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Geometric vs. structural form finding in reciprocal structures

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Abstract

In the present paper the relationship between structural and geometric form-finding in reciprocal structures is investigated, by assessing whether the purely geometric form finding is also capable of delivering structurally optimal structures. The work will use as a case study the reciprocal structure designed by the author D. Parigi for the submission to the international design competition "Sky Forest City" awarded 3rd prize in December 2016 in Chongqing.

Keywords: form finding, adaptable joint, fabrication aware design

1. Introduction

Reciprocal structures are formed by mutually supported elements, joined together according to the principle of reciprocity. The principle of reciprocity is based on the use of load-bearing elements which, supporting one another along their spans and never at the ends, compose a spatial configuration with no clear structural hierarchy (Pugnale and Sassone [1]). According to this definition, reciprocal structures cannot be modelled with the theory of pin-jointed frameworks, composed by bar elements connecting only at their ends with pin (hinge) joints.

The form finding of reciprocal structure differs substantially from classical methods of form finding for discrete networks, particularly relevant in structures that transfer their loads purely through axial or in-plane forces, as unstrained gridshells, cable-nets and tensegrity (Veenendal et al. [2]). The topology of reciprocal structures permits the transfer of loads through bending moments, thus the classical form finding methods would require a reformulation that provide a generalization to an expanded set of internal forces. Furthermore, when a form finding method is applied to reciprocal structures it is necessary to tackle at the same time a more complex geometrical compatibility of the elements, since the elements axes can be non-aligned, and elements does not meet at the ends but along their span. The form finding process of reciprocal structures is in this case purely geometrical and does not take into consideration load conditions; rather it is based on the constraints posed by the geometric compatibility of the bars, allowing the structure to be assembled with a simple adaptable joint (Parigi [3]).

In a particular subset of reciprocal structures, where the eccentricity value is set to 0, i.e. the axis of elements are aligned and an axial adaptable joint can be used (Figure 1), the geometry will be primarily affected by the boundary conditions rather than the geometric parameters at the joint. In such configurations, forces are mainly transferred through in-plane forces, and the resulting shape is primarily governed, similarly to a grid structure form-found through a classical form-finding algorithm, by the boundary and load conditions. Despite the form finding being purely geometrical, the form-found solutions appear to be comparable with the solutions obtained with classical form-finding methods. A re-formulation of the geometric form finding will be provided to allow a better comparison with classic form-finding methods. Furthermore, a geometrical interpretation of the form-finding will be provided. Finally, a set of case study will be used to validate the results.

2. Reciprocal axial connection and universal joint

In this work, the subset of reciprocal structures where the node eccentricity is 0, i.e. the element axis are aligned at the nodes, is considered. The connection of the bars at the nodes will be called axial connection. Despite initial studies have been carried to define the detail of the connection, the technology used to create the joint is not the focus of this work. It is the opinion of the authors however that this typology will allow a fast and efficient method of construction following essentially two alternative solutions:

a) joint using a hinged connector

b) joint using fitting carved bars ends robotically produced and a bolted/screwed connection

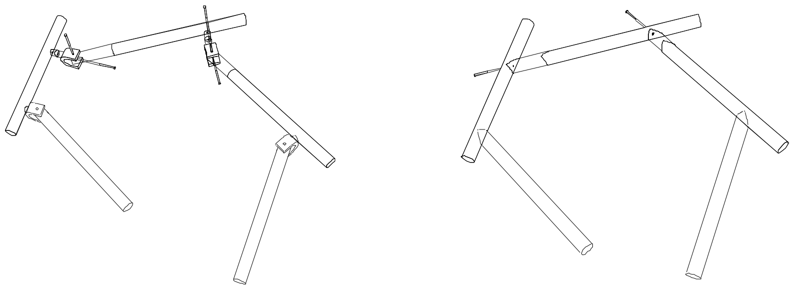


Figure 1: reciprocal configuration with axial connection and example of joint: a) using connectors, b) using carved bars ends and bolted/screwed connection.

3. Classical form finding: Force Density Method (FDM) and Dynamic Relaxation (DR)

Methods of Force Density (FDM) and Dynamic relaxation (DR) have been implemented by the authors in order to compare classical methods of form finding to the form finding method developed for reciprocal structures. The formulation of FDM is described here to allow a comparison with the matrix formulation of reciprocal structures' form-finding.

2.1. The Force Density Method

The Force Density Method is a method of form finding belonging to the family of *Geometric stiffness methods* (Veenendal et al. [2]). It is material independent, with only a geometric stiffness, using force densities to express the equilibrium problem with a system of linear equations.

Accordingly with the theory of pin-jointed assemblies, the equilibrium can be assessed writing *3n* equations, stating the equilibrium of each joint under the external load and the internal forces (axial) in the bars. In such equations, the *m* axial forces and the *3n* external loads are put in relation through a *3n* x *m* matrix of coefficients called the equilibrium matrix. The properties of such a matrix will rule the static determinacy of the structure.

The system of equations is linearized by introducing q as force density S/L, where S is the force in element and L its length. If q and P are known, the result is a set of linear equations in x, y, and z and it expressed in matrix form:

$$\begin{aligned} C^T*Q*C*x &= -\ C^T*Q*Cf*xf \\ C^T*Q*C*y &= -\ C^T*Q*Cf*yf \\ C^T*Q*C*z &= -\ C^T*Q*Cf*zf \end{aligned} \qquad (1)$$

C is a matrix that describes the element connectivity. It has **m** rows and **n** columns, **m** is the number of elements and **n** is the number of free nodes (n) plus the number of fixed nodes (nf). Fixed nodes should come after free nodes in the node numbering. All elements of **C** are zero except nodes connecting elements take a value 1 for element start node or -1 for element end node.

By numbering fixed nodes at the end, **C** is divided into **Ci** and **Cf** (**C** =[**Ci**| **Cf**]).

Q is the diagonal matrix of the vector of force densities **q**.

Linear Equations (1) are solved to get **x**, **y** and **z**.

4. Equilibrium in reciprocal structures

The specificity of reciprocal structures elements being joined not at the ends but in intermediate points makes these structures to behave in a different way from pin-joined assemblies. In order to extend the theory of pin-joint assemblies to planar reciprocal frames, it is necessary to remove two assumptions that are at the basis of such a theory: the use of pins as the unique kind of constraints, and the position of joints only at the ends of the bars.

In order to write the equilibrium equations for reciprocal assemblies an alternative formulation has been developed. The equilibrium equations are written for the bars rather than for the joints, and will include the presence of bending moments (Parigi [4]). The resulting set of equations and associated matrix will have the size of [6m x 2n]: The formulation for the equilibrium of the bars rather than at the nodes renders the equilibrium matrix not directly usable in form finding methods of pin-jointed assemblies used in FDM.

In the following paragraph, a novel formulation for reciprocal form finding based on the geometric compatibility is proposed and described with a similar notation to the traditional form finding methods (Veenendal et al. [2]). The results are then compared with the ones obtained from FDM and DR.

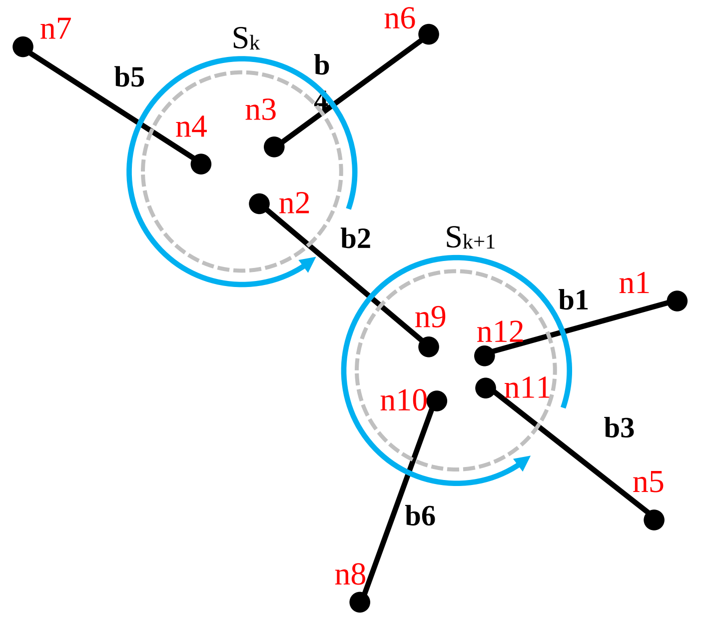


Figure 2: the starting topology of a reciprocal configuration with supernodes S_k and S_{k+1}

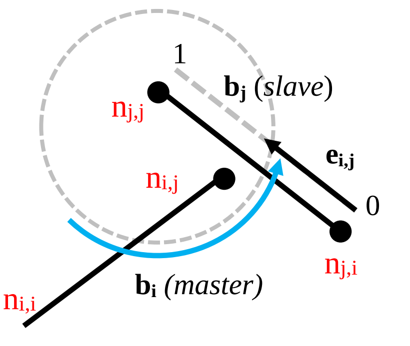


Figure 3: a connection $n_{i,j}$ between bars b_i-b_j

5. Formulation of reciprocal form-finding

A network of reciprocally connected bars (Figure 2) is defined to illustrate the formulation.
A matrix **C'** is defined, describing the elements connectivity: the matrix has **m** rows and **n** columns, where **m** is the number of bars and **n** is the number of nodes, and the [i,j] entry is

1 if the node j is the start node of the bar i
-1 if the node j is the end node of the bar i.

0 in all the other entries

$$\mathbf{C'}=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&-1\\0&1&0&0&0&0&0&0&-1&0&0&0\\0&0&0&0&1&0&0&0&0&0&-1&0\\0&0&1&0&0&-1&0&0&0&0&0&0\\0&0&0&1&0&0&-1&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&-1&0&0\end{bmatrix}$$

For a network of **m** bars and **n** nodes in the three-dimensional space, the matrix **C** is constructed as

$$\mathrm{C}=\begin{bmatrix}\mathrm{C}'&&\\&\mathrm{C}'&\\&&\mathrm{C}'\end{bmatrix}$$

Since the topology in reciprocal structure is defined as connectivity of bars rather than connectivity of nodes an additional matrix **D** is defined. It first requires the identification of a *k* number of "supernodes" S_k, with a clockwise (CW) or counter-clockwise (CCW) direction of the bars' connectivity.
Every bar can be in turn *master* or *slave* in each of the connections $n_{i,j}$ between two bars b_i-b_j..
The matrix **D** has **n** rows and **m** column and the [i,j] entry is

1 if the end node of the slave bar b_j is converging in the supernode center;
-1 if the end node of the slave bar b_j is opposite to the supernode center.

$$\mathbf{D}=\begin{bmatrix}0&0&0&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&1&0\\0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\\0&0&1&0&0&0\\1&0&0&0&0&0\\0&1&0&0&0&0\end{bmatrix}$$

The node $n_{i,j}$ of the master bar b_i connects to the slave bar b_j in an intermediate position determined with the parameter **e** engagement length such as that its value is

0 to define the point at the start of the slave bar b_j i.e. the node $n_{j,i}$
1 to define the point at the end of the slave bar b_j i.e. the node $n_{j,j}$
0<e<1 to define intermediate points in the bar b_j

The parameter *e* value for each b_i-b_j connection is stored in the vector **e**, and in **E** diagonal matrix of **e**.
The [3n x 1] nodal coordinate vector **x** is

$$\boldsymbol{x}=\begin{bmatrix}\boldsymbol{x}'\\\boldsymbol{y}'\\\boldsymbol{z}'\end{bmatrix}$$

where $\mathbf{x}'=(x_1, x_2,\dots x_n)^T$, $\mathbf{y}'=(y_1, y_2,\dots y_n)^T$ and $\mathbf{z}'=(z_1, z_2,\dots z_n)^T$ are vectors containing the nodal coordinates in Cartesian direction.
At each iteration the system (2) is computed

$$\mathbf{x}_{i,t+dt}=\mathbf{x}_{i,t}+\mathbf{D}*\mathbf{C}*\mathbf{E}*\mathbf{x}_{i,t} \tag{2}$$

where *t* is the iteration step. The convergence criteria is determined as the value of **D*C*E*** $\mathbf{x}_{i,t}$ gets smaller then the tolerance set.

6. Geometric interpretation

A geometric interpretation of the algorithm is provided, first with a simple case of two connecting bars, then with a planar reciprocal configuration of three bars.
Given two bars b_i (start point in $n_{i,0}$ and end point in n_i) and b_j (start point in $n_{j,0}$ and end point in to n_j), and given $n_{i,1}$ and $n_{j,1}$ converging in a point O in the initial position, at each step t the following operations are performed:

-node n_i is displaced to $n_{i,t}$ the middle point of the connecting bar b_j
-node n_j is displaced to $n_{j,t}$ the middle point of the connecting bar b_i

The intersection between the bars in the new position gives the point O^t. bars bi and bj are redrawn with the end points converging in O^t. The process is iterated for each step t until convergence.
It is observed that at the last step the position of O^t lies at the center of the line connecting the start nodes of each of the two starting bars $n_{i,i}$ and $n_{j,i}$ (figure 1)

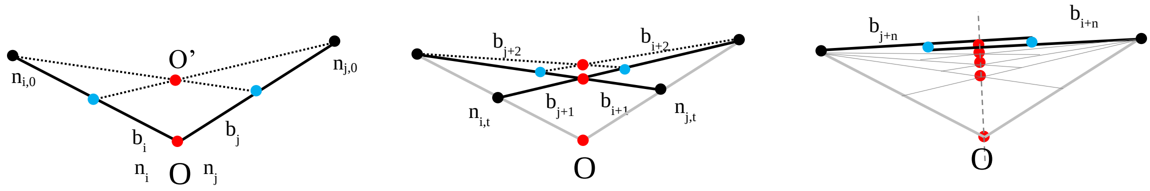


Figure 4: Iterations when bars bi and bj have the same starting length and e=0.5

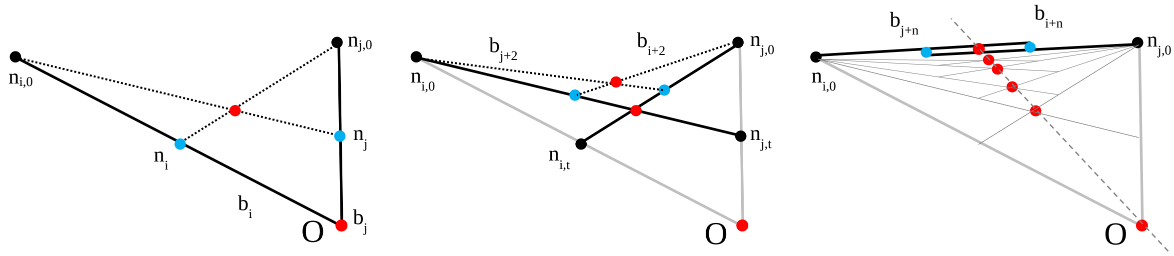


Figure 5: Iteration when bars bi and b_j have different starting length and e=0.5

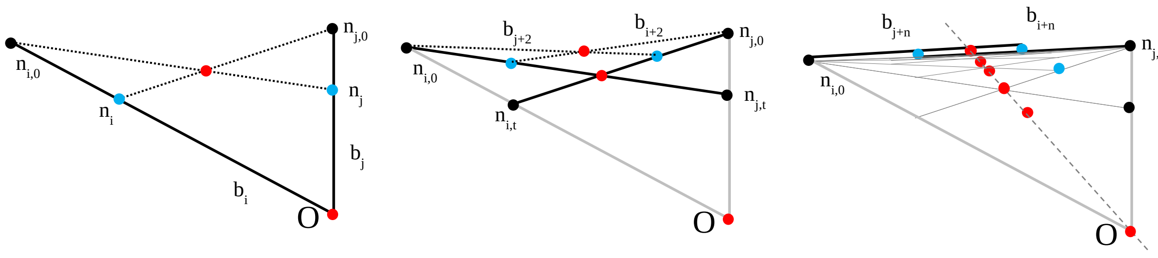
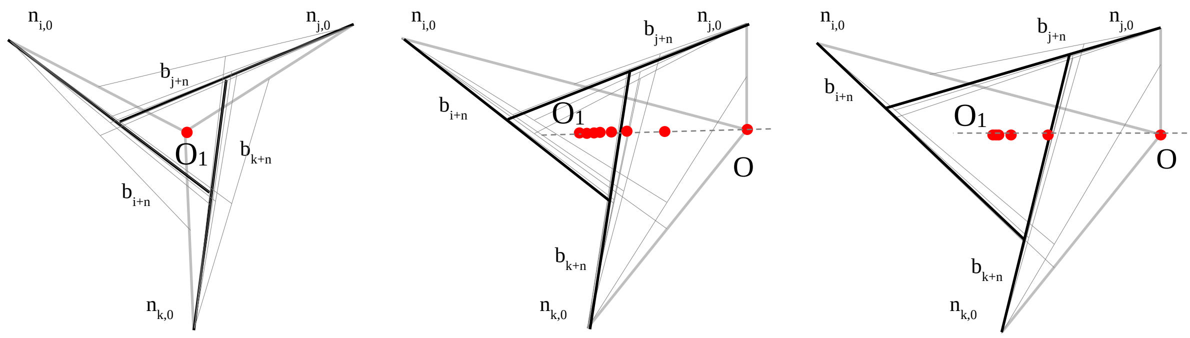


Figure 6: Iteration when bars b_i and b_j have different starting length and e=0.33

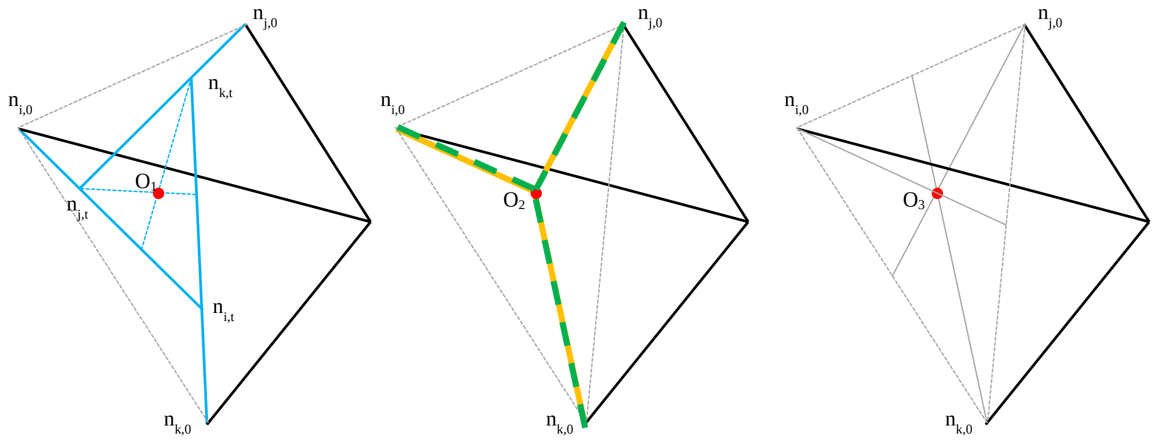
The final position of O^t is independent of the starting length of the bars b_i and b_j (figure 2) and from the value of the parameter e (figure 3). In this latter instance, with e=0.33, the end nodes of the bars are displaced to the third of the span of the connecting bars. The final position of O^t is at the center of the line connecting the nodes at the start node of the two bars $n_{i,0}$ and $n_{j,0}$.

The same applies to a reciprocal unit with three bars and three connections (Figure 7). For this configuration the engagement **e** is set to the same value for every bar, and similarly, the force density **f** for the FDM, and masses **m** for DR are set to be uniform for all bars. The centroid O_1 of the reciprocal configuration is computed as the centroid of the points $n_{i,t}$, $n_{j,t}$, $n_{k,t}$. It lies in the same position of the free node O_2 whose position is computed by running FDM or DR (Figure 8) and of node O_3, centroid of the fixed nodes $n_{i,0}$, $n_{j,0}$, $n_{k,0}$. It is expected that in larger reciprocal configuration the centroids of reciprocal fans will lie in a similar position of the nodes of a correspondent pin-jointed configuration form-found with FDM or DR.



(a) (b) (c)

Figure 7: Iteration on a reciprocal configuration when bars b_i and b_j have equal starting length and e=0.5 (a), different starting length and e=0.5 (b), and different starting length and e=0.33 (c)



(a) (b) (c)

Figure 8: Centroid O_1 of a reciprocal configuration with different starting length (a), position of the free node O_2 obtained by using FDM (yellow line) and DR (green dotted line), and geometric centroid O_3

7. Case studies

As a consequence of the above principles, it is possible to conclude that the parameter *e* will not affect the final shape of the configuration also when applying this algorithm to larger assemblies (Figure 9). This behavior is unexpected in reciprocal structures since the geometry of a configuration is in general determined by the parameter *e* value (Parigi [3]). Since higher values of *e* also determine increased length of the connecting bars, this property can be used at the designer's advantage in order to fine-tune the density of the construction, and in order to optimize the configuration with respect to the structural performance without affecting the overall geometry. The result obtained with the reciprocal form finding is comparable and virtually identical to the one obtained with FDM and DR.

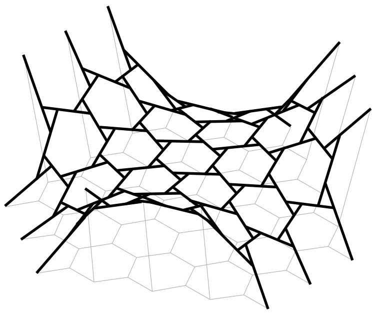


Figure 9a: reciprocal configuration with e=0.33

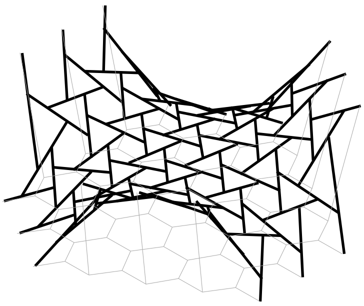


Figure 9b: reciprocal configuration with e=0.66

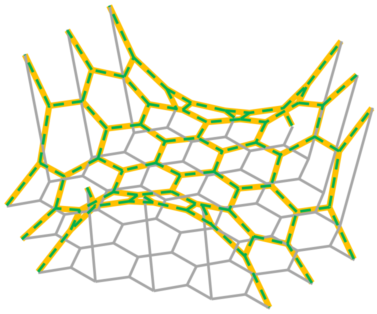


Figure 9c: a correspondent pin-jointed structure obtained with FDM and DR

A larger configuration of 1418 bars is presented (Figure 10). The configuration was designed by the author D. Parigi for the submission to the international design competition "Sky Forest City" awarded 3rd prize in December 2016 in Chongqing. The configuration has a plan dimension of 7x 21 meters. It has top edge support and supports at the ground around three circular footings. A comparable study with FDM and DR confirmed that the resulting shape can be overlapped with a corresponding pin-jointed structure obtained with FDM and DR (Figure 11).

Configuration	**Maximum displacement** [mm]	
	Distributed load (1 KN/m$^{2)}$	*Concentrated load (4 KN)*
Pin-jointed	8	6
Reciprocal	8	5

Table 1: maximum displacemet for the reciprocal and the corresponding pin-jointed configuration

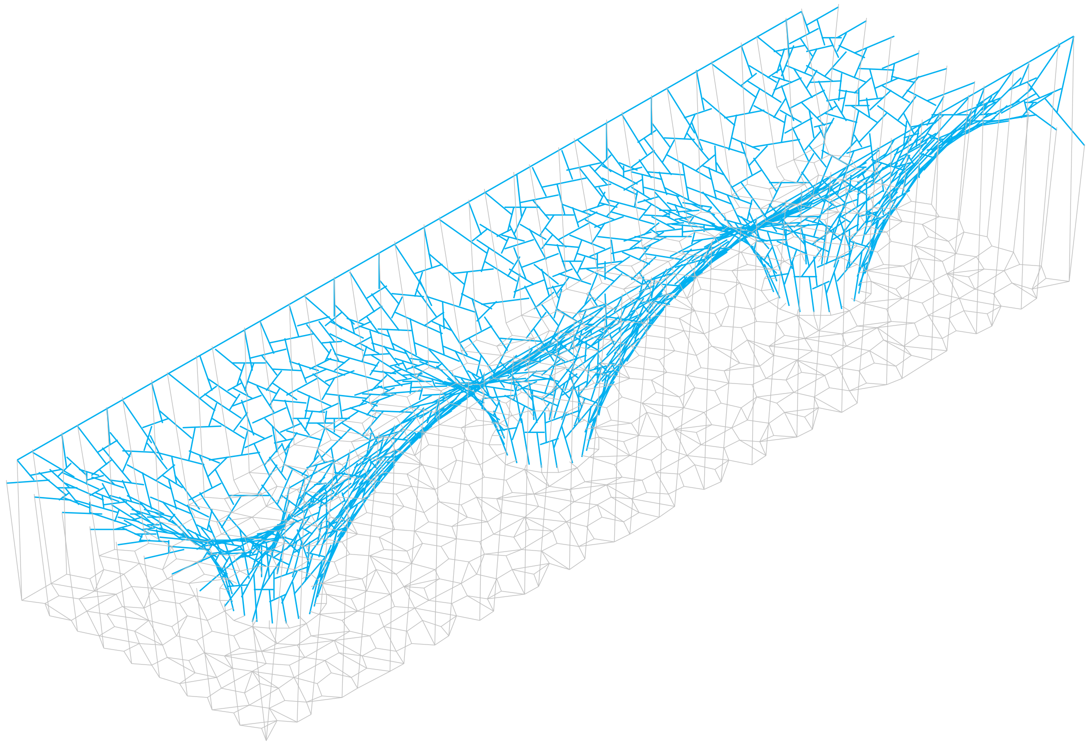


Figure 10: Reciprocal configuration with 1418 bars obtained with the reciprocal form finding. Light gray lines visualize the starting topology of the configuration

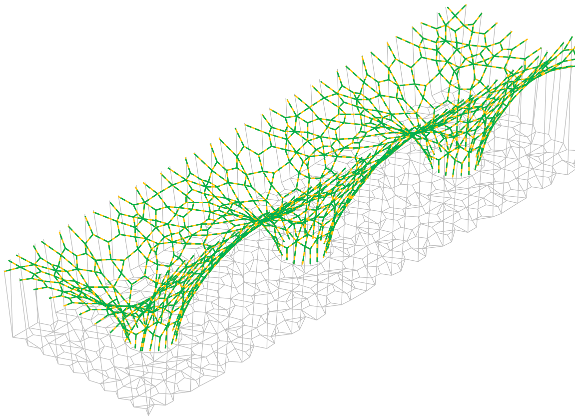


Figure 11: Corresponding pi-jointed configuration obtained with FDM (yellow) and DR (green)

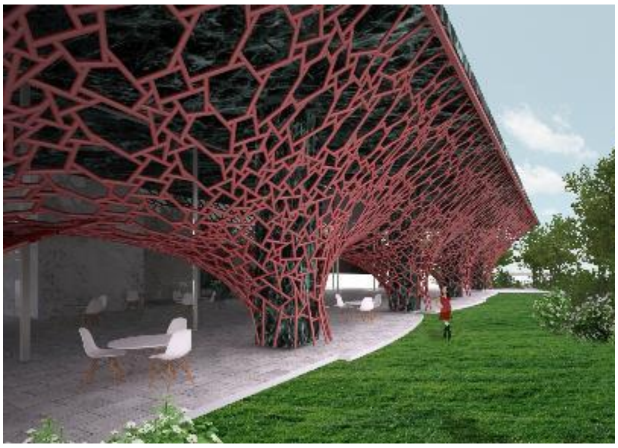


Figure 12: Rendered view of the reciprocal structure

8. On structural and technological efficiency

A preliminary structural analysis shows that, using the same section size for the pin-jointed structure and the reciprocal structure, the stiffness of the assembly is similar. The maximum displacement values with distributed load and concentrated loads are evaluated (Table 1). While the values show a slight advantage of reciprocal structures with the concentrated load, further investigations should determine how to globally assess the efficiency of the structure. The reciprocal structure has approximately an initial 30% larger total bars length, i.e it uses 30% more material. However, as noted in paragraph 6, the stiffness of the reciprocal configuration can be adjusted locally with the parameter e, leading to potential material saving. Another aspect of the evaluation concerns the jointing technique. In free-form pin-jointed configurations the joint has a critical impact on the economy and the construction efficiency. The complexity increases with the increased number of converging bar in one node, with the manufacturing of thousands of complex and unique joints. A reciprocal configuration on the other hand will offer the possibility for a simpler connection technique, since in each connection a maximum of two bars meet. Since the element axis are aligned, and they meet along the *slave* bar span, different types of joint can be conceived (Figure 1), potentially simplifying the construction phase.

9. Conclusions

The paper has assessed the convergence of structural and technological efficiency in a particular subset of reciprocal structures designed with axial connection. While preliminary studies show an initial better material efficiency of pin-jointed structures, a reciprocal structure will allow simpler construction techniques and more opportunities for structural optimization. A broader evaluation, taking in consideration both the structural performance and construction phase, is finally suggested for the assessment of the optimality of a built structure.

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