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An Optimal Market-Oriented Demand Response Model for Price-Responsive Residential Consumers

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Abstract: For many years in the wholesale electricity market, the generation companies would only seek to compete with each other to sell electric energy to customers in a way to make more profit. Moreover, there was no mechanism in such an environment to enable demand-side participation especially for residential building units with relatively high power consumptions. This caused the increasing market power of generation companies and soon to realize that the demand-side would yield to any price to purchase the required energy. Having gradually identified this issue, demand response (DR) programs were introduced as confronting tools to help consumers being away from such situations. This paper proposes an effective market-oriented DR model for residential consumers to change their consumption patterns over the time for getting maximum benefits based on their own utility functions. According to the results of simulated case studies, it is demonstrated that the proposed model is able to adapt to different consumers with different levels of flexibility against the price signals. Moreover, simulation results demonstrate that the residential consumption levels can be easily adjusted during the examined period in a way not only to meet the user's objectives, but also to reshape and smooth the system's aggregated load profile.

Keywords: Demand-side management, economic demand response model, consumer utility function, electricity market restructuring.

Nomenclature

D_{peak}	Demand for the peak times before implementation of DR Programs
$D_{shoulder}$	Demand for shoulder times before implementation of DR Programs
$D_{off\text{-}peak}$	Demand for off-peak times before implementation of DR Programs
D'_{peak}	Demand for the peak times after implementation of DR Programs
$D'_{shoulder}$	Demand for shoulder times after implementation of DR Programs
$D'_{off\text{-}peak}$	Demand for off-peak times after implementation of DR Programs
ρ	Rate of time preference
θ	Coefficient of relative risk aversion
U_t	Utility function

P_{peak} Peak time price

$P_{shoulder}$ Shoulder time price

$P_{off-peak}$ Off-peak time price

B   Budget before implementation of DR Programs

B′   Budget after implementation of DR Programs

1. Introduction

1.1. Motivation of the work

The presentation of Demand Response (DR) models is not only important for retailers, but also for other players in the electricity market. It can also help network operators to obtain necessary information about the responses of consumers to different load scheduling programs to determine the most appropriate tariffs for network congestion management. Most of the existing DR models are based on the concept of price elasticity. In these models, point elasticity (or arc elasticity) is the price elasticity of demand at a specific point (or between two given points) instead of the entire curve. On the other hand, price elasticity at different points on the demand curve is different. Hence, existing DR models cannot be used as a feasible tool for implementing DR in the electricity markets [1]-[2]. This paper employs economic theories and mathematical formulations to introduce a new model for time-of-use (TOU)-based DR programs which not only addresses this issue, but also enables consumers to better modify their consumption behaviors over the time to get higher benefits.

1.2. Literature review

In the beginnings of the electric utility industry deregulation, there was no effective participation from the demand side in the electricity market, but an extensive contribution from the big generation companies (GenCos) as the major players in the market. Therefore, demand-sides were far away from the benefits and the interests of the electricity markets. They had not sufficient knowledge and tools to participate effectively in such competitive environments. This kind of demand-side behavior (passive market player) caused high price spikes and triggered a process which is called "hockey stick bidding" in a single-sided market. Indeed, a hockey stick bidding in power markets is a strategy in which GenCos will increase the energy price considerably over the marginal utility when demand-side for the electric energy is very inelastic [3]. One of the strategies that can change an inelastic demand to an elastic one is called demand side management (DSM) [4]-[5]. This concept implies a supply/demand-side relationship that presents mutual benefits to GenCos and consumers as well as numerous profits to deregulated distribution systems [6]. Moreover, DSM programs encourage customers to shape

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their patterns of electricity consumption for certain goals.

One of the major thrust areas of DSM is known as DR. The US department of energy defines DR as: a tariff or a program established to motivate changes in electricity load shapes by consumers in response to changes in the price of electricity over time, or to give incentive payments designed to induce lower electricity use at times of high market prices or when grid reliability is jeopardized. Similarly, it has been shown in [7] that increasing the capability of demand-side to react to electricity prices can not only decrease the running costs of the system, but also alleviate the rate volatility of prices during peak times. Generally, DR programs fall into two categories [8]: (1) Price-based DR programs and (2) Incentive-based DR programs.

In priced-based DR programs, consumers receive dynamic prices that show the value of electricity in various time periods [9]. With this information, consumers can modify electricity usage when electricity prices are high.

In [10], the authors show that the implementation of dynamic pricing can lead to an increase in consumers' willingness to participate in DR programs, which in turn improves the performance of the electricity markets. In [11], a new approach to determine the effects of DR program on cost of energy and market clearing price is presented. In a similar manner, researchers in [12] present a novel model based on linear price sensitive demand bidding curves to examine the effect of price-based DR programs on market clearing and marginal prices. This model is constructed based on the load shifting scheme, so that the total energy consumption (before and after DR implementation) remains the same over the study period. The results also show that the proposed DR model can alleviate the electricity prices. Authors of [13] study the problems of DR programs under various working conditions and describe how these DR programs can affect the economic dispatch of GenCos and social welfares in power systems. In [14], an algorithm is designed to optimize the DR scheduling for normal operation and during contingency events. In [15], authors present a real-time price-based DR management model for residential appliances. This model can help the residential consumers to administer their appliances in a way to minimize the energy consumption costs over the time. In [16], authors propose a method that the optimal prices during various times in a day are reported to the consumers simultaneously and users would minimize their costs and optimally schedule their power usage accordingly as a part of participation in DR program. Reference [17] designs a novel self-scheduling framework for DR aggregators. This model takes into account customers uncertainties and electricity prices. A new DR model based on fuzzy subtractive cluster approach is presented in [18]. This model can help domestic customers to manage controllable loads and thus improve the load pattern to maximize their utility functions. The results of this paper also show the effectiveness of the proposed model for larger test systems. Another demand management framework is designed in [19] based on DR programs that provides independent system operators (ISOs) with more flexible options for scheduling

the available energy resources in electricity markets. A novel formulation of DR program for retail customers is presented in [20] where authors demonstrate that the proposed model could persuade inherent demand flexibility and reduce operational cost. Likewise, a DR model with elastic economic dispatch is presented in [21] where ISOs can utilize the proposed model to get a clear vision about DR actions for their decision-making process. In [22], coalitions of consumers with varying sensitivity to DR programs are presented in a day-ahead market. An algorithm based on Monte-Carlo is then presented for formulation of multiple purchase offers in this market.

One of the most important concepts in DR programs is electricity demand elasticity, which is defined as the sensitivity of the customer's demand to the changes in price of electricity. This means when the price of electricity increases, the customers will have more motivations to decrease the consumption level. In [23], incentive-based DR models are introduced based on the concept of consumer's utility function and flexible price elasticity of demand. The mathematical model for elasticity of demand is introduced to compute each elasticity of the DR program based on the price of electricity before and after the execution of DR programs. In [24], a model for DR programs is introduced. Consumer demand for electricity is shown to be dependent on the price elasticity of demand, reward and value of the penalty set for DR programs. In [25], a DR model based on the consumers' behavior with the concept of demand-price elasticity is introduced. In this model, the difference between encouragement and punishment is accepted and proves that encouragement is a great way to establish or make changes in a habit than punishment. In [26], a responsive load model is developed based on the concept of price elasticity and customer utility function. The mathematical model to calculate elasticity of the demand is proposed based on the price of electricity before and after the implementation of DR programs. However, the price elasticity of demand prepares a theoretical view on the effects of the DR and cannot be used as a feasible tool for implementing DR in the electricity markets as it doesn't consider customer's features and its preferences [19]. Hence, A good DR model is the one that includes the following features: 1) adaptability to different consumers with different response level (such as Low-Flexible Behavior (LFB), Semi-Flexible Behavior (SFB) ,Highly Flexible Behavior (HFB) [1]-[3]), and 2) adjustability to user's preferences. Consumers' desire to use electricity at certain hours varies within a day which mostly depends on the lifestyle. Therefore, an efficient DR model should also account for the adjustment of consumption level with the aim of satisfying the demand-side objectives.

1.3. Contributions

In view of the discussion presented in previous section, it is clearly understood that an efficient model should be able not only to express the extent of consumers' reactions to price information, but also to account for adjustment of consumption levels in different time periods. To this end, the goal of this paper is to address the mentioned issues on the existing DR models based on the demand-price elasticity concept with the help of an efficient economic DR model. This

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model simulates the residential DR program based on a TOU pricing scheme within which a daily time horizon is divided, based on power consumption levels, into three periods of peak, shoulder and off-peak, each with a unique tariff.

In this regard, with a focus on residential DR programs, as shown in Figure 1, residential loads can be divided into two groups: non-flexible loads and flexible loads. It is notable that flexible loads have a big contribution in peak-time demand and can be used as efficient tools for DR implementation [27]-[28]. In response to a TOU-based DR program, residential consumers may behave in two different ways: 1) they may reduce the consumption in the targeted time periods without making any change in the routine consumption of other periods; or 2) they may shift part of their flexible loads from the targeted time periods to other periods. In the reference [29] has been emphasized that in the long run, participation from the demand side in the electricity market is likely to be almost neutral. This means that consumers only change their demand from one period to another in response to price signals. Therefore, the manner in which consumers adjust their consumption level in different time periods to achieve the highest financial gain (or avoid financial loss) is of particular importance.
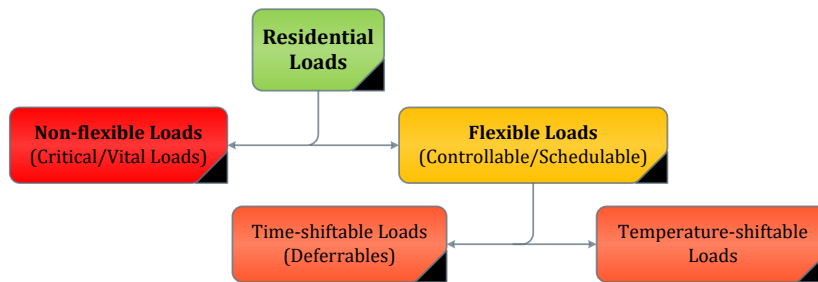

Figure 1: Electrical loads of residential sector [28]

This paper also employs economic theories and mathematical formulations to introduce a new model for time-of-use (TOU)-based DR program. Compared to the existing price-elasticity based DR models, the proposed model in this paper considers the entire demand curve and brings more flexibility in demand management. As will be shown later, this flexibility would cause new parameters to emerge and guarantee the prerequisites for making a good DR model (i.e., adaptability and adjustability). Moreover, to provide a comprehensive DR model with a strong economics-based core, the Diamond's OLG model [30] is applied in this paper as the foundation of a TOU-based DR model with three-time periods. By developing such a model, it is possible not only to study different levels of user's participation in DR actions, but also to consider consumer's preferences during the course of the action. As a whole, the contributions of this paper could be summarized as follows:

- A model for three-period TOU program is proposed based on the mathematical methods and economics theories to enable consumer response to price signals for utility maximization,

- Tendency of customers to reapportionment of their electricity usage over the time periods is included into DR programs,

- An efficient framework is presented for residential load management by taking into account the different levels of participation in DR programs.

The organization of this paper is as follows: Section 2 explains the mathematical formulation of the proposed DR model. Simulation results together with model validation under different test scenarios are presented in Section 3. Finally, Section 4 concludes the paper by summarizing the main results and discussing future work.

2. Market-Oriented DR Model

2.1. Diamond's OLG Model

As previously mentioned, the economic model presented in this paper is based on the Diamond's OLG model. This subsection provides only a brief description of this model but interested readers may find more detailed information in [30]-[31]. Diamond's OLG model assumes that the life of every person can be divided into two periods: 1) working period, and 2) retirement period. In working period, the person earns money and in retirement period he/she spends the money earned in the previous period. So in the first period, he/she spends a portion of his/her income $C_{1,t}$ and saves the rest for the second period $C_{2,t+1}$. This person should adjust his/her consumption in these two periods such that his/her total profit gets maximized. So, to find the best consumption level in each period, he/she must maximize a utility function as follow:

$$U_t = (C_{1,t}, \ \frac{1}{1+\rho} C_{2,t+1}) \tag{1}$$

where ρ is the rate of time preference denoting that the benefits a person earns in the next period is less valuable than the one he/she earns in the current period, and a negative value which is signifying the opposite. This parameter can be adjusted to specify the preference in each period.

Utility function can be expressed through different formulations; however, Diamond's OLG model utilizes a function called constant relative risk aversion. This utility function is the macroeconomists' favorite model and has the following form [30]:

$$U_t = \frac{C^{1-\theta}}{1-\theta} \quad ; \ 0 < \theta < 1 \tag{2}$$

in which, θ is the coefficient of relative risk aversion, which has a direct relationship with the concavity of the utility function, meaning that the more concave the utility function, the more risk averse is the person. In other words, θ is the extra compensation that the person expects to earn in the next period at the expense of a single unit decrease in his/her consumption in the current period. It should be noted that the more concave the utility function, the greater is the value of

θ and the more risk averse is the person, and the higher is the requirement for cutting back in monotonous consumption and saving for the next period. Therefore, as θ increases, the person becomes less inclined to shift his/her consumption from the current period to the next. According to (1) and (2), utility function of the Diamond's OLG model is as follows:

$$U_t = \frac{C_{1,t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_{2,t+1}^{1-\theta}}{1-\theta} \qquad \rho > -1, \, 0 < \theta < 1 \qquad (3)$$

2.2. Proposed DR Model

In view of the definition provided in the previous sections, to develop a TOU-based residential DR program based on load shifting scheme with Diamond's OLG model acting as its foundation, a three-time period model is needed. In this regard, it is assumed that diurnal hours are divided into three periods: 1) peak time, 2) shoulder time, and 3) off-peak time. It is further assumed that in the absence of DR program, energy consumption in these three periods are D_{peak}, $D_{shoulder}$ and $D_{off\text{-}peak}$, and a TOU-based DR program changes these values to D'_{peak}, $D'_{shoulder}$ and $D'_{off\text{-}peak}$ respectively.

The price-responsive demand model developed in this paper has the three following stages:

i.   During the peak time, user consumes D'_{peak} instead of his routine consumption D_{peak} ($D'_{peak} \leq D_{peak}$) and saves the rest (what remains from his/her conceived power budget) for other periods,

ii. During the shoulder time period, user has three options; he/she can consume a portion of savings made during the peak time ($D'_{shoulder} > D_{shoulder}$) or can consume less than he/she normally does and saves the rest for the off-peak time ($D'_{shoulder} < D_{shoulder}$). Alternatively, he/she can follow his/her routine consumption trend ($D'_{shoulder} = D_{shoulder}$),

iii. During the off-Peak time, user consumes his/her routine quota plus the savings made during the past periods.

Based on the above definitions, consumer's utility function for participating in a given DR program will be as follows:

$$U_t = (D'_{peak}, \frac{1}{1+\rho_1} D'_{shoulder}, \frac{1}{1+\rho_2} D'_{off-peak}) \qquad (4)$$

Based on the Diamond's OLG model, this formulation can be rewritten as follows:

$$U_t = \frac{D'^{1-\theta}_{peak}}{1-\theta} + \frac{1}{1+\rho_1} \frac{D'^{1-\theta}_{shoulder}}{1-\theta} + \frac{1}{1+\rho_2} \frac{D'^{1-\theta}_{off-peak}}{1-\theta} \qquad (5)$$

where ρ_i and θ are now described for a consumer in electricity market, according to the Diamond's OLG model. Parameter θ (or coefficient of relative risk aversion) determines the consumer's willingness to participate in DR program. The greater the value of θ, the less inclined will be the consumer to participate in DR program. This parameter provides also the first feature of a good DR model known as *adaptability*. This means that a wide range of consumers in the electricity market can be taken into consideration only by changing/fine-tuning this parameter. In a similar fashion, parameter ρ_i (or the rate of time preference) accounts for adjustment of consumption ratio over the time; i.e., changing ρ_1 parameter provides an

adjustment in the ratio of consumption in the peak-time to the shoulder-time, while changing ρ_2 parameter provides arbitrary ratio tuning for the consumption in the peak-time to the off-peak time. This enables the model to account for adjustment of consumption levels in every time period, which is the second feature of a good DR model known as *adjustability* of consumption levels.

The importance of the preference parameters primarily lies in their association with consumption in the shoulder-time, because without the shoulder-time (i.e., a two-price period model), a portion of power consumption in the peak-time would be automatically shifted to the off-peak time, and to adjust the consumption levels of the two periods one would only need to check the intensity of this transition. Incorporating three-tariff periods into the model, (while paying no attention to the adjustability of consumption level) will result in a shift of consumption from both pricier periods (peak-time and shoulder-time) toward the cheaper period (off-peak time), which might be inconvenient for customer, as he/she may prefer to use a portion of the power saved during the peak-time not in the off-peak time but rather in the shoulder-time. Preference parameters allow the model to address this issue and incorporate different consumption strategies based on the customer's lifestyle and preferences.

From the consumer's perspective and in the presence of a DR program, the goal is to maximize the utility function by adjusting the consumption levels in the three mentioned periods, thus:

$$Max\left\{U_t = \frac{D_{peak}^{'1-\theta}}{1-\theta} + \frac{1}{1+\rho_1}\frac{D_{shoulder}^{'1-\theta}}{1-\theta} + \frac{1}{1+\rho_2}\frac{D_{off-peak}^{'1-\theta}}{1-\theta}\right\}$$

s.t.

$$B^{'} = D^{'}_{peak}.P_{peak} + D^{'}_{shoulder}.P_{shoulder} + D^{'}_{off-peak}.P_{off-peak}$$

(6)

where the equality constraint in the last line indicates that the cost incurred due to purchase of electricity should be equal to the allocated budget. The mentioned optimization problem can be solved using different methods, however in this paper Lagrange multipliers method is adopted as the one to do so [32]. This method is particularly useful for problems with equality constraints as the one presented in (6).

To get a better insight, let's consider the following optimization problem:

$$Max \ \{f(x)\};$$
$$s.t. \ h_j(x) = 0$$

(7)

If x^* is considered as the optimal solution, then there exists a constant λ_j ($j=1, 2, 3…$) such that:

$$\nabla f(x^*) + \sum_{j=1}^{l} \lambda_j \nabla h_j(x^*) = 0$$
$$h_j(x^*) \leq 0 \ ; \ \forall j = 1,2,...,l$$
$$\mu_i \geq 0 \qquad ; \ \forall i = 1,2,...,m$$

(8)

Using the same approach, (6) can be reformulated as follow:

$$L = \frac{D_{peak}^{'1-\theta}}{1-\theta} + \frac{1}{1+\rho_1}\frac{D_{shoulder}^{'1-\theta}}{1-\theta} + \frac{1}{1+\rho_2}\frac{D_{off-peak}^{'1-\theta}}{1-\theta} + \lambda \cdot \left\{ B' - D'_{peak}.P_{peak} - D'_{shoulder}.P_{shoulder} - D'_{off-peak}.P_{off-peak} \right\} \qquad (9)$$

The optimal solution lies on solving the following equations:

$$\frac{dL}{dD'_{peak}} = D_{peak}^{'-\theta} - \lambda P_{peak} = 0 \quad \Rightarrow \quad \lambda = \frac{D_{peak}^{'-\theta}}{P_{peak}} \qquad (10)$$

$$\frac{dL}{dD'_{shoulder}} = \frac{D_{shoulder}^{'-\theta}}{(1+\rho_1)} - \lambda P_{shoulder} = 0 \quad \Rightarrow \quad D_{shoulder}^{'-\theta} = \lambda(1+\rho_1)P_{shoulder} \qquad (11)$$

$$\frac{dL}{dD'_{off-peak}} = \frac{D_{off-peak}^{'-\theta}}{(1+\rho_2)} - \lambda P_{off-peak} = 0 \quad \Rightarrow \quad D_{off-peak}^{'-\theta} = \lambda(1+\rho_2)P_{off-peak} \qquad (12)$$

$$\frac{dL}{d\lambda} = B' - D'_{peak}.P_{peak} - D'_{shoulder}.P_{shoulder} - D'_{off-peak}.P_{off-peak} = 0 \Rightarrow B' = D'_{peak}.P_{peak} - D'_{shoulder}.P_{shoulder} - D'_{off-peak}.P_{off-peak} \qquad (13)$$

Substituting (10) into (11) and (12) gives:

$$D_{shoulder}^{'-\theta} = D_{peak}^{'-\theta}(1+\rho_1)\frac{P_{shoulder}}{P_{peak}} \quad \Rightarrow \quad D'_{shoulder} = D'_{peak}\left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} \qquad (14)$$

$$D_{off-peak}^{'-\theta} = D_{peak}^{'-\theta}(1+\rho_2)\frac{P_{off-peak}}{P_{peak}} \quad \Rightarrow \quad D'_{off-peak} = D'_{peak}\left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}} \qquad (15)$$

and substituting these two equations into (13) gives:

$$D'_{peak} = \frac{B'}{P_{peak} + P_{shoulder}\left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + P_{off-peak}\left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}}} \qquad (16)$$

Substituting the above equation into (14) and (15), $D'_{shoulder}$ and $D'_{off-peak}$ can be calculated as:

$$D'_{shoulder} = \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} \times \frac{B'}{P_{peak} + P_{shoulder}\left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + P_{off-peak}\left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}}} \qquad (17)$$

$$D'_{off-peak} = \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}} \times \frac{B'}{P_{peak} + P_{shoulder}\left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + P_{off-peak}\left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}}} \qquad (18)$$

As mentioned, in the long-term, consumers participation in electricity markets is neutral. In other words, the change will be in the pattern (hours) of consumption in response to price signals, not in the total power consumed, that's mean [29]:

$$D_{peak} + D_{shoulder} + D_{off-peak} = D'_{peak} + D'_{shoulder} + D'_{off-peak} \qquad (19)$$

By substituting (16)-(18) into (19), it can be deduced that:

$$B^{'} = \frac{D_{peak} + D_{shoulder} + D_{off-peak}}{(1 + \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}})} (P_{peak} + P_{shoulder}(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)})^{\frac{1}{\theta}} + P_{off-peak}(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)})^{\frac{1}{\theta}}) \tag{20}$$

Finally, by substituting (20) into (16)-(18), it can be deduced that:

$$\mathrm{D}^{'}_{peak} = \frac{D_{peak} + D_{shoulder} + D_{off-peak}}{(1 + \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}})} \tag{21}$$

$$D^{'}_{shoulder} = \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} \times \frac{D_{peak} + D_{shoulder} + D_{off-peak}}{(1 + \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}})} \tag{22}$$

$$D^{'}_{off-peak} = \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}} \times \frac{D_{peak} + D_{shoulder} + D_{off-peak}}{(1 + \left(\frac{P_{peak}}{P_{shoulder}(1+\rho_1)}\right)^{\frac{1}{\theta}} + \left(\frac{P_{peak}}{P_{off-peak}(1+\rho_2)}\right)^{\frac{1}{\theta}})} \tag{23}$$

where $D^{'}_{peak}$, $D^{'}_{shoulder}$ and $D^{'}_{off-peak}$ represent electric power consumed in each of the three mentioned periods, respectively. To obtain the consumption at each hour of a period one must use the following equations:

$$\mathrm{D}^{'}_{t\,peak} = D_{t\,peak} \times \frac{D^{'}_{peak}}{D_{peak}}$$

$$\mathrm{D}^{'}_{t\,shoulder} = D_{t\,shoulder} \times \frac{D^{'}_{shoulder}}{D_{shoulder}} \tag{24}$$

$$\mathrm{D}^{'}_{t\,off-peak} = D_{t\,off-peak} \times \frac{D^{'}_{off-peak}}{D_{off-peak}}$$

The power purchased at each certain hour can be limited to a threshold to prevent power transactions that may threat consumer welfare.

3. Results and Discussion

In this section, performance of the proposed residential DR model is investigated considering a given load profile for a building unit as shown in Figure 2. As can be observed, daily time horizon is divided into three-time frames including peak-time, shoulder-time, and off-peak time. Electricity tariff is also defined based on TOU pricing scheme, which means each time period has its own unique price. These prices are summarized in Table 1.
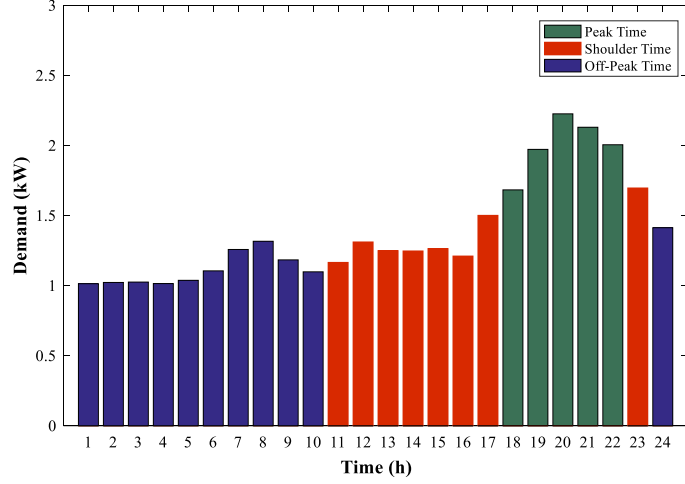
Figure 2: The examined residential load profile [33]

Table 1: Price of electricity in different time periods [1]

Demand level	P_{peak}	$P_{shoulder}$	$P_{off\text{-}peak}$
Time Period	18:00-22:00	11:00-17:00 & 23:00	00:00-10:00
Price (*cents* per *kWh*)	13.8	10	6.9

A new participant in the electricity market is also considered to assist better implementation of DR programs. This participant is called DR provider (DRP) who is in charge of management and implementation of DR programs at demand-side. To implement the proposed DR model for the examined residential building unit, all the equations presented in the previous sections are coded into scripts using MATLAB software. The choice of the suitable ρ and θ parameters are also obtained through a sensitivity analysis where needed.

Results of running the proposed DR program during the peak-time with two different values of ρ and θ are shown in Table 2. These results show that as θ decreases, so does the power consumption; i.e., the lower θ value represents the more consumer's willingness to participate in the DR program. Therefore, in the peak-time period which corresponds to the time range with the highest electricity price, the consumer's willingness to participate in DR actions increases, which in turn decreases the consumption in this period. As can be seen in Figure 3 and can be expected, peak-time electricity consumption after execution of DR program is decreased in all scenarios, meaning that the main aim of the proposed DR program, which is reducing peak-time consumption, has been achieved. Moreover, as shown in this Figure, changes of ρ_1 and ρ_2 can slightly affect the consumption level in this period; however, the general trend (consuming less during peak periods) remains the same; i.e., given a ρ_2 value and changing ρ_1 parameter provides an adjustment in the ratio of consumption in the peak-time to the shoulder-time, while changing ρ_2 parameter (given a fixed ρ_1 value) provides arbitrary ratio tuning for the consumption in the peak-time to the off-peak time.

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Table 2: Changes in the power consumption during the peak time

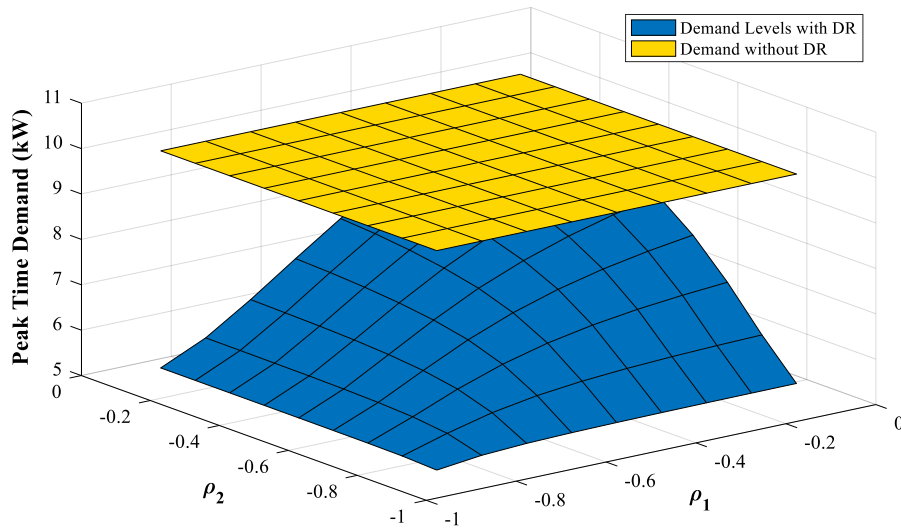| Program	θ	D_{peak} (kWh) (ρ_1= -0.7 & ρ_2= -0.78)	D_{peak} (kWh) (ρ_1= -0.6 & ρ_2= -0.85)
Base Case-Without DR	-	10.0140	
Proposed DR model	0.7	6.2150	6.0647
	0.6	6.0272	5.8667
	0.5	5.8093	5.6461
	0.4	5.5658	5.4135
	0.3	5.3189	5.2068
	0.2	5.1285	5.0872

Figure 3: Consumption levels for different rates of time preferences (peak period)

Changing the values of ρ_1 and ρ_2 have much more effects on the power consumption levels in the shoulder-time than in other time frames as observed from Figure 4. By adjusting the rates of time preferences in this period, the consumer can choose one of the following modes:

1) Energy-saving mode: consumer can select this mode to transfer elastic part of the demand into the off-peak times to avoid high energy consumption costs (the area below the intersection of the load surface without DR and the load surface with DR actions),
2) Consumption mode: consumer can select this mode to consume the elastic part of the demand which saved during the peak time because of the lower prices of this time compared to the peak-time (the area above the intersection of the load surface without DR and the load surface with DR actions)
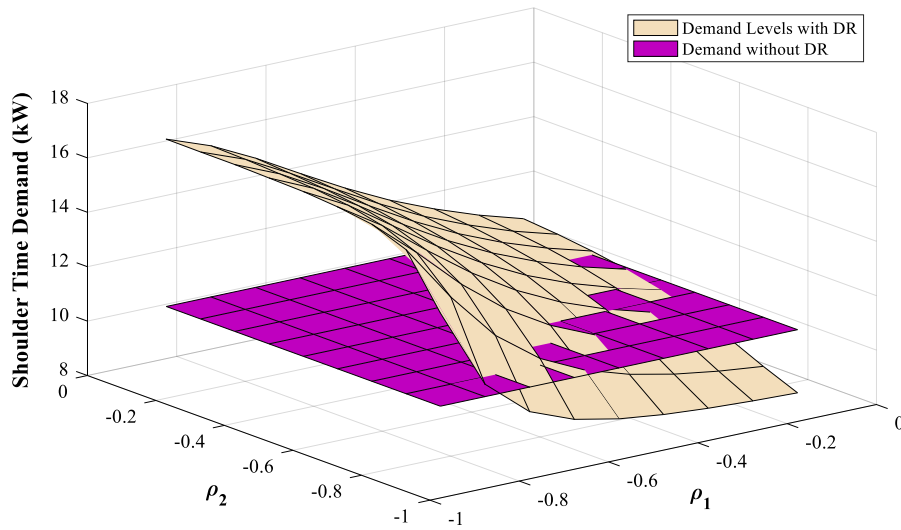
Figure 4: Consumption levels for different rates of time preferences (shoulder period)

On the other hand, simulation results regarding DR implementation during the shoulder-time period demonstrate that given a ρ_i value and changing the relative risk aversion parameter can change the energy consumption trend. For example, when ρ_1=-0.7 and ρ_2=-0.78 as θ decreases, $D_{shoulder}$ increases; but when ρ_1=-0.6 and ρ_2=-0.85 the opposite happens as shown in Table 3.

Table 3: Changes in the power consumption during the shoulder time

Program	θ	$D_{shoulder}(kWh)$ (ρ_1= -0.7 & ρ_2= -0.78)	$D_{shoulder}(kWh)$ (ρ_1= -0.6 & ρ_2= -0.85)
Base Case-Without DR	-	10.6436	
Proposed DR model	0.7	12.1548	10.4391
	0.6	12.2841	10.2606
	0.5	12.4402	10.0087
	0.4	12.6341	9.6394
	0.3	12.8791	9.1236
	0.2	13.2063	8.5094

As can be seen in Figure 5 and can be expected, once θ (i.e., coefficient of relative risk aversion) increases, the user becomes less inclined to shift his/her consumption from the current period to the next one. And so, with the increase in θ, the amount of energy saving decreases with the goal of transferring it to other times.
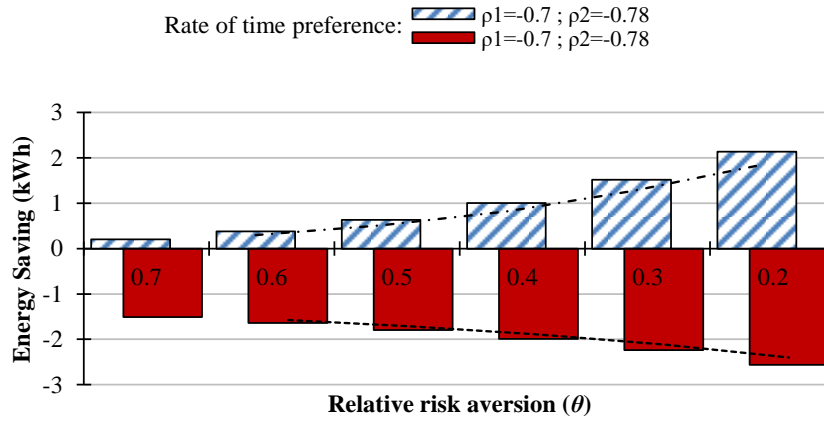
Figure 5: Levels of commitment in DR programs under different scenarios

The results obtained for DR actions during the off-peak time are shown in Table 4. As can be seen in Table 4, with the decrease of θ (and increase of consumers' participation accordingly), there is a growing trend in the power consumption of this period due to the lower electricity prices.

Table 4: Changes in the power consumption during the off-peak time

Program	θ	$D_{off\text{-}peak}$ (kWh) (ρ_1= -0.7 & ρ_2= -0.78)	$D_{off\text{-}peak}$ (kWh) (ρ_1= -0.6 & ρ_2= -0.85)
Base Case-Without DR	-	12.4847	
Proposed DR model	0.7	14.8574	16.7203
	0.6	14.9208	17.0980
	0.5	14.9830	17.5866
	0.4	15.0345	18.1865
	0.3	15.0421	18.9109
	0.2	15.0773	19.6456

In a similar manner, by adjusting the rate of time preference parameters in this period, consumers can choose two modes:

1) Consumption mode: in this mode the user would consume the energy saved during other time intervals (the area above the intersection of the base load surface and the one with DR actions in Figure 6)

2) Energy-saving mode: this mode denotes energy saving during off-peak period (the area below the intersection of the two surfaces in Figure 6) which is not an economic working condition for a well-designed DR program. Indeed, this mode would trigger a point of discontinuity in the consumer's decision pattern, meaning that the goal of the proposed DR model, which is increasing off-peak time consumption, cannot be achieved. Therefore, this mode will not be considered as a valid state.
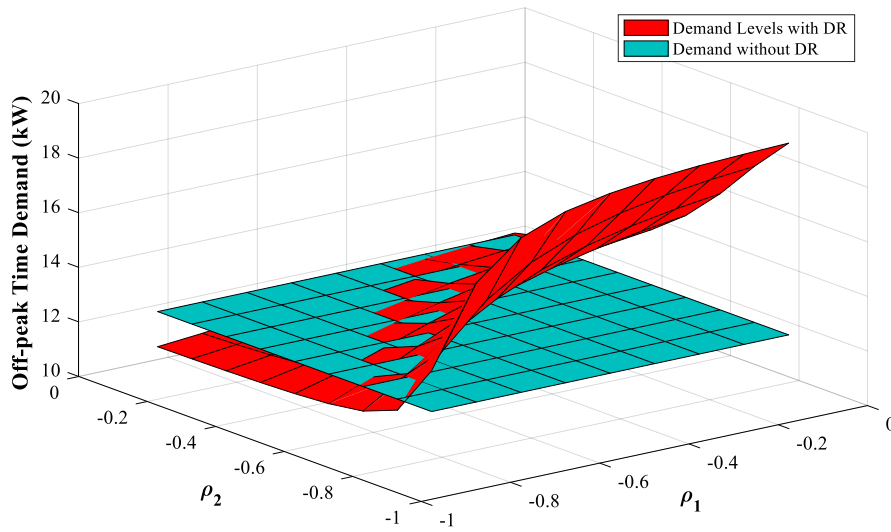
Figure 6: Consumption levels for different rates of time preferences (off-peak period)

To get better insight into the level of commitment in residential DR Programs over the time, the consumption profiles have been plotted in Figure 7 for different scenarios. It can be observed that with the increase of θ (while keeping other parameters unchanged), the consumer's willingness to participate in DR actions especially during the peak-time period decreases. This effect is clearly illustrated in Figure 8 for each time interval. Therefore, this model can combine the willingness of customers (according to the behavior of each consumer) into the DR model to control the consumption over the time in an affordable way. From the results, it can be observed that DR program execution not only reduces consumption at peak times, but also helps to shift electricity demand to other time periods with lower price.
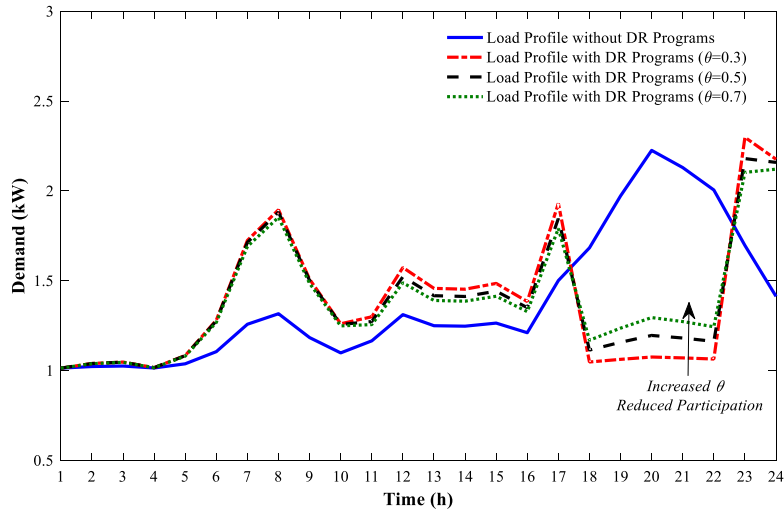

Figure 7: Levels of commitment in residential DR programs under different scenarios
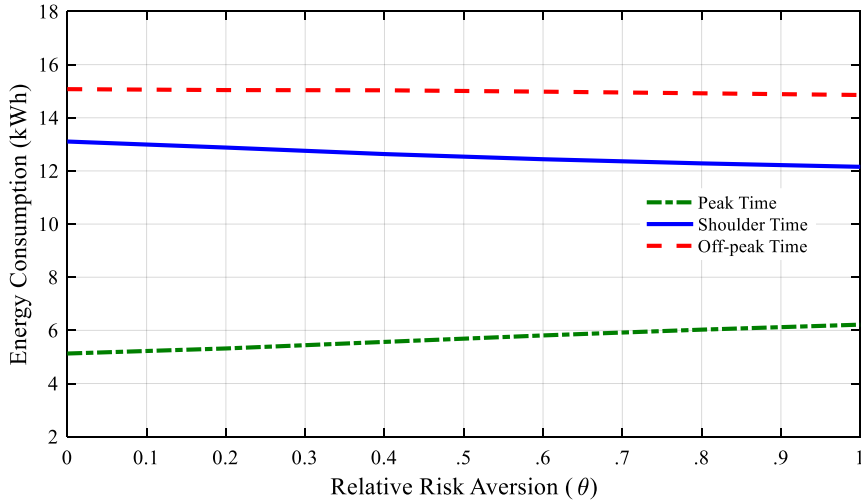
Figure. 8: Changes in power consumption in each period versus changes in θ

Considering the results tabulated in Table 5, it is also observed that for a given θ, the total energy consumed over all time periods (D_{All}) is constant, which means that consumption levels have only shifted between the periods to adapt to new prices. Moreover, it can be easily understood that the budget allocated to the electricity purchase decreases as θ goes down which in turn increases the consumer's willingness to participate in DR programs. This mean that a consumer who participates in a DR program can decrease his/her electricity bill up to 16% without changing his/her overall energy consumption.

Table 5: Results of the model in different scenarios

Program	θ	DAll (kWh)	Budget ($)	DAll (kWh)	Budget ($)
		(ρ1= -0.7 & ρ2= -0.78)		(ρ1= -0.6 & ρ2= -0.85)	
Base Case-Without DR	-	33.1423	3.3077	33.1423	3.3077
Proposed DR model	0.7	33.2273	3.0983	33.2240	3.0345
	0.6	33.2321	3.0897	33.2253	3.0154
	0.5	33.2325	3.0795	33.2414	2.9935
	0.4	33.2343	3.0689	33.2395	2.9659
	0.3	33.2401	3.0598	33.2414	2.9358
	0.2	33.3121	3.0586	33.2422	2.9085

At the end, it should be emphasized that two methods are currently available to implement DR technologies. The first is to deploy autonomous systems that require no interaction from consumers or utilities and automatically sense changing conditions of the power grid and adjust accordingly. The second method is to communicate an incentive signal through the utility to each consumer. In this method, price information is sent to consumers thereby providing sufficient information to modify energy usage. Two signals are used in most implementations. The first is the Transactive Incentive Signal (TIS). The TIS is sent by the utility to each consumer. The TIS contains expected energy prices for upcoming periods of time. The second is the Transactive Feedback Signal (TFS). The TFS is sent from the consumer to the utility and contains

information regarding expected energy use at various prices. The TIS and TFS together comprise an energy market where price information is communicated to consumers. To communicate TCS and TFS information to the consumer, wireless technologies and power line carrier are two common methods [34].

4. Conclusion and Future Works

In this paper, the well-known Diamond's OLG model was used as the basis of an effective responsive load modeling for TOU-based residential demand response (DR) program. The proposed DR model was able to adapt to different consumers with different flexibilities against prices and allowed the consumption levels to be adjusted over different time intervals. It was demonstrated that in the developed model, parameter θ can be used to easily simulate the consumer's response based on his flexibility against prices, and parameter ρ can be used to adjust the consumption levels in different time periods as desired. This model represents the mechanisms used to encourage consumers to shift demand during peak times to other times and could be realized by appropriate coding of related equations and algorithms presented previously into a residential energy management system and enabling two-way communications between utilities and consumers. Also, this model can be used in real time DR models implemented by retailer companies to offer bidding strategy.

Future works will be mainly focused on developing the proposed model of residential DR program based on alternative schemes such as real-time pricing and critical peak pricing and evaluating its applicability and effectiveness in such working conditions. We will also conduct more simulations on larger test systems such as a cluster of residential and office buildings in a smart grid environment and investigate the effectiveness of our proposed model in presence of uncertain parameters (such as price uncertainty and stochastic consumer's behavior).

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