Construction of 12 DOFs spur gear coupling dynamic model

Huanchao Lv[1], Zhengminqing Li[2], Wenlin Zhu[3], Xin Tang[4], Jie Gao[5], Rupeng Zhu[6]
[1, 2, 6]National Key Laboratory of Science and Technology on Helicopter Transmission, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China
[3, 4, 5]Science and Technology on Helicopter Transmission Laboratory, AECC Hunan Aviation Powerplant Research Institute, Zhuzhou, 412002, China
[2]Corresponding author
E-mail: [1]*huanchao_lv@163.com*, [2]*lzmq_cmee@nuaa.edu.cn*, [3]*Zhuwenling12411@163.com*, [4]*tangx4711@163.com*, [5]*gaodongchu7009@sina.com*, [6]*rpzhu@nuaa.edu.cn*

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Abstract. A 12-degree-of-freedom (DOF) spur gear dynamic model is constructed, which is coupled by the mesh gear pair and the gearbox. The construction method of spur gear coupling dynamic model, based on lumped mass method, is better than finite element method, due to higher modeling efficiency. The work would be benefit to spur gear coupling dynamic modeling and analyses.

Keywords: spur gear, coupling dynamic model, 12 DOFs.

1. Introduction

Gear dynamic models are focused by many scholars. There is an extensive body of literatures on it [1-9]. Jin et al. established gear dynamic models coupled with bending-torsion-axis-swing of mesh pairs based on lumped mass method [10]. Zhu et al. constructed finite element models of the gear transmission, and evaluated dynamic behavior of the system [11, 12]. Ren et al. proposed a construction method of gear dynamic models based on substructure method [13-15]. However, the gear coupling dynamic models associated with mesh pairs and gearbox supports are few studied. Thus, in the paper, a 12 DOFs spur gear coupling dynamic model, based on lumped mass method, is proposed. The work would be helpful to the spur gear coupling dynamic analyses.

2. Construction of 12DOFs dynamic model

The gear transmission system is mainly composed of two spur gears, bearings and gearbox supports. When modeling with the finite element method, it is inefficient because of the complexity of the gearbox supports. Therefore, a 12 DOFs coupling dynamic model based on lumped mass method is established, as shown in Fig. 1.

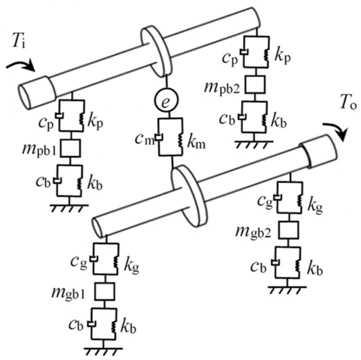


Fig. 1. The coupling dynamic model

As illustrated in Fig. 1, subscript p and g express driving gear and driven gear, respectively, k is a bending stiffness, c is a bending damping, k_m is a mesh stiffness, c_m is a mesh damping, e is a static transmission errors (STE), T_i is the input torsion, T_o is the output torsion, k_b is the support stiffness, c_b is the support damping. Moreover, m_{pb1}, m_{pb2}, m_{gb1} and m_{gb2} are the equivalent masses of the gearbox supports.

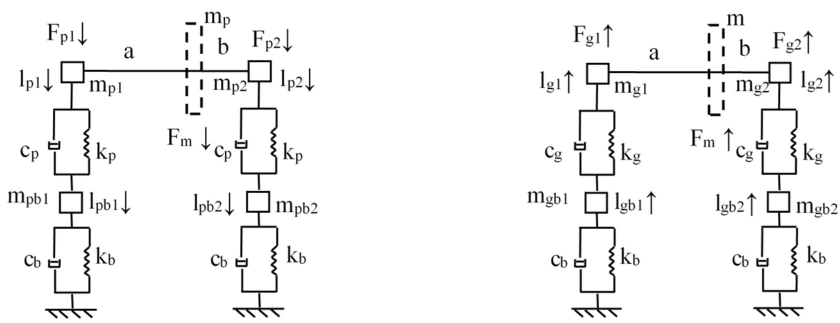
As given in Fig. 1, the mathematical equations of the meshing pair could be derived by:

$$\begin{cases} m_p\ddot{l}_p + c_p\dot{l}_p + k_p l_p - F_m = 0, \\ m_g\ddot{l}_g + c_g\dot{l}_g + k_g l_g + F_m = 0, \\ I_i\ddot{\theta}_i + c_1(\dot{\theta}_i - \dot{\theta}_p) + k_1(\theta_i - \theta_p) = T_i, \\ I_p\ddot{\theta}_p - c_1(\dot{\theta}_i - \dot{\theta}_p) - k_1(\theta_i - \theta_p) - r_p \cdot F_m = 0, \\ I_g\ddot{\theta}_g + c_2(\dot{\theta}_g - \dot{\theta}_o) + k_2(\theta_g - \theta_o) + r_g \cdot F_m = 0, \\ I_o\ddot{\theta}_o - c_2(\dot{\theta}_g - \dot{\theta}_o) - k_2(\theta_g - \theta_o) = -T_o, \end{cases} \tag{1}$$

where subscript i and o express motor and load, respectively, θ is a torsion degree, l is a bending degree, m is a mass, r is a base circle radius, I is a moment of inertia, k_1 and k_2 are torsional stiffness of the shaft, c_1 and c_2 are torsional damping of the shaft, and F_m could be deduced as:

$$F_m = k_m \cdot (r_g\theta_g - r_p\theta_p + e + l_g - l_p) + c_m \cdot (r_g\dot{\theta}_g - r_p\dot{\theta}_p + \dot{e} + \dot{l}_g - \dot{l}_p). \tag{2}$$

The gearbox supports dynamic equivalent model is proposed, as shown in Fig. 2.



a) Equivalent model of the driving gear b) Equivalent model of the driven gear

Fig. 2. The gearbox supports dynamic equivalent model

As illustrated in Fig. 2, the equivalent mass of the gears at the bearing fulcrum could be deduced as:

$$\begin{cases} m_{p1} = m_p \cdot \dfrac{b}{a+b}, \\ m_{p2} = m_p \cdot \dfrac{a}{a+b}, \\ m_{g1} = m_g \cdot \dfrac{b}{a+b}, \\ m_{g2} = m_g \cdot \dfrac{a}{a+b}, \end{cases} \tag{3}$$

where a and b are the distance from the gear to the bearing fulcrum.

As given in Fig. 2, the mathematical equations of the support structure could be derived by:

$$\begin{cases} m_{p1}\ddot{l}_{p1} + c_p(\dot{l}_{p1} - \dot{l}_{pb1}) + k_p(l_{p1} - l_{pb1}) - F_{p1} = 0, \\ m_{pb1}\ddot{l}_{pb1} + c_b\dot{l}_{pb1} + k_b l_{pb1} - c_p(\dot{l}_{p1} - \dot{l}_{pb1}) - k_p(l_{p1} - l_{pb1}) = 0, \\ m_{g1}\ddot{l}_{g1} + c_g(\dot{l}_{g1} - \dot{l}_{gb1}) + k_g(l_{g1} - l_{gb1}) - F_{g1} = 0, \\ m_{gb1}\ddot{l}_{gb1} + c_b\dot{l}_{gb1} + k_b l_{gb1} - c_g(\dot{l}_{g1} - \dot{l}_{gb1}) - k_g(l_{g1} - l_{gb1}) = 0, \\ m_{p2}\ddot{l}_{p2} + c_p(\dot{l}_{p2} - \dot{l}_{pb2}) + k_p(l_{p2} - l_{pb2}) - F_{p2} = 0, \\ m_{pb2}\ddot{l}_{pb2} + c_b\dot{l}_{pb2} + k_b l_{pb2} - c_p(\dot{l}_{p2} - \dot{l}_{pb2}) - k_p(l_{p2} - l_{pb2}) = 0, \\ m_{g2}\ddot{l}_{g2} + c_g(\dot{l}_{g2} - \dot{l}_{gb2}) + k_g(l_{g2} - l_{gb2}) - F_{g2} = 0, \\ m_{gb2}\ddot{l}_{gb2} + c_b\dot{l}_{gb2} + k_b l_{gb2} - c_g(\dot{l}_{g2} - \dot{l}_{gb2}) - k_g(l_{g2} - l_{gb2}) = 0. \end{cases} \tag{4}$$

According to the deformation coordination relationship, as shown in Fig. 3, the deformation coordination equations could be derived by:

$$\begin{cases} l_p = \dfrac{bl_{p1} + al_{p2}}{a+b}, \\ l_g = \dfrac{bl_{g1} + al_{g2}}{a+b}. \end{cases} \tag{5}$$

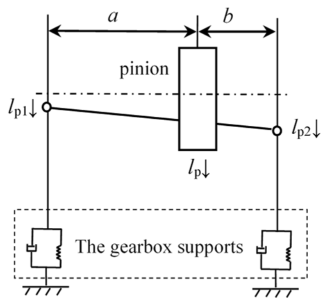


Fig. 3. The coordination relationship

According to the deformation coordination Eq. (5), Eq. (1) and Eq. (4), a 12 DOFs coupling dynamic model, based on lumped mass method, is established.

3. Simulations

In order to verify the accuracy of the proposed method, the parameters of an example case are listed in Table 1.

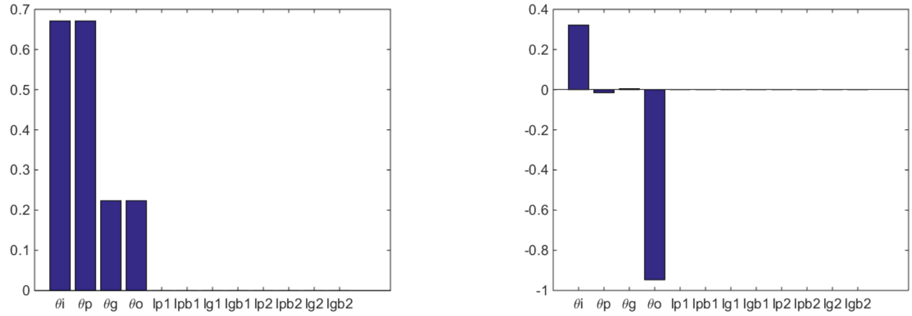
Table 1. Parameters of system

Symbol name	Value	Unit
Modulus / m	4	mm
Pressure angle / α	20	°
Tooth number of driving gear / z_1	23	–
Tooth number of driven gear / z_2	69	–
Addendum coefficient / h_a^*	1	–
Clearance coefficient / c^*	0.25	–

According to the 12 DOFs coupling dynamic model and the parameters listed in Table 1, the natural frequencies of the example case are simulated. Part of the results are shown in Fig. 4.

In the case of Fig. 4, the natural vibration mode vector of the first-order non-zero natural frequency (second frequency: 1403 Hz) is:

$$\phi_A = \{0.3211, -0.0149, 0.0050, -0.9469, -0.0004, -0.0002, -0.0004, -0.0002, -0.0004, -0.0002, -0.0004, -0.0002\} \tag{6}$$



a) First frequency: 0 Hz b) Second frequency: 1403 Hz

Fig. 4. Natural frequency simulations

According to the simulation result based on finite element model, as shown in Fig. 5, the natural vibration mode vector could be expressed as:

$$\phi_B = \{-0.2524, 0, 0, 0.7573, 0, 0, 0, 0, 0, 0, 0, 0\}. \tag{7}$$

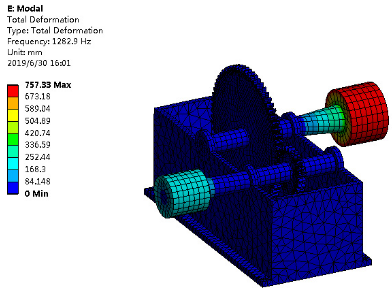


Fig. 5. Natural vibration mode based on FEM (natural frequency: 1282.9 Hz)

According to the modal assurance criterion (MAC), Eq. (6) and Eq. (7), the natural vibration mode vector correlation can be derived by:

$$MAC = \frac{\left|\phi_B^T \phi_A\right|^2}{\phi_B^T \phi_B \phi_A^T \phi_A}. \tag{8}$$

According to Eq. (8), the MAC value of the example case is 0.9997, namely, the natural vibration mode shown in Fig. 4(b) and the natural vibration mode shown in Fig. 5 are the same-order physical mode. The relative error of the natural frequencies between two methods is calculated, as shown in Table 2.

Table 2. The relative error of the natural frequencies between two methods

	Value	Unit
The natural frequency based on lumped mass method	1403	Hz
The natural frequency based on FEM	1282.9	
The relative error	9.36	%

In the case of Table 2, the relative error of the natural frequencies between two methods is 9.36 %, namely, the proposed method is accurate and feasible.

4. Conclusions

In the issue, a 12 DOFs spur gear coupling dynamic model, based on lumped mass method, is proposed. The construction method of spur gear coupling dynamic model is better than finite element method, because it enables rapid modeling of complex gearbox and makes dynamic modeling more efficient. This contribution would be helpful to the spur gear coupling dynamic modeling and analyses.

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