, follows a path which may be described by a curve for a vector function $\vec{r}(t)$.” An expression is provided for the vector function (cf. top of Figure 4), and it is stated that time is measured in seconds and distance in meters. For “Task 6” we are only interested in Emil’s answer to question A, which asks the students to draw the curve for $\vec{r}(t)$.



Opgave 6

En modelbil, som vist på billedet, følger en bane, der kan beskrives ved kurven for en vektorfunktion $\vec{r}(t)$.

Forskriften for $\vec{r}(t)$ er

$$\vec{r}(t)=\begin{pmatrix}\sin(t)+2\\ 3\sin(0.5t)+1\end{pmatrix} \quad t\in[0;4\pi]$$

Tiden måles i sekunder og afstande i meter.

A) Tegn kurven for $\vec{r}(t)$

B) Bestem en forskrift for farten $f(t)=|\vec{v}(t)|$, hvor $\vec{v}(t)$ er hastighedsvektoren.

Foto: Opgavekommissionen

Længden af kurven for en vektorfunktion er givet ved formlen

$$L=\int_a^b |\vec{v}(t)|\,dt \text{, hvor } t\in[a;b]$$

C) Bestem hvor langt bilen har kørt i det givne interval.

D) Bestem bilens mindste fart i intervallet $t\in[\pi;2\pi]$

Figure 4. - “Task 6” which concerns the path of remote control car described by a curve for a vector function. From September 2011, Emil’s 3rd year of General Upper Secondary School.

In Figure 5, Emil provides the following answer to question A: “The curve for $\vec{r}(t)$ is drawn below (attached are arrows illustrating the direction of the movement):” Sometime after having handed in the math assignment involving “Task 6” and getting comments back from the teacher, Emil was interviewed about his answer above.

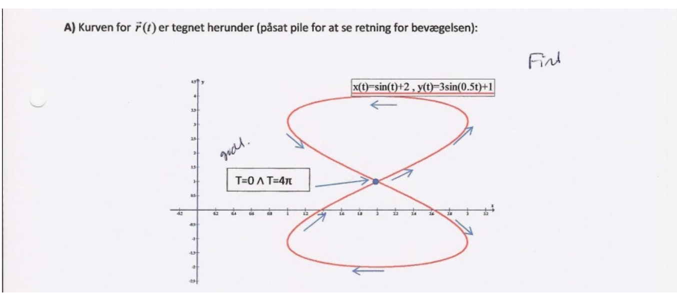


Figure 5. - Emil’s answer to question A of “Task 6”. Teacher’s comments read: “Good” and “Fine”. From September 2011, Emil’s 3rd year of General Upper Secondary School.

Iversen: So, it is a function, a vector function, which you drawn in *Graph* again?
Emil: I’ve drawn the vector function, and then I put it into *Word*, and then I made these arrows here, because our

teacher said that this was a good way of illustrating how a vector function behaves.
Iversen: Okay, so let me understand this correctly. You drew it in *Graph* and then you copy-pasted it into *Word* as an image?
Emil: You could say that *Graph*…
Iversen: It purely technically that I'm asking.
Emil: So technically, I've done it in *Word*. You can insert an object, also if you write in 'equation', then under 'object' there are *Graph* elements too.
Iversen: Okay.
Emil: So you can make your vector functions there, and then when you shut down *Graph* it is inside *Word*, and then I've entered 'insert' and then 'figures' and I've made small arrows around.
Iversen: Okay, afterwards!
Emil: Yes, and then I've made a small box here to describe that it is this point where it begins and ends at, by turning in the interval 0 to 4π, so that is where...
Iversen: Oh yeah, so that's the intervals its doing…
Emil: Yes, it begins here and then it goes like this and it ends here.
Iversen: 2π and 2π, I assume?
Emil: Yes.
Iversen: Let me then ask you again: who is this information directed to? Or what is the purpose of your answer? The purpose of the arrows? It is to illustrate how it goes. But who is this information directed to?
Emil: Well, I think it is directed to the teacher, because it doesn't say in the task that you have to do this. So it was something extra I did to try it out, in another way. And also, I wanted to make some overview for myself, by seeing – it is probably easier for me to work with a function, if I know exactly how it behaves; it can't go in this direction, but it can go in this direction, and it can even begin here… So probably, I made this drawing for me.
(Interview with Emil, November 11, 2011, translated from Danish)

SUB-ANALYSIS 2: IDENTITY AS THE KNOWLEDGEABLE STUDENT

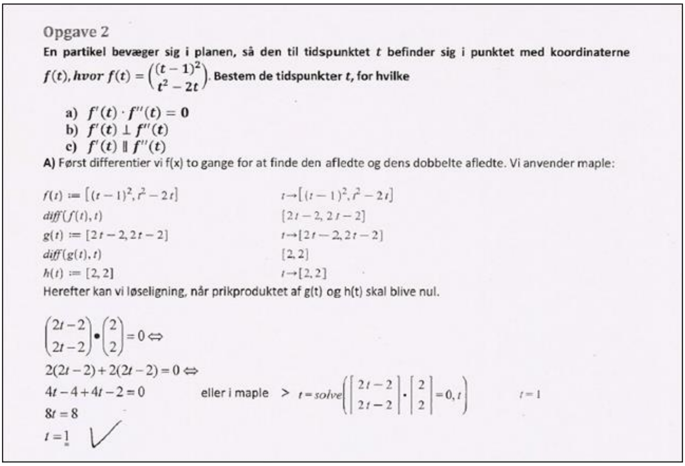
As Emil explains, the visualization in Figure 5 is done in two different processes. Firstly, he has created the graph with a digital tool (*Graph*). Secondly, he has embedded the image into *Word* and drawn arrows in order to illustrate the direction of the curve as t progresses. An interesting aspect of Emil's answer to "Task 6" is that he writes: "(attached are arrows illustrating the direction of the movement)”. He does not say, for example, “I have attached arrows...” Nor does he say for whom the arrows are attached – which is what to some extent comes to drive the interview.

In the interview, Emil explains a number of different reasons for drawing the arrows. He describes that the arrows actually make sense and help him obtain an overview of the mathematical situation represented by the curve. At the same time, the illustration is directed towards the teacher, in the sense that he wants to show the teacher that he has an overview of what is going on. Emil is adding the arrows even though this is not a requirement either from the teacher or from the task. Hence, we witness a synergy between identity work (Gee, 2003), where Emil illustrates that he is a ‘mathematician’ who actually cares about the actual mathematical behavior of the entities involved in the task, and epistemic mediations (Artigue, 2002; Guin et al., 2005), where he is interested in creating a mathematical overview of the situation by means of doing – and benefitting from – the visualization.

SUB-CASE 3: IDENTITY, PRAGMATIC AND EPISTEMIC USE OF TECHNOLOGY

The third example provided in the article combines pragmatic mediations, identity work, and epistemic mediations (p-i-e). The task (“Task 2”) reads: “A particle moves in the plane, so that it at time t is located in the point with coordinates $f(t)$, where $f(t) =$... Find those times t for which a)...; b)...; c)...” We are interested in Emil’s answer to question a, in which he uses that the dot product of the functions $g(t)$ and $h(t)$ should be zero (cf. bottom of Figure 6).

The situation is that Emil answers question A of “Task 2” by first doing a rather long and – at least at this level of education – somewhat complex calculation (bottom of Figure 6, left hand side) and immediately after solving the same task with one line of *Maple* code. What is interesting is his need for including both solutions as his answer to the question.



Opgave 2

En partikel bevæger sig i planen, så den til tidspunktet t befinder sig i punktet med koordinaterne $f(t)$, hvor $f(t) = \begin{pmatrix} (t-1)^2 \\ t^2 - 2t \end{pmatrix}$. Bestem de tidspunkter t, for hvilke

a) $f'(t) \cdot f''(t) = 0$
b) $f'(t) \perp f''(t)$
c) $f'(t) \parallel f''(t)$

A) Først differentier vi f(x) to gange for at finde den afledte og dens dobbelte afledte. Vi anvender maple:

$f(t) := [(t-1)^2, t^2 - 2t]$	$t \to [(t-1)^2, t^2 - 2t]$
$diff(f(t), t)$	$[2t-2, 2t-2]$
$g(t) := [2t-2, 2t-2]$	$t \to [2t-2, 2t-2]$
$diff(g(t), t)$	$[2, 2]$
$h(t) := [2, 2]$	$t \to [2, 2]$

Herefter kan vi løseligning, når prikproduktet af g(t) og h(t) skal blive nul.

$$\begin{pmatrix} 2t-2 \\ 2t-2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0 \Leftrightarrow$$

$2(2t-2) + 2(2t-2) = 0 \Leftrightarrow$

$4t - 4 + 4t - 2 = 0$ eller i maple $> t = solve\left(\begin{bmatrix} 2t-2 \\ 2t-2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 0, t\right)$ $t = 1$

$8t = 8$

$t = 1$

Figure 6. - “Task 2” concerning the movement of a particle in the plane. Notice, Emil’s two different answers at the bottom: a ‘paper-and-pencil’ answer and a CAS answer. From April 2011, Emil’s 2nd year of General Upper Secondary School.

SUB-ANALYSIS 3: CONFLICTING TYPES OF IDENTITIES

In this third example, we see how Emil presents himself and his mathematical work with technology in two different ways at the same time. With the algebraic solution he shows his mathematical overview and conceptual understanding as well as his ability to do mathematics in a competent way without the use of computational tools. Simultaneously he shows an easy, elegant and minimal computer-based *Maple* solution as an alternative to the lengthy and time-consuming algebraic approach. Of course it is always sensible to check one’s paper-and-pencil results by means of technology, which may also be what Emil is doing. Still, from an identity perspective all text production is identity work (Gee, 2003). Hence, the fact that he actively choses to present both solutions invites to identity-related interpretations.

We consider the fact that Emil provides two solutions in the task as being related to the two types of identities witnessed in sub-case 1 (i-p) and sub-case 2 (i-e), respectively. The “CAS-wizard” student (sub-case 1) will of course provide the minimal effort solution reflecting strong skills in technology, and the necessary mathematical overview to map the mathematical problem into just one line of *Maple* code. The “knowledgeable mathematics” student (sub-case 2), on the contrary, wants to show skills in classical approaches to the problem as well as conceptual overview. When Emil presents two so different solutions to the task, it might be because he needs to demonstrate that he actively masters the construction of both types of identities ("CAS-wizard" and "knowledgeable student"). We see identity work going on, and in such a situation, where both types of identities come into play (p-i-e) at the same time, we are witnessing a potential conflict between the two prototypical identities in relation to Emil's identity work in the subject of mathematics. In the case of Emil, one interpretation is that he might be insecure about how to present himself in this situation and thus ends up providing two solutions to one mathematical task. From a broader perspective, sub-case 3 shows student identity as a parameter in the choices and considerations in relation to the use of technology in mathematics. Another interpretation could be that Emil is deliberately constructing a surplus identity of being able to solve the task in several different ways. Either way, we notice that this insight has a number of consequences for the teaching of mathematics and mathematics education research.[1]

CONCLUDING DISCUSSION

The case illustrate that the written work of the upper secondary school student, Emil, and the feedback from his teacher can meaningfully be viewed as identity work (see also Iversen, 2014), and furthermore that the way the student uses technology in his work is a central part of this identity work (Gee, 2003). We see that Emil moves between several meaningful positionings towards the use of technology, reflecting also the well-established continuum between epistemic and pragmatic mediations (Artigue, 2002; Guin et al., 2005). But more crucially, for the argument of the present article, in sub-analysis 3 we see an example where Emil apparently needs to show two very different solutions to a mathematical problem, aligned with the two different types of identity work (i-p and i-e, cf. Figure 1) that we have documented to occur between teacher and student. Sfard and Prusak (2005) use the concept *designated identity* to describe the identity of "the one you want to be" and use this concept to explain students' motivation for learning. Hence, students' identity work with technology relates to their intention and motivation to learn mathematics. In the case study of this article, Emil's use of CAS does indeed relate to his designated identity. As it was clear from the first interview excerpt with Emil presented above, he wants to study mathematics at a technical university and he sees ICT competencies as important for

[1] "Task 2" in Figure 6 may potentially also be analyzed by means of other theoretical constructs. For an example of using that of sociomathematical norms, see Jankvist et al. (accepted). Didactical contract is another construct which clearly offers itself to the situation surrounding this task – for similar examples analyzed from a didactical contract perspective, see Jankvist et al. (2016).

his future. This also means that when Emil sometimes chooses a technique, which within the instrumental approach would be considered as a pragmatic use of CAS, it might not be fairly described as such. Emil is not just trying to “do mathematics tasks”. Rather he is building his identity as a future mathematician. These two ways of looking at Emil’s work show that much more is going on than just the interplay between epistemic and pragmatic mediations and the development of personal instruments. Identity work frames and directs the mathematical work of Emil, and disregarding this might lead to insufficient or faulty understandings of Emil’s use of technology in mathematics education.

Technology such as CAS and other strong tools (e.g. *Wolfram Alpha* and *GeoGebra*) are expanding the possible positionings available to students in their identity work. Of course, students’ socially situated identities are always in play in teaching and learning situations (Gee, 2001; Ivanič, 2006), and therefore also when learning mathematics. But the uses of technology – as we have witnessed above – allows a number of new positionings for students and therefore also new ways of doing identity work. This means that such technologies are potentially changing the entire space of possible – and accepted – socially situated identities in the mathematics classrooms.

In the case of Emil and his work, the teacher is very present in co-constructing Emil’s mathematical identity. There are several reasons for this. First and foremost, the empirical strategy that we have followed almost prescribes the teacher to be present. However, the data do suggest that the interaction with the teacher’s response and feedback are actively shaping and molding Emil’s identity in relation to the use of technology in mathematics. Hence, the role of the teacher in the processes of constructing students’ identities is important. Furthermore, since teachers have different approaches to and values concerning the use of technology (Iversen, 2014), the discussion of classroom norms and values around use of technology may, from an identity perspective, be increasingly important and relevant. The teachers’ governance of rules and values concerning technology makes different positionings on the development of students’ mathematical identities important, and thereby also students’ opportunities to engage meaningfully in the teaching-learning processes of mathematics. Hence, the extensive use of technology in mathematics education pave the road for new types of student identities, and this space of identities is neither well understood nor well negotiated among curriculum developers and teachers.

In conclusion, we suggest that taking the perspective of studying student identity work (as suggested by Gee, 2001) provides new insight to students’ work with technology in mathematics education. This suggestion leaves at least one question of theoretical nature open: Why is it that the existing approaches to understanding students’ use of technology when learning mathematics, does not truly consider the students’ identity work as important? One possible answer could be that the insight (e.g. from the instrumental approach) that doing mathematics should not be considered as work by a "brain in a vat", but as an interaction between artifacts and cognitive beings, has led to a blind spot in the sense that we have considered students as having "identity-free" brains working by means of artifacts in order to obtain mathematical insights and results. In the case of Emil we see that omitting his identity work and focusing on the development of instrumented techniques leads to an analysis not acknowledging the value of the position as "CAS-wizard", which Emil takes, and also how this positioning relates to Emil's foreground (Skovsmose, 2005) or designated identity (Sfard & Prusak, 2005). Hence, the perspective of identity proves to be a very valuable augmentation to the instrumental approach in this case. Or to put it more boldly, students' identities and identity work make up a non-neglectable dimension in relation to technology in mathematics education. From a theoretical standpoint, this indicates that we should not only consider the dualism between pragmatic and epistemic mediations, but the *trinity* of these two types of mediations and students' identity work.

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